Internet Appendix: "A Gap-Filling Theory of Corporate Debt Maturity Choice"*

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This note presents supplementary results and data referred to in "A Gap-Filling Theory of Corporate Debt Maturity Choice."

## 1. Derivations for Proposition 4: Adding Shocks to the Basic Model

What kind of shocks do we need to add to the model to generate a positive coefficient on government debt (*g*) and a negative coefficient on the corporate share (*f*) in a multivariate forecasting regression of excess bond returns?

Below, we show that we require shocks to at least three variables: (1) long-term government supply *g*, (2) target corporate maturity *z*, and (3) some other variable which affects required returns (e.g., arbitrageur risk tolerance *γ*). As pointed out in the paper, if the only shocks are to government supply and investor risk tolerance, then the corporate share is linearly related to expected returns and government supply adds nothing in a multivariate regression (the coefficient on *g* is zero). Intuitively, there needs to be some "noise" in the corporate share; otherwise it would perfectly reveal expected returns in our stylized model. If we only consider shocks to government supply *(g)* and target corporate maturities *(z)*, the coefficients on government suppy *(g)* and the corporate share *(f)* will both be positive. The reason is that as seen in Eq. (5) increases in either *g* or *z* increase risk premia on long-term bonds. Thus, once we control for *g*, increases in *f* are associated with higher values of *z* and, hence, higher returns.

From Equation (5) in the paper the expected excess return on long term bonds is

$$\pi^*(\gamma,g,z)=P^{*-1}-(1+r_1)(1+E[r_2])=\left[\frac{\theta(1+r_1)^2 Var[r_2]}{\gamma\theta+C(1+r_1)^2 Var[r_2]}\right](g+Cz)$$

and the equilibrium long term debt share for corporations is

$$f^*(\gamma,g,z)=z-\left[\frac{(1+r_1)^2 Var[r_2]}{\gamma\theta+C(1+r_1)^2 Var[r_2]}\right](g+Cz)$$

We consider shocks to government supply *(g)*, target corporate maturities *(z)*, and aggregate arbitrageur risk tolerance (*γ*). Specifically, assume that realized excess returns are given by $\pi=\pi^*(\gamma,g,z)+\varepsilon_\pi$. Also assume that $g=\overline{g}+\varepsilon_g$, $g=\overline{g}+\varepsilon_g$, and $g=\overline{g}+\varepsilon_g$. As in the model,

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think of $\varepsilon_\pi, \varepsilon_g, \varepsilon_\gamma$ as being drawn at t=0 before *P* is set and associate $\varepsilon_\pi$ with the realization of $r_2$ which becomes known at t=1. These four shocks, $\varepsilon_\pi, \varepsilon_g, \varepsilon_\gamma$, and $\varepsilon_z$ are assumed to be independent.

Our empirical findings are summarized as follows:

1. A *negative* coefficient from a univariate regression of $f^*$ on $g$.
2. A *positive* coefficient from a univariate regression of $\pi$ on $g$.
3. A *negative* coefficient from a univariate regression of $\pi$ on $f^*$.
4. If we estimate the multivariate regression
$$\pi = a_\pi + b_g \cdot g + b_f \cdot f^* + \varepsilon_\pi.$$
we find $b_g > 0$ and $b_f < 0$. Specifically, neither $g$ nor $f^*$ completely drives the other out in a horse race, although inclusion of both tends to attenuate coefficients from univariate regressions.

To understand these results, let
$$\beta(\gamma) = \frac{\theta(1+r_1)^2 Var[r_2]}{(\overline{\gamma}+\varepsilon_\gamma)\theta + C(1+r_1)^2 Var[r_2]}$$
denote the realized slope of the relationship between expected excess returns and excess long-term supply, $g + C_z$. Note that this slope may depend on shocks to arbitrageur risk tolerance. We can rewrite our two main equations as
$$\pi^*(\gamma, g, z) = \beta(\gamma)(\overline{g} + C\overline{z} + \varepsilon_g + C\varepsilon_z)$$
$$f^*(\gamma, g, z) = \overline{z} + \varepsilon_z - (\pi^*(\gamma, g, z))/\theta$$

Since all the εs are independent we have
$$Cov(f^*, g) = -(E[\beta(\gamma)]\sigma_g^2)/\theta < 0,$$
$$Cov(\pi, g) E[\beta(\gamma)]\sigma_g^2 > 0,$$
$$Cov(f^*, \pi) = E[\beta(\gamma)]C\sigma_z^2 - (Var[\pi^*(\gamma, g, z)])/\theta.$$
Therefore, predictions **(1)** and **(2)** will hold so long as there are shocks to long-term government supply ($\sigma_g^2 > 0$). To generate prediction **(3),** we require $Cov(f^*, \pi) = Cov(f^*, \pi^*) < 0$ This will hold trivially if z is deterministic ($\sigma_z^2 = 0$). However, we will see below that we need $\sigma_z^2 > 0$ to generate prediction 4. More generally, we need $E[\beta(\gamma)]C\sigma_z^2 = Cov(z, \pi^*) < (Var[\pi^*(\gamma, g, z)])/\theta$ so that shocks to z are not a significant driver of expected returns. This will be the case if, for instance, *C* is small, so that the government sector is large relative to the corporate sector. If shocks to *z* are a major driver of expected excess returns, then this positive relationship between shocks to *z* and expected returns will outweigh corporate substitution towards cheaper maturities.

We now ask what configuration of shocks is needed to generate finding **(4)** above. We consider cases where $\sigma_g^2 > 0$ since this is required to generate **(1)** and **(2)**.

**Shocks to *g* alone:** Suppose $\sigma_g^2 > 0$ but that $\sigma_z^2 = \sigma_\gamma^2 = 0.$ In this case $g$ and $f^*$ will be perfectly collinear, so we cannot contemplate a multivariate forecasting regression.

**Shocks to** $g$ **and** $\gamma$**:** Here we have $\pi^*(g,z)=\beta(\gamma)(\overline{g}C+\varepsilon_g)$ and $f^*(g,z)=\overline{z}-\pi^*/\theta$. As seen above, this will generate predictions **(1)**, **(2)**, and **(3)**. With respect to prediction **(4),** $g$ adds nothing to $f^*$ in a multivariate forecasting regression of future excess returns. This is because $f^*$ is a linear function of the best possible forecast of expected returns, $\pi^*$. Specifically, if we were to run the regression $\pi=a_\pi+b_g g+b_f f+\varepsilon_\pi$ we would find $b_g=0$ and $b_f=-\theta$.

**Shocks to** $g$ **and** $z$**:** Now suppose that $\sigma_g^2>0$ and $\sigma_z^2>0$ but $\sigma_\gamma^2=0$. In this case, we have

$$\pi^*(g,z)=\beta_{\overline{\gamma}}(\overline{g}+C\overline{z})+\beta_{\overline{\gamma}}\varepsilon_g+\beta_{\overline{\gamma}}C\varepsilon_z)$$

where $\beta_{\overline{\gamma}}\equiv\beta(\overline{\gamma})$ and

$$f^*(g,z)=\overline{z}-(\beta_{\overline{\gamma}}/\theta)(\overline{g}+C\overline{z})+(1-C(\beta_{\overline{\gamma}}/\theta))\varepsilon_z-(\beta_{\overline{\gamma}}/\theta)\varepsilon_g.$$

To generate prediction **(3)**, we need

$$Cov(f^*,\pi)=\beta_{\overline{\gamma}}C\left(1-(\beta_{\overline{\gamma}}/\theta)C\right)\sigma_z^2-\beta_{\overline{\gamma}}(\beta_{\overline{\gamma}}/\theta)\sigma_g^2<0$$

or $C/(\sigma_g^2/\sigma_z^2+C^2)<\beta_{\overline{\gamma}}/\theta$. This will be the case if $\theta$ is small or when $\sigma_g^2/\sigma_z^2$ is large. Intuitively, we will have $Cov(f^*,\pi)<0$ when $\theta$ is small, so that firms respond elastically to government supply shocks, and when $\sigma_g^2/\sigma_z^2$ is large, so government supply shocks are large relative to corporate maturity shocks. Finally consider the multivariate regression. Using $\mathbf{b}=Var[\mathbf{x}]^{-1}Cov[\mathbf{x},y]$, we have

$$\begin{bmatrix}b_g\\b_f\end{bmatrix}=\frac{\beta_{\overline{\gamma}}}{1-(C\beta_{\overline{\gamma}})/\theta}\begin{bmatrix}1\\C\end{bmatrix}$$

Since $1-\beta_{\overline{\gamma}}C/\theta>0$, we have $b_g>0$ and $b_f>0$. The reason is simple: Any possible negative relationship between $f^*$ and $\pi$ reflects shocks to government debt $(\varepsilon_g)$. However, once we control for government supply shocks, the relationship between $\pi$ and $z$ reflects shocks to target corporate maturities ($\varepsilon_z$) which are associated with higher excess returns. Therefore, in this case we obtain both $b_g>0$ and $b_f>0$.

**Shocks to** $g,\gamma$ **and** $z$**:** Finally suppose that $\sigma_g^2>0$ and $\sigma_\gamma^2>0$ and $\sigma_z^2>0$. Here we have

$$\pi^*(\gamma,g,z)=\beta(\gamma)(\overline{g}+C\overline{z}+\varepsilon_g+C\varepsilon_z)$$

$$f^*(\gamma,g,z)=\overline{z}+\varepsilon_z-(\pi^*(\gamma,g,z))/\theta$$

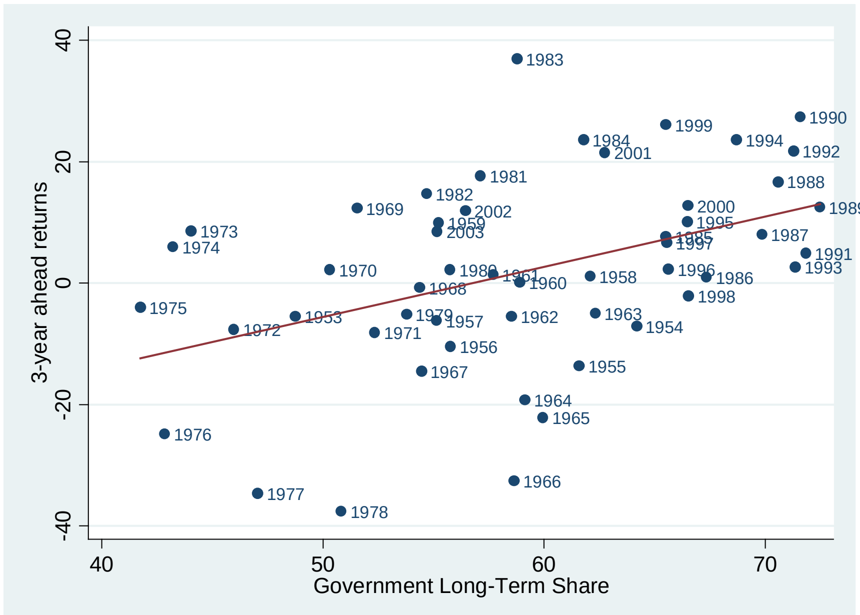
As noted above all the multivariate results will hold as long as $Cov(f^*,\pi)<0$ which holds if $Cov(z^*,\pi)=E[\beta(\gamma)]C\sigma_z^2<Var(\pi^*)/\theta$. Turning to the multivariate results, we have

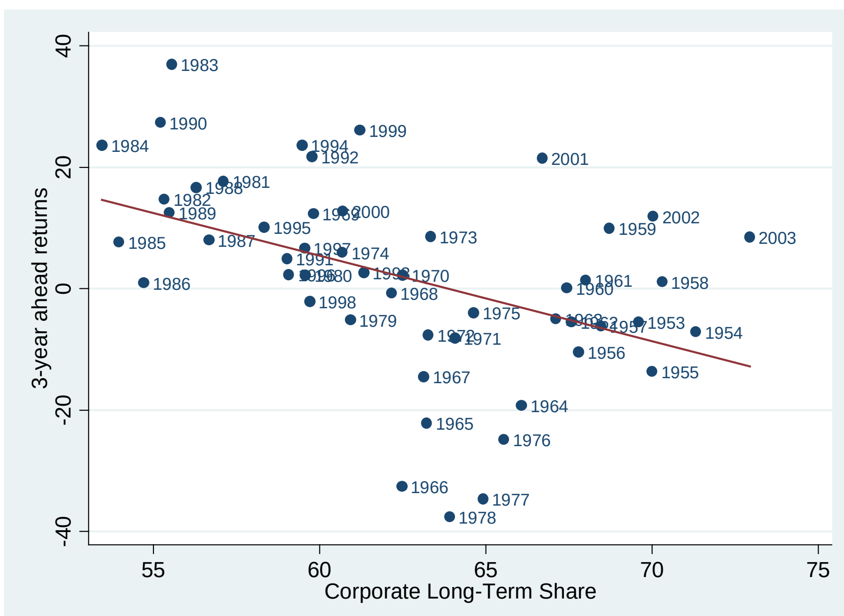
$$\begin{bmatrix}b_g\\b_f\end{bmatrix}=\frac{1}{\Delta}\begin{bmatrix}Cov(\pi^*,g)(\sigma_z^2-\frac{1}{\theta}Cov(\pi^*,z))\\\frac{1}{\theta}\left([Cov(\pi^*,g)]^2-\sigma_g^2Var(\pi^*)\right)+\sigma_g^2\cdot Cov(\pi^*,z)\end{bmatrix}.$$

where $\Delta>0$ is the determinant of the relevant variance matrix. Now $[Cov(\pi^*,g)]^2-\sigma_g^2Var(\pi^*)=\left([Corr(\rho_{\pi^*},g)]^2-1\right)\sigma_g^2Var(\pi^*)<0$, so we see that $b_g>0$ and $b_f<0$ so long as $Cov(z^*,\pi)$ is sufficiently small. As noted above, since $Cov(z^*,\pi)=E[\beta(\gamma)]C\sigma_z^2$, this will be the case if $C$ is small.

## 2. Scatter plots for forecasting results

Excess bond returns are plotted against the government long-term share in the first panel, and against the corporate long-term share in the second panel. As can be seen in the figures, the corporate sector's ability to "time" excess bond returns is largely driven by the positive relation between the government long-term share and subsequent bond returns. These plots correspond to the results in the last table of the paper.



## 3. Vector Auto-Regressions

As discussed in the text, we have explored the lead-lag properties of the relation between government and corporate maturities using Vector Auto-Regressions. In the first two columns below, we estimate a VAR(1) for long-term corporate issues, $d_{L,t-1}^{C} / d_{t-1}^{C}$, and changes in the long-term government level share, $\Delta\left(D_{L,t}^{G} / D_{t}^{G}\right)$. We find a negative and significant relationship between the current corporate issue share and lagged changes in the government level share. However, there is no evidence of relationship between current changes in the government level share and the lagged corporate issue share. That is, changes in government maturities appear to Granger-cause corporate issues, but not vice versa. In columns (3)-(4) and (5)-(6) we repeat this analysis using changes in the FOF and Compustat level share, respectively, in place of the FOF issue share. In both cases, we find a negative relation between current changes in the corporate level share and lagged changes in the long-term government share, although this is only statistically significant for changes in FOF levels. By contrast, there is no evidence that changes in government maturities responds to past changes in corporate maturities. These lead-lag asymmetries further alleviate possible concerns about reverse causation.

**VAR Results.** t-statistics are based on Newey-West (1987) standard errors allowing for two years of lags.

| | FOF Issues | | Change in FOF Levels | | Change in Comp. Levels | |
|---|---|---|---|---|---|---|
| | $d_{L,t}^{C} / d_{t}^{C}$ | $\Delta\left(D_{L,t}^{G} / D_{t}^{G}\right)$ | $\Delta\left(D_{L,t}^{C} / D_{t}^{C}\right)$ | $\Delta\left(D_{L,t}^{G} / D_{t}^{G}\right)$ | $\Delta\left(D_{L,t-1}^{C} / D_{t-1}^{C}\right)$ | $\Delta\left(D_{L,t}^{G} / D_{t}^{G}\right)$ |
| $d_{L,t-1}^{C} / d_{t-1}^{C}$ | 0.550 | -0.053 | | | | |
| | [4.35] | [0.51] | | | | |
| $\Delta\left(D_{L,t-1}^{C} / D_{t-1}^{C}\right)$ | | | 0.182 | -0.136 | 0.035 | -0.040 |
| | | | [1.56] | [-0.69] | [0.34] | [0.18] |
| $\Delta\left(D_{L,t-1}^{G} / D_{t-1}^{G}\right)$ | -0.397 | 0.293 | -0.174 | 0.286 | -0.162 | 0.301 |
| | [-2.60] | [1.75] | [-1.89] | [1.91] | [-1.39] | [1.77] |
| $R^2$ | 0.41 | 0.11 | 0.11 | 0.11 | 0.04 | 0.10 |

## 4. Mortgages

An excerpt from the results below (the three RHS columns) appears in Table IV in the paper. Mortgages are discussed the section "Taking the Model to the Data."

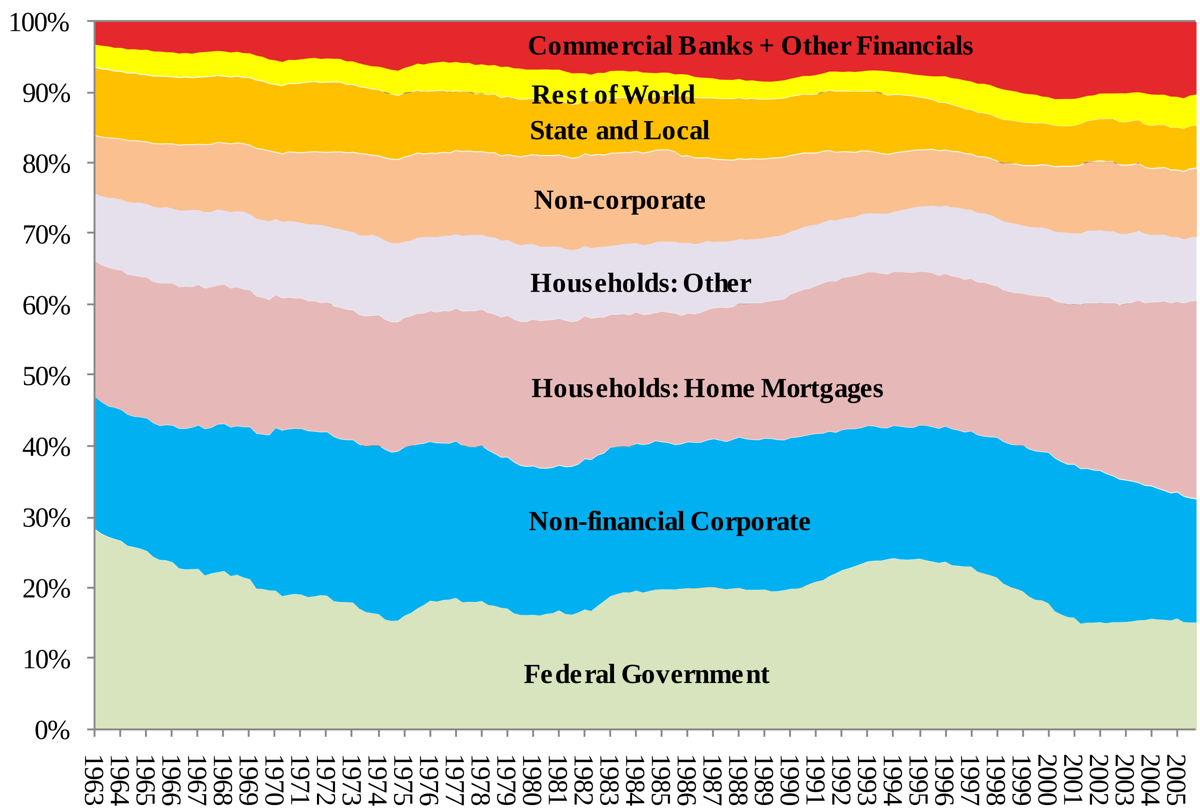
We have experimented with modified measures of long-term debt that include residential mortgage debt. Specifically, we have computed versions of $D_L / D$ that count either (1) all residential mortgage debt or (2) GSE debt and GSE-backed mortgage securities as long-term debt. The latter measure includes only the marketable portion of residential mortgage debt, which probably makes the most sense. To compute the modified measure in the second case we use $D_L / D = (D_L^{GOV} + D^{GSE} + D^{MBS}) / (D^{GOV} + D^{GSE} + D^{MBS})$. (Treating all GSE debt and GSE-backed MBS and as long-term debt is clearly a rough approximation, especially since we know that the effective duration of mortgages varies over time.)

These modified measures work quite well in our baseline multivariate specifications. Due to the growth of the mortgage market relative to the Treasury market, the modified measures have a much stronger time trend than our original series and so we expect the results to be weaker when failing to control for this trend. In fact, this is what we find in the (un-tabulated) univariate specifications. In the table below, standard errors are based on Newey-West standard errors allowing for two years of lags.

| | **Baseline Measure** | | | **Count Home Mortgages as LT** | | | **Count GSE Debt & MBS as LT** | | |
|---|---|---|---|---|---|---|---|---|---|
| | **FOF Level** | **FOF Issues** | **CSTAT Levels** | **FOF Level** | **FOF Issue** | **CSTAT Levels** | **FOF Level** | **FOF Issue** | **CSTAT Levels** |
| $D_L / D$ | -0.387 | -0.318 | -0.228 | -0.834 | -0.774 | -0.599 | -0.509 | -0.434 | -0.293 |
| | [-5.45] | [-5.77] | [-2.33] | [-3.92] | [-4.78] | [-2.17] | [-4.00] | [-5.00] | [-1.87] |
| $y_{St}$ | -1.263 | -0.836 | 0.123 | -1.252 | -0.833 | 0.123 | -1.294 | -0.864 | 0.107 |
| | [-3.55] | [-5.81] | [0.48] | [-2.77] | [-4.14] | [0.45] | [-3.07] | [-4.82] | [0.38] |
| $y_{Lt} - y_{St}$ | -1.257 | -0.486 | 0.504 | -1.433 | -0.671 | 0.352 | -1.263 | -0.497 | 0.504 |
| | [-2.72] | [-1.08] | [0.88] | [-2.15] | [-1.23] | [0.56] | [-2.35] | [-0.96] | [0.80] |
| *Trend* | 0.160 | 0.069 | 0.103 | 0.263 | 0.180 | 0.195 | 0.415 | 0.291 | 0.248 |
| | [2.26] | [2.07] | [1.62] | [2.08] | [2.79] | [1.82] | [2.74] | [4.00] | [1.72] |
| R-squared | 0.73 | 0.61 | 0.34 | 0.55 | 0.49 | 0.28 | 0.61 | 0.52 | 0.25 |

## 5. Components of Long- and Short-term debt

Below we present a breakdown of all Credit Market Debt by sector from Table L.1 of the *Flow of Funds*. Credit market debt includes Treasuries, municipal securities, corporate bonds, commercial paper, corporate bonds, bank and other loans, mortgages, and consumer credit. However, this is not a breakdown of all the liabilities of all major sectors in the U.S. economy. For instance, Credit Market Debt does not include deposits, interbank borrowings, or the operating liabilities of non-financial businesses (e.g. trade payables or taxes payable).[1] We note that (1) Federal Government plus Nonfinancial Corporate debt together average 40% of all credit market debt and range from 47% to 33% over our 1963-2005 sample. These are the two components of credit market debt that we capture in our baseline analysis. (2) If one adds all residential mortgages, as we do in the robustness analysis above, these three components together account for an average of 61% of all credit market debt (if we restrict attention to GSE-debt and GSE-backed MBS the corresponding figure is 50%).

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[1] The *Flow of Funds* also counts GSE debt, GSE-backed MBS, and ABS as separate components of credit market debt. In constructing this figure, we net out these categories from total credit market debt since these instruments are essentially pass-through vehicles for debts of the household sector (and perhaps other sectors in the case of ABS) which are already counted in the FOF Accounts. An often cited statistic is that the U.S. residential mortgage market is a $10.5 trillion market (year-end of 2007). This is the total amount of household home mortgage borrowing of which $7.4 trillion is ultimately owned by investors in the form of GSE debt and GSE-backed MBS. However, we don't want to say that the mortgage market is a $17.9 trillion market.

## 6. Main time-series data

| Year | FOF: Levels | FOF: Issues | Compustat: Levels | Government Share |
|---|---|---|---|---|
| 1963 | 67.11 | 23.38 | 84.61 | 62.33 |
| 1964 | 66.08 | 22.98 | 84.27 | 59.14 |
| 1965 | 63.23 | 20.53 | 82.35 | 59.96 |
| 1966 | 62.48 | 23.60 | 79.75 | 58.66 |
| 1967 | 63.14 | 25.41 | 80.09 | 54.46 |
| 1968 | 62.17 | 22.11 | 80.70 | 54.38 |
| 1969 | 59.82 | 19.30 | 80.90 | 51.54 |
| 1970 | 62.50 | 28.92 | 81.42 | 50.32 |
| 1971 | 64.07 | 24.90 | 83.73 | 52.35 |
| 1972 | 63.28 | 22.87 | 83.69 | 45.98 |
| 1973 | 63.35 | 27.57 | 81.65 | 44.04 |
| 1974 | 60.69 | 19.41 | 78.62 | 43.22 |
| 1975 | 64.64 | 25.20 | 84.95 | 41.74 |
| 1976 | 65.55 | 24.61 | 86.92 | 42.85 |
| 1977 | 64.93 | 26.08 | 86.88 | 47.06 |
| 1978 | 63.92 | 23.67 | 87.30 | 50.83 |
| 1979 | 60.93 | 18.76 | 84.96 | 53.78 |
| 1980 | 59.58 | 17.78 | 85.30 | 55.76 |
| 1981 | 57.11 | 18.30 | 84.52 | 57.14 |
| 1982 | 55.34 | 15.32 | 87.13 | 54.69 |
| 1983 | 55.56 | 18.87 | 88.48 | 58.78 |
| 1984 | 53.46 | 18.67 | 86.96 | 61.80 |
| 1985 | 53.97 | 19.57 | 84.81 | 65.51 |
| 1986 | 54.72 | 20.98 | 86.00 | 67.35 |
| 1987 | 56.67 | 21.86 | 85.05 | 69.86 |
| 1988 | 56.30 | 18.40 | 78.73 | 70.59 |
| 1989 | 55.49 | 15.79 | 78.42 | 72.48 |
| 1990 | 55.22 | 15.05 | 77.01 | 71.60 |
| 1991 | 59.03 | 16.72 | 79.36 | 71.86 |
| 1992 | 59.78 | 15.19 | 79.18 | 71.30 |
| 1993 | 61.33 | 17.98 | 78.03 | 71.38 |
| 1994 | 59.48 | 14.75 | 81.93 | 68.71 |
| 1995 | 58.34 | 17.74 | 82.72 | 66.48 |
| 1996 | 59.08 | 19.15 | 82.26 | 65.63 |
| 1997 | 59.55 | 21.28 | 82.37 | 65.56 |
| 1998 | 59.71 | 22.55 | 82.03 | 66.54 |
| 1999 | 61.22 | 24.68 | 81.70 | 65.51 |
| 2000 | 60.70 | 19.90 | 81.12 | 66.52 |
| 2001 | 66.70 | 30.13 | 85.03 | 62.76 |
| 2002 | 70.04 | 25.35 | 88.19 | 56.45 |
| 2003 | 72.95 | 29.55 | 89.75 | 55.18 |
| 2004 | 73.12 | 26.38 | 88.33 | 54.13 |
| 2005 | 72.62 | 26.46 | 89.27 | 54.60 |