Regression vs. Volatility Tests of the Efficiency of Foreign Exchange Markets

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Volatility tests are an alternative to regression tests for evaluating the joint null hypothesis of market efficiency and risk neutrality. By considering tests based on conditional volatility bounds, we show that if the alternative hypothesis is that one could 'beat the market' using a linear combination of observable variables, then regression tests are at least as powerful as the conditional volatility tests. If the application is to spot and forward markets for foreign exchange, then the most powerful conditional volatility test turns out to be equivalent to the analogous regression test in terms of asymptotic power.

There are two ways to go about testing the joint hypothesis of efficiency and risk neutrality in a particular financial market: regression tests, which compute conditional first moments, and volatility tests, which compute second moments. On the one hand, the regression tests look for predictability, given some information set. For example, in a forward or futures market (e.g., commodities or foreign exchange), the deviation of the next period's realized spot rate from the current one-period forward rate should be uncorrelated with variables known currently. An analogous condition holds in a longer-term asset market (e.g., stocks or bonds): the deviation of the present discounted value, assuming it is observable, of realized future returns (dividends or coupon payments) from the current asset price should again be uncorrelated with variables known currently. On the other hand, the volatility tests introduced by Shiller (1979) and LeRoy and Porter (1981) compare variances. In a forward market this would mean comparing the variance of

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the spot rate to the variance of the forward rate. The joint null hypothesis of market efficiency and risk neutrality implies that the forward rate is less volatile than the spot rate. In a longer-term asset market it would mean comparing the variance of the return with the variance of the asset price. The hypothesis implies that the asset price is less volatile than the return, in a specific sense.

A natural question to ask is which kind of tests, the regression tests or the volatility tests, is more powerful, i.e., is better able to reject the hypothesis in the event that it is false. As is often the case with questions of power, the answer depends on what the alternative hypothesis is. In this paper we take the alternative to be a particular failure of rational expectations or market efficiency. The alternative hypothesis is that one could 'beat the market' on average, using a linear combination of data in a particular information set. We show that in all cases the regression tests are at least as powerful against this alternative as the volatility tests.

In the case of spot and forward rates a comparison of simple unconditional variances tells us very little. Empirically, the unconditional sample variance of the spot rate differs negligibly from the unconditional sample variance of the forward rate. We argue in Section I that such considerations suggest comparing the variances conditional on some particular information set, which is analogous to what one does in regression tests. Thus, we consider a class of variance bound tests similar to those implemented by Mankiw, et al. (1985), which entail computing variance bounds for perfect foresight prices around 'naive' conditional means. The most powerful such volatility test will compute a variance conditional on an optimal linear combination of known variables. One might intuitively suspect that the linear combination would be the same as the estimates one would get from a regression on the same set of variables. It is perhaps more surprising that this most powerful volatility test turns out to be equivalent to the analogous regression test in terms of asymptotic power. That is, as the number of observations becomes large, the volatility test is no more and no less likely to reject the variance inequality than the coefficients in the regression test are to differ significantly from zero. We prove this central result of the paper in Section II.

These results suggest that regression tests are often preferable to volatility tests. This is, however, not always the case. Three important exceptions to our results stand out. First, our argument assumes that the data are correctly aligned. If they are not, as Shiller (1981a) points out, regression tests can be less powerful than volatility tests. Second, like Mankiw, et al. (1985), we assume that the 'perfect foresight' price (here, the future spot exchange rate; in Mankiw, et al.'s (1985) case, the perfect foresight stock price) is observable ex-post. Third, volatility tests that examine the present discounted valuation relation (such as Shiller's (1981b) and LeRoy and Porter's (1981) application to stock prices and dividends) can have greater power than regression tests against certain alternatives: for example, in the context of the term structure of interest rates, Stock (1982) shows that volatility tests can be expected to have greater power than regression tests when individuals prefer smooth consumption streams. However, none of these three exceptions apply to standard tests of efficiency in the foreign exchange market.

I. Volatility Bounds for Spot and Forward Rates

The rational expectations/efficient markets hypothesis is commonly stated as

$$S_{t+1} = F_t + e_{t+1}, \quad E_t e_{t+1} = 0$$

The joint null hypothesis of market efficiency and risk neutrality implies that the forward rate is less volatile than the spot rate. In a longer-term asset market it would mean comparing the variance of the return with the variance of the asset price. The hypothesis implies that the asset price is less volatile than the return, in a specific sense.

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TABLE 1. Variances around the sample mean
(June 1973—April 1982).

<table>
<thead>
<tr>
<th>Currency</th>
<th>Spot rate</th>
<th>Forward rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canadian dollar</td>
<td>0.00566866</td>
<td>0.00561098</td>
</tr>
<tr>
<td>French franc</td>
<td>0.00046331</td>
<td>0.00047980</td>
</tr>
<tr>
<td>German mark</td>
<td>0.0040633</td>
<td>0.00431700</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>0.00000036</td>
<td>0.0000038</td>
</tr>
<tr>
<td>Pound sterling</td>
<td>0.0637202</td>
<td>0.0632473</td>
</tr>
</tbody>
</table>

where $E_r(\cdot) = E(\cdot | I_r)$ is the expectation conditional on the information set $I_r$. In addition, we assume that $\text{var}(\varepsilon_r) = \delta_r^2$ is finite. This implies a simple variance inequality:

$$\text{var}(S_{r+1}) = \text{var}(F_r - Z_r + \varepsilon_{r+1}) = \text{var}(F_r - Z_r) + \text{var}(\varepsilon_{r+1}) + 2 \text{cov}(F_r - Z_r, \varepsilon_{r+1}).$$

Since $\text{cov}(F_r - Z_r, \varepsilon_{r+1}) = 0$ under the null hypothesis, this results in the bound:

$$\text{var}(F_r - Z_r) \leq \text{var}(S_{r+1} - Z_r).$$

For the tests considered in the paper, we take this notion of examining deviations about a nonconstant variable $Z_r$ one step further. From the familiar decomposition that mean square error is variance plus the square of the bias, a reasonable generalization of $\langle 2' \rangle$ is to consider a mean square error bound; that is, to consider a bound based on moments that in general could be noncentral, rather than the simple central moments examined so far. We now consider noncentral moments. Since $S_{r+1} - Z_r = F_r - Z_r + \varepsilon_{r+1}$ and $E_r F_r \varepsilon_r = E_r Z_r \varepsilon_r = 0$, we have $E_r(\varepsilon_{r+1}^2) = E_r(F_r - Z_r + \varepsilon_{r+1})^2 = E_r(F_r - Z_r)^2 + E_r \varepsilon_{r+1}^2$. Thus under the null hypothesis,

$$E_r(F_r - Z_r)^2 \leq E_r(S_{r+1} - Z_r)^2.$$
This inequality provides a basis for developing more exacting volatility tests of (1), since it explicitly employs the assumption that \( Z_i \) is in \( I_i \). Furthermore, the inequality (2) is a special case of (3) in which \( Z_i = \text{E}(S_{t+1}) = \text{E}(F_{t}) \) is constant.

It is interesting to note that (3) can also be arrived at by an altogether different line of reasoning than the motivation of increasing the power of the test. An important cause for concern related to any statistical implementation of the bound (2') falls under the general rubric of nonstationarity. Nonstationarity comes in many flavors: two of the most popular among econometricians are the existence of a time-dependent mean and the nonstationarity associated with a process having unit roots, so that the variance of the process is infinite. These two variants of nonstationarity seem particularly applicable to the foreign exchange data at hand. In the first case, the strong trends exhibited by exchange rates of the 1970s could be modeled as deterministic, although they may logically stem from nondeterministic factors such as inflation. In the second case, Meese and Rogoff (1983) demonstrate that spot exchange rates cannot be modeled better than by a random walk. Even if the spot rate process in reality has finite variance, this suggests difficulty in estimating variances of the process in any finite sample. Both of these concerns suggest deriving bounds with conditional means and computing sample moments around means that vary over the sample period; in other words, the bound (3) can be seen as a simple way to defend against the perils of nonstationarity.

As an example of a volatility bound implied by (3) which also seems to be a reasonable correction for this possible nonstationarity, let \( Z \) be the lagged spot rate. Thus, assuming lagged spot rates are in the information set, (1) implies that

\[
\text{E}(F_t - S_{t+1})^2 \leq \text{E}(S_{t+1} - S_{t+1})^2.
\]

The sample variances associated with this bound are presented in Table 2. For these data, the bound is satisfied in all cases considered, so no formal test of significance is necessary to see that market efficiency as embodied in (4) cannot be rejected.

Can we devise a still more exciting volatility test of market efficiency than (4)? Indeed we can. If we define the test statistic

\[
R(Z) = 1 - \frac{E(S_{t+1} - Z)^2}{E(F_t - Z)^2},
\]

then (3') can be rewritten as

\[
R(Z) \leq 0.
\]

A value of the test statistic significantly above zero would constitute a rejection of the null hypothesis: forward rates would be too volatile relative to spot rates. Since under the null hypothesis today's forward rate is an unbiased predictor of
tomorrow's spot rate, a reasonable choice for \(Z_t\) (which plays the role of the conditional mean of \(S_{r+1}\)) is that \(Z_t = F_t + \beta X_t\), where \(X_t\) is a mean-zero, nonconstant, univariate series with finite variance assumed to belong to \(I_\eta\). Since the bound \(\langle 3' \rangle\) holds for all scalar \(\beta\), we should select the value of \(\beta\) for which a test based on \(\langle 3' \rangle\) is as likely as possible to reject the null hypothesis. Letting
\[
\hat{\beta} = (\sum X_t e_{r+1} / (\sum X_t^2 \sum e_{r+1}^2))^{1/2}
\]
is the sample correlation coefficient.

The proof of \(\langle 6 \rangle\) is easy: since \(\hat{\beta}(F + \beta X_t) = 1 - \sum (e_{r+1} - \beta X_t)^2 / \beta^2 \sum X_t^2\), to solve \(\langle 5 \rangle\) it is merely necessary to solve:
\[
\min_\beta \sum (\beta^{-1} e_{r+1} - X_t)^2
\]
which has the solution \(\beta^{-1} = \sum e_{r+1} X_t / \sum e_{r+1}^2\). Substituting this statistic into the definition of \(\hat{\beta}(F + \beta X)\) yields the result. It thus appears that the most discerning volatility test based on a statistic of the form \(\hat{\beta}(F + \beta X)\) is equivalent to the correlation coefficient, which arises from considering regression tests! The regression \(R^2\) is, of course, a measure of the variability of the dependent variable which is explained by the right-hand variables in the regression. In this sense, a test of the joint significance of the explanatory variables in a regression is a 'volatility test'. This interpretation of regression tests as indications of excess variability (or as variability of a predictable risk premium) is noted by Stataz (1982). This paper makes precise the link between the class of volatility tests based on \(\langle 3' \rangle\) on the one hand, and the particular 'volatility tests' implemented by linear regressions on the other.

The preceding argument is not based on formal power considerations. However, as is shown in the next section, among this class of volatility tests the 'most discerning' test is in fact asymptotically most powerful against the (local) alternative that \(X_t\) and \(e_{r+1}\) are correlated. Intuitively, the question whether the correlation coefficient is significantly different from zero is the same as the question whether the regression coefficient is significantly different from zero.

II. Formal Statement of the Result

In this section we examine the power of the volatility tests of the previous section against the alternative that \(e_{r+1}\) and \(X_t\) are correlated. The proof uses asymptotic statistical arguments. Specifically, it compares asymptotic approximations to the power functions of test statistics based on \(\hat{\beta}(F + \beta X)\), where \(\beta\) is permitted to be any function of data as long as \(\beta^{-1}\), when standardized, has a limiting distribution with all its mass on the real line. Since the power of a test based on the statistic \(\langle 6 \rangle\) will go to one when the covariance between \(X_t\) and \(e_{r+1}\) is bounded away from zero,
we adopt the conventional asymptotic approach of considering a local alternative under which this covariance tends towards zero as the sample size tends towards infinity.

The proof itself has two parts. First, the class of random variables $\tilde{\beta}^{-1}$ that need to be considered is narrowed down to those which tend to zero in probability under the local alternative. Second, it is possible to appeal to the results of the previous section to show that, of the variables with this property, the solution to the maximization problem (4) does indeed yield the asymptotically most powerful test.

For the statement of the result, it is convenient to reparameterize the problem. Let the local alternative be $\sigma_{\tilde{\beta}} = T^{1/2} \delta$, where $T$ is the number of observations and $\delta$ is some nonzero, finite fixed number. Let $\phi = \tilde{\beta}^{-1}$. Let $\Phi$ be the set of all random variables $\phi$ which are functions of the data (possibly degenerate — that is, possibly a constant) and are such that $T^{1/2}(\phi - \phi)$ has a limiting distribution on the real line. In making this assumption we are assuming that both $X$, and $\delta$, are stationary in the sense of not having a unit root in their autoregressive representations. Also, let $\phi^*$ be that element of $\Phi$ such that the one-sided test of the restriction (3) has the greatest local asymptotic power of all the tests of level $\alpha$ based on $\hat{R}(F + \phi^* X)$.

For the proof, first we use the ‘delta method’ to find the limiting distribution of the standardized random variable based on $\hat{R}(F + \phi^* X)$. Let $\phi = \text{plim} \tilde{\beta}$ and let $a = \tilde{\beta} \gamma |_{\gamma = \phi = \phi^*}$. Also, let $r(\phi) = \text{plim}(\hat{R}(F + \phi^* X))$, which will exist by the assumption that when standardized will have a limiting distribution and because $r(\cdot)$ is continuous in $Z$. Then

$$\tau(\phi)^2 = a^\top \Sigma a$$

where $\tau(\phi)^2 = a^\top \Sigma_a$ and $\phi = \text{plim} \tilde{\beta}$. Since $a$ is continuous in $\phi$, $\tau(\phi)^2$ is continuous in $\phi$.

Since the null hypothesis is that $R < 0$, we wish to find the statistic of the form (7) that has the greatest chance of exceeding zero under the local alternative. One approach to this problem is to compute $\tau(\phi)^2$ directly for many statistics $\tilde{\beta}$, and to compare the limiting behavior under the local alternative. However, this would be difficult, since the candidates $\tilde{\beta}$ must be specified in advance.

This problem can be sidestepped by noting that a necessary condition for a test of the form (7) to have nonnegligible power is that $\tau \geq 0$; otherwise $P(\hat{R} > 0) \to 0$ as $T \to \infty$ by definition of convergence in probability. Thus we can restrict our
attention to those $\hat{\phi}$ which result in $T^{1.2}\hat{R}$ having a limit which is bounded in probability away from $-\infty$.

It is easy to see that in fact $T^{1.2}\hat{R}$ must be bounded in probability (be $O^2(1)$). By definition,

$$T^{1.2}\hat{R} = T^{1.2}\left[1-(\phi^c e^c - 2\hat{\phi} e^c X + X^c X)/X^c X\right]$$

$$= 2\phi^c e^c X^{1.2} - (T^{1.2}\hat{\phi})\phi^c e^c X^{1.2}/X^c X/T.$$

By assumption $\hat{\phi} \in \phi$, $X^c X/T \hat{\phi} \sigma^2_X$ and $\phi^c e^c T \hat{\phi} \sigma^2_X$. Also, under the local alternative, $e^c X T^{1.2}$ has a limiting law on the line. Thus, by Slutsky's Theorem, $T^{1.2}\hat{R}$ is bounded above in probability for all $\hat{\phi}$, so $R \leq 0$. Thus we can restrict attention to $\hat{\phi}$ such that $r = 0$, i.e., such that $T^{1.2}\hat{R} = O_p(1)$. But, by (8), this will occur only if $T^{1.2}\hat{\phi} = O_p(1)$ which in turn implies that $\phi = 0$.

The result follows from this requirement, since it implies that, for all $\hat{\phi}$ yielding nonnegligible power against the local alternative, $T^{1.2}\hat{R} \to N(0, \tau(0)^2)$ under the null hypothesis. Furthermore, since $\tau(\phi)$ is continuous, the variance of the limiting distribution of $T^{1.2}\hat{R}$ under the local alternative will be $\tau(0)^2$ for all contenders $\phi$. Thus the problem reduces to finding the function $\hat{\phi}$ such that $R$ is maximal for all $\hat{\phi}$ satisfying $T^{1.2}\hat{\phi} = O_p(1)$. Since $\hat{\phi} = e^c X/e^c e$ was shown to solve this problem among all functions of the data, and since under the local alternative $T^{1.2} e^c X/e^c e = O_p(1)$, we have $\hat{\phi} = e^c X/e^c e$.

The asymptotic equivalence to the regression test follows from noting that, under the null hypothesis, the $t$-statistic for the slope coefficient of the OLS regression satisfies $T^{-1} r = (e^c X)^2/(X^c X)(u'u)$, where $u = e - \hat{\gamma} X$, with $\hat{\gamma} = e^c X/X^c X$. However, under the local alternative, $(e^c e - u'u)/T$ converges to zero in probability. Thus $T^{-1} r$ is asymptotically equivalent to $\hat{\rho}_{e,e}^c = \hat{R}(F + \hat{\phi}^c e^c X)$.

### III. Conclusion

We have examined a second moment bound based on the fact that the variance of a conditional expectation (the forward rate) is no more than the unconditional variance of the random variable (the spot rate). We find that volatility tests of this bound will do no better than conventional regression tests of market efficiency. At best, when the volatility test is approximately modified to be conditional on available information, it does as well as regression tests with the same set of information.

### Notes

1. Other papers on volatility tests include Flavin (1982), Grossman and Shiller (1981), LeRoy and LaCivita (1981), Miehener (1982), Shiller (1981a, 1981b), and Singleton (1980). Geweke (1980) also examines the behavior of volatility tests against an alternative of this type. He demonstrates that there are regions of the parameter space in which regression tests will reject but volatility tests will not. Our results differ from his in two ways. First, we consider an expanded class of volatility bounds (3). Second, we demonstrate that there is a conditional volatility test with the same asymptotic power as the corresponding regression test against this particular alternative. In fact, his conclusion that regression tests dominate unconditional volatility tests is implied by the Proposition in Section II.
2. Our forward rates are 30-day forward. Both spot and forward rates are bid rates, 10 am, last day of the month, in dollars per national currency, obtained originally from Data Resources, Incorporated. Flood (1981, p. 220) comments on the 'striking fact' that spot and forward exchange rates 'have about the same degree of volatility.' However, his computations use a measure of the conditional variance somewhat different from ours.

3. In the terminology of Fama (1970), the larger the information set, the 'stronger form' is the test. For one of many such regression studies of the forward exchange market, and for references to others, see Boothe and Longworth (1986).

4. Meese and Singleton (1980) point out the perils of performing naive comparisons of unconditional sample variances of exchange rates when the theoretical variances may be infinite.

5. For simplicity, we have assumed that the variables have zero means. Repeating the argument leading to (6) with the variates being deviations from their sample averages yields the formula for the $R^2$ with an intercept included in the regression.

6. The results of this paper hold for the case that $X_i$ is one-dimensional. If instead $X_i$ is $k$-dimensional and $\beta$ is a $k$-vector, then a result analogous to that of this section holds: letting $\beta^*$ be the vector which maximizes $R(F + X_i \beta) = R^2$, where $R^2$ is the ratio of the explained to the total sum of squares from the ordinary least squares regression of $e_{i+1}$ on $X_i$. Thus our results generalize in a straightforward way to the multidimensional case. However, for simplicity, we limit the discussion in the paper to the one-dimensional case.

References


