

# Macroeconomic Implications of Covid-19: Can Negative Supply Shocks Cause Demand Shortages?\*

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Motivated by the effects of the Covid-19 pandemic, we present a theory of Keynesian supply shocks: shocks that reduce potential output in a sector of the economy, but that, by reducing demand in other sectors, ultimately push aggregate activity *below* potential. A Keynesian supply shock is more likely when the elasticity of substitution between sectors is relatively low, the intertemporal elasticity of substitution is relatively high, and markets are incomplete. Fiscal policy can display a smaller multiplier, but the insurance benefit of fiscal transfers can be enhanced. Firm exits and job destruction can amplify and propagate the shock.

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# 1 Introduction

An old idea in economics is that “supply creates demand,” that is, that income produced in one sector of the economy creates demand for the output of other sectors.<sup>1</sup> In this paper, we study a situation in which a negative supply shock in one sector—that is, a decrease in potential output in that sector—produces income losses and a drop in aggregate demand so large that the economy’s overall output may fall *below* potential. We label a situation like this a “Keynesian supply shock.”

Our interest in this kind of shock is motivated by the Covid-19 pandemic and its asymmetric effects across sectors. Sectors that rely heavily on personal interaction have been hit directly and have contracted in an unprecedented manner. Businesses in these sectors have experienced long shutdowns or have operated at reduced capacity—due to government regulations or to the health concerns of consumers and workers. Other sectors have not been affected directly and have contracted relatively less. These sectors did not require personal interactions or could be easily re-organized to ensure the safety of the people involved. We can then interpret the pandemic as a large, asymmetric, transitory supply shock: the capacity of the economy to produce output without damaging people’s health has been temporarily but severely curtailed in some parts of the economy, less so in others.

When the economy is hit by a transitory supply shock, a traditional reaction is to think that demand stimulus is unwarranted and possibly damaging: the capacity of the economy to produce goods and services is temporarily low, so trying to stimulate demand is not useful and will only cause inflation or shortages. However, in the presence of an asymmetric shock that line of thinking may be incorrect. As [Rowe \(2020\)](#) notices: “a temporary 100% output cut in 50% of the sectors (what the Coronavirus does) is very different from a 50% output cut in 100% of the sectors.” To study the macroeconomic effects of an asymmetric shock requires thinking about the transmission of the shock from the sectors directly affected to the rest of the economy. Let us take as given that a reduction in activity in the directly affected sectors is a necessary response to a lower capacity to produce goods and services. Can this reduction be transmitted to other sectors and cause an inefficient drop in total output and employment? Which policy instruments can then be used to restore efficiency?

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<sup>1</sup>The sectoral dimension of the problem was very present in early writings. Say’s famous chapter “des débouchés” contains many passages like this: “a good harvest is favourable, not only to the agriculturist, but likewise to the dealers in all commodities generally. The greater the crop, the larger are the purchases of the growers. A bad harvest, on the contrary, hurts the sale of commodities at large.”([Say, 1824](#))

These are the questions we seek to address in this paper. These questions are motivated by the Covid-19 pandemic, but lead us more generally to study the transmission of disturbances in an economy with multiple sectors and the effects of sectoral shocks on aggregate demand.

We analyze a two sector model with nominal wage rigidities and look at the effects of a transitory shock that reduces potential output in one sector, assuming that monetary policy keeps the nominal interest rate unchanged. We then ask if the shock causes a demand shortage and inefficient unemployment. That is, we ask whether a reduction in potential output in the first sector causes aggregate output to fall below potential. We analyze first the extreme case of a complete shutdown and then extend our results to partial shutdowns.

Our first observation is that the shutdown of a sector changes the set of goods available to consumers. The fact that some goods are no longer available has two effects on consumer choice. First, it makes it less attractive to spend overall and induces consumers to postpone spending to the future. Second, it induces consumers to reallocate their spending away from the shut-down sector and in favor of the sector still active. Whether or not full employment is maintained in the active sector depends on the relative strength of these two forces. When the economy features a representative household, we show that a demand deficiency arises if the intertemporal elasticity of substitution—which determines the strength of the first force—is larger than the elasticity of substitution across sectors—which determines the strength of the second.

Our second observation is that the shutdown of a sector causes income losses for workers in that sector. These income losses can depress spending in the rest of the economy. To articulate this view, we need to move away from the representative household case and take into account the role of incomplete markets and limited capacity to borrow. We show that when these ingredients are added to the model, the conditions for a Keynesian supply shock are easier to meet. Incomplete markets and borrowing constraints amplify the Keynesian forces in the model, by increasing the marginal propensity to consume (MPC) of agents, and thus increasing the response of total spending to the income losses of the workers directly hit.

Figure 1 illustrates the structure of our economy. Our two sectors are labelled *A* and *B*. In the representative household setting, agents working in both sectors pool their income and spend it. When sector *A* shuts down in panel (b), agents equally share the income produced in sector *B* and choose how much to spend on sector *B*. Here, the difference between the intertemporal and the cross-sector elasticities determines the change in spending on sector *B*. In panel (c) instead, sector *A* workers receive no income, which

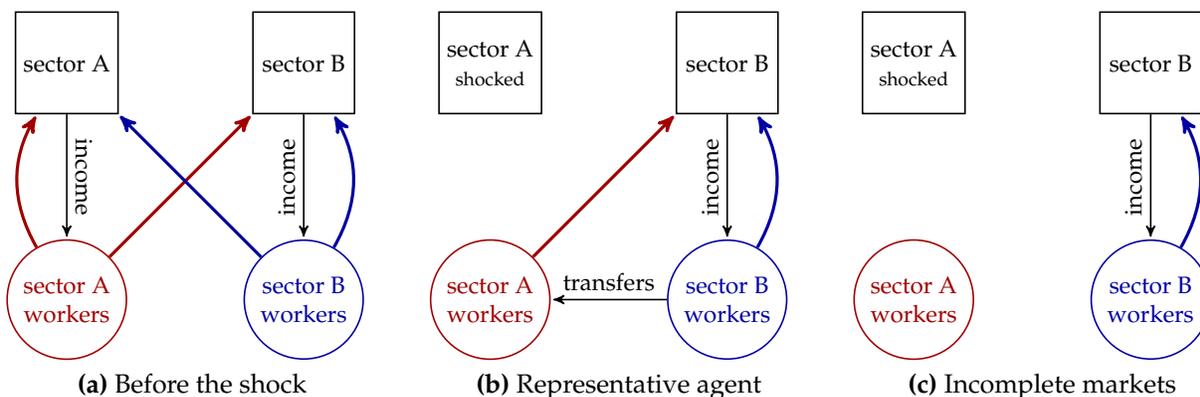


Figure 1: A sectoral shock

has an additional contractionary effect on spending in sector  $B$ .

Our model has surprising implications for fiscal policy. Fiscal policy may be less powerful than usual after an asymmetric sectoral shock. The logic for this result is the following: the workers in the directly affected sector are the ones facing larger income losses and also the ones with the highest MPC. However, an increase in overall spending will not go to their sector and will not lift their incomes, precisely because their sector is supply-constrained. Rather, it will go to other sectors with lower income losses and lower MPCs. This distributional effect weakens the Keynesian multiplier mechanism. In an illustrative baseline case of our model, the multiplier is exactly 1.

While the potency of fiscal policy may be weakened, this does not mean that fiscal interventions are less desirable. In fact, when we focus on transfer programs aimed at replacing lost incomes, we observe that these programs have a double welfare benefit: they reduce inefficient unemployment, in the case of a Keynesian supply shock, and they replace missing insurance markets. We also show that it is possible that the first effect fades out at a replacement rate lower than 1. The reason is that we only need to replace enough income to sustain the pre-pandemic level of spending in the sectors still active. Nonetheless, it may be welfare improving to increase transfers above the level that ensures full employment in the non-directly-affected sectors, as it is still desirable from an insurance perspective to compensate workers for their income losses.

We also consider a variant of our model with health in the utility function and a partial shutdown in sector  $A$ , in which the public-health motive for reducing activity in sector  $A$  is explicit. That variant allows us to make an additional point on fiscal policy. Social distancing policies and fiscal transfers are complements in our context. Absent social

distancing policies, fiscal transfers face a trade off: they have positive benefit of increasing activity in sector  $B$  and improving insurance, but they have negative health side effects, by increasing activity in sector  $A$ . When policies are in place to prevent an increase in activity in sector  $A$ , fiscal transfers are more desirable at the margin.

An important transmission channel for supply and demand shocks across sectors are supply chains. We highlight this issue in an extension of our basic model, in which sector  $A$  uses sector  $B$  goods as an input. In that case, a negative supply shock to sector  $A$  travels upstream to sector  $B$  as a demand shock. We show that this unambiguously increases the likelihood that the supply shock turns into a demand shortage in sector  $B$ .

We also extend our baseline model to a firm-level perspective, introducing individual firms that can decide whether to remain open for business and whether to pay their workers, even when inactive. This extension provides us with a basis to study two important phenomena.

First, we study the decision of sector  $A$  firms to keep paying their workers, i.e., to *hoard labor*. Firms may want to do so due to labor market frictions that make future rehiring costly. We show that labor hoarding unambiguously aids the economy, as it provides insurance to workers hit by the shock. We also show that easy monetary policy can be used to incentivize firms' labor hoarding. Similarly, if hoarding is stymied by liquidity problems, liquidity injections by the central bank can help.

Second, we study the decision of sector  $B$  firms to exit. We identify an amplification mechanism, which we dub the *business exit multiplier*. If the shock in sector  $A$  reduces demand in sector  $B$ , some businesses in that sector may become unwilling, or unable, to stay open. When these businesses close and lay off workers, an additional, endogenous, Keynesian supply shock emerges, amplifying the initial one. This force can increase the reach of the shock and shut down a greater part of the economy than the original shock. We discuss policy proposals to keep businesses running during the pandemic.

Two remarks on terminology. First, we label the shock to the sector directly hit a "supply" shock, because we interpret the effect of the pandemic as technological in nature: some goods cannot be produced without harming people's health. However, that label is not essential. As we explain in Section 2.4, whether the shock in the first sector is interpreted as a technology shock or as a preference shock is not crucial for our argument, as long as potential output, that is, the output level that would arise under flexible prices, is reduced in that sector. Second, we introduce the label "Keynesian supply shock" to describe a specific way in which a supply disturbance is transmitted in a multi-sector economy. But the label is clearly portable and could be used to describe other cases in which supply

shocks have effects on aggregate demand. Some examples appear in the literature review that follows.

## Related Literature

Our paper is related both to recent papers motivated by the Covid-19 pandemic and to earlier work about demand spillovers across sectors and about heterogeneous consumers and incomplete markets.

Among early pandemic-motivated papers two that appeared at the same time as this paper and explore related themes are [Fornaro and Wolf \(2020\)](#) and [Faria e Castro \(2020\)](#). [Fornaro and Wolf \(2020\)](#) consider a standard new Keynesian representative-agent economy and study a pandemic as a negative shock to the growth rate of productivity. In that context, negative news about future productivity can cause a contraction in demand, and, due to an endogenous feedback from current activity to future growth, can even cause a stagnation trap. [Faria e Castro \(2020\)](#) shares our focus on the sectoral dimension and on the role of incomplete markets. Apart from the different way in which the shock is modeled, the main difference is that [Faria e Castro \(2020\)](#) uses a calibrated DSGE to do policy counterfactuals, while we focus more on establishing theoretical conditions under which inefficient unemployment arises or not in the unaffected sector and on the welfare implications.

There is also a number of early policy-oriented papers that have advocated for various forms of fiscal support in response to Covid-19. Some that have formulated arguments related to our analysis of fiscal policy in Section 3 and to our analysis of the costs of match destruction in 5 include [Baldwin and Weder di Mauro \(2020\)](#), [Gopinath \(2020\)](#), [Gourinchas \(2020\)](#), [Hamilton and Veuger \(2020\)](#), [Hanson, Stein, Sunderam and Zwick \(2020\)](#), [Saez and Zucman \(2020\)](#), and [Sahm \(2020\)](#).

Another set of early papers has been building on epidemiological models of contagion, embedding them in an economic environment<sup>2</sup>, and studying the associated economic/health externalities. While these externalities are not central to our analysis, they play a role in our discussion of optimal fiscal policy in Section 3.3.

Since our paper was first circulated, there has been an unprecedented growth in macroeconomic research on the effects of COVID-19. [Woodford \(2020\)](#) studies a multi-sector model with incomplete markets like ours, but allows workers in different sectors to have different preferences. This kind of heterogeneity creates a “circular flow of payments” that

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<sup>2</sup>[Alvarez, Argente and Lippi \(2020\)](#), [Atkeson \(2020\)](#), [Berger, Herkenhoff and Mongey \(2020\)](#), [Eichenbaum, Rebelo and Trabandt \(2020\)](#), and [Kaplan, Moll and Violante \(2020\)](#).

has the potential to amplify the effect of the shock and increase the transfer multiplier. As in our paper, targeted transfers are key to an optimal policy response. [Caballero and Simsek \(2020\)](#) emphasize the role of risk-taking, finding that redistribution through asset prices can induce drops in demand. [Baqae and Farhi \(2020\)](#) explore the implications of a rich input-output network structure, more general than the simple one we explore in Section 4.1. In the more empirical literature, several papers look at the responses of consumer spending in real time, including [Andersen, Hansen, Johannesen and Sheridan \(2020\)](#), [Carvalho, Hansen, Ortiz, Garcia, Rodrigo, Rodriguez Mora and Ruiz de Aguirre \(2020\)](#), [Chetty, Friedman, Hendren and Stepner \(2020\)](#), and [Cox, Ganong, Noel, Vavra, Wong, Farrell and Greig \(2020\)](#). Two papers, [Brinca, Duarte and Faria-e Castro \(2020\)](#) and [Balleer, Zorn, Link and Menkhoff \(2020\)](#), explicitly try to separate demand from supply shocks. There is also a growing set of quantitative/calibrated papers focusing on the effects of fiscal policy, including [Mitman, Rabinovich et al. \(2020\)](#) and [Bayer, Born, Luetticke and Müller \(2020\)](#), and on the sectoral structure, including [Danieli and Olmstead-Rumsey \(2020\)](#).

Our paper is related to the growing literature on new Keynesian models with heterogeneous consumers and incomplete markets. In particular, we employ a simple structure in which there is a zero supply of bonds, as in [Werning \(2015\)](#), and in which, in equilibrium, a group of agents is unconstrained and a group of agents is constrained and behaves endogenously as hand-to-mouth. This makes our analysis close to two agents new Keynesian models, going back to [Galí, López-Salido and Vallés \(2007\)](#). Two recent related papers are [Challe \(2020\)](#) and [Ravn and Sterk \(2021\)](#), which consider cases in which a drop in future productivity can cause an increase in future income risk and, through a precautionary channel, cause a drop in demand, thus pointing to a different channel for Keynesian effects of supply disturbances.<sup>3</sup> Our analysis of fiscal policy in Section 3.1 is also connected to papers in this area that emphasize the joint distribution of MPCs and of the sensitivity of individual incomes to aggregate output movements.<sup>4</sup>

Demand spillovers across sectors play an important role in open economy macro models. In particular, the literature has studied the transmission of a negative productivity shock in the tradable sector causing a reduction in demand in the non-tradable sector, due to low elasticity of substitution between tradables and non-tradables and to the drop in domestic incomes under incomplete markets. For example, in [Tesar \(1993\)](#) a low elasticity of substitution plays a crucial role in producing a low level of cross-country consumption

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<sup>3</sup>For the precautionary channel see also [Ravn and Sterk \(2017\)](#) and [Heathcote and Perri \(2018\)](#).

<sup>4</sup>[Bilbiie \(2008\)](#), [Farhi and Werning \(2016\)](#), [Auclert, Rognlie and Straub \(2018\)](#), [Bilbiie \(2019\)](#), [Patterson \(2019\)](#).

correlations in a complete markets Real Business Cycle model, while in [Corsetti, Dedola and Leduc \(2008\)](#) a low elasticity helps to produce realistic consumption/exchange rate correlations in an incomplete markets model. [Fornaro and Romei \(2019\)](#) study a new Keynesian model where a contraction in tradable output spills over into a lack of demand in the non-tradable sector causing a recession. While there are interesting similarities, there are also important differences between the closed economy and the open economy context, especially because the reduction in domestic tradable output does not affect the ability of domestic consumers to consume tradables (as tradables can be imported) and because of the role of exchange rate adjustments. Another international dimension of demand spillovers is demand spillovers across countries. In particular, when countries produce differentiated goods, the idea that incomes generated in one country provide demand for products of other countries plays an important role in the analysis of fiscal policy coordination (e.g., in [Farhi and Werning, 2017](#)).

Sectoral demand spillovers also play an important role in models of economic growth and sectoral change ([Murphy, Shleifer and Vishny 1989](#), [Matsuyama 2002](#)). Cross-sector effects have been studied in the context of the Great Recession in [Gatti, Gallegati, Greenwald, Russo and Stiglitz \(2012\)](#). They also play a central role in understanding comovement and labor supply responses following demand shocks, as investigated in [Beaudry and Portier \(2014\)](#).

Finally, our analysis with a complete shutdown of one sector connects our paper to work on product varieties and endogenous entry, e.g., [Bilbiie, Ghironi and Melitz \(2012\)](#) and [Bilbiie and Melitz \(2020\)](#), due to the fact that the effect is equivalent to a situation in which a variety becomes unavailable for one period and is expected to be available again next period. This connection is especially relevant for our extension in which we consider a continuum of goods and allow for an entry margin, in Section 6.

## 2 A Two Sector Model

Consider an economy with two sectors  $A$  and  $B$ . The economy is populated by a unit mass of infinitely-lived consumers with preferences represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(c_{At}, c_{Bt}),$$

where  $c_{At}$  and  $c_{Bt}$  denote consumption of the two goods and

$$U(c_{At}, c_{Bt}) = \frac{\sigma}{\sigma - 1} \left( \phi^{\frac{1}{\epsilon}} c_{At}^{\frac{\epsilon-1}{\epsilon}} + (1 - \phi)^{\frac{1}{\epsilon}} c_{Bt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1} \frac{\sigma-1}{\sigma}}, \quad (1)$$

so there is constant elasticity of substitution  $\epsilon$  between the two goods, and constant intertemporal elasticity of substitution  $\sigma$ .

All agents are endowed with one unit of labor, which they supply inelastically. A fraction  $\phi$  of agents work in sector  $A$  and a fraction  $1 - \phi$  in sector  $B$ . For now, we assume that workers are perfectly specialized and cannot move between sectors. In Section 4.2, we consider labor mobility.

The technology to produce both goods is linear,

$$Y_{jt} = N_{jt},$$

for  $j = A, B$ . Competitive firms in sector  $j$  hire workers at the sector-specific wage  $W_{jt}$  and sell good  $j$  at price  $P_{jt}$ . Prices  $P_{jt}$  are flexible and, given the technology above, the price of good  $j$  is equal to the wage,  $P_{jt} = W_{jt}$ , in equilibrium.

Given these assumptions, the steady state of the model is simply characterized by

$$P_A^* = P_B^* = W_A^* = W_B^* = 1, \quad c_A^* = \phi, \quad c_B^* = 1 - \phi,$$

where we normalize all nominal prices to 1. The fact that the relative price of the two goods is 1 comes from the assumption that the parameter  $\phi$  in the utility function (1) is equal to the relative size of the labor supply in sector  $A$ . This assumption can always be satisfied by an appropriate choice of units, so none of our results depend on it.<sup>5</sup>

As mentioned in the introduction, we are interested in shocks that affect the two sectors asymmetrically. We begin by considering an extreme shock leading to a complete shutdown of sector  $A$ . Namely, we consider a one-time, unexpected shock at  $t = 0$  that reduces potential output in sector  $A$  by reducing the labor supply of sector  $A$  workers to 0. The shock is temporary, so all agents' labor supply returns to 1 at  $t = 1$ .

To interpret the shock as a pandemic shock, assume sector  $A$  represents activities that require personal contact—for example, restaurants, hotels, and entertainment venues. This is the sector directly impacted by the shock. Sector  $B$  represents activities that do not require personal contact and are not directly impacted by the shock—for example, services that

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<sup>5</sup>An alternative interpretation is that labor supply in the two sectors is the result of long-run mobility of workers between sectors, leading to wage equalization.

can be supplied by workers working from home, like accounting services, and activities that can be re-organized to limit contacts, like manufacturing. The shock captures the idea that the pandemic makes it unsafe to produce and consume sector  $A$  goods, as doing so would increase the chance of infection for workers and consumers.

We begin from the case of a complete shutdown of sector  $A$ , but also consider other shocks. In Section 2.3, we consider partial reductions in activity in sector  $A$ . In Section 2.4, we consider shocks to preferences that reduce consumption of sector  $A$  goods. In the context of a pandemic shock, this may be due to perceived health risks, as we explicitly illustrate by including health in the utility function.

In the case of a full shutdown, all our derivations in the text assume  $\epsilon > 1$  so the utility function is well defined at  $c_{A0} = 0$ . However, in Section 2.3 we show that our arguments extend to the case  $\epsilon \leq 1$  by looking at a limit case in which  $c_{A0} \rightarrow 0$ .

We analyze the effects of the shock in two versions of the model: first with complete markets and then with incomplete markets. In both cases, we first look at a version of the economy with flexible prices, and ask what happens to output and the real (or natural) interest rate. Next, we consider an economy with downward rigid nominal wages and ask what happens to output and employment if the central bank keeps the nominal interest rate unchanged.

## 2.1 Complete Markets

Consider first the case of complete markets. Namely, suppose the economy is populated by a continuum of families, each composed of  $\phi$  sector  $A$  workers and  $1 - \phi$  sector  $B$  workers. In this way, perfect insurance against sectoral shocks can be achieved by transfers within the family.

**Flexible prices.** When the shock hits at date 0, the equilibrium allocation is

$$c_{A0} = Y_{A0} = 0, \quad c_{B0} = Y_{B0} = 1 - \phi,$$

activity in sector  $A$  shuts down but there is no change in activity in sector  $B$ . The economy returns to the steady state at date  $t = 1$ .

The real interest rate in terms of good  $B$  is defined as

$$1 + r_t \equiv (1 + i_t) \frac{P_{Bt}}{P_{Bt+1}},$$

where  $i_t$  is the nominal interest rate. As we show below, this is the relevant natural interest rate to gauge whether aggregate demand for good  $B$  has increased or fallen.

The Euler equation of the representative family, in terms of good  $B$ , is

$$U_{c_B}(c_{At}, c_{Bt}) = \beta(1 + r_t)U_{c_B}(c_{At+1}, c_{Bt+1}),$$

where  $U_{c_B}$  denotes the partial derivative of  $U$  with respect to  $c_B$ . In steady state, we have  $1 + r_t = 1 + r^* \equiv 1/\beta$  since all consumption levels are constant. After the shock, the natural interest rate is

$$1 + r_0 = \frac{1}{\beta} \frac{U_{c_B}(0, c_B^*)}{U_{c_B}(c_A^*, c_B^*)}. \quad (2)$$

Therefore we have a drop in the natural rate,  $r_0 < r^*$ , if and only if the ratio of marginal utilities on the right-hand side is smaller than 1. Using the functional forms above, this happens if

$$(1 - \phi)^{\frac{1}{\sigma} \frac{\sigma - \epsilon}{\epsilon - 1}} < 1. \quad (3)$$

An immediate implication is the following result.

**Proposition 1.** *In the complete markets economy, a shock that shuts down sector  $A$  reduces the natural interest rate in terms of good  $B$  if and only if*

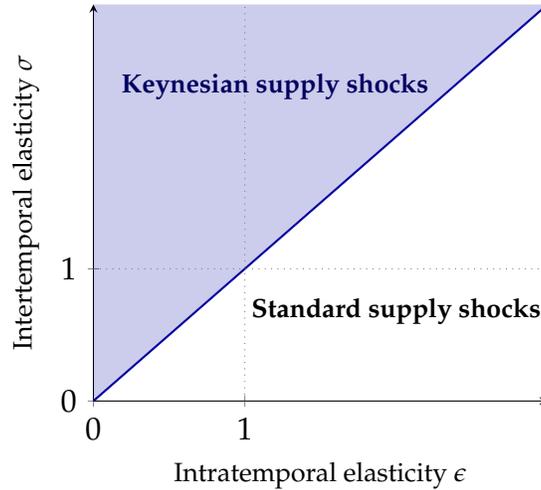
$$\sigma > \epsilon. \quad (4)$$

To interpret this result, notice that consumer choice is driven by inter-temporal and intra-temporal considerations. When good  $A$  becomes unavailable, consumers respond by substituting inter-temporally from present to future consumption since, in the future, both goods will be available again. This reduces demand for  $B$ . Consumers also substitute intra-temporally in favor of good  $B$ . This increases demand for  $B$ . When inequality (4) is satisfied the first force dominates, leading to a drop in demand for  $B$ . To induce consumers to consume enough to keep sector  $B$  at full employment, we need a lower interest rate.<sup>6</sup>

We call a situation in which the reduction in potential output in sector  $A$  causes a decrease in the natural interest rate in sector  $B$  a “Keynesian supply shock” (KSS). We illustrate condition (4) graphically in Figure 2 where we plot the pairs of elasticities  $(\sigma, \epsilon)$  for which we obtain a KSS. Figure 2 includes the case  $\epsilon \leq 1$ , based on the limit argument in Section 2.3, showing that the same condition  $\sigma > \epsilon$  continues to apply in that case.

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<sup>6</sup>For a general utility function, notice that equation (2) implies that a sufficient condition for  $r_0 < r^*$  is simply that the two goods be Edgeworth complements. With our CES preferences, that is equivalent to condition (4).



**Figure 2:** Keynesian supply shocks: complete markets

An immediate corollary of Proposition 1 is that the natural rate never falls when goods  $A$  and  $B$  are perfect substitutes, that is, when  $\epsilon \rightarrow \infty$ . This limit case corresponds to the case of a single-good economy. In a single-good economy, perfect substitutability between goods  $A$  that are no longer available and the remaining goods  $B$  always leads to an increase in demand for the remaining goods, requiring the natural rate to rise temporarily. This highlights that the multi-sector structure plays a crucial role in producing Keynesian supply shocks in our environment. We will return to this result in the incomplete market case.<sup>7</sup>

**Nominal rigidities.** We now introduce nominal rigidities. A convenient and tractable way to do so is to assume that nominal wages  $W_{jt}$  are downward rigid at  $t = 0$ . If labor demand falls below labor supply in sector  $j$ , nominal wages remain unchanged at their steady state value  $W^*$  and there is involuntary unemployment. In case of unemployment, we assume all workers in the sector are equally rationed. We denote by  $n_{B0}$  the employment of workers in sector  $B$ . An equilibrium with nominal wage rigidities must satisfy the following conditions,

$$n_{B0} \leq 1, \quad W_{B0} \geq W^*,$$

with at least one condition holding with equality, and goods market clearing,

$$c_{B0} = Y_{B0} = (1 - \phi) n_{B0}.$$

<sup>7</sup>As mentioned in the introduction, there are other channels by which a supply disturbance can cause a lack of demand, which do not require multiple sectors. In particular, a well-studied channel operates through uncertainty and precautionary effects (see references in the introduction).

In sector  $A$  there is zero labor supply at  $t = 0$ , so there cannot be involuntary unemployment. From period  $t = 1$  on, we assume that prices are flexible and return to their steady state levels. We continue to assume that firms are competitive, so nominal prices are equal to nominal wages,  $P_{jt} = W_{jt}$  in all periods. To derive the effect of the shock absent a monetary policy response, we suppose that the central bank sets the nominal rate  $i_t$  equal to the steady state interest rate  $i^* = 1/\beta - 1$  at all dates  $t$ .

Consider again the effect of a shock that shuts down sector  $A$  at date 0. Now we can use the Euler equation

$$U_{c_B}(0, c_{B0}) = \beta(1 + i^*) \frac{P_{B0}}{P_B^*} U_{c_B}(c_A^*, c_B^*), \quad (5)$$

to solve for  $c_{B0}$ . If there is insufficient demand for good  $B$ , the price of good  $B$  satisfies

$$P_{B0} = W_{B0} = W_B^* = P_B^*.$$

Evaluating (5), we then obtain the following result, which mirrors Proposition 1 in the case of nominal rigidities.

**Proposition 2.** *Consider the complete markets economy with downward rigid wages and assume the central bank keeps the interest rate at  $i_t = i^*$ . If  $\sigma > \epsilon$ , a shock that shuts down sector  $A$  generates involuntary unemployment in sector  $B$ :*

$$n_{B0} = \frac{Y_{B0}}{Y_B^*} = (1 - \phi)^{\frac{\sigma - \epsilon}{\epsilon - 1}} < 1. \quad (6)$$

*If  $\sigma \leq \epsilon$ , sector  $B$  remains at full employment.*

With nominal rigidities, the KSS condition  $\sigma > \epsilon$  implies that, at a given real interest rate, the drop in potential output in sector  $A$  causes output to fall below potential in sector  $B$ . In this case, the economy features two types of job losses: unavoidable job losses in sector  $A$  due to the direct effect of the shock and inefficient job losses in sector  $B$ .

**An interpretation based on shadow prices.** The shutdown of sector  $A$  can be interpreted as a shock sending the shadow price of good  $A$  to infinity. The welfare-based consumer price index (CPI) in this economy is

$$P_t = \left( \phi P_{At}^{1-\epsilon} + (1 - \phi) P_{Bt}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}. \quad (7)$$

This is derived in the usual way from (1), as the utility function  $U(c_{At}, c_{Bt})$  can be expressed as a utility function  $u(c_t) = \frac{\sigma}{\sigma-1} c_t^{1-\frac{1}{\sigma}}$  over a CES consumption bundle  $c_t = \left( \phi^{\frac{1}{\epsilon}} c_{At}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)^{\frac{1}{\epsilon}} c_{Bt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$ .

With  $\epsilon > 1$ , the CPI is well defined even as  $P_{At} \rightarrow \infty$  and is equal to

$$P_0 = \left( (1-\phi) (P_B^*)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} > \left( \phi (P_A^*)^{1-\epsilon} + (1-\phi) (P_B^*)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} = P^*.$$

The demand for the current consumption bundle  $c_0$  can then be derived from the Euler equation

$$u'(c_0) = \beta (1+i^*) \frac{P_0}{P_1} u'(c^*),$$

which yields

$$c_0 = \left( \frac{P_0}{P^*} \right)^{-\sigma} c^*.$$

The demand for good  $B$  is

$$c_{B0} = (1-\phi) \left( \frac{P_B^*}{P_0} \right)^{-\epsilon} c_0.$$

Combining the last two equations, and using  $P_B^* = P^*$  and  $Y_B^* = (1-\phi)$ , finally gives

$$Y_{B0} = \left( \frac{P^*}{P_0} \right)^{-\epsilon} \left( \frac{P_0}{P^*} \right)^{-\sigma} Y_B^*.$$

The interpretation of the last equation is as follows. The unavailability of good  $A$  makes today's consumption basket more expensive relative to future consumption, discouraging consumption today. This effect is captured by the term  $(P_0/P^*)^{-\sigma}$ , which is smaller than 1. At the same time, the unavailability of good  $A$  makes good  $B$  cheaper relative to the total consumption basket, inducing more consumption of good  $B$  for given  $c_0$ . This effect is captured by the term  $(P^*/P_0)^{-\epsilon}$ , which is larger than 1. With CES preferences, the relative strength of the two effects simply depends on comparing  $\sigma$  and  $\epsilon$ .<sup>8</sup>

We can then reinterpret a pandemic shock as a shock that, for a given nominal rate, leads to a sharp temporary increase in the real interest rate in terms of the total consumption basket  $(1+i_0) P_0/P_1$ , causing an increase in savings. The limitation of this interpretation is that the increase in the real interest rate  $(1+i_0) P_0/P_1$  is hard to observe in the data, as it requires using the shadow price of high-contact goods. The reduced availability of

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<sup>8</sup>This interpretation connects to the large literature on endogenous entry and varieties (e.g. [Bilbiie et al. 2012](#)) as our shock is equivalent to a shock that enlarges the number of varieties available in the future.

high-contact goods in the pandemic is reflected not so much in higher traded prices, but in various forms of rationing and trading restrictions.<sup>9</sup>

The discussion above helps clarify the distinction between what is happening here and what happens in a “paradox-of-toil” situation, as described by Eggertsson (2010). In Eggertsson (2010), a persistent, economy-wide negative supply shock, combined with Calvo pricing, causes expected inflation to go up, lowering the real interest rate and thus stimulating demand. In our model instead, a transitory supply shortage in sector  $A$  causes a temporary spike in the shadow price of good  $A$ , causing expected deflation (in terms of the welfare based CPI), increasing the real interest rate and depressing demand for good  $B$ .

## 2.2 Incomplete Markets

We now move to study the role of incomplete markets and borrowing constraints. Market incompleteness introduces an additional transmission channel across sectors: if sector  $A$  workers face income losses and have limited access to credit, they cut back spending in sector  $B$ .

It is helpful to label consumers by  $i \in [0, 1]$ . Each consumer works either in sector  $A$  or in sector  $B$ . There is no family insurance and no market insurance to protect  $A$  workers against the shock that shuts down sector  $A$ . Consumers have access to a one-period bond in zero net supply and their budget constraint is

$$P_{At}c_{iAt} + P_{Bt}c_{iBt} + a_{it} \leq W_{jt}n_{jt} + (1 + i_{t-1})a_{it-1},$$

where  $a_{it}$  denotes bond holdings and  $j$  denotes the sector in which agent  $i$  works. Moreover, a random fraction  $\mu \in [0, 1)$  of agents in each sector has no access to credit and faces the borrowing constraint

$$a_{it} \geq 0. \tag{8}$$

The economy starts in a symmetric steady state with  $a_{it} = 0$ ,  $c_{iAt} = \phi$ ,  $c_{iBt} = 1 - \phi$  for all agents. The pandemic shock is the same as in the complete markets economy: at  $t = 0$ , the labor supply of sector  $A$  workers falls to zero.

**Flexible prices.** Once again, we begin with the flexible price equilibrium. Consider first the mass  $\mu\phi$  of sector  $A$  workers with no access to credit. Since they have no income and

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<sup>9</sup>See Jaravel and O’Connell (2020) for an attempt at measuring a price index which captures the limited availability of varieties during a lockdown.

cannot borrow, their consumption of both goods goes to zero,  $c_{iA0} = c_{iB0} = 0$ . We call this the “constrained” group.

Consider the remaining  $1 - \mu\phi$  agents. This group includes the  $(1 - \mu)\phi$  agents in sector  $A$  who can borrow, and all  $1 - \phi$  agents in sector  $B$ , whether or not they can borrow. All these agents are on their Euler equation,

$$U_{c_B}(c_{iAt}, c_{iBt}) = \beta(1 + r_t)U_{c_B}(c_{iAt+1}, c_{iBt+1}). \quad (9)$$

This follows from the fact that all sector  $B$  workers have the same positive income  $W_{B0}$  and hence are net savers in equilibrium. We call this the “unconstrained” group.

Due to homothetic preferences, and therefore Gorman aggregation, the Euler equation (9) keeps holding if we replace individual consumption levels with the average levels in the group of unconstrained agents, which we denote in bold by  $\mathbf{c}_{At}$  and  $\mathbf{c}_{Bt}$ . We arrive at the following condition for the real interest rate

$$1 + r_0 = \frac{1}{\beta} \frac{U_{c_B}(0, \mathbf{c}_{B0})}{U_{c_B}(\mathbf{c}_{A1}, \mathbf{c}_{B1})}. \quad (10)$$

Let us evaluate this expression at the flexible price equilibrium.

At date  $t = 0$  constrained agents consume zero. Therefore, unconstrained agents must absorb all output of sector  $B$ , which is  $1 - \phi$ . Since there is a mass  $1 - \mu\phi$  of unconstrained consumers, we get

$$\mathbf{c}_{B0} = \frac{1 - \phi}{1 - \mu\phi}.$$

At date  $t = 1$ , average bond holdings among constrained consumers is 0. Thus, by bond market clearing, average bond holdings among unconstrained consumers must also be zero. Moreover, the relative price of good  $A$  is independent of the wealth distribution after  $t = 0$ , due to homothetic preferences, and returns to its original steady state value  $P_A^*/P_B^* = 1$  at  $t = 1$ .<sup>10</sup> Finally, real income returns to 1 for all agents. Together, this means that the average unconstrained agent’s consumption of the two goods is

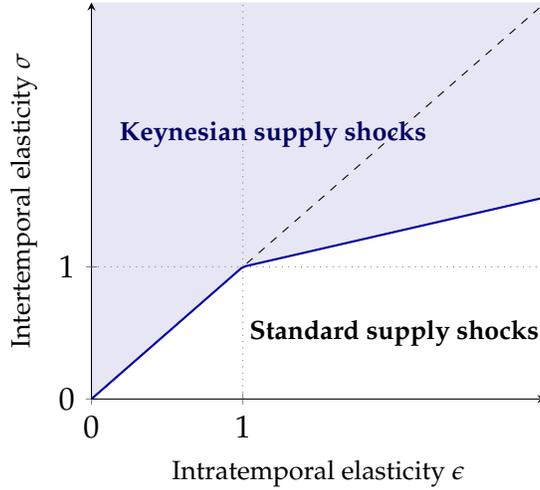
$$\mathbf{c}_{A1} = \phi, \quad \mathbf{c}_{B1} = 1 - \phi. \quad (11)$$

Substituting  $\mathbf{c}_{B0}$ ,  $\mathbf{c}_{A1}$ , and  $\mathbf{c}_{B1}$  into (10), we obtain an expression for the natural interest rate

$$1 + r_0 = \frac{1}{\beta} (1 - \mu\phi)^{\frac{1}{\sigma}} (1 - \phi)^{\frac{1}{\sigma} \frac{\sigma - \epsilon}{\epsilon - 1}}. \quad (12)$$

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<sup>10</sup>This price does not depend on the wealth distribution due to Gorman aggregation.



**Figure 3:** Keynesian supply shocks: incomplete markets

Taking logs and rearranging gives the following result.

**Proposition 3.** *In the incomplete markets economy, a shock that shuts down sector A reduces the natural interest rate in terms of good B if and only if*

$$\sigma > \epsilon + (1 - \epsilon) \frac{\log(1 - \mu\phi)}{\log(1 - \phi)}. \quad (13)$$

Proposition 3 shows that market incompleteness relaxes the condition in Proposition 1, as illustrated in Figure 3 for a  $\mu \in (0, 1)$ . We will discuss the intuition for this result after introducing nominal rigidities.

As in Figure 2, we include the region  $\epsilon \leq 1$  in Figure 3. We construct this region in Section 2.3 as the limit of partial shocks to sector A. When  $\epsilon \leq 1$  the KSS frontier is unaffected by incomplete markets. That is a result of the extreme effects of substitutability at zero consumption: in Section 2.3 we show that with partial shocks, market incompleteness strictly expands the KSS region for any value of  $\epsilon$ .

As in the complete markets case, in the case of perfect substitutes,  $\epsilon \rightarrow \infty$ , condition (13) is necessarily violated. This shows that market incompleteness alone is not enough to generate a KSS in our environment. Sectoral heterogeneity and imperfect substitution remain crucial.

Notice that there are two frictions here: markets are incomplete and there is a borrowing constraint. Both are needed to expand the KSS region. If all agents are allowed to borrow, that is, if  $\mu = 0$ , condition (13) simplifies to  $\sigma > \epsilon$ , which is identical to the complete markets condition (4). Because of Gorman aggregation and the fact that there are no shocks

after date 0, the incomplete-markets economy with  $\mu = 0$  has the same aggregate properties as the complete-markets economy. However, the distribution of individual consumption is different depending on whether we assume complete markets or incomplete markets and  $\mu = 0$ . This distinction will be important when we discuss the welfare effect of transfers in Section 3.2.

**Nominal rigidities.** We next add downward rigid wages, with the central bank keeping the nominal interest rate at  $i^*$ , as in the complete markets case.

Consider again the two groups, constrained and unconstrained consumers. The consumption of constrained consumers is still zero at  $t = 0$ . For unconstrained consumers,  $c_{A1}$  and  $c_{B1}$  are given by (11) as in the case with flexible prices, as real incomes and relative prices return to their steady state values at  $t = 1$ . Setting  $1 + r_0 = 1/\beta$  and solving the Euler equation (10) for  $c_{B0}$ , the average consumption of good  $B$  of unconstrained consumers is then

$$c_{B0} = (1 - \phi)^{\frac{\sigma-1}{\epsilon-1}}.$$

Adding the consumption of constrained and unconstrained consumers, with weights  $\mu\phi$  and  $1 - \mu\phi$ , the total demand for good  $B$  is

$$Y_{B0} = \mu\phi \cdot 0 + (1 - \mu\phi) c_{B0}.$$

Substituting in  $c_{B0}$  and recalling that  $Y_B^* = 1 - \phi$ , proves the following result.

**Proposition 4.** *Consider the complete markets economy with downward rigid wages and assume the central bank keeps the interest rate at  $i_t = i^*$ . If (13) is satisfied, a shock that shuts down sector  $A$  causes involuntary unemployment in sector  $B$  with*

$$n_{B0} = \frac{Y_{B0}}{Y_B^*} = (1 - \mu\phi) (1 - \phi)^{\frac{\sigma-\epsilon}{\epsilon-1}} < 1. \quad (14)$$

*If (13) is not satisfied, sector  $B$  remains at full employment.*

The term  $(1 - \phi)^{\frac{\sigma-\epsilon}{\epsilon-1}}$  in (14) captures the change in individual spending due to limited products' availability. This is the effect analyzed in the complete markets case. With incomplete markets, an additional force is at work, because a fraction of consumers earns zero income and is constrained to spend zero. The term  $1 - \mu\phi < 1$  captures this second force. Even if  $(1 - \phi)^{\frac{\sigma-\epsilon}{\epsilon-1}} > 1$ , so unconstrained consumers spend more on good  $B$  than in steady state, the second force can be sufficiently strong to cause a demand shortage. This explains why market incompleteness enlarges the KSS region in Figure 3.

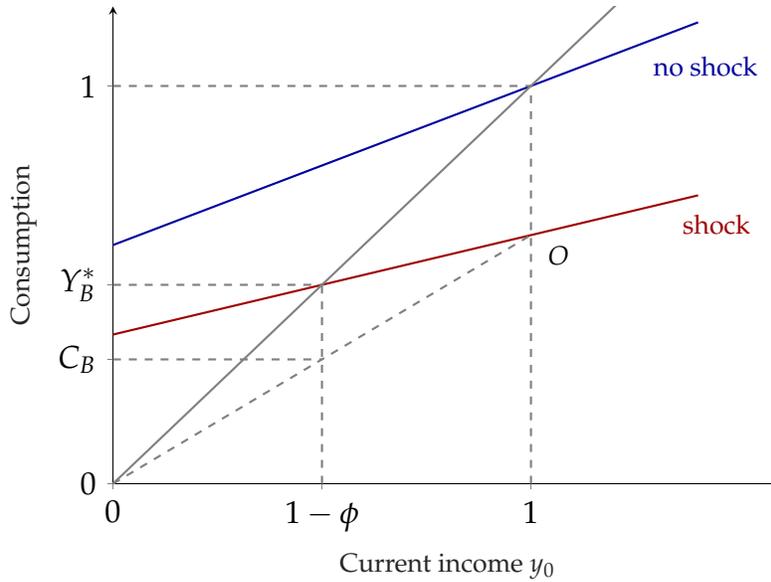


Figure 4: Consumption functions

**A decomposition using consumption functions.** To provide more intuition for this result, consider the following partial equilibrium construction, based on individual consumption functions. Consider an individual consumer, with real income, in units of good  $B$ , equal to  $y_0$  at  $t = 0$  and equal to 1 in all future periods. The consumer faces a constant nominal interest rate of  $i^*$  and all goods prices are at their steady state level. For concreteness, we focus on the case with  $\sigma = \epsilon$ .

A pandemic shock makes good  $A$  temporarily unavailable at  $t = 0$ . In Figure 4 we plot the relation between  $y_0$  and  $c_{A0} + c_{B0}$  before the shock (blue line) and  $c_{B0}$  after the shock (red line). The two lines represent the consumption functions of an unconstrained agent. The consumption function of an agent subject to the borrowing constraint  $a_{it} \geq 0$  can be read on the same graph, looking at the minimum between the unconstrained consumption function and the 45 degree line (gray line). The borrowing constraint makes the consumption functions concave.

With no shock, all consumers have real income  $y_0 = 1$  and the graph shows that their total consumption is equal to 1. The economy is in steady state equilibrium. When sector  $A$  shuts down, two things happen. The unconstrained consumption function shifts down, due to the unavailability of good  $A$ ; and sector  $A$  workers' incomes go to zero.

Suppose the income of sector  $B$  workers remains at its full employment level of 1. With complete markets, the income of the representative family then falls from 1 to  $1 - \phi$ . With  $\sigma = \epsilon$ , the consumption function shifts down by precisely the amount so that, with reduced

family income, demand for good  $B$  is equal to potential  $Y_B^* = 1 - \phi$ . This is why, with  $\sigma = \epsilon$ , there is neither excess demand nor a demand shortage.

With incomplete markets, consumers face heterogeneous income losses that they cannot insure against:  $\phi$  workers earn income 0 and  $1 - \phi$  workers earn income 1. To see how this changes the consumption function in 4, consider the case  $\mu \rightarrow 1$ , so all sector  $A$  workers are at zero consumption, while all sector  $B$  workers are unconstrained, at the point labeled  $O$ . Taking a linear combination of the two points, with weights  $\phi$  and  $1 - \phi$ , we obtain aggregate consumption, which we denote  $C_B$  on the  $y$  axis. Notice that  $C_B < Y_B^*$ , so there is a lack of demand and we cannot have an equilibrium with full employment in  $B$ .

To find the equilibrium level of  $Y_{B0}$ , we need to set the income  $y_0$  of sector  $B$  workers to  $n_{B0} < 1$  and solve for the fixed point in  $n_{B0}$ . This will yield the equilibrium output in (14). To avoid clutter, the general equilibrium values of  $n_{B0}$  and aggregate consumption are not plotted in the graph, as they do not add to the partial equilibrium intuition. The formal details behind Figure 4 can be found in Appendix A.

The construction above shows that the crucial reason why adding incomplete markets can turn a standard supply shock into a Keynesian supply shock is the concavity of the consumption function, which here simply comes from the borrowing constraint. Comparing the complete and incomplete markets cases, we see that reducing the income of  $A$  sector workers from  $\phi$  to 0 has a larger effect on reducing aggregate consumption than increasing the income of  $B$  sector workers from  $1 - \phi$  to 1.

The construction in Figure 4 also shows why in a one sector version of our economy (when  $\epsilon \rightarrow \infty$ ) it is not possible to get a Keynesian supply shock. In that case, the blue line and red line would be the same, so we would only have the effect coming from incomplete markets. The consumption function will still be concave, but the reduction in consumption of the constrained agents can at most be one for one with the drop in income (as the MPC is at most 1). Therefore the total reduction in consumption will always be weaker than the total drop in income.

The logic behind Figure 4 helps us derive a condition for Keynesian supply shocks in terms of measurable features of the consumption functions, instead of model parameters, that takes the following form:

$$\overline{MPC}^A > \frac{\Delta c_B}{\Delta c_A}. \quad (15)$$

The left-hand side  $\overline{MPC}^A$  is the average MPC of sector  $A$  workers. The right-hand side  $\Delta c_B / \Delta c_A$  is the change in consumption of good  $B$  that is associated with a reduction in spending on good  $A$  due to a rising shadow price of good  $A$  at the steady state level

of income.  $\Delta c_B / \Delta c_A$  can be interpreted as a form of cross-good marginal propensity to consume. It answers the question of how consumers would change their consumption  $\Delta c_B$  of good  $B$  if they were to save temporarily  $\Delta c_A$  by spending less on good  $A$ . The right-hand side captures the shift in the consumption function, while the left-hand side captures the movement along the consumption function. In Appendix A, we derive the expressions for  $\overline{MPC}^A$  and  $\Delta c_B / \Delta c_A$  in our model and prove that conditions (13) and (15) are indeed equivalent.

## 2.3 Partial Shutdowns

So far, we have focused on the case of an extreme shock that completely shuts down sector  $A$ . We now show that the analysis naturally extends to the case of a partial shutdown. We model a partial shutdown as labor supply of sector  $A$  workers falling to  $1 - \delta \in (0, 1)$  instead of zero at  $t = 0$ . We directly consider the case with incomplete markets and nominal rigidities. Here we specifically also allow for elasticities  $\epsilon < 1$ .

One implausible implication of a partial shutdown in an economy with elasticity  $\epsilon < 1$  is that the shutdown *raises* the income share of sector  $A$  workers, quite the opposite of a “shutdown”. To mitigate this issue we work in this section with a small modification to our economy, which assumes a completely rigid wage  $W_{A0} = W_{B0} = W^* = 1$  at  $t = 0$  in both sectors.

This assumption means that, in equilibrium, firms in sector  $A$  are rationed on their labor demand. Since the price  $P_{A0}$  still needs to increase in equilibrium to clear the goods market, firms in sector  $A$  will be making positive profits  $(P_{A0} - W^*) Y_{A0}$  at date 0. We assume that firms are symmetrically owned by those consumers who do not face the borrowing constraint  $a_{it} \geq 0$ , implying that the profits go to them. With this assumption, the limit case  $\delta \rightarrow 1$  converges to our analysis of a full shutdown for any  $\epsilon > 1$ .

Sector  $A$  is always supply constrained in equilibrium. We now look for conditions such that sector  $B$  is demand constrained in equilibrium, that is,

$$P_{A0} > W^*, \quad P_{B0} = W^*, \quad n_{A0} = 1 - \delta, \quad n_{B0} < 1.$$

Following steps analogous to those in (2.2), the average consumption of unconstrained consumers can be derived from their Euler equation and is given by

$$\mathbf{c}_0 = \left( \frac{P_0}{P^*} \right)^{-\sigma}.$$

Constrained sector  $A$  workers just consume their income  $W^* (1 - \delta)$ . Moreover, individual demand functions for goods  $A$  and  $B$  are

$$c_{iA0} = \phi \left( \frac{P_{A0}}{P_0} \right)^{-\epsilon} c_{i0}, \quad c_{iB0} = (1 - \phi) \left( \frac{P_{B0}}{P_0} \right)^{-\epsilon} c_{i0},$$

for both constrained and unconstrained consumers.

Combining the results above and using  $P_{B0} = W^* = P^*$ , we find the following aggregate demand equations for goods  $A$  and  $B$

$$\begin{aligned} Y_{A0} &= \phi \left( \frac{P_{A0}}{P_0} \right)^{-\epsilon} \left( \mu \phi \frac{W^*}{P_0} (1 - \delta) + (1 - \mu \phi) \left( \frac{P_0}{W^*} \right)^{-\sigma} \right) \\ Y_{B0} &= (1 - \phi) \left( \frac{W^*}{P_0} \right)^{-\epsilon} \left( \mu \phi \frac{W^*}{P_0} (1 - \delta) + (1 - \mu \phi) \left( \frac{P_0}{P^*} \right)^{-\sigma} \right). \end{aligned} \quad (16)$$

Imposing market clearing for good  $A$ ,  $Y_{A0} = \phi (1 - \delta)$ , yields the following result after a few steps of algebra (see Appendix B).

**Proposition 5.** *Consider the incomplete markets economy with rigid nominal wages and assume  $i_t = i^*$ . If the following condition is satisfied*

$$\sigma > \epsilon - (1 - \epsilon) \frac{\ln \left( 1 - \mu \phi \frac{1 - \delta}{\phi (1 - \delta)^{1 - \frac{1}{\epsilon}} + 1 - \phi} \right) - \ln (1 - \mu \phi)}{\ln \left( \phi (1 - \delta)^{1 - \frac{1}{\epsilon}} + 1 - \phi \right)} \quad (17)$$

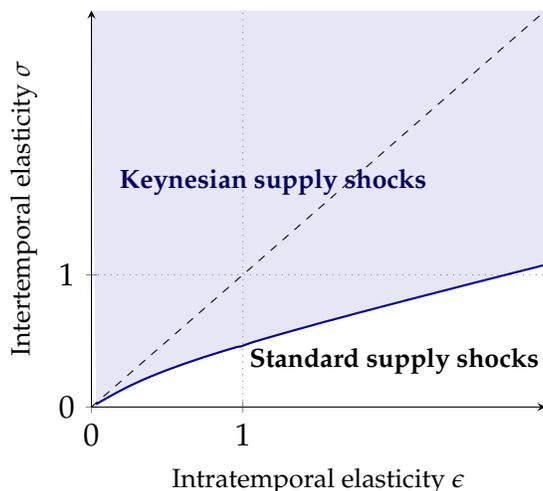
a shock that reduces labor supply in sector  $A$  by  $\delta \in (0, 1)$  causes involuntary unemployment in sector  $B$ . As  $\delta \rightarrow 0$  the condition becomes

$$\sigma > \frac{\epsilon (1 - \mu) - \mu \phi}{1 - \mu \phi}. \quad (18)$$

As  $\delta \rightarrow 1$ , the condition becomes (13) for  $\epsilon > 1$  and  $\sigma > \epsilon$  for  $\epsilon \leq 1$ .

Condition (17) is illustrated graphically in Figure 5. With a partial shutdown, we do not need to impose restrictions on  $\epsilon$ . Moreover, by letting  $\delta \rightarrow 1$  we can recover a KSS region for any  $\epsilon$ . This is how we derive the piecewise linear frontier plotted in Figure 3 (see Appendix B for details).

A somewhat surprising result is that a partial shutdown can lead to a larger KSS region relative to a full shutdown. In particular, keeping all other parameters the same, the frontier



**Figure 5:** Keynesian supply shocks: partial shutdown

with  $\delta \in (0, 1)$  shown in Figure 5 always lies strictly below the frontier for  $\delta \rightarrow 1$  shown in Figure 3.<sup>11</sup> We can get some intuition for this fact by inspecting equation (16). The  $\delta$  shock has two effects on the demand for good  $B$ . A direct effect due to the reduction in sector  $A$  workers' income, captured by the term  $1 - \delta$ . An indirect effect due to the increase in the price of good  $A$ , and thus of the CPI  $P_0$ . The first effect is linear and always goes in the direction of reducing spending in sector  $B$ . The second effect is nonlinear and can be more powerful for high values of  $\delta$ . Therefore, with a large  $\delta$  the second effect can go in the direction of increasing spending in sector  $B$  and dominate the first effect, preventing a KSS.

## 2.4 Preference Shocks and Health Shocks

So far we have focused on aggregate demand spillovers in sector  $B$  caused by a shock to labor supply in sector  $A$ . What if, instead, the shock was not to the supply but to the demand for sector  $A$  goods?

Take the partial shutdown model introduced above and let the per-period utility function be given by

$$U(c_{At}, c_{Bt}) = \frac{\sigma}{\sigma - 1} \left( \phi^{\frac{1}{\epsilon}} \theta_t^{\frac{1}{\epsilon}} c_{At}^{\frac{\epsilon-1}{\epsilon}} + (1 - \phi)^{\frac{1}{\epsilon}} c_{Bt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon-1}{\epsilon} \frac{\sigma-1}{\sigma}}. \quad (19)$$

Consider now the effects of an unexpected, temporary preference shock  $\theta_0 = \theta < 1$  at  $t = 0$  with  $\theta_t = 1$  for any  $t > 0$ . Proposition 8 in Appendix C shows that the logic of

<sup>11</sup>Both figures are drawn using  $\phi = 0.5$  and  $\mu = 0.8$ . Figure 5 uses  $\delta = 0.8$ .

the transmission from sector  $A$  to sector  $B$  is analogous to the case of a supply shock. A negative preference shock to good  $A$  is similar to an increase in the price of good  $A$ . This effective price increase affects an appropriately measured CPI in a similar way as the increase in  $P_{A0}$  caused by a supply shock. Again, a contraction in spending in sector  $B$  will occur if intertemporal substitution and the income effect of constrained agents are stronger than intratemporal substitution across goods.<sup>12</sup> The case of an extreme shock  $\theta = 0$  is especially clean, as it leads to the same condition (13) derived in the case of a supply-induced full shutdown for  $\epsilon > 1$ .

It is useful to consider a variant of the demand shock in sector  $A$  that captures explicitly health in the utility function. For this variant, let us focus, for simplicity, on log preferences and assume the consumers' per-period utility at date 0 is

$$\phi \log c_{A0} + (1 - \phi) \log c_{B0} - h$$

where  $h$  is the probability of being infected during the pandemic. In all future periods, the per-period utility is just  $\phi \log c_{At} + (1 - \phi) \log c_{Bt}$ , as the pandemic only lasts one period. Let  $h$  depend both on the individual and on the aggregate level of activity in sector  $A$  according to

$$h = \eta \log c_{A0} + v(Y_{A0}). \quad (20)$$

The term  $v(Y_{A0})$  captures an externality and introduces a motive for public health interventions.<sup>13</sup> Rearranging, the consumers' date 0 objective is  $(\phi - \eta) \log c_{A0} + (1 - \phi) \log c_{B0}$ , so the pandemic is isomorphic to a temporary preference shock  $\theta = \frac{\phi - \eta}{\phi} < 1$  to the utility of good  $A$ . This model is analyzed in Appendix C and used to discuss interactions between fiscal policy and health policy in Section 3.3.

### 3 Fiscal Policy

We now turn to fiscal policy. We discuss the size of the government spending multiplier, the role of social insurance, and the interactions between fiscal policy and public health measures.

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<sup>12</sup>One dimension in which the supply shock  $\delta$  and the demand shock  $\theta$  are clearly different is the implications for the *measured* price  $P_{A0}$ , which goes up with a supply shock and stays unchanged with a demand shock.

<sup>13</sup>The term  $\eta \log c_{A0}$  ensures that the individual problem remains homothetic.

### 3.1 The Muted Power of Stimulus

To consider the effects of fiscal policy, we add a stylized government sector to the baseline economy of Section 2.2, with incomplete markets, rigid nominal wages, and a full shutdown of sector  $A$ .

At date 0 the government has two instruments: government purchases  $G$  (of good  $B$ ), and real transfers to workers in sectors  $A$  and  $B$  equal to  $\rho(1 - n_{j0})$ . The parameter  $\rho \in [0, 1]$  determines the replacement rate at which the government covers workers' income losses. Total transfers are

$$T = \phi\rho + (1 - \phi)\rho(1 - n_{B0}).$$

The government issues bonds  $D$  to finance spending at date 0, so its budget constraint is

$$D = G + T.$$

In all future periods, the government taxes uniformly the  $1 - \mu$  agents who can borrow. The government budget constraints then requires that each agent who can borrow pays the tax

$$\frac{r^*}{1 - \mu} D$$

from period  $t = 1$  on. The following result is proved in the appendix.

**Proposition 6.** *Consider the incomplete market economy and suppose condition (13) is satisfied. Suppose the central bank keeps the interest rate at  $r_0 = 1/\beta - 1$ . The effects of government purchases and transfers on output, at  $G = \rho = 0$  are given by*

$$\frac{dY_{B0}}{dG} = 1, \quad \frac{dY_{B0}}{dT} = \frac{\mu\phi}{\phi + (1 - \phi)(1 - n_{B0})}. \quad (21)$$

The multiplier of government purchases is 1. The multiplier of transfers is lower than 1 because, given a dollar of transfers, only a fraction goes to constrained agents. The effect per dollar transferred to constrained agents is 1 as in the case of  $G$ .

This is a striking result. The average MPC in the economy is

$$\overline{mpc} = (1 - \mu\phi) \times mpc^U + \mu\phi \times mpc^C, \quad (22)$$

where  $mpc^C = 1$  is the MPC of constrained agents and

$$mpc^U = \frac{1 - \beta}{1 - \beta + \beta(1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}}$$

is the MPC of unconstrained agents. This latter MPC is relatively small and vanishes as  $\beta \rightarrow 1$  and is further reduced due to the unavailability of  $A$  goods when  $\sigma > 1$ .<sup>14</sup> A traditional Keynesian-cross argument would suggest a multiplier of  $1 / (1 - \overline{mpc}) \geq 1$ , which can be large if there is a large fraction of constrained consumers. Several recent papers show that similar Keynesian-cross arguments hold in models with heterogeneity and incomplete markets (Galí et al. 2007, Farhi and Werning 2016, Auclert et al. 2018, Bilbiie 2019). What is different here?

The crucial difference is that sector  $A$  is shut down, so agents cannot spend on sector  $A$ . This severs the usual virtuous cycle behind the Keynesian multiplier. The usual multiplier emerges because the initial stimulus increases incomes, which are then partly spent on all goods; this higher spending creates still higher incomes and so on. However, when sector  $A$  is shut down, money spent by agents or by the government flows into the pockets of sector  $B$  workers, not of sector  $A$  workers with higher MPCs. Thus, traditional fiscal stimulus is less effective dollar-for-dollar. The expression that captures correctly the response of aggregate spending to an increase in output  $Y_{B0}$  is not (22) but

$$(1 - \mu\phi) \times mpc^U \times \frac{dy^U}{dY_{B0}} + \mu\phi \times mpc^C \times \frac{dy^C}{dY_{B0}} = \frac{1 - \beta}{1 - \beta + \beta(1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}} \quad (23)$$

where  $dy^U/dY_{B0}$  and  $dy^C/dY_{B0}$  are the sensitivities of the incomes of each group to aggregate date-0 output  $Y_{B0}$ . These two quantities are equal to  $dy^U/dY_{B0} = 1 / (1 - \mu\phi)$  and  $dy^C/dY_{B0} = 0$  in the model, so the weight on constrained agents goes to zero. The observation that the aggregate effect of shocks depends on the joint distribution of MPCs and income sensitivities is in line with Patterson (2019). However, the pattern of correlation between MPCs and sensitivities is negative due to the asymmetric supply shock, as opposed to positive as estimated by Patterson (2019) during normal times, leading to a dampened effect of fiscal stimulus.<sup>15</sup>

<sup>14</sup>See the derivations of the consumption functions in Appendix A. The notation  $mpc$  used here denotes the marginal propensity to consume for an infinitesimal income change, while the notation  $MPC$  used in (15) denotes the marginal propensity to consume for a discrete income change.

<sup>15</sup>It is useful to clarify that our argument of a muted power of stimulus depends on what is the term of comparison. Our analysis above implicitly compares our economy to an economy with the same marginal distribution of MPCs but without the multi-sector structure. To build such a reference economy, one can

The expression in (23) is small if  $\beta$  is close to 1, but not zero, suggesting a small but positive consumption response to a  $G_0$  shock. Why do we get a unit multiplier in Proposition 6 instead? The answer has to do with the Ricardian behavior of unconstrained agents, who anticipate higher taxes in the future to finance  $G_0$ . This leads them to reduce their consumption, all else equal, exactly canceling the multiplier effect. A similar argument was made by Woodford (2011) for the representative agent case.<sup>16</sup>

### 3.2 The Importance of Social Insurance

While transfers have lower multipliers than usual, this does not imply that they should not be used. Indeed, transfers serve an important role as social insurance and may provide the best form of fiscal support.

For the next result, we generalize slightly the fiscal intervention and assume that the taxes in  $t = 1, 2, \dots$  are distributed as follows: agents who cannot borrow pay  $\frac{\zeta}{\mu} r^* D$  for some  $\zeta \in [0, 1]$ ; agents who can borrow pay  $\frac{1-\zeta}{1-\mu} r^* D$ . The case analyzed above corresponds to  $\zeta = 0$ .

The following result, proved in the appendix, characterizes the effects of changing the replacement rate  $\rho$ .

**Proposition 7.** *Suppose the interest rate is fixed at  $i_0 = i^*$  and condition (13) for a Keynesian supply shock is satisfied. There exists a cutoff  $\hat{\rho} < 1$  such that output  $Y_{B0}$  is increasing in  $\rho$  for  $\rho < \hat{\rho}$  and constant for  $\rho \geq \hat{\rho}$ . Setting  $\zeta = \mu$  and  $\rho = 1$  achieves the complete markets allocation and is optimal for a utilitarian planner.*

In a nutshell, this proposition states that targeted transfers play a dual role. First, they get us closer to full employment, sometimes all the way to full employment. Second, they provide insurance against income losses. Moreover, a utilitarian planner would always

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introduce idiosyncratic income shocks in a one sector model, so that a fraction  $\mu\phi$  of agents has MPC equal to 1, and hit that economy with a uniform shock, for example a shock to the discount factor  $\beta$ , causing a reduction in consumption of the unconstrained agents. The multiplier of  $G$  in that economy would be  $1 / (1 - \overline{mpc}) > 1$ .

However, if one compares our economy to a one sector economy without idiosyncratic shocks, that economy would behave as a representative agent economy. It would display a multiplier of  $G$  equal to 1 and a multiplier of transfers equal to 0. Compared to that economy, our economy displays the same  $G$  multiplier and a *larger* transfer multiplier. That happens because asymmetric shocks can make more agents constrained and increase the average MPC in the economy.

<sup>16</sup>With different assumptions on the distribution of taxes in future periods, we can get a multiplier above 1. For example, if we assume that all agents pay a permanent tax from period  $t = 1$  onwards we obtain a multiplier for  $G_0$  of  $1 + \mu\phi(1 - \phi)^{\frac{\sigma-1}{\sigma-1}} \frac{1-\beta}{\beta}$  instead of 1.

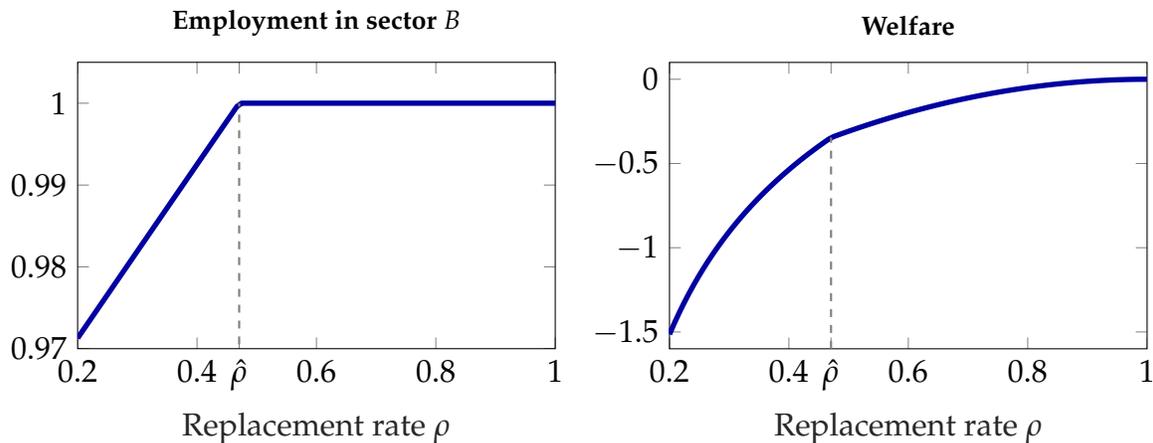


Figure 6: Effects of transfers

prefer full insurance, precisely because of the “double whammy”: it offers maximal macro stimulus while at the same time giving the best interpersonal micro allocation.

Figure 6 shows the effects of changing  $\rho$  in a simple example.<sup>17</sup> The left panel shows that, as the replacement rate increases, employment in sector  $B$  increases—as long as the  $A$  sector workers who can’t borrow are constrained. When  $\rho$  passes the cutoff  $\hat{\rho}$  these workers become unconstrained. At that point, since the economy features Gorman aggregation, the transfer no longer affects aggregate spending. The example uses  $\sigma = \epsilon$ , so at  $\hat{\rho}$  the economy reaches full employment. However, the right panel of Figure 6 shows that increasing the transfer above  $\hat{\rho}$  is welfare improving. The reason is that even though total spending at time 0 is the same, as  $\rho$  increases lifetime resources get reallocated to the  $A$  workers hit directly by the shock. This insurance benefit implies that social welfare increases until  $\rho = 1$ , at which point, given that  $\zeta = \mu$ , lifetime resources are equalized across consumers and we reach the first-best allocation.

Despite the virtues of full insurance, Proposition 7 and Figure 6 also clarify that in order to minimize the output gap it is sufficient to have replacement rates lower than full insurance and lower than 100%. In other words, if the only concern is restoring full employment, then lower levels of transfers will do. The reason for this result is that due to the supply shock, the natural aggregate allocation requires consumers to temporarily reduce their total spending. So to reach that allocation, replacement rates below 100% are sufficient.

<sup>17</sup>The parameters for the example are:  $\phi = 0.5, \mu = 0.1, \beta = 0.8, \epsilon = \sigma = 0.5$ . Welfare is measured in percent distance in consumption equivalent terms from the first best, using a utilitarian social welfare function.

Clearly, the model is highly simplified, so the result that the stimulus effect of the transfer goes to zero after some cutoff is extreme. But we believe there are two broad lessons here, which will extend to richer models. The first is that the stimulus effect of transfers gets weaker as the size of the transfers is increased. The reason is that unconstrained consumers in the economy have high desired saving rates due to the supply shock in sector  $A$ , so at some point transfers will be large enough that consumers will find it optimal to save a substantial fraction of them. The second point is that even if we reach the point at which stimulus effects are weak, it may still be desirable to further increase transfers, to buffer the loss in lifetime income of the consumers directly affected.<sup>18</sup>

### 3.3 Complementarity of Fiscal Policy and Public Health Policy

To conclude this section, we turn to the model with health in the utility function introduced at the end of Section 2.4, in which we analyzed a partial shutdown of sector  $A$ . We now consider an additional policy intervention: a Pigouvian tax  $\tau$  on the consumption of good  $A$ . The aim of the tax is to correct the externality due to the term  $v(Y_{A0})$  that enters the probability of infection (20). We can interpret the tax  $\tau$  broadly as interventions aimed at reducing activity in sector  $A$ , like stay-at-home orders or social distancing measures.

In this environment, we make three remarks on fiscal policy. These remarks can all be made using the illustrative example in Figure 7.<sup>19</sup> The left panels of the figure plot welfare and sector  $A$  employment as a function of the replacement rate  $\rho$ , in the case of a zero Pigouvian tax,  $\tau = 0$ . The right panels plot the same functions when the optimal Pigouvian tax  $\tau = \tau^*$  is in place.

We start with an elementary observation.

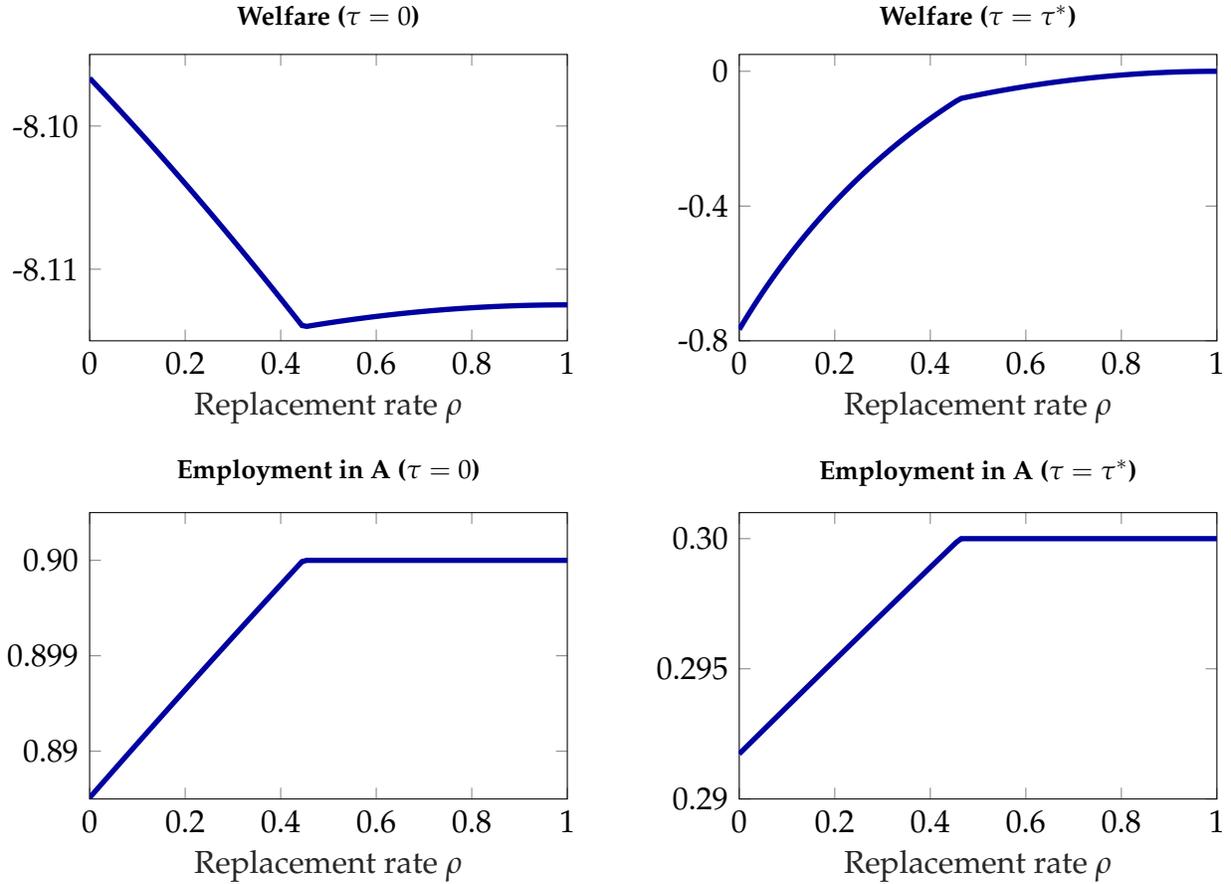
*Remark 1.* Involuntary unemployment is not necessarily socially inefficient in our model.

To make this point, notice that in our example, when  $\tau = 0$ , it is optimal to set a zero transfer  $\rho = 0$  even though both sectors are below full employment (employment in sector  $B$ , not in the graph, is proportional to employment in sector  $A$ ). The reason is that increasing  $\rho$  increases employment in both sectors. While increasing activity in sector  $B$  is welfare improving, increasing it in sector  $A$  is welfare reducing as it increases

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<sup>18</sup>There may be additional reasons why transfers are desirable after a pandemic shock. In particular, Woodford (2020) investigates a situation in which the network structure of the economy is richer than the two sector structure studied here and analyzes the benefit of transfers that help restore the flow of payments through the network.

<sup>19</sup>The parameters for the example are:  $\phi = 0.5, \mu = 0.1, \eta = 0.05, \beta = 0.8$ . The externality term is  $v(Y) = 3 \cdot Y$ .



**Figure 7:** Effects of transfers with and without a Pigouvian tax  $\tau$

the externality term  $v(Y_{A0})$ . There are in fact two benefits from increasing  $\rho$ : increased employment and improved insurance. However, these benefits are more than compensated by the Pigouvian externality. This point is related to the general observation made in one-sector macro-epidemiological models like [Eichenbaum et al. \(2020\)](#), that reducing total activity can be socially desirable in order to slow down infections.

In the example above, there is a trade-off between public health objectives and aggregate demand stabilization. That happens because there is no policy that can differentially discourage activity in sector  $A$ . Once we introduce the tax  $\tau$ , are the social welfare benefits of macro stabilization smaller or larger? That is, are the health policy tool  $\tau$  and the macro policy tool  $\rho$  complements or substitutes?

The next remark shows that in our context they can be complements.

*Remark 2.* Increasing the tax  $\tau$  can increase the marginal social benefit of raising the transfer  $\rho$ . So for some range of parameters, the two policies are complements.

The remark follows from comparing the panels on the left to those on the right of Figure

7. The presence of the tax keeps the level of activity in sector  $A$  low. This increases the marginal benefit of increasing  $\rho$  above zero through two channels: first, the presence of the tax  $\tau$  tends to worsen the Keynesian supply shock by reducing activity and incomes in sector  $A$ , and thus producing a larger spillover on activity in sector  $B$ . Since increasing activity in sector  $B$  is desirable, the presence of the tax  $\tau$  makes the transfer  $\rho$  more desirable at the margin. Second, the tax  $\tau$  tends to lower the incomes of sector  $A$  workers more than those of  $B$  workers, increasing the marginal benefit of the transfer in terms of insurance.

Our last remark is that when all policy tools,  $\rho$ ,  $i_0$  and  $\tau$ , are set optimally the planner can achieve the first best in our economy.

*Remark 3.* In the incomplete markets economy, a combination of public health policies, social insurance policies, and monetary policy can achieve the first best for a utilitarian social planner.

To prove this remark it is enough to see that at  $\rho = 1$  agents incomes are equalized, providing perfect insurance. The interest rate  $i_0$  can then be set to achieve full employment in sector  $B$ , while the tax  $\tau$  is set to achieve the level of activity in sector  $A$  that optimally balances consumption of good  $A$  against the health externality.

In the example just discussed, there is a combination of policies that achieves the first best allocation: a stay-at-home policy that shuts down sector  $A$ , a social insurance policy that compensates the workers in sector  $A$ , and a monetary policy that hits the natural rate. The fact that the social insurance policy makes it easier to achieve the demand stabilization objective is not surprising per se: it is an example of a fiscal policy that makes it easier to do monetary policy. The novel observation is that this type of fiscal policy also makes it less costly for the government to effectively worsen supply shock on the economy to pursue public health objectives.

## 4 Extending the Model

Here we present two simple extensions of our basic Section 2 model: allowing for an input-output network structure of production and allowing for labor mobility across sectors.

### 4.1 Demand Chains

The analysis so far has emphasized the complementarity between the two sectors in consumer preferences. Introducing input-output relations allows for complementarity in

production. The usual argument is that supply chain disruptions cause an amplification of supply shocks occurring upstream in the chain (sector  $A$  as input into sector  $B$ ). In contrast, here we focus on downstream disruptions that reduce demand for upstream sectors (sector  $B$  as input into sector  $A$ ), thus amplifying the induced demand shortage. We use the term “demand chain” to capture this mechanism.

In particular, we consider the possibility that goods produced in sector  $B$  are used as intermediate inputs by the shocked sector  $A$ . Intuitively, restaurants in sector  $A$  may buy goods and services, such as dishwashers and pest control, from sector  $B$  to produce their final good. When restaurants shut down, this impacts sector  $B$  separately from consumer responses. We investigate this idea with a simple input-output structure.

We continue to use the same preferences, but change the technology in sector  $A$  to the constant returns to scale technology

$$Y_A = F(X, N_A),$$

where  $X$  denotes good  $B$  used as intermediate input in sector  $A$ . Good  $B$  is still produced linearly from labor  $Y_B = N_B$ . We choose parameters so that  $p_A^* = p_B^* = w_A^* = 1$ , which is just a convenient normalization. This implies that, as in Section 2, the parameter  $\phi$  in the utility function (1) gives the steady state share of  $A$  in aggregate consumption. Each worker supplies 1 unit of labor and is fully specialized, with fraction  $\tilde{\phi}$  working in sector  $A$  and fraction  $1 - \tilde{\phi}$  in sector  $B$ . Let  $x^* \equiv X^*/Y_B^*$  denote the steady state fraction of good  $B$ 's output used as input in sector  $A$ . Accounting requires that for any parametrization of the model we have  $\tilde{\phi} \leq \phi$  and  $x^* < \phi$ .<sup>20</sup>

Consider now the effect of a temporary shutdown of sector  $A$ . Skipping directly to the incomplete markets case with the real rate at  $1/\beta - 1$ , the recession in sector  $B$  is now given by

$$\frac{Y_{B0}}{Y_B^*} = (1 - (1 - \mu)x^* - \mu\phi) (1 - \phi)^{\frac{\sigma - \epsilon}{\epsilon - 1}}. \quad (24)$$

With greater  $x^*$  the output drop is amplified and the condition for a Keynesian supply shock is relaxed, expanding the region in Figure 3. The expression in (24) reveals a useful intuition. The input share  $x^*$  acts as if it were a separate constrained agent group that loses  $x^*$  income and mechanically lowers its spending on  $A$ . The input-output from production has a marginal propensity to spend of one. In this sense, having demand chains is like

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<sup>20</sup>Total consumption equals total income and total income is 1 in steady state, as wages are 1 in both sectors. Since consumption of  $B$  goods is a share  $1 - \phi$  of total consumption, we have  $c_B^* = (1 - \phi)$ . Moreover, from goods market clearing at full employment we have  $c_B^* = Y_B^* - X^* = (1 - x^*) (1 - \tilde{\phi})$ . It follows that  $1 - \phi = (1 - x^*) (1 - \tilde{\phi})$ . The two inequalities follow from  $x^* \geq 0$  and  $\tilde{\phi} > 0$ .

having a higher fraction of constrained agents.

## 4.2 Labor Mobility

So far we have assumed workers cannot move between sectors. We now extend the analysis to allow a fraction  $\lambda$  of workers in each sector to freely move to the either sector. We revert to the model without demand chains (i.e.  $x^* = 0$ ).

The total fall in output can be decomposed as a drop in natural output and an inefficient output gap due to the lack of demand. A fraction  $(1 - \lambda)\phi$  of workers in  $A$  cannot move to sector  $B$ , so natural output is now  $Y_0^* = Y_{B0}^* = (1 - (1 - \lambda)\phi)$ , and the ratio of natural output to steady state output

$$\frac{Y_0^*}{Y^*} = 1 - \phi + \lambda\phi < 1$$

is increasing in  $\lambda$ . Mobility makes the supply shock less severe. Equilibrium output can be shown to equal  $Y_{B0} = (1 - (1 - \lambda)\phi\mu) (1 - \phi)^{\frac{\sigma-1}{\epsilon-1}}$ , so equilibrium output also rises with mobility.<sup>21</sup> However, the ratio of equilibrium to natural output

$$\frac{Y_0}{Y_0^*} = \frac{1 - \phi\mu + \lambda\phi\mu}{1 - \phi + \lambda\phi} (1 - \phi)^{\frac{\sigma-1}{\epsilon-1}}$$

is decreasing in  $\lambda$ . Thus,  $\lambda > 0$  expands the parameter space  $(\epsilon, \sigma)$  where supply shocks are Keynesian relative to  $\lambda = 0$ .

Complete markets is equivalent to setting  $\mu = 0$  in the above expressions. In this case natural output rises with mobility  $\lambda$ , while equilibrium output is constant. This is illustrated in panel (a) of Figure 8. The gap between the two lines grows with  $\lambda$ . Turning to incomplete markets  $\mu > 0$ , natural output is rising in mobility, just as with complete markets. However, equilibrium output is now also rising in labor mobility. Intuitively, demand is now affected by mobility  $\lambda$  because workers moving out of  $A$  into  $B$  do not lose as much income. Panel (b) in Figure 8 illustrates this situation. According to our expressions the ratio of output to natural output falls with  $\lambda$ , so that the the gap between the two lines grows with  $\lambda$ .

Summing up, labor mobility makes the size of the recession smaller, by buffering income losses, but it raises the level of natural output by still more, thus making the demand deficiency more severe.

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<sup>21</sup>The fraction of workers in  $A$  that cannot move and are thus effectively constrained is  $(1 - \lambda)\mu\phi$ . The remaining fraction of workers  $1 - \phi\mu + \lambda\phi\mu$  consume  $(1 - \phi)^{\frac{\sigma-1}{\epsilon-1}}$  on average, yielding the desired expression.

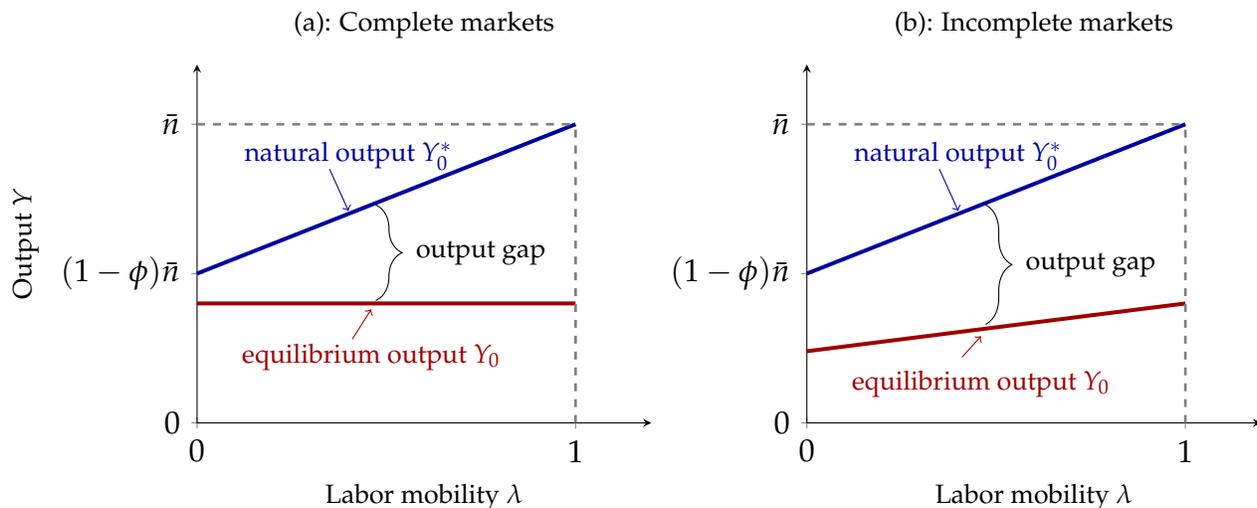


Figure 8: Labor mobility and the output gap.

## 5 Labor Hoarding vs Job Match Destruction

We now turn to an extension in which we model how firms and workers match to produce output. This allows us to discuss how firms in sector  $A$  can provide insurance by retaining workers on the payroll even if they are temporarily unproductive. Moreover, it allows us to discuss potential medium run effects of the shutdown, if destroyed matches are hard to rebuild.<sup>22</sup>

### 5.1 Generalizing the Model

For this extension, we generalize the model of the previous sections, allowing for a continuum of monopolistically competitive firms to produce each sector's output,

$$Y_{At} = \left( \int_0^\phi y_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad Y_{Bt} = \left( \int_\phi^1 y_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

To ensure tractability, we assume that the elasticity of substitution between firms is the same as the one across sectors,  $\epsilon$ . This assumption is not relevant for the results in this section. Each firm  $i$  is matched with a (representative) worker  $i$  who works in that sector. Thus, there are  $\phi$  firms and workers in sector  $A$  and  $1 - \phi$  firms and workers in sector  $B$ .

Due to monopolistic competition, each firm charges a markup  $\frac{1}{\epsilon-1}$  over marginal cost,

<sup>22</sup>A model that also emphasizes the medium-run effects of job destructions is [Gregory, Menzio and Wiczer \(2020\)](#).

so that the real wage is given by

$$w = 1 - \epsilon^{-1}$$

Moreover, firms are subject to the following friction: they can either pay workers for their full hours  $\bar{n}$ , even if their workers' actual hours  $n_{it}$  are below  $\bar{n}$ , or pay them not at all. When a firm decides not to pay and thus lay off their worker, we assume that it loses some future match value with the worker, as replacing the worker costs time. For tractability, we assume that the firm loses all its future match value, and is immediately replaced by an entrant in the next period, hiring the laid off worker.

This implies that the value of firm  $i$  is given by

$$V_{it} = \max \left\{ y_{it} - w\bar{n} + \frac{1}{1+r_t} V_{it+1}, 0 \right\}.$$

In words, firm  $i$  chooses to lay off its worker when its continuation value is below zero, but otherwise keeps paying its worker in full. In steady state, the value of a firm is the same for all firms and given by

$$V^* = \frac{\epsilon^{-1}\bar{n}}{1-\beta}.$$

We assume all firms are owned by unconstrained workers.

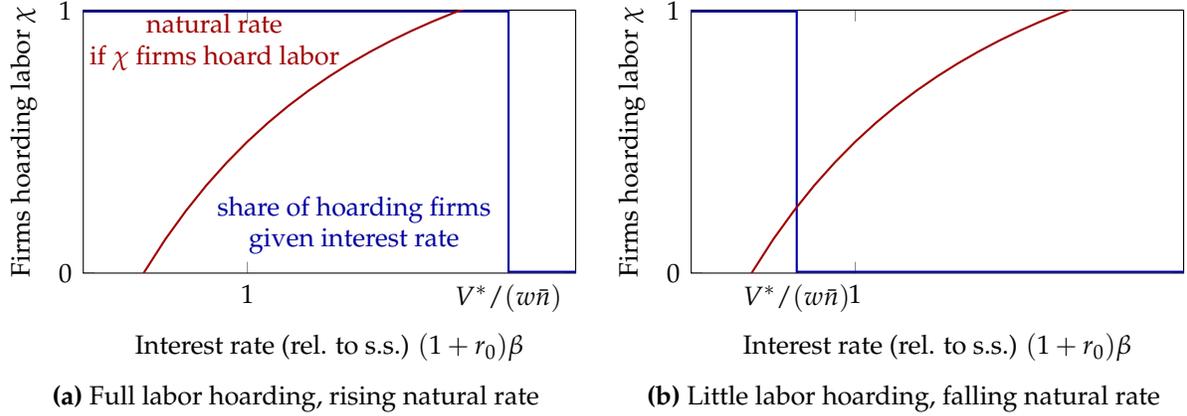
We explore this model in two steps. In this section, we focus on the behavior of firms in sector  $A$ , and accordingly assume that sector  $B$  firms never shut down, which is satisfied if demand in sector  $B$  is close to its steady state value. In the next section, we allow for firms in sector  $B$  to shut down, as well. We focus once more on a shock that stops all economic activity in sector  $A$ .

## 5.2 Insurance Through Labor Hoarding

Firms in sector  $A$  may decide to hoard their workers to preserve future match values, and thus implicitly provide insurance to their workers. In particular, this happens when the firm value is positive despite zero current output, that is,

$$V_0 = -w\bar{n} + \frac{1}{1+r_0} V^* > 0. \quad (25)$$

When, instead,  $V_0$  is negative, workers lose their jobs in sector  $A$ , as we have previously assumed. Finally, when  $V_0$  is equal to zero, an arbitrary share  $\chi \in [0, 1]$  of firms lay off their workers, where  $\chi$  will be determined in equilibrium.



**Figure 9:** Endogenous labor hoarding

Insurance provided to sector  $A$  workers also alters the real interest rate  $r_0$ . In particular, following similar steps to those in Section 2, we find that with  $\chi$  firms hoarding, the natural rate is given by

$$1 + r_0 = \beta^{-1}(1 - \phi)^{\frac{1}{\sigma}\frac{\sigma-\epsilon}{\epsilon-1}} (1 - w\mu\phi)^{\frac{1}{\sigma}} \underbrace{\left( \frac{1 - \phi}{1 - \phi - \chi w\mu\phi} \right)^{\frac{1}{\sigma}}}_{>1} \quad (26)$$

Except for the last term, the expression is virtually identical to (12). The last term is greater than 1 whenever  $\chi > 0$ , capturing the fact that labor hoarding not only acts as insurance for sector  $A$  workers, but also stimulates demand.

As illustrated in Figure 9, the intersection of (25) and (26) describes the equilibrium interest rate  $r_0$  and the equilibrium share of labor hoarding firms in the economy. One can distinguish two possible cases.

Panel (a) shows a situation where at the natural interest rate, firms are fully hoarding labor. In this case, a monetary authority only needs to implement the natural rate; insurance will happen endogenously, without fiscal policy, due to labor hoarding.

Panel (b) shows a situation where firms are not all hoarding labor at the natural interest rate. In that case, an interest rate reduction would induce more firms to hoard labor, as it increases the present value of future match values and therefore makes (25) more likely to hold. One way through which this may happen is through better conditions on business loans that incentivize firm owners not to shut their business down. Thus, there is a role for monetary policy to set the interest rate below the natural rate in order to facilitate insurance through increased labor hoarding, even if output is already at potential. A thorough

quantitative investigation of this “labor hoarding channel” of monetary transmission is left for future research.

### 5.3 Liquidity Problems and Policy Proposals

What happens if firms are liquidity constrained? If firms have some finite amount of liquidity at their disposal, say, because they cannot borrow nor issue equity and have limited past accumulated profits at their disposal, then they no longer maximize the present value of profits in an unconstrained fashion. This distorts firm decisions towards laying workers off, since the current period loss cannot be financed.

In this case, policies that directly affect the liquidity of firms or that insure firms for their loss in revenue, may restore the preferable outcome. In the model, this could be accomplished by a transfer to firms. In practice, these policies could be implemented in a number of ways and through a combination of fiscal and monetary branches of the government.

This discussion lends support to policy proposals at the outset of the economic crises in March 2020 generated by the Covid-19 pandemic in the US and Europe. For example, [Hamilton and Veuger \(2020\)](#) propose emergency loans for small and medium sized firms most affected by liquidity problems facilitated by the Fed, complimented with tax credits on the fiscal side. [Saez and Zucman \(2020\)](#) propose an ambitious insurance policy, or “buyer of last resort”, whereby the government makes up for any loss in revenue by in effect buying up the missing demand. [Hanson et al. \(2020\)](#) propose a “business continuity insurance” policy, providing financial support to businesses to meet fixed obligations. Through the lens of our model, this would help businesses remain alive and able to hire workers.

Even some policy proposals aimed at paying workers directly, through unemployment benefits, emphasize the importance of preserving matches. For example, [Dube \(2020\)](#) calls for incentivizing temporary layoffs, so called furloughs, and the use of worker-sharing provisions, to keeps workers on payroll and allows workers to return easily after the shutdowns.<sup>23</sup>

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<sup>23</sup>[Giupponi and Landais \(2018\)](#) provide some evidence on related policies in Europe and study optimal policy in a model of labor hoarding and work-sharing.

## 5.4 Slow Recoveries

What happens if workers that are let go at  $t = 0$  cannot immediately find a new job at  $t = 1$ ? To be concrete, consider a simple case: suppose no matches can be created at  $t = 1$ , but they can be costlessly created at  $t = 2$ . Then, if the interest rate is not sufficiently low to induce hoarding, the economy will suffer a recession over both periods  $t = 0, 1$ , effectively prolonging the duration of the shock. In period  $t = 0$  the shock is exogenous, but in period  $t = 1$  it results from the loss of job matches. Through an expectations channel, this may also make the recession at  $t = 0$  deeper.<sup>24</sup>

The assumption that matches cannot be created at  $t = 1$  but can be costlessly recreated at  $t = 2$  is extreme, but we expect similar conclusions in a more elaborate model of search and vacancies, where job matches are created in a costly and incremental manner over time.

## 6 Business Exit Cascades

In the previous section we studied the incentives of businesses in sector  $A$  to keep workers employed. Now we shift our focus to sector  $B$  and ask what determines the incentives of businesses in that sector to keep employing workers and to remain open, instead of shutting down and laying off workers. Importantly, if businesses shut down, this reduces demand for all other open businesses, increasing their incentive to shut down as well.

We continue to work with the disaggregated firm-level perspective from the previous section: both sectors consist of a continuum of firms, each employing one worker, who can be laid off at the expense of future surplus. We also continue to assume that firms in both sectors produce goods that are substitutable at the same elasticity  $\epsilon$  that also applies to cross-sectoral substitution.<sup>25</sup>

We make two changes relative to the previous section. First, we fix the real interest rate  $1 + r_0$ , for convenience at the steady state value of  $\beta^{-1}$ , focusing on the implications for

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<sup>24</sup>This mechanism is similar to Anzoategui, Comin, Gertler and Martinez (2019) and Benigno and Fornaro (2018), where growth, rather than future unemployment, is responsible for the amplification. This also differs from most analyses based on search and matching models in which amplification is based on precautionary saving (Ravn and Sterk 2021, Den Haan, Rendahl and Riegler 2018, McKay and Reis 2020, and Broer, Druedahl, Harmenberg and Oberg 2020).

<sup>25</sup>The assumption that cross-firm and cross-sectoral elasticities are equal is not crucial for our analysis. Our results in this section continue to hold in case the cross-firm elasticity is higher, as long as they are still compatible with a Keynesian supply shock, that is, (13) holds for the cross-firm elasticity. More general versions of the model in this section that allow for firm-to-firm linkages in the spirit of Section 4.1 would even allow for larger cross-firm elasticities.

aggregate demand  $Y_{B0}$  in sector  $B$ .

Second, in order to model businesses' exits in a more continuous fashion, we assume that there is a distribution of outstanding (net) liabilities across firms which we denote by  $\zeta$ . In particular, the value of a firm  $i$  is then given by

$$V_{i0} = y_0 - w\bar{n} - \zeta_i + \frac{1}{1+r_0}V^* \quad (27)$$

where  $y_0 = Y_{B0}/(1-\phi)$  is the demand per business in sector  $B$ . We assume each firm's net liability  $\zeta_i$  lies within an interval  $[\underline{\zeta}, \bar{\zeta}]$  and is distributed among sector  $B$  firms according to a cdf  $F\left(\frac{\zeta-\underline{\zeta}}{\bar{\zeta}-\underline{\zeta}}\right)$  where  $F$  is a cdf on  $[0, 1]$ . The boundaries  $\underline{\zeta}$  and  $\bar{\zeta}$  are chosen such that, for simplicity, all firms close when they face zero demand  $y_0 = 0$ ,

$$-w\bar{n} - \underline{\zeta} + \frac{1}{1+r_0}V^* = 0$$

and all firms open with steady state demand  $y_0^* = \bar{n}$ ,

$$(1-w)\bar{n} - \bar{\zeta} + \frac{1}{1+r_0}V^* = 0.$$

## 6.1 The Business Exit Multiplier

This structure implies that there is now an endogenous mass of firms that is active in the economy, which we denote by  $1 - \Phi$ . Previously,  $\Phi$  was equal to  $\phi$ , as only sector  $A$  was shut down. In this section, however, the share of active firms depends on demand for sector  $B$  goods,

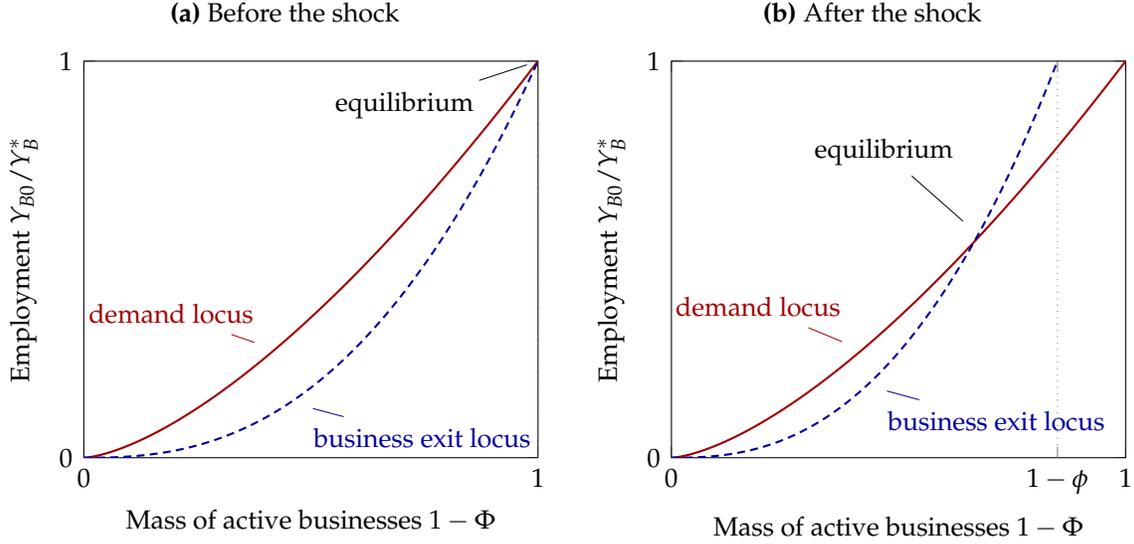
$$1 - \Phi = (1 - \phi)F\left(\frac{Y_{B0}}{Y_{B0}^*}\right). \quad (28)$$

But, as we have shown in the previous sections, demand for sector  $B$  goods precisely depends on the share of open businesses. In particular, a mass  $\Phi$  of workers will be out of work in equilibrium. Based on the Euler equation underlying (26) (with zero labor hoarding among closed businesses) we find

$$\frac{Y_{B0}}{Y_B^*} = (1 - \Phi)^{\frac{\sigma-\epsilon}{\epsilon-1}} (1 - w\mu\Phi). \quad (29)$$

Under the condition for Keynesian supply shocks, (13) (here with  $w\mu$  instead of  $\mu$ ) this is decreasing in  $\Phi$ , capturing that demand falls with more businesses exits.

Figure 10: The business exit multiplier



The relationship between (28) and (29) is illustrated in Figure 10. The horizontal axis represents the mass of active businesses  $1 - \Phi$ . The vertical axis represents the demand relative to potential  $Y_{B0}/Y_B^*$ . Under the condition for Keynesian supply shocks, both (28) and (29) describe positively sloped curves in Figure 10. We call (29) the “demand locus”, as it describes the demand  $Y_{B0}$  for sector  $B$  goods, taking as given how many businesses are still open and workers employed,  $1 - \Phi$ . We call (28) the “(business) exit locus” as it describes the mass of businesses exiting and workers laid off given demand  $Y_{B0}$ .

When there is no shock,  $\phi = 0$ , the two curves necessarily intersect at coordinates  $1 - \Phi = 1$  and  $Y_{B0}/Y_B^* = 1$  (Panel a). However, a positive  $\phi > 0$  shifts the exit locus to the left (Panel b). Interestingly, this shift raises the mass of inactive businesses by more than just  $\phi$ , as additional workers laid off by exiting businesses also stop consuming. There is a cascade of business exits and lay-offs that generates a “business exit multiplier”.<sup>26</sup>

## 6.2 Policies

We use the framework laid out here to discuss the effectiveness of two policies.

**Profit subsidy / employer-side payroll tax cut.** The first policy is a profit subsidy, which, as in Section 3, we assume is paid for by employed agents. The subsidy raises profits by

<sup>26</sup>Figure 10 shows that one can easily get multiple equilibria in this setting, when both curves intersect multiple times. We plan to investigate this case in future research.

$1 + \tau$  for some  $\tau > 0$ . In our model, this is similar to an employer-side cut in payroll taxes. Such a subsidy enters the business exit locus (28), modifying it to

$$1 - \Phi = (1 - \phi) F \left( (1 + \tau) \frac{Y_{B0}}{Y_B^*} \right)$$

and thus shifting the locus to the right. This mitigates some of the consequences of the shock, both in terms of business activity and demand.

**Monetary policy.** Monetary policy can change the real interest rate  $1 + r_0$  away from  $1/\beta$ . This clearly affects the demand locus (29) through the Euler equation. In particular, we have that

$$\frac{Y_{B0}}{Y_B^*} = (1 - \Phi)^{\frac{\sigma - \epsilon}{\epsilon - 1}} (1 - w\mu\Phi) (\beta(1 + r_0))^{-1/\sigma}.$$

Accommodative monetary policy shifts the demand locus up, thereby also reducing the number of business exits in the economy.

Aside from the standard intertemporal substitution channel, however, there is another transmission mechanism that is active here. In particular, observe that, just like in Section 5, the future value of a match is discounted less now in (27), encouraging firms to hoard labor. This modifies the exit locus to

$$1 - \Phi = (1 - \phi) F \left( \frac{Y_{B0}}{Y_B^*} + \left( \frac{1}{\beta(1 + r_0)} - 1 \right) \frac{\beta V^*}{Y_B^*} \right).$$

It shows that monetary policy shifts the exit locus to the right, contributing to the recovery. Observe that this channel of monetary policy depends on the future value  $V^*$  of employment relationships. Viewed through this lens, the channel might have important distributional consequences, encouraging labor hoarding particularly for workers with a large future match surplus. We leave such an exploration for future work.

## 7 Concluding Discussion

The contribution of this paper has been to explore the conditions under which a shock to potential output in some sectors of the economy spills over into an inefficient demand shock in other sectors. Whether the conditions are satisfied depends on the sectoral origins of the shock as well as on the structure of the economy. We now briefly discuss arguments in favor of interpreting the specific case of the Covid-19 pandemic as a Keynesian supply

shock.

The first observation is that, at the outset of the pandemic, there was a widespread reduction in economic activity across all sectors. For example, [Cajner, Crane, Decker, Grigsby, Hamins-Puertolas, Hurst, Kurz and Yildirmaz \(2020\)](#) documents employment losses in almost every sector of the economy, including non-contact intensive ones ([Dingel and Neiman, 2020](#)). Second, data on spending shows that, while spending contracted very severely in some categories, it largely did not pick up in others so as to compensate (e.g. [Ganong, Noel and Vavra, 2020](#)), leading to a large accumulation of savings and pointing to the importance of limited substitutability. Third, the behavior of incomes and spending at the moment in which emergency transfers were introduced, seems consistent with the importance of the income channel, emphasized in our incomplete markets analysis (see the time series patterns for spending in [Chetty et al. \(2020\)](#) and [Cox et al. \(2020\)](#) at the time of stimulus payments).<sup>27</sup>

The fact that CPI dynamics have been subdued, in spite of substantial stimulus, seems broadly consistent with our view of dominant demand forces. However, interpreting inflation data requires a caveat. In sector  $B$  of our model our prediction is simple: if there is a Keynesian supply shock, deflationary forces will be present. Sector  $A$  is trickier. In our baseline model, sector  $A$  is supply constrained, suggesting inflationary pressures in that sector. However, if we explicitly model health as in Section 2.4, the pandemic acts like a preference shock in sector  $A$ , causing prices to fall also in that sector.

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<sup>27</sup>A side remark on the evidence in [Chetty et al. \(2020\)](#). Our simple demand system has homothetic preferences and thus abstracts from luxuries vs. essentials which explains the larger drop in spending by the rich during the pandemic. A straightforward extension of our framework can acknowledge this fact, without affecting our main results.

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Online Appendix to  
**Macroeconomic Implications of COVID-19:  
 Can Negative Supply Shocks Cause Demand Shortages?**

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## A Consumption functions

In this section we derive the individual consumption functions used for the plots in Figures 4, and derive condition (15).

The marginal utility of good  $B$  is

$$U_{c_B}(c_{At}, c_{Bt}) = (1 - \phi)^{\frac{1}{\epsilon}} c_t^{\frac{1}{\epsilon} - \frac{1}{\sigma}} c_{Bt}^{-\frac{1}{\epsilon}},$$

where

$$c_t = \left( \phi^{\frac{1}{\epsilon}} c_{At}^{\frac{\epsilon-1}{\epsilon}} + (1 - \phi)^{\frac{1}{\epsilon}} c_{Bt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}.$$

The Euler equation of unconstrained consumers, with  $c_{A0} = 0$  and  $\beta(1 + r_0) = 1$ , then takes the following form

$$c_0^{\frac{1}{\epsilon} - \frac{1}{\sigma}} c_{B0}^{-\frac{1}{\epsilon}} = c_1^{\frac{1}{\epsilon} - \frac{1}{\sigma}} c_{B1}^{-\frac{1}{\epsilon}}.$$

Substituting  $c_0 = (1 - \phi)^{\frac{1}{\epsilon-1}} c_{B0}$  and  $c_{1B} = (1 - \phi) c_1$  and rearranging, gives

$$c_1 = (1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}} c_{B0}.$$

Taking into account that  $c_t = c_1$  for  $t = 1, 2, \dots$ , the intertemporal budget constraint is

$$c_{B0} + \frac{\beta}{1 - \beta} c_1 = y_0 + \frac{\beta}{1 - \beta}.$$

Solving, we obtain the consumption function

$$c_{B0} = \frac{(1 - \beta) y_0 + \beta}{1 - \beta + \beta (1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}}. \quad (30)$$

For constrained consumers we can follow similar steps, allowing for the Euler equation to hold as an inequality, and obtain

$$c_{B0} = \min \left\{ y_0, \frac{(1 - \beta) y_0 + \beta}{1 - \beta + \beta (1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}} \right\}. \quad (31)$$

The consumption functions without the shock can be derived in similar manner and are

$$c_{B0} = (1 - \phi) ((1 - \beta) y_0 + \beta) \quad (32)$$

for unconstrained consumers, and

$$c_{Bt} = (1 - \phi) \min \{y_0, (1 - \beta) y_t + \beta\} \quad (33)$$

for constrained consumers. Notice that in the last expression the factor  $(1 - \phi)$  appears before the min operator, because before the shock the consumers allocate a fraction  $\phi$  of their spending to good  $A$ , whether or not the constraint is binding. These are the consumption functions plotted in Figure 4.

Suppose now that the income of the consumers in sector  $B$  remains at 1. The total change in consumption following the shock is

$$\frac{(1 - \beta) (1 - \phi) + \beta (1 - \mu \phi)}{1 - \beta + \beta (1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}} - (1 - \phi). \quad (34)$$

This expression is negative iff condition (13) holds.

The expression above can be decomposed in three terms:

1. The shift in the consumption function at income  $y_0 = 1$ :

$$\frac{1}{1 - \beta + \beta (1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}} - (1 - \phi);$$

2. The change in consumption of the unconstrained consumers hit by the shock, due to the income loss:

$$- (1 - \mu) \phi \left( \frac{1 - \beta}{1 - \beta + \beta (1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}} \right);$$

3. The change in consumption of the constrained consumers hit by the shock, due to the income loss:

$$- \mu \phi \left( \frac{1}{1 - \beta + \beta (1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}} \right).$$

The marginal propensities to consume are

$$\frac{1 - \beta}{1 - \beta + \beta (1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}}$$

for the first group and

$$\frac{1}{1 - \beta + \beta (1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}}$$

for the second groups, so the average MPC of  $A$  workers is

$$\overline{MPC}^A \equiv (1 - \mu) \frac{1 - \beta}{1 - \beta + \beta(1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}} + \mu \frac{1}{1 - \beta + \beta(1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}}.$$

The reduction in consumption of  $A$  good is equal to  $\phi$  for all agents so

$$\left[ \frac{\Delta C_B}{\Delta C_A} \right]^{shutdown} = \frac{\frac{1}{1 - \beta + \beta(1 - \phi)^{-\frac{\sigma-1}{\epsilon-1}}} - (1 - \phi)}{\phi}.$$

We conclude that the expression in (34) is negative iff

$$\phi \left[ \frac{\Delta C_B}{\Delta C_A} \right]^{shutdown} - \phi \overline{MPC}^A < 0$$

which gives (15) in the main text.

## B Partial shutdown

### B.1 Proof of Proposition 5

Let us rewrite the equilibrium conditions derived in the text, using the notation  $p = P_{A0}/W^*$  and  $P = P_0/W^*$  and dropping time subscripts:

$$Y_A = \phi(1 - \delta) = \phi p^{-\epsilon} \left( \mu \phi P^{\epsilon-1} (1 - \delta) + (1 - \mu \phi) P^{\epsilon-\sigma} \right), \quad (35)$$

$$Y_B = (1 - \phi) \left( \mu \phi P^{\epsilon-1} (1 - \delta) + (1 - \mu \phi) P^{\epsilon-\sigma} \right). \quad (36)$$

Taking ratios side by side and using  $Y_B^* = 1 - \phi$  yields

$$n_B = \frac{Y_B}{Y_B^*} = p^\epsilon (1 - \delta). \quad (37)$$

From the CPI (7) we get

$$P = \left( \phi p^{1-\epsilon} + 1 - \phi \right)^{\frac{1}{1-\epsilon}}.$$

The equilibrium value of  $p$  can then be found substituting  $P$  in (35) and solving:

$$1 - \delta = p^{-\epsilon} \left( \mu \phi \left( \phi p^{1-\epsilon} + 1 - \phi \right)^{-1} (1 - \delta) + (1 - \mu \phi) \left( \phi p^{1-\epsilon} + 1 - \phi \right)^{\frac{\epsilon-\sigma}{1-\epsilon}} \right). \quad (38)$$

It can be shown that this equation has a unique solution  $p$ , strictly increasing in  $\delta$ . Substituting in (37) gives  $n_B$  and  $Y_B$ .

To complete the equilibrium characterization, we need to check that sector  $A$  workers

with no credit access are indeed constrained, that is, that their Euler equation holds as an inequality, which requires

$$\frac{1 - \delta}{P} < P^{-\sigma}.$$

Aggregating (35) (multiplied by  $p$ ) and (36) side by side and using the definition of the CPI yields the following

$$\phi n_A + (1 - \phi) n_B + (p - 1) Y_A = p Y_A + Y_B = \mu \phi (1 - \delta) + (1 - \mu \phi) P^{1-\sigma}.$$

Using  $n_A = 1 - \delta$  and  $n_B = p^\epsilon (1 - \delta)$  we then get

$$p^{1-\sigma} = \frac{\phi (1 - \mu) (1 - \delta) + (1 - \phi) p^\epsilon (1 - \delta) + (p - 1) Y_A}{1 - \mu \phi} > 1 - \delta,$$

where the second inequality follows from  $p > 1$ .

To derive the frontier of the KSS region, we impose  $n_B = 1$  in (37) to obtain

$$p = (1 - \delta)^{-\frac{1}{\epsilon}}.$$

Substituting in (38) yields

$$1 = \mu \phi \frac{1 - \delta}{\phi (1 - \delta)^{1-\frac{1}{\epsilon}} + 1 - \phi} + (1 - \mu \phi) \left( \phi (1 - \delta)^{1-\frac{1}{\epsilon}} + 1 - \phi \right)^{\frac{\epsilon - \sigma}{1 - \epsilon}}.$$

Solving this equation for  $\sigma$  gives the level  $\hat{\sigma}$  that yields exactly  $n_B = 1$ , given all other parameters. The expression for  $\hat{\sigma}$  is equal to the right-hand side of (17).

To complete the argument, we need to show that when  $\sigma > \hat{\sigma}$  the pair  $(n_B, p)$  that solves (37) and (38) satisfies  $n_B < 1$ . To do so we keep all parameters fixed and do comparative statics with respect to  $\sigma$ . Inspecting (38) shows that increasing  $\sigma$  reduces  $p$ . It follows that  $n_B$  from (37) is decreasing in  $\sigma$ , completing the argument.

To derive the limit case for  $\delta \rightarrow 0$  notice that a linear approximation of (38) at  $\delta = 0$  gives

$$-d\delta = -\epsilon dp - \mu \phi \delta + \mu \phi (\epsilon - 1) dP + (1 - \mu \phi) (\epsilon - \sigma) dP,$$

substituting  $dP = \phi dp$  and rearranging gives

$$dp = \frac{1 - \mu \phi}{(1 - \phi) \epsilon + \phi (\mu \phi + (1 - \mu \phi) \sigma)} d\delta.$$

Approximating (37) and substituting  $dp$  gives

$$dn_B = \epsilon dp - d\delta = \left( \epsilon \frac{1 - \mu \phi}{(1 - \phi) \epsilon + \phi (\mu \phi + (1 - \mu \phi) \sigma)} - 1 \right) d\delta.$$

Therefore we get  $dn_B < 0$  iff the expression in parenthesis is negative, which gives

$$\sigma > \frac{\epsilon(1-\mu) - \mu\phi}{1 - \mu\phi}.$$

The same expression can be obtained by applying L'Hopital's rule to (17).

## B.2 Derivation for the limit case $\delta \rightarrow 1$

Notice that as  $\delta \rightarrow 1$  we have  $p \rightarrow \infty$ . If  $\epsilon < 1$  we also have  $P \rightarrow \infty$ . Inspecting the expression (36) shows that the term with  $P^{\epsilon-1}$  goes to zero. The term with  $P^{\epsilon-\sigma}$  goes to zero if  $\epsilon < \sigma$ , in which case we have a KSS that leads to a complete shutdown of both sectors  $A$  and  $B$ . The term  $P^{\epsilon-\sigma}$  goes to  $\infty$  if  $\epsilon > \sigma$ , in which case we have full employment in sector  $B$ . Using this limit argument, in the case  $\epsilon < 1$ , the frontier of the KSS region is  $\sigma = \epsilon$ , as plotted in Figure (3).

# C Preference shocks and health

## C.1 Preference shocks

We want to characterize an equilibrium in which both sector  $A$  and sector  $B$  are demand constrained so  $P_{A0} = P_{B0} = P^*$ . The Euler equations of the unconstrained consumers are then

$$\begin{aligned} \phi^{\frac{1}{\epsilon}} \theta^{\frac{1}{\epsilon}} \mathbf{c}_0^{\frac{1}{\epsilon} - \frac{1}{\sigma}} \mathbf{c}_{A0}^{-\frac{1}{\epsilon}} &= 1, \\ (1 - \phi)^{\frac{1}{\epsilon}} \mathbf{c}_0^{\frac{1}{\epsilon} - \frac{1}{\sigma}} \mathbf{c}_{B0}^{-\frac{1}{\epsilon}} &= 1, \end{aligned}$$

which can be solved to give

$$\begin{aligned} \mathbf{c}_{A0} &= \phi\theta (\phi\theta + 1 - \phi)^{\frac{\epsilon-\sigma}{1-\epsilon}}, \\ \mathbf{c}_{B0} &= (1 - \phi) (\phi\theta + 1 - \phi)^{\frac{\epsilon-\sigma}{1-\epsilon}}. \end{aligned}$$

For constrained agents with income  $n_{A0}$  we get

$$\begin{aligned} c_{A0} &= \phi\theta (\phi\theta + 1 - \phi)^{-1} n_{A0}, \\ c_{B0} &= (1 - \phi) (\phi\theta + 1 - \phi)^{-1} n_{A0}. \end{aligned}$$

Aggregating, we obtain

$$\begin{aligned}\frac{Y_{A0}}{Y_A^*} &= \theta \left[ \mu\phi (\phi\theta + 1 - \phi)^{-1} \frac{Y_{A0}}{Y_A^*} + (1 - \mu\phi) (\phi\theta + 1 - \phi)^{\frac{\epsilon - \sigma}{1 - \epsilon}} \right], \\ \frac{Y_{B0}}{Y_B^*} &= \mu\phi (\phi\theta + 1 - \phi)^{-1} \frac{Y_{A0}}{Y_A^*} + (1 - \mu\phi) (\phi\theta + 1 - \phi)^{\frac{\epsilon - \sigma}{1 - \epsilon}}.\end{aligned}$$

It immediately follows that

$$\frac{Y_{A0}}{Y_A^*} = \theta \frac{Y_{B0}}{Y_B^*}, \quad (39)$$

and, in particular,  $\frac{Y_{A0}}{Y_A^*} < \frac{Y_{B0}}{Y_B^*}$ . We can substitute (39) into the second equation to arrive at

$$\frac{Y_{B0}}{Y_B^*} = \frac{(1 - \phi(1 - \theta))^{\frac{1 - \sigma}{1 - \epsilon}}}{1 - \phi \frac{1 - \mu}{1 - \mu\phi} (1 - \theta)}.$$

Notice that  $1 - \phi \frac{1 - \mu}{1 - \mu\phi} (1 - \theta) > 0$  because  $\theta > 0$ , so the expression above is always positive. We have an equilibrium in which both sectors are demand constrained iff the expression on the right-hand side is less than 1. This proves the following result.

**Proposition 8.** *Consider the incomplete markets economy, with rigid wages and the nominal rate set at  $i_0 = i^*$ . A temporary preference shock  $\theta_0 = \theta < 1$  causes a contraction in activity in both sectors A and B, with a larger contraction in sector A, if*

$$\sigma > \epsilon - (1 - \epsilon) \frac{\ln \left( 1 - \mu\phi \frac{\theta}{\phi\theta + 1 - \phi} \right) - \ln(1 - \mu\phi)}{\ln(\phi\theta + 1 - \phi)}. \quad (40)$$

Notice the similarity with the condition for a supply shock causing a partial shutdown in Proposition 5. In particular, if we define  $p = \theta^{-1/(\epsilon - 1)}$  as the effective price of good A in terms of future consumption we can define the effective CPI (in terms of future consumption) as

$$P_0 = W^* (\phi\theta + 1 - \phi)^{\frac{1}{1 - \epsilon}}.$$

Output in sector B can then be written as

$$\frac{Y_{B0}}{Y_B^*} = \left( \frac{W^*}{P_0} \right)^{-\epsilon} \left( \mu\phi \frac{W^*}{P_0} \frac{Y_{A0}}{Y_A^*} + (1 - \mu\phi) \left( \frac{P_0}{P^*} \right)^{-\sigma} \right),$$

which mirrors the expression (16) for the partial shutdown model and captures the three forces at work: intratemporal substitution, intertemporal substitution, income losses of constrained consumers. The only difference, when solving for condition (40), is that the ratio of output gaps in the two sectors  $\frac{Y_{A0}/Y_A^*}{Y_{B0}/Y_B^*}$  is  $\theta$  instead of  $p^{-\epsilon}$ .

## C.2 Health model

We characterize the model with health in the utility function. Assume  $\phi - \eta > 0$  so there is positive consumption in sector  $A$  and define

$$\theta \equiv \frac{\phi - \eta}{\phi}.$$

To set the stage for the analysis in Section 3.3, we introduce the transfer  $\rho(1 - n_{j0})$  as in Section 3 financed by government debt

$$D = \rho [\phi(1 - n_{A0}) + (1 - \phi)(1 - n_{B0})],$$

and we introduce a tax  $\tau$  on consumption of good  $A$ , which is rebated lump sum. From the Euler equations, the average consumption of unconstrained consumers is now

$$\begin{aligned} c_{A0} &= \frac{\theta\phi}{1 + \tau} \left( 1 + \frac{\mu\phi}{1 - \mu\phi} rD \right), \\ c_{B0} &= (1 - \phi) \left( 1 + \frac{\mu\phi}{1 - \mu\phi} rD \right), \end{aligned}$$

where  $r = 1/\beta - 1$ . For constrained consumers we get

$$\begin{aligned} c_{A0} &= \frac{\theta\phi}{1 + \tau} \frac{n_{A0} + \rho(1 - n_{A0})}{\frac{\theta\phi}{1 + \tau} + 1 - \phi}, \\ c_{B0} &= (1 - \phi) \frac{n_{B0} + \rho(1 - n_{B0})}{\frac{\theta\phi}{1 + \tau} + 1 - \phi}, \end{aligned}$$

as long as the borrowing constraint is binding, which happens iff the following condition holds

$$n_{A0} + \rho(1 - n_{A0}) < \left( \frac{\theta\phi}{1 + \tau} + 1 - \phi \right) (1 - rD).$$

If the borrowing constraint above is binding, the goods market equilibrium conditions are

$$\begin{aligned} Y_{A0} &= \frac{\theta\phi}{1 + \tau} \left[ \mu\phi \frac{n_{A0} + \rho(1 - n_{A0})}{\frac{\theta\phi}{1 + \tau} + 1 - \phi} + 1 - \mu\phi + \mu\phi rD \right], \\ Y_{B0} &= (1 - \phi) \left[ \mu\phi \frac{n_{A0} + \rho(1 - n_{A0})}{\frac{\theta\phi}{1 + \tau} + 1 - \phi} + 1 - \mu\phi + \mu\phi rD \right]. \end{aligned}$$

Combining the conditions above shows that the equilibrium features binding borrowing constraints and unemployment in sector  $B$ ,  $n_{B0} < 1$ , if

$$\rho < \hat{\rho} \equiv \frac{1}{1 + r\phi \left( \frac{\theta\phi}{1+\tau} + 1 - \phi \right)} \frac{1 - n_{A0}^* + \frac{\theta\phi}{1+\tau} - \phi}{1 - n_{A0}^*} < 1,$$

where  $n_{A0}^* = \frac{\theta}{1+\tau}$ . If  $\rho \geq \hat{\rho}$  the borrowing constraint is not binding for any consumer, the goods market equilibrium conditions are

$$Y_{A0} = \frac{\theta\phi}{1 + \tau}, \quad Y_{B0} = 1 - \phi,$$

and there is full employment in sector  $B$ . The fact that the conditions for a non-binding constraint and for full employment in  $B$  coincide is due to the fact that the log case satisfies condition  $\sigma = \epsilon$ , so the natural rate is equal to  $1/\beta - 1$  under complete markets.

## D Fiscal Policy

We characterize an equilibrium with fiscal policy. Consider first an equilibrium in which the borrowing constraint of sector  $A$  workers with no credit access is binding, which requires

$$\rho < (1 - \phi)^{\frac{\sigma-1}{\epsilon-1}} \left( 1 - \frac{\zeta}{\mu} r^* D \right).$$

The average consumption of unconstrained consumers in periods  $t = 1, 2, \dots$  is

$$c_1 = 1 + \frac{r^* D}{1 - \mu\phi} - \frac{\phi(1 - \mu) \frac{1-\zeta}{1-\mu} r^* D + (1 - \phi)(1 - \mu) \frac{1-\zeta}{1-\mu} r^* D + (1 - \phi) \mu \frac{\zeta}{\mu} r^* D}{1 - \mu\phi}$$

given that the stock of debt  $D$  is entirely held by the group of  $1 - \mu\phi$  unconstrained consumers. Rearranging gives

$$c_1 = 1 + \frac{\zeta\phi}{1 - \mu\phi} r^* D.$$

Using the Euler equation their consumption of good  $B$  at date 0 is

$$c_{B0} = (1 - \phi)^{\frac{\sigma-1}{\epsilon-1}} \left( 1 + \frac{\zeta\phi}{1 - \mu\phi} r^* D \right),$$

and total demand in sector  $B$  is

$$Y_{B0} = G + \mu\phi\rho + (1 - \mu\phi)(1 - \phi)^{\frac{\sigma-1}{\epsilon-1}} \left( 1 + \frac{\zeta\phi}{1 - \mu\phi} r^* D \right), \quad (41)$$

where

$$D = G + \phi\rho + (1 - \phi)\rho(1 - n_{B0}).$$

### D.1 Proof of Proposition (6)

Set  $\zeta = 0$ . At  $G = \rho = 0$  we get

$$dY_{B0} = dG + \mu\phi d\rho,$$

and the effect of  $d\rho$  on total transfers is

$$dT = [\phi + (1 - \phi)(1 - n_{B0})] d\rho.$$

The expressions for the multipliers follow from these two equations.

### D.2 Proof of Proposition 7

Substituting the expression for  $D$  in (41) and rearranging we get

$$n_{B0} = \frac{\mu\phi\rho}{1 - \phi} + (1 - \phi)^{\frac{\sigma - \epsilon}{\epsilon - 1}} [1 - \mu\phi + \zeta\phi r^* (\phi\rho + (1 - \phi)\rho(1 - n_{B0}))].$$

It is possible to show that as  $\rho$  varies in  $[0, 1]$  the value of  $n_{B0}$  that solves this equation is increasing in  $\rho$  and so is the expression

$$\rho - (1 - \phi)^{\frac{\sigma - 1}{\epsilon - 1}} \left(1 - \frac{\zeta}{\mu} r^* D\right), \quad (42)$$

and both are continuous. Let the cutoff  $\hat{\rho}$  be the smallest  $\rho$  for which either  $n_{B0} = 1$  or (42) is zero. Notice that if (42) is negative, then all consumers are unconstrained and demand for good  $B$  is given by

$$Y_{B0} = (1 - \phi)^{\frac{\sigma - 1}{\epsilon - 1}}.$$

Therefore, by the definition of  $\hat{\rho}$ , when  $\rho > \hat{\rho}$  the equilibrium value of  $n_{B0}$  is constant and either equal to 1 or equal to  $(1 - \phi)^{\frac{\sigma - \epsilon}{\epsilon - 1}} < 1$ .

Setting  $\zeta = \mu$  and  $\rho = 1$  implies that all agents receive income after transfers equal to 1 in period 0 and pay tax  $r^*D$  in all future periods. Therefore, it achieves perfect insurance and replicates the complete market allocation. If  $\sigma < \epsilon$  the complete market allocation also achieves full employment in sector  $B$ , so it is first best optimal. If  $\sigma > \epsilon$  the complete market allocation with real rate equal to  $1/\beta$  is not first best optimal as there is unemployment in sector  $B$ . However, the planner cannot increase output above  $(1 - \phi)^{\frac{\sigma - 1}{\epsilon - 1}}$  with the fiscal instruments allowed ( $\rho$  and  $\zeta$ ) and the complete market allocation maximizes the utilitarian planner objective conditional on  $Y_{B0} \leq (1 - \phi)^{\frac{\sigma - 1}{\epsilon - 1}}$ . So in both cases setting  $\zeta = \mu$  and  $\rho = 1$  is optimal.