

# Monetary Policy in Times of Structural Reallocation\*

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We characterize optimal monetary policy in response to asymmetric shocks that shift demand from one sector to another, a condition arguably faced by many economies emerging from the Covid-19 crisis and its aftermath. We show that the asymmetry naturally manifests itself as an endogenous cost-push shock, breaking divine coincidence, and resulting in inflation optimally exceeding its target despite elevated unemployment. In fact, there is no simple, possibly re-weighted, inflation index that is optimally targeted. When labor is mobile between sectors, the benefits of monetary easing can be even higher, if it translates into wage increases in the expanding sector.

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# 1 Introduction

The Covid-19 crisis has been a stark reminder that some macroeconomic shocks can have uneven effects across sectors. Uneven shocks pose important challenges to policy, given that different sectors can suffer from opposite problems: some may be experiencing insufficient demand and unemployment, while others may be subject to supply constraints, causing shortages and inflationary pressures. How should monetary policy respond to this type of situation? Is the optimal response to target economy-wide average measures of inflation and of the output gap, or do the asymmetries across sectors require a deviation from standard recommendations, in one direction or another?

A specific issue that we address here is how monetary policy interacts with the process of sectoral reallocation. When uneven shocks have a persistent nature, a natural concern is that the economy should readjust by moving productive resources from declining sectors, that suffer from insufficient demand, towards growing sectors where demand is expanding.

In the context of the pandemic shock, [Barrero, Bloom and Davis \(2020\)](#) have pointed out the long-lasting reallocation forces set in motion by the pandemic. Sectors and businesses that are more able to take advantage of various forms of remote work have shown great capacity to expand, while other sectors and businesses that rely more on personal interaction may experience a long lasting decline even after the pandemic recedes. This requires a reallocation of factors of production in favor of the growing sectors. In particular, [Barrero et al. \(2020\)](#) document high rates of job creation and gross hiring activity during the pandemic (see also [Cajner, Crane, Decker, Grigsby, Hamins-Puertolas, Hurst, Kurz and Yildirmaz 2020](#)), and present survey evidence that suggests that pandemic-induced shifts in work arrangements, consumer spending patterns, and business practices will not fully reverse after the pandemic.<sup>1</sup>

A concern that can arise in this situation is that excessively easy monetary policy may hamper the reallocation process. The logic is that some businesses and some jobs that get destroyed in a recession are not going to be viable after the recession is over ([Caballero and Hammour, 1991](#)). By stimulating demand in the aggregate, monetary policy ends up stimulating activity in those sectors, possibly slowing down the reallocation process. Is this concern justified? Should optimal monetary policy be less expansionary because of it?

As is well known, monetary policy must balance various goals. The macroeconomic lit-

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<sup>1</sup>[Barrero et al. \(2020\)](#), in line with the previous literature on job reallocation, stress that a lot of reallocation takes place within industries. While we use the label “sector” throughout the paper, our arguments can easily be extended to reallocation at a finer level, as long as there is some degree of segmentation and imperfect mobility across the labor markets different businesses tap into.

erature on optimal monetary policy has developed insights into navigating these goals. The most influential and celebrated idea provides conditions under which inflation targeting can obtain both price and macroeconomic stability—in some situations there is no tradeoff. It is well appreciated that we may have to depart from this benchmark. This paper explores scenarios that fall quite some distance away from this ideal “divine coincidence.” We build a stylized model that departs from workhorse macroeconomic models in important ways, incorporating realistic features such as multiple sectors, downward wage rigidities and costly labor reallocation. We then consider a reallocation shock and study optimal monetary policy.

In more detail, our model features two sectors. Monetary policy controls aggregate demand, which determines demand for the goods of two sectors,  $A$  and  $B$ . Both sectors are subject to downward nominal wage rigidities as well as sticky prices.<sup>2</sup> We assume the economy is in steady state and we hit it with an asymmetric preference shock, reducing the demand for sector  $A$  goods while simultaneously raising the demand for sector  $B$  goods. We then analyze the positive and normative implications for monetary policy.

We first consider the case without reallocation, that is, without labor mobility across sectors. In that case, the asymmetric shock causes inflation in the expanding sector  $B$  and unemployment in sector  $A$ , where wages and prices are prevented from falling by the downward rigidity. This highlights that an asymmetric shock, such as ours, in an environment with downwardly rigid wages, operates like a cost-push shock in a textbook New-Keynesian model, causing simultaneously unemployment and inflation. This poses a well-known trade-off for monetary policy, and we show that optimal monetary policy generally allows for some inflation in excess of its target, and some unemployment above its natural level.

We then introduce the possibility of reallocation, by allowing for costly labor mobility across sectors. In our model, the presence of nominal rigidities and unemployment affects the decision to move in two ways. Workers are induced to move from  $A$  to  $B$  either because the probability of finding a job is higher in sector  $B$  or because the real wage is higher in sector  $B$ . A more expansionary monetary policy affects these margins differently. On the one hand, by reducing unemployment in  $A$ , expansionary monetary policy discourages reallocation. On the other hand, by promoting wage inflation in the expanding sector, expansionary policy encourages reallocation. We show two versions of our model, one in

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<sup>2</sup>Downward wage rigidity has not figured prominently in studies of optimal monetary policy. There is a long history of empirical evidence for an asymmetric response of wages. Most recently, [Hazell and Taska \(2020\)](#) employ micro data on posted wages and find that they adjust upwards in states where unemployment falls but do not fall in states where unemployment rises.

which the first effect is stronger and expansionary policy stifles reallocation, and one in which the second effect dominates and expansionary policy favors reallocation.

How do these different effects influence the optimal degree of monetary accommodation? Since unemployment and inflation are both costly in our model, the amount of factor reallocation across sectors may be inefficiently low. The reason is that when workers move from sector  $A$  to sector  $B$ , their private choice has social benefits that they do not internalize: it makes it easier for other workers to find a job in sector  $A$ , by reducing congestion in a demand-constrained labor market, and it eases the supply constraints in sector  $B$ , reducing inflation in that sector. This means that if easy policy discourages reallocation, it is optimal to choose a more contractionary stance, while if easy policy encourages reallocation, it is then optimal to choose a more expansionary stance. Therefore, the two versions of the model with opposite positive predictions about the effects of monetary policy on reallocation also end up having opposite normative predictions.

Our analysis is related to three strands of literature. First, our analysis builds on the study of optimal monetary policy in response to asymmetric shocks. Seminal contributions in this area include [Aoki \(2001\)](#), [Woodford \(2003\)](#), and [Benigno \(2004\)](#), that extend the analysis of optimal monetary policy in the standard New Keynesian framework to multi-sectoral models where different sectors are hit by asymmetric shocks and, possibly, also differ in terms of their degree of price rigidity.<sup>3</sup> What we have in common with those papers is the idea that monetary policy should not only be concerned with average inflation and the average output gap, but also with getting relative prices across sectors close to their frictionless level, so as to reduce inefficiencies in the composition of output. The main difference with those papers is that we build our analysis in a model with downward rigid nominal wages, which introduces non-linear Phillips curves at the sectoral level. The main implication of this difference, is that to get relative prices right it is easier to get inflation in the expanding sectors than to get deflation in the contracting sectors, imparting an inflationary bias to optimal policy.

Our work is also related to the large literature on reallocation over the business cycle, going back to [Caballero and Hammour \(1991\)](#) and [Davis, Haltiwanger, Schuh et al. \(1998\)](#). In particular, [Caballero and Hammour \(2005\)](#) discuss two potential views: a “liquidationist” view that holds that recessions are periods more favorable to reallocation, because non-viable jobs and businesses are efficiently destroyed, and a “reverse liquidationist” view that holds that booms are more favorable to reallocation, since in booms high demand helps

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<sup>3</sup>A recent paper that analyzes a rich version of this type of optimal policy problems, in an economy with a general production network and input-output structure is [Rubbo \(2020\)](#).

new sectors grow. This paper makes a similar distinction in a new Keynesian framework with unemployment and nominal rigidities, and discusses forces by which a monetary easing can be damaging for reallocation—a liquidationist argument—and forces that can produce the opposite result—a reverse-liquidationist argument. We then argue that the relative importance of these forces matters for the design of optimal monetary policy. Some empirical evidence consistent with our model of a reallocation shock is in [Chodorow-Reich and Wieland \(2020\)](#), who show that reallocation shocks have different effects depending on the state of the business cycle: when the economy is expanding reallocation shocks appear to have no effects on unemployment, but they do increase unemployment if the economy is in recession.

Finally, our focus on the desirability for relative prices and wage changes that are hindered by downward rigidity relates to an older literature on structural inflation which also featured these concerns ([Olivera, 1964](#)). Despite this important point of contact, there are many differences. First and foremost, this literature did not study the optimal conduct of monetary policy, but instead assumed it to be entirely passive. Second, it did not consider costly reallocation of labor or feature unemployment, since monetary policy was assumed expansive enough. Finally, many of the structuralist sought to explain persistent inflation fueled initially by these shocks but perpetuated by indexation and the dynamics implied by adaptive expectations.

## 2 A Model

Consider an economy populated by a unit mass of infinitely-lived households with preferences represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(C_t).$$

Consumption  $C_t$  is the aggregate of two goods  $A$  and  $B$ ,

$$C_t = G(C_{At}, C_{Bt}, \omega_t),$$

where  $G$  is a homothetic aggregator of  $C_{At}$  and  $C_{Bt}$ , and  $\omega_t$  is a preference shock that determines the relative demand for the two goods. Each good  $A$  and  $B$  is the composite of

a continuum of goods produced by monopolistically competitive firms

$$C_{jt} = \left( \int_0^1 (C_{jt}(i))^{1-\frac{1}{\epsilon}} di \right)^{\frac{1}{1-\frac{1}{\epsilon}}}$$

where  $C_{jt}(i)$  denotes consumption of variety  $i$  in sector  $j \in \{A, B\}$  and  $\epsilon > 1$  is the elasticity of substitution between varieties.

The technology to produce each variety is linear,

$$Y_{jt}(i) = L_{jt}(i),$$

where  $L_{jt}(i)$  is the labor input of firm  $i$  in sector  $j \in \{A, B\}$ .

We begin in a setup with immobile labor where a fraction  $N_A$  of workers are fully specialized in producing good  $A$  and  $N_B$  are fully specialized in good  $B$ . Each worker supplies a single unit of labor to the sector of specialization. We introduce (costly) labor mobility across sectors in Section 4.

To allow for the possibility of inefficient unemployment and costly inflation, we introduce nominal rigidities in our model. We will consider different versions of nominal rigidities in the paper. In our baseline, we combine downward rigid nominal wages and costly price adjustments a la [Rotemberg \(1982\)](#).

Firms hire workers on the labor market at the nominal wage  $W_{jt}$ . Wages are downward rigid. Letting  $L_{jt}$  denote labor demand in sector  $j$ , the equilibrium conditions in the labor market in sector  $j$  are

$$W_{jt} \geq W_{jt-1}, \quad L_{jt} \leq N_j$$

with at least one strict inequality. If labor demand is lower than labor supply in equilibrium, sector  $j$ 's workers are rationed proportionally and the probability of finding employment for a  $j$  sector worker is

$$\rho_{jt} = \frac{L_{jt}}{N_j} < 1.$$

Firms are monopolistically competitive. The firm selling variety  $i$  in sector  $j$  sets its price  $P_{jt}(i)$  subject to quadratic adjustment costs a la [Rotemberg \(1982\)](#), where the cost of changing the price is given by

$$\phi \left( \frac{P_{jt}(i) - P_{jt-1}(i)}{P_{jt-1}(i)} \right)^2 Y_{jt},$$

with  $Y_{jt} = C_{jt}$  denoting aggregate output in sector  $j$ . The price adjustment cost is a real cost in terms of the final consumption good and  $\phi > 0$  is a constant.

We assume that all households are composed of  $N_A$  workers specialized in sector  $A$  and  $N_B$  workers specialized in sector  $B$ , which implies that the household's labor income is  $W_{At}L_{At} + W_{Bt}L_{Bt}$ . Each household also owns a representative sample of firms in each sector and receives their profits. These assumptions imply that all the analysis can be done in terms of a representative household.

As it is commonly done, we assume that the producer of each variety  $i$  receives a proportional subsidy  $\sigma = 1/(\epsilon - 1)$ , financed with a lump sum tax on the representative consumer, which corrects the monopolistic distortion in steady state.

### 3 Policy Options with an Asymmetric Shock

We next characterize the behavior of the economy when it is suddenly hit by an asymmetric shock. To do so, we first describe the steady state of the model and then introduce the shock.

#### 3.1 Steady state and shock

We assume that, in the steady state of the model, the taste parameter is constant at some level  $\bar{\omega}$ , there is zero inflation and full employment in both sectors. We assume that, in the initial steady state, the labor allocation  $N_A, N_B$  satisfies the following conditions

$$G_{C_A}(N_A, N_B, \bar{\omega}) = G_{C_B}(N_A, N_B, \bar{\omega}) = 1, \quad (1)$$

so all steady state prices are the same and can be normalized to 1. Using bars to denote steady state values, we have

$$\bar{W}_A = \bar{W}_B = \bar{P}_A = \bar{P}_B = 1.$$

This initial allocation arises endogenously in the long run when mobility across sectors is allowed, as is the case in Section 4.

At some time  $t$ , we suppose the economy is hit by a one time, unexpected, and permanent shock that shifts preferences in favor of good  $B$ . What is the set of feasible allocations and inflation rates that monetary policy can implement after this shock? What is the optimal policy response?

For simplicity, we assume that, from time  $t + 1$  onwards, the economy features perfectly flexible prices and wages. Under this assumption we can focus on the allocation at time  $t$  and on the effects of monetary policy on welfare at  $t$ .

### 3.2 Equilibrium conditions

An equilibrium allocation at time  $t$  is characterized by the following three conditions. For ease of notation we drop the subscript  $t$ .

First, the homothetic aggregator  $G$  implies that the consumer's optimality conditions yield a relative demand function

$$\frac{Y_A}{Y_B} = f\left(\frac{P_B}{P_A}, \omega\right),$$

where  $f$  is increasing in its first argument. In the initial steady state we have  $f(1, \bar{\omega}) = \bar{N}_A / \bar{N}_B$ , which follows from our assumption (1). Since the preference shock  $\omega_t = \omega$  shifts demand in favor of good  $B$  relative to the initial steady state, we have  $f(1, \omega) < \bar{N}_A / \bar{N}_B$ .

Second, there is a price setting condition, which is identical for all producers in sector  $j$  and takes the form

$$P_j (P_j - 1) = \frac{\epsilon}{\phi} \left( \frac{W_j}{P} - \frac{P_j}{P} \right),$$

where  $P$  is the consumer price index. This optimality condition is derived from producers' first order conditions. Its interpretation is standard: when the marginal cost of producing good  $j$  is greater than the price of good  $j$ , producers have an incentive to raise nominal prices. Else, they lower prices.

Third, we have the labor market equilibrium conditions, which take the form

$$W_j \geq 1, \quad Y_j \leq \bar{N}_j,$$

with at least one equality.

### 3.3 Optimal policy

Let  $M$  denote total nominal spending

$$M \equiv P_A Y_A + P_B Y_B.$$

We assume that monetary policy directly chooses  $M$ . It is easy to show that this is equivalent to choosing the interest rate at date  $t$ . Depending on the sector(s) in which the downward rigidity is binding, the equilibrium at date  $t$  falls into one of the following three cases. They are distinguished by the level of  $M$  relative to two cutoffs  $M' < M''$ , which we define below.

We first consider  $M < M'$ . In that case, demand is sufficiently low for the equilibrium to feature unemployment in both sectors,  $A$  and  $B$ . In this case, the downward wage rigidity is binding in both sectors, and firms are happy to keep their nominal prices at the steady state values:

$$W_A = W_B = P_A = P_B = 1.$$

Relative demand is simply  $Y_A/Y_B = f(1, \omega)$  and the equilibrium output levels are

$$Y_A = \frac{f(1, \omega)}{1 + f(1, \omega)} M, \quad Y_B = \frac{1}{1 + f(1, \omega)} M.$$

In this region, increasing  $M$  leads to higher output in both sectors and no inflation. If  $M$  rises above

$$M' \equiv (1 + f(1, \omega)) N_B$$

sector  $B$  reaches full employment, while, by construction, sector  $A$  still has positive unemployment since  $N_A > N_B f(1, \omega)$ . This gives us the left cutoff point  $M'$ .

At the opposite extreme, if  $M > M''$ , the equilibrium features full employment and positive inflation in both sectors. Output levels in the two sectors are just  $Y_A = N_A$  and  $Y_B = N_B$  and the relative price  $P_B/P_A$  is pinned down by the condition

$$f\left(\frac{P_B}{P_A}, \omega\right) = \frac{N_A}{N_B}. \quad (2)$$

Therefore, as  $M$  increases, the two prices  $P_A$  and  $P_B$  grow proportionally, while the relative price  $P_A/P_B$  remains constant.

We can now turn to the most interesting case, which is the intermediate case where  $M' \leq M \leq M''$ . In this case, the equilibrium features unemployment in sector  $A$  and inflation in sector  $B$ . Wages and prices in sector  $A$  are unchanged,  $W_A = P_A = 1$ , while in sector  $B$  we have full employment and positive inflation

$$P_B (P_B - 1) = \frac{\epsilon}{\phi} \left( \frac{W_B}{P} - \frac{P_B}{P} \right) > 0. \quad (3)$$

Relative demand is now given by  $Y_A/Y_B = f(P_B, \omega)$ , which, combined with full employment in sector  $B$ ,  $Y_B = N_B$ , implies that sector  $A$  output is determined by

$$Y_A = f(P_B, \omega) N_B. \quad (4)$$

In this region, increasing  $M$  leads to higher output in sector  $A$  and to higher inflation in sector  $B$ , as long as  $Y_A < N_A$ . This is equivalent to the relative price  $P_B$  remaining below the marginal rate of substitution at full employment,  $\frac{G_{C_B}(N_A, N_B, \omega)}{G_{C_A}(N_A, N_B, \omega)}$ , which we can translate into a condition on  $M$ ,

$$M < M'' \equiv N_A + \frac{G_{C_B}(N_A, N_B, \omega)}{G_{C_A}(N_A, N_B, \omega)} N_B$$

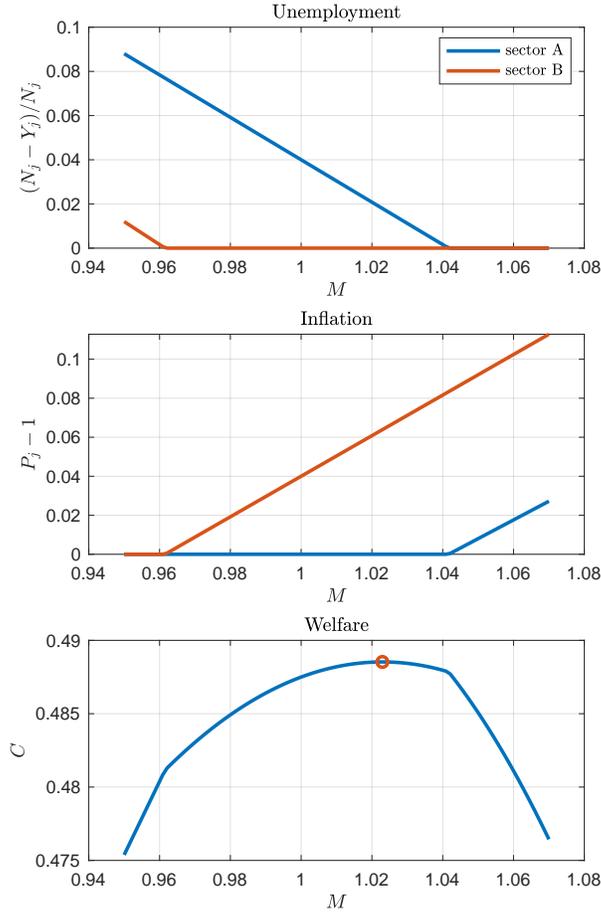
Notice that the positive relation between output and inflation in this region is not governed by the logic behind a traditional new Keynesian Phillips curve, where higher activity pushes up marginal costs and leads firms to increase prices. Here, instead, greater  $M$  increases activity in sector  $A$ —where marginal costs are fixed at 1—while inflation is showing up in sector  $B$ —where marginal costs are increasing even though activity is constant in  $M$ . The reason why this region displays an increasing relation between inflation and output is because inflation in sector  $B$  helps to shift demand in favor of sector  $A$ . In other words, the relevant relation in this region is the relative demand curve (4), not the price setting condition (3).<sup>4</sup>

The crucial observation here is that inflation in this economy can help get relative prices right. In a flexible price equilibrium the preference shock  $\omega$  requires the relative price of good  $B$  to increase to match the relative supply of goods  $A$  and  $B$ . This requires  $P_A/P_B$  to satisfy condition (2). With flexible prices that relative price can be achieved either by letting nominal prices and wages fall in sector  $A$  or by letting prices and wages increase in sector  $B$ . With nominal rigidities, and, in particular, with asymmetric rigidities that make wage reductions harder to achieve than wage increases, this adjustment cannot be achieved costlessly. In particular, an adjustment up of  $P_B/P_A$  requires costly inflation in sector  $B$ .

This logic is directly reflected in the model's welfare implications. In Figure 1, we show how  $M$  affects prices, allocations, and welfare in a simple example. The two cutoffs are

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<sup>4</sup>The clean dichotomy between the two forces behind the Phillips curve is special to the model used here, with inelastic labor supply and a downward wage rigidity. In more general models, with multiple sectors and heterogeneous firms the slope of the Phillips curve at any point will capture both standard elements and elements of relative price adjustments that shift demand from relatively supply-constrained sectors to demand-constrained ones.

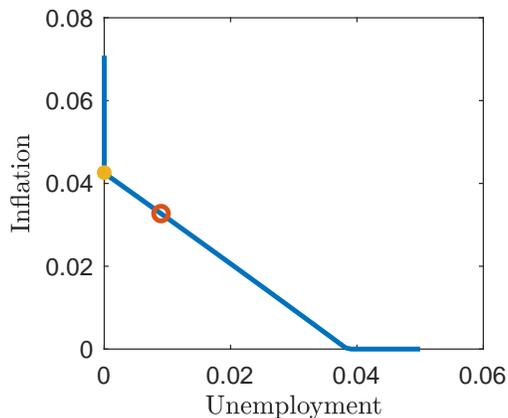


**Figure 1:** Optimal policy with an asymmetric shock

*Note.* The example uses Cobb-Douglas preferences  $G(C_A, C_B, \omega) = C_A^{1-\omega} C_B^\omega$  and the following parameters:  $\omega = 0.52, N_B = 0.5, \phi = 3.5$ .

given by  $M' = 0.962$  and  $M'' = 1.041$ . In the top panel, we show the relation between  $M$  and unemployment in the two sectors. Unemployment in sector  $B$  falls more rapidly, since the preference shock shifts demand towards that sector. In the middle panel we show inflation in the two sectors. Inflation in sector  $B$  rises more rapidly, as it is at full employment and sees wage increases. In the bottom panel we plot welfare in consumption-equivalent units, that is  $C$ , of the representative consumer.

The optimal level of demand  $M$  always falls into the intermediate interval  $[M', M'']$ . To the left of  $M'$ , greater  $M$  only has the favorable effect of lowering unemployment, without causing costly inflation in either sector. To the right of  $M''$ , greater  $M$  has no favorable effects on employment, causing only increased inflation. In the interval  $[M', M'']$ , optimal



**Figure 2:** The Phillips curve in the model with no mobility

policy trades off inflation costs in sector  $B$  against unemployment in sector  $A$ . In the example plotted in Figure 1, there is an interior optimum  $M \in (M', M'')$ .<sup>5</sup>

What is remarkable about the optimum in Figure 1, is that it illustrates how, in our economy, the planner has an inflationary bias after a preference shift. Indeed, inflation helps by shifting demand from sector  $B$  to sector  $A$ , and reducing unemployment there. Because of this trade-off, an asymmetric preference shock acts as an endogenous cost-push shock. We illustrate this in Figure 2, which shows the effective inflation-unemployment trade-off in our model. As can be seen, divine coincidence fails: the point of zero inflation and zero unemployment is unattainable, and the planner optimally picks a point on the downward sloping portion of the relationship. Optimal policy is identified by the red circle.

The inflationary bias identified here is related to the literature on optimal monetary policy and relative prices.

In the classic model of Aoki (2001), there is a sector with sticky prices and one with flexible prices. Aoki (2001) shows that optimal policy in this context consists of perfectly targeting inflation in the sticky sector, which, in fact, achieves divine coincidence. In our model, an exercise similar in spirit to Aoki (2001) comes from reducing the price setting cost to zero,  $\phi \rightarrow 0$ , as in that case prices are flexible in sector  $B$ , while they remain endogenously sticky in sector  $A$  due to downward wage rigidity. In Figure 2, we plot the optimal policy when the cost of inflation  $\phi \rightarrow 0$ . The shape of the Phillips curve is not affected by  $\phi$ , as we observed earlier. However, the objective of the monetary authority

<sup>5</sup>It can be shown that if  $\phi$  is low enough, the optimum is necessarily reached at the cutoff  $M''$ . By contrast, the optimum is never at  $M'$  because inflation costs are second order at that point.

changes, as the inflation cost go down. Reducing  $\phi$  moves the optimal point up and to the left, until, when  $\phi$  is low enough, the optimum corresponds to the kink with zero unemployment and positive inflation, shown by the yellow asterisk. Of course, in our context it is harder to translate this result into a recommendation to target inflation in one sector or the other, as the identity of the sector to target depends on the sign of the shock.

In the model of [Rubbo \(2020\)](#), preference shifts such as hours only affect the allocation of labor across sectors, but do not cause any changes in prices or wages. Divine coincidence is also preserved in this model.

The settings in [Woodford \(2003\)](#) (Chapter 6) and [Benigno \(2004\)](#) are closer to that in this section, allowing for heterogeneous price rigidities and immobile labor across sectors (or equivalently, other fixed factors). These papers argue that inflation of the relatively more sticky sector should be weighted more heavily at the optimum. Since in our model, one sector has perfectly sticky prices downward (sector  $A$ ), these papers' results apply to our economy only in a degenerate form. The main gist, however, carries over, namely that allowing for more inflation in the less sticky sector can be optimal in order to correct relative prices.

## 4 Costly Mobility

An economy has two ways of adjusting to asymmetric shocks like the one we are studying here: either shift demand from sectors with excess supply to sectors with lack of demand, or to shift productive capacity in the opposite direction. This raises two important questions. Is this process of factor reallocation efficient? Should concerns with factor reallocation influence the conduct of monetary policy, and, if so, in what direction?

To address these two questions, we add costly labor mobility to the model of the previous sections. Namely, we assume that the representative household can choose to move workers from sector  $A$  to sector  $B$ . Since the sum of workers per household remains 1, the flow of workers can be equivalently expressed as  $N_{Bt} - N_{Bt-1} = -(N_{At} - N_{At-1})$ . We assume a simple quadratic cost of adjustment, in terms of consumption goods,

$$\frac{\psi}{2} (N_{Bt} - N_{Bt-1})^2.$$

To derive the optimality condition for labor mobility, we need to take into account the possibility of labor rationing, that is, the fact that the probability of employment of a

worker in sector  $j$  is

$$\rho_{jt} = \frac{Y_{jt}}{N_{jt}}$$

and thus possibly smaller than 1. Therefore, the benefit of having a worker specialized in sector  $j$  in terms of consumption goods is  $\rho_{jt} \frac{W_{jt}}{P_t}$ .

The first order condition characterizing optimal mobility then takes the simple form

$$\psi(N_{Bt} - N_{Bt-1}) = \rho_{Bt} \frac{W_{Bt}}{P_t} - \rho_{At} \frac{W_{At}}{P_t} + \frac{1}{1+r_t} \psi(N_{Bt+1} - N_{Bt})$$

where  $r_t$  is the real interest rate. The interpretation of this condition is straightforward: the left-hand side represents the marginal cost of moving workers today, the right-hand side captures the marginal benefit, which includes the net flow benefit of shifting a worker from sector  $A$  to  $B$  plus the discounted lower cost of moving workers in the future.

To simplify the analysis, we continue to assume that the economy is in steady state, that an unexpected one-time shock hits at time  $t$ , and that from time  $t+1$  onwards all adjustment costs are zero—not only the adjustment cost of nominal prices but also the mobility cost. The condition above is then replaced by

$$\psi(N_B - \bar{N}_B) = \rho_B \frac{W_B}{P} - \rho_A \frac{W_A}{P}, \quad (5)$$

where we dropped time subscripts as before, and where  $\bar{N}_B$  denotes the initial steady state labor allocation.

We can again characterize the equilibrium in period  $t$  in terms of total nominal spending  $M$ . The remaining equilibrium conditions are analogous to those of the no-mobility case, they are presented and analyzed in detail in the appendix. In particular, as in the case with no mobility, there are three regions for  $M$ , characterized by different combinations of unemployment and inflation in the two sectors. However, the values of  $N_A$  and  $N_B$  are now endogenous and vary with  $M$ . The three regions for  $M$  will be shown in an illustrative example in Figure 4 below.

## 5 Optimal Policy and Reallocation

We now turn to the main question of our paper which is whether reallocation objectives impart a contractionary or expansionary bias to monetary policy.

To frame the question let us define the following objects—which apply to our model

but can also be derived for a broader class of models. Let  $V(M, N_B)$  denote the utility of the consumer at time  $t$  as a function of nominal spending  $M$  and the mass of workers specialized in sector  $B$ . Let  $\mathcal{N}(M)$  denote the equilibrium mapping between nominal spending  $M$  and the mass of workers specialized in sector  $B$ , which comes from the optimal mobility choice analyzed in the last section.

The optimal monetary policy problem can then be described compactly as

$$\max_M V(M, \mathcal{N}(M)). \quad (6)$$

The model with no mobility is a special case in which monetary policy simply solves  $\max_M V(M, \bar{N}_B)$ .

Assuming an interior optimum the first order condition for optimal monetary policy is

$$\frac{\partial V(M, \mathcal{N}(M))}{\partial M} + \frac{\partial V(M, \mathcal{N}(M))}{\partial N_B} \mathcal{N}'(M) = 0. \quad (7)$$

Our question can then be posed formally as asking what is the sign of the second term of this sum. Namely, does the presence of sectoral mobility add a negative or a positive social benefit to increasing  $M$ ? In the first case, we say that reallocation concerns induce a contractionary bias in monetary policy, in the second, an expansionary bias.

## 5.1 Fully rigid wages: a case of contractionary bias

It is useful to analyze first a variant of our model with a simpler form of nominal rigidity, in which the terms above can be derived particularly easily. This case will give us an example in which monetary policy shows a contractionary bias due to reallocation.

Let us modify our assumptions on pricing. Assume that nominal wages are completely rigid, both upwards and downwards. Assume also that if in the market for good  $j$  there is excess demand, the price of good  $j$  adjusts up to the point where demand equals supply. With these assumptions, we still can have our intermediate equilibrium configuration, with unemployment in  $A$  and full employment in  $B$ , with prices satisfying  $W_A = P_A = 1$  and  $P_B > W_B = 1$ . The profits of the  $B$  producers are simply rebated to the representative household.

Let us assume Cobb-Douglas preferences

$$G(C_A, C_B, \omega) = C_A^{1-\omega} C_B^\omega,$$

so that demand in the two sectors as a function of nominal spending  $M$  is

$$Y_A = (1 - \omega) M, \quad Y_B = \omega \frac{M}{P_B}.$$

Imposing  $Y_B = N_B$ , we can derive the function  $V$  in closed form

$$V(M, N_B) = ((1 - \omega) M)^{1-\omega} N_B^\omega - \frac{\psi}{2} (N_B - \bar{N}_B)^2. \quad (8)$$

The optimal mobility equation (5) now becomes

$$\psi (N_B - \bar{N}_B) = \frac{1}{P} - \rho_A \frac{1}{P'}, \quad (9)$$

given that wages are equal to  $1/P$  in both sectors and  $\rho_B = 1$ .

Let us study separately the terms  $\partial V / \partial N_B$  and  $\mathcal{N}'(M)$  in equation (7) in the context of this model.

Given a value of  $M$  in the intermediate region, we can differentiate (8) at the equilibrium value  $N_B = \mathcal{N}(M)$  and, using the mobility equation (9), we obtain

$$\frac{\partial V}{\partial N_B} = \omega ((1 - \omega) M)^{1-\omega} N_B^{\omega-1} - \psi (N_B - \bar{N}_B) = \frac{P_B - W_B}{P} + \frac{W_A}{P} \rho_A > 0. \quad (10)$$

This equation shows that in equilibrium mobility between sectors  $A$  and  $B$  is inefficiently low. We call the expression in (10) a mobility wedge.

Let us give intuition for the two terms in the last expression. First, agents undervalue the benefit of moving to sector  $B$  due to the wedge between wages and prices in  $B$ ,  $P_B - W_B > 0$ . Second, agents overvalue the benefit of staying in sector  $A$ , as they do not internalize the fact that by leaving sector  $A$  they increase the probability of finding a job for the other workers in sector  $A$ .

To expand the interpretation of the last term in (8) as a congestion externality, notice that  $Y_A = (1 - \alpha) M$  is fixed, for given  $M$ , so we can differentiate

$$\rho_A N_A = Y_A$$

and obtain

$$N_A d\rho_A + \rho_A dN_A = N_A d\rho_A - \rho_A dN_B = 0.$$

This implies that the effect of changing  $N_B$  on the incomes of other workers is

$$\frac{W_A}{P} N_A \frac{d\rho_A}{dN_B} = \frac{W_A}{P} \rho_A,$$

which is precisely the last term in (10). This is a basic idea that will come back later: when workers leave a sector with insufficient demand they underestimate the social benefit that comes from reducing the queue of applicants for a demand-constrained number of jobs.

Notice a connection between the externality identified here and a similar externality arising in models with geographic mobility across different regions, affected by different shocks. In particular, [Farhi and Werning 2014](#) identify a closely related externality in a model of a monetary union in which workers can move between countries.

Next we turn to the term  $\mathcal{N}'(M)$ . This term is negative for two reasons: easier monetary policy increases  $Y_A$  and, for given  $N_A$ , it increases the job finding probability  $\rho_A$ , moreover easier monetary policy increases the price of good  $B$  and so the price index  $P$ , reducing real wages. Inspecting the right-hand side of (9) shows that both forces tend to reduce labor mobility.<sup>6</sup>

In this economy the monetary authority faces the following trade off: increasing  $M$  increases activity in sector  $A$ , reducing wasteful unemployment, but, at the same time, it reduces the incentive of workers to move from sector  $A$  to sector  $B$ . Since workers do not fully internalize the benefit of moving from  $A$  to  $B$ , the monetary authority has to balance a Keynesian wedge against the mobility wedge (9).

Figure 3 shows a numerical example in which this trade-off leads to an interior solution for  $M$  with  $\rho_A < 1$ .<sup>7</sup> The top panel shows the equilibrium relation  $N_B = \mathcal{N}(M)$ , which is decreasing in line with the derivations above. The next two panel shows the effects of  $M$  on unemployment (in  $A$ ) and inflation (in  $B$ ). The bottom panel shows welfare, measured in terms of consumption at  $t$ . Notice that unlike in our baseline model, there

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<sup>6</sup>The equilibrium condition for good  $B$  gives the price index

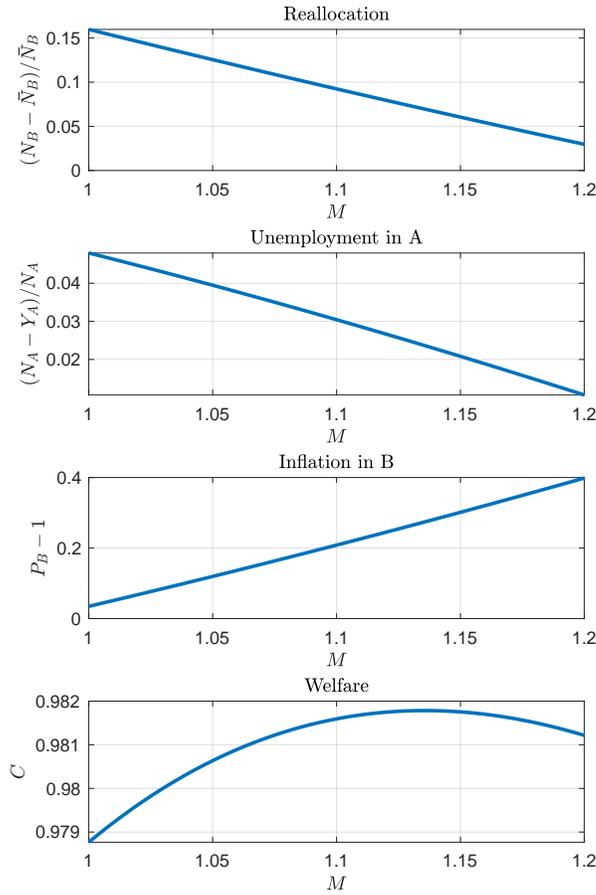
$$P = \omega^{-\omega} (1 - \omega)^{-(1-\omega)} P_B^\omega = (1 - \omega)^{-(1-\omega)} \left( \frac{M}{N_B} \right)^\omega.$$

Using the equilibrium condition for good  $A$ , equation (9) can then be rewritten as follows

$$\psi(N_B - \bar{N}_B) = (1 - \omega)^{1-\omega} \left( \frac{N_B}{M} \right)^\omega \left( 1 - \frac{1 - \omega}{1 - N_B} M \right)$$

and implicit differentiation proves  $\mathcal{N}'(M) < 0$ .

<sup>7</sup>Since the cost of unemployment is first order, it is also possible that the optimum is at the upper boundary of the intermediate region, where  $\rho_A = 1$ .

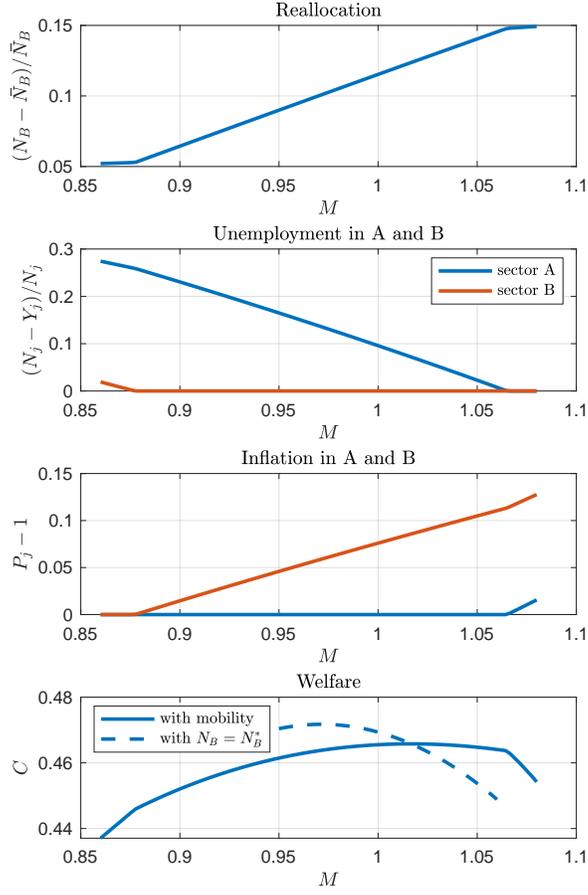


**Figure 3:** Optimal policy in the economy with rigid nominal wages

is no direct cost of inflation in this model, so, absent reallocation motives, optimal policy would simply reach the boundary point where  $\rho_A = 1$  and there is zero unemployment in  $A$ . Therefore, this is a case where the reallocation motive imparts a contractionary bias to policy. Even though unemployment in  $A$  is wasteful, it plays a useful social role in stimulating reallocation towards  $B$ . So this example captures well a liquidationist view.

## 5.2 Expansionary bias

We now go back to our baseline model with costly price adjustment. Once more, we focus on characterizing the second term in the optimal policy condition (7) and analyze separately the two factors in it. For the following derivations, we assume Cobb-Douglas



**Figure 4:** Optimal policy in the economy with downward wage rigidity and sticky prices

preferences.

Also in this model, reallocation is generally inefficient. The expression for the mobility wedge is now

$$\frac{\partial V}{\partial N_B} = \frac{\epsilon - 1}{\epsilon} \phi (P_B - 1) P_B - \frac{\phi}{2} (P_B - 1)^2 + \rho_A \frac{W_A}{P}. \quad (11)$$

This equation is derived in Appendix (B).

The three terms in this expression can be interpreted as follows. The first two terms capture the effect of  $N_B$  on inflation costs. In particular, the first term is due to the following externality in price setting: an individual price setter (in sector  $B$ ) does not internalize that by increasing the price of its variety it induces all other price setters to increase their price, leading to higher price adjustment costs collectively. When a worker moves from sector  $A$  to sector  $B$  it tends to lower the equilibrium price  $P_B$ . Due to the externality just described,

lowering the equilibrium value of  $P_B$  has a positive social value, so this term is positive.

The second term is more mechanical. In the standard formulation of Rotemberg price adjustment costs, which we adopted, these costs are scaled by the aggregate level of activity, which gives rise to a negative externality: increasing activity in the sector with positive inflation costs, sector  $B$  here, raises total inflation costs. Therefore, the first two terms capture two externalities associated to inflation costs. Typically the elasticity of substitution among varieties  $\epsilon$  is set to be larger than 2, which is a sufficient condition for the sum of these two terms to be positive.

The third term is the same congestion externality discussed above: moving a worker out of sector  $A$  increases the job finding probability for the workers who remain.

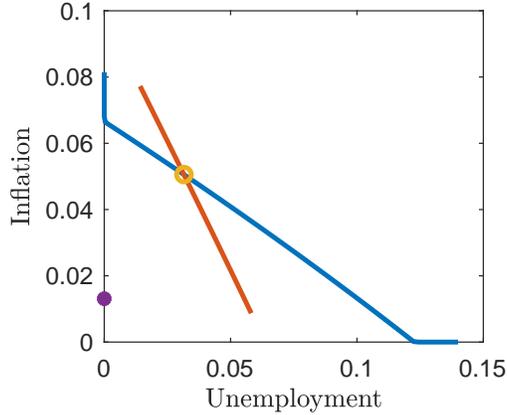
Summing up, also in our baseline model, with  $\epsilon > 2$ , the sign of the mobility wedge is positive: in equilibrium there is inefficiently low mobility from sector  $A$  to sector  $B$ . So far, the model has similar features as the fully rigid one.

Let us turn now to the effect of monetary policy on mobility, that is, to the sign of  $\mathcal{N}'(M)$ . Derivations in Appendix (B) show that in our baseline model with Cobb-Douglas preferences  $\mathcal{N}'(M) > 0$ . So we get opposite predictions that in the fully rigid model: a monetary authority concerned with reallocation should have a more expansionary bias. Why? The reason is that now the mobility condition takes the more general form

$$\psi(N_B - \tilde{N}_B) = \rho_B \frac{W_B}{P} - \rho_A \frac{W_A}{P},$$

and now both the job finding probability  $\rho_j$  and the sectoral real wages are endogenous. A more expansionary monetary policy in this case leads to a real wage increase in sector  $B$  stronger than in sector  $A$ , given that sector  $B$  is supply constrained, and the relative change in  $\frac{W_B}{P}$  and  $\frac{W_A}{P}$  more than undoes the effect of a higher job finding probability  $\rho_A$ .

There are two forces that induce workers to move towards sector  $B$ : a low probability of finding a job in  $A$  and a large wage premium when moving from  $A$  to  $B$ . The model with fully rigid wages mutes the second force, so the only way for monetary policy to induce more mobility is to be less aggressive in contrasting unemployment in  $A$ . The model with upward flexible wages allows another channel to be at work: increasing inflation in  $B$  helps, as it allows wages in sector  $B$  to raise, sending the right price incentive to workers to move. Notice that, as in Section (3), in an economy with downward rigid wages, a degree of inflationary bias helps to get relative prices right. The interesting thing is that here the relative price adjustments helps both on the demand *and* on the supply side of the adjustment process: it reallocates demand in favor of sector  $A$ , and it helps reallocate



**Figure 5:** The Phillips curve in the model with endogenous mobility

factors of production towards sector  $B$ .

Figure 4 shows a numerical example that illustrates our argument. The top panel shows an increasing relation between  $M$  and  $N_B$ , in contrast to the top panel of Figure 3. The middle panels show inflation and unemployment in the two sectors, these panels are qualitatively similar to those of the model with no mobility illustrated in Figure 1, and, in particular display the presence of three regions.

The bottom panel of Figure 4 show the model's welfare implications. As in the model with no mobility of Section 3 the optimum is in the intermediate region, and features positive inflation in sector  $B$  and unemployment in  $A$ . To visualize the bias introduced in monetary policy by the presence of endogenous mobility, in the same panel we also plot welfare as a function of  $M$ , keeping  $N_B$  fixed at the level  $N_B^* \equiv \mathcal{N}(M^*)$ , where  $M^*$  is the solution to (6). That is we plot the function  $V(M, N_B^*)$ , the dashed blue line, in addition to the function  $V(M, \mathcal{N}(M))$ , the solid blue line. The two functions intersect, as they should, at  $M = M^*$ . However, while  $V(M, \mathcal{N}(M))$  reaches its maximum at  $M^*$ , the function  $V(M, N_B^*)$  is decreasing at  $M = M^*$  and reaches its maximum for some  $M < M^*$ . The interpretation of this fact is that, if monetary policy could keep  $N_B$  at  $N_B^*$ , it would choose a less expansionary stance. In other words, the concern with facilitating sectoral restructuring leads the monetary authority to be more expansionary.

### 5.3 Phillips curves

As in the model with no mobility, it is instructive to look at the optimal policy problem in terms of the Phillips curve. Figure 5 shows the Phillips curve for the numerical example of

Figure 4. The red line represents the Phillips curve with a fixed level  $N_B = N_B^*$ , constructed as the welfare curve in the bottom panel of Figure 4. The Phillips curve provides an alternative interpretation for the expansionary bias: the presence of endogenous adjustment of  $N_B$  makes the Phillips curve flatter, inducing the central bank to choose a larger reduction in unemployment in  $A$ .<sup>8</sup>

Notice also that the red line is simply one of a family of Phillips curves which represent the inflation-unemployment trade-off for every value of  $N_B$ . A larger value of  $N_B$  shifts the Phillips curves inwards, leaving the central bank with a more desirable frontier. In other words, workers' mobility allows to partially undo the endogenous "cost-push" shock caused by the asymmetric preference shock.

The last observation leads to an additional point: if the policy maker has access to additional tools to encourage labor mobility (or equivalently to policies that remove obstacles to mobility) these policies allow the central bank, all else equal, to achieve a better mix of inflation and unemployment. This is illustrated by the purple asterisk in Figure 5, which corresponds to the pair  $M^{**}, N_B^{**}$  that solve the unconstrained maximization problem  $\max V(M, N_B)$ . This pair can be achieved if the policy maker has access to a subsidy to mobility which can be used to correct the mobility wedge identified above, without distorting monetary policy.

## 6 Conclusion

The paper has explored the optimal conduct of monetary policy in presence of shocks that require a permanent reallocation of resources among sectors. In particular, we have shown that a desire to facilitate the reallocation process can lead to favor a more expansionary monetary policy. The reason is that higher inflation can facilitate the adjustment of relative wages, so as to provide the right price signals to encourage mobility.

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<sup>8</sup>This intuition is incomplete because, unlike in the model with no mobility, in the current model it is not possible to map welfare solely in terms of inflation (in  $B$ ) and unemployment (in  $A$ ) due to the presence of mobility costs.

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Appendix to  
**Monetary Policy in Times of Structural Reallocation**

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## A Equilibrium with costly mobility

In this appendix, we give a full characterization of the equilibrium with costly mobility of Section 4 in the case of Cobb-Douglas preferences. Let us first summarize the equilibrium conditions

$$\begin{aligned} \frac{Y_A}{Y_B} &= f\left(\frac{P_B}{P_A}, \omega\right), \\ \epsilon\left(\frac{W_j}{P} - \frac{P_j}{P}\right) &= \phi P_j (P_j - 1), \\ \psi(N_B - \bar{N}_B) &= \frac{W_B}{P} \frac{Y_B}{N_B} - \frac{W_A}{P} \frac{Y_A}{N_A}, \\ N_A + N_B &= 1, \end{aligned}$$

and the labor market complementary inequalities

$$Y_j \leq N_j, \quad W_j \geq 1.$$

The price index  $P$  is defined as usual as  $P = \min_{y_A, y_B} \{P_A y_A + P_B y_B : G(y_A, y_B, \omega) = 1\}$ .

In the case of Cobb-Douglas preferences  $G(Y_A, Y_B) = Y_A^{1-\omega} Y_B^\omega$ , the equilibrium can be solved using the following simple algorithm. For each labor allocation  $N_A, N_B$ , we can find the equilibrium prices and quantities solving

$$\begin{aligned} P_A &= \max\left\{(1-\omega) \frac{M}{N_A}, 1\right\} & Y_A &= \min\{(1-\omega) M, N_A\}, \\ P_B &= \max\left\{\omega \frac{M}{N_B}, 1\right\} & Y_B &= \min\{\omega M, N_B\}. \end{aligned}$$

The real wages are then derived from

$$\frac{W_j}{P} = \frac{P_j}{P} + \frac{\phi}{\epsilon} P_j (P_j - 1).$$

These values can then be substituted in

$$\psi(N_B - \bar{N}_B) = \frac{Y_B}{N_B} \frac{W_B}{P} - \frac{Y_A}{N_A} \frac{W_A}{P}.$$

It is possible to show that this defines an equation in  $N_B$  with a unique solution and derive the cutoffs  $M'$  and  $M''$  that characterize the three possible regimes.

## B Derivations for Section

### B.1 Deriving $\partial V / \partial N_B$

We focus on an intermediate equilibrium with  $Y_A < N_A$  and  $P_B > 1$ . With log preferences output in  $A$  is

$$Y_A = (1 - \alpha) M$$

and prices in  $B$  are

$$P_B = \alpha \frac{M}{N_B}. \quad (12)$$

Differentiating the last equation we obtain

$$\frac{dP_B}{dN_B} = -\frac{P_B}{N_B}.$$

Equilibrium consumption can be written as follows

$$V = Y_A^{1-\alpha} N_B^\alpha - \frac{\phi}{2} (P_B - 1)^2 N_B - \frac{\psi}{2} (N_B - \bar{N}_B)^2.$$

Differentiating with respect to  $N_B$  and substituting  $dP_B/dN_B$  then yields

$$\frac{\partial V}{\partial N_B} = \alpha Y_A^{1-\alpha} N_B^{\alpha-1} + \phi (P_B - 1) P_B - \frac{\phi}{2} (P_B - 1)^2 - \psi (N_B - \bar{N}_B).$$

Remember that

$$\psi (N_B - \bar{N}_B) = \frac{W_B}{P} - \rho_A \frac{W_A}{P}$$

and

$$\frac{W_B}{P} = \frac{P_B}{P} + \frac{\phi}{\epsilon} P_B (P_B - 1) = \alpha Y_A^{1-\alpha} N_B^{\alpha-1} + \frac{\phi}{\epsilon} P_B (P_B - 1).$$

Substituting gives expression (11) in the text.

### B.2 Deriving $\mathcal{N}'(M)$

From  $P_A = W_A$  and consumer demand for good  $A$  we obtain

$$\frac{W_A}{P} = \frac{W_P}{P} = \alpha^\alpha (1 - \alpha)^{1-\alpha} P_B^{-\alpha}.$$

Wages in sector  $B$  can be similarly derived as

$$\frac{W_B}{P} = \frac{P_B}{P} + \frac{\phi}{\epsilon} P_B (P_B - 1) = \alpha^\alpha (1 - \alpha)^{1-\alpha} P_B^{1-\alpha} + \frac{\phi}{\epsilon} P_B (P_B - 1).$$

Substituting these expressions and the job finding rate  $\rho_A = (1 - \alpha) M/N_A$ , the optimal mobility condition is

$$\psi (N_B - \bar{N}_B) = \alpha^\alpha (1 - \alpha)^{1-\alpha} P_B^{1-\alpha} + \frac{\phi}{\epsilon} P_B (P_B - 1) - (1 - \alpha) \frac{M}{N_A} \alpha^\alpha (1 - \alpha)^{1-\alpha} P_B^{-\alpha}.$$

Substituting (12) gives the following implicit relation between  $N_B$  and  $M$

$$\psi (N_B - \bar{N}_B) = (1 - \alpha)^{1-\alpha} \left( \frac{\alpha}{N_B} - \frac{1 - \alpha}{1 - N_B} \right) M^{1-\alpha} N_B^\alpha + \frac{\phi}{\epsilon} \alpha \frac{M}{N_B} \left( \alpha \frac{M}{N_B} - 1 \right).$$

Since the right-hand side is increasing in  $M$ , implicit differentiation shows that  $\mathcal{N}'(M) > 0$ .