A Dynamic Theory of Lending Standards*

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July 20, 2020

Abstract

We develop a tractable dynamic model of credit markets in which lending standards and the quality of potential borrowers are endogenous. Competitive banks privately choose their lending standards: whether to pay a cost to screen out some unprofitable borrowers. Lending standards have negative externalities and are dynamic strategic complements: tighter screening worsens the future pool of borrowers for all banks and increases their incentives to screen in the future. Lending standards can amplify and prolong temporary downturns, affecting lending volume, credit spreads, and default rates. We characterize constrained-optimal policy which can generally be implemented as a government loan insurance program. When markets recover, they may do so only slowly, a phenomenon we call "slow thawing." Finally, we show that limits on lending such as from capital constraints naturally incentivize tight lending standards, further amplifying shocks to credit markets.

JEL codes: D82, G21, G01, G10

Keywords: Lending standards; Credit cycle; Strategic complementarity; Amplification; Persistence; Policy response

*We thank Marco Bassetto, Doug Diamond, Aaron Goodman, Brett Green, Kinda Hachem, Robert Marquez, Will Mullins, Rodolfo Rigato, Jeremy Stein, Andrew Winton and participants in seminars at the Federal Reserve Bank of Boston, Duke, MIT, UNC, UVA, the Minnesota Corporate Finance Conference, the 2019 Annual Meetings of the SED, the NBER 2019 Summer Institute (Corporate Finance and Capital Markets and the Economy Meetings), the 2019 Wharton Conference on Liquidity and Financial Fragility, and the Harvard Finance Reading Group. Ludwig Straub appreciates support from the Macro-Financial Modeling Group. Jiageng Liu provided excellent research assistance.

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First version: February 2019.

1 Introduction

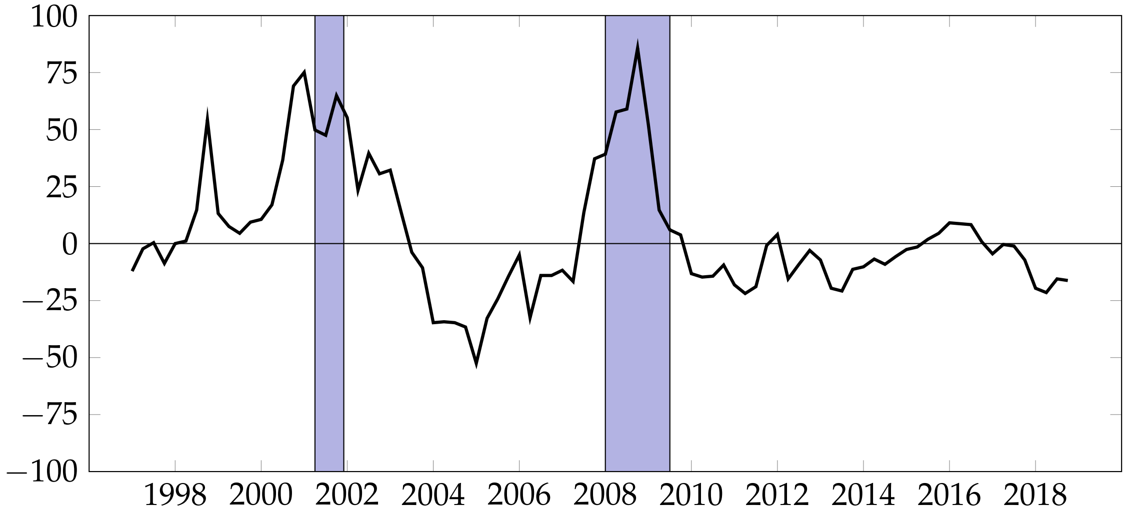
Through the allocation of external financing, lending standards play a key role in the economy, determining which entrepreneurs get initial funding, which firms grow, and which consumers buy houses for example. Figure 1 plots a measure of lending standards in the market for commercial and industrial (C&I) loans from banks. Lending standards, by this metric and others, are highly countercyclical, tightening in recessions and loosening in booms. For example, the recent credit boom-bust cycle was associated with relatively loose lending standards in the lending boom of the mid-2000's, when credit spreads and default rates were low, and relatively tight lending standards during the credit crunch and recession that followed, when spreads and default rates were high. Notably, the relaxation of lending standards following the crisis has been slow and limited.

We develop a dynamic model of a credit market in which lending standards are endogenous, both influencing and responding to economic circumstances. We define lending standards in a standard way as the extent to which banks acquire information about potential borrowers and condition lending on this information. We derive four main results. First, lending standards can lead to hysteresis: temporary changes in market fundamentals, such as the share of good borrowers approaching banks, can set in motion a self-reinforcing dynamic culminating in permanent differences in lending volumes, credit spreads, and default rates. Second, lending standards can be inefficiently tight, and it can be socially optimal to relax lending standards, such as through a government loan guarantee program. Third, temporary constraints that limit lending by banks, such as capital constraints, incentivize tight lending standards, and thus can inefficiently cause long-lasting tight lending standards. Finally, banks may temporarily limit lending and slow credit market recoveries when lending standards are loose and credit conditions are endogenously improving, a situation we call slow thawing.

Our credit market model consists of competitive banks and a pool of potential borrowers, who are identical conditional on public or readily-available information (e.g. conditional on credit score). Each instant, borrowers randomly have the opportunity to approach banks in search of a loan to fund an investment project. Projects differ by borrower type: projects of high-quality (low-quality) borrowers have positive (negative) net present value. To focus on the interesting dynamics of our model, we assume that new borrowers who flow into the pool have projects with positive expected net present value, meaning they are mostly good or that losses are small when they are bad.

Banks set lending standards by choosing whether to acquire costly private information about borrowers and to condition lending on this information. Information acquisition can

Figure 1: Change in Bank Lending Standards for C&I Loans



Note: Figure shows the shows the percent of banks reporting tightening less the share reporting loosening. Banks can also report no change in standards. *Source:* Senior Loan Officer Opinion Survey on Bank Lending Practices, Board of Governors of the Federal Reserve System (2018).

take many forms, for example an interview with the applicant, verification of reported employment or income, appraisal of collateral, or an analysis of a business plan. Importantly, in our model, the additional information collected is private and borrowers whose loan applications are declined at one bank may apply at another bank later. Thus, lending standards are *dynamic strategic complements*: tight lending standards today imply that banks are confronted with a more adversely-selected pool in the future, which raises their incentive to impose tight lending standards.

Our first main result is that credit markets exhibit hysteresis: markets with the same set of fundamentals may see persistently different lending volumes, credit spreads, default rates, and lending standards, depending on their specific history (i.e. their initial conditions). At any point in time, the equilibrium of our model is unique. However, as long as information acquisition is not too costly or too cheap, there are multiple steady states in the single state variable in our model—the share of high-quality potential borrowers in the pool, the *pool quality*. In the *pooling steady state,* the pool quality is high enough that banks do not find it worth the cost to check the quality of applicants just to avoid the occasional low-quality borrower. Thus, low-quality borrowers are funded along with high-quality borrowers, which keeps the pool quality high. In this steady state, the volume of lending is high and loan spreads are low. Conversely, in the *screening steady state,* the pool quality is low enough that banks choose to collect costly private information on borrowers and deny loans to low-quality borrowers which keeps the pool quality low. The volume of lending is low and loan spreads are high.

As a result of the multiplicity of steady states, temporary changes in market fundamentals, e.g. a worsening of borrowers' projects or of the composition of potential borrowers, can set in motion a self-reinforcing feedback loop between deteriorating pool quality and tighter lending standards, culminating in a *permanent* shift in the credit market equilibrium. This feature of our model is consistent with the limited relaxation of lending standards following the Great Recession documented in Figure 1.

Our second main result is that government intervention to relax lending standards can improve on private market outcomes, a result that follows from the fact that tight lending standards have *negative externalities.* Any bank that tightens its lending standard today increases the share of low-quality borrowers in the pool in the future which makes the credit market less efficient. As a result, the pooling steady state Pareto dominates the screening steady state. Further, solving a dynamic planning problem for the socially optimal lending standards, we show that socially optimal lending standards are weakly looser than privately optimal ones at all pool qualities. In our model with linear costs of information acquisition, this inequality is strict for intermediate levels of pool quality.[1]

Thus, in response to a temporary decline in the quality of new borrowers, the optimal policy response can be an intervention that ensures that banks do not tighten lending standards, consistent with the fact that governments often support high-quality credit markets in downturns. Because the pool of potential borrowers is a common resource, there is no way for individual banks to recover the short-term losses from later profits absent collective, i.e. government, actions. We show that such optimal interventions can be implemented by a loan guarantee programs funded by lump sum taxes which relax lending standards and increase lending volume. Finally our results have implications for the timing of interventions. Support for lending markets has short-run costs and long-run benefits. Thus delayed policy interventions are more costly than immediate interventions, and too much delay can make it optimal to not intervene at all.

While policies that target lending standards improve outcomes, they do not achieve the first-best. A first-best policy would eliminate the externality associated with screening by making any private information acquired by banks public. We discuss why credit bureaus commonly do not correct this externality.

Our third main result is that binding capital constraints naturally incentivize banks to

[1]There is also a region of low enough pool quality in which it is efficient to stop lending entirely. Because this type of lending standard does not have the externality associated with screening, the private threshold for this type of lending standard can be inefficiently high or low, depending on incidental modeling choices.

tighten lending standards, setting in motion declines in pool quality, which can ultimately lead to hysteresis and suboptimal market outcomes as just described. When capital constraints bind, the shadow value of banks' capital increases. Thus, banks have a greater incentive to screen potential borrowers in order to lend their limited capital to the most profitable borrowers. This implies that downturns that are accompanied by financial crises are more likely to lead to the emergence and persistence of tight lending standards or to benefit from government policies to relax lending standards.

Finally, our fourth main result is the possibility of an intermediate range of pool quality in which banks restrict lending by rationing credit instead of imposing tight information-based lending standards, a situation we refer to as *slow thawing*. The logic behind this credit rationing is quite different from the typical credit rationing due to adverse selection (Stiglitz and Weiss 1981, Mankiw 1986). During slow thawing, lending rates are falling sufficiently quickly that high-quality borrowers are indifferent between getting funded right away and waiting for the next funding opportunity. This indifference reduces the surplus from bank lending today, leading to credit rationing as some banks stop lending, and reducing the speed of improvement in lending volumes and credit spreads. Convergence to the pooling steady state is thus non-monotonic, and during the initial slow thawing period, the typical effects of many parameters on lending volumes and interest rates are reversed.

Related literature. In our analysis, lenders face an adversely selected pool of borrowers when lenders employ tight lending standards, as in the static models of Fishman and Parker (2015) and Bolton, Santos and Scheinkman (2016) in which the strategic complementarity leads to multiple equilibria. Ruckes (2004), Dell'Ariccia and Marquez (2006), and Hachem (2020) analyze static models of lending standards which do not feature a strategic complementarity in lending standards. In Ruckes (2004), lenders simultaneously acquire private information about borrowers and then simultaneously quote loan rates. In that setting, lending standards are strategic substitutes. In Dell'Ariccia and Marquez (2006) there is cream skimming by informed lenders but these lenders are endowed with their information. Hachem (2020) studies lending standards in a static model in which banks can also exert search effort to attract potential borrowers. When banks are resource constrained, banks put too much effort into searching for borrowers and too little effort into checking them upon arrival, so that lending standards are inefficiently loose.

As in our model, in both Hu (2018) and Farboodi and Kondor (2020) banks choose lending standards and the quality of the borrower pool evolves over time. In these papers, tighter lending standards raise the average quality of newly-entering borrowers and, in

Farboodi and Kondor (2020) rejected borrowers go bankrupt and leave the pool of potential borrowers, which imply that lending standards are dynamic strategic substitutes. We discuss the endogenous determination of the quality of new borrowers in Subsection 7.1.[2] In Hu (2018) economic recoveries can have interesting dynamics such as double-dip recoveries, while Farboodi and Kondor (2020) shows that credit markets can exhibit endogenous cycles in lending standards and borrower quality.

A number of papers study dynamic adverse selection models without information acquisition. Daley and Green (2012, 2016) and Malherbe (2014) analyze models where current markets can break down when high-quality sellers have the incentive to wait for market prices to improve over time as the composition of sellers improves over time. In contrast during slow thawing in our model, the equilibrium *composition* of borrowers does not change, only the *speed* of lending is reduced.[3]

Finally, Our paper is also related to information acquisition and adverse selection in secondary markets. Zou (2019) analyzes a dynamic model of trade in which an agent's incentive to collect information is higher if agents in the future are expected to collect information. In Asriyan, Fuchs and Green (2017), if future market liquidity and hence prices are expected to be high (low), then prices today are high (low) and the adverse selection problem will be less (more) severe. Gorton and Pennacchi (1990), DeMarzo and Duffie (1999), and Dang, Gorton and Holmstrom (2015), among others, analyze how debt securities minimize adverse selection problems in secondary markets. While the issues of information acquisition are similar, our model is designed to address adverse selection at origination (in primary markets) and largely abstracts from issues of security design.

2 A Model of Lending Standards

Time is continuous and runs from t to infinity, $t \in [0, \infty)$. There are two sets of agents: a unit mass of potential borrowers who have no capital and are looking to fund projects and a large mass $\mathcal{J}$ of competitive banks. All agents are risk neutral with discount rate $\rho > 0$. The main state variable in the model is the quality composition of the pool of borrowers,

[2] It turns out that in our model, it is theoretically ambiguous whether the endogenous composition of new borrowers mitigates or amplifies our mechanism. The reason for this is that tighter lending standards can lower the incentive for high-quality borrowers to enter as tighter standards go hand in hand with higher lending rates, which disproportionately discourage high-quality borrowers. See Subsection 7.1 for details.

[3] Related, Zryumov (2015) and Caramp (2017) study models where bad sellers strategically enter when market prices are good. This, in and of itself, does not lead to a market shutdown (lower prices positively select entrants), but as Caramp (2017) emphasizes, the bigger presence of bad sellers can raise the likelihood of adverse selection induced market failures in the future.

defined below and denoted by x_t, which both determines and is influenced by the main control variables, banks' lending standards, denoted by z_{jt}.

Borrowers and banks

Borrowers. At Poisson rate $\kappa > 0$, a potential borrower receives an investment opportunity, a project that requires an up-front investment of 1. Borrowers have no capital and must fund the investment externally. If the borrower raises the funds and makes the up-front investment at time t, then the project returns both a pledgeable cash flow at time $t + T$ and a non-pledgeable private benefit $u > 0$ (in present value) to the borrower. Given this private benefit, all borrowers have the incentive to finance their project, even if they know they will receive no monetary benefit.

There are two types of borrowers: type H ("high quality") and type-L ("low quality"). Type-H borrowers always have investment opportunities with positive net present value (NPV). The pledgeable cash flow of a type-H borrower's project is D_H, with excess return $r_H \equiv e^{-\rho T} D_H - 1 > 0$.[4] Type-$L$ borrowers always have projects with negative net present value. Their pledgeable cash flow is D_L, and their excess return is $r_L \equiv e^{-\rho T} D_L - 1 < 0$. A borrower's type is permanent, always type H or always type-L. We refer to $r^\Delta \equiv \frac{r_H - r_L}{-r_L} > 0$ as the (normalized) *return difference* between the investments opportunities of the two types. It captures how much greater the return on a type H project compared to a type-L project is, relative to the negative return on type-L projects.

When an investment opportunity arises, borrowers choose whether to apply to the competitive banking sector for a unit of funding to implement their project. A borrower who applies for funding is approved or denied depending on whether she satisfies the bank's lending standards. If a borrower is funded, she invests in her project and exits the pool to run the project. Alternatively, if the borrower does not apply for funding or is denied funding, she returns to the pool where at rate $\kappa > 0$ a new investment opportunity arises.Besides exiting the pool if funded, at Poisson rate $\delta > 0$ borrowers exit the pool by "dying," i.e. no longer receiving investment opportunities.

Borrowers who exit the pool because they are funded or because they die are immediately replaced by new borrowers who enter the pool as type H with exogenous probability λ and as type-L with probability $1 - \lambda$. Our exit/entry assumption implies that the borrower pool size is constant at 1. Partly as a result of this assumption, it will suffice to keep track of the fraction of type-H borrowers in the pool of potential borrowers at time t,

[4] It is an excess return because the per-period log return is $\frac{1}{T}\ln(1 + r_H) = \frac{1}{T}\ln(D_H) - \rho$.

$x_t \in [0,1]$, which helps make our main results more tractable and transparent.[5]

We assume throughout that the average project of a borrower entering the pool has positive net present value based on the pledgeable cash flow.

Assumption 1. *The average investment project has a positive excess return* $\lambda r_H + (1-\lambda) r_L > 0$.

While we have not explicitly modeled collateral, we can interpret the loan as a collateralized loan and u as the private benefit net of the loss of collateral (e.g. u is the net benefit of purchasing and living in a house until foreclosure; see Section 7.2).[6]

Banks and lending standards. Banks make two decisions. First, they decide whether to be *active* or *inactive*. Second, conditional on being active, they choose their *lending standard*, that is, how aggressively to screen potential borrowers.

At any instant t, a bank may choose to be *active*, in which case it enters a competitive lending market, where it may receive a loan application by a borrower. Alternatively a bank may choose to be inactive in which case it makes no loans and consequently receives no loan applications. Let θ_{jt} denote the probability that bank j is active at time t. While generally all banks are active in our model, e.g., at all steady-states where x is constant, there may be a region in the state space with equilibrium *credit rationing*, $\theta_{jt} < 1$, where banks offer fewer loans than borrowers demand.

An active bank j chooses a *lending standard* $z_{jt} \in [0, \bar{z}]$, where $\bar{z} \in (0,1]$ is a fixed parameter. With lending standard z_{jt}, a type-L borrower is identified as such with probability z_{jt}, in which case her loan is denied. Otherwise, the borrower's loan is approved.[7] A bank's cost of utilizing the lending standard z_{jt} is $\tilde{c} z_{jt}$, where $c \equiv \frac{\tilde{c}}{-r_L} > 0$ is the (normalized) marginal cost. The most lax lending standard corresponds to $z_{jt} = 0$, in which case all loan applications are deemed to meet bank j's lending standard. Banks choose lending standards to maximize expected profit. Given a lending standard z_{jt}, banks offer to lend 1 in exchange for a promised loan payment at time $t+T$ equal to D_{jt}.

Due to symmetry and competition, it is without loss of generality to assume that all banks choose the same probability of being active, θ_t, the same lending standard z_t, and the same required loan payment D_t, paid at $t+T$. With a loan face value of D_t, repayment

[5] In Appendix B we prove that our main insights carry over to an environment with a constant inflow rather than a constant pool size. The only exception are the results in Section 3.3 on slow thawing, which are less tractable to study in a constant-inflow setting.

[6] With minor modifications, one can also re-interpret the model as capturing a secondary market in which "borrowers" are selling assets of unknown value in order to raise funds to make an investment.

[7] A type-H borrower is never misidentified as a type-L, a modeling assumption only important for tractability, as discussed in Section 7.4.

is $\min\{D_t, D\}$, where D is the payoff on the investment, D_L or D_H depending on borrower type. Since type-L borrowers have negative NPV investments, $D_t > D_L$ for a bank to break even in expectation. Thus, type-L borrowers always default. The repayment D_t is without loss of generality bounded above by D_H since any higher D_t will not generate additional repayment. So if a loan is made (meaning the bank can break even) then type-H borrowers will not default. We define $r_t \equiv e^{-\rho T} D_t - 1$ as the *credit spread* charged by the bank since $\rho + \frac{1}{T} \ln(1 + r_t)$ is per-period (log) return on a loan that does not default. We note that r_t always lies in (r_L, r_H).

Information structure. Borrowers have no private information about their type when they enter the pool. And so long as a borrower has no private information, we call her an *average borrower*. Some borrowers learn that they are type-L after failing to meet a bank's lending standard. We call these borrowers *rejected borrowers*.[8] Whether a borrower is average or rejected, a bank cannot distinguish between a type-H and a type-L borrower unless it is probabilistically revealed by paying $z\tilde{c}$.[9] The shares of average and rejected borrowers are endogenously determined. For instance, the lower are past lending standards, the fewer rejected borrowers will be in the pool. All agents have common knowledge of the structural parameters of the market and the initial fraction of type-H borrowers in the pool, $x_0 \in [0, \lambda]$.[10] All agents can infer past, current, and future x_t.

A borrower's problem

Taking the path of credit spreads $\{r_t\}$ as given, borrowers with investment opportunities choose whether to apply for a loan at each time t. Let φ_t^a denote the probability that an average borrower with an investment opportunity applies for a loan—as opposed to waiting in hope of an improvement in borrowing opportunities. Let φ_t^r denote the probability that a rejected borrower with an investment opportunity applies for a loan. Letting J_t^a and J_t^r denote the value functions of an average borrower and a rejected borrower, respectively, the optimal strategies for the two satisfy the following Hamilton-Jacobi-

[8] Thus there are three types of borrowers in the pool at any time: average borrowers who are actually type H, average borrowers who are actually type-L, and rejected borrowers (all type-L). We show however that optimal behavior depends only on the share of type-H borrowers, x_t, because the behavior of all type-L borrowers is the same.

[9] For each individual bank, previously screened loan applicants will represent a zero mass in the pool of borrowers and can therefore be ignored.

[10] Here, $x_0 = \lambda$ corresponds to a pool consisting entirely of average borrowers.

Bellman equations:

$$\rho J_t^a = \max_{\varphi_t^a \in [0,1]} \kappa \theta_t \varphi_t^a \left\{ \lambda \left(r_H - r_t + u \right) + (1-\lambda)(1-z_t)u + (1-\lambda) z_t J_t^r - J_t^a \right\} + \dot{J}_t^a - \delta J_t^a \tag{1a}$$

$$\rho J_t^r = \max_{\varphi_t^r \in [0,1]} \kappa \theta_t \varphi_t^r \{ (1-z_t)(u - J_t^r) \} + \dot{J}_t^r - \delta J_t^r, \tag{1b}$$

where J_t^a and J_t^r satisfy the transversality conditions $\lim_{t\to\infty} e^{-(\rho+\delta)t} J_t^a = \lim_{t\to\infty} e^{-(\rho+\delta)t} J_t^r = 0$. For an average borrower, (1a) reflects three possible outcomes when she has an investment opportunity, is matched with an active bank, and applies for financing: with probability λ she is type H and is funded, receiving a monetary payoff of $r_H - r_t$ and u in private benefits; with probability $(1-\lambda)(1-z_t)$, she is type-L but satisfies the lending standard, receiving a payoff of u in private benefits; and with probability $(1-\lambda)z_t$, she is type-L and does not satisfy the lending standard (is rejected), receiving a payoff of J_t^r. For a rejected borrower who has an investment opportunity, is matched with an active bank, and applies for a loan, (1b) reflects the fact that with probability $1 - z_t$, she satisfies the lending standard and receives a payoff of u in private benefits; otherwise, she continues as a rejected borrower.

With strategies $\{\varphi_t^a, \varphi_t^r\}$, there is a flow of

$$\kappa_{Ht} \equiv \kappa \varphi_t^a x_t \tag{2}$$

type-H borrowers applying for loans. Note that all of the type-H borrowers belong to the sub-pool of average borrowers. There is a flow of

$$\kappa_{Lt} \equiv \kappa \varphi_t^a \frac{1-\lambda}{\lambda} x_t + \kappa \varphi_t^r \frac{\lambda - x_t}{\lambda} \tag{3}$$

type-L borrowers applying for loans. For the derivation of (3), let A_t denote the share of average borrowers at time t, with $1 - A_t$ being the share of rejected borrowers at time t. The fraction of type-H borrowers in the whole pool is $x_t = A_t \lambda$. The flow of type-L borrowers equals $\kappa \varphi_t^a A(1-\lambda) + \kappa \varphi_t^r (1-A)$. Substituting in $A_t = x_t/\lambda$ yields (3). In equilibrium, it will be the case that $\varphi_t^a = \varphi_t^r = 1$, so that $\kappa_{Ht} = \kappa x_t$ and $\kappa_{Lt} = \kappa(1-x_t)$.

A bank's problem

With a flow $\kappa_{Ht} + \kappa_{Lt}$ of loan applications at time t, it is without loss to assume that there are at most a flow of $\kappa_{Ht} + \kappa_{Lt}$ active banks at time t. As will be seen below, there are cases where some banks remain inactive in equilibrium, leaving only $\theta_t\left(\kappa_{Ht}+\kappa_{Lt}\right)$ active banks, with $\theta_t \in [0,1]$. A fraction θ_t of the flow $\kappa_{Ht}+\kappa_{Lt}$ of loan applications is then received by the $\theta_t(\kappa_{Ht}+\kappa_{Lt})$ active banks.

Conditional on flows κ_{Ht}, κ_{Lt} and credit spread r_t, an active bank's lending standard z solves

$$\Pi_t(r_t) \equiv \max_{z\in[0,\bar{z}]} \kappa_{Ht} r_t + \kappa_{Lt}(1-z) r_L - (\kappa_{Ht}+\kappa_{Lt})\tilde{c}z. \tag{4}$$

as type-H borrowers and a fraction $1-z$ of type-L borrowers are funded. Taking z as given, Bertrand competition among banks then determines r_t by

$$\Pi_t(r_t) = 0. \tag{5}$$

Whenever (5) cannot be satisfied by any r_t, no bank will find it profitable to lend. In this case, we set $\theta_t = 0$ (and $r_t = r_H$).

Evolution of the borrower pool

The evolution of the fraction of type-H borrowers in the pool is given by

$$\dot{x}_t = \theta_t \kappa_{Lt}(1-z_t)\lambda - \theta_t \kappa_{Ht}(1-\lambda) + \delta(\lambda - x_t), \tag{6}$$

which is the combination of three distinct forces: the first term accounts for the $\theta_t\kappa_{Lt}(1-z_t)$ type-L borrowers who are funded and leave the pool (improving pool quality); the second term accounts for the $\theta_t\kappa_{Ht}$ type-H borrowers who are funded and leave the pool (reducing pool quality); and the third term accounts for the birth and death of borrowers (moving pool quality towards λ).

Equilibrium

We define an equilibrium as follows:

Definition 1. Given an initial share of type-H borrowers $x_0 \in [0,1]$ in the pool, an *equilibrium* consists of a path of the fraction of type-H borrowers $\{x_t\}$, credit spreads $\{r_t\}$, shares of active banks $\{\theta_t\}$, borrowers' application decisions $\{\varphi_t^a, \varphi_t^r\}$, implied application flows of type-H and type-L borrowers $\{\kappa_{Ht}, \kappa_{Lt}\}$, and screening choices $\{z_t\}$ such that

- $\{\varphi_t^a, \varphi_t^r\}$ solve each type's maximization problem (1) given $\{r_t, z_t, \theta_t\}$,
- $\{\kappa_{Ht}, \kappa_{Lt}\}$ are determined by (2) and (3),

- z_t solves the bank's maximization problem (4) given $\{r_t, \kappa_{Ht}, \kappa_{Lt}\}$,
- r_t is determined by the zero profit condition for banks (5) given κ_{Ht}, κ_{Lt} whenever possible; if not, $\kappa_{Ht} = 0$ (and w.o.l.o.g. $r_t = r_H$),
- $\{x_t\}$ follows the law of motion (6),
- at no time t can a bank raise its profit Π_t by being active, charging a rate $\tilde{r} < r_t$ that average borrowers would weakly prefer over waiting, and lending to the entire set of borrower applicants (a flow of κ_{Ht} type-H and κ_{Lt} type-L borrowers; see 2 and 3).

A *steady state (equilibrium)* is an equilibrium in which all equilibrium objects $\{x_t, r_t, \theta_t, \varphi_t^a, \varphi_t^r, z_t\}$ are constant over time.

To study variation in lending standards, we make the following parameter assumptions:

Assumption 2. *The cost of bank screening c is not too low or too high:*

$$1 - \lambda < c < 1 - x^s + \bar{z}^{-1} \min\left\{x^s r^\Delta - 1, 0\right\},$$

where $x^s = \lambda - \lambda \frac{(1-\lambda)\bar{z}}{(1-\lambda\bar{z})+\delta\kappa^{-1}}$.

The first inequality in Assumption 2 ensures that the screening cost c is high enough that the lending standard $z = \bar{z}$ does not strictly dominate $z = 0$. The second inequality ensures that c is not so high as to rule out a steady state with $z = \bar{z}$.

Given a linear screening cost, the choice of lending standards will be at a corner, $z = 0$ or $z = \bar{z}$. As we will show, the tendency will be for banks to choose too high of lending standards, $z = \bar{z}$ instead of $z = 0$. Henceforth we will refer to $z = 0$ as a "normal" lending standard and $z = \bar{z}$ as a "tight" lending standard.

3 Equilibrium characterization

The model's tractability allows for an analytical characterization of the set of equilibria, starting with steady-state equilibria.

3.1 Steady-state equilibria

Borrowers, facing the same interest rate r and lending standard z at all times, have no incentive to wait and therefore choose $\varphi^a = \varphi^r = 1$.[11] Rejected borrowers never have an

[11] Since prices and quantities are constant, we drop the time subscripts for this subsection.

incentive to wait. Average borrowers also strictly prefer borrowing to waiting,

$$\lambda (r_H - r + u) + (1-\lambda)(1-z)u + (1-\lambda)zJ^r - J^a > 0, \tag{7}$$

because, for any bank to be active, the loan rate needs to be weakly below the highest pledgeable payoff, $r \leq r_H$. In that case, an average borrower's continuation value is strictly positive, $J^a > 0$, as they cannot lose any money upon receiving funding and always receive the private benefit. Using (1a), $J^a > 0$ is equivalent to (7) in a steady state. Under these conditions, all banks are active in a steady state, $\theta = 1$.

The steady-state quality of the pool x and the steady-state lending standard z are jointly determined by the interaction of two forces. On the one hand, the law of motion of x, (6), implies that when $\dot{x} = 0$,

$$x = \lambda - \lambda \frac{(1-\lambda)\, z}{(1-\lambda z) + \delta\kappa^{-1}}. \tag{8}$$

This equation highlights that tighter lending standards—higher z—are associated with a lower steady-state quality of the pool of borrowers x, as more low-quality borrowers are rejected by banks. This effect is greater when the effects of lending standards on the pool are more persistent (low death rate δ) or when opportunities to invest arise more frequently (high κ) and so potential investors are evaluated more frequently.

On the other hand, banks solve (4) and choose tighter lending standards precisely when the pool is more adversely selected,

$$z = \begin{cases} 0 & \text{if } x > \overline{x} \\ [0, \overline{z}] & \text{if } x = \overline{x} \\ \overline{z} & \text{if } x < \overline{x} \end{cases}, \quad \text{where } \overline{x} \equiv 1 - c. \tag{9}$$

The combination of the two equations (8) and (9) is illustrated in Figure 2. Both represent downward-sloping relationships between x and z, and given Assumption 2 admit three intersections, each of which represents a steady-state equilibrium. This logic is summarized in the following proposition.

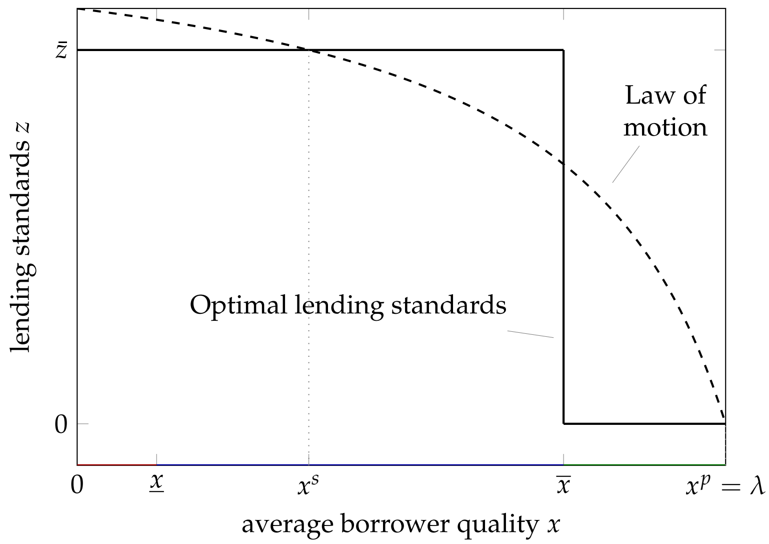
Proposition 1 (Steady state equilibria)**.** *There exist three steady-state equilibria:*

(i) A pooling *steady state with normal lending standards $z = 0$ and $x = x^p \equiv \lambda$.*

(ii) A screening *steady state with tight lending standards $z = \overline{z}$ and $x = x^s \equiv \lambda - \lambda \frac{(1-\lambda)\overline{z}}{(1-\lambda\overline{z}) + \delta\kappa^{-1}}$.*

(iii) A mixed *steady state with $z = \frac{\lambda - \overline{x}}{\lambda - \lambda\overline{x}}\left(1 + \delta\kappa^{-1}\right) \in (0, \overline{z})$ and $x = \overline{x}$.*

Figure 2: The two forces shaping steady-state equilibria.



Note: This figure shows two curves whose intersections yield the steady-state pool quality x and the steady-state lending standard z. The solid line represents the optimal choice of the lending standard, (9). The dashed line represents the pool quality x that is caused by any given lending standard z through the law of motion.

The root of the multiplicity is a *dynamic strategic complementarity* among banks. By (9), banks naturally respond to a lower quality pool by tightening their lending standards; however, according to (8), tighter lending standards worsen the pool itself, creating an even bigger incentive for banks to tighten their standards in the future. This reasoning rationalizes the existence of the pooling and screening equilibria, see Figure 2. The mixed steady state formally exists but will turn out to be unstable and therefore play no role in the remainder of the analysis.

The pooling and screening steady states have the following important characteristics.

Corollary 1 (Quality of funded borrowers)**.** *Compared to the screening steady state, in the pooling steady state:*

1. *the credit spread r is lower,* $r(x^p) = -\frac{r_L}{x^p}(1 - x^p) < -\frac{r_L}{x^s}\{c\bar{z} + (1 - \bar{z})(1 - x^s)\} = r(x^s)$
2. *more projects are funded,* $\kappa > \kappa x + \kappa(1 - x)(1 - \bar{z})$
3. *the default rate is higher,* $1 - x^p > \frac{(1-x^s)(1-\bar{z})}{x^s+(1-x^s)(1-\bar{z})}$

The first point follows from the fact that a lower pool quality, ceteris paribus, hurts banks' profits, and therefore requires larger credit spreads for banks to break even. This is true even though banks choose tight lending standards.[12] The second point follows from

[12]Proposition 3 has a complete characterization of interest rate spreads. .

the fact that screening reduces the flow of borrowers that receive funding. The third point follows from the fact that tight lending standards improve the quality of funded borrowers. But there is a subtlety here. The result relies on rejected borrowers "dying", that is, exiting the pool at a rate $\delta > 0$. If instead $\delta = 0$, that is, no death and rebirth in the pool of potential borrowers, then the default rate would equal $1 - \lambda$ in any steady state, irrespective of the screening decision. In that case, the imposition of tight lending standards in the screening steady state exactly balances the low average project quality in the pool, yielding the same default rate.[13]

These patterns match the stylized facts about credit booms and busts we discussed in the Introduction (see e.g. Greenwood and Hanson 2013): booms feature higher lending volumes, looser lending standards, lower quality of funded borrowers, and lower credit spreads conditional on default probability.

3.2 Transitional dynamics

An important factor that simplifies the steady state analysis is that banks are always active in a steady state, $\theta = 1$. This is no longer true in equilibria with dynamics. In particular, there are now two regions in which banks may choose to remain inactive. Naturally, this is the case when the pool quality x is very low, so that even the maximum loan rate does not make profits for a bank. Thus, $\Pi(r_H) = 0$ defines an $\underline{x}$ such that

$$\theta(x) = \begin{cases} 0 & \text{if } x < \underline{x} \\ [0,1] & \text{if } x = \underline{x} \end{cases}, \quad \text{where } \underline{x} \equiv \frac{1 - \bar{z} + c\bar{z}}{r^{\Delta} - \bar{z}}. \tag{10}$$

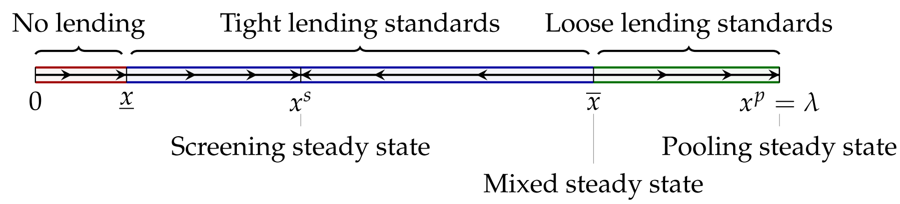
More surprising, banks may also remain inactive and so reduce total lending for a range of x just above $\bar{x}$ when lending standards are otherwise normal. Since in this region the speed of convergence to the pooling steady state is slow and increasing, we refer to this region as the "slow-thawing" region and it is described in detail in Section 3.3. Until then, we assume parameters are such that there is no such region:[14]

Assumption 3 (No slow thawing). *Assume that there is no slow-thawing region, that is,* $\theta(x) = 1$ *for all* $x \geq \underline{x}$.

[13]The results in Corollary 1 are robust to alternative assumptions on the dynamics of the borrower pool, e.g. assuming a constant inflow, rather than a constant pool size. See Appendix B.

[14]For the sake of exposition, this assumption is stated in terms of endogenous objects. The analytical condition is stated in the next section.

Figure 3: State space and banks' optimal strategies.



Under Assumption 3, Proposition 2 completely characterizes the equilibrium transitional dynamics of x. Despite the multiplicity of steady states (Proposition 1), there is a unique equilibrium for any x_0, giving unambiguous model predictions.

Proposition 2 (Transitional dynamics without slow thawing)**.** *Suppose Assumption 3 holds and $x_0 \in [0, \lambda]$ is the initial fraction of type-H borrowers in the pool. There is a unique equilibrium, in which banks' activity policy satisfies* (10) *for $x \leq \underline{x}$, their lending standards are given by* (9)*, and borrowers never wait, $\varphi_t^a = \varphi_t^r = 1$. As $t \to \infty$, the credit market converges to*

(i) the screening steady state, $x_t \to x^s$, if $x_0 < \overline{x}$.

(ii) the mixed steady state, $x_t \to \overline{x}$, if $x_0 = \overline{x}$.

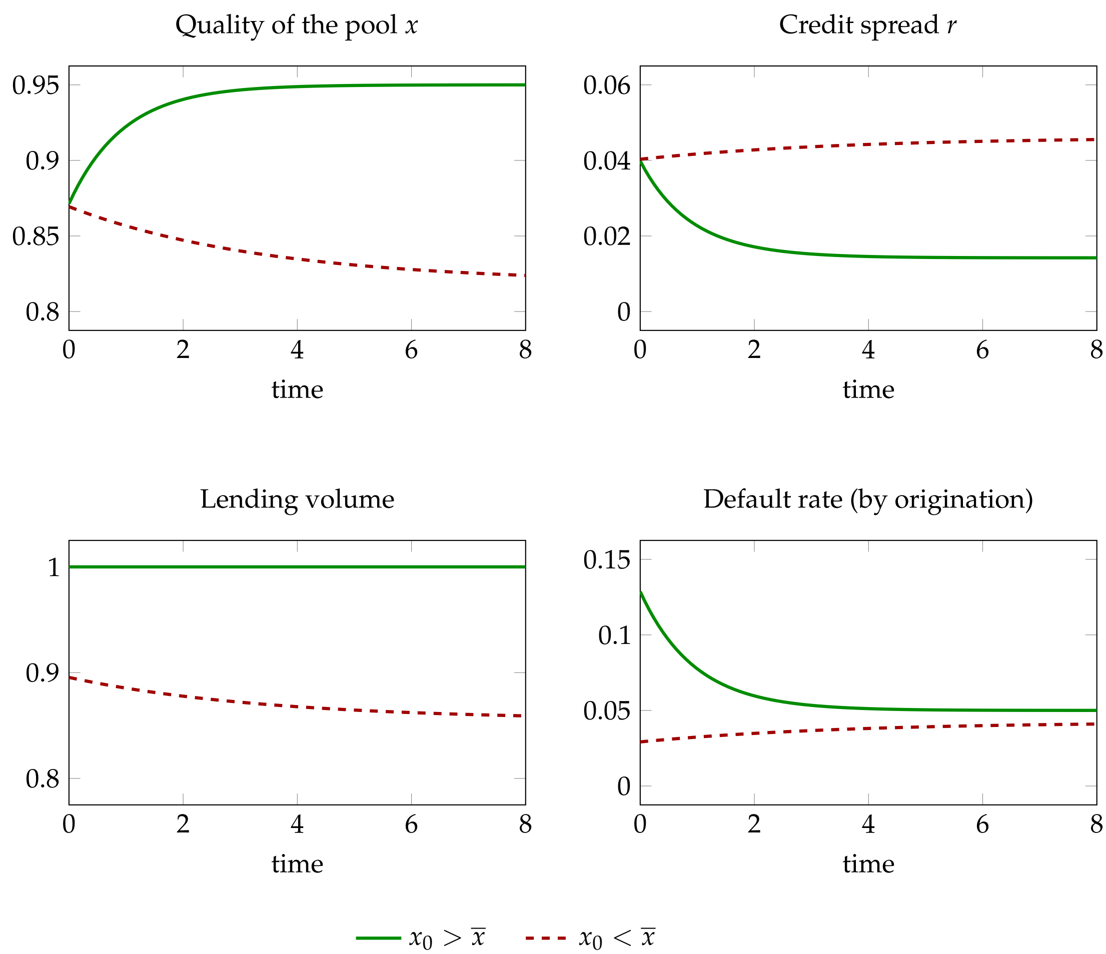
(iii) the pooling steady state, $x_t \to x^p$, if $x_0 > \overline{x}$.

Figure 3 illustrates the state space of the credit market and highlights the transitional dynamics in the three different regions of bank behavior: the "no lending" region for low pool qualities, where banks are inactive ($\theta_t = 0$) and the pool quality improves only due to death and birth; the "tight lending standards" region, where banks screen borrowers $z_t = \overline{z}$ and the market approaches the screening steady state; and the "normal lending standards" region where banks choose $z_t = 0$ and the market returns to the pooling steady state.

A crucial part of the diagram is at $x = \overline{x}$. This point represents a sharp boundary between the tight and normal lending standards regions and gives rise to an important model prediction, a "bifurcation" property: when x_0 lies above $\overline{x}$, the credit market converges to the pooling steady state with normal lending standards; and when x_0 lies below $\overline{x}$, the self-reinforcing nature of tight lending standards pushes the market to the screening steady state.

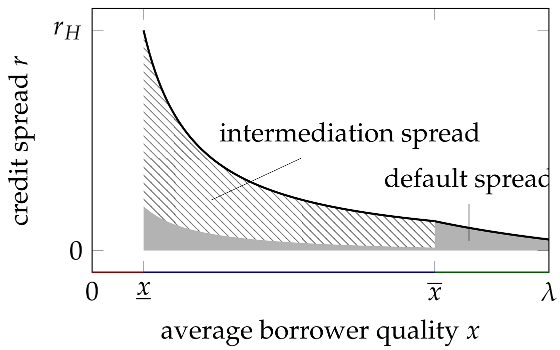
The bifurcation property also comes out in Figure 4 where we simulate the credit market with two different initial values for x_0, one just above $\overline{x}$ (green, solid) and one just below $\overline{x}$ (red, dashed). As can be seen, a small difference in initial conditions leads to quite different

Figure 4: The self-reinforcing property of lending standards.



Note. This figure shows two sets of transitional dynamics in a credit market without slow thawing. Green and solid is a market starting at $x_0 = \overline{x} + \epsilon$ and therefore banks have normal lending standards; red and dashed is a market starting at $x_0 = \overline{x} - \epsilon$ and therefore banks impose tight lending standards. The parameters used for this simulation are as follows: $\rho = \delta = 0.05$, $\lambda = 0.95$, $r_L = -0.27$, $r_H = 0.30$, $\tilde{c} = 0.035$, $\kappa = 1$, $\overline{z} = 0.8$.

Figure 5: Break-even credit spread as function of pool quality x.



Note: Grey is the component of the credit spread that is due to default risk (the *default spread*). Hatched is the component of the credit spread that is due to intermediation costs (the *intermediation spread*).

evolutions of pool qualities x, credit spreads r, and lending volumes $\kappa_{Ht} + \kappa_{Lt}(1-z)$. The final panel of Figure 4 shows the evolution of default rates, $\kappa_{Lt}(1-z)/\left(\kappa_{Ht} + \kappa_{Lt}(1-z)\right)$, by lending cohorts. The market with the slightly *lower* initial pool quality initially has a much lower lending volume and default rate, as banks are imposing tight lending standards. Interestingly, the two markets initially have similar credit spreads. Over time, and foreshadowing our results on efficiency and optimal policy, the slightly lower initial pool quality causes convergence to a steady-state with much higher credit spreads and lower lending volume but similar default rates (see our discussion of Corollary 1).

We can characterize equilibrium credit spreads r_t across credit markets. In particular, how do credit spreads vary with pool quality x_t? Markets with higher x_t have lower default rates for any given lending standard, suggesting lower credit spreads. But markets with higher x_t have normal lending standards, suggesting higher default rates and higher credit spreads. As the following proposition shows, the former effect dominates and r_t is inversely related to x_t.

Proposition 3 (Equlibrium credit spread)**.** *The equilibrium credit spread $r_t = r(x_t)$ is decreasing in the fraction of type-H borrowers, x, and is given by*

$$r_t = r(x_t) = \begin{cases} \infty & \text{if } x_t < \underline{x} \\ (-r_L)x_t^{-1}\left\{c\overline{z} + (1-\overline{z})(1-x_t)\right\} & \text{if } \underline{x} \le x_t < \overline{x} \\ (-r_L)x_t^{-1}\left\{1-x_t\right\} & \text{if } x_t \ge \overline{x} \end{cases}. \tag{11}$$

Using (11), we can decompose $r(x)$ into a *default spread*, $-r_L x^{-1}(1-z(x))(1-x) > 0$ where $z(x)$ is the optimal screening choice given x; and into an *intermediation spread*

$-r_L x^{-1} cz(x) > 0$. Figure 5 plots the credit spread $r(x)$ and these two components over the state space. The shaded areas in Figure 5 highlight that the default spread changes discretely at $x = \overline{x}$ as banks switch between tight and normal lending standards, but this change is offset by an equally large change in the intermediation spread. The spread rises significantly due to intermediation costs at lower pool qualities x. The decoupling of credit spreads and credit risk in this region of the state space provides a rationale for why, at times, credit spreads may appear to be high given the credit risk. He and Milbradt (2014) attribute such high credit spreads to low liquidity. Alternatively one might rely on risk aversion as an explanation. Here the high credit spread derives from intermediation costs.

The monotonicity of $r(x)$ is also reflected in Figure 4, with a rising loan rate for the credit market with the lower quality of potential borrowers and a falling loan rate for the market with higher quality. A falling loan rate raises a question: would average borrowers have an incentive to wait for lower loan rates? The answer is yes in certain cases. There may be a reduced demand for credit even with normal lending standards. In this case, lending volume and the improvement of the borrower pool are slowed, and credit markets recover - “thaw” - much more slowly than otherwise.

3.3 Slow thawing

Say x_0 is just above $\overline{x}$ and conjecture that all banks are active, $\theta_t = 1$, and average borrowers do not wait to apply for loans, $\varphi_t^a = 1$. Then it is possible that the resulting increase in pool quality, x_t, over time would lead to so rapid a decline in r_t that average borrowers would prefer not to borrow as conjectured. Rather they would wait for lower credit spreads before applying for credit $\varphi_t^a = 0$. Hence, in this case, our conjecture is not an equilibrium. Instead, equilibrium must exhibit a slower speed of transition. The improvement in the borrower pool and the decline in borrowing rates must occur more slowly to induce average borrowers to apply for loans in equilibrium. For this transition to be slower, it must be that not all banks are active, $\theta_t < 1$, which can only be the case if there are no profits to be made from making a new loan (see Definition 1). This is precisely the case when borrowers are also indifferent between waiting and applying for loans.

The following proposition establishes that these strategies are indeed an equilibrium. To keep the derivations and exposition clear, we focus on the case where the private benefit from running the project, u, is vanishingly small, $u \to 0$ (in which case $J_t^r \to 0$).

Proposition 4 (Slow thawing)**.** *There exists a threshold $\hat{x} \in (0, x^p)$, such that: (i) if $\hat{x} \leq \overline{x}$, there is no slow thawing region; if (ii) $\hat{x} > \overline{x}$, then for any $x \in [\overline{x}, \hat{x})$, a positive fraction of banks are* *inactive*

$$\theta(x) = \frac{(\rho+\delta)\left(r_H - r(x)\right)}{-\kappa r'(x)(\lambda - x)} - \delta\kappa^{-1} < 1 \tag{12}$$

where $r(x) = -r_L x^{-1}\{1-x\} > 0$*. Borrowers are indifferent and apply for loans,* $\varphi_t^a = \varphi_t^r = 1$*, and* $\hat{x}$ *is determined as the unique solution to* $\theta(\hat{x}) = 1$ *in* $(0, x^p)$*.*

The intuition for the expression in (12) comes directly from the indifference condition of average borrowers. The HJB of an average borrower is given by

$$\rho J_t^a = \max_{\varphi_t^a \in [0,1]} \kappa\theta_t\varphi_t^a \left\{\lambda\left(r_H - r_t\right) - J^a\right\} + \dot{J}^a - \delta J^a$$

with indifference between applying for a loan or not requiring that $J^a(x) = \lambda\left(r_H - r(x)\right)$. Substituting this back into the HJB yields an equation for the speed $\dot{x}$ at which the pool needs to improve for average borrowers to be indifferent,

$$\underbrace{-\lambda r'(x)\dot{x}}_{\text{benefit of waiting}} = \underbrace{(\rho+\delta)\lambda\left(r_H - r(x)\right)}_{\text{opportunity cost of waiting}}. \tag{13}$$

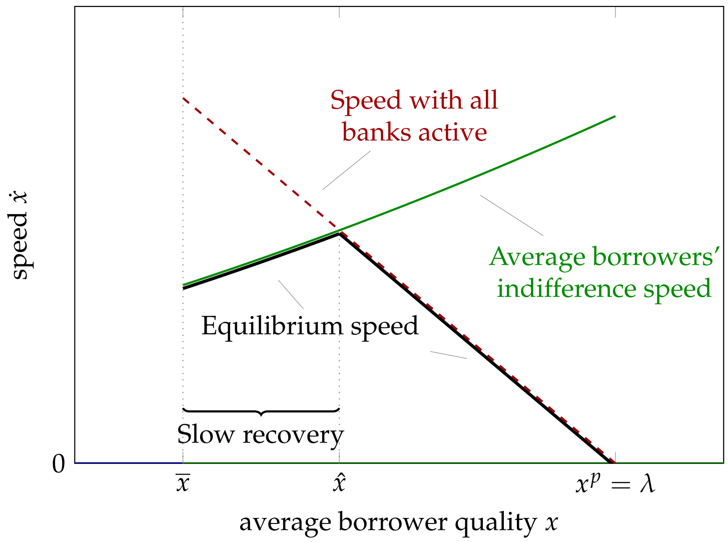
When is $\dot{x}$ the equilibrium speed? Precisely when θ_t is such that $\dot{x}$ satisfies the law of motion of x, (6). Together, (13) and (6) give (12).

Figure 6 schematically illustrates this logic. The green solid line represents the speed $\dot{x}$ at which average borrowers are indifferent between borrowing now and waiting for the pool to improve. This is an increasing line as the benefit of waiting declines the closer x is to the pooling steady state. The red dashed line represents the speed at which the pool quality improves when all banks choose to be active. Clearly, where this line lies below the green solid line of indifference, it is also equal to the equilibrium speed, shown in black thick solid line. However, for $x < \hat{x}$, the growth of $\dot{x}$ with all banks active lies above the solid green indifference curve so that borrowers would prefer to wait which cannot be an equilibrium. In this region, a fraction $1 - \theta(x)$ of banks choose to be inactive, bringing down the equilibrium speed to match the one along the green solid indifference curve. This leads to a hump-shaped thawing speed: initially little lending due to the threat of average borrowers waiting, a period of slow thawing as lending volume and the pool quality accelerate, followed by a period of normal convergence to the steady state.[15]

Slow thawing is thus a mechanism by which lending is slow to recover after crises,

[15] Note that Figure 6 does not show $\dot{x}$ just to the left of $\bar{x}$ because it is negative. By Proposition 3, $\dot{x} < 0$ implies $\dot{r} < 0$. With spreads decreasing over time, there is no incentive to delay and so no region of slow thawing.

Figure 6: Slowly thawing credit markets.



Note. This figure illustrates when there exists a region with "slow thawing" where credit markets recover only very slowly from a crisis. The green solid line represents the speed at which the pool quality needs to improve for average borrowers to be exactly indifferent between applying for loans (strictly preferred below the curve) and waiting (strictly preferred above). The red dashed line represents the speed of improvement when all banks are active. The equilibrium speed (black solid line) is the minimum of both curves.

based on two intuitive ideas. First, credit spreads rise during crises and come down afterwards. Second, the more financially sound (average) borrowers have an incentive to "wait out" crises until credit spreads come down, while low-quality borrowers are unwilling or unable to do so. These two ideas amplify each other, as banks increase credit spreads in response to a worse pool of borrowers, further incentivizing average borrowers to delay borrowing.

What determines how likely or how strong this period of slow thawing is? How could it be sped up? The following corollary reveals the roles of interest rates, project payoffs, and meeting frequencies.

Corollary 2 (Credit market recovery with normal lending standards)**.** *Fix a quality of the borrower pool $x \in (\overline{x}, x^p)$ and let $\dot{x}$ denote the speed of improvement in the pool's quality. Then in the slow thawing region:*

1. *Worse projects always slow down the recovery: $\dot{x}$ falls with lower r_L, r_H.*

2. *Increasing the rate at which borrowers apply for loans does not speed up the recovery in the slow thawing region: for $x < \hat{x}$, $\dot{x}$ does not rise with κ; for $x > \hat{x}$, $\dot{x}$ rises with κ.*

3. *More patient borrowers slow down the recovery: $\dot{x}$ falls with lower ρ if $x < \hat{x}$ (holding fixed r_L, r_H).*

When the rate κ at which borrowers receive investment opportunities increases (part 2 of Corollary 2), the red line in Figure 6 increases. This naturally increases the speed of the recovery towards the steady state outside the slow-thawing region. Inside that region, however, it has no effect. In fact, even when $\kappa \to \infty$, the transition towards the pooling steady state is slow and entirely determined by the indifference condition (13). The reason for this is that banks are not finding it profitable to lend more, as the pool of borrowers is adversely selected, and are thus holding back lending.

Greater patience, lower ρ, makes average borrowers more willing to wait, shifting down the indifference curve in Figure 6 and slowing down the recovery. In practice, the patience parameter captures the borrowers' internal rate of return, reflecting how timely is the borrower's need for funding. Thus, paradoxically, when borrowers are less desperate for funding, the recovery takes longer.

Figure 7 juxtaposes the transitional dynamics with slow thawing (dashed red line) and the transitional dynamics without slow thawing (solid green line). The latter was computed by ruling out slow thawing by assumption, imposing $\varphi_t^a = 1$, $\theta_t = 1$, and dropping equilibrium equation (7) and instead assuming that potential borrowers are myopic in the sense that when they have the opportunity to invest, they approach the competitive banking sector and accept the loan and invest rather than optimally choosing whether instead to wait for their next opportunity to borrow. As is visible in the first panel, slow thawing can greatly slow the transition back to the pooling steady state and lead to a relatively low lending volume.

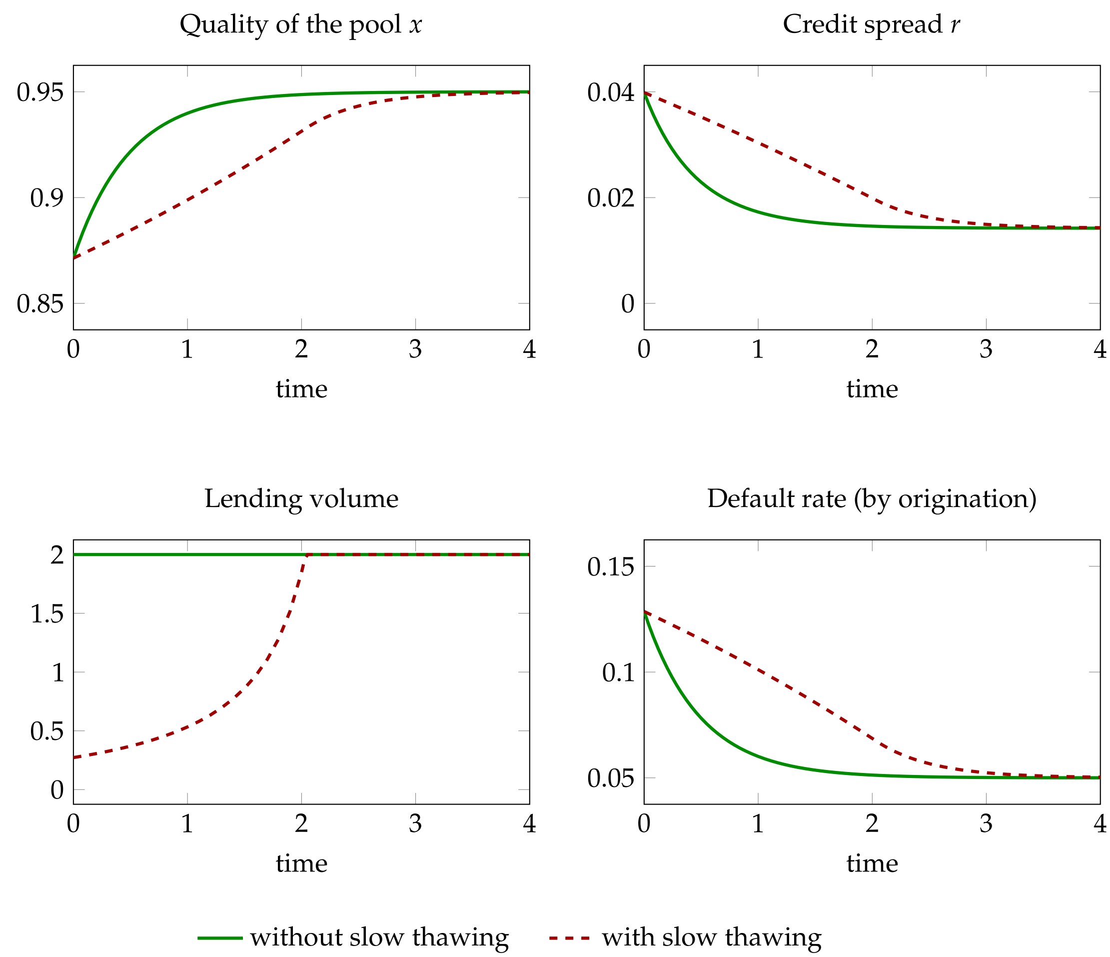
Note that a similar region with slow thawing can also appear in the region between $\underline{x}$ and x^s and slow down the convergence to the screening steady state from the left.

4 Efficiency

Because one bank's lending standard affects future banks' borrower pools, equilibrium lending standards will, in general, not be efficient. This section characterizes the constrained efficient outcomes.[16]

[16]The unconstrained efficient allocation (first-best) in our model would allow the planner to fund only average borrowers, which, given Assumption 2, would be done without screening.

Figure 7: Slowly thawing credit markets.



Note. The plots compare two transitions back to the pooling steady state. Green solid is a transition without "slow thawing", where average borrowers always accept current loan offers and banks do not ration credit; red dashed is a transition with slow thawing, where banks ration credit in equilibrium. The parameters used for this simulation are as follows: $\rho = \delta = 0.05$, $\lambda = 0.95$, $r_L = -0.27$, $r_H = 0.13$, $\tilde{c} = 0.035$, $\kappa = 2$, $\bar{z} = 0.8$.

4.1 Constrained efficient policy

Our concept of constrained efficiency allows the planner to control banks' activity and screening decisions, subject to borrowers' application decisions, so as to maximize the sum of agents' utilities.[17] Throughout this section, we continue to focus on the algebraically simpler case where $u \to 0$. In this section, it is further assumed that the planner can set the path of market interest rates $\{r_t\}$, and therefore prevent average borrowers from waiting, i.e. there is no slow thawing. We discuss relaxing this assumption below.

The constrained efficient planning problem is given by

$$\max_{z_t \in [0,\bar{z}], \theta_t \in [0,1]} \int_0^\infty e^{-\rho t} \kappa \theta_t \left\{ x_t r_H + (1 - z_t)(1 - x_t) r_L - \tilde{c} z_t \right\} dt \tag{14}$$

subject to the law of motion of x_t, (6). The solution to this problem can be characterized as follows.

Proposition 5 (Second-best policy)**.** *There exists a threshold $\overline{x}^* \in [0, \overline{x})$ such that the second-best planner sets:*

$$z_t = \begin{cases} \bar{z} & \text{if } x_t < \overline{x}^* \\ 0 & \text{if } x_t > \overline{x}^* \end{cases} \tag{15}$$

For any $x_t \in (\overline{x}^*, \overline{x})$, equilibrium lending standards are (second-best) inefficiently tight.
For any $\overline{x}^* > x^s$, the optimal policy for bank activity is given by

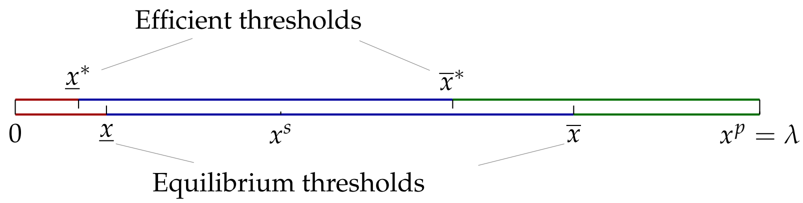
$$\theta_t = \begin{cases} 0 & \text{if } x_t < \underline{x}^* \\ 1 & \text{if } x_t > \underline{x}^* \end{cases}$$

for some $\underline{x}^* \in [0, \overline{x}^*)$.

Proposition 5 shows that the optimal lending standard takes a similar form to the privately-optimal policy: when the quality of the pool is relatively high, $x > \overline{x}^*$, normal lending standards, $z = 0$, are optimal; and when $x < \overline{x}^*$, tight lending standards, $z = \bar{z}$, are optimal. But the cutoffs for the optimal policy and for the market equilibrium differ: There exists a region in the state space, $(\overline{x}^*, \overline{x})$, where equilibrium lending standards are too tight relative to the constrained-efficient outcome.

[17] Since borrowers and banks are risk-neutral, this is without loss when transfers between agents are feasible.

Figure 8: Constrained efficient vs. equilibrium lending standards.



In the proof, we show that $\overline{x}^*$ is the largest x that satisfies

$$\underbrace{\frac{\rho x + \alpha^s x^s}{\rho + \alpha^s} r_H + (1 - \overline{z}) \left(1 - \frac{\rho x + \alpha^s x^s}{\rho + \alpha^s}\right) r_L - \tilde{c}\overline{z}}_{\text{Average social benefit of screening}} \geq \underbrace{\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p} r_H + \left(1 - \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}\right) r_L}_{\text{Average social benefit of pooling}} \quad (16)$$

To develop an intuition for this finding, say the current pool quality is x and banks operate normal lending standards, $z = 0$, in all periods from now on so that the credit market ultimately converges to the pooling steady state x^p. In (16), one can think of $\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}$ as the time-averaged fraction of type-H borrowers funded. The weight on current x is ρ, as with greater discounting the present becomes relatively more important; the weight on the (long-run) steady state x^p is $\alpha^p = \kappa + \delta$, which is the speed at which x converges to x^p. The average social benefit of $z = 0$ is therefore the weighted average surplus from lending to each type of borrower,

$$\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p} r_H + \left(1 - \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}\right) r_L.$$

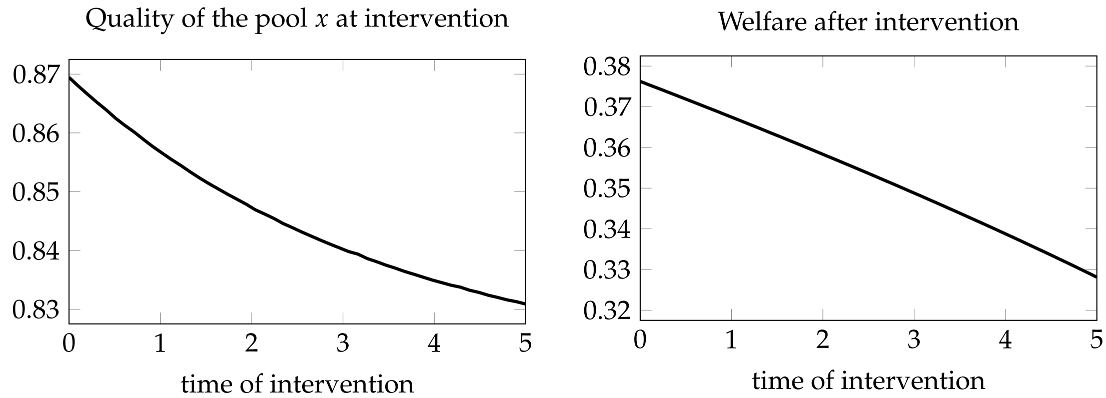
An analogous expression describes the social benefit of tight lending standards, where we additionally account for both the costs of screening and the fact that banks successfully screen a fraction $\bar{z}$ of of low-quality borrowers, giving rise to (16).

By contrast, the private cut-off, $\overline{x}$, is the largest value satisfying

$$\underbrace{xr + (1 - \overline{z})(1 - x) r_L - \tilde{c}\overline{z}}_{\text{Average private benefit of screening}} \geq \underbrace{xr + (1 - x) r_L}_{\text{Average private benefit of pooling}} \quad (17)$$

which is calculated using the *current* fraction x of type-H borrowers and ignores the

Figure 9: Early interventions dominate late ones.



Note. This figure shows how intervention policies affect a credit market that is transitioning towards the screening steady state. The horizontal axis shows the time at which an intervention starts (where 0 corresponds to the immediate, constrained efficient intervention). The parameters used for this simulation are as in Figure 4.

dynamic consequences from screening and pooling. In particular, since in the relevant region it holds that

$$\frac{\rho x + \alpha^s x^s}{\rho + \alpha^s} < x < \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}$$

agents privately ignore the dynamic costs from screening relative to pooling. Therefore, $\overline{x}^* < \overline{x}$. The private and social thresholds are shown in Figure 8.[18]

Another way to highlight the differential dynamic consequences of pooling and screening is to compare steady states.

Corollary 3. *When both steady states exist (a result of Assumption 2), the screening steady state has strictly lower welfare than the pooling steady state.*

If $\delta = 0$, this result would be a simple consequence of the fact that screening potential borrowers is costly and the quality of funded borrowers is independent of the steady state (see discussion below Corollary 1). But with $\delta > 0$, screening borrowers has a social benefit because a share of them are never funded. Still, the corollary shows that welfare in the pooling steady state is higher.

There are two important practical implications from the existence of a non-empty interval $(\overline{x}^*, \overline{x})$ where the market equilibrium diverges from the constrained optimum.

[18]The logic that lending standards are only inefficiently tight in the region $(\overline{x}^*, \overline{x})$ follows from the linearity of the cost function. If banks' screening costs were nonlinear such that the optimal $z^*(x)$ were continuous and strictly decreasing, then lending standards could be generically too tight.

1. Intervention timing matters. Figure 9 illustrates the welfare consequences of intervention in a credit market that starts at a given $x_0 \in (\overline{x}^*, \overline{x})$ for various times when an intervention starts (on the horizontal axis). The later the time of intervention is, the lower is the quality of the borrower pool when the policy switches from screening to pooling (left panel). Later intervention times thus increase the short-run losses incurred at the start of the intervention and are therefore welfare-inferior to early interventions. In fact, after a sufficiently long time, if x_t has fallen below $\overline{x}^*$, intervening may even be welfare-dominated by not intervening at all and allowing the market to converge to the screening steady state, despite its having lower welfare than the pooling steady state. That a late intervention may be worse than no intervention underscores the importance of the timing of interventions in our model.

2. Better screening technology may be detrimental to welfare. Say the cost $\tilde{c}$ of operating tight lending standards falls. While a cost reduction necessarily raises efficiency in any steady-state equilibrium, it can decrease welfare because it raises thresholds both for the market convergence to a screening equilibrium and for the efficient intervention, $\overline{x}$ and $\overline{x}^*$. Therefore, if a market is just recovering from a crisis, with x_0 just above $\overline{x}$, such a technological improvement may cause $\overline{x}$ to rise above x_0 and thereby prevent a recovery and lead to a reduction in welfare. If $\overline{x}^*$ also rises above x_0 then it is too costly for policy to mitigate this decline in welfare.

A decrease in cost $\tilde{c}$ represents an improvement in private information technology. What happens if instead public information technology (e.g. credit reporting) improves? A crude way to capture such a change is as an increase in δ, the probability that rejected borrowers die. While the death of average borrowers has no effect on equilibrium as they are replaced in the pool by an equal measure of new average borrowers, a greater death rate of rejected borrowers does matter for equilibrium. A larger δ increases (decreases) the speed of convergence when x is increasing (decreasing), and raises the pool quality in the screening steady state, so therefore unambiguously increases welfare. Thus, the welfare effects of improving public information are unambiguously positive, a point that leads naturally into a discussion of the first-best policy.

4.2 First-best policy

While inconsistent with our modeling assumptions, the first-best policy would be to eliminate the externality associated with screening, for example by making the outcome of any borrower screening public.[19] One might expect therefore that market participants might design institutions to track and make public such information in situations where the externalities associated with lending standards are severe. We discuss in Section 7.3 (and Appendix C) the extent to which credit bureaus in some countries may mitigate this externality in some markets by reporting credit checks. But credit bureaus do not track rejections. Our model suggests why eliminating the externality is difficult: it is not privately optimal for a borrower/bank pair to report a rejection.

4.3 Implementation of the constrained optimum

There are several ways a government or a regulator could implement the constrained efficient outcome, that is, normal lending standards when $x \in (\overline{x}^*, \overline{x})$. We continue to assume that there is no region of slow thawing. Since such an intervention entails short-run losses and the model's banking sector is competitive, either the government or type-H borrowers have to bear these losses.

An example of such a policy is a government-funded loan insurance program in which the government provides an insurance benefit $b > 0$ (in present value) to be paid to a bank when a borrower defaults. This policy incentivizes banks to use normal lending standards as long as

$$\frac{b}{-r_L} > 1 - \frac{c}{1-x}.$$

This condition is satisfied for $b = 0$ in the region $x > \overline{x}$ where pooling is privately optimal. It requires nonzero insurance benefits $b = b(x) > 0$ when $x < \overline{x}$. As a function of the pool quality, $b(x)$ is decreasing in x. This means a typical intervention starting from some $x_0 < \overline{x}$ requires large insurance benefits early on, which are then phased out over time.

Our model is thus consistent with the ability of government loan guarantees to increase the efficiency of credit markets by decreasing lending standards and interest rates. Examples of such loan guarantees in the US include mortgage markets (e.g. the FHA), student loans, and credit for international trade. All these loan guarantees require that the borrower meet eligibility requirements and/or condition rates on readily-available, verifiable information. Also notably, the retraction of a loan guarantee arguably characterized the start of

[19] A partial step in this direction would be information on how long the borrowers have been in the market for a loan. Since bank borrowing rates are publicly observed, such information would be informative regarding whether borrowers have been denied credit and are thus type-L and not worth financing as in Daley and Green (2012, 2016) – in which the history of offers received by the informed party is not observable – or Chari, Shourideh and Zetlin-Jones (2014) – in which this history is observed and the equilibrium features a partial separation of the high- and low-quality types.

the US financial crisis of 2008-2009. By allowing Lehman Brothers to fail, the US government retracted an implicit guarantee of the short-term debt of large financial institutions (as exhibited in the resolution of Bear Sterns in March 2018). In response, lenders (buyers of short-term commercial paper of financial institutions) tightened lending standards and interest rates rose and lending volumes declined. The government responded with a set of policies – explicit and implicit guarantees, and liquidity provision – that decreased the probability of losses and so reduced interest rate spreads and increased lending volumes to large financial institutions.

A second way to implement the constrained optimum would be to *require* that, whenever $x \in (\overline{x}^*, \overline{x})$, all loans made at each point in time are placed into a common pool from which each bank receives a proportionate payout as the loans mature. Such mandated securitization requires only that a loan origination is observable and contractible, not that a rejection is observable. Under this policy, no individual bank has the incentive to tighten lending standards when $x \in (\overline{x}^*, \overline{x})$ since they receive no benefits from placing a higher-quality loan into the securitized pool.

We assume that banks cannot observe which borrowers are rejected. But if the government could monitor banks and measure either lending standards or rejections (but, like the private sector, not observe the identity of the rejected agent), then another policy that implements the constrained optimum is to tax lending standards or rejections at a high enough rate to ensure normal lending standards (when $x \in (\overline{x}^*, \overline{x})$). This policy is the most direct: taxing the activity – tight lending standards – that has the negative externality. Interestingly, this policy increases equilibrium spreads even though no tax is collected because the profits from type-H borrowers have to cover the losses from lending to more type-L projects.

Note finally, that all such policies are not privately optimal without market-wide collusion. Thus, there is a role for government in a competitive market.

4.4 Limits to constrained efficiency

In practice, policies like government-funded loan subsidies or insurance programs are rarely undertaken for the entire financial sector, but instead usually apply only to certain types of institutions, such as traditional banks but not money market mutual funds or shadow banks.[20] So consider a setting where the government can affect the lending

[20] In the U.S., bank deposits are insured, but not other short-term "deposit"-taking institutions such as money market mutual funds (MMMFs). In the financial crisis of 2008-2009, the government actually did extend deposit guarantees from traditional banks to MMMFs, but did not extend the guarantees to other

decisions of banks but not shadow banks, where the former comprise a fraction $\eta \in [0,1)$ of financial institutions and the latter comprise a fraction $1-\eta$. What is the optimal policy under such circumstances? For this section, we focus on the case without slow thawing or bank inactivity, i.e., $\theta = 1$. We further assume that banks always charge the same interest rates as their shadow bank competitors.

To state the new planning problem for a given x_0, denote by $z^p(x)$ the optimal screening action of a shadow bank, that is, $z^p(x) = 0$ for $x > \overline{x}$ and $z^p(x) = \overline{z}$ for $x < \overline{x}$. Then, the planner has the same objective as before,

$$\max_{z_t} \int_0^{\infty} e^{-\rho t} \kappa \left\{ x_t r_H + (1-z_t)(1-x_t) r_L - \tilde{c} z_t \right\} dt$$

subject to the same law of motion of x_t,

$$\dot{x}_t = (\kappa + \delta)(\lambda - x_t) - \kappa z_t \lambda (1 - x_t),$$

but now with a constraint on the average lending standard z_t,

$$z_t \in [(1-\eta) z^p(x_t), (1-\eta) z^p(x_t) + \eta \overline{z}], \tag{18}$$

rather than $z_t \in [0, \overline{z}]$. This is because only a fraction η of overall lending standards can be controlled by the government.

Constraint (18) significantly changes optimal policy. When $x \in (\overline{x}^*, \overline{x})$, normalizing lending standards for banks entails a slower improvement in credit conditions because shadow banks continue to apply tight lending standards, reducing $\dot{x}$. The optimal policy still takes a threshold form but the threshold $\overline{x}^*(\eta)$ now depends on the share of banks η. For low levels of η, $\overline{x}^*(\eta) = \overline{x}$, and the planner wants to implement the competitive equilibrium. Only when banks comprise a large enough share of the lending sector,

$$\eta > 1 - \frac{(\kappa + \delta)(\lambda - \overline{x})}{\kappa \overline{z} \lambda (1 - \overline{x})}, \tag{19}$$

is it optimal to intervene in some region of the state space, that is $\overline{x}^*(\eta) < \overline{x}$.

Why is not optimal for the planner to intervene for low η? The motivation for intervention is to shift from convergence to the screening steady state to convergence to the pooling steady state. If, however, η is below the threshold in (19), the planner is unable to

short-term debt markets. Similarly, the government took over the government-sponsored mortgage lending agencies and traditional banks but allowed private label securitizers and mortgage brokers to fail.

relax overall lending standards enough to induce the state x_t to move towards pooling for values of x_t close to the private threshold $\overline{x}$. Further even when achieving $\dot{x} > 0$ is possible, credit conditions might improve too slowly to make it worthwhile. Thus, the government can lack the "firepower" to get to pooling and optimally choose not to intervene at all. This discussion illustrates that a large but hard to regulate shadow banking sector constrains optimal government policy.

If the government intervenes for low η, it is forced to cover increasingly large losses as shadow banks impose tight lending standards, reducing the pool quality. Banks then face a more adversely selected pool. This point may have been relevant for the failure of the government sponsored mortgage agencies in the US in 2008.

5 Economic fluctuations and hysteresis

In this section we explore the implications of our model for economic fluctuations. In particular, we study a slowdown caused by a temporary decline in the inflow of type-H borrowers. We show how lending standards can tighten in such periods and propagate a temporary downturn into permanent increases in interest rate spreads and intermediation costs and declines in lending volume. Such hysteresis does not follow from shorter or smaller declines in quality, consistent with lending standards normalizing following the milder 2001 US recession but not the 2008-2009 Great Recession, as displayed in Figure 1.

5.1 Downturn in fundamentals

We feed into the model a temporary decline in the size and quality of the pool of potential borrowers. More specifically, we reduce the quantity of type-H new borrowers entering the pool of potential borrowers from time 0 to time T. Our model in Section 2 involves a fixed pool size $N = 1$ and therefore needs to be amended to allow for dynamics in N. However, as one can easily show, since the law of motion of N is exogenous in the model, the model still applies to a "normalized" version of the credit market, where all absolute quantities (volume of loans, welfare, profit, etc.) are to be thought of as normalized by N.

For the period of the lending slowdown, we reduce the inflow of new type-H borrowers into the pool by μ. During this period, only a share $(1-\mu)\lambda$ of new potential borrowers are type-H. As a result, the fraction of type-H borrowers, x, evolves according to

$$\dot{x}_t = \theta_t\kappa(1-x_t)(1-z_t)\lambda_t - \theta_t\kappa x_t(1-\lambda_t) + \delta N_t^{-1}\left(\lambda - x - \mu 1_{\{t\leq T\}}\lambda(1-x)\right) \tag{20}$$

where λ_t is the average quality of new borrowers entering into the pool,

$$\lambda_t \equiv \begin{cases} \frac{\lambda(1-\mu)}{\lambda(1-\mu)+(1-\lambda)} & t \leq T \\ \lambda & t > T \end{cases}$$

The last term in (20) represents the modified impact of the birth-death process of borrowers for pool quality. It is modified to allow for a reduction in pool quality when $\mu > 0$.

Following the slowdown, when λ_t returns to λ, the inflow of new borrowers increases to return the pool to its original size of 1, as shown by the law of motion of the total pool size N_t

$$\dot{N}_t = \delta(1 - N_t) - \delta\lambda\mu 1_{\{t \leq T\}}. \tag{21}$$

For the simulations in this section, we keep the model parameters from Figure 4 and choose $\mu = 0.66$. Under this parameterization the market has no slow thawing region.

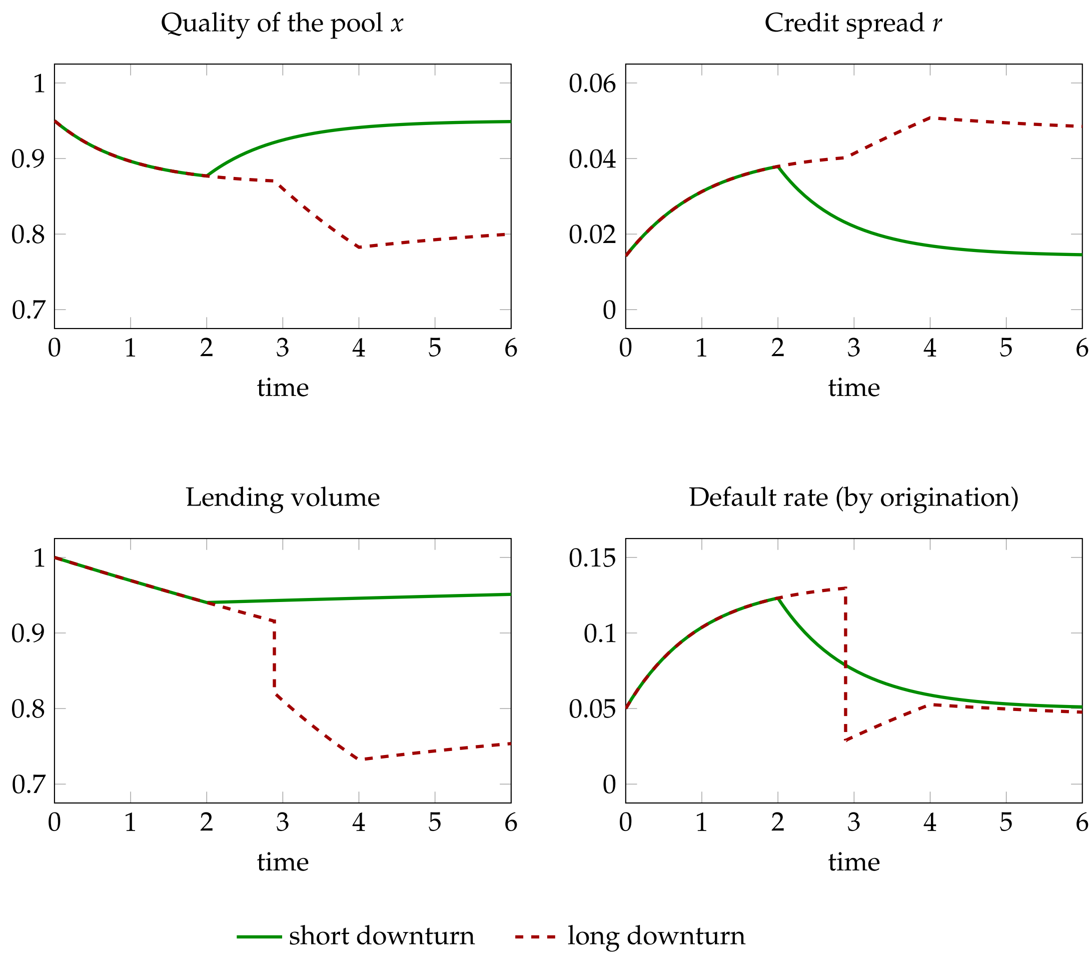
5.2 Lending cycle and hysteresis

We simulate the response to two different lengths of the downturn, $T = 2$ period and 4 period. The results are shown in Figure 10. As the solid green line shows, the short downturn is associated with a decrease in lending volume, as fewer borrowers seek loans, and an increase in interest rate spreads, as the pool quality declines and the ex post default rate increases. When the downturn in fundamentals ends, the increase in the demand for loans and the increase in the quality of potential borrowers both lead to increased lending and decreased interest rate spreads. After a short downturn, the credit market returns to its previous equilibrium with normal lending standards.

By contrast, the longer downturn (dashed red line) is associated with a larger decline in the quality of the average potential borrowers so that, roughly at $T = 3$, banks tighten lending standards leading to an abrupt decline in lending. Following this tightening, pool quality deteriorates more rapidly, credit spreads increase, and the lending volume continues to contract. When the slowdown ends at $T = 4$ and more higher-quality borrowers start to enter the pool of potential borrowers again, the lending volume increases, but interest rate spreads remain high and lending never fully recovers.

While it is individually optimal for banks to tighten lending standards in the long downturn, it is not socially optimal. For the parameters in Figure 10, optimal policy is to maintain normal lending standards and can be implemented, for example, with a loan guarantee program as described in Section 4.3. Under optimal policy, the long downturn in

Figure 10: Recession and hysteresis.



Note. This figure shows a credit market in response to two shocks: the green solid line represents a 2-period downturn, the red dashed line a 4-period downturn. A "downturn" is modeled as a reduction in the inflow of type-H borrowers. The transitions were computed using $\mu = 0.66$. The other parameters used for this simulation are as in Figure 4.

Figure 10 would instead look like an extended version of the short one. With no tightening of lending standards at the end of period 3, the lending volume would continue its smooth decline during the fourth year and then slowly rise back to one following the end of the recession, like a longer version of the two-year downturn. Similarly, the rise in credit spreads and (ex post) defaults continues to slow, and starts to recover rapidly at the end of year 4. In short, the credit market deteriorates by less and recovers more quickly.

In Figure 10, we modeled the downturn as a decline in λ, and studied downturns of different lengths. However, similar dynamics would follow from a downturn in which recovery rates after loans default, r_L, worsened. In this case, banks would maintain normal lending standards for small declines in r_L, leading to no lasting damage to credit markets. In contrast, deep downturns, in which r_L falls enough to trigger tight lending standards, could trigger permanent declines in lending volume and increases in interest rate spreads due to the costs of intermediation.

We now turn to another source of tightening of lending standards and propagation of fluctuations in the real economy—capital constraints.

6 Capital constraints and lending standards

In this section, we show how regulatory or market-based limits on banks' capacity to lend, such as from balance sheet constraints, raise lending standards and propagate and amplify credit crunches through the mechanisms just analyzed.

6.1 Balance sheet constraints

We consider a stylized model of bank balance sheet constraints in which a capital constraint, $\bar{V}_t$, restricts the amount of lending proportionally by each bank so that

$$V_t \leq \overline{V}_t \tag{22}$$

where $V_t \equiv \theta_t\{\kappa_{Ht} + \kappa_{Lt}(1 - z_t)\}$ denotes total bank lending. We assume that $\bar{V}_t$ grows exogenously over time so that banks eventually become unconstrained which implies that the bank optimization problem remains static. We also assume that $\overline{V}_t$ grows sufficiently quickly such that, even in the face of fluctuations in exogenous variables, once banks are unconstrained they remain unconstrained thereafter. We can thus separately analyze periods in which the constraint, (22), binds and periods in which it is slack.[21]

[21] As in Section 3.3, we work here in the limit $u \to 0$ to simplify the exposition.

When the constraint stops binding, the equilibrium is as described in Sections 2 and 3. While the constraint binds, each bank charges the interest rate that makes average borrowers indifferent as to whether to i) apply for a loan when given the opportunity and so receive $r_H - \tilde{r}_t$ with probability λ, or to instead ii) return to the pool and wait for a future borrowing opportunity at a lower rate and so get J_t^a. We denote this rate by $\tilde{r}_t$ so

$$r_t = \tilde{r}_t \tag{23}$$

and indifference implies that $J_t^a = \lambda (r_H - \tilde{r}_t)$. Substituting into (1a), implies $\dot{J}_t^a = (\rho + \delta) J_t^a$ so that $\tilde{r}_t$ evolves according to

$$\dot{\tilde{r}}_t = -(\rho + \delta)(r_H - \tilde{r}_t) \tag{24}$$

with the terminal condition that at a time τ', when banks become unconstrained, $\tilde{r}_{\tau'} = r(x_{\tau'})$ with $r(x)$ as in (11) in Proposition 3. Equation (23) replaces the bank zero-profit condition in our definition of equilibrium. Since (24) implies that $\tilde{r}_t$ will always lie strictly below r_H, all borrowers accept loans in equilibrium and $\kappa_{Ht} = \kappa x_t$ and $\kappa_{Lt} = \kappa(1 - x_t)$.

To understand how capital constraints affect lending standards, consider first the effect of lending standards on the profits from making one and only one loan which is given by

$$\Pi_t^{\text{single}}(z, r) \equiv \frac{x_t r + (1 - x_t)(1 - z) r_L}{x_t + (1 - x_t)(1 - z)} - \frac{\tilde{c} z}{x_t + (1 - x_t)(1 - z)} \tag{25}$$

The first term is the expected return on the loan, where $\frac{x_t}{x_t + (1 - x_t)(1 - z)}$ is the probability that the borrower is type-H and thus that the bank earns r on the loan, and $\frac{(1 - x_t)(1 - z)}{x_t + (1 - x_t)(1 - z)}$ is the probability that the borrower is type-L and the bank loses r_L. The second term is the expected cost of lending standards and exceeds $\tilde{c}$ because several potential borrowers may have to be checked before finding one that passes the lending standard. The bank chooses tight lending standards if $\Pi_t^{\text{single}}(\bar{z}, r) > \Pi_t^{\text{single}}(0, r)$ which collapses to the condition

$$x_t(1 - x_t)(r - r_L) > \tilde{c}. \tag{26}$$

Given the ability to make one loan, the bank prefers tight lending standards when costs of screening are low, when the information uncovered has high variance (when $x_t(1 - x_t)$ is large), and when the losses from lending to type$-L$ borrowers are large (r_L large negative). Most importantly, the propensity to impose tight lending standards is greater the higher r. Thus binding capital constraints—which endogenously lead to higher interest rates—also incentivize tighter lending standards.

The arguments of the previous paragraph apply when the capital constraint is tight in the sense that it binds even when lending standards are tight and so the desired lending volume is low. When the capital constraint is at an intermediate level so that it would not bind if $z = \bar{z}$ but would bind if $z = 0$, banks have three possible strategies for lending standards. First, they can impose tight standards, which generates profits

$$\kappa \left(x_t + (1 - x_t)(1 - \bar{z})\right) \Pi_t^{\text{single}}(\bar{z}, r_t).$$

Since the capital constraint does not bind when banks impose tight standards, the interest rate equals that in the competitive economy. Thus tight lending standards only occur for $x < \bar{x}$. Second, banks can impose normal lending standards in which case total profits are

$$\overline{V}_t \Pi_t^{\text{single}}(0, r_t).$$

Finally, banks can set intermediate lending standards, $z_t = \frac{1-\overline{V}_t/\kappa}{1-x_t}$, such that applying these standards to all potential borrowers leads banks to make $\overline{V}_t$ loans.[22] At this intermediate lending standard, bank profits are

$$\kappa x_t r + (\overline{V}_t - \kappa x_t) r_L - \frac{1 - \overline{V}_t/\kappa}{1 - x_t} \tilde{c}.$$

Optimal lending standards are then characterized as follows:

Proposition 6 (Optimal lending standard with capital constraints)**.**

1. (Loose constraint). *If $\overline{V}_t > \kappa$, the capital constraint does not bind: all current and future variables are identical to those in the market without constraints.*

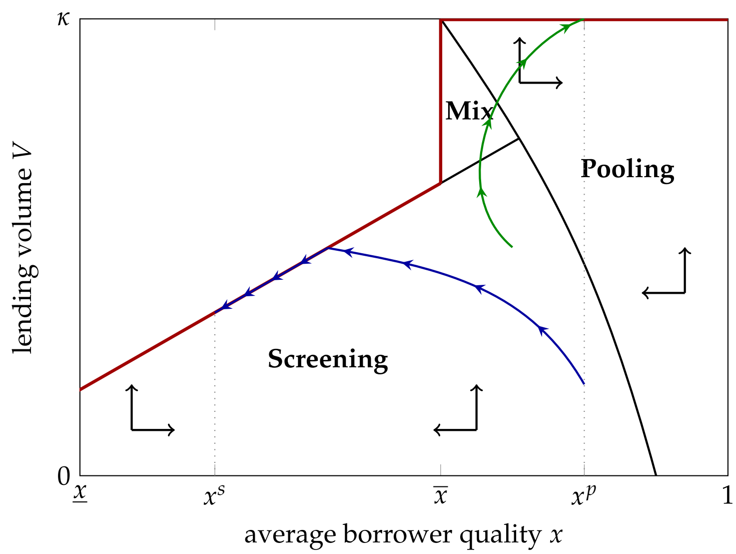
2. (Tight constraint; Screening region). *If $\overline{V}_t < \kappa x_t + \kappa(1 - x_t)(1 - \bar{z})$, the capital constraints binds: There exists a threshold*

$$\bar{x}_t^C = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4\tilde{c}}{r_t - r_L}}\right) \geq \bar{x}$$

such that banks impose tight lending standards, $z = \bar{z}$, if $x_t < \bar{x}_t^C$, and impose normal lending standards, $z = 0$, if $x_t > \bar{x}_t^C$.

[22]This strategy has the same payoff as applying the lending standard $\bar{z}$ to a share $\frac{1-V_t/\kappa}{(1-x_t)\bar{z}}$ of borrowers and lending to the remaining borrowers at $z = 0$.

Figure 11: Optimal lending standards with capital constraints



3. (Intermediate constraint). *For* $\kappa x_t + \kappa(1 - x_t)(1 - \bar{z}) < \overline{V}_t < \kappa$:

   (a) (Pooling region). *For* $x_t > \bar{x}_t^C$ *banks lend with normal lending standards and the constraint binds*

   (b) (Mixed region). *For* $x_t \in (\bar{x}, \bar{x}_t^C)$ *banks impose intermediate lending standards,* $z_t = \frac{1-\overline{V}_t/\kappa}{1-x_t}$*, and the capital constraint binds.*

   (c) (Unconstrained region) *For* $x < \bar{x}_t$*, banks impose tight lending standards and the constraint does not bind.*

We illustrate Proposition 6 in Figure 11. The figure shows the volume of bank lending on the vertical axis and the quality of the pool of potential borrowers on the horizontal axis. The thick (red) line which increases from $\underline{x}$ to $\bar{x}$ and then rises to remain at κ from $\bar{x}$ to 1 displays the equilibrium volume of lending when the constraint does not bind (Proposition 6.1 and 6.3.c). Following Proposition 2, when the credit market is unconstrained, to the right of $\overline{x}$, the lending volume equals $V_t = \kappa$ and x_t rises towards the pooling steady state; to the left of $\overline{x}$, the economy has volume $V_t = \kappa x_t + \kappa(1 - x_t)(1 - \overline{z})$ and x_t declines towards the screening steady state.

Strictly below this thick (red) line there are three regions that the credit market can be in when capital constraints bind. In all three regions, the lending volume is always rising as capital constraints are (exogenously) relaxing. Similarly, in these constrained regions, interest rate spreads are always (weakly) declining, following (24).

In the right "pooling" region (bounded on the left by the curved solid line which intersects $x = x^p$), the pool quality is high enough that even with binding constraints, banks set normal lending standards ($z = 0$) because (26) is not met. Note that credit spreads are higher than if banks were unconstrained, but not so high that lending standards are tightened.

By contrast, within the "screening" region, x is low enough given $\bar{V}$ that lending standards are tight. For $x < \bar{x}$, lending standards are simply tight as before without lending constraints, although again, interest rate spreads are higher than without lending constraints. However, the more interesting region is the screening region where $x > \bar{x}$. Banks would impose normal lending standards if not constrained. But in this region the volume of lending is sufficiently constrained that banks impose tight lending standards. As shown in Proposition 6, this region always exists for some range of $\bar{V} < \kappa$ and $x > \bar{x}$.

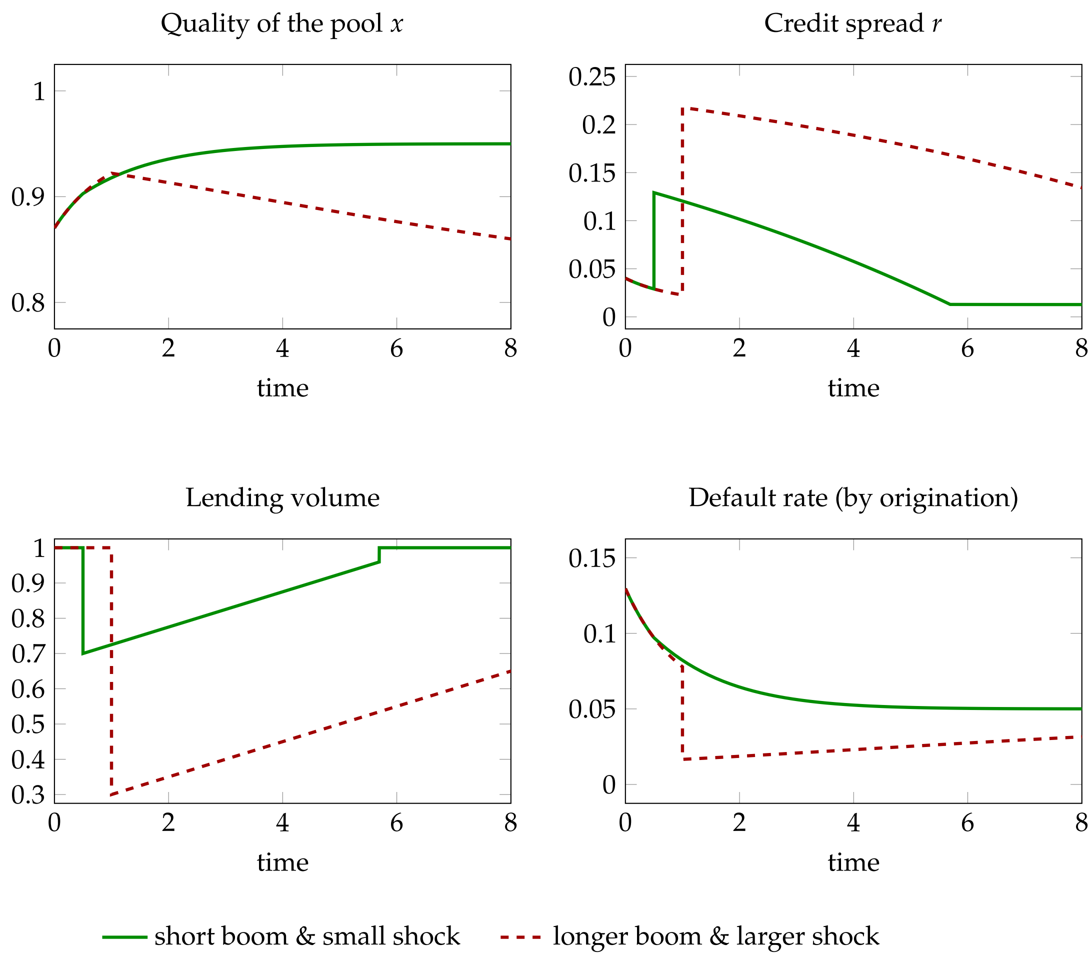
Focusing on x just above $\bar{x}$, when $\bar{V}$ is not too large the credit market is in the screening region (Proposition 6.2) and banks set tight lending standards. If on the other hand, $\bar{V}$ is large enough that banks would be unconstrained if $z = \bar{z}$, then banks instead set intermediate lending standards, $z \in (0, \bar{z})$, which are either tightened or loosened over time as pool quality improves or declines at the chosen lending standards, depending on the tightness of the constraint (Proposition 6.3.b). The higher (green) convergence path illustrated in Figure 11 shows an example dynamic path for a credit market which passes through this "Mix" region.

Figure 11 provides a first illustration of how the effects of balance sheet constraints can be amplified and propagated by lending standards. For a tight enough constraint on lending, banks impose tight lending standards not only for x just above $\bar{x}$ but also at much higher levels of pool quality such as that at the pooling steady state, x^p, and even higher, $x > x^p$. Thus, even when the market starts at a high pool quality at or above the pooling steady state x^p, tight balance sheet constraints can lead to tight lending standards, declines in pool quality, and ultimately to convergence to the screening steady state x^s that persists even when the constraint on lending is fully relaxed. An example of this dynamic is depicted in the lower (blue) convergence path in the Figure 11 and explored in the next subsection.

6.2 Booms and busts in lending standards and balance sheets

This subsection presents a simulation of a credit market hit by a balance sheet shock to illustrate the dynamic interaction of lending standards and balance sheet constraints. We start the economy just to the right of $\bar{x}$. Thus, unperturbed, it converges towards the

Figure 12: Balance sheet constraints and lending standards



This figure simulates two economies starting at $x_0 = \bar{x}$. One experiences a shock at $t = 0.5$, reducing available capital for loans to $\overline{V} = 0.7$. The other experiences a shock at $t = 1$, reducing capital to $\overline{V} = 0.3$. In both economies, capital improves with $\dot{\overline{V}} = 0.05$. The other parameters used for this simulation are as in Figure 4.

pooling steady state. One may think of this convergence as a "lending boom."

After some time T, we assume that an unanticipated negative balance sheet shock hits banks, reducing the volume of loans that they can make to some low level $\overline{V}_T$, after which this constraint monotonically increases at exogenous speed $\dot{\overline{V}}$. We further assume that $\overline{V}_T$ is decreasing in T. These assumptions capture the idea of an economic shock at T that causes greater one-time losses (bigger $-r_L$) for type-L borrowers which have received loans in the past. These (un-modeled) defaults would then cause banks' balance sheet constraints to bind. Further, the longer the "lending boom", i.e. the convergence towards the pooling steady state has progressed, the more type-L projects have been funded shortly before T and the tighter the balance sheet constraints are at T when the shock hits.

Figure 12 depicts the dynamics for the credit market for $T = 0.5$ and for $T = 1$. When the "short boom", $T = 0.5$, bursts, depicted by the solid (green) lines in Figure 12, balance sheet constraints bind, both reducing the volume of lending and tightening lending standards. Credit credit spreads rise, and the quality of lending rises (default rates fall). Tight lending standards cause the pool quality to start declining over time, an effect which amplifies the initial downturn in the credit market. However, because the exposure to the aggregate shock $\overline{V}_T$ is not yet that large, this amplification is temporary and goes away as the balance sheet constraint relaxes.As banks grow out of the balance sheet constraint, they start to relax lending standards, partially at first, but then more over time as the constraint relaxes further and as pool quality starts to increase. Ultimately, the economy converges to the pooling equilibrium as the balance sheet constraint on lending ceases to bind. These dynamics are those depicted in the upper (green) convergence path in Figure 11.

When the "long boom", $T = 1$, bursts, as depicted by the dashed (red) lines in Figure 12, lending standards increase as when the short boom ends, but lending volume declines further because the capital constraint $\overline{V}_T$ is tighter than following the short boom. Credit spreads increase by more than in the short boom both because it will be longer until lending is unconstrained and because when it is unconstrained, interest rates are higher because the pool quality is lower. As following the short boom, tight lending standards amplify the credit downturn, but after the long boom there is a sufficiently long period with tight lending standards that the credit crunch becomes becomes permanent. Persistent tight lending standards reduce the pool quality sufficiently such that the market converges to and remains at the screening steady state even when the balance sheet constraints have passed. As pictured in Figure 12, because lending standards are never relaxed, lending never returns to its pre-crisis level and credit spreads remain permanently high. These dynamics are those depicted in the lower (blue) convergence path in Figure 11.

These experiments emphasizes that lending standards can amplify and propagate the negative effects of capital losses on credit markets. Capital constraints on lending not only lead to declines in lending and increases in spreads, but by incentivizing tighter lending standards, they further increase credit spreads, and can slow recovery or even cause self-reinforcing dynamics that perpetuate high interest rate spreads, low lending volumes, and tight lending standards.

6.3 Constrained-efficient policy when capital constraints bind

While we assume that the evolution of $\overline{V}_T$ is exogenous, our example illustrates that a richer version of our model, which links $\overline{V}_T$ to the amount of type-L loans on banks' balance sheets, would similarly imply that the longer an economy is in the pooling region, the more constrained banks will be upon impact of the shock. In such a model with an explicit capital constraint, the planner may find it optimal to pursue policies to loosen lending standards if capital constraints tighten lending standards in a crash. To be slightly more precise in the context of our model, there is a region in Figure 12 just to the left of the cutoff between pooling and screening or mixing, in which the planner who can control only lending standards prefers normal lending standards. This policy prescription follows for the same arguments as in section 4.1 and because, conditional on lending, the interest rate charged by banks only determines the division of surplus.

However, there are two interesting and informative differences between optimal lending standards policy with and without capital constrains (as described in Section 4). First, with a constraint on lending, the optimal cutoff for normal vs. tight lending standards from the planner perspective is not the same as $\bar{x}^*$, and in fact a planner facing capital constraints chooses to intervene in a wider range of situations for the following reason. Lending volume is lower when capital constraints bind relative to a situation without a constraint on lending, so that the costs of imposing normal lending standards in the present are lower. Lending volume rises over time, so that the benefits of normal standards later are more valuable. Thus, a constraint on lending increases the private incentive to raise lending standards but also reduces the cost of polices to normalize them again.

The second difference of note is that when lending standards are tight due to only to the capital constraint, an interest rate cap can increase efficiency by normalizing lending standards at no fiscal cost. This optimal policy can be implemented by entering the market and lending with normal standards so as to relax the market-wide lending constraints. Examples of this type of program include asset purchases aimed at normalizing spreads associated with new loans, such as those pursued by the U.S. policymakers following the 2008 financial crisis (e.g. the first round of quantitative easing in 2008-09) and the pandemic recession (the Main Street lending program in 2020).

While we have discussed only policies to change lending standards when a constraint on lending binds, optimal policy may be to increase capital requirements prior to the crash so that the constraint is less binding and lending standards are not tightened during a crash. Our model of capital constraints is of course too simple to capture the benefits and costs of policies to recapitalize banks with any realism. However, our analysis does highlight another benefit of recapitalization. When $\bar{x}^C(x_t) > x_t > \bar{x}$ and lending standards are tight, recapitalizing banks not only increases lending volume but also has the benefit of decreasing lending standards.

7 Discussion of assumptions and extensions

We discuss the importance of several modeling assumptions and sketch model extensions.

7.1 Borrower effort to raise project quality

Suppose that borrowers could exert effort to improve the quality of their projects. Will this change our model's predictions? To investigate this question, we sketch a simple extension of our model.

Suppose that after an average borrower is matched with a bank, but before he is screened, the borrower chooses private costly effort, where more effort increases the likelihood of being a type-H borrower. The effective share of type-H borrowers in the pool now depends on borrower effort. Let x denote this effective pool quality. Banks take x as given so that a bank's problem is unchanged. A type-H borrower with an opportunity to fund a project receives a payoff $r_H - r_t(x_t) + u$ while a type-L borrower receives an expected payoff of $(1-z)u + zJ_t^r$. With small u (and hence small J_t^r), the payoff to a type-L borrowers is independent of lending standards (their payoff is always zero). Thus, in this case, the payoff to a type-H borrowers, $r_H - r_t(x_t)$, determines the incentive to be type-H relative to type-L. This incentive is decreasing in x through higher interest rates. So a larger x associated with tighter lending standards is associated with a *lower* incentive to be type-H relative to type-L. [23]

Note that if tight lending standards increase the incentive to be type-L, then our strategic complementarity would be even stronger, possibly to the extent of leading to equilibrium multiplicity. The reason for this is that whenever the effective pool quality x_t is low, not only are banks screening, further reducing *future* pool quality; but due to higher credit spreads, fewer borrowers now exert effort, reducing *current* effective pool quality. Thus, the complementarity at the core of this paper is amplified.

7.2 Screening with contracts

Rejected borrowers know they are type-L. Our modeling assumptions imply that banks cannot use contract terms like fees or covenants to screen out these low-quality borrowers.

Consider instead a situation in which banks use collateral requirements to screen potential borrowers. Let borrowers be endowed with a collateral asset and suppose banks

[23] When $u > 0$, there is a countervailing force that increases the incentive to be type-H. This is that a type-L borrower gets funded sooner when lending standards are loose, but this incentive is proportional to the small utility benefit.

require borrowers to post their collateral to receive a loan. As long as the value of the collateral exceeds the private benefits, previously rejected borrowers would not apply for loans if all banks require collateral. In this situation, the strategic complementarity remains, but the negative externality is eliminated.

However, in practice, several issues arise. First, when private benefits are unknown to banks, as long as the private value is greater than the collateral value for some type-L borrowers, they still apply for loans. In this case, collateral does not resolve the negative externality. Second, collateral values vary, and banks may be able to acquire costly private information about this value (e.g. appraise the collateral). In this case, the share of borrowers with good collateral would determine whether banks applied normal or tight lending standards to the collateral of borrowers, and these lending standards would act much like those in our model, exhibiting strategic complementarities and negative externalities among banks.

7.3 Credit bureaus

One reason underlying the negative externality from tight lending standards is the assumption that information on previous rejections is unobservable and non-verifiable. Does this mean our model is inapplicable to credit markets in which credit bureaus track potential borrowers?

First, note that our model applies to information above and beyond publicly available information. As we described, it applies to potential borrowers within a given credit score bracket, which summarizes the past credit information contained in the credit bureau. Second, credit bureaus typically do *not* track much of the information that lenders might investigate prior to making a loan, and that might be uncovered by tight lending standards in our model. Appendix C contains extensive details about the prevalence, coverage, and information provided by credit bureaus around the world. None of the countries that we investigated have credit bureaus that report whether credit was denied or instead turned down by the potential borrower. While many credit bureaus, like consumer bureaus in the US, delineate whether a credit check is *hard*, meaning associated with an application for credit, or *soft*, due to account review, marketing, or possibly hiring, only about half of the credit bureaus report the purpose of previous credit checks. Even with information on hard credit checks, lenders typically cannot tell whether a potential borrowers who recently applied for a loan, applied for a mortgage, a car loan, or a credit card (again, see Appendix C for details).[24]

[24] Further, a past credit check without a subsequent loan does not indicate that a given borrower failed a

Finally, we note that our model may apply even in situations in which credit bureaus accurately track borrowers if lending standards are applied to collateral instead of borrowers. That is, if tight lending standard evaluate and reject on the basis of collateral (e.g. an appraisal of a house), then credit bureaus do not address the externality of tight lending standards because they track borrowers, not the assets they wish to fund.

Given the negative externality in our model, why do credit bureaus not arise to track rejections fix the negative externality? Our model suggests that bureaus do not track credit rejections because it is not incentive compatible for a bank borrow pair to report a negative evaluation or to report a rejection.[25] Statements from credit bureaus are consistent with this reasoning and suggest that credit bureaus are unable to enforce the reporting of soft information that it is not privately optimal to report (see Appendix C).

We conclude that mature credit markets in legal environments with low-cost enforcement mechanisms may exhibit various mechanisms that mitigate but unlikely eliminate the negative externality associated with tight lending standards.

7.4 Further non-essential assumptions

We simplified the analysis by assuming that the banking sector is competitive so that banks make zero profits. We conjecture that the qualitative features of the steady-states, dynamics and welfare results would remain if banks shared the surplus of a match with a given potential borrower.

Do our main results rely on our specific screening technology? Our screening technology never mistakes a type-H for a type-L. Such mistakes would imply that borrowers who are screened do not learn their type with certainty, and so there would be a distribution of beliefs among potential borrowers, with beliefs depending on the number of times a borrower had been screened and rejected for a loan. Such complexity would change the

past lending standard. The borrower may have applied for a job, or may have simply decided not to take the loan (decided not to buy that house or car, or decided to pick a different credit card). Importantly, in practice, lenders can evaluate potential borrowers before verifying their information via a credit report and leave no trail of credit checks for rejected borrowers. That is, as is common in mortgage markets for example, banks can fully apply their lending standards on the basis of information reported by the borrower and additional information gathered by the bank, and only verify information with a credit report for borrowers that pass the lending standard. Thus many applications are not recorded.

[25] Other models suggest different reasons. For example Axelson and Makarov (2019) show the striking result that introducing a credit registry that tracks borrowers' loan application histories but not the borrowing rates offered can lead to more adverse selection and quicker market breakdown. In that model, acquiring information on a borrower is costless and the result follows from the fact that a lender who knows that a borrower's offer was rejected does know whether the borrower was bad or whether the borrower made a too-low interest rate offer.

exact formula for $\dot{x}$. And it would complicate the analysis of the slow thawing region by potentially making possible regions with different speeds of slow thawing. Apart from this region however, since borrowers with different beliefs behave identically in the model (outside of any slow thawing region), our main results would remain intact.

Other changes to the screening technology are less consequential. Our model can easily incorporate a screening technology with a cost that is non-linear in the lending standard z. Concavity replicates our current results. Convexity would imply that rather than necessarily screening at a level of $\bar{z}$, banks might choose a lower level instead, equating the marginal benefit and cost of screening. Then a bank's lending standard would be increasing in x. There is also the possibility of more than two steady states, which occurs when the optimal lending standards line in Figure 2 decreases more smoothly and so has more intersections with the $\dot{x} = 0$ line. We assumed that screening produces a binary signal, and it would be inconsequential to instead assume a continuous signal as banks would simply choose a cutoff value for their binary lending decision. Lastly, if screening were correlated across banks, this would increase the strategic complementarity at the heart of our model since when one bank screens and rejects a potential borrower, it makes it easier for the next bank to detect that borrower as bad and so raises the private value to screening.

Finally, our model has debt contracts. But there is an equity contract that delivers exactly the same payoffs to banks and borrowers of each type. This equivalence arises because the model has only two types of borrowers. With more types, our model could become significantly more complex in general. While the degree of complexity would depend on how well the screening technology detected different types, the extensions we have considered have all involved more state variables, which raises the possibility of non-linear dynamics that can occur in such systems.

8 Concluding remarks

We develop a dynamic theory of lending standards, based on two intuitive ideas. First, tighter lending standards lead to the rejection of unprofitable loan applications. And second, it is not costless for banks to identify unprofitable applicants, even those previously rejected by other banks. These two ideas give rise to a dynamic strategic complementarity between banks, which leads to more persistence in lending standards than in business cycles themselves. The ideas also generate negative externalities of tight lending standards, implying that lending standards are too tight for too long after negative shocks.

Which markets does our theory apply to? These principle ideas provide some guidance. The first idea is likely true for *any* kind of lending standard. The second suggests that markets in which borrowers are likely to shop for loans from multiple lenders are particularly exposed to the results in this paper. This includes markets in which borrowers have pre-existing relationships with many lenders (such as corporate lending markets) and markets in which borrowers need to roll over loans (as this raises the likelihood of looking for loans after rejection). In addition, the second idea requires lenders to rely on costly private information when applying lending standards. In contrast, markets in which borrowers have limited ability or need to approach multiple banks, or in which the outcome of lending depends purely on public information, are unlikely to be subject to the conclusions from our theory.

Our paper opens up several avenues for future research that build on our model and analysis of the negative externality associated with tight lending standards. For example, the government intervention that is analyzed effectively assumes that the negative shock that put the market on the path to the inefficient steady state is expected to never recur. But suppose it might recur. Is government intervention still optimal? Addressing this question requires a specification of the social cost of a government policy, e.g., a subsidized loan guarantee. We expect that it can be shown that if the likelihood that the shock recurs is low enough, then government intervention will be value enhancing. This is because with a low probability of a negative shock, the cost of an intervention can be amortized over a longer period of time.

Another interesting extension is to investigate the interaction of lending standards with other factors that drive bank profits and dynamic choices, such as variation in the competitiveness of the banking sector or dynamic choices of capacity to perform careful evaluation of borrowers. Similarly, we study the effect of simple, exogenous capital constraints on lending volume, and there may be interesting feedback dynamics from lending standards back into bank balance sheets and constraints. Finally, our analysis is partial equilibrium, an analysis of a single lending market. An interesting question is how and whether lending standards interact across credit markets, and how general equilibrium effects through interest rates in particular feed back into lending standards.

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A Proofs and derivations

A.1 Steady state equilibria: Proof of Proposition 1

The three pairs (x, z) mentioned in Proposition 1 are solutions to (8) and (9) if $\lambda > \overline{x}$, $x^s < \overline{x}$, and $\frac{\lambda-\overline{x}}{\lambda-\lambda\overline{x}}\left(1+\delta\kappa^{-1}\right) < \overline{z}$. The first two of these hold by Assumption 2 and the third is a straight consequence of the second.

We claim that the three pairs indeed constitute equilibria, with $\theta = 1$, $\varphi^a = \varphi^r = 1$ and with r pinned down by Proposition 3. To prove this, first note that the law of motion (6) as well as the bank's maximization problem (4) are satisfied due to (8) and (9). The zero profit condition (5) pins down the interest rate (see our proof to Proposition 3). Finally, in any steady state the average borrower strictly prefers a loan today, that is,

$$\lambda\left(r_H - r + u\right) + (1-\lambda)(1-z)u + (1-\lambda)zJ^r - J^a > 0,$$

and since $r \leq r_H$ (which holds since $x^s \geq \underline{x}$ with $\underline{x}$ as in (10) due to Assumption 2) we have that $\theta = 1$ and $\varphi^a = \varphi^r = 1$.

A.2 Proof of Corollary 1

The flow of projects being funded in the pooling steady state is κ, compared to $\kappa x^s + \kappa(1 - x^s)(1-\overline{z})$ in the screening steady state. The credit spread result follows directly from Proposition 3 and the fact that $r(x)$ is strictly decreasing in x. The equilibrium default rate is given by

$$\frac{\kappa(1-x)(1-z)}{\kappa(1-x)(1-z)+\kappa x} = \left(1 + \frac{x}{(1-x)(1-z)}\right)^{-1}$$

which can further be simplified to

$$(1-\lambda)\left(1 + \frac{\lambda z\delta\kappa^{-1}}{(1+\delta\kappa^{-1})(1-z)}\right)^{-1}.$$

Thus, when $\delta = 0$, the equilibrium default rate is always equal to $1-\lambda$, irrespective of the steady state.

A.3 Proof of Proposition 2

Begin with $x_0 \in (\overline{x}, \lambda]$. In that case, $z = 0$ is is the optimal bank strategy (see (4)), and therefore the law of motion of x, (6), reads

$$\dot{x}_t = \kappa(1 - x_t)\lambda - \kappa x_t(1 - \lambda) + \delta(\lambda - x_t) = (\kappa + \delta)(\lambda - x_t) > 0$$

which is positive for any $x_t < \lambda$, implying convergence to the pooling steady state.

Next turn to $x_0 \in [\underline{x}, \overline{x})$. In that case, $z = \overline{z}$ is the optimal bank strategy (see (4)), and therefore the law of motion of x, (6), reads

$$\dot{x}_t = \kappa(1 - x_t)(1 - \overline{z})\lambda - \kappa x_t(1 - \lambda) + \delta(\lambda - x_t) = (\delta - \kappa\overline{z}\lambda + \kappa)\left(x^s - x_t\right)$$

implying convergence to the screening steady state.

For $x_0 < \underline{x}$, note that $\theta_t = 0$ and so

$$\dot{x}_t = \delta(\lambda - x_t) > 0$$

implying that the pool quality improves until it crosses $\underline{x}$ and thereafter converges to the screening steady state.

The case of $x_0 = \overline{x}$ is straightforward as $\overline{x}$ is already a steady state.

A.4 Proof of Proposition 3

The zero profit condition (5) implies that

$$\Pi(R) = \kappa_H r + \kappa_L(1 - z)r_L - (\kappa_H + \kappa_L)\tilde{c}z = 0.$$

Reformulating this we obtain

$$\kappa x r / r_L + \kappa(1 - x)(1 - z) + \kappa c z = 0$$

$$r = -r_L \frac{cz + (1 - x)(1 - z)}{x}$$

which proves Proposition 3.

A.5 Proof of Proposition 4

Define $\theta(x)$ as in (12) and define $\hat{x}$ implicitly as the unique value of $x < \lambda$ with $\theta(x) = 1$. Such a value exists since $\theta(x)$ is strictly increasing and continuous in x with $\theta(0) = -\delta\kappa^{-1} < 0$ and $\lim_{x\to\lambda}\theta(x) = \infty$.

Assume $\hat{x} > \overline{x}$. Conjecture for any $x_0 \in [\overline{x}, \hat{x})$ that the equilibrium is one with $\theta_t = \theta(x_t)$. To verify the conjecture, we need to show that average borrowers are indifferent between taking a loan and waiting. Assuming $u \to 0$ in (1a), this is equivalent to

$$J_t^a = \lambda(r_H - r(x_t))$$

with

$$\rho J_t^a = \dot{J}_t^a - \delta J_t^a.$$

Putting the two together, we obtain (13),

$$-\lambda r'(x)\dot{x} = (\rho + \delta)\lambda\left(r_H - r(x)\right).$$

The law of motion for x with $\theta < 1$ is $\dot{x}_t = (\kappa\theta + \delta)(\lambda - x)$, which, together with (13) yields (12) and therefore confirms that average borrowers are, by construction, precisely indifferent.

A.6 Proof of Corollary 3

By Assumption 2, $c \geq 1 - \lambda$. Therefore, welfare in the screening steady state is bounded above,[26]

$$W^s = x^s r_H - (1-\overline{z})(1-x^s)r_L - \tilde{c}\overline{z} = x^s\frac{r_H}{-r_L} - (1-\overline{z})(1-x^s)r_L - c\overline{z}$$

$$\leq x^s\frac{r_H}{-r_L} - (1-x^s)(1-\overline{z}) - (1-\lambda)\overline{z} = x^s\left(\frac{r_H}{-r_L} + 1 - \overline{z}\right) - (1-\lambda\overline{z})$$

Welfare in the pooling steady state is $W^p = \lambda(\frac{r_H}{-r_L} + 1) - 1$. Observe that W^s increases in x^s, so W^s can only ever be above W^p if x^s is as large as possible. Clearly, given the formula

[26] We define all welfare expressions here as multiples of κ, for expositional clarity. κ multiplies both W^s and W^p equally.

for x^s, x^s is largest as a function of δ if $\delta = \infty$ where $x^s = \lambda$. In that case, we find

$$W^s \leq x^s \left(\frac{r_H}{-r_L} + 1 - \bar{z} \right) - (1 - \lambda \bar{z}) < \lambda \left(\frac{r_H}{-r_L} + 1 - \bar{z} \right) - (1 - \lambda \bar{z}) = \lambda \left(\frac{r_H}{-r_L} + 1 \right) - 1 = W^p$$

Therefore, welfare of the pooling steady state always dominates that of the screening steady state.

A.7 Proof of Proposition 5

We prove Proposition 5 in two steps. First, we determine the efficient screening policy $z^*(x)$ conditional on banks operating. Then we determine the optimal behavior for banks to operate $\theta^*(x)$.

A.7.1 Optimal screening policy $z^*(x)$

To do so, let $V(x,z)$ denote the present value of welfare if the current state of the market is x and the screening policy is z from hereafter, that is,

$$V(x,z) \equiv \frac{\rho x + \alpha^z x^z}{\rho + \alpha^z} r_H + (1-z) \left(1 - \frac{\rho x + \alpha^z x^z}{\rho + \alpha^z} \right) r_L - \tilde{c} z. \tag{27}$$

where $\alpha^z \equiv \kappa + \delta - \lambda \kappa z$ and $x^z \equiv \lambda - \lambda \frac{(1-\lambda)z}{(1-\lambda z) + \delta \kappa^{-1}}$. Also, denote by

$$v(x,z) \equiv \rho \left\{ x r_H + (1-z)(1-x) r_L - \tilde{c} z \right\} \tag{28}$$

the flow value of policy z at state x. Finally, we call

$$d(x,z) \equiv \kappa(1-x)(1-z)\lambda - \kappa x(1-\lambda) + \delta(\lambda - x) \tag{29}$$

the derivative of x at state x under policy z (see the law of motion in (6)). Observe that

$$\rho V(x,z) = v(x,z) + V_x(x,z) d(x,z) \tag{30}$$

as well as

$$d(x^s, \bar{z}) = 0 \quad d(x^p, 0) = 0. \tag{31}$$

We first prove the following helpful lemma.

Lemma 1. *We have:*

1. If $\lambda\kappa r^{\Delta} \geq \rho + \kappa + \delta$, pooling is strictly optimal for any state x, i.e. $z^*(x) = 0$.
2. If $\lambda\kappa r^{\Delta} < \rho + \kappa + \delta$, $V(x,z)$ has negative cross-partials, $V_{xz} < 0$.
3. If $\lambda\kappa r^{\Delta} < \rho + \kappa + \delta$ and $V(x,0) > V(x,z_1)$ for some $z_1 > 0$, then also $V(x,0) > V(x,z_2)$ for any $z_2 \in (0,z_1)$.

Proof. Assume $\lambda\kappa r^{\Delta} \geq \rho + \kappa + \delta$. Suppose pooling were not strictly optimal for every state x. First, if $d(x,z^*(x))$ is ever negative for some $x < \lambda$, there must be a steady state at some $x^0 \in [0,\lambda)$ with some $z^0 = z^*(x^0) > 0$. This cannot be optimal since

$$V(x^0,z^0) < V(x^0,0)$$

is equivalent to (after a few lines of algebra)

$$-\left(\rho + (1-\lambda)\alpha^p - \rho x^0\right)\left(r^{\Delta}\kappa\lambda - (\rho+\kappa+\delta)\right) < (\rho+\alpha^s)(\rho+\alpha^p)c$$

which is true since the left hand side is negative. Second, assume $d(x,z^*(x))$ is positive everywhere. Then, $x^p = \lambda$ is still the unique steady state. Let $\mathbf{V}(x)$ be the optimal value function. It has to hold that

$$r\mathbf{V}(x) = v(x,z^*(x)) + \mathbf{V}'(x)d(x,z^*(x)). \tag{32}$$

Rearranging,

$$\mathbf{V}'(x) = \frac{r\mathbf{V}(x) - v(x,z^*(x))}{d(x,z^*(x))} \equiv \mathbf{F}(\mathbf{V}(x),x).$$

Compare this to the ODE describing the value of pooling,

$$V_x(x,0) = \frac{rV(x,0) - v(x,0)}{d(x,0)} = \mathbf{F}^0(V(x,0),x)$$

Observe that $\mathbf{F}(V,x) > \mathbf{F}^0(V,x)$ for any x for which $z^*(x) > 0$.[27] Since $\mathbf{V}(x^p) = V(x^p,0)$, it must be that $\mathbf{V}(x) < V(x,0)$ for some x if there is a positive measure where $z^* > 0$. This contradicts our assumption that $\mathbf{V}(x)$ is the optimal value function. Thus, pooling is optimal for every state.

Assume $\lambda\kappa r^{\Delta} < \rho + \kappa + \delta$. Simple algebra based on (27) implies that

$$V_x = \frac{\rho}{\rho+\alpha^z}r_H + (1-z)\frac{r}{\rho+\alpha^z}r_L > 0$$

[27]Note that $\mathbf{V}'(x) > 0$ by a simple envelope argument.

and

$$V_{xz} = \rho \frac{\lambda \kappa r^{\Delta} - (\rho + \kappa + \delta)}{(\rho + \alpha^{z})^{2} r_L} < 0.$$

For point 3 in the lemma, fix $x \in [0, \lambda]$. Define the positive constants

$$c_0 \equiv \frac{\rho x + \lambda \kappa + \lambda \delta}{\lambda \kappa}, \quad c_1 \equiv \frac{\rho + \kappa + \delta}{\lambda \kappa}, \quad c_2 \equiv r^{\Delta}, \quad c_3 = -r_L$$

Then, after some algebra, we can write

$$\begin{aligned} V(x,z) &= (1-z) r_L - \tilde{c} z + c_3 \frac{c_0 - z}{c_1 - z} (c_2 - z) \\ &= r_L - \tilde{c} z + c_3 (c_0 + c_2 - c_1) + c_3 \frac{(c_1 - c_2)(c_1 - c_0)}{c_1 - z} \end{aligned}$$

c_1 is always greater than c_0. Also, given $\lambda \kappa r^{\Delta} < \rho + \kappa + \delta$, c_1 is greater than c_2. Therefore, $V(x,z)$ is a convex function in z, and thus in particularly quasi-convex, from which the stated property follows. □

In the next lemma, we narrow down the set of optimal policies using the necessary (but not sufficient) first order conditions of (14).

Lemma 2. *Describe an efficient screening policy $z^*(x)$ by the following general form: Let $I_1, I_2, I_3 \subset [0, \lambda]$ be (possibly empty) connected intervals such that $I_1 \leq I_2 \leq I_3$, $I_1 \cup I_2 \cup I_3 = [0, \lambda]$, and*

- $z^*(x) = \bar{z}$ *for* $x \in I_1$
- $z^*(x) \in [0, \bar{z}]$ *for* $x \in I_2$
- $z^*(x) = 0$ *for* $x \in I_3$

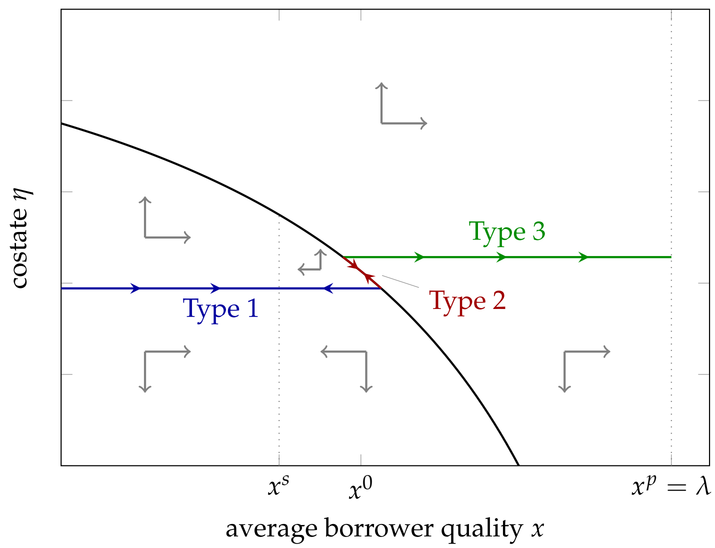
where we construct I_1 to be the largest connected interval where $z^*(x) = \bar{z}$, and similarly I_3 for $z^*(x) = 0$.

Then: If I_2 is non-empty (and thus $z^*(x)$ not bang-bang), there exists a $x^0 \in I_2$ with $d(x^0, z^*(x^0)) = 0$.

Proof. We begin by writing down the necessary first order conditions of (14). Denoting by η the costate of x, we have the law of motion

$$\dot{\eta} = \rho \eta - \kappa \left\{ r_H - (1 - z_t) r_L \right\}$$

Figure 13: Phase diagram for constrained efficient problem



as well as the first order condition for z, showing that $z_t = \bar{z}$ if

$$\kappa \left\{-(1 - x_t)r_L - \tilde{c}\right\} - \eta \left(\kappa(1 - x_t)\lambda\right) > 0 \tag{33}$$

and $z_t = 0$ if (33) hold with "<" inequality; with equality, z_t can be anywhere in $[0, \bar{z}]$.

Together with the law of motion of x in (6), this gives a system of two ODEs. We first note that there are three possible steady states. The two steady states for $z = 0$ and $z = \bar{z}$, as well as a third one, $z = z^0$ pinned down by $\dot{\eta} = \dot{x} = 0$ and (33) holding with equality,

$$\frac{-r_L}{\lambda} - \frac{\tilde{c}/\lambda}{1 - x^0} = \frac{\kappa}{\rho} \left\{ r_H - (1 - z^0) r_L \right\} \tag{34}$$

where

$$x^0 = \lambda \frac{1 - z^0 + \delta\kappa^{-1}}{(1 - \lambda z^0) + \delta\kappa^{-1}}. \tag{35}$$

Observe that, after substituting (35) into (34), the left hand side of (34) is increasing in z^0, while the right hand side is decreasing, so there is at most a single solution to (34).

Now consider the phase diagram in Figure 13. As can be seen, there are 3 types of candidate optimal paths. Type 1 converges to x^s, with $z = \bar{z}$ along the path and constant η; Type 3 converges to x^p, with $z = 0$ along the path and constant η; and finally type 2 converges to x^0 as in (35). Observe that the second type of paths only works if z^0, x^0 exist, solving (34) and (35).

This implies that, unless the optimal policy $z^*(x)$ is bang-bang, there has to exist a x^0 with $d(x^0, z^*(x^0)) = 0$. $\square$

With this result in mind, we assume in the following that $\lambda\kappa r^\Delta < \rho + \kappa + \delta$ and characterize $z^*(x)$.

Lemma 3. *Assume $\lambda\kappa r^\Delta < \rho + \kappa + \delta$. The efficient screening policy $z^*(x)$ is to screen if $x < \overline{x}^*$ and to pool if $x > \overline{x}^*$, where*

$$V(\overline{x}^*, 0) = V(\overline{x}^*, \overline{z}) \tag{36}$$

as long as the solution to that equation is greater or equal to x^s. Otherwise, $\overline{x}^*$ is determined by

$$v_z(\overline{x}^*, 0) + V_x(\overline{x}^*, 0) d_z(\overline{x}^*, 0) = 0. \tag{37}$$

Proof. First, notice that $\overline{x}^*$ is indeed well-defined, in that if the solution to (36) is x^s, then (37) is also solved by x^s. Assume

$$V(x^s, 0) = V(x^s, \overline{z}).$$

Combining (30) and (31), we can rewrite $V(x^s, 0)$ and $V(x^s, \overline{z})$ and obtain

$$v(x^s, 0) + V_x(x^s, 0) d(x^s, 0) = v(x^s, \overline{z}) + V_x(x^s, \overline{z}) d(x^s, \overline{z}).$$

Since $d(x^s, \overline{z}) = 0$, this can be combined into

$$v(x^s, \overline{z}) - v(x^s, 0) + V_x(x^s, 0) \left(d(x^s, \overline{z}) - d(x^s, 0) \right) = 0 \tag{38}$$

which is equivalent to (37) as v and d are linear in z. Moreover, going these steps backwards, if $\overline{x}^* < x^s$, then (38) holds with inequality and therefore

$$V(x^s, 0) > V(x^s, \overline{z}). \tag{39}$$

Now we proceed to our main argument, a proof by contradiction. We distinguish four possible cases.

Case 1: There exists $x > \overline{x}^*$ with $x \geq x^s$ where pooling is not optimal. If true, by Lemma 2, this would require there to be at least one point $x^0 \in [\overline{x}^*, \lambda)$ where the planner strictly prefers to remain at x^0 forever (by choosing strategy $z^0 \in (0, \overline{z}]$ such that $d(x^0, z^0) =$

0) over pooling. In math,

$$V(x^0, z^0) > V(x^0, 0).$$

Since V has a negative cross-partial $V_{xz} < 0$ (Lemma 1), this implies that $V(\overline{x}^*, z^0) > V(\overline{x}^*, 0)$ and $V(x^s, z^0) > V(x^s, 0)$, which, by point 3 in Lemma 1, is contradicting either (36) or (39).

Case 2: There exists $x < \overline{x}^*$ with $x \geq x^s$ where screening is not optimal. If true, by Lemma 2, this would require there to be at least one point $x^0 \in (x^s, \overline{x}^*]$ where the planner strictly prefers to remain at x^0 forever (by choosing strategy $z^0 \in [0, \overline{z})$ such that $d(x^0, z^0) = 0$) over screening. In math,

$$V(x^0, z^0) > V(x^0, \overline{z}).$$

Since V has a negative cross-partial $V_{xz} < 0$ (Lemma 1), this implies that $V(\overline{x}^*, z^0) > V(\overline{x}^*, \overline{z})$, which by point 3 in Lemma 1, contradicts (36).

Case 3: There exists $x > \overline{x}^*$ with $x \leq x^s$ where screening is optimal. If true, this would require there to be at least one point $x^0 \in [\overline{x}^*, x^s]$ where the planner strictly prefers to screen with some intensity $z^0 > 0$ in the current instant while pooling is chosen thereafter. That is,

$$v(x^0, z^0) + V_x(x^0, 0)d(x^0, z^0) > v(x^0, 0) + V_x(x^0, 0)d(x^0, 0).$$

Due to linearity of this equation, it also has to hold with $z^0 = \overline{z}$, and therefore also expressed as derivative,

$$v_z(x^0, 0) + V_x(x^0, 0)d_z(x^0, 0) > 0. \tag{40}$$

Since this is a linear equation in x^0, to be consistent with (37), it must be that (40) in fact holds for any $x^0 > \overline{x}^*$, including $x^0 = x^p = \lambda$. In that case, however, (40) simplifies to $v_z(x^p, 0) + V_x(x^p, 0)d_z(x^p, 0) > 0$, which is false, since $V_x(x, 0) > 0$, $d_z(x, 0) < 0$ and $v_z(x^p, 0) = -\kappa r_L\,(c - (1 - \lambda)) < 0$ by Assumption 2.

Case 4: There exists $x < \overline{x}^* \leq x^s$ where pooling is optimal. Let $\mathbf{V}(x)$ be our conjectured value function left of $\overline{x}^*$. By design, $\mathbf{V}(x)$ solves

$$\rho\mathbf{V}(x) = v(x, \overline{z}) + \mathbf{V}'(x)d(x, \overline{z})$$

where $d(x,\overline{z}) = \alpha^{\overline{z}}(x^s - x)$ and $\mathbf{V}'(x)$ solves

$$(r + \alpha^{\overline{z}})\mathbf{V}'(x) = v_x(x,\overline{z}) + \mathbf{V}''(x)d(x,\overline{z}).$$

This ODE can be solved explicitly, giving[28]

$$\mathbf{V}'(x) = \rho r_L \left(\frac{r^\Delta}{\rho + \alpha^p} - \frac{r^\Delta - \overline{z}}{\rho + \alpha^{\overline{z}}}\right)\left(\frac{x^s - x}{x^s - \overline{x}^*}\right)^{-\beta} + \rho r_L \frac{r^\Delta - \overline{z}}{\rho + \alpha^{\overline{z}}}$$

where $\beta = 1 + \frac{\rho}{\alpha^{\overline{z}}}$. The coefficient on the first term is positive, since we assumed $r^\Delta \lambda\kappa < \rho + \kappa + \delta$. Thus, $\mathbf{V}'(x)$ is bounded above by

$$\mathbf{V}'(x) \leq \mathbf{V}'(\overline{x}^*) = r(1 - R_L)\frac{r^\Delta}{\rho + \alpha^p}. \tag{41}$$

Could it ever be that the planner prefers pooling in this region? If so, we would have an $x < \overline{x}^*$ with

$$v_z(x,0) + \mathbf{V}'(x)d_z(x,0) < 0$$

which due to (41) and the fact that $d_z(x,0) = -\kappa\lambda(1-x) < 0$ implies that

$$v_z(x,0) + \mathbf{V}'(\overline{x}^*)d_z(x,0) < 0.$$

Using the expressions in (28) and (29) we then see that this cannot hold as the left hand side is zero at $\overline{x}^*$ (by definition), and has a negative slope throughout,

$$v_{xz} + \mathbf{V}'(\overline{x}^*)d_{xz} = \rho r_L \left[-1 + \frac{r^\Delta \lambda\kappa}{\rho + \alpha^p}\right] < 0$$

where again we used $r^\Delta\lambda\kappa < \rho + \kappa + \delta$. This is a contradiction: there cannot be an $x < \overline{x}^*$ where pooling is optimal. □

A.7.2 Optimal bank operation policy $\theta^*(x)$

Next we focus on the optimal policy $\theta^*(x)$ for banks to operate. We prove the following result.

Lemma 4. *If it is strictly optimal to have banks operate at $\overline{x}^*$, the optimal policy describing when*

[28] Note that $v_x(x,\overline{z})$ is a constant in x.

banks operate is bang-bang, that is,

$$\theta^*(x) = \begin{cases} 0 & x < \underline{x}^* \\ 1 & x > \underline{x}^* \end{cases} \tag{42}$$

The threshold $\underline{x}^*$ is the supremum of all $x \in [0, \lambda]$ that solve

$$v(x, z^*(x)) + \mathbf{V}'(x) \left(\kappa \left(\lambda - x\right) - \kappa(1-x)z^*(x)\lambda\right) < 0 \tag{43}$$

where $\mathbf{V}(x)$ is the value function associated with the optimal screening policy $z^*(x)$.

Proof. Let $\underline{x}^*$ be defined as in (43) and let $\mathbf{V}(x)$ be the value function conditional on banks operating with screening policy $z^*(x)$. If it is optimal for the planner to follow the bang-bang policy (42), then its value function for $x \geq \underline{x}^*$ is given by $\mathbf{V}(x)$, whereas for $x < \underline{x}^*$ the value function solves

$$\rho \mathbf{V}(x) = \mathbf{V}'(x)\delta(\lambda - x)$$

which can be solved to express the marginal value in state x as

$$\mathbf{V}'(x) = \mathbf{V}'(\underline{x}^*) \left(\frac{\lambda - x}{\lambda - \underline{x}^*}\right)^{-1-\rho/\delta}.$$

Observe that this is increasing in x. To prove that the bang-bang policy (42) is indeed optimal, we need to prove that

$$\max_{z \in [0, \bar{z}]} v(x, z) + \mathbf{V}'(x) d(x, z, 1) \leq \max_{z \in [0, \bar{z}]} \mathbf{V}'(x) d(x, z, 0) \tag{44}$$

for $x < \underline{x}^*$, where

$$d(x, z, \theta) \equiv \theta\kappa(1-x)(1-z)\lambda - \theta\kappa x(1-\lambda) + \delta(\lambda - x)$$

is the speed at which the pool improves given (x, z, θ). Simplifying (44), we obtain

$$\max_{z \in [0, \bar{z}]} v(x, z) + \mathbf{V}'(x) \left[\kappa\lambda(1-z) - \kappa x(1 - z\lambda)\right] \leq 0.$$

The left hand side of this inequality has a negative cross-partial in (x, z), since $v_{xz} < 0$ and $\mathbf{V}'(x)(1-x) \propto (\lambda - x)^{-\rho/\delta} \frac{1-x}{\lambda - x}$ increases in x. Thus, given that $z = \bar{z}$ is optimal for $x = \underline{x}^*$, it is also optimal for any $x < \underline{x}^*$.

The problem then reduces from (44) to showing that for $x < \underline{x}^*$

$$F(\lambda - x) \equiv v(x, \bar{z}) + \mathbf{V}'(x)\left[\kappa\lambda(1-\bar{z}) - \kappa x(1-\bar{z}\lambda)\right] < 0. \tag{45}$$

To see this, we first show that $F(y)$ is quasi-concave (only has a single local maximum) and therefore can at most have two roots. $F(y)$ is of the form

$$F(y) = -F_0 y + F_1 y^{-\alpha-1}(y - y_0) + const$$

where $\alpha = \rho/\delta > 0$, $F_0 = \rho r_H + \rho(1-\bar{z})r_L > 0$, $F_1 = \kappa(1-\lambda\bar{z})\mathbf{V}'(\underline{x}^*)(\lambda - \underline{x}^*)^{1+\alpha} > 0$, $y_0 = \lambda - x^s > 0$. F can only ever have a single local maximum as long as these parameters are positive:

$$F'(y) = 0 \quad \Leftrightarrow \quad y^{-\alpha-2}\left[(1+\alpha)y_0 - \alpha y\right] = F_1/F_0$$

The left hand side of this equation is strictly decreasing for $y \in (0, (1+\alpha)y_0/\alpha)$ with range $(0, \infty)$ and thus admits a unique solution for any $F_1/F_0 > 0$. This establishes that $F(y)$ is quasi-concave.

Since $F(y)$ is quasi-concave, it admits at most two roots, $y_1 < y_2$, in between which $F(y)$ is positive, and negative outside of $[y_1, y_2]$. Root y_2 must correspond to $\lambda - \underline{x}^*$: if y_1 were to correspond to $\lambda - \underline{x}^*$, $\underline{x}^*$ would not be the supremum of x with $F(\lambda - x) < 0$ since for any $\epsilon > 0$ small enough, $F(\lambda - (\underline{x}^* - \epsilon)) > 0$. But if $y_2 = \lambda - \underline{x}^*$, then $F(\lambda - x) < 0$ for any $x < \underline{x}^*$, which proves (45). □

A.8 Proof of Proposition 6

There exists $\bar{x}^C \geq \bar{x}$ such that banks screen if $x < \bar{x}^C$ and $\bar{V}_t < \kappa x_t + \kappa(1-x_t)(1-\bar{z})$. Banks prefer to screen at rate $z > 0$ iff

$$\frac{x_t r_t + (1-x_t)(1-z)r_L}{1-(1-x_t)z} - \frac{\tilde{c}z}{1-(1-x_t)z} > x_t r_t + (1-x_t)r_L$$

which simplifies to

$$0 > (r_t - r_L)x_t^2 - (r_t - r_L)x + \tilde{c}.$$

The roots are

$$x_t^C = \frac{1}{2}\left[1 \pm \sqrt{1 - \frac{4\tilde{c}}{r_t - r_L}}\right] \tag{46}$$

where we denote the larger one by

$$\overline{x}_t^C = \frac{1}{2}\left[1 + \sqrt{1 - \frac{4\tilde{c}}{r_t - r_L}}\right].$$

As $\tilde{c} \to 0$, the two roots (46) converge to 0 and 1, meaning tight standards everywhere.

Proof that $\bar{x}_t^C \geq \bar{x}$. We first prove a useful property. The interest rate at $\overline{x}$ in the unconstrained equilibrium is given by

$$r(\overline{x}) = (-r_L)\frac{1 - \overline{x}}{\overline{x}}.$$

With this interest rate we find a cutoff $\overline{x}^C$ of

$$\overline{x}^C(r = r(\overline{x})) = \frac{1}{2}\left[1 + \sqrt{1 - \frac{4\tilde{c}}{r(\overline{x}) - r_L}}\right] = 1 - c = \overline{x}. \tag{47}$$

Next we prove that $\bar{x}_t^C \geq \bar{x}$ more generally. We proceed by contradiction. Suppose there existed an $x_0 < \overline{x}$ and a continuous unbounded increasing function $\overline{V}_t$ starting with $\overline{V}_0 < \kappa x_0 + \kappa(1 - x_0(1 - \overline{z})$ such that pooling is optimal, that is, $x_0 > \overline{x}_t^C$. Observe that since $\tilde{r}_t$ declines monotonically along the transition and x_t increases, this also implies that $x_t > \overline{x}_t^C$ at any future time t. Thus, pooling remains optimal.

Let T be the time at which banks become unconstrained. It has to be the case that $x_T > \overline{x}$. If not, we would have a contradiction since $\overline{x}_T^C < x_T$ per the discussion above, but at the same time

$$\overline{x}_T^C = \overline{x}^C(r = r(x_T)) > \overline{x}^C(r = r(\overline{x})) = \overline{x}$$

following from (47) and the fact that $r(\cdot)$ is monotone (Proposition 3). Thus, $x_T > \overline{x}$.

Now compare the path x_t that ultimately reaches x_T to a different path that reaches x_T. This second path has unconstrained banks and begins an ϵ to the right of $\overline{x}$. On its way to the pooling steady state, it crosses x_T. Observe that this second path has a slower decline in the interest rate r_t as average borrowers are not indifferent along the path, $0 > \dot{r}_t > \dot{\tilde{r}}_t$. Moreover, the transition along the second path is faster as banks are unconstrained. Together, this implies that the initial interest rate $\tilde{r}_0$ along the first path must have been greater than the initial interest rate $r(\overline{x})$ along the second path. But this means that

$$\overline{x}^C(r = \tilde{r}_0) > \overline{x}^C(r = r(\overline{x})) = \overline{x}$$

which is a contradiction to our assumptions $x_0 < \overline{x}$ and $x_0 > \overline{x}^C(r = \tilde{r}_0) = \overline{x}_0^C$.

The case $\kappa x_t + \kappa(1-\bar{z})(1-x_t) < \overline{V}_t < \kappa$. Consider first $x_t > \bar{x}$. If banks were to impose tight lending standards, $z_t = \bar{z}$, then they would be unconstrained and the interest rate equals the interest rate in the competitive equilibrium. At the competitive interest rate, banks prefer tight lending standards only if $x_t \leq \bar{x}$. Thus for $x_t > \bar{x}$, it can only be that $z_t < \bar{z}$ and we need only consider whether normal lending standards dominate intermediate ones. The largest z_t that does not make banks unconstrained is $z_t = \frac{1-\overline{V}_t/\kappa}{1-x_t}$.

Given our derivation in Section 6 and the fact that $\bar{z}$ did not enter (26), pooling is preferable if and only if $x_t > \overline{x}_t^C$. For $x_t < \overline{x}_t^C$, $z_t = \frac{1-\overline{V}_t/\kappa}{1-x_t}$ is the optimum. For $x_t \leq \overline{x}$, banks can screen without binding constraints. Given $x_t < x_t^C$, this is their optimal policy.

B Model with non-constant pool size

For this section, we assume that the pool size is not constant. We demonstrate that this economy gives rise to the exact same steady states and the exact same welfare predictions.[29]

B.1 Equilibria

Without a constant pool size, there are two state variables: m_H, the number of type-H borrowers in the pool and m_L, the number of type-L borrowers in the pool. The laws of motion of the state variables are given by

$$\dot{m}_H = \delta\lambda - \delta m_H - \kappa m_H \tag{48}$$

$$\dot{m}_L = \delta(1-\lambda) - \delta m_L - \kappa(1-z_t)m_L \tag{49}$$

The first term in both laws of motion stems from the constant inflow of $\delta\lambda$ type-H borrowers and $\delta(1-\lambda)$ type-L borrowers. The second term captures the constant exit probability of borrowers in the pool. The final term is the flow rate of borrowers who receive a loan.

Observe that the first equation is independent of z_t. We can thus treat m_H as if it was at its steady state forever,

$$m_H = m_H^* = \frac{\delta\lambda}{\delta+\kappa} \tag{50}$$

We continue to denote the share of type-H borrowers by $x_t \equiv m_H/(m_H + m_L)$. Given (50),

[29]For simplicity, we focus on the case of active banks, $\theta_t = 1$, throughout this section.

the law of motion of x_t can then be shown to be given by

$$\frac{\dot{x}}{x/\lambda} = \kappa(1-x)(1-z)\lambda - \kappa x(1-\lambda) + \delta(\lambda - x) \tag{51}$$

The right hand side of (51) is identical to the one of (6) after substituting out κ_{Ht} and κ_{Lt} and setting $\theta_t = 1$. Thus, the only difference between the constant-pool-size model and the one in this section is that the speed in this one is altered by a factor x/λ, on the left hand side of (51). In particular, all results on steady states and their properties in Sections 3 carry over one-for-one to the model in this section.[30]

B.2 Welfare

The social planning problem now becomes

$$\max_{z_t \in [0,\overline{z}]} \int_0^\infty e^{-\rho t} \kappa \left\{ m_H^* r_H + (1-z_t) m_L r_L - \tilde{c} z_t (m_L + m_H) \right\} dt$$

subject to the law of motion for m_L, (49). One can show that it has the exact same properties as the planning problem in Section 4. Relative to the privately optimal threshold $\overline{x} = 1 - c$, which corresponds to

$$\overline{m}_L = \frac{\lambda}{1+\kappa\delta^{-1}} \frac{c}{1-c}$$

there exists a socially optimal threshold $\overline{x}^* \equiv \frac{m_H^*}{m_H^* + \overline{m}_L^*}$ where $\overline{m}_L^*$ is determined by

$$\underbrace{(1-\overline{z}+c\overline{z}) \frac{\rho \overline{m}_L^* + \alpha^s m_L^s}{\rho + \alpha^s} + c\overline{z} m_H^*}_{\text{Average social cost from lending to type-}L\text{ when screening}} = \underbrace{\frac{\rho \overline{m}_L^* + \alpha^p m_L^p}{\rho + \alpha^p}}_{\text{Average social cost from lending to type-}L\text{ when pooling}} \tag{52}$$

Here, we define the transition speeds for m_L under pooling and screening by $\alpha^p \equiv \kappa + \delta$ and $\alpha^s \equiv \kappa(1-\overline{z}) + \delta$. The associated steady state values for m_L are given by $m_L^p = \frac{\delta(1-\lambda)}{\delta+\kappa}$ and $m_L^s = \frac{\delta(1-\lambda)}{\delta+\kappa(1-\overline{z})}$. Similar to Section 4, one can show here, too, that the social planner marginally prefers more pooling, that is,

$$\overline{m}_L^* > \overline{m}_L$$

The reason is identical to that in Section 4.

[30]What becomes harder to analyze with a non-constant pool size is slow thawing, since $\theta_t < 1$ affects both type-H and type-L borrowers, i.e. m_H is no longer constant at m_H^*.

An especially simple welfare result is the comparison of steady state welfares across steady states. Here, the question is whether it is the case that welfare in the pooling steady state exceeds that in the screening steady state. In the context of this model, this is satisfied if

$$(1 - \bar{z} + c\bar{z})m_L^s + c\bar{z}m_H^* > m_L^p$$

After some algebra, this simplifies to

$$1 + \delta^{-1}\kappa > (1 - \lambda)c^{-1} + \lambda\delta^{-1}\kappa\bar{z}$$

which is necessarily the case given our Assumption 2: $c > 1 - \lambda$. Thus, Corollary 3 also carries over to this model.

C Credit bureaus

This appendix provides additional information on credit bureaus around the world. The on-line Appendix D provides the data that underlie this Appendix.

Credit bureaus, as opposed to credit registries, track potential borrowers and provide information about them to potential lenders.[31] When a potential borrower approaches a lender that is a member of a credit bureau, the lender can perform a *credit check* before making a loan, which involves getting a *credit report* from the bureau. Credit reports provide information on potential borrowers including existing credit and payment histories. In addition, many credit bureaus keep track of information about past credit checks and include this information on credit reports. Table 1 describes credit reports for credit bureaus in different countries around the world (underlying data sources are provided in on-line Appendix D).

In most countries, a bank that conducts a credit check can generally observe past credit checks and whether the potential borrower subsequently did or did not receive a loan. The information in the bureaus tends to be available only to entities in the bureau's network, although some countries' bureaus sell the information to entities outside the credit market. In some countries like Japan and Germany, bureau members are required to report in

[31] Credit registries are more widespread than credit bureaus, but registries only track the history of outstanding credit and/or loan payments and delinquencies. In our model, and probably in reality, outstanding loans do not assist banks in discriminating among borrowers who have recently been rejected. Credit registries seem to serve the purpose of providing information to assist a bank in setting loan terms, such as loan amount and interest rate based on payments-to-income ratio and/or pre-existing liens on collateral.

Table 1: Data captured by credit bureaus

	Coverage? (consumers/firms)	Reporting required?	Credit checks: On report for ≤ months	Credit checks: Hard check labeled?	Credit checks: Who requested?	Credit checks: Purpose?	Rejections reported?
Advanced Economies							
Australia	Both		60	Yes	Yes	Yes	No
Canada	Both		72	Yes	Yes	No	No
European Union (AnaCredit)	Firms	By law	0	NA	No	No	No
France	Firms	By law	0	NA	No	No	No
Germany	Both	For access	12	Yes	No	No	No
Ireland	Both	By law	60	Yes	Yes	Yes	No
Italy	Both		6	Yes		Yes	No
Japan	Consumers	For access	6	Yes	Yes	Yes	No
Singapore	Both		24	Yes	Yes	Yes	No
South Korea	Both		0	NA	No	No	No
Taiwan	Both		3	Yes	Yes	Yes	No
United Kingdom	Both		24	Yes	Yes	Yes	No
United States	Both	Voluntary	24	Yes	Yes	Yes	No
Emerging Economies							
China	Both	By law	24	Yes	Yes	Yes	No
India	Both		24	Hard only	Yes	Yes	No
Malaysia	Both		12	Hard only	Yes	No	No

Blank cells are missing data.
Note: All information is from consumer credit reports and Bureau FAQs, except for EU and France, see Appendix for sources.

exchange for access, but in other countries reporting is voluntary or only required by bureau members (second column of Table 1,). Most credit bureaus, like consumer bureaus in the US, delineate whether a credit check is *hard*, meaning associated with an application for credit, or *soft*, due to account review, marketing, or possibly hiring. Records of credit inquiries stay in credit report from 2 months in Taiwan to 24 months in the U.S. to 60 months in Ireland.

Importantly, however, none of the countries that we investigated have credit bureaus that report whether credit was denied or turned down by the potential borrower (final column of Table 1).[32] Further, credit bureaus generally contain only rudimentary information about the initiator of previous credit checks, such as whether they were banks, mortgage brokers, utilities, etc., and some in some countries, such as South Korea, France, and Germany, even this information is not recorded (fifth column). And only about half of the credit bureaus report the purpose of previous credit checks (sixth column), so that a credit card issuer for example does not know if a previous credit check was associated with an application for a credit card, mortgage, car loan or job.

A past credit check without a subsequent loan does not indicate that a given borrower failed a past lending standard. The borrower may have simply decided not to take the loan (decided not to buy that house or car, or decided to pick a different credit card). Importantly

[32] For example, Experian UK states "Here's what our role doesn't involve: - We aren't told which applications are successful or refused. - We don't know why you may have been refused credit."A possible exception is Experian Italy.

in practice, lenders can evaluate potential borrowers before verifying their information via a credit report and leave no trail of credit checks for rejected borrowers. That is, as is common in mortgage markets for example, banks can fully apply their lending standards on the basis of information reported by the borrower and additional information gathered by the bank, and only verify information with a credit report for borrowers that pass the lending standard. Thus many applications are not recorded.

As noted, our theoretical model suggests that bureaus do not track credit rejections because it is not incentive compatible for banks to report rejections. Statements from credit bureaus are consistent with this reasoning and suggest that credit bureaus are not able to enforce the reporting of soft information that it is not privately optimal to report. First, bureaus state that they want to avoid arbitrating arguments over rejections. Rejection is easy to hide (e.g. just offer unfavorable loan terms) and hard to verify (consistent with our assumptions). Second, bureaus store only verifiable information due to privacy and legal concerns. Credit checks are hard information, rejections are not. Every credit bureau lists data verification and correction measures on their website.

Finally, we re-emphasize that our model may apply even in situations in which credit bureaus accurately track borrowers if lending standards are applied to collateral instead of borrowers. That is, if tight lending standard evaluate and reject on the basis of collateral (e.g. an appraisal of a house), then credit bureaus do not address the externality of tight lending standards because they track borrowers not the assets they wish to fund.

We conclude that mature credit markets in legal environments with low-cost enforcement mechanisms may exhibit various mechanisms for mitigating, but maybe not eliminating, the negative externality associated with tight lending standards.