Indebted Demand
and Economic Policy in a Post-Covid World

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Virtual Macro Seminar
April 2020
Rise in debt and decline in $r^*$ — especially relevant post-Covid!

- How did this happen? Do the two plots interact? What are the implications?
Rise in debt and decline in $r^*$ — especially relevant post-Covid!

- How did this happen? Do the two plots interact? What are the implications?
Rise in debt driven by households and government
The rich lend to the non-rich

- “Saving glut of the rich and the rise in household debt”
Why might this matter? — Rich & wealthy save more

- **Dynan Skinner Zeldes (2004)**: saving rates increase in current income

![Bar chart showing saving rates by income percentile](chart)

From Dynan, et al, Table 3, column 2
Why might this matter? — Rich & wealthy save more

- **Straub (2019):** consumption has elasticity < 1 w.r.t. average income
Why might this matter? — Rich & wealthy save more

- **Fagereng Holm Moll (2019):** saving rate across the wealth distribution

![Graph showing saving rates across wealth distribution](image)

**Figure 6:** Saving rates across the wealth distribution.
The indebted demand framework

• Introduce **non-homothetic consumption-saving behavior** into conventional two-agent endowment economy
  → the rich have a higher saving rate
The indebted demand framework

- Introduce **non-homothetic consumption-saving behavior** into conventional two-agent endowment economy
  - the rich have a higher saving rate

- Main insight: “**Indebted demand**”
  - shifts & policies that stimulate demand today through debt creation, reduce demand in the future by shifting resources from borrowers to savers
The indebted demand framework

- Introduce **non-homothetic consumption-saving behavior** into conventional two-agent endowment economy
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- Main insight: **“Indebted demand”**
  - shifts & policies that stimulate demand today through debt creation, reduce demand in the future by shifting resources from borrowers to savers

- Implications:
  - rising inequality depresses \( r \), amplified by rising debt levels
  - monetary + fiscal policy have **limited ammunition** when they create debt
  - economies can fall into a **“debt trap”** — liquidity trap driven by too much debt
  - once in it, **debt-financed stimulus deepens recession** in the future
  - redistributive policies help
At the center of our analysis is a simple diagram


4. **Inequality and debt (empirics)**: Cynamon Fazzari (2015), Mian Straub Sufi (2019)


Outline

1. Model
2. Equilibria & indebted demand
3. Inequality & financial liberalization
4. Fiscal & monetary policy
5. Debt trap
6. Indebted demand post-Covid
7. Extensions & conclusion
Model
Model of indebted demand

- Deterministic $\infty$-horizon endowment economy with real assets (“trees”)
- Populated by two separate dynasties
- Same preferences, but different endowments of trees
  - mass 1 of borrowers $i = b$: endowment $\omega^b$
  - mass 1 of savers $i = s$: endowment $\omega^s > \omega^b$
  - total endowment $\omega^b + \omega^s = 1$
- Trees are nontradable, dynasties trade debt contracts
- Agents within a dynasty die at rate $\delta > 0$, wealth inherited by offspring
Preferences

- Dynasty $i$ consumes $c_t^i$, owns wealth $a_t^i$. 

\[
\int_0^\infty e^{-\left(\rho + \delta\right)t} \left\{ \log c_t^i + \delta \rho \cdot v(a_t^i) \right\} dt
\]

- Budget constraint 

\[
c_t^i + \dot{a}_t^i \leq r_t a_t^i
\]

- $v(a_t^i) =$ utility from bequest 

- Future consumption, "status" benefits from wealth, 

- Artwork, gifts (to relatives or charities), adjustment frictions in illiquid accounts

- Key object: 

\[
\eta(a_t^i) = \frac{av'(a_t^i)}{\log a_t^i}
\]

- Homothetic model: 

\[
\eta(a_t^i) = \text{const} \Rightarrow v(a_t^i) \propto \log a_t^i
\]

- Non-homothetic model: 

\[
\eta(a_t^i) \text{ increases in } a_t^i
\]
Preferences

• Dynasty $i$ consumes $c^i_t$, owns wealth $a^i_t$. Preferences:

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\int_0^\infty e^{-(\rho+\delta)t} \left\{ \log c^i_t + \frac{\delta}{\rho} \cdot v(a^i_t) \right\} dt
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• Budget constraint

$$c^i_t + \dot{a}^i_t \leq r^i_t a^i_t$$

• $v(a)$ = utility from bequest [future consumption, “status” benefits from wealth, artwork, gifts (to relatives or charities), adjustment frictions in illiquid accounts]

• Key object: $\eta(a) \equiv a v'(a)$ — marginal utility of $v(a)$ relative to log

  • homothetic model: $\eta(a) = \text{const} \Rightarrow v(a) \propto \log a$

  • non-homothetic model: $\eta(a)$ increases in $a$
Borrowing constraint & asset market

• Total wealth = real asset wealth net of debt

\[ a_t^i = \omega^i p_t - d_t^i \]

where \( p_t = \) price of a Lucas tree: \( r_t p_t = 1 + \dot{p}_t \)
Borrowing constraint & asset market

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  where \( p_t \) = price of a Lucas tree: \( r_t p_t = 1 + \dot{p}_t \)

- Agents can pledge \( \ell \) trees each to borrow \( d_t^i \)
  
  \[ d_t^i \leq p_t \ell \]
Borrowing constraint & asset market

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- Agents can pledge \( \ell \) trees each to borrow \( d_t^i \) (\( \lambda \equiv \) bond “decay rate”) 
  \[ \dot{d}_t^i + \lambda d_t^i \leq \lambda p_t \ell \]
  new debt issuance
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  new debt issuance

- steady state: \( d^i \leq p\ell \)  [paper: generalize to \( \ell = \ell(\{r_s\}_{s \geq t}) \)]
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- Market clearing \( d^s_t + d^b_t = 0 \) pins down interest rate \( r_t \)

- Focus on **debt of borrowers**: \( d_t \equiv d^b_t \) **(state variable)**
Scale invariance

• Non-homothetic model is typically **not scale invariant** in aggregate
  • economic growth ⇒ $28’000 today is like $200’000 around 1900
  • so …someone with $28’000 today should save a ton?!
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- economic growth ⇒ $28’000 today is like $200’000 around 1900
- so … someone with $28’000 today should save a ton?!

In reality, savings preferences probably closer to \( v(a/A) \) or \( v(a/Y) \)

**We work with** \( v(a/Y) \), where so far \( Y = 1 \) (total endowment = 1)
Equilibria & indebted demand
Saving supply curves

• Savers’ Euler equation

\[
\frac{\dot{c}_t}{c_t} = r_t - \rho - \delta + \delta \frac{c_t}{\rho a_t} \cdot \eta(a_t^s)
\]
Saving supply curves

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• Setting \( \dot{c} = 0 \) in Euler and use \( c^s = ra^s \) \( \Rightarrow \)

\[
r = \rho \cdot \frac{1 + \rho/\delta}{1 + \rho/\delta \cdot \eta(a^s)}
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r = \rho \cdot \frac{1 + \rho/\delta}{1 + \rho/\delta \cdot \eta(a_s)}
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- This is a long-run saving supply curve:
  - \( r \) necessary for which saver keeps wealth constant at \( a^s \)
  - \( \eta(a^s) \) determines the shape of the saving supply curve
Long-run saving supply curves

\[ \eta(a) \downarrow \text{ in } a \text{ (saving is necessity)} \]

\[ \eta(a) = \text{const} \text{ (homothetic)} \]

\[ \eta(a) \uparrow \text{ in } a \text{ (saving is luxury)} \]
• If $\eta(a^s)$ increasing: **larger wealth** $a^s$ requires **lower return on wealth** $r$ for saver to be indifferent about saving!
Steady state equilibria

- **Steady state**: intersect long-run supply curve with debt demand curve

\[ r = \rho \cdot \frac{1 + \rho/\delta}{1 + \rho/\delta \cdot \eta(c_s/c + d)} \]

\[ d = \frac{\ell}{r} \]
Steady state equilibria

- **Steady state**: intersect long-run supply curve with debt demand curve

\[ r = \rho \cdot \frac{1 + \rho/\delta}{1 + \rho/\delta \cdot \eta(\omega^s/r + d)} \]

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Steady state equilibria

- **Steady state**: intersect long-run **supply curve** with **debt demand curve**

\[ r = \rho \cdot \frac{1 + \rho/\delta}{1 + \rho/\delta \cdot \eta(\omega^s/r + d)} \]

\[ d = \frac{\ell}{r} \]
• Start from a steady state & **raise debt service costs** by some $dx$

• What is **response of aggregate spending**? (partial equilibrium, $r$ fixed)
Indebted demand

• Start from a steady state & **raise debt service costs** by some $dx$

• What is **response of aggregate spending**? (partial equilibrium, $r$ fixed)

$$dC = dc^s + dc^b = -\frac{\rho + \delta}{r} \frac{1}{2} \left( 1 - \sqrt{1 - 4 \left( 1 - \frac{r}{\rho + \delta} \right) \frac{\eta'(a) a}{\eta(a)} } \right) dx$$

$\Rightarrow$ Thus increase in debt service costs weighs on aggregate demand

• $dC < 0$ if $\eta' > 0$
Indebted demand

• Start from a steady state & **raise debt service costs** by some $dx$

• What is **response of aggregate spending**? (partial equilibrium, $r$ fixed)

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⇒ Thus increase in debt service costs weighs on aggregate demand

• $dC < 0$ if $\eta' > 0$

• Call this phenomenon **“indebted demand”**
Equilibrium transitions
The indebted demand diagram

- **Saving supply curve** = how low does $r$ have to be given % resources controlled by savers
- **Debt demand** = how much do borrowers want to borrow given $r$
• **Saving supply curve** = how low does \( r \) have to be given % resources controlled by savers

• **Debt demand** = how much do borrowers want to borrow given \( r \)
Inequality & financial liberalization
Rising inequality $\omega^s \uparrow$: lowers $r$ and raises debt

Homothetic model

- Effects of rising inequality $\omega^s \uparrow$ in non-homothetic model:
  - One. $\omega^s \uparrow \Rightarrow$ more saving by the rich $\Rightarrow r \downarrow \Rightarrow$ debt $\uparrow$
  - Two. Debt $\uparrow$ first raises demand, pushing against decline in $r$
  - Three. High debt eventually lowers demand, aggravating decline in $r$
Rising inequality $\omega^s \uparrow$: lowers $r$ and raises debt

**Homothetic model**

Old and new steady state

**Non-homothetic model**

Old steady state

New steady state

Effects of rising inequality $\omega^s \uparrow$ in non-homothetic model:

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Inequality and debt across 14 advanced economies
Financial liberalization: raising pledgability $\ell$

Homothetic model

Mechanism in non-homothetic model:
1. osf. raises debt & demand, pushing $r$ up (short-run saving supply slopes up)
2. osf. ultimately high debt weighs on demand, lowering $r$, stimulating further debt → resolves puzzle in literature [e.g. Justiniano Primiceri Tambalotti]
Financial liberalization: raising pledgability $\ell$

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Financial liberalization: raising pledgability $\ell$

**Mechanism** in non-homothetic model:

1. **raises debt & demand**, pushing $r$ up (short-run saving supply slopes up)
2. ultimately **high debt weighs on demand**, lowering $r$, **stimulating further debt**!

→ resolves puzzle in literature [e.g. Justiniano Primiceri Tambalotti]
Fiscal & monetary policy
Fiscal policy implications

- Gov’t spends $G_t$, has debt $B_t$, raises income taxes $\tau^s_t, \tau^b_t$, subject to

$$G_t + r_t B_t \leq \dot{B}_t + \tau^s_t \omega^s + \tau^b_t \omega^b$$

- Total demand for debt now $d_t + B_t$
Fiscal policy implications

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- **Result:** In the long run
  1. larger gov’t debt $B \uparrow$: depresses interest rate $r \downarrow$, crowds in household debt $d \uparrow$
  2. tax-financed spending $G \uparrow$: raises $r \uparrow$, crowds out $d \downarrow$
  3. fiscal redistribution $\tau^s \uparrow, \tau^b \downarrow$: raises $r \uparrow$, crowds out $d \downarrow$

- With homothetic preferences none of these policies change $r$ or $d$!
Deficit-financed fiscal policy

- Caveat: this assumed gov't pays same interest rate
- In many advanced economies, gov't actually pays a lower rate
- e.g. when investors derive other benefits from their debt (safety, convenience)
- In that case, what matters is how those benefits affect savers' investments
  → paper: natural case where things are unchanged

\[\text{Plot}\]

\[r\quad d\]
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  → paper: natural case where things are unchanged
Imagine inequality falls exogenously. How much does the interest rate rise?

**Low** $B$  

**High** $B$
Imagine inequality falls exogenously. How much does the interest rate rise?

**Low B**

Strong recovery of \( r \) with low gov’t debt

**High B**

With higher \( B \), any given increase in \( r \) weighs down more on aggregate demand
“Japanification” — how high public debt makes $r$ less likely to rise

Imagine inequality falls exogenously. How much does the interest rate rise?

**Low $B$**

Strong recovery of $r$ with low gov’t debt

**High $B$**

Little recovery of $r$ with high gov’t debt
Imagine inequality falls exogenously. How much does the interest rate rise?

**Low** \( B \)

Strong recovery of \( r \) with low gov’t debt

**High** \( B \)

Little recovery of \( r \) with high gov’t debt

With **higher** \( B \), any given increase in \( r \) **weighs down more on aggregate demand**
Monetary policy has limited ammunition when it raises debt

- Can extend our setup to include nominal rigidities (see paper)
- Monetary policy sets path of interest rates $\{r_t\}$, output is endogenous

Main result:

![Graph showing possible natural interest rate paths and monetary intervention](image-url)
Debt trap
Introducing the lower bound

- Consider lower bound \( r \) on interest rate \( r \)
  - \( r > 0 \) if \( r \) is return on wealth (e.g. \( r \approx 3.5\% \) during recent US ZLB)
Introducing the lower bound

- Consider lower bound $r$ on interest rate $r$
  - $r > 0$ if $r$ is return on wealth (e.g. $r \approx 3.5\%$ during recent US ZLB)

- What happens if the steady state natural rate falls below $r$?
The debt trap (≡ a debt-driven liquidity trap)

• **Result**: if natural rate $< r$, get **stable** liquidity trap steady state: “debt trap”
  → **Output persistently below potential**
  
  $$
  \hat{Y} = Y \frac{r}{(1 - \tau^s)\omega^s + \ell} \cdot \left[ \eta^{-1} \left( \frac{\rho}{r} (1 + \rho/\delta) - \rho/\delta \right) - B \right] < Y
  $$

• **Liquidity trap more likely if**
  
  • income inequality $\omega^s$ is high, low taxes on savers $\tau^s$
  
  • pledgability $\ell$ high, gov. debt $B$ high
How does an economy fall into the debt trap? (i) Rising inequality

- Anticipation of the liquidity trap pulls the economy in even faster
How does an economy fall into the debt trap? (ii) Credit boom-bust cycle
Fighting debt with debt? Deficit financing in the liquidity trap

Gov. spending

Interest rate

Output gap

Here, deficit financing is only a temporary remedy against a chronic disease. Lessons for Covid crisis?
Fighting debt with debt? Deficit financing in the liquidity trap

- Here, deficit financing is only **temporary remedy** against a **chronic disease**
- lessons for Covid crisis?
Indebted demand post-Covid
Covid shock set to further raise debt
Modeling Covid in our framework

• Assume agents work in two sectors, “social” and “distant”

• Assume borrowers are over-represented in “social”

• Shock:
  • potential output falls $Y \downarrow$ and inequality rises $\omega^s \uparrow, \omega^b \downarrow$
  • assume this induces negative demand shock in “distant” sectors
Covid in the indebted demand diagram

Effective lower bound
Covid in the indebted demand diagram

- Effective lower bound
- Induced demand shock
Covid in the indebted demand diagram

- Reduced borrowing capacity
- Effective lower bound
- Induced demand shock
Covid in the indebted demand diagram

Covid shock: $r \downarrow, \text{debt} \uparrow$

Reduced borrowing capacity

Induced demand shock

Effective lower bound
Three “archetypes” of policies in response to Covid shock

(A) Stimulating (non-productive) private debt to buffer the shock
   • e.g. Fed’s lending facilities via SPV’s
     → model as increase in credit limit

(B) Government funds transfers using public debt, paid for by all taxpayers
   • e.g. stimulus checks, UI, grants to businesses
     → model as increase in government debt

(C) Government funds transfers by taxing (now or later) very progressively
   • e.g. Landais-Saez-Zucman, Greenwood-Thesmar
     → model as saver-financed increase in government debt
Three “archetypes” of policies in response to Covid shock

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Different across (A), (B), (C): whether there is a transfer from savers to borrowers
Policies in the indebted demand diagram

Covid shock: $r \downarrow$, debt $\uparrow$

Effective lower bound
Policies in the indebted demand diagram

Policy (A) — Stagnation post-Covid

Effective lower bound

Covid shock + (A): \( r \downarrow \downarrow, \text{debt } \uparrow \uparrow \)
Policies in the indebted demand diagram

Effective lower bound

Covid shock + (B): $r \downarrow$, debt $\uparrow$

Policy (B) — Softer stagnation post-Covid

Bottom line: Transfers $> \frac{\text{Debt}}{\text{long term}} \rightarrow \text{address any structural problems leading to greater inequality}$
Policies in the indebted demand diagram

Covid shock + (C): $r \uparrow$, debt $\uparrow$

Policy (C) — No stagnation!
Policies in the indebted demand diagram

Bottom line: Transfers > Debt

(long term → address any structural problems leading to greater inequality)
Extensions & conclusion
Extensions

- Redistribution (e.g. wealth tax) = Pareto improvement in debt trap
- Investment can help, especially if it complements borrowers’ labor
- Similar results when there is gov’t bond pay lower rate
- Intergenerational mobility helps
- Sufficient statistic exercise

In paper:
- Open economy model
- Uzawa preferences, relative wealth preferences
**Indebted Demand:**

Demand decreases in $r \times \text{debt}$

 Particularly relevant post-Covid!
Indebted Demand:

Demand decreases in $r \times \text{debt}$

Particularly relevant post-Covid!
Extra slides
Inequality and debt

- Top 1% income share
- Interest rate
- Household debt / GDP
- Debt service / GDP

Homothetic model
Non-homothetic model
Deficit spending causes indebted (government) demand

**Gov. debt / GDP**

- 0% to 20% over 0-20 years

**Interest rate**

- 0% to 6% to 0% over 0-20 years

**Household debt / GDP**

- 0% to 90% to 80% to 75% over 0-20 years
But ... what about $r < g$? (here: $g$ normalized to zero)

- Our $r$ is **return on wealth** so always $r > g$. But what if gov’t pays $r^B < g$?
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- Our $r$ is **return on wealth** so always $r > g$. But what if gov’t pays $r^B < g$?

- Our model points to **two objects that matter** (see paper for details)
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1. Derivative of debt service cost of $(r^B - g)B$ w.r.t. $B$

   \[
   \frac{\partial (r^B - g)B}{\partial B} = r^B - g + \frac{\partial r^B}{\partial B} \quad \begin{cases} < 0 \quad \text{if} \\ > 0 \quad \text{if} \end{cases} 0
   \]
But ... what about $r < g$? (here: $g$ normalized to zero)

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- Our model points to **two objects that matter** (see paper for details)

1. **Derivative of debt service cost** of $(r^B - g)B$ w.r.t. $B$

$$\frac{\partial (r^B - g)B}{\partial B} = r^B - g + \frac{\partial r^B}{\partial B} \quad ? \quad 0$$

2. Where does the spread $r - r^B$ come from? Investors really like $B!$

   - $B$ is **not negative for savers** just because $(r^B - g)B < 0$

   - $B \uparrow$ still makes savers wealthier, $\alpha^S \uparrow$, lowering required return on wealth $r$
Redistribution and welfare

- What policy mitigates a debt trap? → redistribution

- Example: wealth tax of $τ^a > 0$ on saver’s wealth, redistributed to borrowers

- Saver’s budget constraint becomes

$$c_t^s + \dot{a}_t^s = (r_t - τ^a) a_t^s$$

→ Wealth tax reduces return on wealth at ZLB to $r - τ^a$, raising $\hat{Y}$

- What about welfare?
  - borrower clearly benefits: lower $r$ + wealth tax transfers + higher incomes
  - saver also benefits: greater incomes (& asset prices) more than compensate for tax!

- Thus: Redistribution mitigates debt trap, at no welfare cost!
Introducing investment

• Assume goods are now produced from capital and both agents’ labor

\[ Y = F(K, L^b, L^s) \]

• \( F \) is net-of-depreciation production, \( K \) pinned down by \( F_K = r \)

• \( \sigma \equiv \) (Allen) elasticity of substitution between \( K \) and \( L^b \)
Introducing investment

- Assume goods are now produced from capital and both agents’ labor

\[ Y = F(K, L^b, L^s) \]

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- \( \sigma \equiv \) (Allen) elasticity of substitution between \( K \) and \( L^b \)

- Key: savers’ income share \( \omega^s = \omega^s(r) \) now a function of \( r \! \)

\[ \omega^s(r) \equiv \frac{F_K K}{F} + \frac{F_L L^s}{F} = 1 - \frac{F_L L^b}{F} \]
• Assume goods are now produced from capital and both agents’ labor
  \[ Y = F(K, L^b, L^s) \]

  • \( F \) is net-of-depreciation production, \( K \) pinned down by \( F_K = r \)
  
  • \( \sigma \equiv \) (Allen) elasticity of substitution between \( K \) and \( L^b \)

• Key: savers’ income share \( \omega^s = \omega^s(r) \) **now a function of** \( r \)!
  \[
  \omega^s(r) \equiv \frac{F_K K}{F} + \frac{F_{L^s} L^s}{F} = 1 - \frac{F_{L^b} L^b}{F}
  \]

  • \( \omega^s(r) \) independent of \( r \) if \( \sigma = 1 \) [e.g. Cobb-Douglas]
  
  • \( \omega^s(r) \uparrow \text{ as } r \downarrow \text{ iff } \sigma > 1 \) [e.g. capital-skill complementarity, robots]
• **Main result:** Our results are unchanged if $\sigma = 1$. Amplified if $\sigma > 1$. 

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![Graph showing the relationship between r and d with different values of \(\sigma\).](image-url)
**Main result:** Our results are unchanged if $\sigma = 1$. Amplified if $\sigma > 1$.

**Related Q:** Can corporate debt also cause indebted demand?
- yes, if $\sigma > 1$! but always weaker indebted demand than household debt
- why? corporate debt productive, raising $Y$, easier to repay
Government yield spread

• Allow for benefits from gov’t bonds [cf Krishnamurthy Vissing-Jorgensen (2012)]

\[
\log (c_t^s + \xi B_t) + \frac{\delta}{\rho} \cdot v (a_t^s + \xi B_t/r)
\]

• Implies fixed spread \( \xi > 0 \)

\[
r^B = r - \xi
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  \]
- Implies fixed spread \( \xi > 0 \)
  \[
  r^B = r - \xi
  \]
- Define effective wealth as including benefits \( \xi B_t \) from bonds. In steady state:
  \[
  a_{eff} \equiv \frac{\omega^s}{r} + d + \frac{r^B B}{r} + \frac{\xi B}{r}
  \]
  \[
  = B
  \]
- Savings supply curve unchanged in effective wealth
  \[
  r = \rho \frac{1 + \rho/\delta}{1 + \rho/\delta \cdot \eta(a_{eff})}
  \]
• With probability $q > 0$, savers turn into borrowers and vice versa

• Saver-turned-borrowers consume down their wealth instantly

• Borrower-turned-savers get transfer from other savers to raise wealth
• With probability $q > 0$, savers turn into borrowers and vice versa

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• **Saving supply curve becomes flatter** with $q$

\[ r = \rho \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(a)} + q\gamma\delta \frac{\delta/\rho \cdot \eta(a)}{1 + \delta/\rho \cdot \eta(a)} \]

  - contribution of mobility

• $q \uparrow$ thus **mitigates indebted demand**, especially if high **income inequality** $\gamma$

\[ \gamma \equiv 1 - \frac{\omega^b - \ell}{\omega^s + \ell} \]
• Consumption function of rich \( c(r, a) \). Along curve:

\[
c(r(a), a) = r(a)a
\]
• Consumption function of rich $c(r, a)$. Along curve:

$$c(r(a), a) = r(a)a \Rightarrow \frac{c_r}{c} \frac{dr}{a d \log a} + \frac{c_a}{MPC\text{cap. gains}} = \frac{dr}{d \log a} + r$$

semi-elast. $\epsilon_r$ wrt $r$
Is this first order? What is the slope of savings supply in the data?

- Consumption function of rich $c(r, a)$. Along curve:

$$c(r(a), a) = r(a)a \Rightarrow \frac{dr}{d \log a} = \frac{MPC^{\text{cap. gains}} - r}{1 - \epsilon_r \frac{c}{a}}$$
• Consumption function of rich $c(r, a)$. Along curve:

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• Standard PIH model: $\text{MPC}^{\text{cap. gains}} = r$ \quad log preferences: $\epsilon_r = 0$
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- Assume $\epsilon_r = 0$, $r \approx 0.06$, $MPC_{\text{cap. gains}} \approx 0.025$
  
  [Farhi-Gourio, Di Maggio-Kermani-Majluf, Baker-Nagel-Wurgler, Chodorow-Reich Nenov Simsek]
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  [Farhi-Gourio, Di Maggio-Kermani-Majluf, Baker-Nagel-Wurgler, Chodorow-Reich Nenov Simsek]

\[
\frac{dr}{d \log a} = -0.035
\]

• In words: if wealth ↑ by 10%, required \( r \) ↓ by 35bps
Bottom 90% did not accumulate assets

Bottom 90% reduced saving

Relative to 63–82

Contributions into housing ($N_h$)
Contributions into non-housing ($N_{nh}$)
Change in debt ($\Delta D$)
Saving ($\Theta$)
Thought experiment: How large is $dC$ implied by current levels of household & government debt, had interest rates not come down?
How indebted is US demand?

- Thought experiment: How large is $dC$ implied by current levels of household & government debt, had interest rates **not** come down?
- Counterfactual debt service burden, holding $r$ constant:
• Thought experiment: How large is $dC$ implied by current levels of household & government debt, had interest rates **not** come down?

• Counterfactual debt service burden, holding $r$ constant:

$$dC \approx \underbrace{-15\%}_{\text{borrower debt service}} + \underbrace{\frac{\text{MPC cap. gains}}{r} \cdot 15\%}_{\text{partial offset by savers}} = -8\%$$