Parallel Trends and Dynamic Choices

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Abstract

Difference-in-differences (or DiD) is a commonly used method for estimating treatment effects, and parallel trends is its main identifying assumption: the trend in mean untreated outcomes must be independent of the observed treatment status. In observational settings, treatment is often a dynamic choice made or influenced by rational actors, such as policy-makers, firms, or individual agents. This paper relates the parallel trends assumption to economic models of dynamic choice, which allow for dynamic selection motives such as learning or optimal stopping. In these cases, we clarify the implications of parallel trends on agent behavior, and we show how such features can lead to violations of parallel trends even in simple settings where anticipation concerns are ruled out or mean untreated outcomes are stationary. Finally, we consider some identification results under alternative assumptions that accommodate these features of dynamic choice.

Key words: Difference-in-differences, parallel trends, dynamic choice models, treatment effects, causal inference

JEL codes: C21, C23

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1 Introduction

Difference-in-differences (or DiD) is a popular research design for causal inference with panel data or repeated cross-section data. Though econometricians have estimated panel data regressions for decades, the causal interpretation of coefficients from these regressions is subtle and forms the basis of this paper. On the one hand, interpretations with non-experimental data using structural choice models\(^1\) with optimizing agents depend on what one assumes about how these choices (about treatment, or inputs, or prices) were made. On the other hand, and without a behavioral model, identifying assumptions are used to provide causal interpretations that allow for evaluations of various treatments. Chief among them is an assumption of parallel trends, namely that the untreated trend is independent of the realized sequence of treatment. Such assumptions indirectly restrict the kinds of choice behavior that agents are allowed to have. This is especially important in the dynamic settings where these data and designs are mainly used.

In this paper, we clarify the connections between the central DiD causal identifying assumption — parallel trends — and dynamic choice behavior. We do so mainly by highlighting through simple examples the kinds of choice behaviors that are or are not compatible with the assumption.\(^2\) As a second contribution, we consider identification under alternative assumptions that accommodate various features of our discussion on dynamic choice.

A complicating factor that raises various potential selection issues in DiD models is time. This is especially so when treatments are indexed by time in observational data. In that case, a variety of dynamic considerations — such as future discounting, option values, learning, and anticipation — may all play an important role in determining choices. While these selection and information issues and their role in empirical model building and interpretations are well known to econometricians, the purpose of this paper again is to shed light on exactly how these behaviors intersect with commonly used assumptions in DiD models, as well as what

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\(^1\)In some settings, structural behavioral model are termed “generative models” in that choices and the way these arise are directly modeled. We do not use this terminology in this paper.

\(^2\)We do note that panel data regressions are always meaningful as a way to summarize data via variance decomposition for instance, and are commonly (and successfully) used for predictive purposes. The literature here is vast and covers almost all textbook treatment of panel regressions as a predictive exercise, just as least squares in static linear regressions is viewed as solving a well posed prediction problem. Our only concern is when one uses these regressions for causal interpretations and the kinds of economic models or behaviors that these regressions allow.
classes of economic choice models of optimizing behavior are consistent with the behaviorally agnostic DiD model.

For example, we show that some dynamic features, such as discounting or time-varying costs of treatment, need not lead to violations of parallel trends per se, whereas other phenomena such as learning, optimal stopping, or repeated Roy-model-like behavior can (but need not always be) more problematic. We also elucidate how and why parallel trend assumptions among some subsequences of observed treatment may be more robust to dynamic selection concerns than others. Finally, we translate these observations into alternative identification assumptions and strategies. These include considering weakened parallel trend assumptions, time stationarity assumptions that can circumvent dynamic selection concerns, and partially identifying assumptions motivated by our structural models of choice.

We contribute primarily to a recent yet already substantial literature clarifying aspects of DiD assumptions and estimators in settings that extend the basic two-period DiD model with no treatment in the first period. Much of this work has focused on i) interpreting typical regression-based slope parameters and what these estimate when heterogeneity in effects is allowed over multiple time periods, and ii) deriving estimators that target a causal parameter of interest under parallel trends.\(^3\) In addition, other work has studied the identified features of the model under weaker assumptions by deriving bounds on parameters of interest.\(^4\) One takeaway from this literature is that a variety of identification and estimation complications arise in the empirically common settings where the basic DiD model is enriched to fuzzy designs (where some individuals are treated in the “pre-treatment” period) or multiple time periods. Yet, it is precisely also in these richer settings that we might expect richer dynamic selection considerations to arise. Thus, we complement this literature by focusing on the relationship between parallel trends and structural models of dynamic choice.

Our exercise is closest in spirit to the recent and concurrent work of Ghanem, Sant’Anna, and Wüthrich (2022), who study the relationship between selection and the parallel trends

\(^3\)See for instance De Chaisemartin and d’Haultfoeuille (2018), De Chaisemartin and d’Haultfoeuille (2020), Borusyak, Jaravel, and Spiess (2021), Callaway and Sant’Anna (2021), Goodman-Bacon (2021), Sun and Abraham (2021), and Athey and Imbens (2022) to name a few. With the exception of the first two papers, most of this work considers staggered designs where the sequence of realized treatment is monotonic across time.

\(^4\)See for example Manski and Pepper (2018) and Rambachan and Roth (2019).
assumption. They consider the basic design where no unit is treated initially and selection on
untreated outcomes in period 1 can be a function of an individual fixed effect or time-varying
errors in the untreated outcome equation. We extend the class of selection mechanisms
under consideration in two ways. First, we consider selection in fuzzy designs, which allows
us to further explore the dynamics of selection with two periods of treatment. In addition, we
consider selection models with imperfect information, which allows us to consider models with
temporal resolution of uncertainty and learning. Finally, we consider alternative identification
approaches in the case where the parallel trends assumption remains in question.

In addition, we draw on a large literature in applied economics that explicitly models
dynamic decisions capturing incentives, option values, and/or learning. See for instance
Heckman (1976), Rust (1994), and Keane and Wolpin (1997). These models are central to
the literature on structural estimation of dynamic decision processes.\footnote{These models are
directly related to the literature on reinforcement learning and causal bandits. See
Igami (2020) for more on this connection.} See also the work of
Taber (2001), Heckman and Navarro (2007) and Abbring (2010). This structural literature
embeds within it causal parameters because it builds a model of behavior that is based on
economic optimizing agents. Hence, causal relations are provided by construction through
model specification that can be used to address counterfactuals and policy impacts. The
literature on DiD on the other hand focuses on identifying parameters that are given a causal
interpretation under some identifying restrictions without the need for full specification of
a behavioral model. We think this is a strength of the approach and one that is worth
highlighting. However, we also point out that care should be taken in particular setups when
these “design” assumptions come at odds with simple notions of dynamic selection that may
be relevant in some data applications.

Finally, the exercise that we undertake is familiar to econometricians.\footnote{For a similar exercise on the role theory plays, more precisely
general equilibrium, in formulating empirical models, see Acemoglu (2010).} In standard static
causal setups, Vytlacil (see Vytlacil (2002)) provided a class of choice models that are con-
sistent with the LATE assumptions of Imbens and Angrist (1994). In particular, Vytlacil
showed that a separable choice model with a univariate unobservable is (observationally)
equivalent to the monotone causal model underlying LATE. A univariate unobservable de-
terminating choices may well be a reasonable model of behavior for many applications. Still, the existence of such a choice model clarifies the types of behavior under which causal interpretations are warranted. While our paper does not attempt a choice-theoretic characterization of the DiD model, it takes an otherwise similar approach in the context of dynamic panel/DiD regressions.

The paper proceeds as follows. In Section 2 we review the two-period fuzzy DiD model and the parallel trends assumption that the mean untreated trend is independent of the entire realized sequence of treatment. In Section 3 we begin by introducing a simple but useful equivalent formalization of the “full” parallel trends assumption as a set of conditional or “partial” assumptions on subsequences of treatment, and then use this to study a sequence of choice models and their compatibility with parallel trends. In Section 4 we then provide some other suggestive approaches that either weaken parallel trends or replace it with more selection-robust or selection-motivated assumptions. The emphasis throughout is on stylized models that retain key features of dynamic selection in a simple framework that allow one to focus on exact channels of interest. Section 5 concludes.

2 Model

For an individual $i$ in a pair of time periods $t \in \{0, 1\}$, a researcher observes a sequence of realized treatments $D_{it} \in \{0, 1\}$ and realized outcomes $Y_{it} \in \mathbb{R}$. We restrict our analysis to two time periods to clearly highlight linkages and for ease of exposition.

Following the DiD literature, we consider two assumptions on potential outcomes. First, potential outcomes are indexed by own treatment in the present period only. Under this assumption, individuals can be thought of as elements $\omega$ of the sample space $\Omega$, and so we henceforth suppress the individual index $i$ for simplicity of notation. Since we maintain this assumption throughout and it is not our object of interest, we index it by zero.

Assumption 0 (No Anticipation, History-Dependence, or Dependence on the Treatments

\footnote{This not only requires the SUTVA assumption, but more crucially requires that outcomes in a period depend only on treatment in that period.}

\footnote{Of course, anticipation is also an important challenge to identification in dynamic choice models, but it is one that has already received more attention in the literature. See for instance Abbring and Van den Berg}
of Others in Potential Outcomes). Potential outcomes at time $t$ depend only on own treatment at time $t$, i.e. $Y_t(d_0, d_1) = Y_t(d_t)$ almost surely for $t \in \{0, 1\}$.

Second, the mean untreated outcomes, i.e., $Y_t(0)$, satisfy parallel trends across each treatment sequence $(D_0, D_1)$.

**Assumption 1 (Multi-Period Parallel Trends).** The mean untreated trend is constant across all realized sequences of treatment:

$$E[Y_1(0) - Y_0(0) | (D_0, D_1) = (d_0, d_1)] = \tau$$

for some $\tau \in \mathbb{R}$ and all $(d_0, d_1) \in \{0, 1\}^2$.

Thus expressed, Assumption 1 is identical to the Strong Exogeneity condition of De Chaisemartin and d’Haultfoeuille (2020) for two time periods and a given group. In the basic case of a pre-treatment period where $D_0 = 0$ for all individuals, Assumption 1 conditions only on treatment in period 1. In that case and under this assumption, the “causal effect of treatment on the treated” in period 1 is identified by the basic DiD estimator:

$$E[Y_1(1) - Y_1(0) | D_1 = 1] = E[Y_1 - Y_0 | D_1 = 1] - E[Y_1 - Y_0 | D_1 = 0]$$

A recent literature studies estimators that identify weighted averages of such DiD estimators (and hence treatment effects) in more general cases where some units are treated in period 0 or there are more than two periods. These papers impose versions of Assumptions 0 and 1, in combination with further assumptions such as monotonicity of $D_t$ in $t$ (staggered research design) or some versions of parallel trends in treated outcomes.

### 3 Simple Examples

Our first aim is to explore when Assumption 1 is (in)consistent with dynamic models of choice. In doing so, we restrict to models satisfying Assumption 0 to isolate issues of selection.

(2003), Heckman and Navarro (2007) and Heckman, Humphries, and Veramendi (2016), and in the particular context of DiD models Malani and Reif (2015).

See for example, De Chaisemartin and d’Haultfoeuille (2018), De Chaisemartin and d’Haultfoeuille (2020), Borusyak, Jaravel, and Spiess (2021), Callaway and Sant’Anna (2021), Goodman-Bacon (2021), Sun and Abraham (2021), and Athey and Imbens (2022) to name a few.
To start, notice that Assumption 1 can be equivalently expressed as a set of pairwise equalities, which will be useful in analyzing subsequent examples.\textsuperscript{10}

**Observation 1.** Assumption 1 is equivalent to the two joint statements below:

\begin{align}
E[Y_1(0) - Y_0(0)|D_0 = d_0] & \text{ is constant in } d_0; \\
E[Y_1(0) - Y_0(0)|D_0 = d_0, D_1 = d_1] & \text{ is constant in } d_1 \text{ for each } d_0.
\end{align}

(Note that by construction the two constants mentioned in conditions 1 and 2 must be the same.) In turn, each pairwise parallel trend equality can be rearranged and reinterpreted as a stationarity condition in the magnitude of selection.

**Observation 2.** Conditions (1) and (2) above are respectively equivalent to:

\begin{align}
E[Y_t(0)|D_0 = 1] - E[Y_t(0)|D_0 = 0] & \text{ is constant in } t; \\
E[Y_t(0)|D_0 = d_0, D_1 = 1] - E[Y_t(0)|D_0 = d_0, D_1 = 0] & \text{ is constant in } t \text{ for each } d_0.
\end{align}

Conditions (3) and (4) clarify how Assumption 1 allows for arbitrary selection on the untreated outcome in a given period but requires the magnitude of this selection (in terms of the mean untreated potential outcome) to be fixed across periods $t$ given a treatment history. In other words, under parallel trends, the selection terms:

\begin{align}
E[Y_t(0)|D_r = 1] - E[Y_t(0)|D_r = 0]
\end{align}

must be constant in $t$ but may vary over $r$.

Observation 2 implies that the parallel trends condition in Assumption 1 requires static selection in the following sense: the discrepancy between the expected potential untreated outcome at time $t$ conditional on different observed treatments remains constant across $t$. Yet, parallel trends also allow differential selection over time due to the changes in observed treatments at different times. This is evident because the two constants mentioned in (3) and (4) respectively are allowed to be different. In the examples that follow, we contrast such

\textsuperscript{10}This decomposition is not unique, but it maps most closely into our considerations of dynamic choice. It is also worth noting that Assumption 1 is not implied by conditioning just on the marginal events $\{D_0 = d_0\}$ and $\{D_1 = d_1\}$.
differential selection with other sources of dynamic selection that violate the assumption of parallel trends.

Finally, to assess the viability of parallel trends in various models of dynamic choice, it is useful to note that — regardless of selection into treatment — Assumption 1 is satisfied if untreated outcomes are equal almost surely over time, i.e., \( Y_0(0) = Y_1(0) \). The extent to which this is also deemed necessary for parallel trends can be interpreted as a benchmark of the (parallel trend) assumption’s compatibility with various determinants of dynamic choice. We begin with two stark examples to simply illustrate some of the ideas.

### 3.1 Selection on Past Outcomes

First, consider a simple example in which no one is treated in period 0 (a pre-treatment period) and there is perfect selection on the past outcome in period 1. With a pre-treatment period, Assumption 1 reduces to the familiar assumption that \( E[Y_1(0) - Y_0(0)|D_1 = d_1] \) is constant in \( d_1 \).

**Example 1.** Potential outcomes are binary, i.e., \( Y_t(0), Y_t(1) \in \{0, 1\} \). At \( t = 0 \), everyone is untreated, i.e., \( D_0 = 0 \). At \( t = 1 \), there is perfect selection on the previous period’s realized outcome, i.e., \( D_1 = Y_0 \), with \( P(D_1 = 1) \in (0, 1) \). Then parallel trends is satisfied if and only if untreated outcomes are equal almost surely:

\[
Y_0(0) = Y_1(0) \quad a.s.
\]

We provide a proof of the claimed equivalence in Appendix A. In this example, present decisions depend on past outcomes. This provides some justification for the inclusion of lagged outcomes in panel models. In a classic example, Ashenfelter (1978) finds that participants in job training programs experience a decline in earnings in the year prior to training. As he observes, “In retrospect this is not very surprising since the Department of Labor was instructed to enroll unemployed workers in the MDTA [training] programs in this period and it is just such workers who would be most likely to want to enter a training program.”

Our simple example above suggests that parallel trends (Assumption 1) is likely to fail when there is backward-looking dynamic selection: the present propensity to enroll in a training
program depends on unemployment in the previous period. More generally, when current
decisions depend directly (or indirectly) on past outcomes, parallel trends will be suspect.

3.2 Selection on Present Outcomes: A Repeated Roy Model

Given the nature of the parallel trends assumption, similar reasoning applies to the compat-
ibility of parallel trends with selection on outcomes in the present period, in particular when
such selection is based on information specific to each period. For example, consider a re-
peated version of a basic static Roy model where decision-makers perfectly observe potential
outcomes in each period and choose treatment if and only if the observed treated outcome
weakly exceeds the observed untreated outcome.

Example 2 (Repeated Static Roy Model). Potential outcomes $Y_t(d)$ are binary and choices
satisfy a repeated Roy model where $D_t = I\{Y_t(1) \geq Y_t(0)\}$. Additionally, there is an interior
probability of never-treated observations, $P(D_0 = D_1 = 0) \in (0,1)$. Then Assumption 1 is
satisfied if and only if the following hold jointly:

1. Untreated outcomes are stationary: $E[Y_0(0)] = E[Y_1(0)]$

2. Untreated outcomes are degenerate at 1 among the ever-untreated:

$$P(Y_0(0) = Y_1(0) = 1|D_0D_1 = 0) = 1$$

If in addition potential outcomes $(Y_t(0), Y_t(1))$ are independent over time, then Assumption 1
requires $Y_t(0)$ to be degenerate at 1 for both periods $t = 0, 1$.

The stark version of the Roy model we consider in this example imposes strong assumptions
on the observed data as a function of treatment: if $D_t = 0$ for either period $t = 0, 1$, then
$Y_t(0) = 1$. However, this assumption on the treatment rule imposes no restrictions on the
unconditional distribution of potential outcomes per se. In contrast, parallel trends is only
satisfied for distributions of potential outcomes exhibiting stationarity (condition 1) and
sufficient serial correlation (condition 2). The first condition follows because the Roy model
requires a zero trend among the never-treated:

$$E[Y_1(0) - Y_0(0)|D_0 = D_1 = 0] = 1 - 1 = 0.$$
Under Assumption 1, this implies $E[Y_0(0)] = E[Y_1(0)]$. The second condition follows because parallel trends then requires that if $D_t = 0$ for either $t = 0, 1$, then also $Y_{1-t}(0) = 1$ (almost surely). While condition 2 is expressed in terms of the treatment rule for simplicity, this restriction of other-period outcomes on treatment also restricts the distribution of potential outcomes. For example, to be consistent with our Roy model and parallel trends, the probability of strict switches in potential outcomes across time must be zero:

$$P(Y_t(0) > Y_t(1), Y_{1-t}(0) < Y_{1-t}(1)) = 0$$

for $t = 0, 1$. Such requirements are in turn inconsistent with independence of potential outcomes over time, except in extreme cases.

The Roy model we consider takes a strong stance on how ties are broken when $Y_t(0) = Y_t(1)$. Similar results would hold in the alternative specification where the weak inequality is replaced with a strict one, $D_t = I\{Y_t(1) > Y_t(0)\}$. However, our strong conclusions rely on the fact that treatment in each period is a deterministic function of potential outcomes and a non-trivial function of untreated outcomes.\footnote{For an analysis of the static, single-period Roy model without assumptions on tie-breaking, see Mourifie, Henry, and Meango (2020).} As in the preceding section, the aim was to elucidate a basic selection mechanism in tension with parallel trends. In the following subsections, we enrich this analysis in more structural models of dynamic choice.

### 3.3 Dynamic Choice with Persistent Information

Next, we consider a structural dynamic choice model where treatment decisions are made by forward-looking individuals who maximize the present value of their expected utility over time. To define the maximization problem, we introduce two additional components: a utility function $v_t(d_t, y_t)$ representing decision-makers’ preferences over realized treatment and outcome sequences in each period $t$, and a vector of state variables $U = (U_0, U_1(0), U_1(1))$ summarizing decision-makers’ information sets at each time and potential treatment history. In particular, decision-makers observe $U_0$ in period 0 and $U_1 \equiv U_1(D_0)$ in period 1 conditional on decision $D_0$ in period 0; they learn about potential outcomes in each period given the information set available in that period. For simplicity, we focus on the case $U_1(D_0) \equiv (U_0, Y_0(D_0))$.
where past realized outcomes are observed and are the only source of information accrual. The econometrician observes the realized sequence of outcomes \((D_0, D_1, Y_0, Y_1)\) but not the state variables \(U\). The dependence of state variables on counterfactual treatment histories allows for motives of endogenous information acquisition. We assume that decision-makers rationally form and update beliefs given knowledge of the joint distribution of unobservables and potential outcomes \((U(\cdot), Y(\cdot))\). This will be made more precise in the context of the next example.

**Example 3** (Dynamic Utility Maximization). Given an initial state variable \(U_0\) and continuation state variables \((U_0, Y_0(0))\) or \((U_0, Y_0(1))\) summarizing decision-makers’ information sets at each time and potential treatment history, treatment decisions maximize the sum of expected discounted utility:

\[
V_1(d_0) \equiv \max_{d_1 \in \{0, 1\}} E[v_1(d_1, Y_1(d_1))|U_0, Y_0(d_0)] 
\tag{6}
\]

\[
D_1(d_0) \in \arg\max_{d_1 \in \{0, 1\}} E[v_1(d_1, Y_1(d_1))|U_0, Y_0(d_0)] 
\tag{7}
\]

\[
D_0 \in \arg\max_{d_0 \in \{0, 1\}} E[v_0(d_0, Y_0(d_0)) + \beta V_1(d_0)|U_0] 
\tag{8}
\]

where per-period utility is equal to the realized outcome net a time-varying cost \(k_t\) of choosing treatment:

\[
v_t(d_t, y_t) = \nu_t(y_t) - k_t d_t 
\tag{9}
\]

Throughout we focus on the optimal decision rule that chooses treatment in cases of indifference. We begin with a case of this model that satisfies Assumption 1 (and hence DiD) despite the presence of dynamic selection motives. In the subsequent section, we then show how relaxing attributes of this case can yield violations of Assumption 1 in the spirit of the preceding counterexamples.

**Case 1** (of Example 3). Initial information is sufficient for mean outcomes and costs. For \(d_0, d_1 \in \{0, 1\},\)

\[
E[Y_1(d_1)|U_0, Y_0(d_0)] = E[Y_1(d_1)|U_0] 
\tag{10}
\]

\[
E[K_t|U_0] = K_t 
\tag{11}
\]
Untreated outcomes given initial information satisfy conditional parallel trends:

\[ E[Y_1(0) - Y_0(0) | U_0] = \tau \]  

(12)

Case 1 of Example 3 is a particular model that is consistent with the following simple coin experiment. No treatment and treatment are coins whose outcome probabilities are summarized in \( U_0 \), and potential outcomes in each period are independent draws according to these probabilities. Conditional parallel trends (12) allows for these outcome probabilities to shift by a known trend \( \tau \); in the simplest case, the coins are fixed across time and \( \tau = 0 \). However, the coin probabilities can still differ arbitrarily between no treatment and treatment and across decision-makers through \( U_0 \), and costs may also vary across decision makers and time.

As in condition (10) and (11), the outcomes of previous coin tosses may be observed prior to the decision at \( t = 1 \), but they are uninformative about future outcomes or costs since the decision-maker already knows the coin probabilities and costs from \( U_0 \).

Under utility (9) and sufficiency of initial information (10) and (11), an optimal treatment rule is:

\[ D_t = I \{ E[\nu_t(Y_t(1)) - \nu_t(Y_t(0)) - K_t | U_0] \geq 0 \} \]  

(13)

in both periods \( t = 0, 1 \). This is because (10) implies the second-period value function \( V_1(d_0) \) is constant over \( d_0 \). Thus, agents have a persistent type \( U_0 \) that is sufficient for their choices and allows for time to affect costs or expected returns to treatment. Formally, the optimal treatment rule (13) defines sets \( \mathcal{U}(d_0, d_1) \) such that:

\[ \{ (D_0, D_1) = (d_0, d_1) \} = \{ U_0 \in \mathcal{U}_0(d_0, d_1) \} \]  

(14)

Combining (14) with the law of iterated expectations and (12) then yields the desired result:

\[ E[Y_1(0) - Y_0(0) | (D_0, D_1) = (d_0, d_1)] = E[Y_1(0) - Y_0(0) | U_0 \in \mathcal{U}_0(d_0, d_1)] \]

\[ = E[E[Y_1(0) - Y_0(0) | U_0] | U_0 \in \mathcal{U}_0(d_0, d_1)] \]

\[ = \tau. \]

Hence, parallel trends (Assumption 1) is satisfied in Case 1 of Example 3. This is an example of a model where the determinant of treatment in every period depends only on initial information \( U_0 \) but not any updated information from \( Y_0(D_0) \). Nevertheless, the time
dependence of the threshold on cost $K_t$ allows for rich selection incentives into and out of treatment across individuals over time. The compatibility with parallel trends depends on the particular model of choice. This model requires the sufficiency of initial information assumption that satisfies a conditional parallel trend assumption, (12). Next, we show how Assumption 1 may be violated when we relax the properties of the information set to allow for dynamic phenomena such as learning.

### 3.4 Learning

Another avenue for selection in dynamic settings is one where selection is based on information accrued at the end of the first period, such as the past outcome in period $t = 0$. This basic selection issue was illustrated in Example 1. The following case shows how similar issues may arise in the richer context of Example 3.

**Case 2** (of Example 3). Potential outcomes are binary $Y_t(d_t) \in \{0, 1\}$, and so without further loss $\nu_t(y_t) = y_t$. Relax the restrictions on learning in (10) as follows. There is no learning across no treatment and treatment arms by decision makers:

$$E[Y_1(d_1)|U_0, Y_0(d_0)] = E[Y_1(d_1)|U_0] \quad \text{when } d_1 \neq d_0,$$

but the past outcome can provide an informative signal about the own mean return of an arm in the next period:

$$E[Y_1(d_1)|U_0, Y_0(d_0) = 0] \leq E[Y_1(d_1)|U_0] \leq E[Y_1(d_1)|U_0, Y_0(d_0) = 1] \quad \text{when } d_1 = d_0$$

Costs of treatment are known at the outset (11) and untreated outcomes continue to satisfy parallel trends conditional on the initial selection term (12), as in Case 1.

The main change in Case 2 relative to Case 1 is that we relax sufficiency of initial information (10) to allow for learning within the no treatment and treatment arms. Potential outcomes are assumed binary to crystallize the basic selection issue, as evidenced in (16). In the same spirit, in what follows we assume for simplicity of notation that there is a pre-treatment period, $D_0 = 0$.\(^{12}\) Finally, to highlight the source of the failure from relaxing (10), we

\(^{12}\)For example, this would arise if $K_0 = \infty$, and recall from Case 1 of Example 3 that changes in cost over time do not lead to violations of parallel trends per se. Also, by Observation 1, the same kind of violation of parallel trends (Assumption 1) would arise conditional on choice in period 0 if we allowed for a fuzzy design.
continue assuming that costs are known and that parallel trends is satisfied conditional on the initial selection term $U_0$.\(^\text{13}\)

For example, treatment and no treatment are bandit arms and there is learning about the arms’ mean returns, net of a known trend: decision makers hold non-degenerate prior beliefs which are updated based on past realized outcomes. If learning is valuable to a decision maker, then this past information affects future decisions.

In period 1, the (maximal) optimal treatment rule in (7) now is:

$$D_1 = D_1(0) = I \{ E[Y_1(1)|U_0] - E[Y_1(0)|U_0, Y_0(0)] \geq k_1 \}$$

(17)

Since no one is treated initially, the mean untreated outcome in period 1 conditions on the information that is learned in period 0 from the safe arm (and hence the conditioning on both $U_0$ and $Y_0(0)$). Combining (16) and (17), we can partition the sample space into three groups as a function of the response to additional information from the past realized outcome $Y_0(0)$: those that are always ($a$) or never ($n$) treated in period 1 regardless of $Y_0(0)$, and those for whom $D_1$ depends on $Y_0(0)$ because there is valuable learning ($vl$):

$$\Omega_a \equiv \{ \omega \in \Omega : E[Y_1(1)|U_0(\omega)] - k_1 \geq E[Y_1(0)|U_0(\omega), Y_0(0) = 1] \};$$

$$\Omega_n \equiv \{ \omega \in \Omega : E[Y_1(1)|U_0(\omega)] - k_1 < E[Y_1(0)|U_0(\omega), Y_0(0) = 0] \};$$

$$\Omega_{vl} \equiv \{ \omega \in \Omega : E[Y_1(0)|U_0(\omega), Y_0(0) = 0] \leq E[Y_1(1)|U_0(\omega)] - k_1 < E[Y_1(0)|U_0(\omega), Y_0(0) = 1] \};$$

Note that there may still be learning among the always- and never-treated (where here the terms are used in reference to treatment as a function of realized outcomes), but it does not affect their future decisions and hence is not valuable. Also, the groups are defined in terms of counterfactual expectations — that is, for values of $Y_0(0)$ that may differ from those realized.\(^\text{14}\) We can further partition the “valuable learning” group into their realized period

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\(^{\text{13}}\)For nonzero trends, the combination of (12) with binary outcomes imposes additional support restrictions on $E[Y_0(0)|U_0]$. We presume these are satisfied since they are of limited economic interest. As in the preceding Case 1, the salient example is a zero trend, but we consider the general case to also understand the role of the trend in our conclusions.

\(^{\text{14}}\)For simplicity of notation we assume the expectations are well-defined. This is so except where potential outcomes are degenerate given $U_0$. At such unobservables there is no rational learning, in which case the ill-defined history-conditional expectations $E[Y_1(0)|U_0, Y_0(0)]$ can be replaced with the well-defined history-unconditional expectations $E[Y_1(0)|U_0]$. 

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0 outcomes:

\[ \Omega^0_{vl} = \{ \omega \in \Omega_{vl} : Y_0(0, \omega) = 0 \} \]

\[ \Omega^1_{vl} = \{ \omega \in \Omega_{vl} : Y_0(0, \omega) = 1 \} \]

Let \( P^j_i \equiv \Pr\{\Omega^j_i\} \) for \( i \in \{a, n, vl\} \) and \( j \in \{0, 1\} \). Then we can write

\[
E[Y_1(0) - Y_0(0)|D_1 = 1, D_0 = 0] = E[Y_1(0) - Y_0(0)|\Omega_a \cup \Omega^0_{vl}]
\]

\[
= \frac{P_a}{P_a + P^0_{vl}} E[Y_1(0) - Y_0(0)|\Omega_a] + \frac{P^0_{vl}}{P_a + P^0_{vl}} E[Y_1(0) - Y_0(0)|\Omega^0_{vl}]
\]

\[
= \frac{P_a}{P_a + P^0_{vl}} \tau + \frac{P^0_{vl}}{P_a + P^0_{vl}} (E[Y_1(0)|\Omega^0_{vl}] - 1)
\]

By analogous reasoning,

\[
E[Y_1(0) - Y_0(0)|D_1 = 0, D_0 = 0] = E[Y_1(0) - Y_0(0)|\Omega_n \cup \Omega^1_{vl}]
\]

\[
= \frac{P_n}{P_n + P^1_{vl}} E[Y_1(0) - Y_0(0)|\Omega_n] + \frac{P^1_{vl}}{P_n + P^1_{vl}} E[Y_1(0) - Y_0(0)|\Omega^1_{vl}]
\]

\[
= \frac{P_n}{P_n + P^1_{vl}} \tau + \frac{P^1_{vl}}{P_n + P^1_{vl}} (E[Y_1(0)|\Omega^1_{vl}] - 1)
\]

Because \( E[Y_1(0) - Y_0(0)] = \tau \) by (12), parallel trends is satisfied if and only if both preceding equations equal \( \tau \). This in turn requires at least one of the following to hold. Either there is zero probability of valuable learning, \( P_{vl} = P^0_{vl} + P^1_{vl} = 0 \), or valuable learning occurs where untreated outcomes are identical over time almost surely, \( P(Y_0(0) = Y_1(0)|\Omega_{vl}) = 1 \), and there is a zero trend, \( \tau = 0 \).  

The first possibility is obvious. For the second, observe that if \( P^0_{vl}, P^1_{vl} > 0 \), then parallel trends requires that:

\[
E[Y_1(0)|\Omega^0_{vl}] = E[Y_1(0)|\Omega^1_{vl}] - 1 = \tau
\]

Because \( Y_1(0) \) is binary, this is only possible if \( \tau = 0 \), which yields the desired result. Finally, in the remaining cases where either \( P^0_{vl} = 0 \) or \( P^1_{vl} = 0 \) (but not both), it follows that \( E[Y_0(0)|\Omega_{vl}] \in \{0, 1\} \). In that case \( Y_0(0) \) is known by decision makers in \( \Omega_{vl} \) in period 0, and so there is essentially no rational learning from the untreated outcome.

\[ ^{15} \text{This still allows for learning about the untreated arm. For example, there is initial uncertainty about whether the process is one of two degenerate arms, so that } E[Y_1(0)|\Omega_{vl}] \in (0, 1). \text{ However, once the first return is observed, this uncertainty is fully resolved, i.e., } E[Y_1(0)|\Omega_{vl}, Y_0(0)] \in \{0, 1\}. \]
Summarizing, among the always and never-treated groups, parallel trends is satisfied by the same arguments as in Subsection 3.3 because the treatment sequence is a function of $U_0$. However, parallel trends is violated among the group of valuable learners because of backward-looking dynamic selection, similarly to what we saw in Example 1 before. Thus, parallel trends is violated if there is a positive probability of valuable learning on a subset of the sample space where untreated outcomes are not identical over time almost surely.

Given complex dynamic incentives such as the endogenous value of experimentation, it is perhaps more surprising that a partial parallel trends condition defined in (1) remains generally satisfied in period 0 even when there is learning and selection into treatment in period 0, where we now allow $D_0 \neq 0$. As in Case 1, the optimal decision rule in period 0:

$$D_0 = I\{E[Y_0(1) - Y_0(0) + \beta(V_1(1) - V_1(0)) - K_0|U_0] \geq 0\}$$

(18)

is a function of $U_0$. Therefore, combining with (12), similar reasoning as in Subsection 3.3 yields:

$$E[Y_1(0) - Y_0(0)|D_0 = d_0] = E[Y_1(0) - Y_0(0)|U_0 \in U_0(d_0, 0) \cup U(d_0, 1)]$$

$$= E[E[Y_1(0) - Y_0(0)|U_0]|U_0 \in U_0(d_0, 0) \cup U(d_0, 1)]$$

$$= \tau$$

where the second equality follows from (12). This implies partial parallel trends (1) from the standpoint of period 0. Intuitively, while decision makers can anticipate the expected value of future information and even internalize the endogenous motive for learning in their first-period decisions, the important distinction is that the first-stage decisions cannot condition differentially on first versus second stage potential outcomes, since both outcomes are identically uncertain at the time of decision making. In contrast, future decisions can condition on past outcomes, and may optimally do so when past outcomes convey valuable information about the future (which would generally invalidate Assumption 2).

3.5 Learning Only From a Risky Treatment Arm

It is worth noting an example of learning that is compatible with Assumption 1 in the classic bandit model of exploration versus exploitation. This is the case where the untreated arm is
safe, in the sense that its mean return is known at the outset, and provides no information about the risky treatment arm.

**Case 3** (of Example 3). *Everything remains as in Case 2, except that we additionally exclude the possibility of learning from the untreated arm:*

\[
E[Y_1(0)|U_0, Y_0(0)] = E[Y_1(0)|U_0] \tag{19}
\]

and assume the treated outcome provides no information about the untreated outcome in period 0, conditional on initial information.

\[
E[Y_0(0)|U_0, Y_0(1)] = E[Y_0(0)|U_0] \tag{20}
\]

Combined with the restriction (15) that there is no learning across no treatment and treatment arms, (19) still relaxes (10) because it allows for information to be acquired through experimentation with the risky treatment arm,\(^{16}\)

\[
E[Y_1(1)|U_0, Y_0(1)] \neq E[Y_1(1)|U_0]
\]

Because our model now allows for selection on determinants beyond \(U_0\), with (20) we additionally rule out information across treatment arms within the same period, as would arise if the outcomes were conditionally independent. This does not rule out correlation in the arms across decision makers since the assumption conditions on \(U_0\). Thus, from the econometrician’s standpoint, an observation from one arm could still be informative about the other.

Under (15) and (19), an optimal treatment rule in period 1 is:

\[
D_1(0) = I\{E[Y_1(1) - Y_1(0) - K_1|U_0] \geq 0\} \tag{21}
\]

\[
D_1(1) = I\{E[Y_1(1)|U_0, Y_0(1)] - E[Y_1(0) + K_1|U_0] \geq 0\} \tag{22}
\]

Thus, one also recovers partial parallel trends (2) in \(D_1 = d_1\) conditional on \(D_0 = 0\) by the now-familiar argument from Subsection 3.3 that \(D_1(0)\) and \(D_0\) are functions of \(U_0\), in which

\(^{16}\)Again, (10) maintains that \(E[Y_1(d_1)|U_0, Y_0(d_0)] = E[Y_1(d_1)|U_0]\) for all \(d_0, d_1 \in \{0, 1\}\), and so the mean belief about period 1 outcomes remains the same.
untreated outcomes satisfy conditional parallel trends by (12). Recalling the definition of $U_0(d_0, d_1)$ from (14), formally we have:

$$E[Y_1(0) - Y_0(0)|D_0 = 0, D_1 = d_1] = E[Y_1(0) - Y_0(0)|D_0 = 0, D_1(0) = d_1]$$

$$= E[Y_1(0) - Y_0(0)|U_0 \in U_0(0, d_1)]$$

$$= E[E[Y_1(0) - Y_0(0)|U_0]|U_0 \in U_0(0, d_1)]$$

$$= \tau$$

which implies partial parallel trends (2) in period 1 conditional on $D_0 = 0$.

Finally, since the decision rule $D_1(1)$ is a function of $(U_0, Y_0(1))$, recovering partial parallel trends (2) in period 1 conditional on $D_0 = 1$ requires the additional assumption (20) that the risky arm return provides no feedback to the decision maker about the past outcome under the safe arm. In that case, combining with (19), selection on the risky arm in $D_1(1)$ does not induce selection on the untreated trend.

The optimal treatment rules (18) in period 0 and (22) in period 1 define sets $W(1, d_1)$ such that:

$$\{(U_0, Y_0(1)) \in W(1, d_1)\} = \{D_0 = 1, D_1(1) = d_1\}$$

Then:

$$E[Y_1(0) - Y_0(0)|D_0 = 1, D_1 = d] = E[Y_1(0) - Y_0(0)|D_0 = 1, D_1(1) = d]$$

$$= E[Y_1(0) - Y_0(0)|(U_0, Y_0(1)) \in W(1, d)]$$

$$= E[E[Y_1(0) - Y_0(0)|(U_0, Y_0(1))]|(U_0, Y_0(1)) \in W(1, d)]$$

$$= E[E[Y_1(0) - Y_0(0)|U_0]|(U_0, Y_0(1)) \in W(1, d)]$$

$$= \tau$$

where the fourth equality invoked (15) and (20). Combining with the preceding instances of partial parallel trends, by Observation 1 we then recover full parallel trends (Assumption 1) in the classic exploitation-exploration stationary bandit model. The important difference from the preceding counter-example is that learning only arose from experimentation with the treatment arm; there was no learning by decision makers about or from the untreated outcome.
The cases of Example 3 we have considered are meant to illustrate how basic selection issues can — but need not — arise in a structural model of dynamic choice. Of course, such selection issues may not be based on learning or even rooted in information. For example, other selection issues could arise based on selection on present information (as in Example 2), or in a model where the utility function depends on past realized outcomes. We conclude with a final example where we investigate the plausibility of parallel trends in the context of optimal stopping.

### 3.6 Optimal Stopping

To further investigate the relation between parallel trends and choices in a dynamic setting, we now study an optimal stopping problem where decision makers are rational and forward-looking. We examine when such behavior is consistent with parallel trends.

Optimal stopping has a long history in empirical economics. Early examples are the two classic works by Mortensen (1970) and Lippman and McCall (1976). A key feature of these models is that current decisions depend on previous ones in the sense that an agent continues to search only if this agent did not accept a job offer in the past. Once the agent accepts an offer, then this agent cannot revert back to the non-employment state.

In an optimal stopping problem, the decision to exit at time $t$, denoted by $D_t = 0$, is irreversible. This is akin to the “staggered” design in event studies (i.e., $D_t = 0$ implies $D_{t'} = 0$ for all $t' > t$). For simplicity, we consider a two-period optimal stopping problem where individuals choose whether to receive a binary treatment in each period $t \in \{0, 1\}$.

The decision to exit ($D_t = 0$) terminates the optimal stopping process with a constant payoff which is normalized to 0. The time discount factor is $\beta \in (0, 1)$.

At $t = 1$, a rational individual chooses to stay ($D_1 = 1$) if and only if $v_1(Y_0(1)) - \varepsilon_1 \geq 0$, where the left-hand side is the instantaneous utility from staying, with $Y_0(1)$ being the potential outcome from staying, which is realized after the choice to stay at $t = 0$.

At $t = 0$, the expected return from staying ($D_0 = 1$) is

$$v_0 - \varepsilon_0 + \beta \int \max\{v_1(Y_0(1)) - \varepsilon_1, 0\} dF(Y_0(1), \varepsilon_1|\varepsilon_0),$$

17 Other important contributions in economics are the works of Pakes (1986), Rust (1987), and Wolpin (1987). More generally, this problem is part of the dynamic Markovian decision problem literature that includes reinforcement learning problems.
where \( v_0 \) is a constant component in the instantaneous payoff from staying at \( t = 0 \), and \( F(\cdot|\varepsilon_0) \) denotes the potential transition to future states \((Y_0(1), \varepsilon_1)\) from \( \varepsilon_0 \) under the choice of staying at \( t = 0 \). Thus, a rational and forward-looking individual chooses to stay at \( t = 0 \) if and only if (23) is greater than zero, the normalized payoff from exiting. We denote the set of realized \( \varepsilon_0 \) that leads to \( D_0 = 1 \) by \( \omega_0 \).

In this model, parallel trends requires that the following conditional moments be identical:

\[
E[Y_1(0) - Y_0(0)|D = (0, 0)] = E[Y_1(0) - Y_0(0)|\varepsilon_0 \notin \omega_0];
\]
\[
E[Y_1(0) - Y_0(0)|D = (1, 0)] = E[Y_1(0) - Y_0(0)|\varepsilon_0 \in \omega_0, \varepsilon_1 > v_1(Y_0(1))];
\]
\[
E[Y_1(0) - Y_0(0)|D = (1, 1)] = E[Y_1(0) - Y_0(0)|\varepsilon_0 \in \omega_0, \varepsilon_1 \leq v_1(Y_0(1))].
\]

Thus, Assumption 2 would not hold if the trend in untreated potential outcomes \( Y_1(0) - Y_0(0) \) is generally correlated with the selection errors \((\varepsilon_0, \varepsilon_1)\) as well as \( Y_0(1) \), the potential outcome from staying at \( t = 0 \).

### 4 Some Identification Approaches Motivated by Choice

Based on insights from the models in Section 3, this section considers a menu of approaches for identification when full parallel trends (Assumption 1) is violated. In developing alternative identification approaches to (full) parallel trends, we emphasize that there is typically not a single solution because different models of the data-generating process yield different results. This is similar to the framework\(^{18}\) of Manski and Pepper (2018).

Here, we focus on a single target parameter, the average treatment effect at \( t = 1 \) among switchers into treatment:

\[
E[Y_1(0) - Y_1(0)|D_0 < D_1]
\]  \( (24) \)

The treatment effect among switchers \((24)\) coincides with the usual treatment effect on the treated in period \( t = 1 \), i.e., \( E[Y_1(1) - Y_1(0)|D_1 = 1] \), in the case with a pre-treatment period (where \( D_0 = 0 \) with probability one). The parameter defined in \((24)\) provides a natural generalization under fuzzy designs, where \( P(D_0 = 1) > 0.\)

\(^{18}\)See also Rambachan and Roth (2019) for interesting recent work in this direction.

\(^{19}\)Alternatively, the usual ATT \( E[Y_1(1) - Y_1(0)|D_1 = 1] \) in the fuzzy design would average over treated and untreated observations in period 0, whereas the ATT on the persistently treated \( E[Y_1(1) - Y_1(0)|D_0 = 0] \)
effect among switchers also serves as a building block for the DID-M estimator proposed in De Chaisemartin and d’Haultfoeuille (2020).

Despite a seeming resemblance to the LATE with time as an instrument, there are some important differences. In typical panel data settings, time is not randomly assigned or excluded from the potential outcome equation, and treatment may not be monotonic in time. Additionally, treatment is observed in each time period, whereas it is typically only observed for one instrument realization. We return to the conceptual relations between the two parameters and their identification in Subsection 4.2. Throughout we restrict ourselves to settings where a positive mass of never-treated and switchers into treatment exists, $P(D_0 = D_1 = 0) > 0, P(D_1 > D_0) > 0$.

4.1 Partial Parallel Trends

One takeaway from the choice models in Section 3 is that parallel trend comparisons across some treatment sequences may be more robust to selection concerns than others. This motivates the question: what can be identified under relaxations of Assumption 1 that impose parallel trends only among subsets of the treatment sequence realizations?

First, recall the following partial parallel trends assumption from Observation 1, which relaxes full parallel trends by only imposing parallel trend restrictions between switchers into treatment and the never-treated.

**Assumption 2** (Partial Parallel Trend Between Switchers and Never-Treated).

$$E[Y_1(0) - Y_0(0)|D_0 = 0, D_1 = d_1] = \tau_{0,1} \text{ for } d_1 \in \{0, 1\}$$

Assumption 2 only requires that the untreated trend is independent of the first period treatment conditional on $D_0 = 0$; it imposes no restriction on trend-based selection in the initial period 0. For example, we showed in Section 3.5 how Assumption 2 is consistent with a model of dynamic choices with learning under the condition (19) that the untreated arm is $D_1 = 1$ would likely require stronger assumptions, since among this subgroup the untreated outcome is never observed.

20 For an analysis where time is a monotonic instrument, see De Chaisemartin and d’Haultfoeuille (2018).
“safe.” Now we show that Assumption 2 is sufficient for recovering the period 1 average treatment effect for switchers into treatment.

**Proposition 1.** Let Assumption 2 hold. Then, the following is identified:

\[
E[Y_1(1) - Y_1(0)|D_0 < D_1] = E[Y_1 - Y_0|D_0 < D_1] - E[Y_1 - Y_0|D_0 = D_1 = 0] \quad (25)
\]

The result follows immediately from the usual logic with a pre-treatment period applied to the subgroup where \( D_0 = 0 \).\(^{21}\) Furthermore, it is straightforward to show that (25) is one of only two basic treatment effects among \( E[Y_r(1) - Y_r(0)|(D_0, D_1) = (d_0, d_1)] \) that is identified even under full parallel trends.\(^{22}\) In summary, violations of full parallel trends need not lead to failures of point identification per se.

Further motivated by the more general learning example in Subsection 3.4, we also consider the content of a weakened parallel trend restriction that only conditions on past realized treatments.

**Assumption 3** (Forward Parallel Untreated Trend). The mean untreated trend is constant across realized treatment in period 0:

\[
E[Y_1(0) - Y_0(0)|D_0 = d_0] = \tau_0 \quad \text{for } d_0 \in \{0, 1\}. \quad (26)
\]

As shown in Subsection 3.4, such an assumption is consistent with choice in environments where decision-makers can learn but have no differential information about untreated outcomes across period 0 and period 1 at the time of deciding in period 0. Assumption 3 relaxes

\(^{21}\)Relating to the existing literature, Proposition 1 captures the basic intuition behind the DID\(_+\) estimator (in turn underlying the DID\(_M\) estimator) of De Chaisemartin and d’Haultfoeuille (2020) but shows that it holds under a logically weaker assumption than full parallel trends (Assumption 1). A similar intuition also underlies the “building block” estimators of Callaway and Sant’Anna (2021) and Sun and Abraham (2021) in staggered settings. In that case, however, our simple two-period framework does not distinguish between whether the control group \( \{D_0 = D_1 = 0\} \) is never-treated or not-yet treated. For elaboration of the distinction, see Callaway and Sant’Anna (2021). Of course, it is worth noting that in each of these papers, the primary contribution is arguably to provide methods for aggregating over such basic effects. Additionally, our relaxation of full to partial parallel trends is different in spirit from bound-based, partially identifying relaxations considered in Manski and Pepper (2018) and Rambachan and Roth (2019).

\(^{22}\)By symmetry, the other is the period 0 average treatment effect among switchers out of treatment,

\[
E[Y_0(1) - Y_0(0)|D_0 > D_1]
\]
multi-period parallel trends (Assumption 1) by only imposing parallel trends conditional on treatment in period 0. This allows for selection on past outcomes in the treatment decision of period 1.\footnote{A common approach in such settings is to use estimators that condition or match on lagged dependent variables (e.g. Abadie, Diamond, and Hainmueller (2010) and Dehejia and Wahba (1999), respectively); see also Angrist and Pischke (2008) and Ding and Li (2019) for circumstances where the treatment effect is bounded between the DiD and lagged dependent variable estimators. However, conditions for consistent estimation with lagged dependent variables may be necessarily strong (e.g. Nickell (1981)).}

In contrast to Assumptions 1 and 2, Assumption 3 does not point identify the average treatment effect among switchers into treatment (and thereby, as a weakening of Assumption 1, any other basic treatment effects). Consider the identity:

\[
E[Y_1(1) - Y_1(0) | D_1 > D_0] = E[Y_1(1) - Y_0(0) | D_1 > D_0] - E[Y_1(0) - Y_0(0) | D_1 > D_0] \tag{27}
\]

The first term is identified, and so it remains to identify the second term, namely the untreated trend among the switchers. Decomposing (26) into its constituent treatment sequences, Assumption 3 is equivalent to:

\[
\sum_{d_1} P(D_1 = d_1 | D_0 = 0) E[Y_1(0) - Y_0(0) | D_0 = 0, D_1 = d_1] = \sum_{d'_1} P(D_1 = d'_1 | D_0 = 1) E[Y_1(0) - Y_0(0) | D_0 = 1, D_1 = d'_1]
\]

Thus, Assumption 3 only allows us to express the untreated trend in (27) in terms of other unobserved untreated trends among the always-treated and switchers out of treatment:

\[
E[Y_1(0) - Y_0(0) | D_0 < D_1] = \frac{P(D_1 = 1 | D_0 = 1)}{P(D_1 = 1 | D_0 = 0)} E[Y_1(0) - Y_0(0) | D_0 = D_1 = 1] + \frac{P(D_1 = 0 | D_0 = 1)}{P(D_1 = 1 | D_0 = 0)} E[Y_1(0) - Y_0(0) | D_0 > D_1] - \frac{P(D_1 = 0 | D_0 = 0)}{P(D_1 = 1 | D_0 = 0)} E[Y_1(0) - Y_0(0) | D_0 = D_1 = 0]
\]

A stark research design that circumvents this issue is one where \(P(D_1 = 0) = 1\), but in that case there is no selection in the second period and so full parallel trends (Assumption 1) is trivially recovered. Next, we consider other assumptions — in some cases nested by the above — that allow at least partial identification of the desired target parameter.
4.2 Mean Stationarity

We now revisit the role of mean stationarity of untreated outcomes. In Section 3, mean stationarity arose as a necessary condition for parallel trends in Examples 1 and 2 and as a salient case of our dynamic choice model in Example 3, where agents learned about the productivity of fixed bandit arms. Next we show how, when applicable, such mean stationarity can also bypass selection concerns by providing an alternative path to identification. Recall the following treatment-invariant notion of untreated stationarity:

**Assumption 4** (Mean Stationarity of Untreated Outcomes). The mean untreated outcome is constant across time:

\[ E[Y_1(0) - Y_0(0)] = 0 \]

Assumption 4 is not nested by parallel trends (Assumption 1). Parallel trends does not restrict the trend to zero, and so it does not imply stationarity. Conversely, stationarity is independent of the realized treatment sequence \( D = d \), and so it does not imply any version of parallel trends.\(^{24}\) Even though unconditional stationarity does not imply parallel trends, it also identifies the average treatment effect on the treated in the basic setting with a pre-treatment period.\(^{25}\) Note that a similar result would hold as a function of any assumed unconditional trend \( E[Y_1(0) - Y_0(0)] \) other than zero.

**Proposition 2.** Suppose \( D_0 = 0 \). Under Assumption 4, the treatment effect on the treated in period 1 is identified:

\[
E[Y_1(1) - Y_1(0)|D_1 = 1] = E[Y_1(1) - Y_1(0)|D_0 < D_1] = \frac{E[Y_1] - E[Y_0]}{P(D_1 = 1)}
\]

The proof is provided in Appendix A. Since \( D_1 \geq D_0 = 0 \), this appears related to the

\(^{24}\)In the basic DiD setting, Roth and Sant’Anna (2021) show that a stronger, distributional version of parallel trends in the untreated outcome holds if and only if the distribution of untreated outcomes is a mixture of a distribution that is invariant in time and a distribution that is invariant in treatment. This seems to suggest that either stationarity or random assignment of untreated outcomes is a sufficient condition for parallel trends. However, their notion of stationarity also conditions on treatment. Example 1 provides a simple case where mean stationarity can hold while parallel trends does not.

\(^{25}\)A related version of time invariance is considered previously by Manski and Pepper (2018). The main difference between our assumptions is that theirs is imposed by indexed observation rather than in expectation; in our setting, an elementwise version of time invariance, i.e., \( Y_0(0) = Y_1(0) \), would also imply parallel trends.
LATE theorem of Imbens and Angrist (1994) with time as an instrument. However, recall a few important differences from the previous discussion of the target parameter. First, the time index violates the exclusion restriction since the potential outcome $Y_t(d)$ can depend on $t$. Time invariance (Assumption 4) can be interpreted as a mean exclusion restriction on the untreated outcomes, but no such assumption is imposed on mean treated outcomes. Instead, mean treated outcomes in period 0 are by assumption never realized. Finally, both $D_0$ and $D_1$ are everywhere observed.

A stronger, treatment-conditional version of stationarity that identifies an average treatment effect on switchers in the fuzzy design, where some observations are initially treated $P(D_0 = 1) > 0$, is the following.

**Assumption 5** (Forward Mean Stationarity of Untreated Outcomes). The mean untreated outcome is constant across time, conditional on past treatment:

$$E[Y_1(0) - Y_0(0)|D_0 = d_0] = 0 \quad \text{for} \quad d_0 \in \{0, 1\}.$$  

Assumption 5 is jointly an assumption about stationarity (because it implies Assumption 4) and about selection (because it conditions on treatment and implies Assumption 3). Thus, it is similar to a mean version of conditional stationarity in Roth and Sant’Anna (2021). However, it differs by conditioning on past treatment in period 0, rather than treatment in period 1. As illustrated by the examples in Section 3 that violate parallel trends, conditioning on past treatment may be more credible in dynamic choice contexts where decision-makers are less likely to have (and thereby select on) information about the future than information about the past. In particular, under zero trend, Assumption 5 is satisfied in the learning example of Subsection 3.4, whereas its analog conditioning on treatment in period 1 is not.

Under Assumption 5, an analogous identification result is immediate upon applying Proposition 2 in the subpopulation where $D_0 = 0$.

**Corollary 1.** Under Assumption 5, the treatment effect in period 1 for the switchers into treatment is identified:

$$E[Y_1(1) - Y_1(0)|D_0 < D_1] = \frac{E[Y_1|D_0 = 0] - E[Y_0|D_0 = 0]}{P(D_1 = 1|D_0 = 0)}$$  

26For previous results interpreting time as an instrument, see also De Chaisemartin and d’Haultfoeuille (2018).
It is worth briefly comparing this to the weaker Assumption 3 result, which allowed for a non-zero conditional trend $\tau_0 \neq 0$. In that case, identification was not possible without further assumptions. However, as in the discussion of Assumption 4, there is nothing special about assuming that the conditional trend is equal to zero; a similar result would hold as a function of any assumed conditional trend $E[Y_1(0) - Y_0(0)|D_0 = d_0] = \tau_0$ other than zero. Finally, we turn to the possibility of partial identification when such assumptions on trends are not viable.

### 4.3 Bounds from Economic Structure

We now consider partial identification of $E[Y_1(1) - Y_1(0)|D_0 < D_1]$ under assumptions stemming from our model(s) of dynamic selection. First, observe that the treated outcome $E[Y_1(1)|D_0 < D_1]$ is identified from the data, and so it suffices to focus on bounding the untreated outcome in period 1 among switchers into treatment, $E[Y_1(0)|D_0 < D_1]$. Alternatively, expanding the treatment effect as in (25),

$$E[Y_1(1) - Y_1(0)|D_0 < D_1] = E[Y_1(1) - Y_0(0)|D_0 < D_1] - E[Y_1(0) - Y_0(0)|D_0 < D_1],$$

it suffices to bound the switchers’ untreated trend, $E[Y_1(0) - Y_0(0)|D_0 < D_1]$. In both approaches, we can partially identify the average treatment effect on the switchers into treatment ($D_0 < D_1$) using some version of monotone treatment on selection assumptions, similar to those introduced by Manski (1997) and Manski and Pepper (2000) but generalized to our multi-period setting.

For example, consider:

**Assumption 6 (MTS).** *Monotone Treatment Selection (or MTS) on Level or Trend of Untreated Potential Outcomes:*

- **Level:** $E[Y_1(0)|D_0 < D_1] \leq E[Y_1(0)|D_0 = D_1 = 0]$

- **Trend:** $E[Y_1(0) - Y_0(0)|D_0 < D_1] \geq E[Y_1(0) - Y_0(0)|D_0 = D_1 = 0]$

---

27 For related but more model-agnostic approaches, see also Manski and Pepper (2013) and Rambachan and Roth (2019).
Although one could consider more general MTS assumptions across treatment sequences, we focus on these specific comparisons to the never-treated because they i) partially identify our target parameter and ii) have a basis in our learning model of Subsection 3.4. The proof of the following identification result is immediate by assumption and the preceding discussion.

**Proposition 3.** Under Assumption 6, the identified set for the treatment effect \( E[Y_1(1) - Y_1(0)|D_0 < D_1] \) is:

\[
\begin{align*}
&[E[Y_1|D_1 > D_0] - E[Y_1|D_0 = D_1 = 0], \\
&E[Y_1 - Y_0|D_1 > D_0] - E[Y_1 - Y_0|D_0 = D_1 = 0]
\end{align*}
\]

We now motivate the identifying assumptions with the learning model presented in Subsection 3.4. We begin by showing that, among decision-makers whose period 1 decision depends on the past realized outcome, those who continue without treatment are over-sampled from higher-mean untreated arms. This follows from two observations. First, recalling (16), past untreated outcomes provide further information about the true returns of the bandit arm, and thereby about future outcomes:

\[
E[Y_1(0)|U_0, Y_0(0) = 1] \geq E[Y_1(0)|U_0, Y_0(0) = 0].
\]  

(28)

Second, for initial information sets \( U_0 = u^* \) of “valuable learners” where \( D_0 = 0 \) and the period 1 treatment depends on the past outcome, (17) implies:

\[
\{U_0 = u^*, D_0 = 0, D_1 = d_1\} = \{U_0 = u^*, Y_0(0) = 1 - d_1\} \quad \text{for } d_1 \in \{0, 1\}
\]  

(29)

Combining (28) and (29) leads to higher untreated outcomes \( Y_1(0) \) in period 1 among the never-treated than among those who switch into treatment:

\[
E[Y_1(0)|U_0 = u^*, D_0 = D_1 = 0] \geq E[Y_1(0)|U_0 = u^*, D_0 < D_1].
\]  

(30)

Additionally, (29) implies that:

\[
E[Y_0(0)|U_0 = u^*, D_0 = 0, D_1 = d_1] = 1 - d_1
\]  

(31)

and the fact that \( Y_1(0) \) was assumed binary implies:

\[
E[Y_1(0)|U_0 = u^*, D = d] \in [0, 1]
\]  

(32)
Combining (31) and (32) yields the opposite direction of inequality on the untreated trend:

$$E[Y_1(0) - Y_0(0)|U_0 = u^*, D_0 = D_1 = 0] \leq 0 \leq E[Y_1(0) - Y_0(0)|U_0 = u^*, D_0 < D_1].$$  (33)

The directions of inequality in (30) and (33) are those of Assumption 6. However, these inequalities also condition on a period 0 unobservable realization $U_0 = u^*$, for which the choice of period 1 treatment is a function of the period 0 realized outcome.

In the special case where there is no ex ante variation in unobservables from the standpoint of decision-makers, i.e., $P(U_0 = u^*) = 1$, the inequalities (30) and (33) imply Assumption 6. Otherwise, one solution is to impose (or ideally, derive) additional structure between treatment sequences, untreated outcomes, and unobservables. For example, consider:

**Assumption 7.** The period 0 unobservable can be taken as scalar $U_0 \in \mathbb{R}$ that satisfies:

1. For all $u$ where the conditional expectations exist,
   $$E[Y_1(0)|U_0 = u, D_0 < D_1] \leq E[Y_1(0)|U_0 = u, D_0 = 0]$$
2. $E[Y_1(0)|U_0 = u, D_0 = 0]$ is nondecreasing in $u$.
3. $(U_0|D_0 < D_1) \preceq_{FOSD} (U_0|D_0 = D_1 = 0)$

Assumption 7a generalizes (30) across all unobservables $U_0 = u$: selection into treatment in period 1 is indicative of lower untreated outcomes in expectation. Assumption 7b imposes that higher unobservables correspond to higher untreated outcomes; for example, the unobservable could be an average return from a previous, fixed-length sequence of untreated outcome realizations observed by decision-makers but not the econometrician. Note that this has no content on its own, since unobservables $U_0$ can always be relabeled and arranged such that this condition is satisfied. However, Assumption 7c imposes that unconditional selection into treatment oversamples lower unobservables, which by Assumption 7b correspond to
lower mean expected returns. Under Assumption 7, we recover Assumption 6a as follows:

\[
E[Y_1(0)|D_0 < D_1] = E[E[Y_1(0)|U_0, D_0 < D_1]|D_0 < D_1] \\
\leq E[E[Y_1(0)|U_0, D_0 = 0]|D_0 < D_1] \\
\leq E[E[Y_1(0)|U_0, D_0 = 0]|D_0 = D_1 = 0] \\
\leq E[E[Y_1(0)|U_0, D_0 = D_1 = 0]|D_0 = D_1 = 0] \\
= E[Y_1(0)|D_0 = D_1 = 0]
\]

where the equalities follow by the law of iterated expectations, the first and third inequalities follow from Assumption 7a, and the second inequality follows from combining Assumption 7b and c. Alternatively, one could attempt to (partially) identify a marginal group \(U_0 = u^*\) with other assumptions or an additional source of exogenous variation, such as an instrument. We leave this to future work.

5 Conclusion

In this paper, we made connections between the commonly used parallel trends assumption and models of dynamic rational choice in economics. In particular, we highlight models with time-varying treatment costs, learning, correlated utilities, and optimal stopping. Our aim is to focus cleanly on channels of dynamic behavior that are at work in these models, in order to understand the way that these channels can validate or invalidate design-based identifying assumptions. The examples we provide are deliberately stylized and simple. In cases when parallel trends may be violated, we provide pointers to inference approaches based on simple and familiar economic restrictions that are motivated by economic concerns. These include either relaxing or considering alternatives to parallel trends.

Our hope is that the paper provides a canvas by which further work on the econometrics of treatment or causal inference with observational data can be examined. This is particularly important with dynamic decisions in rich environments where a variety of preference- and information-based dynamic considerations play a role and where it is helpful to relate choice models based on these considerations to examine what behavior is allowed and what is not.
We conclude with a sentiment consistent with our approach in this paper, namely that “models are most useful when they are used to challenge existing formulations, rather than to validate or verify them” (Oreskes, Shrader-Frechette, and Belitz (1994) p. 644). In that sense, using choice models to shed light on DiD regressions is a common and worthy use of modeling.
A Proofs

Proof of Observation 1. The fact that Assumption 1 implies the set of pairwise equalities (1) and (2) is immediate. Conversely, suppose (1) holds and define \( \tau = E[Y_1(0) - Y_0(0)|D_0 = d_0] \), which is by assumption independent of \( d_0 \). It suffices to show that also \( E[Y_1(0) - Y_0(0)|D_0 = d_0, D_1 = d_1] = \tau \) for each \((d_0, d_1)\). Define \( p(d_0) = E[D_1|D_0 = d_0] \). Then for each \( t \) the law of total probability implies:

\[
E[Y_t(0)|D_0 = d_0] = p(d_0)E[Y_t(0)|D_0 = d_0, D_1 = 1] + (1 - p(d_0))E[Y_t(0)|D_0 = d_0, D_1 = 0]
\]

and thus:

\[
E[Y_1(0) - Y_0(0)|D_0 = d_0] = p(d_0)E[Y_1(0) - Y_0(0)|D_0 = d_0, D_1 = 1] + (1 - p(d_0))E[Y_1(0) - Y_0(0)|D_0 = d_0, D_1 = 0]
\]

By (2) each difference term on the right is equal to the term on the left, which in turn is equal to \( \tau \).

Proof of the claim in Example 1. We begin by showing that equality \( Y_0(0) = Y_1(0) \) a.s. is necessary for parallel trends. First, observe that \( D_0 = 0, D_1 = Y_0 \) and \( P(D_1 = 1) \in (0, 1) \) imply:

\[
E[Y_0(0)|D_1 = d] = d \quad (34)
\]

Substituting into the deviation from parallel trends,

\[
E[Y_1(0) - Y_0(0)|D_1 = 1] - E[Y_1(0) - Y_0(0)|D_1 = 0] = E[Y_1(0)|D_1 = 1] - E[Y_1(0)|D_1 = 0] - 1.
\]

Because outcomes \( Y_1(0) \) are assumed binary, this deviation is zero only if:

\[
E[Y_1(0)|D_1 = d] = d \quad \text{for } d = 0, 1.
\]

Combining with (34), this in turn requires \( Y_0(0) = Y_1(0) \) almost surely because the potential outcomes are binary. Conversely, equality \( Y_0(0) = Y_1(0) \) a.s. is sufficient for parallel trends.

\[\Box\]
**Proof of the claims in Example 2.** That conditions 1 and 2 are sufficient for Assumption 1 is immediate from the fact they jointly imply:

\[ E[Y_1(0) - Y_0(0)|(D_0, D_1) = (d_0, d_1)] = 0 \]

for all \((d_0, d_1)\) occurring with positive probability. Conversely, suppose the Roy model holds, so that:

\[ D_t = 0 \implies Y_t(0) = 1. \] (35)

Given an assumed interior probability of never-treated observations,

\[ E[Y_1(0) - Y_0(0)|D_0 = D_1 = 0] = 1 - 1 = 0 \]

Therefore Assumption 1 requires stationarity. Again invoking (35), parallel trends also requires that:

\[ E[Y_{1-t}(0)|D_t = 0, D_{1-t} = 1] = 1 \text{ for } t \in \{0, 1\} \]

Aggregating over the set of ever-untreated \(D_0D_1 = 0\) yields Condition 2.

Next, suppose that potential outcomes are also independent across time:

\((Y_0(0), Y_0(1)) \perp (Y_1(0), Y_1(1))\)

Since treatments \(D_t\) are a function of \((Y_t(0), Y_t(1))\) by definition of the Roy model, it follows that \(D_t \perp (Y_{1-t}(0), D_{1-t})\), and thus \(D_t \perp Y_{1-t}(0)|D_{1-t}\), for \(t \in \{0, 1\}\). In that case, we have:

\[ E[Y_1(0) - Y_0(0)|(D_0, D_1) = (d_0, d_1)] = E[Y_1(0)|D_1 = d_1] - E[Y_0(0)|D_0 = d_0] \]

for all \((d_0, d_1)\) occurring with positive probability. By assumption that \(P(D_0 = D_1 = 0) \in (0, 1)\), we have \(P(D_t = 0) > 0\) for all \(t\) and \(P(D_t = 1) > 0\) for some \(t\), say \(t = 0\). Then the following expectations are well-defined and Assumption 1 requires:

\[ E[Y_1(0)|D_1 = 0] - E[Y_0(0)|D_0 = 0] = E[Y_1(0)|D_1 = 0] - E[Y_0(0)|D_0 = 1] = \tau \]

and therefore:

\[ E[Y_0(0)|D_0 = 0] = E[Y_0(0)|D_0 = 1] \]

\(^{28}\text{By preceding arguments, } \tau = 0, \text{ but this is not necessary for what follows.}\)
However, from Condition 2 we know that $E[Y_0(0) | D_0 = 0] = 1$. Therefore $E[Y_0(0) | D_0 = 1] = 1$, and thereby $E[Y_0(0) | D_0 = D_1 = 1] = 1$. Finally, stationarity (Condition 1) then requires $E[Y_1(0) | D_0 = D_1 = 1] = 1$, yielding the desired conclusion. Conversely, recall that almost sure equality of untreated outcomes always satisfies Assumption 1.

\[
\boxed{\text{Proof of Proposition 2. We have:}}
\]

\[
E[Y_1(1) - Y_1(0) | D_0 < D_1] \\
= E[Y_1(1) - Y_1(0) | D_1 = 1] \\
\leq E[Y_1 | D_1 = 1] - E[Y_1(0) | D_1 = 1] \\
= \frac{E[Y_1 | D_1 = 1]P(D_1 = 1) - E[Y_1(0) | D_1 = 1]P(D_1 = 1)}{P(D_1 = 1)} \\
\leq \frac{E[Y_1 | D_1 = 1]P(D_1 = 1) - \{E[Y_1(0)] - E[Y_1(0) | D_1 = 0]P(D_1 = 0)\}}{P(D_1 = 1)} \\
\leq \frac{E[Y_1 | D_1 = 1]P(D_1 = 1) - E[Y_0(0)] + E[Y_1 | D_1 = 0]P(D_1 = 0)}{P(D_1 = 1)} \\
= \frac{E(Y_1) - E(Y_0)}{P(D_1 = 1)}
\]

The first equality follows because no one is treated in period 0, i.e. $D_0 = 0$ almost surely. The fourth equality follows by the law of iterated expectation. The fifth equality follows by Assumption 4. The sixth equality follows by the law of iterated expectation and the fact that $D_0 = 0$ almost surely.
References


