Uncertainty and the Signaling Channel of Monetary Policy

Jenny Tang*
Federal Reserve Bank of Boston

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Abstract

This paper studies an environment where policy actions provide a signal of economic fundamentals to imperfectly informed agents. I give closed-form solutions for optimal discretionary policy which explain how this signaling channel can improve outcomes by leading the policymaker to maintain lower inflation. The extent of this effect depends on relative uncertainty levels. I compare various direct communication strategies and show why less communication may be beneficial. I also find new empirical evidence in favor of the signaling channel based on a previously untested theoretical prediction of an interaction between uncertainty and responses of inflation forecasts to interest rate surprises.

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1 Introduction

It has become widely accepted that expectations play a key role in the decisions that drive economic fluctuations. How these expectations are formed has been a subject of much debate. With a few exceptions, the majority of macroeconomic models feature private agent expectations of economic fundamentals that are formed independently of policy actions. However, there is a growing body of both anecdotal and empirical evidence supporting the view that monetary policy actions, in fact, communicate information about the economy to the public, and thereby affect agents' expectations. Thus, it follows that optimal policy may be altered when policy actions also influence the economy through this channel.

In this paper, I study both theoretically and empirically, a setting where asymmetric information exists between the policymaker and private agents. I assume that the policymaker has more information about the state of the economy than private agents. This assumption captures the central bank's private information about policy targets and its access to some confidential data. A central bank can also be better informed due to an advantage in processing data that's available to all agents[1]. In this environment, rational private agents gain information from observations of monetary policy actions that respond to these fundamentals. This process through which policy affects private agents' beliefs about the state of the economy is what I refer to as the signaling channel.

My first key theoretical result is that, for a given monetary policy, the model can produce positive responses of inflation and output forecasts to positive interest rate surprises. Second, I show that when the interest rate conveys information about an output target, the signaling channel tilts the inflation-output tradeoff in a way that allows a discretionary policymaker to credibly implement an equilibrium closer to the one possible under commitment. This is one driver behind my third key result showing that it can be beneficial for the policymaker to withhold information from the public. I also consider alternate setups that illuminate more general features of an environment where the interest rate has signaling effects. Lastly, I present new empirical evidence that there is a positive effect of interest rate surprises on inflation forecasts that is concentrated in periods when prior uncertainty about inflation is high. This interaction effect is a previously untested prediction of the noisy information setup in this paper and it strengthens the case for a signaling effect of policy beyond the existing evidence which looks only at the overall effect.

The analysis is conducted using a standard New Keynesian model with a representative household and firms who have homogeneous, but imperfect information about exogenous shocks. In the baseline model, there are two exogenous shocks: government demand and a time-varying

[1] With costly information processing, a central bank that devotes more resources to information processing, relative to private agents, is ultimately better informed about relevant economic fundamentals.

target for the gap between actual output and the flexible-price level (or "output target", for brevity)[2]. The output target summarizes exogenous variation in the wedge between the efficient and flexible-price levels of output coming from real imperfections not otherwise captured by the model. It can also represent exogenous variation in a politically-motivated output target that differs from the socially efficient level. Variations in the output target prevent the central bank from achieving the first-best and create a tradeoff between deviations of inflation and output from their targets. The policymaker is better informed than private agents about both shocks and sets the nominal interest rate conditional on this extra knowledge, thus making it a signal to private agents about these fundamentals. This setup reflects a narrative often seen in the popular press: upon seeing a negative interest rate surprise, private agents can interpret this as a countercyclical response to weakness in the economy (lower demand) or a central bank's desire to further boost activity (a higher output target).

Private agents' signal extraction results in a signaling effect of policy actions on beliefs which depends on the relative uncertainty over the two shocks. When uncertainty about demand is high relative to uncertainty about the policy target, interest rate surprises lead to larger belief revisions about demand and smaller revisions to beliefs about the output target. The recent crisis provides a good example of a time when uncertainty about economic strength was particularly high and indeed, the press has interpreted many recent policy actions as indicators of economic strength[3].

My first key result is that when the policy response to demand shocks is inadequate and positive interest rate surprises are a strong enough signal of higher demand, the model produces a positive response of inflation and output gap forecasts to these surprises. For this result, the output target shock merely acts as a source of noise preventing agents from perfectly inferring the demand shock from the interest rate. This mechanism can explain the empirical patterns documented by Romer and Romer (2000), Campbell, Evans, Fisher, and Justiniano (2012), and Nakamura and Steinsson (2013) which show small increases in forecasts of inflation and real economic activity following positive federal funds rate surprises. The model further predicts that the responses of inflation and output gap forecasts to interest rate surprises will vary with uncertainty levels in the economy and I find new empirical evidence of this fact.

Turning to the question of optimal discretionary interest rate policy, I show in closed form that the interest rate's signaling effect on private agents' beliefs about the output target makes accommodation of these target shocks more costly. This change in the inflation-output tradeoff results in the central bank choosing an optimal allocation that features smaller inflation fluc-

[2]Similar policy target shocks have been used by Faust and Svensson (2001) and Mertens (2011).

[3]A particularly interesting example is the Financial Times article entitled "Fed Discount Rate Rise Sends Recovery Signal" following the February 2010 decision to raise the discount rate. This article came despite the Federal Reserve's press release explicitly stating that "the modifications [...] do not signal any change in the outlook for the economy or for monetary policy".

tuations than in the perfect information case. In the language of New Keynesian models, the signaling effect reduces the stabilization bias typically associated with a lack of policymaker commitment.

To better understand the intuition, note that the inflation-output tradeoff is summarized by a New Keynesian Phillips curve linking inflation to the output gap and expected future inflation. Inflation expectations can be further split into expectations of two components: (i) future fundamentals and (ii) future policy responses to those fundamentals. In a perfect information setting, a policymaker who cannot commit to future policy has no effect on inflation expectations through either component. However, when the interest rate provides a signal of the output target, lowering the interest rate in response to a positive output target shock results in an upward revision of agents' belief about the output target which pushes inflation expectations up and results in a higher level of current inflation. This higher inflation cost of keeping output close to its target leads the policymaker to respond less to output target shocks, thus insulating inflation from these fluctuations more than he would if policy actions had no effect on beliefs. Thus, a signaling effect of interest rates on beliefs about the future output target alters optimal policy in a way that's similar to the case where policy affects agents' beliefs about future policy through commitment. In fact, I show that as the signaling effect strengthens, the optimal discretionary policy approaches the policy under commitment to a forward-looking interest rate rule[4]. Therefore, maintaining an information advantage imposes welfare-improving discipline on discretionary interest rate policy which contributes to my next result on the value of communication.

Using this model, I examine whether direct communication of the policymaker's additional information to the public improves welfare. In addition to the baseline no direct communication case, I consider noiseless communication of both or either one of the exogenous states to private agents. I assume that the interest rate follows the optimal discretionary policy corresponding to each case. Here, I find that the welfare is lowest under full communication of both states so that there is a benefit of maintaining some information advantage. Gains from less communication come from two sources: (i) a reduction in the stabilization bias as discussed above, and (ii) smaller overall fluctuations under imperfect information even absent a reduction in the stabilization bias. Shocks affect equilibrium outcomes directly and through expectations. Keeping information away from agents reduces overall fluctuations in the economy by reducing the correlation between the direct and expectational effects of shocks. Thus,

[4]The signaling channel will generally not enable optimal policy under discretion to achieve the welfare possible under an unrestricted commitment. In particular, optimal discretionary policy continues to be forward-looking with the interest rate responding to past shocks only through their effect on current beliefs. This contrasts with an explicit commitment of responding to lagged shocks to improve the set of achievable outcomes intertemporally which can lead to higher welfare in the perfect information setting (Woodford (2003)).

some form of intransparency is always beneficial in this setting[5]. The effect on current period welfare of choosing partial versus no communication will depend on the current realizations of shocks. Therefore, the communication policy problem in this environment will generally exhibit time-inconsistency.

The paper then presents theoretical results under several other setups. One is an invariance result in which I show that if the policymaker only has superior information about shocks that do not generate an inflation-output tradeoff (i.e., shocks to the natural rate of interest), the optimality condition characterizing interest rate policy is always the same as the perfect information case. I also explore cases where the central bank has an information advantage about a time-varying inflation target or a cost-push shock rather than an output target. A particularly interesting result from this exercise is that, in contrast to an output target, it is actually beneficial for private agents to be informed about an inflation target. However, rather than directly communicating the inflation target to the public, it is best for the central bank to allow agents to infer the inflation target from the interest rate when interest rates are set under discretion.

In the empirical part of the paper, I present new evidence of a signaling role of the federal funds rate. This analysis focuses on inflation forecasts from the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia. I estimate a slightly positive effect of federal funds rate surprises[6] on inflation forecasts which echoes a result from an earlier sample in Romer and Romer (2000). I then decompose this overall effect and show that the response is more positive in periods when forecasters had high uncertainty regarding their previous forecast. In contrast to the overall effect, this interaction with uncertainty is not naturally generated by competing explanations, such as a cost channel where higher interest rates raise firms' financing costs[7].

In another set of empirical results, I estimate time-varying gain coefficients measuring the response of inflation forecasts to general news about inflation. I show that there is substantial variation in this coefficient over time and that it is negatively correlated with forecast dispersion and positively correlated with subjective uncertainty in a way that is consistent with the noisy information framework. This adds to the evidence found in Coibion and Gorodnichenko (2012a) and Coibion and Gorodnichenko (2012b) in support of this framework.

The next subsection reviews the related literature. Section 2 sets up the model. I discuss equilibrium dynamics under a general linear interest rate rule in Section 3 to build intuition about the interest rate's signaling effect. I turn to the main question of optimal discretionary

[5]Note that I show this under the assumption that direct communication by the central bank is noiseless and costless. Gains from intransparency would only increase if this communication were obscured by signal noise or a friction such as sticky information a la Mankiw and Reis (2002) or rational inattention as in Sims (2003).

[6]These are measured using futures prices following Kuttner (2001).

[7]I show below that forecasters' subjective uncertainty is not highly correlated with economic activity.

interest rate policy in Section 4 with a discussion on the value of information in Section 5. Section 6 explores alternate shock configurations while other extensions are included in Appendix C. In Section 7, I present empirical results supporting monetary policy's signaling role and Section 8 concludes.

1.1 Related literature

My theoretical results complement previous work on the signaling effect of monetary policy actions. Cukierman and Meltzer (1986), Faust and Svensson (2001), and Geraats (2007) focus on the effect of the signaling channel on the average inflation bias when the central bank has a positive average output target. In this paper, I show how the signaling channel can lessen the stabilization bias present when there is no average inflation bias. Walsh (2010) and Berkelmans (2011) study the signaling channel under dispersed information using numerical methods. The paper closest to mine is Mertens (2011) where monetary policy is a signal only of policy objectives. By allowing the interest rate to affect agents' beliefs about the level of demand, my framework is able to produce the empirical results found in Romer and Romer (2000), Campbell, Evans, Fisher, and Justiniano (2012), and Nakamura and Steinsson (2013). Furthermore, this paper sharpens the intuitions behind the numerical results in Mertens (2011) by providing closed-form solutions and discussing the exact mechanisms at work.

My result on the benefits of central bank intransparency are consistent with Faust and Svensson (2001), Walsh (2010), and Mertens (2011) while I also precisely characterize the sources of these benefits. The findings differ from those in models where lack of perfect information is the only friction such as Lucas Jr. (1972) and Barro (1976)[8]. The result can be interpreted through the lens of Angeletos and Pavan (2007) which shows in a more stylized setting that less information can be beneficial when the perfect information equilibrium is inefficient.

Romer and Romer (2000), Campbell, Evans, Fisher, and Justiniano (2012), and Nakamura and Steinsson (2013) show that positive interest rate surprises can have positive effects on inflation and output forecasts. I present more direct support of the signaling effect of monetary policy by showing a positive interaction between these responses and subjective uncertainty over previous forecasts. My results also relate to the estimation of noisy information models in Coibion and Gorodnichenko (2012a) and Coibion and Gorodnichenko (2012b). I estimate a time-varying response of inflation forecasts to news and show that it correlates with forecast dispersion and prior uncertainty in ways consistent with noisy information models.

Lastly, Ellingsen and Söderström (2001), Erceg and Levin (2003), and Gürkaynak, Sack, and Swanson (2005) use an interest rate signaling effect to explain features of macroeconomic

[8]Even when information frictions are the only frictions, full communication may be suboptimal if the central bank cannot give perfect, homogeneous information to all agents (Adam (2007), Baeriswyl and Cornand (2010)).

data including inflation persistence and the yield curve's response to monetary policy. Melosi (2013) structurally estimates a dispersed information DSGE model where monetary policy is follows a Taylor rule that responds to aggregate variables not observed by individual firms. However, the estimation does not allow for time-varying uncertainty.

2 Model

2.1 Setup

I study the signaling channel of monetary policy in a standard New Keynesian economy with monopolistically competitive firms and sticky prices in the style of Calvo (1983). Fluctuations are driven by an exogenous government spending shock and a shock to the policy target for the output gap. I assume that the monetary authority has perfect information while the representative household and firms have homogeneous but imperfect information regarding these shocks. Private agents observe shocks perfectly with a one-period lag and get information about current values from observations of a nominal interest rate that responds linearly to current state variables. I first describe the model structure and then detail the information structure and belief formation.

2.1.1 Households

The representative household maximizes utility that is additively separable in time, labor, and consumption of a composite good made up of a continuum of varieties

$$\max E \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t\right) - V\left(L_t\right)\right], \text{ where } C_t \equiv \left[\int_0^1 C_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \varepsilon > 1$$

The economy is cashless. The household gets profits from all firms, pays a lump sum tax, and can trade in a riskless nominal one-period bond so that the budget constraint is

$$\int_0^1 P_{jt} C_{jt} dj + B_t \leq R_{t-1} B_{t-1} + W_t L_t - T_t + \int_0^1 \Pi_{jt} dj$$

household optimization results in a standard intertemporal Euler equation and an intratemporal labor supply relation involving the price of the composite good

$$U_{C,t} = \beta R_t E\left[U_{C,t+1} \frac{P_t}{P_{t+1}} \middle| \mathcal{I}_t\right]$$
$$\frac{V_{L,t}}{U_{C,t}} = \frac{W_t}{P_t}$$

where $\mathcal{I}_t$ is a time-t information set to be defined below.

The resulting household demand for each variety j is

$$C_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} C_t$$

and the price of the composite good becomes

$$P_t = \left[\int_0^1 P_{jt}^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}$$

2.1.2 Firms

There is a continuum of firms producing differentiated goods that each maximize profits subject to demand from the household and government. I assume that the government consumes the same composite good and allocates their demand across varieties in the same way as the household. Then, firm j faces total demand of

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} Y_t$$

where Y_t is aggregate real output defined as

$$Y_t \equiv \frac{1}{P_t} \int P_{jt} \left(C_{jt} + G_{jt}\right) dj = C_t + G_t$$

Production technologies are identical across firms and linear in each firms' labor

$$Y_{jt} = AL_{jt}$$

The labor market is perfectly competitive while firms also receive a constant proportional subsidy τ on their wage bills so that each firm's total cost of production is

$$\psi\left(Y_{jt}\right) = (1-\tau) \frac{W_t}{A} Y_{jt}$$

Each firm faces a $1-\theta$ probability of being able to reset their prices in each period. Firms who cannot reset prices charge their previous price. Each resetter maximizes the net present value of profits discounted according to the household's stochastic discount factor $\beta^k \frac{\lambda_{t+k}}{\lambda_t}$ where λ_{t+k} is the Lagrange multiplier on the household's budget constraint which reflects the shadow value of wealth in period $t+k$.

$$P_{jt}^{*} = \arg\max_P \sum_{k=0}^{\infty} (\theta\beta)^k E\left[\frac{\lambda_{t+k}}{\lambda_t}\left[PY_{j,t+k} - \psi\left(Y_{j,t+k}\right)\right] \middle| \mathcal{I}_t\right]$$

Since firms employ identical technologies and hire workers from a centralized labor market, all resetters choose the same optimal price in a given period (i.e., $P_{jt}^* = P_t^* \ \forall j$). Then, the aggregate price level evolves as

$$P_t = \left[(1-\theta) \left(P_t^*\right)^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

2.2 Equilibrium conditions

Unless otherwise noted, let lower-case letters represent log deviations from steady-state values (i.e., $x_t \equiv \ln(X_t/X)$) and let private agents' expectations be denoted by $x_{t'|t} \equiv E\left[x_{t'}|\mathcal{I}_t\right]$. Then, log-linearizing the above optimality conditions around the deterministic steady state leads to two equations characterizing aggregate output and inflation dynamics.

$$\tilde{y}_t = \tilde{y}_{t+1|t} - \frac{1}{\sigma}\left[i_t - \pi_{t+1|t} - \sigma\left(d_t - d_{t+1|t}\right)\right] \tag{1}$$

$$\pi_t = \beta \pi_{t+1|t} + \kappa \tilde{y}_t \tag{2}$$

$$\text{where } d_t = \frac{\varphi}{\sigma+\varphi}\left(1 - \frac{C}{Y}\right) g_t, \ \tilde{y}_t \equiv y_t - y_t^n, \text{ and } y_t^n = \frac{\sigma}{\varphi} d_t$$

d_t is an aggregate demand shock that originates from government spending. $\tilde{y}_t$ represents the gap between output and its natural (i.e., flexible-price) level which is determined by the level of this exogenous source of demand. The coefficients in this log-linearized model are functions of steady-state values and structural parameters.

$$\sigma \equiv -\frac{U_{cc}Y}{U_c}, \quad \varphi \equiv \frac{V_{ll}L}{V_l}, \quad \kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}(\sigma+\varphi) \tag{3}$$

The first equilibrium condition, (1), stems from the resource constraint and the household's Euler equation. The New Keynesian Phillips curve, (2), is derived from firms' pricing behavior, labor supply, the resource constraint and the evolution of aggregate prices.

The natural real rate of interest and the real rate gap in this model are

$$r_t^n \equiv \sigma\left(d_t - d_{t+1|t}\right) \quad \text{and} \quad \tilde{r}_t \equiv i_t - \pi_{t+1|t} - r_t^n$$

When the real rate gap is kept at zero, output stays at its natural level. In this model, this also gives zero inflation.

The model is closed with specifications for the nominal interest rate $i_t \equiv \ln(R_t/R)$ and the shocks. For now, I assume that the interest rate responds linearly to the demand shock, an output target shock $\bar{y}_t$ and private agents' beliefs.

$$i_t = f_d d_t + f_{d,b} d_{t|t} + f_{\bar{y}} \bar{y}_t + f_{\bar{y},b} \bar{y}_{t|t} \tag{4}$$

$\bar{y}_t$ is the policymaker's time-varying target for the output gap. The role of this target will be clarified when I present the optimal policy problem. For now, it should be apparent that this shock affects equilibrium output and inflation in a way similar to an exogenous interest rate shock since it only enters the model's equilibrium conditions through the interest rate. I will first characterize the equilibrium under general policy coefficients and later show a case where optimal discretionary monetary policy results in interest rate setting behavior that matches the form in (4).

I assume that both shocks follow AR(1) processes

$$
\begin{aligned}
d_t &= \rho_d d_{t-1} + \epsilon_{d,t}, \quad \epsilon_{d,t} \sim N\left(0, \sigma^2_{d,t-1}\right) && (5)\\
\bar{y}_t &= \rho_{\bar{y}} \bar{y}_{t-1} + \epsilon_{\bar{y},t}, \quad \epsilon_{\bar{y},t} \sim N\left(0, \sigma^2_{\bar{y},t-1}\right) && (6)
\end{aligned}
$$

$\epsilon_{d,t}$ and $\epsilon_{\bar{y},t}$ are each serially uncorrelated and uncorrelated with each other. I do not restrict the stochastic properties of $\sigma^2_{d,t-1}$ and $\sigma^2_{\bar{y},t-1}$ for now. This timing of the variances is chosen so that the one-period-ahead conditional distributions of the levels remain normal with known variances. This timing is also used in the uncertainty shock literature by Bloom (2009).

2.3 Information structure and belief formation

I assume that agents know the structure of the model and the true values of all parameters, including those in the interest rate rule. However, they do not see the true current values of shocks. This implies that private agents cannot see the true current values of $\tilde{y}_t$ and π_t (otherwise, they can infer d_t). My preferred explanation of this setup is that it describes a situation where individuals face idiosyncratic shocks and are not aware of current aggregate conditions. They also do not see current aggregate outcomes as these are based on decisions made simultaneously by other individuals. The Appendix provides a derivation of the equilibrium conditions for aggregate variables in this type of environment and shows that the only differences are extra terms in the aggregate inflation equation which depend on the exogenous shocks $\epsilon_{d,t}$ and $\epsilon_{\bar{y},t}$. I choose not to proceed with a setup using idiosyncratic shocks in order to abstract from the additional issues involved when an interest rate provides public information to private agents with dispersed information.[9]

I assume that agents observe lagged state variables perfectly (perhaps through observations of lagged aggregate outcomes) which mimics the information setup used in Lucas (1973). They also observe i_t which gives an additional piece of information about the current state. Formally,

[9]Morris and Shin (2002), Angeletos and Pavan (2007), and Lorenzoni (2010) examine these issues in other settings.

the information set of private agents in period t is

$$\mathcal{I}_t = \left\{ i^t, d^{t-1}, \bar{y}^{t-1}, \sigma_d^t, \sigma_{\bar{y}}^t \right\}$$

Meanwhile, I assume that the central bank has perfect information about the entire history of exogenous variables up to time t. Thus, the central bank's information advantage is captured by knowledge of the current shocks $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$. A benefit of assuming that agents can see lagged true values is that it limits the signaling effect of the interest rate to current beliefs and allows me to focus on changes to the short-run incentives that are central to the optimal discretionary policy problem. I discuss the case where lagged true values cannot be seen as an extension in Appendix C.2.

Since the shocks are AR(1) and past shocks are perfectly observed, beliefs are optimally formed through a static Gaussian signal extraction problem. There is a slight departure due to the dependence of the interest rate on current private agent beliefs. This introduces circularity into the belief formation problem which I resolve using the method outlined in Svensson and Woodford (2003). The basic approach is to posit a form of beliefs and then to re-express the belief formation problem in terms of errors from expectations made absent the interest rate signal. In this form, there is no circularity issue and beliefs can be found using standard signal extraction results. Here, I posit that beliefs take the form

$$d_{t|t} = \rho_d d_{t-1} + K_{d,t} \left(i_t - f_d \rho_d d_{t-1} - f_{d,b} d_{t|t} - f_{\bar{y}} \rho_{\bar{y}} \bar{y}_{t-1} - f_{\bar{y},b} \bar{y}_{t|t} \right) \tag{7}$$

$$\bar{y}_{t|t} = \rho_{\bar{y}} \bar{y}_{t-1} + K_{\bar{y},t} \left(i_t - f_d \rho_d d_{t-1} - f_{d,b} d_{t|t} - f_{\bar{y}} \rho_{\bar{y}} \bar{y}_{t-1} - f_{\bar{y},b} \bar{y}_{t|t} \right) \tag{8}$$

for some $K_{d,t}, K_{\bar{y},t}$ that I will later solve for. Then, writing the evolution of the shocks and the interest rate in terms of expectational errors defined as $x_t^{err} \equiv x_t - E\left[x_t | \mathcal{I}_t \setminus i_t\right]$ yields a standard Gaussian signal extraction problem without the signal being a function of current beliefs.

$$\begin{aligned} d_t^{err} &= \epsilon_{d,t} \\ \bar{y}_t^{err} &= \epsilon_{\bar{y},t} \\ i_t^{surp} &= \left(1 + f_{d,b} K_{d,t} + f_{\bar{y},b} K_{\bar{y},t}\right) \left(f_d d_t^{err} + f_{\bar{y}} \bar{y}_t^{err}\right) \end{aligned} \tag{9}$$

The expectational error for the nominal interest rate is denoted by i_t^{surp} since it corresponds to an interest rate surprise defined as the difference between the observed interest rate and the one expected based on all period t information *except* for the interest rate itself. This signal extraction problem gives

$$d_{t|t}^{err} = \frac{f_d \sigma_{d,t-1}^2}{f_d^2 \sigma_{d,t-1}^2 + f_{\bar{y}}^2 \sigma_{\bar{y},t-1}^2} \frac{1}{1 + f_{d,b} K_{d,t} + f_{\bar{y},b} K_{\bar{y},t}} i_t^{surp}$$
$$\bar{y}_{t|t}^{err} = \frac{f_{\bar{y}} \sigma_{\bar{y},t-1}^2}{f_d^2 \sigma_{d,t-1}^2 + f_{\bar{y}}^2 \sigma_{\bar{y},t-1}^2} \frac{1}{1 + f_{d,b} K_{d,t} + f_{\bar{y},b} K_{\bar{y},t}} i_t^{surp}$$

Since $x_{t|t} = x_{t|t}^{err} + E\left[x_t | \mathcal{I}_t \setminus i_t\right]$, beliefs will fit the form assumed above so that, in equilibrium, they depend on lagged true states and current shocks

$$d_{t|t} = \rho_d d_{t-1} + K_{d,t} \left(f_d \epsilon_{d,t} + f_{\bar{y}} \epsilon_{\bar{y},t}\right), \quad K_{d,t} = \frac{f_d \frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}}{f_d^2 \frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2} + f_{\bar{y}}^2} \tag{10}$$

$$\bar{y}_{t|t} = \rho_{\bar{y}} \bar{y}_{t-1} + K_{\bar{y},t} \left(f_d \epsilon_{d,t} + f_{\bar{y}} \epsilon_{\bar{y},t}\right), \quad K_{\bar{y},t} = \frac{f_{\bar{y}}}{f_d^2 \frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2} + f_{\bar{y}}^2} \tag{11}$$

The AR(1) form of d_t and $\bar{y}_t$ then implies that $d_{t+h|t} = \rho_d^h d_{t|t}$ and $\bar{y}_{t+h|t} = \rho_{\bar{y}}^h \bar{y}_{t|t}$.

Note the following properties of belief formation.

1. $f_d d_{t|t} + f_{\bar{y}} \bar{y}_{t|t} = f_d d_t + f_{\bar{y}} \bar{y}_t$: Since the sum on the right can be perfectly inferred from i_t, the belief formation process can be understood as agents observing a linear combination of two unknown shocks and assigning a portion of this value to each shock. The relative fraction assigned to each underlying shock depends on the relative importance of that shock in the sum.

2. $\frac{K_{d,t}}{K_{\bar{y},t}} = \frac{f_d}{f_{\bar{y}}} \frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}$: This states that relatively more of the observed sum is attributed to a demand shock when the interest rate rule responds relatively more to demand shocks ($\frac{f_d}{f_{\bar{y}}}$ is high) or when the demand shock is more variable ($\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}$ is high). When agents are relatively more unsure about the current demand level versus the central bank's output target, then they find it likely that the policy surprise is due mostly to a change in demand conditions.

3 Equilibrium dynamics

The model is described by the system of equations summarizing private agent optimization ((1), (2)), policy (equation (4)), shock evolution ((5), (6)), and beliefs ((10) and (11)). This can be solved by conjecturing that $\tilde{y}_t$ and π_t are linear in the true states and current private agent beliefs $\left\{d_t, \bar{y}_t, d_{t|t}, \bar{y}_{t|t}\right\}$ with unknown coefficients[10]. This allows $\tilde{y}_{t+1|t}$ and $\pi_{t+1|t}$ to be

[10] An interest rate rule of the form given in (4) will not guarantee that this equilibrium is unique. The latter part of Corollary 1 illustrates how the interest rate rule can be rewritten to guarantee uniqueness while

expressed in terms of current beliefs. Then, substituting (4) into (1) and (2) gives two equations in terms of $\left\{d_t, \bar{y}_t, d_{t|t}, \bar{y}_{t|t}\right\}$ which are used to solve for the unknown coefficients.

With this linear solution, the response of a given outcome x_t to the two structural shocks can each be broken down into three parts

$$\frac{dx_t}{d\epsilon_{\bar{y},t}} = \frac{\partial x_t}{\partial \bar{y}_t} + \frac{\partial x_t}{\partial \bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \frac{\partial x_t}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}}$$
$$\frac{dx_t}{d\epsilon_{d,t}} = \frac{\partial x_t}{\partial d_t} + \frac{\partial x_t}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{d,t}} + \frac{\partial x_t}{\partial \bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}}$$

The first term captures the direct effects of shocks on equilibrium conditions or the interest rate. The last two terms capture an indirect expectational effect which works through both the interest rate's response to private agents' beliefs as well as forward-looking terms in the equilibrium conditions stemming from consumption smoothing, Calvo pricing mechanisms, and the autocorrelated nature of the exogenous states. This expectational effect is altered when information becomes imperfect. In the perfect information case, beliefs are correct so that $\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} = \frac{d\bar{y}_t}{d\epsilon_{\bar{y},t}} = 1$ and $\frac{dd_{t|t}}{d\epsilon_{d,t}} = \frac{dd_t}{d\epsilon_{d,t}} = 1$ while $\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} = \frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}} = 0$. With a monetary policy signaling channel, the expectational effects of the two shocks "spill over" into each other. Thus, one shock can have effects that resemble the other shock(s).

The marginal responses of forecasts behave similarly

$$\frac{dx_{t+1|t}}{d\epsilon_{\bar{y},t}} = \rho_{\bar{y}}\left(\frac{\partial x_t}{\partial \bar{y}_t} + \frac{\partial x_t}{\partial \bar{y}_{t|t}}\right)\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \rho_d\left(\frac{\partial x_t}{\partial d_t} + \frac{\partial x_t}{\partial d_{t|t}}\right)\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}}$$
$$\frac{dx_{t+1|t}}{d\epsilon_{d,t}} = \rho_d\left(\frac{\partial x_t}{\partial d_t} + \frac{\partial x_t}{\partial d_{t|t}}\right)\frac{dd_{t|t}}{d\epsilon_{d,t}} + \rho_{\bar{y}}\left(\frac{\partial x_t}{\partial \bar{y}_t} + \frac{\partial x_t}{\partial \bar{y}_{t|t}}\right)\frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}}$$

In the remainder of this section, I examine the comovement between current outcomes, forecasts, and interest rate surprises. I build intuition for the general case by first examining two benchmark cases.

3.1 Benchmark 1: Perfect information

The model above can be made isomorphic to a perfect information model with an exogenous interest rate shock by allowing agents to perfectly observe the current value of d_t. Then, with $f_{\bar{y}} \neq 0$, the interest rate perfectly reveals $\bar{y}_t$ so that beliefs are correct in equilibrium and interest rate behavior simplifies to

$$i_t = (f_d + f_{d,b})\, d_t + (f_{\bar{y}} + f_{\bar{y},b})\, \bar{y}_t$$

maintaining the same equilibrium behavior vis-à-vis the state variables.

Then, the interest rate surprise reduces to a scaled output target shock

$$i_t^{surp} = \left(f_{\bar{y}} + f_{\bar{y},b}\right) \epsilon_{\bar{y},t}$$

Since agents are perfectly informed after observing i_t, the resulting responses of outcomes to the interest rate surprise are the same as those under perfect information. In other words, this case gives a model that's isomorphic to a perfect information model in which $\left(f_{\bar{y}} + f_{\bar{y},b}\right) \bar{y}_t$ is an autocorrelated exogenous component of the nominal interest rate. To get impulse responses with the usual signs, I make the following assumption that the shocks are not too persistent[11]

Assumption 1 $\rho_d, \rho_{\bar{y}} \in [0, \bar{\rho})$ *where* $\bar{\rho} \leq \theta$. *(See Appendix for the exact expression for* $\bar{\rho}$*.)*

Under Assumption 1, the familiar perfect information channels of a positive interest rate surprise are at work. First, it raises the current real rate gap which lowers the current output gap and inflation holding expectations fixed.

$$\frac{d\tilde{r}_t}{di_t^{surp}} = \frac{\left(1 - \rho_{\bar{y}}\right)\left(1 - \beta\rho_{\bar{y}}\right)}{\left(1 - \rho_{\bar{y}}\right)\left(1 - \beta\rho_{\bar{y}}\right) - \frac{\kappa}{\sigma}\rho_{\bar{y}}} > 0$$

Secondly, the persistent nature of the output target shock means that future real interest rate gaps also increase following a positive interest rate surprise which lowers expectations of future output gaps and inflation

$$\frac{d\tilde{y}_{t+1|t}}{di_t^{surp}} = -\rho_{\bar{y}} \frac{\frac{1}{\sigma}\left(1 - \beta\rho_{\bar{y}}\right)}{\left(1 - \rho_{\bar{y}}\right)\left(1 - \beta\rho_{\bar{y}}\right) - \frac{\kappa}{\sigma}\rho_{\bar{y}}} \leq 0$$

$$\frac{d\pi_{t+1|t}}{di_t^{surp}} = -\rho_{\bar{y}} \frac{\frac{\kappa}{\sigma}}{\left(1 - \rho_{\bar{y}}\right)\left(1 - \beta\rho_{\bar{y}}\right) - \frac{\kappa}{\sigma}\rho_{\bar{y}}} \leq 0$$

In sum, both the current real rate gap and future expectations channels lower the current output gap and inflation after a positive interest rate surprise

$$\frac{d\tilde{y}_t}{di_t^{surp}} = -\frac{\frac{1}{\sigma}\left(1 - \beta\rho_{\bar{y}}\right)}{\left(1 - \rho_{\bar{y}}\right)\left(1 - \beta\rho_{\bar{y}}\right) - \frac{\kappa}{\sigma}\rho_{\bar{y}}} < 0$$

$$\frac{d\pi_t}{di_t^{surp}} = -\frac{\frac{\kappa}{\sigma}}{\left(1 - \rho_{\bar{y}}\right)\left(1 - \beta\rho_{\bar{y}}\right) - \frac{\kappa}{\sigma}\rho_{\bar{y}}} < 0$$

The properties of this benchmark that contrast with the cases below are that: (1) both current and forecasted output gap and inflation respond negatively to an interest rate surprise, (2) responses do not vary with $\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}$, and (3) responses do not depend on policy response coefficients.

[11] This assumption becomes unnecessary if the interest rate is written in a form that guarantees determinacy as in the latter part of Corollary 1.

3.2 Benchmark 2: The policymaker perfectly offsets r_t^n

For this case, recall that fluctuations in the natural real rate affect the equilibrium output gap and inflation only if they are passed through to fluctuations in the real rate gap. The policymaker can prevent this by setting $f_d = \sigma$ and $f_{d,b} = -\sigma\rho_d$ so that the nominal interest rate follows

$$i_t = r_t^n + f_{\bar{y}}\bar{y}_t + f_{\bar{y},b}\bar{y}_{t|t}$$

This creates an equilibrium where there are no fluctuations directly related to changes in d_t or $d_{t|t}$ (i.e., $\frac{\partial \tilde{y}_t}{\partial d_t} = \frac{\partial \tilde{y}_t}{\partial d_{t|t}} = \frac{\partial \pi_t}{\partial d_t} = \frac{\partial \pi_t}{\partial d_{t|t}} = 0$). Endogenous variables will depend only on changes in the output target and agents' belief about its current level. However, demand shocks still move the nominal interest rate and therefore affect outcomes indirectly through agents' belief about the output target.

Here, the responses of a given outcome x_t to the shocks become

$$\frac{dx_t}{d\epsilon_{\bar{y},t}} = \frac{\partial x_t}{\partial \bar{y}_t} + \frac{\partial x_t}{\partial \bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} \quad \text{and} \quad \frac{dx_t}{d\epsilon_{d,t}} = \frac{\partial x_t}{\partial \bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}}$$

while the interest rate surprise is linear in the two shocks

$$i_t^{surp} = \iota_d\epsilon_{d,t} + \iota_{\bar{y}}\epsilon_{\bar{y},t}$$

Since the interest rate surprise is now made up of two independent shocks, there are two ways to characterize how the output gap or inflation move with interest rate surprises. I can look at the "response" of some outcome x_t to an interest rate surprise conditional on a shock to $s \in \{d, y\}$ using the ratio $\frac{dx_t/d\epsilon_{s,t}}{di_t^{surp}/d\epsilon_{s,t}}$. Alternatively, I can also look at the statistic $\frac{Cov_{t-1}\left(x_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}$ for a given outcome variable x_t. This scaled covariance is analogous to the statistic estimated by OLS regressions of x_t on interest rate surprises with the exception that one-period-ahead conditional moments are used due to the presence of time-varying uncertainty.

The following three additional coefficient restrictions help me to sign responses.

Assumption 2 $f_{\bar{y}} \leq 0,\ f_{\bar{y}} + f_{\bar{y},b} \leq 0$

Assumption 3 $f_{\bar{y}} \leq 0,\ f_{\bar{y},b} \leq -\rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right) f_{\bar{y}},\ \rho_d \in \left(0, \rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right)\right)$

Assumption 4 $f_{\bar{y}} \leq 0,\ f_{\bar{y},b} \leq -\rho_{\bar{y}}\left(1 + \frac{\frac{\kappa}{\sigma}}{1-\beta\rho_{\bar{y}}}\right) f_{\bar{y}},\ \rho_d \in \left(0, \rho_{\bar{y}}\left(1 + \frac{\frac{\kappa}{\sigma}}{1-\beta\rho_{\bar{y}}}\right)\right)$

The first of these assumptions can be understood as policy responding the "right way" to output target shocks. Holding beliefs constant, $f_{\bar{y}} < 0$ means that the nominal interest rate is lower when the output target is high. Additionally, $f_{\bar{y}} + f_{\bar{y},b} \leq 0$ ensures that an output target shock positively affects the interest rate surprise. The second and third assumptions place successively tighter bounds on the nominal rate's response to agents' beliefs about the output target and analogous bounds on ρ_d which are needed to sign some of the responses.

Turning first to the responses under each individual shock, I obtain the following:

1. Under Assumption 2, $\frac{di_t^{surp}}{d\epsilon_{\bar{y},t}} = \iota_{\bar{y}} < 0 < \iota_d = \frac{di_t^{surp}}{d\epsilon_{d,t}}$
2. Under Assumptions 1 and 3, $\frac{d\pi_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} < 0$ and $\frac{d\pi_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} < 0$; both increase with $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.
3. Under Assumptions 1 and 4, $\frac{d\tilde{y}_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} < 0$ and $\frac{d\tilde{y}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} < 0$; both increase with $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.

The main departure from the perfect information benchmark is the responses' dependence on the relative uncertainty $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$. In this case, interest rate policy ensures that the true level and agents' belief about demand have no direct impact on current or future outcomes. Thus, a positive interest rate surprise (stemming from either shock) is attributed by private agents partly to an increase in demand which has no effect in equilibrium, and partly to a decrease in the output target, which has a persistent contractionary effect. Then, the net effect is always negative but is weaker when more of the interest rate surprise is attributed to a change in demand. In this information structure, this occurs when uncertainty about demand is high relative to uncertainty about the output target.

Turning to the scaled conditional covariance between outcomes and interest rate surprises, I obtain the following under Assumptions 1 and 2:

1. $\frac{Cov_{t-1}\left(\pi_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} < 0$ and is increasing in $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$. $\frac{Cov\left(\pi_t, i_t^{surp}\right)}{Var\left(i_t^{surp}\right)} \to 0$ as $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} \to \infty$. The same is true for the output gap.
2. $\frac{Cov\left(\pi_{t+h|t}, i_t^{surp}\right)}{Var\left(i_t^{surp}\right)} < 0$ and is increasing in $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$. $\frac{Cov\left(\pi_{t+h|t}, i_t^{surp}\right)}{Var\left(i_t^{surp}\right)} \to 0$ as $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} \to \infty$. The same is true for output gap forecasts.

This statistic is a weighted average of the conditional responses to individual shocks so the above intuition continues to hold. Furthermore, a higher $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ also results in greater weight on the responses to $\epsilon_{d,t}$ in this statistic. As $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} \to \infty$, $\frac{Cov_{t-1}\left(\pi_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}$ approaches $\frac{d\pi_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}}$ which is zero in this limit. The same logic applies to the output gap.

3.3 The general case

For the general case, I use the following restrictions on the interest rate's response to demand and agents' belief about the current demand level.

Assumption 5 $f_d \geq 0,\ f_d + f_{d,b} \in (0, \sigma(1-\rho_d))$

Assumption 6 $f_d \geq 0,\ f_d + f_{d,b} \in \left(0, \sigma\left(\frac{\frac{\kappa}{\sigma}\rho_d}{(1-\rho_d)(1-\beta\rho_d)} - \rho_d\right)\right)$

The additional feature present under Assumption 5 is that the policy response to demand shocks is inadequate. Then, demand shocks retain an expansionary effect in equilibrium. Since the signaling effect of an interest rate surprise is a weighted average of the effects of each of the underlying shocks, this allows a positive interest rate surprise to produce an expansionary signaling effect. This can overtake the direct contractionary effect and result in a net expansion following a positive interest rate surprise when the interest rate is a strong enough signal of demand shocks. Again, this occurs when uncertainty about demand is relatively high.

Proposition 1 *Given Assumptions 1, 2, and 5*

1. $\frac{di_t^{surp}}{d\epsilon_{\bar{y},t}} = \iota_{\bar{y}} < 0 < \iota_d = \frac{di_t^{surp}}{d\epsilon_{d,t}}$

2. $\frac{d\tilde{y}_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}}$ *and* $\frac{d\pi_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}}$ *can both be positive for large* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.

3. $\frac{d\tilde{y}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ *and* $\frac{d\pi_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ *can both be positive for large* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ *under Assumption 6.*

4. $\frac{Cov_{t-1}\left(\pi_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}$ *is increasing in* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ *and can be positive for a large enough* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$. *The same is true for the output gap.*

5. $\frac{d\tilde{y}_{t+h|t}/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} = \frac{d\tilde{y}_{t+h|t}/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ *and* $\frac{d\pi_{t+h|t}/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} = \frac{d\pi_{t+h|t}/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ *can all be positive and are increasing in* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.

6. $\frac{Cov\left(\pi_{t+h|t}, i_t^{surp}\right)}{Var\left(i_t^{surp}\right)}$ *is increasing in* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ *and can be positive for a large enough* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$. *The same is true for output gap forecasts.*

Proof. See Appendix D. ∎

This mechanism has been discussed as one reason behind the expansionary responses of inflation and unemployment forecasts to positive interest rate surprises found in Romer and Romer (2000) and Campbell, Evans, Fisher, and Justiniano (2012). The theory presented here also implies that this is particularly likely to be the case when (i) the policy response to demand shocks is inadequate and (ii) private agents are relatively more uncertain about the strength of the economy than they are about policy objectives. The recent recession was a period of time where these conditions were plausibly present since the federal funds target effectively reached zero at the end of 2008 and there is also evidence of high economic uncertainty prior to and during the recession, as in the influential work by Bloom (2009). Section 7.3 also presents new empirical evidence that the response of inflation forecasts to interest rate surprises is indeed more positive when forecasters' subjective uncertainty is high.

4 Optimal discretionary interest rate policy

In this section, I turn to the question of optimal discretionary interest rate policy. For now, I do not allow the central bank to directly communicate their additional information to the public aside from the information embodied in the interest rate. To retain tractability, I limit attention to the case where variances are constant parameters and consider comparative statics with respect to the relative variance $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$. I discuss the implications of time-varying uncertainty for the optimal policy problem in Appendix C.3. I also assume that a constant wage bill subsidy τ offsets the average monopolist pricing inefficiency and that there is no inherited initial price dispersion so that the nonstochastic steady state is undistorted. Then, a second-order log approximation around this steady state gives that the household's lifetime utility from date t_0 onwards is proportional to

$$\mathbb{U}_{t_0,\infty} = -\sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\tilde{y}_t^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right) + h.o.t.$$

where I've omitted constants and terms independent of policy.

I then consider a monetary authority that maximizes welfare derived from household utility but with an exogenous time-varying target for the output gap. A similar time-varying target has been used in other papers studying optimal policy in an imperfect information context such as Mertens (2011) and Faust and Svensson (2001). My preferred interpretation of this shock is that it summarizes short-run deviations of the efficient level of output from the natural flexible-price level of output which are not captured by the above microfoundations. Then, $\tilde{y}_t - \bar{y}_t$ represents the deviation of actual output from the efficient level. The policymaker's objective is to minimize the following loss

$$\mathcal{L}_{t_0} = E_{t_0}^{CB} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\left(\tilde{y}_t - \bar{y}_t \right)^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right) \tag{12}$$

where the expectation is evaluated according to his own information set[12].

[12] The model equations can be rearranged into the canonical form studied in Clarida, Galí, and Gertler (1999) where the output target shock shows up as both a positive cost-push shock and a negative component of the demand shock.

$$L_{t_0} = E_{t_0}^{CB} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\left(\tilde{y}_t^{CB} \right)^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right)$$

$$\tilde{y}_t^{CB} = \tilde{y}_{t+1|t}^{CB} - \frac{1}{\sigma} \left[i_t - \pi_{t+1|t} - r_t^{CB} \right]$$

$$\pi_t = \beta \pi_{t+1|t} + \kappa \tilde{y}_t^{CB} + v_t$$

$$\text{where } \tilde{y}_t^{CB} \equiv \tilde{y}_t - \bar{y}_t,\ r_t^{CB} = \sigma \left[\left(d_t - \bar{y}_t \right) - \left(d_{t+1|t} - \bar{y}_{t+1|t} \right) \right] \quad \text{and} \quad v_t = \kappa \bar{y}_t$$

In the imperfect information case, a policymaker who cannot commit chooses the interest rate level in each period to minimize this loss subject to equilibrium conditions (1) and (2) while taking private agents' beliefs regarding future policy and the form of current policy as given.

Beliefs regarding future policy affect the expectations $\left\{\tilde{y}_{t+1|t}, \pi_{t+1|t}\right\}$. Since the equilibrium of this model is linear in $\left\{d_t, d_{t|t}, y_t, y_{t|t}\right\}$ while beliefs satisfy $d_{t+1|t} = \rho_d d_{t|t}$ and $\bar{y}_{t+1|t} = \rho_{\bar{y}} \bar{y}_{t|t}$, these expectations can be written in matrix form as

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M} \begin{bmatrix} d_{t+1|t} \\ \bar{y}_{t+1|t} \end{bmatrix} \tag{13}$$

In equilibrium, the coefficients in the matrix $\mathbf{M}$ are determined by the behavior of future nominal interest rates. Then, taking private agents' beliefs about future policy as given amounts to the policymaker recognizing that his current choice does not have an effect on this $\mathbf{M}$ matrix. However, the policymaker does recognize that his choice impacts $\left\{d_{t+1|t}, \bar{y}_{t+1|t}\right\}$ and therefore has a marginal effect on current outcomes through $\left\{\tilde{y}_{t+1|t}, \pi_{t+1|t}\right\}$. This is in contrast to the discretionary policy problem under perfect information where the interest rate level chosen today has zero impact on these expectations.

Unlike the perfect information case, private agents' beliefs about the form of current policy are now relevant since it determines private agents' belief formation process. When private agents suppose that the current equilibrium interest rate can be described by

$$i_t = f_d d_t + f_{d,b} d_{t|t} + f_{\bar{y}} \bar{y}_t + f_{\bar{y},b} \bar{y}_{t|t} \tag{14}$$

then beliefs follow the forms in (7) and (8) with constant K_d and $K_{\bar{y}}$. This can be solved to yield the following expressions of beliefs as functions of the nominal interest rate i_t and lagged states

$$d_{t|t} = \frac{f_{\bar{y}} + f_{\bar{y},b}}{1 + K_{\bar{y}} f_{\bar{y},b} + K_d f_{d,b}} \left(K_{\bar{y}} \rho_d d_{t-1} - K_d \rho_{\bar{y}} \bar{y}_{t-1}\right) + \frac{K_d}{1 + K_{\bar{y}} f_{\bar{y},b} + K_d f_{d,b}} i_t \tag{15}$$

$$\bar{y}_{t|t} = \frac{f_d + f_{d,b}}{1 + K_{\bar{y}} f_{\bar{y},b} + K_d f_{d,b}} \left(K_d \rho_{\bar{y}} \bar{y}_{t-1} - K_{\bar{y}} \rho_d d_{t-1}\right) + \frac{K_{\bar{y}}}{1 + K_{\bar{y}} f_{\bar{y},b} + K_d f_{d,b}} i_t \tag{16}$$

as shown above, where K_d and $K_{\bar{y}}$ take the forms given in (10) and (11) with $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$ now being constant. Then, a policymaker who takes private agents' beliefs about current policy as given faces the following effects on beliefs $d_{t|t}$ and $\bar{y}_{t|t}$.

$$
\begin{aligned}
\frac{dd_{t|t}}{di_t} &= \frac{K_d}{1+K_{\bar{y}}f_{\bar{y},b}+K_d f_{d,b}} = \frac{f_d \frac{\sigma_d^2}{\sigma_{\bar{y}}^2}}{f_d\left(f_d+f_{d,b}\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)} \\
\frac{d\bar{y}_{t|t}}{di_t} &= \frac{K_{\bar{y}}}{1+K_{\bar{y}}f_{\bar{y},b}+K_d f_{d,b}} = \frac{f_{\bar{y}}}{f_d\left(f_d+f_{d,b}\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)}
\end{aligned}
$$

To summarize, a policymaker who can only choose the interest rate level today and cannot make credible commitments about policy does not internalize the effect of equilibrium interest rate behavior on the following: (i) the $\mathbf{M}$ matrix which captures the relationship between beliefs about state variables and expectations $\left\{\tilde{y}_{t+1|t}, \pi_{t+1|t}\right\}$ as well as (ii) the coefficients governing belief formation. This is consistent with the notion that the policymaker chooses the current *level* of the nominal interest rate but cannot commit to implementing a particular interest rate *rule*. The main difference from the perfect information discretionary policy problem is that the policymaker recognizes that the current interest rate choice can influence expectations of future outcomes through the beliefs in the vector $\left[d_{t|t} \quad \bar{y}_{t|t}\right]'$ in (13).

Because the policymaker minimizes a quadratic loss function subject to linear constraints of the same form in each period, the optimal interest rate ends up having the same form as (14). Solving for an equilibrium under optimal policy then consists of finding a solution to the set of linear stochastic difference equations given by (1), (2), (5), (6), (15), (16), and the policymaker's optimality condition.

Proposition 2 *The policymaker's optimality condition is*

$$
\tilde{y}_t - \bar{y}_t = -\mathcal{R}\frac{\varepsilon}{\kappa}\pi_t \tag{17}
$$

$$
\text{where } \mathcal{R} \equiv \frac{\frac{d\pi_t}{di_t}}{\frac{d\tilde{y}_t}{di_t}} = \frac{\frac{\partial \pi_t}{\partial i_t} + \frac{\partial \pi_t}{\partial \bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{di_t}}{\frac{\partial \tilde{y}_t}{\partial i_t} + \frac{\partial \tilde{y}_t}{\partial \bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{di_t}} \text{ in equilibrium}
$$

$\mathcal{R}$ is itself a function of interest rate response coefficients and is therefore determined in equilibrium. There may be multiple equilibrium values for $\mathcal{R}$ but those that are real and nonnegative exhibit the following properties when $\beta\rho_{\bar{y}} > 0$:

1. *$\mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{y}}}\right]$*

2. *$\mathcal{R}$ is decreasing in $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$*

    - *As $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to \infty$, $\frac{d\bar{y}_{t|t}}{di_t} \to 0$ and $\mathcal{R} \to \kappa$. In this limit, the interest rate has no effect on $\bar{y}_{t|t}$ and the optimality condition for policy becomes equivalent to that in the case of perfect information.*

- *As* $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to 0$, $\frac{d\bar{y}_{t|t}}{di_t} \to \frac{1}{f_{\bar{y}}+f_{\bar{y},b}}$ *and* $\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}}$. *In this limit, the interest rate has its largest possible effect on* $\bar{y}_{t|t}$ *and the optimality condition for policy becomes equivalent to that in the case of commitment to a rule of the form*

$$i_t = r_t^n + f_{\bar{y}}^c \bar{y}_t + f_{\bar{y},b}^c \bar{y}_{t|t}$$

3. *When* $\beta = 0$ *or* $\rho_{\bar{y}} = 0$, $\mathcal{R} = \kappa$ *in equilibrium for any value of* $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$.

This optimality condition is the one obtained under any initial supposed private sector belief about current policy that results in beliefs $d_{t|t}$ *and* $\bar{y}_{t|t}$ *that are linear in* i_t. *More specifically, the same condition is obtained if (14) is replaced with a belief that the current interest rate may also respond linearly to the entire history of past fundamentals.*

Proof. See Appendix D. ■

The optimal policy results in this environment can be understood by noting that the signaling channel tilts the policymaker's short-run tradeoff between inflation and deviations of the output gap from its target. To better understand this, note that the policymaker's problem can be recast as one in which he chooses $\tilde{y}_t$ since there is a one-to-one mapping between the nominal interest rate and $\tilde{y}_t$ through (1). Then, the only remaining constraint imposed on the policymaker is the inflation equation in (2), which I rewrite here in terms of the output gap deviation from its target.

$$\pi_t = \beta\pi_{t+1|t} + \kappa\left(\tilde{y}_t - \bar{y}_t\right) + \kappa\bar{y}_t$$

This New Keynesian Phillips curve then summarizes the policymaker's tradeoff between π_t and $\tilde{y}_t - \bar{y}_t$. In the perfect information setting, the discretionary policymaker has no impact on $\pi_{t+1|t}$. Therefore, the slope of this constraint is

$$\mathcal{R}^{PI} = \frac{\partial\pi_t/\partial i_t}{\partial\tilde{y}_t/\partial i_t} = \kappa$$

When the policymaker has an information advantage, any change in $\tilde{y}_t$ now requires a change in i_t which impacts the expectation $\pi_{t+1|t}$ through beliefs. This changes the slope of the policymaker's constraint to

$$\mathcal{R} = \frac{\frac{\partial\pi_t}{\partial i_t} + \frac{\partial\pi_t}{\partial d_{t|t}}\frac{dd_{t|t}}{di_t} + \frac{\partial\pi_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{di_t}}{\frac{\partial\tilde{y}_t}{\partial i_t} + \frac{\partial\tilde{y}_t}{\partial d_{t|t}}\frac{dd_{t|t}}{di_t} + \frac{\partial\tilde{y}_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{di_t}}$$

Thus, the policymaker's optimality condition retains the same form as the perfect information setting where the goal is to maintain an optimal ratio between output and inflation deviations. The key difference is that the slope $\mathcal{R}$ governing this ratio now depends crucially on the size of the effects that the interest rate has on beliefs.

In equilibrium, $\mathcal{R}$ depends only on the effect that interest rates have agents' belief about the output target and not their belief about demand. This is because the demand level only directly impacts the natural real rate which the policymaker is able to perfectly neutralize so that in equilibrium, it will be the case that $\frac{\partial \pi_t}{\partial d_{t|t}} = \frac{\partial \tilde{y}_t}{\partial d_{t|t}} = 0$. Then, the interest rate still affects $d_{t|t}$, but this does not translate into an effect on inflation expectations. On the other hand, changes in the true level and belief about the output target will affect inflation expectations under the optimal policy. Thus, what ultimately matters for optimal policy is how much influence policy actions have on this belief.

Solving for the equilibrium value of $\mathcal{R}$ reveals that $\mathcal{R} \geq \mathcal{R}^{PI}$, meaning that it's optimal to maintain smaller inflation deviations relative to output deviations when policy has a signaling effect on $\bar{y}_{t|t}$. This reduces the usual stabilization bias that occurs in perfect information New Keynesian models where short-run inflation fluctuations are inefficiently large when a policymaker is not able to commit. As uncertainty about the output target grows relative to uncertainty about demand shocks, policy's signaling effect on $\bar{y}_{t|t}$ becomes larger and this stabilization bias is further reduced. In a more general setting where there are exogenous interest rate shocks, the key measures are uncertainty about the output target relative to uncertainty about all other unobserved components of i_t.

At the limits of the interest rate's influence on beliefs, the optimal discretionary policy in this imperfect information model corresponds with some familiar benchmarks. When $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to \infty$, the interest rate has no effect on beliefs about the output target shock and the optimal discretionary policy under imperfect information coincides with that under perfect information. When $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to 0$, the interest rate has its largest possible effect on beliefs about the output target shock and the optimal discretionary policy coincides with the policy chosen by a policymaker can commit to an interest rate rule of the form given above. In other words, there is no benefit to this type of commitment at this limit. In this particular example, the optimal discretionary policy at this limit also coincides with the optimal policy under perfect information when the policymaker can commit to a rule of the form considered in section 4.2.1 of Clarida, Galí, and Gertler (1999) which is

$$i_t = r_t^n + f_{\bar{y}}^c \bar{y}_t$$

Lastly, the equilibrium ratio $\mathcal{R}$ does not depend on relative variance levels in the special cases where $\beta\rho_{\bar{y}} = 0$.

1. When $\rho_{\bar{y}} = 0$, future output target levels become unforecastable. The interest rate's signaling effect is then only on agents' belief about the current output target which has no direct impact on current outcomes.

2. When $\beta = 0$, inflation expectations no longer affect the current policy tradeoff since prices

are set by firms who only consider current profits. Note that the key discount factor that β is capturing in this special case is the one used by firms in their price-setting decision. This result still holds if I assume that firm managers use a discount factor that differs from the one of the household which owns the firms.

The stationary equilibrium under this optimality condition features an output gap and inflation which are linear in $\bar{y}_t$ and $\bar{y}_{t|t}$

$$\tilde{y}_t - \bar{y}_t = -\frac{\mathcal{R}\varepsilon}{1+\mathcal{R}\varepsilon}\bar{y}_t - \frac{\mathcal{R}\varepsilon\beta\rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}\bar{y}_{t|t} \tag{18}$$

$$\pi_t = \frac{\kappa}{1+\mathcal{R}\varepsilon}\bar{y}_t + \frac{\kappa\beta\rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}\bar{y}_{t|t} \tag{19}$$

The next result characterizes the interest rate that implements this equilibrium.

Corollary 1 *A nominal interest rate which can implement this policy is given by*

$$i_t^* = r_t^n + f_{\bar{y}}^*\left(\mathcal{R}\right)\bar{y}_t + f_{\bar{y},b}^*\left(\mathcal{R}\right)\bar{y}_{t|t}$$

The interest rate moves one-for-one with the natural rate of interest while $f_{\bar{y}}^*$ *and* $f_{\bar{y},b}^*$ *are functions of* $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$ *through* $\mathcal{R}$. *This interest rate behavior matches that assumed in the second benchmark case above with coefficients on* $\bar{y}_t$ *and* $\bar{y}_{t|t}$ *that satisfy Assumption 3. The exact expressions for the functions* $f_{\bar{y}}^*(\cdot)$ *and* $f_{\bar{y},b}^*(\cdot)$ *are given in Appendix D.*

This can be compared to the nominal interest rate under optimal discretionary policy in the perfect information case which can be written as

$$i_t^{*,PI} = r_t^n + \left(f_{\bar{y}}^*\left(\kappa\right) + f_{\bar{y},b}^*\left(\kappa\right)\right)\bar{y}_t$$

To ensure unique implementation, the interest rate specification can be augmented by a term that reacts more than one-for-one to deviations of inflation from its intended path

$$i_t^* = r_t^n + \left(f_{\bar{y}}^*\left(\mathcal{R}\right) - \phi_\pi\Gamma_{\bar{y}}\right)\bar{y}_t + \left(f_{\bar{y},b}^*\left(\mathcal{R}\right) - \phi_\pi\Gamma_{\bar{y},b}\right)\bar{y}_{t|t} + \phi_\pi\pi_t$$

where $\Gamma_{\bar{y}}, \Gamma_{\bar{y},b}$ *are the coefficients on* $\bar{y}_t$ *and* $\bar{y}_{t|t}$ *in the equilibrium solution for* π_t. *Choosing* $\phi_\pi > 1$ *ensures that the intended equilibrium is the unique solution in the system of equations defined by this expression along with (1), (2), (5), (6), (15), and (16).*

Proof. See Appendix D. ■

A necessary element in these results is that the policymaker has an information advantage regarding a state variable that has some persistence and creates an inflation-output trade-off for the policymaker. Without these features, the current interest rate level cannot affect expectations $\left\{\tilde{y}_{t+1|t}, \pi_{t+1|t}\right\}$ and optimal policy becomes invariant to the signaling channel.

To be precise, consider a model with a more general set of shocks. I denote the set of exogenous state variables with a vector $\mathbf{z}_t$ that evolves as a VAR(1) process with independent shocks

$$\mathbf{z}_t = \boldsymbol{\Upsilon}\mathbf{z}_{t-1} + \mathbf{e}_t,\ \mathbf{e}_t \sim \text{iid } N\left(0, \boldsymbol{\Sigma}\right) \text{ where } \boldsymbol{\Sigma} \text{ is diagonal}$$

I partition this vector into two subvectors $\mathbf{z}_{1,t}, \mathbf{z}_{2,t}$ where $\mathbf{z}_{1,t}$ is perfectly observed by private agents while they can only see the true value of $\mathbf{z}_{2,t}$ with a lag. I impose $\Upsilon_{12} = 0$ so that forecasts $\mathbf{z}_{1,t+h|t}$ do not depend on $\mathbf{z}_{2,t|t}$. I also assume that the eigenvalues of $\boldsymbol{\Upsilon}$ are less than one in absolute value.

Again, the central bank's information advantage is that they can observe the current $\mathbf{z}_{2,t}$ while private agents cannot. I then let private agents suppose that the interest rate i_t is linear in $\left\{\mathbf{z}_{1,t}, \mathbf{z}_{2,t}, \mathbf{z}_{2,t|t}\right\}$ which is the case under the optimal discretionary policy. Let the equilibrium conditions in this model be

$$\begin{aligned}
\tilde{y}_t^{CB} &= \tilde{y}_{t+1|t}^{CB} - \frac{1}{\sigma}\left(i_t - \pi_{t+1|t}\right) + \boldsymbol{\Xi}_{\tilde{y}}\mathbf{z}_t + \boldsymbol{\Xi}_{\tilde{y},b}\mathbf{z}_{2,t|t} \\
\pi_t &= \beta\pi_{t+1|t} + \kappa\tilde{y}_t^{CB} + \boldsymbol{\Xi}_{\pi}\mathbf{z}_t + \boldsymbol{\Xi}_{\pi,b}\mathbf{z}_{2,t|t}
\end{aligned}$$

where I now use $\tilde{y}_t^{CB}$ to denote the welfare-relevant output gap under this alternate configuration of shocks. Then, I obtain the following

Proposition 3 *Suppose that changes in $\mathbf{z}_{2,t}$ and $\mathbf{z}_{2,t|t}$ do not impose an output-inflation trade-off. That is, suppose that $\boldsymbol{\Xi}_{\pi,b} = 0$ and $\boldsymbol{\Xi}_{\pi}\mathbf{z}_t = \boldsymbol{\Xi}_{\pi,1}\mathbf{z}_{1,t}$ so that only $\mathbf{z}_{1,t}$ enters into the inflation equilibrium condition. Then the equilibrium under the discretionary optimal policy features $\frac{d\tilde{y}_t^{CB}}{d\mathbf{z}_{2,t}} = \frac{d\pi_t}{d\mathbf{z}_{2,t}} = \frac{d\tilde{y}_t^{CB}}{d\mathbf{z}_{2,t|t}} = \frac{d\pi_t}{d\mathbf{z}_{2,t|t}} = 0$ while the policymaker's optimality condition is the same as the perfect information case*

$$\tilde{y}_t^{CB} = -\varepsilon\pi_t$$

Proof. See Appendix D. ■

In the language of New Keynesian models, this result shows that if the policymaker only has an information advantage regarding shocks to demand or the natural real rate while not having superior knowledge regarding cost-push-type shocks, then the policymaker optimally maintains the same ratio between output gap and inflation deviations as in the perfect information case. While changes in the interest rate still have an effect on private agents' beliefs $\mathbf{z}_{2,t|t}$, the presence of this signaling channel does not impact optimal discretionary policy when the information advantage is limited to this class of shocks.

5 The value of information

In this section, I consider whether it would be beneficial for the policymaker to directly communicate information to private agents. I will first compare the welfare losses under the two extremes of no communication and full communication. Later in this section, I examine the case of partial communication.

The no communication case is the one analyzed above where the policymaker can only choose the interest rate under the given asymmetric information structure. Under full communication, the central bank costlessly and noiselessly discloses the true values of both current exogenous states $\{d_t, \bar{y}_t\}$ to all private agents so that the setting is equivalent to the perfect information case. In each case, I presume that the central bank is implementing the optimal discretionary interest rate policy.

The loss under no communication can be evaluated using the equilibrium shown in the previous section. Meanwhile, optimal discretionary policy under full communication is

$$\tilde{y}_t^{PI} - \bar{y}_t = -\varepsilon \pi_t^{PI}$$

Substituting this into (2) and solving forward gives the equilibrium solutions

$$\tilde{y}_t^{PI} - \bar{y}_t = \frac{-\varepsilon\kappa}{1 - \beta\rho_{\bar{y}} + \varepsilon\kappa}\bar{y}_t \quad \text{and} \quad \pi_t^{PI} = \frac{\kappa}{1 - \beta\rho_{\bar{y}} + \varepsilon\kappa}\bar{y}_t$$

The period t welfare loss consists of a current period loss and an expected future loss

$$\mathcal{L}_t = \underbrace{\frac{1}{2}\left[(\tilde{y}_t - \bar{y}_t)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right]}_{l_t} + \underbrace{E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-t}\frac{1}{2}\left((\tilde{y}_s - \bar{y}_s)^2 + \frac{\varepsilon}{\kappa}\pi_s^2\right)}_{\beta E_t^{CB}\mathcal{L}_{t+1}}$$

Proposition 4 *Under an equilibrium where $\mathcal{R} \geq 0$,*

1. *The expected future loss is always higher under full communication*

$$E_t^{CB}\mathcal{L}_{t+1} \leq E_t^{CB}\mathcal{L}_{t+1}^{PI}$$

2. *The current period loss under no communication may be higher or lower than the full communication case. The difference depends on the current realizations of shocks $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$.*

Proof. See Appendix D. ■

The gains from no communication relative to full communication come from two sources. The first is the reduction in the stabilization bias when the interest rate's signaling effect on inflation expectations leads a discretionary policymaker to be tougher on inflation. The second benefit comes from imperfect information resulting in smaller inflation and output fluctuations even absent a reduction in the stabilization bias. To understand this better, first note that the policymaker is always able to fully offset the effects of changes in demand. Now, consider a positive shock to the output target which leads the policymaker to boost output by lowering the interest rate. The inflation fluctuations created by this action depend on both its impact on firms' current marginal costs as well as their forecasts of future marginal costs where the latter depends on firms' beliefs. In the perfect information setting, these components move in tandem. When firms are imperfectly informed, their forecasts of future marginal costs depend on their beliefs about the output target which now moves less than one-for-one with true output target shocks while also moving with demand shocks. Thus, for a given deviation of output away from its efficient level, the resulting inflation fluctuation is spread across both shocks and ends up being smaller on average. As an extreme example, suppose that after setting the interest rate, the central bank can directly manipulate beliefs by choosing any value of $\bar{y}_{t|t}$. Then, it's clear from the equilibrium in (18) and (19) that it will choose $\bar{y}_{t|t}$ in a way that offsets $\bar{y}_t$. Maintaining imperfect information helps the policymaker to get closer to this ideal.

I can also show that these two benefits of no communication operate independently.

Corollary 2 *To isolate the benefit from a smaller stabilization bias, I exogenously impose that $\bar{y}_{s|s} = \bar{y}_s$ for $s > t$ in evaluating the welfare losses. In this case,*

$$E_t^{CB}\mathcal{L}_{t+1} \leq E_t^{CB}\mathcal{L}_{t+1}^{PI} \quad \textit{for } \mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{y}}}\right]$$

To isolate the benefit of beliefs that do not correlate perfectly with true states, I exogenously impose $\mathcal{R} = \kappa$. In this case,

$$E_t^{CB}\mathcal{L}_{t+1} \leq E_t^{CB}\mathcal{L}_{t+1}^{PI}$$
$$\textit{when } Var_t^{CB}\left(\bar{y}_{s|s}\right) \leq Var_t^{CB}\left(\bar{y}_s\right) \ \textit{ and } Corr_t^{CB}\left(\bar{y}_{s|s}, \bar{y}_s\right) \leq 1 \textit{ for } s > t$$

which is satisfied in this model.

Proof. See Appendix D. ■

As a second exercise, I now consider partial communication where the central bank perfectly communicates the true value of one of the current exogenous states to private agents. The true value of the remaining exogenous state is then perfectly inferred from the interest rate so that all agents are perfectly informed in equilibrium. The key difference from the full communication case is that the interest rate retains a signaling effect on private agents' beliefs since it is used to infer the remaining exogenous state which was not directly communicated.

I will first consider the case of the central bank communicating the true current state of demand to agents. Then their belief about the current level of the output target is inferred from the interest rate as

$$\bar{y}_{t|t} = \frac{1}{f_{\bar{y}} + f_{\bar{y},b}} \left(i_t - \left(f_d + f_{d,b} \right) d_t \right)$$

Thus, a discretionary policymaker still faces a signaling effect of $\frac{d\bar{y}_{t|t}}{di_t} = \frac{1}{f_{\bar{y}}+f_{\bar{y},b}}$ when choosing the interest rate though beliefs will be correct in equilibrium. This maximizes the marginal effect of the discretionary policymaker's interest rate choice on inflation expectations and results in an inflation-output tradeoff characterized by $\mathcal{R} = \frac{\kappa}{1-\beta\rho_{\bar{y}}}$. This achieves the largest possible reduction in the stabilization bias through the signaling channel and raises welfare compared to both the no communication and full communication cases. However, because agents are perfectly informed in equilibrium, beliefs about the output target will now move in sync with true shocks which lowers welfare compared to the no communication case. On net, partial communication of only the demand shock is always preferable to full communication but is not unambiguously preferable to no communication.

Proposition 5 *In an equilibrium where $\mathcal{R} \geq 0$ and with partial communication of only the demand shock denoted by a ^{d} superscript,*

1. *Both the current and expected future welfare losses are higher under full communication than under partial communication of only the demand shock*

$$E_t^{CB}\mathcal{L}_{t+1}^{d} \leq E_t^{CB}\mathcal{L}_{t+1}^{PI} \quad \text{and} \quad l_t^d \leq l_t^{PI} \quad \text{for any realization of shocks } \{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$$

2. *The expected future welfare loss under no communication may be higher or lower than under partial communication of only the demand shock. The difference cannot be unambiguously signed and depends on parameter values.*

3. *The current period loss under no communication may be higher or lower than under partial communication of only the demand shock. The difference depends on the current realizations of shocks $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$ even for a fixed set of parameter values.*

Proof. See Appendix D. ■

Partial communication of only the true current output target results in the same optimal discretionary interest rate policy and welfare loss as full communication. In this case, the interest rate's signaling effect is only on agents' beliefs about demand and therefore, optimal interest rate policy is unaffected as discussed in Section 4.

The fact that the current period loss is not unambiguously lower under either no communication or partial communication of only the demand shock implies that this choice features time inconsistency. For a fixed set of parameter values, the central bank always wants to commit to one of these communication policies for future periods. However, there may be realizations of shocks that make the alternate communication policy preferable after taking into account current welfare, which would go against the policymaker's commitment. This property also suggests that a full analysis of optimal discretionary communication policy in this setting would involve private agents' beliefs that are formed by a non-Gaussian signal extraction problem. When it's optimal for the policymaker to communicate only in certain states, then a decision to withhold information is itself informative.

6 Alternate setups

6.1 Signaling about an inflation target

Here, I will show the case of an inflation target $\bar{\pi}_t$ rather than the time-varying output target. That is, suppose that the policy objective in (12) is replaced with

$$\mathcal{L}_{t_0} = E_{t_0}^{CB} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\tilde{y}_t^2 + \frac{\varepsilon}{\kappa} \left(\pi_t - \bar{\pi}_t\right)^2 \right)$$
$$\text{where } \bar{\pi}_t = \rho_{\bar{\pi}} \bar{\pi}_{t-1} + \epsilon_{\bar{\pi},t}, \quad \epsilon_{\bar{\pi},t} \sim N\left(0, \sigma_{\bar{\pi},t-1}^2\right) \tag{20}$$

with $\epsilon_{\bar{\pi},t}$ being serially uncorrelated and uncorrelated with $\epsilon_{d,t}$. All other aspects of the setup remain parallel with the baseline case of an output target. In particular, the central bank continues to have perfect information while the information set of private agents is $\mathcal{I}_t = \{i^t, d^{t-1}, \bar{\pi}^{t-1}, \sigma_d^t, \sigma_{\bar{\pi}}^t\}$.

For equilibrium dynamics under a general linear interest rate rule, suppose that the interest rate in (4) is replaced with the following expression which is now linear in the inflation target along with beliefs about the inflation target

$$i_t = f_d d_t + f_{d,b} d_{t|t} + f_{\bar{\pi}} \bar{\pi}_t + f_{\bar{\pi},b} \bar{\pi}_{t|t}$$

Then, belief formation mirrors the baseline case with

$$d_{t|t} = \rho_d d_{t-1} + \underbrace{\frac{f_d \frac{\sigma_{d,t-1}^2}{\sigma_{\bar{\pi},t-1}^2}}{f_d^2 \frac{\sigma_{d,t-1}^2}{\sigma_{\bar{\pi},t-1}^2} + f_{\bar{\pi}}^2}}_{K_{d,t}} \left(f_d \epsilon_{d,t} + f_{\bar{\pi}} \epsilon_{\bar{\pi},t}\right)$$

$$\bar{\pi}_{t|t} = \rho_{\bar{\pi}} \bar{\pi}_{t-1} + \underbrace{\frac{f_{\bar{\pi}}}{f_d^2 \frac{\sigma_{d,t-1}^2}{\sigma_{\bar{\pi},t-1}^2} + f_{\bar{\pi}}^2}}_{K_{\bar{\pi},t}} \left(f_d \epsilon_{d,t} + f_{\bar{\pi}} \epsilon_{\bar{\pi},t}\right)$$

The equilibrium is now characterized by the system of equations given by (1), (2), (5) and (20) along with the above policy rule and belief formation equations. Since $\bar{\pi}_t$ and $\bar{\pi}_{t|t}$ enter into this system of equations in the exact same way as $\bar{y}_t$ and $\bar{y}_{t|t}$ in the baseline model, the results related to the output target in Section 3 continue to hold here with the inflation target.

For the optimal discretionary policy problem, assuming that the variances of shocks are constant and following the same steps as in Section 4 yields the following optimality condition

$$\tilde{y}_t = -\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}\left(\pi_t - \bar{\pi}_t\right), \quad \text{where } \mathcal{R}_{\bar{\pi}} \equiv \frac{\frac{d\pi_t}{di_t}}{\frac{d\tilde{y}_t}{di_t}} = \frac{\frac{\partial \pi_t}{\partial i_t} + \frac{\partial \pi_t}{\partial \bar{\pi}_{t|t}}\frac{d\bar{\pi}_{t|t}}{di_t}}{\frac{\partial \tilde{y}_t}{\partial i_t} + \frac{\partial \tilde{y}_t}{\partial \bar{\pi}_{t|t}}\frac{d\bar{\pi}_{t|t}}{di_t}} \text{ in equilibrium}$$

It can again be shown that $\mathcal{R}_{\bar{\pi}} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{\pi}}}\right]$ where $\mathcal{R}_{\bar{\pi}}$ approaches its lower bound as $\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} \to \infty$ so that private agents attribute any change in the interest rate to a demand shock. When $\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} \to 0$, interest rate changes have their largest possible effect on inflation target beliefs and $\mathcal{R}_{\bar{\pi}}$ approaches its largest possible equilibrium value. In fact, since this optimality condition is identical to (17) with $\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}\bar{\pi}_t$ in place of $\bar{y}_t$, the implied equilibrium interest rate behavior will also mirror the case of an output target shock with this change of variable.

This optimal interest rate policy result still relies on the signaling effect tilting the inflation-output tradeoff in favor of smaller inflation deviations, but the logic differs slightly from the output target case. Here, maintaining a marginally smaller output gap requires a smaller interest rate reaction to an inflation target shock. The implied signaling effect is a smaller revision of agents' inflation target beliefs which results in inflation being closer to zero (unlike the case of an output target). However, less inflation in this case translates into a larger inflation deviation from its target for a given reduction in the output gap.

The stationary equilibrium under this optimality condition is given by

$$\tilde{y}_t = \frac{\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}}{1+\mathcal{R}_{\bar{\pi}}\varepsilon}\bar{\pi}_t - \frac{\frac{1}{\kappa}\left(\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\beta\rho_{\bar{\pi}}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)}\bar{\pi}_{t|t}$$
$$\pi_t - \bar{\pi}_t = -\frac{1}{1+\mathcal{R}_{\bar{\pi}}\varepsilon}\bar{\pi}_t + \frac{\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)}\bar{\pi}_{t|t}$$

The results so far have coincided with the output target case. The main difference in these two cases comes when I consider the value of communication. In the case of an inflation target, partial communication of only demand now becomes unambiguously optimal for the expected future loss. The best communication strategy for the current period loss will still depend on the realizations of shocks. The following proposition states these results where I again denote the case of partial communication of only the demand shock by a superscript ^{d}.

Proposition 6 *In an equilibrium where $\mathcal{R}_{\bar{\pi}} \geq 0$,*

1. *The expected future loss is always lowest under communication of only d_t, that is*

$$E_t^{CB}\mathcal{L}_{t+1}^{d} \leq E_t^{CB}\mathcal{L}_{t+1}^{PI} \quad and \quad E_t^{CB}\mathcal{L}_{t+1}^{d} \leq E_t^{CB}\mathcal{L}_{t+1}$$

2. *For the current period loss, communication of only d_t is always preferable to full communication.*

$$l_t^d \leq l_t^{PI} \quad \textit{for any realization of shocks } \{\epsilon_{d,t}, \epsilon_{\bar{\pi},t}\}$$

However, whether it is preferable to no communication depends on the current realizations of shocks $\{\epsilon_{d,t}, \epsilon_{\bar{\pi},t}\}$.

Proof. See Appendix D. ■

The intuition for this difference is that, in contrast to the output target case, it's easier for the central bank to bring inflation closer to its target when the target is known by private agents in equilibrium. To better understand the contrast, consider first the case of a positive shock to an output target. If firms are aware of this higher target, they will raise prices more today in anticipation of equilibrium output being higher for some time. This increased inflation will have a negative effect on demand, thus undermining the central bank's effort to boost output towards the higher target. In the case of a positive shock to an inflation target, firms that are aware of this elevated target will raise prices more today for a given level of current output which is now beneficial to the central bank's goal of achieving a higher inflation target. However, in the context of discretionary policy, it is also beneficial for interest rate changes to have a signaling effect on inflation target beliefs. Thus it becomes best to create a situation where the inflation target is perfectly revealed indirectly through the interest rate.

In summary, when interest rate changes have an effect on private agents' beliefs about either an output target and or inflation target, it remains possible to observe increases in inflation and output following interest rate surprises. In addition, signaling effects about either type of shock will lead a discretionary policymaker to choose to maintain smaller inflation deviations from target than he would under perfect information, thus resulting in a reduction in the stabilization bias arising from a lack of commitment. However, the implications differ for communication policy in that the central bank is better able to achieve its stabilization goals when private agents know the true inflation target in equilibrium but not the true output target. In this setting, a central bank that allows private agents to fully infer the inflation target from their observations of the interest rate will fully capture both the beneficial disciplining effect of the signaling channel on discretionary interest rate policy as well as the benefit of private agents knowing the inflation target in equilibrium.

6.2 Signaling about a cost-push shock

In this section, I show how implications for optimal policy can differ when the interest rate serves as a signal about cost-push shocks rather than policy targets. That is, suppose that the inflation equation is now augmented by a shock (which can be microfounded as time-variation in the elasticity of substitution between varieties)

$$\pi_t = \beta\pi_{t+1|t} + \kappa\tilde{y}_t + v_t$$
$$\text{where } v_t = \rho_v v_{t-1} + \epsilon_{v,t} \text{ with } \epsilon_{v,t} \sim \text{iid } N\left(0, \sigma_v^2\right) \text{ and } \rho_v \in [0, \bar{\rho})$$

The policy objective is now the standard one which can be derived using a second-order approximation to the representative household's lifetime utility.

$$\mathcal{L}_{t_0} = E_{t_0}^{CB}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\tilde{y}_t^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right)$$

There continues to be a demand shock and the information setup also mirrors the baseline case. In this setting, it's possible to show that the signaling effect actually tilts the inflation-output tradeoff in the opposite direction as the baseline case and results in larger inflation fluctuations.

Proposition 7 *When the interest rate is a signal about a demand and cost-push shock, then the optimal discretionary interest rate choice is characterized by*

$$\tilde{y}_t = -\mathcal{R}_v\frac{\varepsilon}{\kappa}\pi_t$$
$$\text{where } \mathcal{R}_v \equiv \frac{\frac{d\pi_t}{di_t}}{\frac{d\tilde{y}_t}{di_t}} = \frac{\frac{\partial\pi_t}{\partial i_t} + \frac{\partial\pi_t}{\partial v_{t|t}}\frac{dv_{t|t}}{di_t}}{\frac{\partial\tilde{y}_t}{\partial i_t} + \frac{\partial\tilde{y}_t}{\partial v_{t|t}}\frac{dv_{t|t}}{di_t}} \text{ in equilibrium}$$

$\mathcal{R}_v$ is itself a function of interest rate response coefficients and is therefore determined in equilibrium. There may be multiple equilibrium values for $\mathcal{R}_v$ but nonnegative real solutions satisfy the following properties when $\beta\rho_v > 0$:

1. *$\mathcal{R}_v \leq \kappa$*

2. *As $\frac{\sigma_d^2}{\sigma_v^2} \to \infty$, $\frac{dv_{t|t}}{di_t} \to 0$ and $\mathcal{R}_v \to \kappa$. In this limit, the interest rate has no effect on $v_{t|t}$ and the optimality condition for policy becomes equivalent to that in the case of perfect information.*

3. *As $\frac{\sigma_d^2}{\sigma_v^2} \to 0$, $\frac{dv_{t|t}}{di_t} \to \frac{1}{f_v + f_{v,b}}$ and $\mathcal{R}_v < \kappa$. In this limit, the interest rate has its largest possible effect on $v_{t|t}$ and the optimality condition for policy allows larger inflation fluctuations relative to output fluctuations when compared to the perfect information case.*

4. When $\beta = 0$ or $\rho_v = 0$, $\mathcal{R}_v = \kappa$ in equilibrium for any value of $\frac{\sigma_d^2}{\sigma_v^2}$.

Proof. See Appendix D. ■

This result is similar to the "opacity bias" found by Walsh (2010). The intuition behind this result can again be understood through the signaling effect's impact on the inflation cost of keeping output gap fluctuations marginally smaller. As in the time-varying inflation target case, this requires the central bank to respond less to the cost-push shock. The signaling effect of this action is a smaller revision to agents' belief about the cost-push shock which results in a smaller impact of the shock on inflation expectations. Therefore, the inflation cost of achieving a smaller output gap is lower in the presence of a signaling effect.

Because the signaling effect is associated with an "opacity bias", it's clear that the value of communication will also differ when the interest rate is a signal about cost-push shocks. More specifically, intransparency essentially worsens the stabilization bias. However, there is still some benefit of intransparency due to fluctuations being smaller on average when agents' beliefs about v_t do not correlate perfectly with the true state. Thus, it should be ambiguous whether full communication is better than no communication. However, it will be the case that partial communication of only the demand level gives the largest welfare loss since it creates this opacity bias while agents still become perfectly informed in equilibrium. Partial communication of only v_t combined with the optimal discretionary interest rate policy will again result in an equilibrium equivalent to full communication.

An important lesson of the results under the previous two alternative setups is that, in the presence of a signaling effect, understanding the implications for interest rate and communication policy requires a finer grouping of shocks beyond the basic division into those that the central bank can perfectly offset by changing nominal interest rates and those that prevent the central bank from always achieving his first-best allocation.

7 Empirical evidence

7.1 Empirical model

The regressions below are motivated using a model that assumes an AR(1) reduced form for inflation along with a Taylor-style interest rate rule that responds directly to inflation[13]. Coibion and Gorodnichenko (2012a) and Coibion and Gorodnichenko (2012b) show that this type of reduced-form framework characterizes inflation forecast data well.

[13]I show in Appendix E that the New Keynesian structural model above can be modified slightly to give similar empirical relationships as the ones tested below.

Suppose that inflation is described by the following AR(1) process

$$\pi_t = \rho_\pi \pi_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \sigma^2_{\varepsilon,t-1}\right)$$

Agents observe π_t with a one-period lag and also receive two signals about current inflation: one from the observed interest rate which responds to true inflation and another composite signal which contains idiosyncratic noise.

$$\begin{aligned} i_t &= \phi \pi_t + u_t \\ s_{jt} &= \pi_t + e_{jt} \end{aligned}$$

I assume $\phi > 0$ and that the two signal noise terms $\{u_t, e_{jt}\}$ are uncorrelated across time, with each other, and with ε_t. They are normally distributed with variances that are identical across agents and possibly time-varying. This departs from empirical models used in previous studies in two ways. First, most other models generally assume that agents cannot see true inflation at any lag. Another difference is the explicit inclusion of an interest rate signal containing additional information about inflation. The interest rate's response to true inflation is key. If the interest rate was a function only of private beliefs about π_t, then it would not convey any additional information to private agents.

Each agent j has the information set $\mathcal{I}_{jt} = \left\{\pi^{t-1}, i^t, s^t_j, \sigma^t_\varepsilon\right\}$ and forms his conditional expectation of current inflation via a static Gaussian signal extraction problem which yields

$$\pi_{t|jt} = \rho_\pi \pi_{t-1} + K^i_t \left(i_t - E\left[i_t|\pi_{t-1}\right]\right) + K^s_t \left(s_{jt} - E\left[s_{jt}|\pi_{t-1}\right]\right)$$

where $K^i_t \in \left(0, \phi^{-1}\right)$ and $K^s_t \in (0,1)$ are increasing in $\sigma^2_{\varepsilon,t-1}$, which captures prior uncertainty. This expression can be transformed into two different testable relationships.

First, the model makes predictions about the effect of interest rate surprises on inflation forecast revisions. With data on aggregate interest rate surprises, one can test the following relationship for aggregate forecast revisions

$$\begin{aligned} \overline{\pi_{t+h|t}} - \overline{\pi_{t+h|t-1}} = {} & \rho^h_\pi K^i_t \left(i_t - \overline{E\left[i_t|\pi_{t-1}\right]}\right) + \rho^h_\pi K^s_t \left(\pi_t - \overline{\pi_{t|t-1}}\right) \\ & + \rho^{h+1}_\pi \left(1 - K^s_t\right)\left(\pi_{t-1} - \overline{\pi_{t-1|t-1}}\right) + error_{ht} \end{aligned} \tag{21}$$

where the error term is a function of the average noise in s_t and not correlated with the other RHS terms. My focus will be on the implication that the response of forecast revisions to interest rate surprises will be increasing in prior uncertainty. Note also that this regression equation is nearly identical to equation (5) in Romer and Romer (2000). The main difference is that while they use the Federal Reserve's forecasts to control for other inflation-related news, all relevant news in this model is captured by the lagged forecast and nowcast errors.

The second implication of this model can be seen by combining the news from both the interest rate and s_{jt} into a current nowcast error term which reflects all current period news. This gives the following equation for forecast revisions for different horizons $h \geq 0$

$$\begin{aligned} \pi_{t+h|jt} - \pi_{t+h|j,t-1} &= K_t \rho_\pi^h \left(\pi_t - \pi_{t|j,t-1}\right) + (1 - K_t)\, \rho_\pi^{h+1} \left(\pi_{t-1} - \pi_{t-1|j,t-1}\right) + error_{jht} \quad (22) \\ \text{where } K_t &\equiv \phi K_t^i + K_t^s \in (0,1) \end{aligned}$$

K_t is decreasing in signal noise and increasing in prior uncertainty $\sigma^2_{\varepsilon,t-1}$. The error term may be correlated across individuals and horizons but is uncorrelated across time and with the other RHS variables. This expression says that higher prior uncertainty should result in a greater effect of current nowcast errors on inflation forecast revisions and a smaller effect of lagged forecast errors.

7.1.1 Extensions of the empirical model

I can allow for a standard direct negative effect of i_t on π_t of the following form

$$\pi_t = \rho_\pi \pi_{t-1} - \delta i_t + \varepsilon_t$$

where $\delta > 0$ and the expressions for i_t and s_{jt} continue to be those given above. This yields a solution for π_t that is similar to the above model

$$\pi_t = \check{\rho}_\pi \pi_{t-1} + \frac{1}{1+\delta\phi}\varepsilon_t - \frac{\delta}{1+\delta\phi} u_t \quad \text{where } \check{\rho}_\pi \equiv \frac{\rho_\pi}{1+\delta\phi}$$

Now forecast revisions are given by

$$\begin{aligned} \overline{\pi_{t+h|t}} - \overline{\pi_{t+h|t-1}} = \check{\rho}_\pi^h \check{K}_t^i \left(i_t - \overline{E\left[i_t|\pi_{t-1}\right]}\right) + \check{\rho}_\pi^h \check{K}_t^s \left(\pi_t - \overline{\pi_{t|t-1}}\right) \\ + \check{\rho}_\pi^{h+1}\left(1 - \check{K}_t^s\right)\left(\pi_{t-1} - \overline{\pi_{t-1|t-1}}\right) + error_{ht} \end{aligned}$$

where $\check{K}_t^i$ may now take on negative values but both $\check{K}_t^i$ and $\check{K}_t^s$ are still increasing in $\sigma^2_{\varepsilon,t-1}$.

If I do not allow agents to agents observe lagged inflation, then agents' forecasts are described by a Kalman filter[14]. In this case, aggregate forecast revisions evolve as

$$\overline{\pi_{t+h|t}} - \overline{\pi_{t+h|t-1}} = \rho_\pi^h \hat{K}_t^i \left(i_t - \overline{i_{t|t-1}}\right) + \rho_\pi^h \hat{K}_t^s \left(\pi_t - \overline{\pi_{t|t-1}}\right) + error_{ht}$$

where $\hat{K}_t^i \in \left(0, \phi^{-1}\right)$ and $\hat{K}_t^s \in (0,1)$ are now increasing in prior uncertainty, $Var_{t-1}(\pi_t)$, which itself is increasing in $\sigma^2_{\varepsilon,t-1}$. The lagged nowcast term drops out of the regression equation. However, this term enters significantly in the regressions below, suggesting that the assumption

[14] This is the linear least-squares forecast which is also optimal if I additionally assume that agents' prior beliefs about the initial state π_0 are normally distributed.

that agents can see lagged inflation is reasonable approximation of the data.

7.2 Data

For aggregate inflation forecasts, I use median quarterly forecasts of the GNP/GDP deflator (GDP starting in 1992) from the Survey of Professional Forecasters provided by the Federal Reserve Bank of Philadelphia. The survey starts in 1968Q4 and is quarterly with about 40 respondents in each quarter. A unique feature of the SPF is that, in addition to point forecasts, it also asks respondents to report forecasted probability distributions for annual inflation. This allows me to impute a measure of subjective uncertainty over inflation.

For actual data, I use real-time data from the Federal Reserve Bank of Philadelphia taking values from a two-quarters ahead vintage (e.g., the 2001Q1 observation for inflation is taken from the 2001Q3 vintage). This timing is chosen to correspond to the final published NIPA estimates prior to annual or benchmark revisions.

To measure policy surprises, I use prices for 30-day federal funds futures obtained from Bloomberg which start in December 1988. I use the method described in Kuttner (2001) to construct surprises on policy news days. I define these as days when the target rate changed or scheduled Federal Open Market Committee meeting days starting in 1994 (some dating adjustments were made following Kuttner (2003)). As described in Swanson (2006), the FOMC only began issuing post-meeting press releases in 1994. Additionally, rate changes were not strongly associated with meeting days prior to 1994. For instance, only 31% of actual target changes from the start of 1989 to the end of 1993 occur within one day before or after a scheduled meeting compared to 86% starting in 1994 until the target effectively hit zero in late 2008. Thus, pre-1994 meeting days when no change was made are not categorized as news days, but the results are not sensitive to this choice. To get a measure of policy surprises that corresponds to the quarterly SPF timing, I sum one-day policy surprises between SPF deadlines[15].

Finally, in the regressions estimating the effect of news from interest rate surprises, I exclude dates after 2011Q1 due the Fed's decision to begin regularly releasing economic projections of Federal Reserve Board members and Bank presidents in conjunction with post-meeting press releases. The results are not sensitive to this choice.

7.2.1 Imputing subjective uncertainty

I proxy subjective uncertainty using the SPF's probability forecasts for the GNP/GDP deflator where agents report probabilities of inflation being in pre-defined ranges. Starting in 1981Q3,

[15] Deadline dates are available starting in 1990Q2. Prior to that, I use the 15th of the middle month of each quarter.

the survey consistently contains these reports for both the current and following years' inflation as measured by the percentage change in the annual averages of the price index. To impute the variance associated with these forecasts, I minimize the sum of squared differences between the reports and probabilities for the same ranges implied by a normal distribution following Giordani and Söderlind (2003) and Lahiri and Liu (2006). More formally, for a given set of reported probabilities $\{q_n\}_{n=1}^{N}$ corresponding to ranges $\{[a_n, b_n)\}_{n=1}^{N}$, I solve

$$\min_{\mu,\sigma} \sum_{n=1}^{N} \left\{ q_i - \left[\Phi\left(\frac{b_n - \mu}{\sigma}\right) - \Phi\left(\frac{a_n - \mu}{\sigma}\right) \right] \right\}^2$$

I remove individual-level post-1991 means from these variances to account for a switch from GNP to GDP measures and a change in the number of ranges provided in the survey from 6 to 10. In the analysis below, I use the median of the adjusted variances of forecasts for the next year's inflation as a proxy of subjective forecast uncertainty, denoted as $\overline{Std_t^\pi}$. The following table shows that this measure is not highly correlated with macroeconomic variables or other measures of uncertainty commonly used in the literature on uncertainty shocks[16]. This low correlation with other uncertainty measures is not surprising since they capture many aspects of economic uncertainty and not just those related to inflation. The low correlation with macroeconomic variables indicates that regressions containing interactions with this measure of subjective uncertainty are unlikely to be picking up nonlinearities or state-dependence related to the business cycle.

Table 1: Correlations between $\overline{Std_t^\pi}$ and macro variables

x	x_{t-1}	x_t	x_{t+1}
Macro Variables			
Inflation	−0.02	0.12	−0.09
Real GNP/GDP growth	−0.08	0.02	0.10
Uncertainty Measures			
Google econ uncertainty index	0.24**	0.13	0.12
Stock volatility	0.02	−0.11	−0.10
Policy uncertainty index	0.07	−0.05	−0.05

Notes: These correlations are computed with the longest samples available in the data. The sample sizes vary between 110 and 124 quarters. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

[16] Uncertainty measures are from the dataset accompanying Bachmann, Elstner, and Sims (2013) as well as the policy-related economic uncertainty described in Baker, Bloom, and Davis (2013) and available at www.policyuncertainty.com.

7.3 Effect of interest rate surprises on inflation forecasts

In this section, I estimate the impact of interest rate news on inflation forecasts and present the main empirical result in support of the interest rate's signaling effect.

My first set of baseline estimates echo the findings in Table 8 of Romer and Romer (2000) which shows that monetary policy tightening seems to have a mildly positive (though not statistically significant) effect on inflation forecasts. This can be seen as estimating a version of (21) with constant coefficients. My analysis differs from theirs in several ways. First, my sample period is 1989:Q1 to 2011:Q1 which has little overlap with their sample of 1974:Q3 to 1991:Q4 with the Volcker years removed. Secondly, I use lagged forecast and nowcast errors as my summary measures of "other news" as implied by the above empirical model while they used changes in the Federal Reserve's Greenbook forecast. Lastly, they used federal funds rate changes or a dummy variable based on articles in the *Wall Street Journal* following Cook and Hahn (1989a) and Cook and Hahn (1989b) to measure monetary policy actions. For my regressions, I instead use interest rate surprises measured using daily federal funds futures prices which arguably has less of an endogeneity problem.

Despite these differences, the main results are remarkably similar. In fact, my estimates show a positive effect of surprise interest rate tightening on inflation forecast revisions that is actually significant at a 10% or better level for all forecast horizons. The coefficients are larger than those estimated by Romer and Romer (2000) since the average magnitude of interest rate surprises is only about one-third the average size of target changes.

Table 2: Baseline effect of federal funds rate surprises on inflation forecasts

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	0.304*	0.267**	0.332***	0.181*
	[1.81]	[2.14]	[2.76]	[1.79]
$\pi_t - \overline{\pi_{t\|t-1}}$	0.101***	0.020	0.028	0.030
	[2.69]	[0.89]	[1.27]	[1.32]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}}$	0.191***	0.143***	0.067***	0.095***
	[3.79]	[4.30]	[2.94]	[3.55]
Adjusted R^2	0.325	0.278	0.204	0.216
N	88	88	88	88

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 and 1996:Q1 dropped due to switches in the SPF from the GNP to GDP deflator and then subsequently to the GDP price index making the lagged forecast unavailable in those periods. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

Now, to test the main prediction that K_t^i is higher when agents have more uncertainty over the last forecast they made, I interact the news variables in this regression with the measure of subjective prior uncertainty described above. Table 3 shows the results of interacting each news variable with a dummy indicating whether $\overline{Std^{\pi}_{t-1}}$ is below or above its median.

Table 3: Effect of federal funds rate surprises on inflation forecasts with a high vs low prior uncertainty interaction

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.081	0.110	0.114	0.144
	[0.45]	[0.85]	[1.20]	[1.49]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.666**	0.428**	0.756***	0.212
	[2.37]	[2.05]	[4.52]	[0.84]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.064	−0.023	−0.007	0.026
	[1.01]	[−0.61]	[−0.21]	[0.73]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.116**	0.043	0.039	0.029
	[2.35]	[1.52]	[1.54]	[1.11]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.0.230***	0.199***	0.097***	0.112***
	[3.13]	[4.45]	[3.21]	[3.11]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.141**	0.071*	0.042	0.066
	[2.60]	[1.93]	[1.49]	[1.65]
$\overline{Std^{\pi}_{t-1}}$ high	0.113*	0.068	0.082**	0.022
	[1.82]	[1.64]	[2.26]	[0.57]
Adjusted R^2	0.335	0.313	0.276	0.189
N	88	88	88	88
P-value of F-test of difference in $i_t - \overline{i_{t\|t-1}}$ coef	0.083	0.199	0.001	0.801

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 and 1996:Q1 dropped due to switches in the SPF from the GNP to GDP deflator and then subsequently to the GDP price index making the lagged forecast unavailable in those periods. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

Compared to the baseline results, the coefficient on interest rates surprises in periods of low prior uncertainty are smaller and not statistically significant while the coefficients in periods of high uncertainty are higher and statistically significant (save for the farthest horizon). F-tests also show some statistical significance of the differences in these coefficients. In addition, the interactions on the news captured by the lagged forecast and nowcast errors also go in the predicted directions.

Table 4 shows that estimating a continuous interaction with prior uncertainty produces the same qualitative results. Here, the prior uncertainty measure is standardized to have zero mean and standard deviation of one. Thus, the coefficients on the main effects of each news term can be interpreted as the average effect when prior uncertainty is at its mean value. In this set of results, it's evident that the interaction effect is stronger at shorter horizons. one candidate explanation of this is that the Federal Reserve's information advantage in forecasting inflation is stronger at lower horizons. Some evidence supporting this possibility is presented in Table 4 of Sims (2003) which shows results of a test of whether the Federal Reserve's inflation forecast has a lower RMSE than the SPF's average forecast. The evidence presented there is stronger for one-quarter-ahead forecasts than for four-quarter-ahead forecasts. Lastly, comparing the adjusted R^2 values to the baseline case indicates that allowing for this interaction improves the model's ability to explain forecast revisions.

Table 4: Effect of federal funds rate surprises on inflation forecasts with a continuous prior uncertainty interaction

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	0.452***	0.254	0.352**	0.147
	[2.92]	[1.63]	[2.19]	[1.07]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$	0.422**	0.235*	0.187	−0.098
	[2.07]	[1.70]	[1.64]	[−0.77]
$\pi_t - \overline{\pi_{t\|t-1}}$	0.091**	0.022	0.028	0.034
	[2.60]	[0.99]	[1.31]	[1.48]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$	0.070*	0.062**	0.038**	0.005
	[1.73]	[2.38]	[2.15]	[0.20]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}}$	0.215***	0.144***	0.065***	0.090***
	[3.99]	[4.30]	[2.84]	[3.07]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$	−0.048	−0.071*	−0.027	0.023
	[−0.79]	[−1.73]	[−0.93]	[0.63]
$\overline{Std^{\pi}_{t-1}}$	0.015	0.019	0.046***	0.004
	[0.41]	[0.88]	[2.69]	[0.22]
Adjusted R^2	0.347	0.296	0.239	0.193
N	88	88	88	88

Notes: $\overline{Std^{\pi}_{t-1}}$ is standardized to have zero mean and standard deviation of one. The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 and 1996:Q1 dropped due to switches in the SPF from the GNP to GDP deflator and then subsequently to the GDP price index making the lagged forecast unavailable in those periods. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

In summary, the results of this section show that surprise federal funds rate increases are associated with positive revisions in median inflation forecasts and that the effect is especially positive when the median reported subjective uncertainty in last quarter's inflation forecasts was high. A signaling effect of interest rate surprises naturally leads to this interactive effect while alternative explanations such as a cost channel do not. More evidence consistent with a signaling effect of interest rates can be found in Ozdagli (2013). In particular, he finds that a surprise increase in the federal funds rate has a larger contractionary effect on the S&P 500 Index on days when the market has received news about the macroeconomy prior to the FOMC announcement. The signaling story presented here can explain this result since agents will place less weight on the federal funds rate surprise as an indicator of the strength of the economy if economic news earlier in the day has reduced their uncertainty. Thus, the possibly expansionary signaling effect will play a smaller role on those days and the overall effect will predominantly be driven by the direct contractionary effect of an interest rate increase.

7.3.1 Robustness checks

One might be concerned that forecasters take into account other variables when making inflation forecasts. To address this issue, I also run specifications with added measures of news about either real GNP/GDP growth or unemployment. These news terms are proxied analogously with lagged forecast and nowcast errors. The tables given in Appendix F show that the results remain unchanged. In fact, with these additional controls, the interaction effect of prior uncertainty on the response to interest rate surprises becomes stronger.

I get similar results using revisions of the Federal Reserve's Greenbook GNP/GDP deflator forecasts as the proxy for other news (following Romer and Romer (2000)) though I lose some observations due to the Greenbook's five-year publication lag[17]. The estimates are also almost identical with the lagged SPF forecast on the right hand side with a coefficient that is not constrained to one.

Appendix G presents the same estimation for real GNP/GDP growth rather than inflation. The results are qualitatively similar though the estimates are less precise.

7.4 Time-variation in sensitivity of inflation forecasts to news

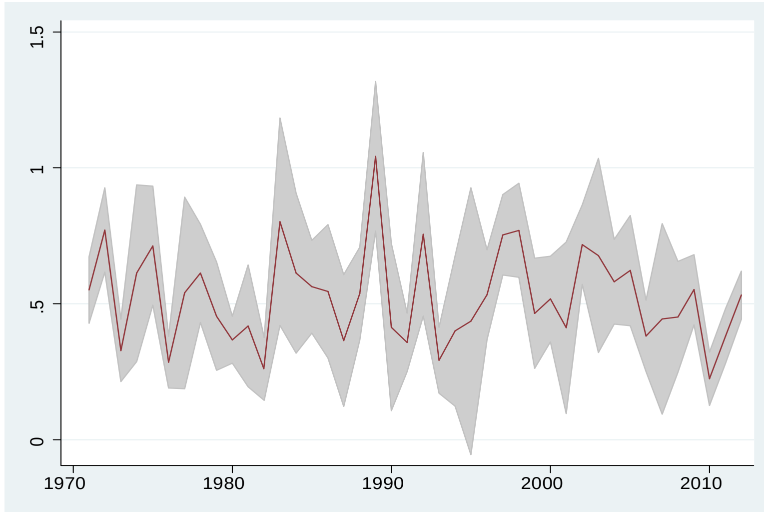
In this final section, I examine the overall effect of all inflation news on forecasts given in (22). Using 17,716 observations of individual level quarterly data over the period 1971-2012, I obtain annual estimates using a nonlinear least squares estimation of the following equation with standard errors clustered within quarters[18].

$$\pi_{t+h|jt} - \pi_{t+h|j,t-1} = \alpha_{ht} + K^{FE}_{year_t}\rho^h_\pi\left(\pi_t - \pi_{t|j,t-1}\right) + K^{NE}_{year_t}\rho^{h+1}_\pi\left(\pi_{t-1} - \pi_{t-1|j,t-1}\right) + error_{jht}$$

Figure 1 shows estimates of my main coefficients of interest which are the time-varying responses of inflation forecasts to current news.

[17] The Greenbook switches to forecasting the GDP deflator measure five months after the SPF switched so these observations are excluded.

[18] Coibion and Gorodnichenko (2012a) also estimates time-varying sensivity of forecasts to news using a different empirical approach. They discuss low frequency changes in this parameter associated with the Great Moderation.



Notes: Gray area represents 90% confidence intervals

Figure 1: Annual estimates of $K^{FE}_{year_t}$

There is substantial time-variation in this coefficient. Table 5 shows that the estimates correlate negatively with forecast dispersion (an imperfect proxy for idiosyncratic signal noise[19]) and positively with my measure of prior uncertainty as predicted by the model.

Table 5: Correlations between $\hat{K}^{FE}_{year_t}$ and signal noise or prior uncertainty

Variable	Correlation
Dispersion: $h = 0$	-0.39**
Dispersion: $h = 1$	-0.30*
Dispersion: $h = 2$	-0.36**
Dispersion: $h = 3$	-0.15
Dispersion: $h = 4$	-0.13
Lagged current year uncertainty	0.40**
Lagged next year uncertainty	0.38**

Notes: Correlations are calculated between annual coefficient estimates and annual means of the variables. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

Meanwhile, time-variation in these estimates does not seem to be associated with macroeconomic variables or other common measures of uncertainty as shown in Table 6. The fact that these correlations are lower than the ones in Table 5 suggests that the variation in inflation forecast sensitivity to news is more related to an information story than other explanations.

[19] The proxy is imperfect due to a nonmonotonic relationship between idiosyncratic signal noise and forecast dispersion. If variation in s_{jt} is dominated by noise, agents optimally ignore these signals and forecast dispersion approaches zero.

Table 6: Correlations between $\hat{K}^{FE}_{year_t}$ and macro variables

x	x_{year_t-1}	x_{year_t}	x_{year_t+1}
Macro Variables			
Inflation	−0.03	−0.07	−0.09
Real GNP/GDP growth	−0.05	0.28*	0.21
Uncertainty Measures			
Google econ uncertainty index	−0.18	−0.07	−0.14
Stock volatility	0.20	0.00	−0.05
Policy uncertainty index	−0.02	−0.22	−0.18

Notes: Correlations are calculated between annual coefficient estimates and annual means of the variables. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

8 Conclusion

In this paper, I explored the impact of a signaling channel on the conduct of optimal interest rate policy as well as equilibrium responses to policy surprises. I found that a discretionary policymaker who is better informed about an output target can influence inflation expectations in a way that tilts the short-run inflation-output tradeoff toward a policy that maintains smaller inflation fluctuations. This effect is stronger when the policymaker has a larger impact on inflation expectations. As this influence grows, the optimal discretionary policy approaches the optimal policy under commitment to a forward-looking interest rate rule. Compared to the perfect information case, the signaling effect reduces the stabilization bias which typically exists when the policymaker is unable to commit. This contributes to the finding that it is optimal for the policymaker to maintain an information advantage when it comes to output target fluctuations. Considering the signaling effect in alternate setups reveals some additional nuances. A particularly interesting implication is that it is beneficial for private agents to be informed about the inflation target in equilibrium, but that it's best to indirectly reveal this information through the interest rate since this will lead to a smaller stabilization bias under a discretionary interest rate policy.

For a general interest rate rule, I showed that when the policymaker is better informed about demand shocks and the policy response to these shocks is inadequate, then it is possible to see positive responses of current economic activity and forecasts to positive interest rate surprises. This matches the empirical patterns found in the present paper as well as previous work on this topic. Furthermore, I present evidence of a previously untested prediction of this information setup which is that responses of inflation forecasts to positive interest rate surprises are more positive when prior uncertainty about inflation is high.

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A Aggregate equilibrium conditions with idiosyncratic government spending shocks

In this section, I derive equilibrium conditions for an economy where firms face idiosyncratic government spending shocks so that it is consistent for households and firms not to have information about current aggregate outcomes. This yields a condition for the aggregate output gap which is identical to (1) in the model in the main text. The inflation condition differs from (2) in a few ways which I outline at the end of the section.

A.1 Setup

The setup shares many features with Lorenzoni (2010). There is a continuum of yeoman farmer households with identical preferences and technology who produce differentiated goods and face a Calvo friction.

Each period contains three stages. In stage 1, the policymaker sees the entire history of aggregate government spending and output target levels $\left\{g^t, \bar{y}^t\right\}$ and sets the nominal interest rate i_t conditional on these aggregate states. In the private sector, all households have the same beginning-of-period information which contains true realizations of past state variables and the current nominal interest rate so that their Stage 1 information set is $\mathcal{I}_t^1 = \left\{i^t, g^{t-1}, \bar{y}^{t-1}\right\}$. In this stage, pre-commitments are made regarding aggregate nominal consumption.

In stage 2, each worker-firm j now realizes his firm-specific government demand shock, g_{jt}, where the idiosyncratic component of g_{jt} is iid. Firms who are able to reset prices then choose prices based their updated Stage 2 information sets $\mathcal{I}_{jt}^2 = g_{jt} \cup \mathcal{I}_t^1$. I do not include past observations of g_{jt} in these information sets since they are irrelevant for current and future payoffs once g^{t-1} is known. All prices are set simultaneously without knowledge of the resulting aggregate price. The household receives no further information about $\bar{y}_t$.

In stage 3, all prices are revealed and households optimally allocates the pre-committed amount of nominal spending across varieties j. The revelation of prices in this stage also reveals the true aggregate states and households carry this knowledge into Stage 1 of the next period.

Prior to the realizations of $\{g_{jt}\}$, ex-ante risks are the same across households. I assume that households perfectly risk-share by trading in a complete set of contingent claims in Stage 1. These claims pay out at the beginning of Stage 1 of the next period so that the beginning-or-period wealth is the same across households.

I assume that the idiosyncratic component of government spending is such that the resulting log-linearized total demand faced by each firm j is given by

$$\begin{aligned} y_{jt} &= \frac{C}{Y} c_t + \left(1 - \frac{C}{Y}\right) g_{jt} - \varepsilon \left(p_{jt} - p_t\right) \\ &= y_t + \left(1 - \frac{C}{Y}\right) \omega_{jt} - \varepsilon \left(p_{jt} - p_t\right) \\ \text{since } y_t &= \frac{C}{Y} c_t + \left(1 - \frac{C}{Y}\right) g_t \text{ by market clearing} \\ \text{where } g_{jt} &= g_t + \omega_{jt}, \quad \omega_{jt} \sim \text{iid } N\left(0, \sigma_\omega^2\right) \end{aligned}$$

Meanwhile, I continue to assume AR(1) forms for the aggregate shocks

$$\begin{aligned} g_t &= \rho_g g_{t-1} + \epsilon_{g,t}, \quad \epsilon_{g,t} \sim \text{iid } N\left(0, \sigma_g^2\right) \\ \bar{y}_t &= \rho_{\bar{y}} \bar{y}_{t-1} + \epsilon_{\bar{y},t}, \quad \epsilon_{\bar{y},t} \sim \text{iid } N\left(0, \sigma_{\bar{y}}^2\right) \end{aligned} \tag{23}$$

A.2 Consumption

Preferences are identical across households and the same as the model in the main text

$$\max E\sum_{t=0}^{\infty}\beta^t\left[U\left(C_t\right)-V\left(L_t\right)\right],\ \text{where } C_t\equiv\left[\int_0^1 C_{jt}^{\frac{\varepsilon-1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}},\varepsilon>1$$

All households have access to the same full basket of goods in stage 3 so there's only one relevant aggregate inflation rate. Then, since all households pre-commit nominal spending in Stage 1 based on the same information set, beginning-of-period wealth, and idiosyncratic risks, they all choose the same aggregate nominal consumption which yields the following Euler equation in log-linearized form

$$c_t=E\left[c_{t+1}|\mathcal{I}_t^1\right]+\frac{U_c}{U_{cc}C}\left(i_t-E\left[\pi_{t+1}|\mathcal{I}_t^1\right]\right)$$

Note that combining this consumption Euler equation with the resource constraint yields the same condition for the aggregate output gap as in (1) since I can write

$$\tilde{y}_t=E\left[\tilde{y}_{t+1}|\mathcal{I}_t^1\right]-\frac{1}{\sigma}\left(i_t-E\left[\pi_{t+1}|\mathcal{I}_t^1\right]\right)+d_t-E\left[d_{t+1}|\mathcal{I}_t^1\right] \tag{24}$$

$$\text{where } \tilde{y}_t\equiv y_t-y_t^n=\frac{C}{Y}c_t+\frac{\varphi}{\sigma+\varphi}\left(1-\frac{C}{Y}\right)g_t\ \text{ and } d_t\equiv\frac{\varphi}{\sigma+\varphi}\left(1-\frac{C}{Y}\right)g_t$$

as in the main text and importantly, the information set $\mathcal{I}_t^1$ is also the same as the one used in the main text. This definition of the aggregate demand shock d_t also gives

$$d_t=\frac{\varphi}{\sigma+\varphi}\left(1-\frac{C}{Y}\right)g_t=\rho_d d_{t-1}+\epsilon_{d,t},\ \text{ where } \rho_d=\rho_g\ \text{ and }\ \epsilon_{d,t}=\frac{\varphi}{\sigma+\varphi}\left(1-\frac{C}{Y}\right)\epsilon_{g,t} \tag{25}$$

Purchases of individual varieties are made in Stage 3 after prices are revealed so that

$$c_{jt}=c_t-\varepsilon\left(p_{jt}-p_t\right)$$

A.3 Production and price-setting

In Stage 2, a worker-firm j learns the government portion of their demand g_{jt} so their information set is $\mathcal{I}_{jt}^2\equiv\left\{i^t,g^{t-1},\bar{y}^{t-1},g_{jt}\right\}$. They face the demand function

$$y_{jt}=\frac{C}{Y}c_t+\left(1-\frac{C}{Y}\right)g_{jt}-\varepsilon\left(p_{jt}-p_t\right)$$

However, they do not see aggregate prices and so they do not know how much they'll ultimately sell for a given price p_{jt}.

Technology is again linear for each worker-firm

$$Y_{jt}=AL_{jt}$$

where the nominal cost of labor is given by the MRS multiplied by the aggregate price index which has the following log-linear form (where φ, σ retain the definitions in (3))

$$w_{jt} = \sigma \frac{C}{Y} c_t + \varphi l_{jt} + p_t$$

The log-linearized pricing condition for a firm is then the following

$$\begin{aligned} p_{jt}^* &= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k E\left[w_{j,t+k} | \mathcal{I}_{jt}^2\right] \\ &= (1-\theta\beta)\left(\sigma \frac{C}{Y} c_t + E\left[\varphi y_{jt}^* + p_t | \mathcal{I}_{jt}^2\right]\right) + \theta\beta E\left[p_{j,t+1}^* | \mathcal{I}_{jt}^2\right] \end{aligned}$$

where I use a star on y_{jt}^* to highlight the fact that at reset prices depend on output-dependent labor costs among price resetters which will differ from that of non-resetters. Using the firms' demand function, this can be transformed to

$$p_{jt}^* = (1-\theta\beta)\left((\sigma+\varphi)\frac{C}{Y} c_t + \varphi\left(1 - \frac{C}{Y}\right) g_{jt} - \varphi\varepsilon p_{jt}^* + (1+\varphi\varepsilon) E\left[p_t | \mathcal{I}_{jt}^2\right]\right) + \theta\beta E\left[p_{j,t+1}^* | \mathcal{I}_{jt}^2\right]$$

I assume that the Calvo shock is independent of the idiosyncratic component of government spending such that the average government spending shock among price resetters is equal to the average among all the firms. That is, I assume the following where I order firms so that the set of price resetters are those indexed by $j \in [\theta, 1]$

$$\frac{1}{1-\theta} \int_{\theta}^{1} g_{jt} dj = g_t$$

Then, as long as p_{jt}^* is linear in the variables in $\mathcal{I}_{jt}^2$, this gives

$$\frac{1}{1-\theta} \int_{\theta}^{1} p_{jt}^* dj = p_t^* \equiv \int_{0}^{1} p_{jt}^* dj$$

Secondly, I note that the iid nature of the idiosyncratic component of government spending shocks along with the posited linearity of p_{jt}^* implies that

$$E\left[p_{j,t+1}^* | \mathcal{I}_{jt}^2\right] = E\left[p_{t+1}^* | \mathcal{I}_{jt}^2\right]$$

Then, the aggregate price index implies the usual log-linearized first-order dynamics

$$p_t = \theta p_{t-1} + \int_{\theta}^{1} p_{jt}^* dj = \theta p_{t-1} + (1-\theta) p_t^* \tag{26}$$

so that expectations must satisfy

$$E\left[p_t | \mathcal{I}_{jt}^2\right] = \theta p_{t-1} + (1-\theta) E\left[p_t^* | \mathcal{I}_{jt}^2\right]$$

The aggregate price relation also gives the following property

$$(1-\theta) E\left[p_{t+1}^* | \mathcal{I}_{jt}^2\right] = E\left[\pi_{t+1} | \mathcal{I}_{jt}^2\right] + (1-\theta) E\left[p_t | \mathcal{I}_{jt}^2\right]$$

Aggregating the individual reset prices over resetters $j \in [\theta, 1]$ and using these properties then gives

$$(1-\theta)\, p_t^* = \frac{(1-\theta)(1-\theta\beta)(\sigma+\varphi)}{1+(1-\theta\beta)\,\varepsilon\varphi}\tilde{y}_t + \frac{\theta\beta}{1+(1-\theta\beta)\,\varepsilon\varphi} E\left[\pi_{t+1}|\mathcal{I}_t^2\right] + (1-\theta)\, E\left[p_t|\mathcal{I}_t^2\right] \tag{27}$$

where with a slight abuse of notation, I denote aggregate expectations with

$$E\left[x|\mathcal{I}_t^2\right] \equiv \int_0^1 E\left[x|\mathcal{I}_{jt}^2\right] dj$$

Some further manipulation delivers the Phillips curve in this setting

$$\begin{aligned}\pi_t = &\frac{\beta}{1+(1-\theta\beta)\,\varepsilon\varphi} E\left[\pi_{t+1}|\mathcal{I}_t^1\right] + \frac{\kappa}{1+(1-\theta\beta)\,\varepsilon\varphi}\tilde{y}_t \\ &+ \frac{\beta}{1+(1-\theta\beta)\,\varepsilon\varphi}\left(E\left[\pi_{t+1}|\mathcal{I}_t^2\right] - E\left[\pi_{t+1}|\mathcal{I}_t^1\right]\right) + \frac{(1-\theta)^2}{\theta}\left(E\left[p_t^*|\mathcal{I}_t^2\right] - p_t^*\right)\end{aligned} \tag{28}$$

This aggregate inflation condition along with (23), (24), (25), (26), (27), and an interest rate that's linear in $\left\{g^t, y^t\right\}$ give a set of linear stochastic difference equations that define the equilibrium. Thus, it will be the case that agents' choices will be linear in the variables in their information sets as I conjectured earlier[20].

In particular, behavior of the aggregate output gap and inflation are given by (24) and (28) which are the counterparts to the key equilibrium conditions (1) and (2) from the main text. The only differences in equilibrium behavior of aggregate variables comes from the differences in the inflation equation. Looking at (28), it's clear that explicitly accounting for idiosyncratic shocks yields a Phillips curve that differs from (2) in the main text in two ways:

1. The coefficients are scaled down by a multiplicative factor $\frac{1}{1+(1-\theta\beta)\varepsilon\varphi} < 1$ due to the yeoman farmer decentralized labor market setup.

2. There are two new terms due specifically to the idiosyncratic shocks and information sets.

   - $E\left[\pi_{t+1}|\mathcal{I}_t^2\right] - E\left[\pi_{t+1}|\mathcal{I}_t^1\right]$ reflects the difference in aggregate beliefs that comes from individual agents having the idiosyncratic signals $\{g_{jt}\}_{j\in[0,1]}$. $E\left[\pi_{t+1}|\mathcal{I}_t^1\right]$ will be a prior based on the histories $\left\{g^{t-1}, \bar{y}^{t-1}\right\}$ plus a term reflecting news from i_t. $E\left[\pi_{t+1}|\mathcal{I}_t^2\right]$ will be the same prior plus a term incorporating the same news from i_t as well as another term capturing news from the idiosyncratic signals whose noise averages out to zero in aggregate. Hence, the difference between these beliefs will be linear in the news terms with coefficients that are related to the informativeness of the extra signals $\{g_{jt}\}_{j\in[0,1]}$. In equilibrium, these news terms, and hence $E\left[\pi_{t+1}|\mathcal{I}_t^2\right] - E\left[\pi_{t+1}|\mathcal{I}_t^1\right]$, are linear in $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$.
   - $E\left[p_t^*|\mathcal{I}_t^2\right] - p_t^*$ will be linear in the aggregate belief errors $E\left[g_t|\mathcal{I}_t^2\right] - g_t$ and $E\left[\bar{y}_t|\mathcal{I}_t^2\right] - \bar{y}_t$ which are themselves linear in $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$.

In summary, the inflation condition differs from the one used in the main text due to a change of coefficients and extra direct effects of the shocks $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$. In particular, both shocks now enter into the NKPC, thus giving them additional properties akin to a cost-push shock. The intuitions behind the main results should remain unaffected.

[20] Lorenzoni (2010) proves this in a model that has a similar structure.

B Solution under arbitrary policy coefficients

Rearranging equilibrium conditions (1) and (2) gives the following system

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} - \begin{bmatrix} 1 \\ \kappa \end{bmatrix} d_{t+1|t} + \begin{bmatrix} 1 \\ \kappa \end{bmatrix} d_t - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} i_t$$

Conjecturing that the output gap and inflation are both linear in $\left\{d_t, d_{t|t}, \bar{y}_t, \bar{y}_{t|t}\right\}$ leads to the following implied form for expectations

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M} \begin{bmatrix} d_{t+1|t} \\ \bar{y}_{t+1|t} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

Combining the previous two expressions along with (4) then gives

$$\begin{aligned} \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = & \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \mathbf{M} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} - \begin{bmatrix} 1 \\ \kappa \end{bmatrix} \rho_d d_{t|t} \\ & + \begin{bmatrix} 1 \\ \kappa \end{bmatrix} d_t - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} \left(f_d d_t + f_{\bar{y}} \bar{y}_t + f_{d,b} d_{t|t} + f_{\bar{y},b} \bar{y}_{t|t} \right) \end{aligned}$$

Using this to evaluate the one-period-ahead expectation and matching coefficients gives the solution for $\mathbf{M}$

$$\mathbf{M} = - \begin{bmatrix} \frac{1}{\sigma}\Omega_d \left(1 - \beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1 - \beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right) \\ \frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) \end{bmatrix}$$

$$\text{with } \Omega_d \equiv \frac{1}{\left(1-\rho_d\right)\left(1-\beta\rho_d\right) - \frac{\kappa}{\sigma}\rho_d} \quad \text{and} \quad \Omega_{\bar{y}} \equiv \frac{1}{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right) - \frac{\kappa}{\sigma}\rho_{\bar{y}}}$$

This immediately gives the solution for one-period-ahead expectations and substituting this back into the above expression gives the solution for current outcomes, both as functions of current beliefs and true states

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = - \begin{bmatrix} \frac{1}{\sigma}\Omega_d \left(1 - \beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1 - \beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}} \\ \frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = & \begin{bmatrix} -\frac{1}{\sigma}\Omega_d \left(1 - \beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) - \left(1 - \frac{1}{\sigma}f_d\right) \\ -\frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) - \kappa\left(1 - \frac{1}{\sigma}f_d\right) \end{bmatrix} d_{t|t} \\ & + \begin{bmatrix} -\frac{1}{\sigma}\Omega_{\bar{y}}\left(1 - \beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{1}{\sigma}f_{\bar{y}} \\ -\frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{\kappa}{\sigma}f_{\bar{y}} \end{bmatrix} \bar{y}_{t|t} + \begin{bmatrix} 1 - \frac{1}{\sigma}f_d & -\frac{1}{\sigma}f_{\bar{y}} \\ \kappa\left(1 - \frac{1}{\sigma}f_d\right) & -\frac{\kappa}{\sigma}f_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} \end{aligned} \tag{29}$$

Longer horizon forecasts then evolve as

$$\begin{bmatrix} \tilde{y}_{t+h|t} \\ \pi_{t+h|t} \end{bmatrix} = - \begin{bmatrix} \frac{1}{\sigma}\Omega_d \left(1 - \beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d^h & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1 - \beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}}^h \\ \frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d^h & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}}^h \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

Setting $d_{t|t} = d_t$ and $\bar{y}_{t|t} = \bar{y}_t$ leads to the perfect information responses in Section 3.1.

$$\begin{bmatrix} \tilde{y}_t^{PI} \\ \pi_t^{PI} \end{bmatrix} = - \begin{bmatrix} \frac{1}{\sigma}\Omega_d \left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right) \\ \frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix}$$

$$\begin{bmatrix} \tilde{y}_{t+h|t}^{PI} \\ \pi_{t+h|t}^{PI} \end{bmatrix} = - \begin{bmatrix} \frac{1}{\sigma}\Omega_d \left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d^h & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}}^h \\ \frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d^h & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}}^h \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix}$$

Responses for $\tilde{r}_t$ can be obtained using these solutions and the definition $\tilde{r}_t \equiv i_t - \pi_{t+1|t} - \sigma\left(d_t - d_{t+1|t}\right)$.

The signs of responses depend crucially on the signs of Ω_d and $\Omega_{\bar{y}}$. In particular, these coefficients need to be positive to ensure that responses go the intuitive way (i.e., the perfect information responses of the output gap and inflation to a positive interest rate surprise are negative). Assumption 1 achieves this since for a given $\rho \in \left\{\rho_d, \rho_{\bar{y}}\right\}$, the corresponding Ω has the same sign as

$$(1-\rho)(1-\beta\rho) - \frac{\kappa}{\sigma}\rho = \beta\rho^2 - \left(1+\beta+\frac{\kappa}{\sigma}\right)\rho + 1$$

This is an upward-facing parabola with 2 real roots. The larger root is greater than one.

$$\frac{1+\beta}{2\beta} + \frac{\frac{\kappa}{\sigma} + \sqrt{\left(1+\beta+\frac{\kappa}{\sigma}\right)^2 - 4\beta}}{2\beta} \geq 1 \text{ for } \beta \leq 1$$

Then, since $\rho_d, \rho_{\bar{y}} < 1$ must hold in order for the exogenous states to be stationary, ρ_d and $\rho_{\bar{y}}$ must be below the smaller root of the parabola for $\Omega_d, \Omega_{\bar{y}}$ to be positive. Thus, I impose

$$\rho_d, \rho_{\bar{y}} < \bar{\rho} \equiv \frac{1+\beta+\frac{\kappa}{\sigma} - \sqrt{\left(1+\beta+\frac{\kappa}{\sigma}\right)^2 - 4\beta}}{2\beta}$$

$$\text{where } \frac{\kappa}{\sigma} = \frac{(1-\theta)(1-\theta\beta)}{\theta}\left(1+\frac{\varphi}{\sigma}\right)$$

Rearranging this shows that $\bar{\rho} = \theta$ for $\varphi = 0$. Combining this with the fact that

$$\frac{\partial\bar{\rho}}{\partial\frac{\kappa}{\sigma}} = \frac{1}{2\beta}\left[1 - \frac{1+\beta+\frac{\kappa}{\sigma}}{\sqrt{\left(1+\beta+\frac{\kappa}{\sigma}\right)^2 - 4\beta}}\right] < 0$$

shows that $\bar{\rho} < \theta$ for $\varphi > 0$.

C Extensions

C.1 Adding more structural shocks

In this section, I explore how the above results may change in environments with additional shocks. The optimal discretionary policy is affected by the existence of a signaling channel only through a change in the slope of the short-run inflation-output tradeoff which, in turn, determines the optimal ratio maintained between output gap and inflation deviations. An immediate consequence of this property is that the interest rate should still perfectly offset shocks that affect only the natural real rate of interest regardless of whether the policymaker possesses an information advantage on these shocks.

On the other hand, the presence of additional cost-push-type shocks, which the policymaker cannot perfectly offset, produces more interesting results. Consider the case of adding a shock v_t to the firms' price-setting equation so that it becomes

$$\pi_t = \beta\pi_{t+1|t} + \kappa\tilde{y}_t + v_t$$
$$\text{where } v_t = \rho_v v_{t-1} + \epsilon_{v,t}, \;\; \epsilon_{v,t} \sim \text{iid } N\left(0, \sigma_v^2\right) \text{ and } \rho_v \in [0, \bar{\rho})$$

I first assume that both private agents and the policymaker can see the entire history v^t at time t so that the policymaker has no information advantage regarding this shock. Then, I obtain the following

Proposition 8 *The optimal interest rate under discretionary policy with an additional cost-push shock which the policymaker does not have an information advantage for is*

$$i_t^* = r_t^n + f_{\bar{y}}^*(\mathcal{R})\,\bar{y}_t + f_{\bar{y},b}^*(\mathcal{R})\,\bar{y}_{t|t} + f_v^*(\mathcal{R})\,v_t$$

where $\mathcal{R}$ depends on underlying parameters in the same way as in the baseline model.

This can be compared to the optimal interest rate under perfect information

$$i_t^{*,PI} = r_t^n + \left(f_{\bar{y}}^*(\kappa) + f_{\bar{y},b}^*(\kappa)\right)\bar{y}_t + f_v^*(\kappa)\,v_t$$

$f_v^*(\cdot)$ is an increasing function and the exact expression is in Appendix D.

Furthermore, in the limit $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to 0$ where $\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}}$, the optimal interest rate policy no longer corresponds to the optimal commitment to a rule of the form

$$i_t = r_t^n + f_{\bar{y}}^c\bar{y}_t + f_{\bar{y},b}^c\bar{y}_{t|t} + f_v^c v_t$$

In this limit, the response coefficients for $\bar{y}_t$ and $\bar{y}_{t|t}$ coincide with those under commitment to the above rule, but the response to v_t differs. That is,

$$f_{\bar{y}}^{*,c} = f_{\bar{y}}^*\left(\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right) \quad \textit{and} \quad f_{\bar{y},b}^{*,c} = f_{\bar{y},b}^*\left(\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right)$$
$$\textit{but } f_v^{*,c} = f_v^*\left(\frac{\kappa}{1-\beta\rho_v}\right) \neq f_v^*\left(\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right)$$

Proof. See Appendix D. ■

Despite the policymaker not having an information advantage about the cost-push shock v_t, the optimal response to this shock is still influenced by the signaling effect that the interest rate has on private agents' belief about the output target. The presence of that signaling effect tilts the short-run inflation-output tradeoff in a way that leads the policymaker to enforce smaller inflation deviations *conditional on any shock* to the economy. Since $f_v^*(\cdot)$ is increasing in its argument, then when $\rho_v < \rho_{\bar{y}}$ and the interest rate is a signal about the output target, a discretionary policymaker could actually choose an interest rate that *overreacts* to the cost-push shock v_t relative to a policymaker who can commit to a rule of the form given above. Due to this overreaction, full communication may become welfare-improving depending on the relative importance of the different shocks.

I can also consider the case where the policymaker has an information advantage about v_t in addition to $\{d_t, \bar{y}_t\}$. Moreover, beliefs are formed under the following supposed current interest rate behavior which replaces

(14)

$$i_t = f_d d_t + f_{d,b} d_{t|t} + f_{\bar{y}} \bar{y}_t + f_{\bar{y},b} \bar{y}_{t|t} + f_v v_t + f_{v,b} v_{t|t}$$

Now there are three private agent beliefs $\{d_{t|t}, \bar{y}_{t|t}, v_{t|t}\}$ all of which will again be linear in i_t. Then the optimal discretionary policy can be shown to be equivalent to the one derived above in the baseline model with the exception that now, the equilibrium $\mathcal{R}$ depends on $\frac{dv_{t|t}}{di_t}$ as follows:

$$\mathcal{R} \equiv \frac{\frac{d\pi_t}{di_t}}{\frac{d\tilde{y}_t}{di_t}} = \frac{\frac{\partial \pi_t}{\partial i_t} + \frac{\partial \pi_t}{\partial \bar{y}_{t|t}} \frac{d\bar{y}_{t|t}}{di_t} + \frac{\partial \pi_t}{\partial v_{t|t}} \frac{dv_{t|t}}{di_t}}{\frac{\partial \tilde{y}_t}{\partial i_t} + \frac{\partial \tilde{y}_t}{\partial \bar{y}_{t|t}} \frac{d\bar{y}_{t|t}}{di_t} + \frac{\partial \tilde{y}_t}{\partial v_{t|t}} \frac{dv_{t|t}}{di_t}}$$

where $\frac{d\bar{y}_{t|t}}{di_t}$ and $\frac{dv_{t|t}}{di_t}$ will now depend on $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$, $\frac{\sigma_v^2}{\sigma_{\bar{y}}^2}$, and the equilibrium interest rate coefficients.

C.2 Lagged states not observed

When agents cannot see the true lagged states, then beliefs are formed through a Kalman filter rather than a static signal extraction problem. This is the information structure which is more commonly found in the recent literature studying imperfect information in New Keynesian models such as Lorenzoni (2009), Mertens (2011), Berkelmans (2011). The same technique from Svensson and Woodford (2003) used above to deal with the circularity issue present in the belief formation problem can also be applied here. With $\rho_d, \rho_{\bar{y}} < 1$ and constant variances, this Kalman filter converges to a steady state where beliefs are given by

$$\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} = \begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} + \begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix} \left(i_t - f_{d,b} d_{t|t} - f_{\bar{y},b} \bar{y}_{t|t} - f_d d_{t|t-1} - f_{\bar{y}} \bar{y}_{t|t-1} \right)$$

where $d_{t+1|t} = \rho_d d_{t|t}$ and $\bar{y}_{t+1|t} = \rho_{\bar{y}} \bar{y}_{t|t}$. In this steady state, $\hat{K}_d, \hat{K}_{\bar{y}}$ are functions of $\{\rho_d, \rho_{\bar{y}}, f_d, f_{\bar{y}}, \sigma_d^2, \sigma_{\bar{y}}^2\}$. Again, beliefs can be expressed as a function of prior beliefs and the current interest rate.

$$d_{t|t} = \frac{f_{\bar{y}} + f_{\bar{y},b}}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} \left(\hat{K}_{\bar{y}} d_{t|t-1} - \hat{K}_d \bar{y}_{t|t-1} \right) + \frac{\hat{K}_d}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} i_t$$
$$\bar{y}_{t|t} = \frac{f_d + f_{d,b}}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} \left(\hat{K}_d \bar{y}_{t|t-1} - \hat{K}_{\bar{y}} d_{t|t-1} \right) + \frac{\hat{K}_{\bar{y}}}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} i_t$$

The main difference now is that agents' prior beliefs are no longer based on observations of the true lagged values in each period. Rather, beliefs from period t form the prior belief for $t+1$. In essence, this change in the information structure turns private agents' beliefs into additional endogenous state variables which policy influences.

This adds another dimension to the interest rate's signaling effect. When agents can see lagged true fundamentals, the interest rate's signaling effect is limited to private agents' current expectations. When agents cannot see lagged fundamentals, the policymaker's choice of the current interest rate now also affects future beliefs and thereby, future outcomes. This additional effect adds a set of new terms to the policymaker's optimality condition

$$\tilde{y}_t - \bar{y}_t = -\mathcal{R} \frac{\varepsilon}{\kappa} \pi_t - \beta \frac{d\tilde{y}_{t+1}/di_t}{d\tilde{y}_t/di_t} \left(E_t^{CB} \left[\tilde{y}_{t+1} - \bar{y}_{t+1} \right] + \frac{d\pi_{t+1}/di_t}{d\tilde{y}_{t+1}/di_t} \frac{\varepsilon}{\kappa} E_t^{CB} \left[\pi_{t+1} \right] \right)$$

In equilibrium, this optimality condition still implies a forward-looking optimal interest rate level which is linear in $\{d_t, d_{t|t}, \bar{y}_t, \bar{y}_{t|t}\}$. When expressed in this form, the optimal interest rate no longer moves one-for-one with the natural real rate and a part that's linear in $\left\{\bar{y}_t, \bar{y}_{t|t}\right\}$. To be precise, I denote the optimal interest rate and policy coefficients under this altered information structure by a superscript ** and show that

Proposition 9 *In general, when agents cannot see lagged true states*

$$i_t^{**} \neq r_t^n + f_{\bar{y}}^{**}\bar{y}_t + f_{\bar{y},b}^{**}\bar{y}_{t|t} \quad \textit{for any } f_{\bar{y}}^{**}, f_{\bar{y},b}^{**}$$

Proof. See Appendix D. ■

To understand the intuition behind this property, suppose instead that the interest rate continues to respond one-for-one to $r_t^n = \sigma\left(d_t - \rho_d d_{t|t}\right)$. This offsets the contemporaneous effects of the natural real rate on outcomes so that ultimately, $\tilde{y}_t$ and π_t move only with variations in the true level and belief about the output target. However, now that agents cannot see lagged true states, the current forecast error made about demand carries through to the next period and affects future outcomes through $\bar{y}_{t+1|t+1}$. Thus, d_t and $d_{t|t}$ have a new intertemporal effect on future outcomes through the forecast error $d_t - d_{t|t}$. A policymaker with an information advantage can detect this forecast error and foresee this effect. This introduces a new element to the tradeoff he faces when deciding how to respond to d_t and $d_{t|t}$, which alters the resulting optimal response. The following corollary gives special cases where this new consideration does not apply and the policymaker again finds it optimal to set a nominal interest rate that moves one-for-one with the natural real rate.

Corollary 3 *(i) Under $\hat{K}_d = 0$, $\hat{K}_{\bar{y}} = 0$, or $\rho_{\bar{y}} = \rho_d$, the interest rate does not affect future beliefs and the optimal interest rate behavior from the case where agents could see true states with a lag is again optimal here.*

(ii) When $\rho_{\bar{y}} = 0$, future output target levels become unforecastable and the policymaker's optimality condition becomes equivalent to the perfect information case, as it also does when agents see true states with a lag.

(iii) When $\rho_d = 0$, the optimal interest rate responds one-for-one to the natural real rate, but responses to the output target and private agents' belief about it differ. That is,

$$i_t^{**} = r_t^n + f_{\bar{y}}^{**}\bar{y}_t + f_{\bar{y},b}^{**}\bar{y}_{t|t}, \textit{ where } f_{\bar{y}}^{**} \neq f_{\bar{y}}^{*}(\mathcal{R}) \textit{ and } f_{\bar{y},b}^{**} \neq f_{\bar{y},b}^{*}(\mathcal{R})$$

Proof. See Appendix D. ■

In the first set of special cases, beliefs become a function only of the current interest rate in equilibrium so there is no effect of a marginal change in the interest rate on future outcomes. In the second special case with $\rho_d = 0$, though the current interest rate still affects future outcomes through prior beliefs that agents carry into the next period, the current forecast error for the demand shock has no intertemporal effect on future beliefs. Then, the tradeoff with respect to d_t and $d_{t|t}$ becomes equivalent to the case above where they only have contemporaneous effects.

C.3 Optimal policy under dynamic time-varying uncertainty

Here, I consider optimal policy under time-varying uncertainty of the kind assumed in Section 2. To review, the exogenous states are AR(1) processes with serially uncorrelated shocks that have time-varying variances

$$d_t = \rho_d d_{t-1} + \epsilon_{d,t}, \quad \epsilon_{d,t} \sim N\left(0, \sigma_{d,t-1}^2\right)$$
$$\bar{y}_t = \rho_{\bar{y}}\bar{y}_{t-1} + \epsilon_{\bar{y},t}, \quad \epsilon_{\bar{y},t} \sim N\left(0, \sigma_{\bar{y},t-1}^2\right)$$

Private agents' information sets are $\mathcal{I}_t = \left\{i^t, d^{t-1}, \bar{y}^{t-1}, \sigma_d^t, \sigma_{\bar{y}}^t, \mathbf{f}^t\right\}$ where $\mathbf{f}_t$ denotes the vector of time t interest rate responses to the state variables $\left\{d_t, d_{t|t}, \bar{y}_t, \bar{y}_{t|t}\right\}$.

$\epsilon_{d,t}$ and $\epsilon_{\bar{y},t}$ are serially uncorrelated and uncorrelated with each other. With static variances, I showed that the optimal $f_{\bar{y}}^*$ and $f_{\bar{y},b}^*$ depend on the relative variance $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$. Because of this, I conjecture an equilibrium where policy coefficients are now time-varying via a dependence on the time-varying relative variance. I assume that private agents know the entire history of variances including the current values so that they know the current policy coefficients. Then, their beliefs can be derived in the same way as in Section 2.3 with the only difference being time subscripts on policy coefficients. Due to this time dependence, I conjecture that equilibrium $\tilde{y}_t$ and π_t are linear in $\left\{d_t, \bar{y}_t, d_{t|t}, \bar{y}_{t|t}\right\}$ with time-varying coefficients. Then, agents' expectations of future outcomes will be linear in beliefs that depend on future policy coefficients which the policymaker takes as given.

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M}_t \begin{bmatrix} d_{t+1|t} \\ \bar{y}_{t+1|t} \end{bmatrix}$$

Beliefs are then given by

$$\begin{aligned} d_{t|t} &= \frac{f_{\bar{y},t} + f_{\bar{y},b,t}}{1 + K_{\bar{y},t} f_{\bar{y},b,t} + K_{d,t} f_{d,b,t}} \left(K_{\bar{y},t}\rho_d d_{t-1} - K_{d,t}\rho_{\bar{y}}\bar{y}_{t-1}\right) + \frac{K_{d,t}}{1 + K_{\bar{y},t} f_{\bar{y},b,t} + K_{d,t} f_{d,b,t}} i_t \\ \bar{y}_{t|t} &= \frac{f_{d,t} + f_{d,b,t}}{1 + K_{\bar{y},t} f_{\bar{y},b,t} + K_{d,t} f_{d,b,t}} \left(K_{d,t}\rho_{\bar{y}}\bar{y}_{t-1} - K_{\bar{y},t}\rho_d d_{t-1}\right) + \frac{K_{\bar{y},t}}{1 + K_{\bar{y},t} f_{\bar{y},b,t} + K_{d,t} f_{d,b,t}} i_t \\ &\text{where } K_{d,t} = \frac{f_{d,t}\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}}{f_{d,t}^2\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2} + f_{\bar{y},t}^2} \quad \text{and} \quad K_{\bar{y},t} = \frac{f_{\bar{y},t}}{f_{d,t}^2\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2} + f_{\bar{y},t}^2} \end{aligned}$$

and the policymaker also takes $K_{d,t}$ and $K_{\bar{y},t}$ as given. Longer horizon forecasts continue to be $d_{t+h|t} = \rho_d^h d_{t|t}$ and $\bar{y}_{t+h|t} = \rho_{\bar{y}}^h \bar{y}_{t|t}$.

In this setting, the policymaker's optimality condition has the same form as before

$$\tilde{y}_t - \bar{y}_t = -\mathcal{R}_t \frac{\varepsilon}{\kappa} \pi_t$$

where $\mathcal{R}_t$ is now characterized by a nonlinear stochastic difference equation whose forcing variable is $\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}$. Furthermore, the optimal interest rate is

$$i_t^* = r_t^n + f_{\bar{y},t}^* \bar{y}_t + f_{\bar{y},b,t}^* \bar{y}_{t|t}$$

where $f_{\bar{y},t}^*$ is a function of $\mathcal{R}_t$ alone and $f_{\bar{y},b,t}^*$ can be written as

$$f_{\bar{y},b,t}^* = E\left[\mathcal{F}\left(\mathcal{R}_t, \mathcal{R}_{t+1}, ...\right)|\mathcal{I}_t\right]$$

To see this, note that $\tilde{y}_t - \bar{y}_t$ and π_t can again be written in terms of exogenous states and i_t with time-varying coefficients

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \frac{\mathbf{\Psi}_t \begin{bmatrix} K_{\bar{y},t}\left(f_{\bar{y},t} + f_{\bar{y},b,t}\right) & -K_{d,t}\left(f_{\bar{y},t} + f_{\bar{y},b,t}\right) \\ -K_{\bar{y},t}\left(f_{d,t} + f_{d,b,t}\right) & K_{d,t}\left(f_{d,t} + f_{d,v,t}\right) \end{bmatrix}}{1 + K_{\bar{y},t} f_{\bar{y},b,t} + K_{d,t} f_{d,b,t}} \begin{bmatrix} \rho_d d_{t-1} \\ \rho_{\bar{y}} \bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ \kappa & 0 \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} + \begin{bmatrix} H_{\tilde{y},i,t} \\ H_{\pi,i,t} \end{bmatrix} i_t$$

$$\text{where } \mathbf{\Psi}_t \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \mathbf{M}_t \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} - \begin{bmatrix} \rho_d & 0 \\ \kappa\rho_d & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} H_{\tilde{y},i,t} \\ H_{\pi,i,t} \end{bmatrix} \equiv \frac{\mathbf{\Psi}_t \begin{bmatrix} K_{d,t} \\ K_{\bar{y},t} \end{bmatrix}}{1 + K_{\bar{y},t} f_{\bar{y},b,t} + K_{d,t} f_{d,b,t}} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix}$$

In this form, it's again true that the discretionary policymaker has no control over time $t+1$ or later outcomes and the problem simplifies to

$$\min_{i_t} \frac{1}{2}\left(\left(\tilde{y}_t - \bar{y}_t\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right) \quad \text{subject to the preceding equation}$$

Thus, the FOC is analogous to the constant variances case but with a time-varying $\mathcal{R}_t$

$$\tilde{y}_t - \bar{y}_t = -\mathcal{R}_t \frac{\varepsilon}{\kappa}\pi_t, \;\; \text{where } \mathcal{R}_t = \frac{H_{\pi,i,t}}{H_{\tilde{y},i,t}}$$

Using this FOC and the structural equations to back out the optimal equilibrium i_t, gives

$$\pi_t = \beta\pi_{t+1|t} - \mathcal{R}_t\varepsilon\pi_t + \kappa\bar{y}_t = \frac{\kappa}{1+\mathcal{R}_t\varepsilon}\bar{y}_t + \frac{\kappa\beta\rho_{\bar{y}}}{1+\mathcal{R}_t\varepsilon} E\left[\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{\left(1+\mathcal{R}_{t+1}\varepsilon\right)\left(1+\mathcal{R}_{t+2}\varepsilon\right)} + ... \middle| \mathcal{I}_t\right] \bar{y}_{t|t}$$

$$\tilde{y}_t = \bar{y}_t - \mathcal{R}_t \frac{\varepsilon}{\kappa}\pi_t = \frac{1}{1+\mathcal{R}_t\varepsilon}\bar{y}_t - \frac{\mathcal{R}_t\varepsilon\beta\rho_{\bar{y}}}{1+\mathcal{R}_t\varepsilon} E\left[\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{\left(1+\mathcal{R}_{t+1}\varepsilon\right)\left(1+\mathcal{R}_{t+2}\varepsilon\right)} + ... \middle| \mathcal{I}_t\right] \bar{y}_{t|t}$$

when $\lim_{T\to\infty}\left(\prod_{k=0}^{T} \frac{\beta}{1+\mathcal{R}_{t+k}\varepsilon}\right)\pi_{t+T|t} = 0$. Then, expectations are

$$\pi_{t+1|t} = \kappa E\left[\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{\left(1+\mathcal{R}_{t+1}\varepsilon\right)\left(1+\mathcal{R}_{t+2}\varepsilon\right)} + ... \middle| \mathcal{I}_t\right] \rho_{\bar{y}}\bar{y}_{t|t}$$

$$\tilde{y}_{t+1|t} = \left\{1 - E\left[\mathcal{R}_{t+1}\varepsilon\left(\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{\left(1+\mathcal{R}_{t+1}\varepsilon\right)\left(1+\mathcal{R}_{t+2}\varepsilon\right)} + ...\right) \middle| \mathcal{I}_t\right]\right\} \rho_{\bar{y}}\bar{y}_{t|t}$$

By taking $\bar{y}_{t|t}$ out of the expectations, I'm assuming (and later show) that $\mathcal{R}_t$ will be a function of current and past relative variances which are not informative about future levels of the output target.

Then, this implies that the interest rate can be written in terms of $\left\{d_t, d_{t|t}, \bar{y}, \bar{y}_{t|t}\right\}$

$$\begin{aligned} i_t &= r_t^n + \pi_{t+1|t} + \sigma\left(\tilde{y}_{t+1|t} - \tilde{y}_t\right) \\ &= \sigma d_t - \sigma\rho_d d_{t|t} - \sigma\frac{1}{1+\mathcal{R}_t\varepsilon}\bar{y}_t \\ &+ \underbrace{\sigma E\left[1 + \left(\frac{\kappa}{\sigma} - \mathcal{R}_{t+1}\varepsilon + \frac{\mathcal{R}_t\varepsilon\beta}{1+\mathcal{R}_t\varepsilon}\right)\left(\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{\left(1+\mathcal{R}_{t+1}\varepsilon\right)\left(1+\mathcal{R}_{t+2}\varepsilon\right)} + ...\right) |\mathcal{I}_t\right]}_{f^*_{\bar{y},b,t}} \rho_{\bar{y}}\bar{y}_{t|t} \end{aligned}$$

In addition, the above expressions for $\pi_{t+1|t}, \tilde{y}_{t+1|t}$ gives an expression for the equilibrium M_t

$$\mathbf{M}_t = \left[\begin{array}{cc} 0 & 1 - E\left[\mathcal{R}_{t+1}\varepsilon\left(\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{(1+\mathcal{R}_{t+1}\varepsilon)(1+\mathcal{R}_{t+2}\varepsilon)} + ...\right)\middle| \mathcal{I}_t\right] \\ 0 & \kappa E\left[\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{(1+\mathcal{R}_{t+1}\varepsilon)(1+\mathcal{R}_{t+2}\varepsilon)} + ...\middle| \mathcal{I}_t\right] \end{array} \right]$$

Using this in the expression for $[H_{\tilde{y},i,t} \quad H_{\pi,i,t}]'$ and combining this with the expressions for $f^*_{\bar{y},b,t}$, and $K_{\bar{y},t}$ gives a non-linear stochastic difference equation implicitly relating $\mathcal{R}_t$ to future $\{\mathcal{R}_{t+k}\}_{k\geq 1}$ where the driving variable is the relative variance level $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.

$$\begin{aligned}
\mathcal{R}_t &= \frac{H_{\pi,i,t}}{H_{\tilde{y},i,t}} \\
\left[\begin{array}{c} H_{\tilde{y},i,t} \\ H_{\pi,i,t} \end{array}\right] &= \left(\left[\begin{array}{cc} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{array}\right] \mathbf{M}_t \left[\begin{array}{cc} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{array}\right] - \left[\begin{array}{cc} 0 & \frac{1}{\sigma} f^*_{\bar{y},b,t} \\ 0 & \frac{\kappa}{\sigma} f^*_{\bar{y},b,t} \end{array}\right]\right) \left[\begin{array}{c} K_{d,t} \\ K_{\bar{y},t} \end{array}\right] - \left[\begin{array}{c} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{array}\right] \\
\text{where } f^*_{\bar{y},b,t} &= \sigma E\left[1 + \left(\frac{\kappa}{\sigma} + \frac{\mathcal{R}_t\varepsilon\beta}{1+\mathcal{R}_t\varepsilon} - \mathcal{R}_{t+1}\varepsilon\right)\left(\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{(1+\mathcal{R}_{t+1}\varepsilon)(1+\mathcal{R}_{t+2}\varepsilon)} + ...\right) |\mathcal{I}_t\right]\rho_{\bar{y}} \\
\mathbf{M}_t &= \left[\begin{array}{cc} 0 & 1 - E\left[\mathcal{R}_{t+1}\varepsilon\left(\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{(1+\mathcal{R}_{t+1}\varepsilon)(1+\mathcal{R}_{t+2}\varepsilon)} + ...\right)\middle| \mathcal{I}_t\right] \\ 0 & \kappa E\left[\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{(1+\mathcal{R}_{t+1}\varepsilon)(1+\mathcal{R}_{t+2}\varepsilon)} + ...\middle| \mathcal{I}_t\right] \end{array} \right] \\
K_{\bar{y},t} &= -\frac{1}{\sigma}\frac{1+\mathcal{R}_t\varepsilon}{(1+\mathcal{R}_t\varepsilon)^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + 1}
\end{aligned}$$

If the relative variance $\frac{\sigma^2_{d,t}}{\sigma^2_{\bar{y},t}}$ is Markov, then it may be possible to show that the key variable $\mathcal{R}_t$ should depend only on $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ and $\frac{\sigma^2_{d,t}}{\sigma^2_{\bar{y},t}}$. Likewise, $f^*_{\bar{y},b,t}$ would also have this property.

D Proofs

D.1 Proposition 1

To arrive at the results under imperfect information, I first express the interest rate surprise as a function of the policy coefficients and the relative variance

$$\begin{aligned}
i^{surp}_t &\equiv i_t - E\left[x_t|\mathcal{I}_t \setminus i_t\right] = \left(1 + f_{d,b}K_{d,t} + f_{\bar{y},b}K_{\bar{y},t}\right)\left(f_d\epsilon_{d,t} + f_{\bar{y}}\epsilon_{\bar{y},t}\right) \\
&= \underbrace{\frac{f_d\left(f_d + f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}}_{\iota_d} f_d\epsilon_{d,t} + \underbrace{\frac{f_d\left(f_d + f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}}_{\iota_{\bar{y}}} f_{\bar{y}}\epsilon_{\bar{y},t}
\end{aligned}$$

Then, under Assumptions 2 and 5, it's clear that

$$\frac{di^{surp}_t}{d\epsilon_{d,t}} = \iota_d > 0 > \iota_{\bar{y}} = \frac{di^{surp}_t}{d\epsilon_{\bar{y},t}}$$

From here, impulse responses for $\tilde{y}_t$ and π_t can be obtained from the equilibrium given above and belief formation which gives

$$\frac{dd_{t|t}}{d\epsilon_{d,t}} = f_d K_{d,t}, \quad \frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} = f_{\bar{y}} K_{d,t}, \quad \frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}} = f_d K_{\bar{y},t}, \quad \frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} = f_{\bar{y}} K_{\bar{y},t}$$

$$\text{where } K_{d,t} = \frac{f_d \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2} \quad \text{and} \quad K_{\bar{y},t} \frac{f_{\bar{y}}}{f_d^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}$$

Putting this all together gives the following relative responses to the exogenous shocks

$$\begin{aligned}
\frac{d\tilde{y}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} &= \frac{1}{\iota_{\bar{y}}} \left[\frac{\partial \tilde{y}_t}{\partial \bar{y}_t} + \frac{\partial \tilde{y}_t}{\partial \bar{y}_{t|t}} \frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \frac{\partial \tilde{y}_t}{\partial d_{t|t}} \frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} \right] \\
&= -\frac{1}{\sigma} \frac{\Omega_{\bar{y}} \left(1 - \beta\rho_{\bar{y}}\right) \left(f_{\bar{y}} + f_{\bar{y},b}\right) f_{\bar{y}} + \Omega_d \left[\left(f_d + f_{d,b}\right) \left(1 - \beta\rho_d\right) - \kappa\rho_d \right] f_d \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d \left(f_d + f_{d,b}\right) \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}} \left(f_{\bar{y}} + f_{\bar{y},b}\right)} \\
\frac{d\pi_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} &= \frac{1}{\iota_{\bar{y}}} \left[\frac{\partial \pi_t}{\partial \bar{y}_t} + \frac{\partial \pi_t}{\partial \bar{y}_{t|t}} \frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \frac{\partial \pi_t}{\partial d_{t|t}} \frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} \right] \\
&= -\frac{\kappa}{\sigma} \frac{\Omega_{\bar{y}} \left(f_{\bar{y}} + f_{\bar{y},b}\right) f_{\bar{y}} + \Omega_d \left[f_d + f_{d,b} - \sigma\beta\rho_d \left(1 - \rho_d\right) - \kappa\rho_d \right] f_d \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d \left(f_d + f_{d,b}\right) \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}} \left(f_{\bar{y}} + f_{\bar{y},b}\right)}
\end{aligned}$$

$$\begin{aligned}
\frac{d\tilde{y}_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} &= \frac{1}{\iota_d} \left[\frac{\partial \tilde{y}_t}{\partial d_t} + \frac{\partial \tilde{y}_t}{\partial \bar{y}_{t|t}} \frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}} + \frac{\partial \tilde{y}_t}{\partial d_{t|t}} \frac{dd_{t|t}}{d\epsilon_{d,t}} \right] \\
&= \frac{1}{\sigma} \frac{1}{f_d} \frac{-\Omega_{\bar{y}} \left(1 - \beta\rho_{\bar{y}}\right) \left(f_{\bar{y}} + f_{\bar{y},b}\right) f_{\bar{y}} f_d + \sigma f_{\bar{y}}^2 - \Omega_d \left(1 - \beta\rho_d\right) \left(f_d + f_{d,b} - \sigma \left(1 - \rho_d\right)\right) f_d^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d \left(f_d + f_{d,b}\right) \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}} \left(f_{\bar{y}} + f_{\bar{y},b}\right)} \\
\frac{d\pi_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} &= \frac{1}{\iota_d} \left[\frac{\partial \pi_t}{\partial d_t} + \frac{\partial \pi_t}{\partial \bar{y}_{t|t}} \frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}} + \frac{\partial \pi_t}{\partial d_{t|t}} \frac{dd_{t|t}}{d\epsilon_{d,t}} \right] \\
&= \frac{\kappa}{\sigma} \frac{1}{f_d} \frac{-\Omega_{\bar{y}} \left(f_{\bar{y}} + f_{\bar{y},b}\right) f_{\bar{y}} f_d + \sigma f_{\bar{y}}^2 - \Omega_d \left(f_d + f_{d,b} - \sigma \left(1 - \rho_d\right)\right) f_d^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d \left(f_d + f_{d,b}\right) \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}} \left(f_{\bar{y}} + f_{\bar{y},b}\right)}
\end{aligned}$$

Assumption 1 gives $\Omega_d, \Omega_{\bar{y}} > 0$ as discussed in the previous section. For the relative responses to $\epsilon_{\bar{y},t}$, Assumption 2 ensures that the sign is opposite of the sign of the numerators. For the numerators, the same assumption ensures that the first term is positive while the second terms are negative as long as Assumption 6 holds since

$$\begin{gathered}
\left(f_d + f_{d,b}\right) \left(1 - \beta\rho_d\right) - \kappa\rho_d < 0 \quad \text{and} \quad f_d + f_{d,b} - \sigma\beta\rho_d \left(1 - \rho_d\right) - \kappa\rho_d < 0 \\
\Leftrightarrow f_d + f_{d,b} < \min \left\{ \frac{\kappa\rho_d}{1 - \beta\rho_d}, \rho_d \left(\sigma\beta \left(1 - \rho_d\right) + \kappa\right) \right\} = \frac{\kappa\rho_d}{1 - \beta\rho_d}
\end{gathered}$$

where the last equality comes from the fact that $\Omega_d > 0$. Meanwhile, this same fact gives

$$\frac{\kappa\rho_d}{\left(1 - \rho_d\right)\left(1 - \beta\rho_d\right)} - \sigma\rho_d < \frac{\kappa\rho_d}{1 - \beta\rho_d} \quad \text{and} \quad \frac{\kappa\rho_d}{\left(1 - \rho_d\right)\left(1 - \beta\rho_d\right)} \leq \sigma$$

Thus, Assumption 6 is sufficient to guarantee that these second terms in the numerators of $\frac{d\tilde{y}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ and $\frac{d\pi_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ are negative while the last fact shows that this assumption places a tighter condition than the one in Assumption 5. Then, it's clear that $\frac{d\tilde{y}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ and $\frac{d\pi_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ can be positive if the second terms in the numerator are large (i.e., when $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ is large). For the relative responses to $\epsilon_{d,t}$, the first terms are negative while the last 2 terms are positive under Assumption 5. Then, it's clear that they can be positive if the last two terms in the numerator are large (i.e., when $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ is large).

The scaled covariance between an outcome x_t and the interest rate surprise is given by

$$\frac{Cov_{t-1}\left(x_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = \frac{\frac{dx_t}{d\epsilon_{d,t}}\iota_d\sigma^2_{d,t-1} + \frac{dx_t}{d\epsilon_{\bar{y},t}}\iota_{\bar{y}}\sigma^2_{\bar{y},t-1}}{\iota_d^2\sigma^2_{d,t-1} + \iota_{\bar{y}}^2\sigma^2_{\bar{y},t-1}} = \frac{dx_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}}\frac{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2} + \frac{dx_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}\frac{f_{\bar{y}}^2}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}$$

so that

$$\frac{Cov_{t-1}\left(\pi_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = -\frac{\kappa}{\sigma}\frac{\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)f_d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d\left(f_d + f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)}$$

$$\frac{Cov_{t-1}\left(\tilde{y}_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = -\frac{1}{\sigma}\frac{\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)f_d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d\left(f_d + f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)}$$

Then, Assumptions 2 and 5 are sufficient to show that

$$\frac{d\frac{Cov_{t-1}\left(\pi_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}}{d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}} = \frac{\kappa}{\sigma}\frac{\Omega_{\bar{y}}\left(f_d + f_{d,b}\right) - \Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)}{\left[f_d\left(f_d + f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\right]^2}f_d f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) > 0$$

$$\frac{d\frac{Cov_{t-1}\left(\tilde{y}_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}}{d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}} = \frac{1}{\sigma}\frac{\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_d + f_{d,b}\right) - \Omega_d\left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)}{\left[f_d\left(f_d + f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\right]^2}f_d f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) > 0$$

These 2 assumptions are also sufficient to ensure that these scaled covariances are positive for large enough $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.

The responses of forecasts of horizons $h \geq 1$ and the real interest rate gap can be signed in a similar manner.

$$\begin{aligned}
\frac{d\tilde{y}_{t+h|t}}{d\epsilon_{\bar{y},t}} &= \frac{\partial\tilde{y}_{t+h|t}}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \frac{\partial\tilde{y}_{t+h|t}}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} \\
&= -\frac{1}{\sigma}f_{\bar{y}}\frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\rho_d^h\left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)f_d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}
\end{aligned}$$

$$\frac{d\pi_{t+h|t}}{d\epsilon_{\bar{y},t}} = \frac{\partial\pi_{t+h|t}}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \frac{\partial\pi_{t+h|t}}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} = -\frac{\kappa}{\sigma}f_y\frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h\left(f_{\bar{y}} + f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\rho_d^h\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)f_d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}$$

$$\frac{dx_{t+h|t}}{d\epsilon_{d,t}} = \frac{f_d}{f_{\bar{y}}}\frac{dx_{t+h|t}}{d\epsilon_{\bar{y},t}} \text{ for } x_{t+h|t} \in \left\{\tilde{y}_{t+h|t}, \pi_{t+h|t}\right\}$$

$$\frac{d\tilde{r}_t}{d\epsilon_{d,t}} = \frac{di_t}{d\epsilon_{d,t}} - \frac{d\pi_{t+1|t}}{d\epsilon_{d,t}} - \sigma\frac{dd_t}{d\epsilon_{d,t}} + \sigma\rho_d\frac{dd_{t|t}}{d\epsilon_{d,t}}$$

$$= \frac{\Omega_{\bar{y}}\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}}+f_{\bar{y},b}\right)f_{\bar{y}}f_d - \sigma f_{\bar{y}}^2 + \Omega_d\left(1-\rho_d\right)\left(1-\beta\rho_d\right)\left(f_d+f_{d,b}-\sigma\left(1-\rho_d\right)\right)f_d^2\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}}{f_d^2\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}+f_{\bar{y}}^2}$$

$$\frac{d\tilde{r}_t}{d\epsilon_{\bar{y},t}} = \frac{di_t}{d\epsilon_{\bar{y},t}} - \frac{d\pi_{t+1|t}}{d\epsilon_{\bar{y},t}} - \sigma\frac{dd_t}{d\epsilon_{\bar{y},t}} + \sigma\rho_d\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}}$$

$$= \frac{\Omega_{\bar{y}}\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}}+f_{\bar{y},b}\right)f_{\bar{y}}^2 + \left[\sigma+\Omega_d\left(1-\rho_d\right)\left(1-\beta\rho_d\right)\left(f_d+f_{d,b}-\sigma\left(1-\rho_d\right)\right)\right]f_y f_d\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}}{f_d^2\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}+f_{\bar{y}}^2}$$

Since the responses of forecasts under the individual shocks are proportional to each other, the scaled covariance between forecasts and the interest rate surprise can be found by looking just at the relative response to the output target shock

$$\frac{Cov_{t-1}\left(x_{t+h|t}, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = \frac{dx_{t+h|t}/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}}\frac{f_d^2\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}}{f_d^2\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}+f_{\bar{y}}^2} + \frac{dx_{t+h|t}/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}\frac{f_{\bar{y}}^2}{f_d^2\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}+f_{\bar{y}}^2} = \frac{dx_{t+h|t}/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$$

so that

$$\frac{Cov_{t-1}\left(\pi_{t+h|t}, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = -\frac{\kappa}{\sigma}\frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h\left(f_{\bar{y}}+f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\rho_d^h\left(f_d+f_{d,b}-\sigma\left(1-\rho_d\right)\right)f_d\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}}{f_d\left(f_d+f_{d,b}\right)\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}+f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)}$$

$$\frac{Cov_{t-1}\left(\tilde{y}_{t+h|t}, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = -\frac{1}{\sigma}\frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}}+f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\rho_d^h\left(1-\beta\rho_d\right)\left(f_d+f_{d,b}-\sigma\left(1-\rho_d\right)\right)f_d\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}}{f_d\left(f_d+f_{d,b}\right)\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}+f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)}$$

Assumptions 2 and 5 are again sufficient to ensure that these scaled covariances are positive for large enough $\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}$ and that

$$\frac{d\frac{Cov_{t-1}\left(\pi_{t+h|t}, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}}{d\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}} = \frac{\kappa}{\sigma}\frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h\left(f_d+f_{d,b}\right) - \Omega_d\rho_d^h\left(f_d+f_{d,b}-\sigma\left(1-\rho_d\right)\right)}{\left[f_d\left(f_d+f_{d,b}\right)\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}+f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)\right]^2}f_d f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right) > 0$$

$$\frac{d\frac{Cov_{t-1}\left(\tilde{y}_{t+h|t}, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}}{d\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}} = \frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h\left(1-\beta\rho_{\bar{y}}\right)\left(f_d+f_{d,b}\right) - \Omega_d\rho_d^h\left(1-\beta\rho_d\right)\left(f_d+f_{d,b}-\sigma\left(1-\rho_d\right)\right)}{\sigma\left[f_d\left(f_d+f_{d,b}\right)\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}+f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)\right]^2}f_d f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right) > 0$$

Looking back at the equilibrium solution, it's clear that setting $f_d = \sigma$ and $f_{d,b} = -\sigma\rho_d$ results in the coefficients on $d_{t|t}$ and d_t being zero. Using these parameter values in the responses immediately gives the properties presented in Section 3.2.

D.2 Proposition 2

Here, I repeat the equations summarizing the policymaker's problem described in Section 4

$$\min_{i_t,\tilde{y}_t,\pi_t} E^{CB}_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left((\tilde{y}_t-\bar{y}_t)^2+\frac{\varepsilon}{\kappa}\pi_t^2\right)$$

$$\text{subject to } \tilde{y}_t=\tilde{y}_{t+1|t}-\frac{1}{\sigma}\left(i_t-\pi_{t+1|t}\right)+d_t-d_{t+1|t}\ \text{ and }\ \pi_t=\beta\pi_{t+1|t}+\kappa\tilde{y}_t$$

where

$$\begin{bmatrix}\tilde{y}_{t+1|t}\\ \pi_{t+1|t}\end{bmatrix}=\mathbf{M}\begin{bmatrix}\rho_d & 0\\ 0 & \rho_{\bar{y}}\end{bmatrix}\begin{bmatrix}d_{t|t}\\ \bar{y}_{t|t}\end{bmatrix}$$

$$d_{t|t}=\frac{f_{\bar{y}}+f_{\bar{y},b}}{1+K_{\bar{y}}f_{\bar{y},b}+K_df_{d,b}}\left(K_{\bar{y}}\rho_d d_{t-1}-K_d\rho_{\bar{y}}\bar{y}_{t-1}\right)+\frac{K_d}{1+K_{\bar{y}}f_{\bar{y},b}+K_df_{d,b}}i_t$$

$$\bar{y}_{t|t}=\frac{f_d+f_{d,b}}{1+K_{\bar{y}}f_{\bar{y},b}+K_df_{d,b}}\left(K_d\rho_{\bar{y}}\bar{y}_{t-1}-K_{\bar{y}}\rho_d d_{t-1}\right)+\frac{K_{\bar{y}}}{1+K_{\bar{y}}f_{\bar{y},b}+K_df_{d,b}}i_t$$

with $\{\mathbf{M},K_d,K_{\bar{y}},f_d,f_{d,b},f_{\bar{y}},f_{\bar{y},b}\}$ taken as given. Though the coefficients in $\{K_d,K_{\bar{y}},f_d,f_{d,b},f_{\bar{y}},f_{\bar{y},b}\}$ must be consistent with the resulting interest rate behavior in equilibrium, they appear in the above equations as agents' beliefs regarding current policy which the policymaker takes as given.

Then, I can write the output gap deviation and inflation in matrix form as the following function of current beliefs and i_t

$$\begin{aligned}\begin{bmatrix}\tilde{y}_t-\bar{y}_t\\ \pi_t\end{bmatrix}&=\begin{bmatrix}1 & \frac{1}{\sigma}\\ \kappa & \frac{\kappa}{\sigma}+\beta\end{bmatrix}\mathbf{M}\begin{bmatrix}\rho_d & 0\\ 0 & \rho_{\bar{y}}\end{bmatrix}\begin{bmatrix}d_{t|t}\\ \bar{y}_{t|t}\end{bmatrix}-\begin{bmatrix}\rho_d\\ \kappa\rho_d\end{bmatrix}d_{t|t}\\ &+\begin{bmatrix}1 & -1\\ \kappa & 0\end{bmatrix}\begin{bmatrix}d_t\\ \bar{y}_t\end{bmatrix}-\begin{bmatrix}\frac{1}{\sigma}\\ \frac{\kappa}{\sigma}\end{bmatrix}i_t\end{aligned}\tag{30}$$

By plugging in beliefs, this can be transformed into the following function of exogenous states and i_t

$$\begin{bmatrix}\tilde{y}_t-\bar{y}_t\\ \pi_t\end{bmatrix}=\frac{\mathbf{\Psi}\begin{bmatrix}K_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right) & -K_d\left(f_{\bar{y}}+f_{\bar{y},b}\right)\\ -K_{\bar{y}}\left(f_d+f_{d,b}\right) & K_d\left(f_d+f_{d,b}\right)\end{bmatrix}}{1+K_{\bar{y}}f_{\bar{y},b}+K_df_{d,b}}\begin{bmatrix}\rho_d d_{t-1}\\ \rho_{\bar{y}}\bar{y}_{t-1}\end{bmatrix}+\begin{bmatrix}1 & -1\\ \kappa & 0\end{bmatrix}\begin{bmatrix}d_t\\ \bar{y}_t\end{bmatrix}+\begin{bmatrix}H_{\tilde{y},i}\\ H_{\pi,i}\end{bmatrix}i_t\tag{31}$$

$$\text{where } \mathbf{\Psi}\equiv\begin{bmatrix}1 & \frac{1}{\sigma}\\ \kappa & \frac{\kappa}{\sigma}+\beta\end{bmatrix}\mathbf{M}\begin{bmatrix}\rho_d & 0\\ 0 & \rho_{\bar{y}}\end{bmatrix}-\begin{bmatrix}\rho_d & 0\\ \kappa\rho_d & 0\end{bmatrix}$$

$$\text{and } \begin{bmatrix}H_{\tilde{y},i}\\ H_{\pi,i}\end{bmatrix}\equiv\frac{\mathbf{\Psi}\begin{bmatrix}K_d\\ K_{\bar{y}}\end{bmatrix}}{1+K_{\bar{y}}f_{\bar{y},b}+K_df_{d,b}}-\begin{bmatrix}\frac{1}{\sigma}\\ \frac{\kappa}{\sigma}\end{bmatrix}=\begin{bmatrix}\frac{\partial\tilde{y}_t}{\partial i_t}+\frac{\partial\tilde{y}_t}{\partial d_{t|t}}\frac{dd_{t|t}}{di_t}+\frac{\partial\tilde{y}_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{di_t}\\ \frac{\partial\pi_t}{\partial i_t}+\frac{\partial\pi_t}{\partial d_{t|t}}\frac{dd_{t|t}}{di_t}+\frac{\partial\pi_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{di_t}\end{bmatrix}$$

In this form, it's clear that the discretionary policymaker has no control over time $t+1$ or later outcomes and the problem and accompanying optimality condition are:

$$\min_{i_t}\frac{1}{2}\left((\tilde{y}_t-\bar{y}_t)^2+\frac{\varepsilon}{\kappa}\pi_t^2\right)\quad\text{subject to (31)}$$

$$\Rightarrow\quad(\tilde{y}_t-\bar{y}_t)H_{\tilde{y},i}+\frac{\varepsilon}{\kappa}\pi_tH_{\pi,i}=0\Rightarrow\tilde{y}_t-\bar{y}_t=-\mathcal{R}\frac{\varepsilon}{\kappa}\pi_t$$

matching the form given in the proposition with $\mathcal{R} = \frac{H_{\pi,i}}{H_{\bar{y},i}} = \frac{\frac{\partial \pi_t}{\partial i_t} + \frac{\partial \pi_t}{\partial d_{t|t}} \frac{d d_{t|t}}{d i_t} + \frac{\partial \pi_t}{\partial \bar{y}_{t|t}} \frac{d \bar{y}_{t|t}}{d i_t}}{\frac{\partial \tilde{y}_t}{\partial i_t} + \frac{\partial \tilde{y}_t}{\partial d_{t|t}} \frac{d d_{t|t}}{d i_t} + \frac{\partial \tilde{y}_t}{\partial \bar{y}_{t|t}} \frac{d \bar{y}_{t|t}}{d i_t}}$.

Solving for $\tilde{y}_t$ using this optimality condition and substituting this into the inflation condition gives

$$\pi_t = \beta \pi_{t+1|t} - \mathcal{R}\varepsilon \pi_t + \kappa \bar{y}_t$$

By restricting attention to nonnegative values of $\mathcal{R}$, I can iterate this forward while using the fact that $\bar{y}_{t+h|t} = \rho_{\bar{y}}^h \bar{y}_{t|t}$ to get the stable solution for the path of π_t in terms of $\left\{\bar{y}_t, \bar{y}_{t|t}\right\}$. Substituting that expression for π_t back into the optimality condition gives the solution for $\tilde{y}_t$ in terms of the same state variables

$$\begin{aligned}
\pi_t &= \frac{\kappa}{1+\mathcal{R}\varepsilon} \bar{y}_t + \frac{\beta \rho_{\bar{y}} \kappa}{\left(1-\beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)} \bar{y}_{t|t} \\
\tilde{y}_t &= \frac{1}{1+\mathcal{R}\varepsilon} \bar{y}_t - \frac{\mathcal{R}\varepsilon \beta \rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)} \bar{y}_{t|t}
\end{aligned}$$

Then, this gives expressions for expectations $\tilde{y}_{t+1|t}$ and $\pi_{t+1|t}$ which immediately reveals the equilibrium value of $\mathbf{M}$ as a function of $\mathcal{R}$

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1-\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon} \\ 0 & \frac{\kappa}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon} \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

These can be used along with (1) to back out the implied nominal interest rate in terms of $\left\{d_t, d_{t|t}, \bar{y}_t, \bar{y}_{t|t}\right\}$

$$\begin{aligned}
i_t &= \sigma\left(d_t - d_{t+1|t}\right) + \pi_{t+1|t} + \sigma\left(\tilde{y}_{t+1|t} - \tilde{y}_t\right) \\
&= \underbrace{\sigma d_t - \sigma \rho_d d_{t|t}}_{r_t^n} \underbrace{- \sigma \frac{1}{1+\mathcal{R}\varepsilon}}_{f_{\bar{y}}^*(\mathcal{R})} \bar{y}_t + \underbrace{\sigma\left(\frac{1}{1+\mathcal{R}\varepsilon} - \frac{1}{\Omega_{\bar{y}}} \frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon}\right)}_{f_{\bar{y},b}^*(\mathcal{R})} \bar{y}_{t|t}
\end{aligned} \tag{32}$$

Substituting these optimal response coefficients along with $\mathbf{M}$ into the equilibrium condition for $\mathcal{R}$ and rearranging gives

$$\mathcal{R} = \kappa \frac{\frac{\beta\rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)} K_{\bar{y}} - \frac{1}{\sigma}}{\frac{-\beta\rho_{\bar{y}}\mathcal{R}\varepsilon}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)} K_{\bar{y}} - \frac{1}{\sigma}} \quad \text{where } K_{\bar{y}} = -\frac{1}{\sigma} \frac{1+\mathcal{R}\varepsilon}{\left(1+\mathcal{R}\varepsilon\right)^2 \frac{\sigma_d^2}{\sigma_{\bar{y}}^2} + 1} \tag{33}$$

Here, it's clear that when $\beta\rho_{\bar{y}} = 0$, the terms involving $K_{\bar{y}}$ drop out of this expression and it gives $\mathcal{R} = \kappa$.

To focus on equilibrium values for $\mathcal{R}$ which give finite policy response coefficients, I impose $1+\mathcal{R}\varepsilon \neq 0$ and $1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon \neq 0$ which allows me to write (33) as this third-order polynomial

$$\begin{aligned}
0 &= \mathcal{R}\left(1-\beta\rho_{\bar{y}}\right) - \kappa + \left(\mathcal{R}-\kappa\right)\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \\
&= \varepsilon^2 \frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \mathcal{R}^3 + \varepsilon\left(2-\beta\rho_{\bar{y}}-\varepsilon\kappa\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}\mathcal{R}^2 + \left[\left(1-\beta\rho_{\bar{y}}\right)\left(1+\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}\left(1-\varepsilon\kappa\right)\right) - \varepsilon\kappa\right]\mathcal{R} - \kappa\left(1+\left(1-\beta\rho_{\bar{y}}\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}\right)
\end{aligned} \tag{34}$$

For $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} > 0$, $\varepsilon^2 \frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \geq 0$ while $-\kappa\left(1+\left(1-\beta\rho_{\bar{y}}\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}\right) < 0$ so that there must be at least one positive root for any values of the other parameters according to Descartes' rule of signs.

Again, attention is limited to real nonnegative solutions for $\mathcal{R}$. To see that $\mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{y}}}\right]$, note that (34) says that $\mathcal{R}$ must satisfy

$$\mathcal{R}\left(1-\beta\rho_{\bar{y}}\right)-\kappa=(\kappa-\mathcal{R})\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)(1+\mathcal{R}\varepsilon)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$$

$\mathcal{R} \in [0,\kappa)$ violates this condition since the LHS would be negative while the RHS is positive. $\mathcal{R} > \frac{\kappa}{1-\beta\rho_{\bar{y}}}$ would give a positive LHS and negative RHS.

Implicitly differentiating (34) gives

$$\frac{d\mathcal{R}}{d\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}}=-\frac{(\mathcal{R}-\kappa)\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)(1+\mathcal{R}\varepsilon)}{1-\beta\rho_{\bar{y}}+\left[(\mathcal{R}-\kappa)\left[\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)+(1+\mathcal{R}\varepsilon)\right]\varepsilon+\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)(1+\mathcal{R}\varepsilon)\right]\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}}<0$$

Now, I look at the cases given by the limits of $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$.

- When $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to \infty$: In this case, referring back to (33), it's clear that $K_{\bar{y}} \to 0$ and $\mathcal{R} = \kappa$ is the unique solution in this limit. To see that this is the solution of the perfect information case, note that the policymaker's problem in that setting is

$$\min_{i_t}\frac{1}{2}\left(\left(\tilde{y}_t-\bar{y}_t\right)^2+\frac{\varepsilon}{\kappa}\pi_t^2\right)$$

  subject to (31) but with $d_{t|t} = d_t$ and $\bar{y}_{t|t} = \bar{y}_t$. Then, it's clear that the optimality condition is the same as the one given in the proposition with $\mathcal{R} = \kappa$.

- When $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to 0$: (33) shows that

$$\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}} \quad \text{since } K_{\bar{y}} \to -\frac{1+\mathcal{R}\varepsilon}{\sigma}$$

  Now, I show that this is equivalent to the case of a commitment to a rule of the form

$$i_t = r_t^n + f_{\bar{y}}^c\bar{y}_t + f_{\bar{y},b}^c\bar{y}_{t|t}$$

  First, I substitute these coefficients into the solution under a given rule derived earlier in the Appendix and given in (29))

$$\begin{bmatrix}\tilde{y}_t-\bar{y}_t\\ \pi_t\end{bmatrix}=\begin{bmatrix}-\frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}}^c+f_{\bar{y},b}^c\right)+\frac{1}{\sigma}f_{\bar{y}}^c\\ -\frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}}^c+f_{\bar{y},b}^c\right)+\frac{\kappa}{\sigma}f_{\bar{y}}^c\end{bmatrix}\bar{y}_{t|t}+\begin{bmatrix}-\frac{1}{\sigma}f_{\bar{y}}^c-1\\ -\frac{\kappa}{\sigma}f_{\bar{y}}^c\end{bmatrix}\bar{y}_t$$

  where equilibrium beliefs in this limit are given by

$$\bar{y}_{t|t}=\bar{y}_t+\frac{\sigma}{f_{\bar{y}}^c}\epsilon_{d,t}$$

Then, the policymaker who can commit to this rule solves

$$\min_{f^c_{\bar{y}}, f^c_{\bar{y},b}} E^{CB}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left((\tilde{y}_t - \bar{y}_t)^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right)$$

$$\text{where} \begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sigma}\Omega_{\bar{y}} \left(1 - \beta\rho_{\bar{y}}\right) \left(f^c_{\bar{y}} + f^c_{\bar{y},b}\right) - 1 \\ -\frac{\kappa}{\sigma}\Omega_{\bar{y}} \left(f^c_{\bar{y}} + f^c_{\bar{y},b}\right) \end{bmatrix} \bar{y}_t + \begin{bmatrix} -\Omega_{\bar{y}} \left(1 - \beta\rho_{\bar{y}}\right) \left(1 + \frac{f^c_{\bar{y},b}}{f^c_{\bar{y}}}\right) + 1 \\ -\kappa\Omega_{\bar{y}} \left(1 + \frac{f^c_{\bar{y},b}}{f^c_{\bar{y}}}\right) + \kappa \end{bmatrix} \epsilon_{d,t}$$

Then, the two optimality conditions are given by

$$0 = \frac{\partial}{\partial f^c_{\bar{y}}} E^{CB}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left((\tilde{y}_t - \bar{y}_t)^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right)$$

$$\Rightarrow 0 = E^{CB}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left((\tilde{y}_t - \bar{y}_t) \left(1 - \beta\rho_{\bar{y}}\right) + \varepsilon\pi_t \right) \left[-\frac{1}{\sigma}\bar{y}_t + \frac{f^c_{\bar{y},b}}{\left(f^c_{\bar{y}}\right)^2} \epsilon_{d,t} \right]$$

$$0 = \frac{\partial}{\partial f^c_{\bar{y},b}} E^{CB}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left((\tilde{y}_t - \bar{y}_t)^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right)$$

$$\Rightarrow 0 = E^{CB}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left((\tilde{y}_t - \bar{y}_t) \left(1 - \beta\rho_{\bar{y}}\right) + \varepsilon\pi_t \right) \left[-\frac{1}{\sigma}\bar{y}_t - \frac{1}{f^c_{\bar{y}}} \epsilon_{d,t} \right]$$

Both conditions are satisfied by a policy that maintains

$$\tilde{y}_t - \bar{y}_t = -\frac{\varepsilon}{1 - \beta\rho_{\bar{y}}} \pi_t \quad \forall t$$

which is equivalent to the optimality condition of the discretionary policy with $\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}}$ in this limit.

Lastly I show that the same discretionary optimal policy condition is obtained if I start with agents who suppose that current policy responds linearly to the entire history of shocks $\left\{d^t, \bar{y}^t\right\}$[21]. That is, I replace the supposed behavior of current policy in (14) with

$$i_t = \sum_{k=0}^{\infty} f^{hist}_d (k)\, d_{t-k} + \sum_{k=0}^{\infty} f^{hist}_{\bar{y}} (k)\, \bar{y}_{t-k} \tag{35}$$

Then, beliefs are given by a static Gaussian signal extraction problem where

$$\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} = \begin{bmatrix} \rho_d d_{t-1} \\ \rho_{\bar{y}} \bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} K^{hist}_d \\ K^{hist}_{\bar{y}} \end{bmatrix} \left[i_t - E\left[i_t | \mathcal{I}_t \setminus i_t\right] \right] \tag{36}$$

where $E\left[i_t|\mathcal{I}_t \setminus i_t\right] = \left[f^{hist}_d (0)\, \rho_d + f^{hist}_d (1)\right] d_{t-1} + \left[f^{hist}_{\bar{y}} (0)\, \rho_{\bar{y}} + f^{hist}_{\bar{y}} (1)\right] \bar{y}_{t-1} + \sum_{k=2}^{\infty} \left[f^{hist}_d (k)\, d_{t-k} + f^{hist}_{\bar{y}} (k)\, \bar{y}_{t-k}\right]$

$$\text{and } K^{hist}_d = \frac{f^{hist}_d (0)\, \sigma_d^2}{\left(f^{hist}_d (0)\right)^2 \sigma_d^2 + \left(f^{hist}_{\bar{y}} (0)\right)^2 \sigma_{\bar{y}}^2}, \quad K^{hist}_{\bar{y}} = \frac{f^{hist}_{\bar{y}} (0)\, \sigma_{\bar{y}}^2}{\left(f^{hist}_d (0)\right)^2 \sigma_d^2 + \left(f^{hist}_{\bar{y}} (0)\right)^2 \sigma_{\bar{y}}^2}$$

[21] In equilibrium, a rule that also includes current and lagged private agent beliefs can be written in this form since private agent beliefs would be a function of lagged and current state variables.

To proceed, I now conjecture that the equilibrium solution for the endogenous outcomes $\tilde{y}_t$ and π_t are linear in the full history of shocks, thus resulting in expectations of the form

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M}^{hist} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} + \sum_{k=1}^{\infty} \mathbf{M}_d^{hist}(k)\, d_{t-k} + \sum_{k=1}^{\infty} \mathbf{M}_{\bar{y}}^{hist}(k)\, \bar{y}_{t-k}$$

Again, this allows me to write the output gap deviation and inflation as

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \sum_{k=0}^{\infty} H_d^{hist}(k)\, d_{t-k} + \sum_{k=0}^{\infty} H_{\bar{y}}^{hist}(k)\, \bar{y}_{t-k} + \begin{bmatrix} H_{\tilde{y},i}^{hist} \\ H_{\pi,i}^{hist} \end{bmatrix} i_t \tag{37}$$

$$\text{where } \begin{bmatrix} H_{\tilde{y},i}^{hist} \\ H_{\pi,i}^{hist} \end{bmatrix} \equiv \left(\begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \mathbf{M}^{hist} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} - \begin{bmatrix} \rho_d & 0 \\ \kappa\rho_d & 0 \end{bmatrix} \right) \begin{bmatrix} K_d^{hist} \\ K_{\bar{y}}^{hist} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix}$$

and $\left\{ H_d^{hist}(k), H_{\bar{y}}^{hist}(k) \right\}_{k=0}^{\infty}$ are functions of $\mathbf{M}^{hist}, K_d^{hist}, K_{\bar{y}}^{hist}, \left\{ f_d^{hist}(k), f_{\bar{y}}^{hist}(k), \mathbf{M}_d^{hist}(k), \mathbf{M}_{\bar{y}}^{hist}(k) \right\}_{k=0}^{\infty}$

Then, the discretionary policy problem and accompanying optimality condition are

$$\min_{i_t} \frac{1}{2}\left((\tilde{y}_t - \bar{y}_t)^2 + \frac{\varepsilon}{\kappa}\pi_t^2 \right) \quad \text{subject to (37)}$$

$$\tilde{y}_t - \bar{y}_t = -\mathcal{R}^{hist}\frac{\varepsilon}{\kappa}\pi_t \quad \text{where } \mathcal{R}^{hist} = \frac{H_{\pi,i}^{hist}}{H_{\tilde{y},i}^{hist}}$$

This is equivalent to the solution above as long as the equilibrium condition for $\mathcal{R}^{hist}$ is the same. The rest of this section proves this.

Using the equilibrium conditions gives the following expression for expectations

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1-\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon} \\ 0 & \frac{\kappa}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon} \end{bmatrix}}_{\mathbf{M}^{hist}} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

and an interest rate that responds only to current true states and beliefs

$$i_t^* = \sigma d_t - \sigma\rho_d d_{t|t} - \sigma\frac{1}{1+\mathcal{R}^{hist}\varepsilon}\bar{y}_t + \sigma\left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon} \right)\bar{y}_{t|t}$$

Combining (35) and (36) shows that equilibrium beliefs are a function only of time t and $t-1$ fundamentals

$$\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} = \begin{bmatrix} \rho_d d_{t-1} \\ \rho_{\bar{y}}\bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} K_d^{hist} \\ K_{\bar{y}}^{hist} \end{bmatrix} \left[f_d^{hist}(0)\left(d_t - \rho_d d_{t-1}\right) + f_{\bar{y}}^{hist}(0)\left(\bar{y}_t - \rho_{\bar{y}}\bar{y}_{t-1}\right) \right]$$

Then, comparing (35) to the optimal interest rate proves that $f_d^{hist}(k) = f_{\bar{y}}^{hist}(k) = 0$ for $k \geq 2$. Using these equilibrium beliefs in the expression for i_t^* allows me to obtain the remaining coefficients $\left\{f_d^{hist}(0), f_d^{hist}(1), f_{\bar{y}}^{hist}(0), f_{\bar{y}}^{hist}(1)\right\}$

$$\begin{aligned}
i_t^* &= \sigma d_t - \sigma\rho_d\left[\rho_d d_{t-1} + K_d^{hist}\left[f_d^{hist}(0)\left(d_t - \rho_d d_{t-1}\right) + f_{\bar{y}}^{hist}(0)\left(\bar{y}_t - \rho_{\bar{y}}\bar{y}_{t-1}\right)\right]\right] - \sigma\frac{1}{1+\mathcal{R}^{hist}\varepsilon}\bar{y}_t \\
&+ \sigma\left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)\left[\rho_{\bar{y}}\bar{y}_{t-1} + K_{\bar{y}}^{hist}\left[f_d^{hist}(0)\left(d_t - \rho_d d_{t-1}\right) + f_{\bar{y}}^{hist}(0)\left(\bar{y}_t - \rho_{\bar{y}}\bar{y}_{t-1}\right)\right]\right] \\
&= \sigma\left(1 - \rho_d K_d^{hist} f_d^{hist}(0) + \left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist} f_d^{hist}(0)\right)d_t \\
&- \sigma\left[\rho_d\left(1 - K_d^{hist} f_d^{hist}(0)\right) - \left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist} f_d^{hist}(0)\right]\rho_d d_{t-1} \\
&- \sigma\left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} + \rho_d K_d^{hist} f_{\bar{y}}^{hist}(0) - \left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist} f_{\bar{y}}^{hist}(0)\right)\bar{y}_t \\
&+ \sigma\left[\rho_d K_d^{hist} f_{\bar{y}}^{hist}(0) + \left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)\left[1 - K_{\bar{y}}^{hist} f_{\bar{y}}^{hist}(0)\right]\right]\rho_{\bar{y}}\bar{y}_{t-1}
\end{aligned}$$

which gives

$$f_d^{hist}(0) = \frac{\sigma}{1 + \sigma\rho_d K_d^{hist} - \sigma\left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist}} \quad \text{and} \quad f_{\bar{y}}^{hist}(0) = -\frac{f_d^{hist}(0)}{1+\mathcal{R}^{hist}\varepsilon}$$

Substituting this into the expression for $K_{\bar{y}}^{hist}$ gives $\rho_d K_d^{hist}$ as a function of $K_{\bar{y}}^{hist}$.

$$\begin{aligned}
K_{\bar{y}}^{hist} &= -\frac{1}{\sigma}\frac{\left(1+\mathcal{R}^{hist}\varepsilon\right)}{\left(1+\mathcal{R}^{hist}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} + 1}\left[1 + \sigma\rho_d K_d^{hist} - \sigma\left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist}\right] \\
\Rightarrow \rho_d K_d^{hist} &= -\left(\frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon} + \left(1+\mathcal{R}^{hist}\varepsilon\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}\right)K_{\bar{y}}^{hist} - \frac{1}{\sigma}
\end{aligned}$$

Then, using the expression for $\mathcal{R}^{hist}$ and the equilibrium expression for $\mathbf{M}^{hist}$ gives

$$\mathcal{R}^{hist} = \kappa\frac{\frac{\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}}+\frac{\kappa}{\sigma}+\beta\right)}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}K_{\bar{y}}^{hist} - \rho_d K_d^{hist} - \frac{1}{\sigma}}{\frac{\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}}+\frac{\kappa}{\sigma}\right)}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}K_{\bar{y}}^{hist} - \rho_d K_d^{hist} - \frac{1}{\sigma}} = \kappa\frac{\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon} + \left(1+\mathcal{R}^{hist}\varepsilon\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}}{\frac{1-\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon} + \left(1+\mathcal{R}^{hist}\varepsilon\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}}$$

where I again restrict attention to finite interest rate coefficients by looking only for solutions where $1+\mathcal{R}^{hist}\varepsilon \neq 0$ and $1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon \neq 0$. Rearranging this gives

$$0 = \mathcal{R}^{hist}\left(1-\beta\rho_{\bar{y}}\right) - \kappa + \left(\mathcal{R}^{hist} - \kappa\right)\left(1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon\right)\left(1+\mathcal{R}^{hist}\varepsilon\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$$

which indeed matches equilibrium condition (34) derived above for $\mathcal{R}$ thus showing that the equilibrium is the same when I generalize private agents' belief about current policy to the form in (35).

D.2.1 Corollary 1

The proof above of Proposition 2 gave the forms of $f_{\bar{y}}^{*}(\mathcal{R})$ and $f_{\bar{y},b}^{*}(\mathcal{R})$ in (32). There, it was also shown that the perfect information discretionary policy optimality condition is

$$\tilde{y}_t^{PI} - \bar{y}_t = -\varepsilon \pi_t^{PI}$$

Again, using this condition along with the NKPC in (2) gives

$$\pi_t^{PI} = \frac{\kappa}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\bar{y}_t \text{ and } \tilde{y}_t^{PI} = \frac{1-\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\bar{y}_t$$

Then, this gives expressions for expectations

$$\pi_{t+1|t}^{PI} = \frac{\kappa\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\bar{y}_t \text{ and } \tilde{y}_{t+1|t}^{PI} = \frac{\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\bar{y}_t$$

which can again be used along with (1) to back out the implied optimal nominal interest rate in terms of $\{d_t, \bar{y}_t\}$

$$i_t^{*,PI} = \underbrace{\sigma\left(1-\rho_d\right)d_t}_{r_t^n} - \sigma\frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\bar{y}_t = r_t^n + \left(f_{\bar{y}}^{*}(\kappa) + f_{\bar{y},b}^{*}(\kappa)\right)\bar{y}_t$$

Returning to the imperfect information case, I next show how the interest rate behavior can be altered to ensure determinacy so that the equilibrium in equations (18) and (19) is the unique path in this model. To do this, I add a term to the interest rate that reacts to deviations of π_t from its intended equilibrium path

$$\begin{aligned} i_t^{*} &= r_t^n + f_{\bar{y}}^{*}(\mathcal{R})\bar{y}_t + f_{\bar{y},b}^{*}(\mathcal{R})\bar{y}_{t|t} + \phi_\pi\left(\pi_t - \pi_t^{*}\right) \\ &= r_t^n + \left(f_{\bar{y}}^{*}(\mathcal{R}) - \phi_\pi\Gamma_{\bar{y}}\right)\bar{y}_t + \left(f_{\bar{y},b}^{*}(\mathcal{R}) - \phi_\pi\Gamma_{\bar{y},b}\right)\bar{y}_{t|t} + \phi_\pi\pi_t \\ \text{where } \pi_t^{*} &= \underbrace{\frac{\kappa}{1+\mathcal{R}\varepsilon}}_{\Gamma_{\bar{y}}}\bar{y}_t + \underbrace{\frac{\beta\rho_{\bar{y}}\kappa}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}}_{\Gamma_{\bar{y},b}}\bar{y}_{t|t} \text{ is the intended equilibrium} \end{aligned}$$

Clearly, along the intended stationary equilibrium path, $\pi_t = \pi_t^{*}$ so that the response of i_t^{*} to state variables is the same as without this extra term. What this term does change are the dynamics of $[\tilde{y}_t \;\; \pi_t]'$ since the system of equilibrium conditions now becomes

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{1+\phi_\pi\frac{\kappa}{\sigma}} & \frac{1-\beta\phi_\pi}{\sigma+\phi_\pi\kappa} \\ \frac{\kappa}{1+\phi_\pi\frac{\kappa}{\sigma}} & \frac{\frac{\kappa}{\sigma}+\beta}{1+\phi_\pi\frac{\kappa}{\sigma}} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma+\phi_\pi\kappa} \\ \frac{\kappa}{\sigma+\phi_\pi\kappa} \end{bmatrix} \left(\left(f_{\bar{y}}^{*}(\mathcal{R}) - \phi_\pi\Gamma_{\bar{y}}\right)\bar{y}_t + \left(f_{\bar{y},b}^{*}(\mathcal{R}) - \phi_\pi\Gamma_{\bar{y},b}\right)\bar{y}_{t|t}\right)$$

Then, determinacy of $[\tilde{y}_t \;\; \pi_t]'$ is guaranteed by the largest eigenvalue of $\mathbf{A}$ being less than one

$$\max\left\{eig\left(\mathbf{A}\right)\right\} = \frac{\frac{1+\beta+\frac{\kappa}{\sigma}}{1+\phi_\pi\frac{\kappa}{\sigma}} \pm \sqrt{\left(\frac{1+\beta+\frac{\kappa}{\sigma}}{1+\phi_\pi\frac{\kappa}{\sigma}}\right)^2 - 4\frac{\beta}{1+\phi_\pi\frac{\kappa}{\sigma}}}}{2} < 1 \Leftrightarrow \phi_\pi > 1$$

D.3 Proposition 3

Here, the equilibrium conditions in matrix form are

$$\begin{bmatrix} \tilde{y}_t^{CB} \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix} \begin{bmatrix} \tilde{y}_{t+1|t}^{CB} \\ \pi_{t+1|t} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} i_t + \begin{bmatrix} \boldsymbol{\Xi}_{\tilde{y}} \\ \boldsymbol{\Xi}_{\pi} \end{bmatrix} \mathbf{z}_t + \begin{bmatrix} 1 \\ \kappa \end{bmatrix} \boldsymbol{\Xi}_{\tilde{y},b} \mathbf{z}_{2,t|t} \tag{38}$$

where the shocks are given by

$$\begin{bmatrix} \mathbf{z}_{1,t} \\ \mathbf{z}_{2,t} \end{bmatrix} = \begin{bmatrix} \Upsilon_{11} & 0 \\ \Upsilon_{21} & \Upsilon_{22} \end{bmatrix} \begin{bmatrix} \mathbf{z}_{1,t-1} \\ \mathbf{z}_{2,t-1} \end{bmatrix} + \mathbf{e}_t,\ \mathbf{e}_t \sim \text{iid } N\left(0, \boldsymbol{\Sigma}\right)$$

with $\boldsymbol{\Sigma}$ diagonal and the eigenvalues of $\boldsymbol{\Upsilon}$ being less than one in absolute value.

In the perfect information case, $\mathbf{z}_{t|t} = \mathbf{z}_t$ and the discretionary policy problem is

$$\min_{i_t} \frac{1}{2}\left(\left(\tilde{y}_t^{CB}\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right) \quad \text{subject to (38) where } \tilde{y}_{t+1|t}^{CB} \text{ and } \pi_{t+1|t} \text{ are taken as given}$$
$$\Rightarrow \tilde{y}_t^{CB} = -\varepsilon \pi_t$$

Private agents suppose that the interest rate i_t is

$$i_t = \mathbf{F}_1 \mathbf{z}_{1,t} + \mathbf{F}_2 \mathbf{z}_{2,t} + \mathbf{F}_{2,b}\mathbf{z}_{2,t|t}$$

while their information set is $\left\{i^t, \mathbf{z}_1^t, \mathbf{z}_2^{t-1}\right\}$. The same process described in Section 2.3 shows that beliefs are the following function of i_t and exogenous lagged variables

$$\begin{aligned} \mathbf{z}_{2,t|t} &= \boldsymbol{\Upsilon}_{\text{row } 2}\mathbf{z}_{t-1} + \mathbf{K}_z\left(i_t - \mathbf{F}_1\mathbf{z}_{1,t} - \mathbf{F}_2 \begin{bmatrix} \Upsilon_{21} & \Upsilon_{22} \end{bmatrix} \mathbf{z}_{t-1} - \mathbf{F}_{2,b}\mathbf{z}_{2,t|t}\right) \\ &= \left(\mathbf{I} + \mathbf{K}_z\mathbf{F}_{2,b}\right)^{-1}\left(\mathbf{I} - \mathbf{K}_z\mathbf{F}_2\right) \begin{bmatrix} \Upsilon_{21} & \Upsilon_{22} \end{bmatrix} \mathbf{z}_{t-1} + \left(\mathbf{I} + \mathbf{K}_z\mathbf{F}_{2,b}\right)^{-1}\mathbf{K}_z\left(i_t - \mathbf{F}_1\mathbf{z}_{1,t}\right) \end{aligned}$$

Then, conjecturing a linear solution for $\tilde{y}_t^{CB}$ and π_t again leads to a linear conjecture for expectations

$$\begin{bmatrix} \tilde{y}_{t+1|t}^{CB} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M}_1\mathbf{z}_{1,t+1|t} + \mathbf{M}_2\mathbf{z}_{2,t+1|t} = \left(\mathbf{M}_1\Upsilon_{11} + \mathbf{M}_2\Upsilon_{21}\right)\mathbf{z}_{1,t} + \mathbf{M}_2\Upsilon_{22}\mathbf{z}_{2,t|t}$$

The current outcomes can then be written in terms of exogenous states and i_t

$$\begin{bmatrix} \tilde{y}_t^{CB} \\ \pi_t \end{bmatrix} = \left(\begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix}\left(\mathbf{M}_1\Upsilon_{11} + \mathbf{M}_2\Upsilon_{21}\right) - \boldsymbol{\Psi}\left(\mathbf{I} + \mathbf{K}_z\mathbf{F}_{2,b}\right)^{-1}\mathbf{K}_z\mathbf{F}_1\right)\mathbf{z}_{1,t} + \begin{bmatrix} \boldsymbol{\Xi}_{\tilde{y}} \\ \boldsymbol{\Xi}_{\pi} \end{bmatrix}\mathbf{z}_t \tag{39a}$$

$$+ \boldsymbol{\Psi}\left(\mathbf{I} + \mathbf{K}_z\mathbf{F}_{2,b}\right)^{-1}\left(\mathbf{I} - \mathbf{K}_z\mathbf{F}_2\right) \begin{bmatrix} \Upsilon_{21} & \Upsilon_{22} \end{bmatrix} \mathbf{z}_{t-1} + \begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} i_t \tag{39b}$$

where $\boldsymbol{\Psi} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix}\mathbf{M}_2\Upsilon_{22} + \begin{bmatrix} 1 \\ \kappa \end{bmatrix}\boldsymbol{\Xi}_{\tilde{y},b}$ and $\begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} \equiv \boldsymbol{\Psi}\left(\mathbf{I} + \mathbf{K_z}\mathbf{F_{2,b}}\right)^{\mathbf{-1}}\mathbf{K}_z - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix}$

Then, the discretionary policy problem and resulting optimality condition are

$$\min_{i_t} \frac{1}{2}\left(\left(\tilde{y}_t^{CB}\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right) \text{ subject to (39)}$$
$$\Rightarrow \tilde{y}_t^{CB} = -\frac{H_{\pi,i}}{H_{\tilde{y},i}}\frac{\varepsilon}{\kappa}\pi_t$$

I again limit attention to equilibrium solutions where $\frac{H_{\pi,i}}{H_{\tilde{y},i}} \geq 0$. Then, substituting this into the inflation equation and solving forward for π_t gives

$$\pi_t = \beta\pi_{t+1|t} - \frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon\pi_t + \boldsymbol{\Xi}_{\pi,1}\mathbf{z}_{1,t} = \frac{\boldsymbol{\Xi}_{\pi,1}}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon}\left[\mathbf{I} - \frac{\beta}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon}\Upsilon_{11}\right]^{-1}\mathbf{z}_{1,t}$$

Then, the optimality condition gives

$$\tilde{y}_t^{CB} = -\frac{H_{\pi,i}}{H_{\tilde{y},i}}\frac{\varepsilon}{\kappa}\frac{\boldsymbol{\Xi}_{\pi,1}}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon}\left[\mathbf{I} - \frac{\beta}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon}\Upsilon_{11}\right]^{-1}\mathbf{z}_{1,t}$$

This shows that fluctuations in the welfare-relevant outcomes $\tilde{y}_t^{CB}$ and π_t are only caused by $\mathbf{z}_{1,t}$ and changes in $\mathbf{z}_{2,t}$ and $\mathbf{z}_{2,t|t}$ do not affect these outcomes in equilibrium and so

$$\frac{d\tilde{y}_t^{CB}}{d\mathbf{z}_{2,t}} = \frac{d\pi_t}{d\mathbf{z}_{2,t}} = \frac{d\tilde{y}_t^{CB}}{d\mathbf{z}_{2,t|t}} = \frac{d\pi_t}{d\mathbf{z}_{2,t|t}} = 0$$

These expressions also reveal that $\mathbf{M}_2 = 0$ and give the equilibrium expression for $\mathbf{M}_1$.

$$\begin{bmatrix} \tilde{y}_{t+1|t}^{CB} \\ \pi_{t+1|t} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{H_{\pi,i}}{H_{\tilde{y},i}}\frac{\varepsilon}{\kappa} \\ 1 \end{bmatrix}\frac{\boldsymbol{\Xi}_{\pi,1}}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon}\left[\mathbf{I} - \frac{\beta}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon}\Upsilon_{11}\right]^{-1}}_{\mathbf{M}_1}\Upsilon_{11}\mathbf{z}_{1,t}$$

Then, the discretionary policy optimality condition is equivalent to the perfect information case since

$$\begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} = \begin{bmatrix} 1 \\ \kappa \end{bmatrix}\left(\boldsymbol{\Xi}_{\tilde{y},b}(\mathbf{I} + \mathbf{K_z F_{2,b}})^{\mathbf{-1}}\,\mathbf{K}_z - \frac{1}{\sigma}\right) \Rightarrow \frac{H_{\pi,i}}{H_{\tilde{y},i}} = \kappa$$

D.4 Proposition 4

I repeat the equilibrium conditions here for convenience

$$\tilde{y}_t = \tilde{y}_{t+1|t} - \frac{1}{\sigma}\left(i_t - \pi_{t+1|t}\right) + d_t - d_{t+1|t}$$
$$\pi_t = \beta\pi_{t+1|t} + \kappa\tilde{y}_t$$

The optimal discretionary interest rate policy under perfect information implements $\tilde{y}_t^{PI} - \bar{y}_t = -\varepsilon\pi_t^{PI}$ which yields the solution

$$\begin{bmatrix} \tilde{y}_t^{PI} - \bar{y}_t \\ \pi_t^{PI} \end{bmatrix} = \begin{bmatrix} -\varepsilon\kappa \\ \kappa \end{bmatrix}\frac{1}{1-\beta\rho_{\bar{y}} + \varepsilon\kappa}\bar{y}_t$$

The optimal discretionary interest rate policy under imperfect information implements $\tilde{y}_t - \bar{y}_t = -\mathcal{R}\frac{\varepsilon}{\kappa}\pi_t$ which yields the following solution (as shown in the proof of Proposition 2)

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} -\mathcal{R}\varepsilon \\ \kappa \end{bmatrix} \frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon}\left(\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\bar{y}_{t|t}-\bar{y}_t\right)+\bar{y}_t\right)$$

The equilibrium belief error is

$$\bar{y}_{t|t}-\bar{y}_t = \left(K_{\bar{y}}f^*_{\bar{y}}\left(\mathcal{R}\right)-1\right)\epsilon_{\bar{y},t}+K_{\bar{y}}\sigma\epsilon_{d,t} = -\frac{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1}\epsilon_{\bar{y},t}-\frac{1+\mathcal{R}\varepsilon}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1}\epsilon_{d,t}$$

which gives

$$\begin{aligned} E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)^2\right] &= \frac{\left(1+\mathcal{R}\varepsilon\right)^2\sigma_d^2}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1} \quad \text{for } s>t \\ E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)\bar{y}_s\right] &= -\frac{\left(1+\mathcal{R}\varepsilon\right)^2\sigma_d^2}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1} \quad \text{for } s>t \end{aligned}$$

Thus, in equilibrium

$$\begin{aligned} l_t^{PI} &\equiv \frac{1}{2}\left[\left(\tilde{y}_t^{PI}-\bar{y}_t\right)^2+\frac{\varepsilon}{\kappa}\left(\pi_t^{PI}\right)^2\right] = \frac{1}{2}\frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\bar{y}_t^2 \\ l_t &\equiv \frac{1}{2}\left[\left(\tilde{y}_t-\bar{y}_t\right)^2+\frac{\varepsilon}{\kappa}\pi_t^2\right] = \frac{1}{2}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\left(\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\bar{y}_{t|t}-\bar{y}_t\right)+\bar{y}_t\right)^2 \end{aligned}$$

$$\begin{aligned} E_t^{CB}\mathcal{L}_{t+1}^{PI} &\equiv E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\frac{1}{2}\left(\left(\tilde{y}_s^{PI}-\bar{y}_s\right)^2+\frac{\varepsilon}{\kappa}\left(\pi_s^{PI}\right)^2\right) = \frac{1}{2}\frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{y}_s^2\right] \\ E_t^{CB}\mathcal{L}_{t+1} &\equiv E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\frac{1}{2}\left(\left(\tilde{y}_s-\bar{y}_s\right)^2+\frac{\varepsilon}{\kappa}\pi_s^2\right) \\ &= \frac{1}{2}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\left(\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\bar{y}_{s|s}-\bar{y}_s\right)+\bar{y}_s\right)^2\right] \\ &= \frac{1}{2}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\left\{\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{y}_s^2\right]-\frac{1}{1-\beta}\frac{2\left(1+\mathcal{R}\varepsilon\right)-\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\frac{\left(1+\mathcal{R}\varepsilon\right)^2\sigma_d^2}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1}\right\} \end{aligned}$$

The difference in the expected future welfare loss is then

$$\begin{aligned} E_t^{CB}\left[\mathcal{L}_{t+1}-\mathcal{L}_{t+1}^{PI}\right] &= \frac{1}{2}\left(\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}-\frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\right)\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{y}_s^2\right] \\ &\quad -\frac{1}{2}\frac{\beta\rho_{\bar{y}}}{1-\beta}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\frac{\left[2\left(1+\mathcal{R}\varepsilon\right)-\beta\rho_{\bar{y}}\right]\sigma_d^2}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1} \end{aligned}$$

To see that the first term is negative, note that Proposition 2 showed that $\mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{y}}}\right]$. Then,

$$\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2} = \frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2} \text{ for } \mathcal{R}=\kappa$$

$$\text{and } \frac{d}{d\mathcal{R}}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2} = 2\varepsilon^2\frac{\left(1-\beta\rho_{\bar{y}}\right)\mathcal{R}-\kappa}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^3} \leq 0 \quad \text{for } \mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{y}}}\right]$$

$$\Rightarrow \frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2} \leq \frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2} \quad \text{for } \mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{y}}}\right]$$

The second term is clearly negative since $2\left(1+\mathcal{R}\varepsilon\right)-\beta\rho_{\bar{y}} \geq 1+2\mathcal{R}\varepsilon \geq 0$.

The difference in the current period loss is

$$\begin{aligned} l_t - l_t^{PI} &= \frac{1}{2}\left(\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2} - \frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\right)\bar{y}_t^2 \\ &+ \frac{1}{2}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\bar{y}_{t|t}-\bar{y}_t\right)^2 + 2\left(\bar{y}_{t|t}-\bar{y}_t\right)\bar{y}_t\right) \end{aligned}$$

Again, the first term is negative, but the second term may be positive and larger than the first term.

D.4.1 Corollary 2

If I exogenously impose that $\bar{y}_{s|s} = \bar{y}_s$, then this is equivalent to setting

$$E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)^2\right] = E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)\bar{y}_s\right] = 0$$

which gives

$$\begin{aligned} E_t^{CB}\left[\mathcal{L}_{t+1}-\mathcal{L}_{t+1}^{PI}\right] &= \frac{1}{2}\left(\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2} - \frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\right)\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{y}_s^2\right] \\ &\leq 0 \quad \text{if } \mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{y}}}\right] \end{aligned}$$

If I exogenously impose $\mathcal{R} = \kappa$, then the difference in the expected future welfare loss is then

$$\begin{aligned} E_t^{CB}\left[\mathcal{L}_{t+1}-\mathcal{L}_{t+1}^{PI}\right] &= \frac{1}{2}\frac{\varepsilon\kappa\beta\rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\left\{\frac{\beta\rho_{\bar{y}}}{1+\varepsilon\kappa}E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)^2\right] + 2E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)\bar{y}_s\right]\right\} \\ &= \frac{1}{2}\frac{\varepsilon\kappa\beta\rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\frac{\beta\rho_{\bar{y}}}{1+\varepsilon\kappa}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\left(E_t^{CB}\left[\bar{y}_{s|s}^2\right] - E_t^{CB}\left[\bar{y}_s^2\right]\right) \\ &+ \frac{\varepsilon\kappa\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\frac{1}{1+\varepsilon\kappa}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\left(E_t^{CB}\left[\bar{y}_{s|s}\bar{y}_s\right] - E_t^{CB}\left[\bar{y}_s^2\right]\right) \end{aligned}$$

This is clearly weakly negative if

$$E_t^{CB}\left[\bar{y}_{s|s}^2\right] \leq E_t^{CB}\left[\bar{y}_s^2\right] \quad \text{and} \quad E_t^{CB}\left[\bar{y}_{s|s}\bar{y}_s\right] \leq E_t^{CB}\left[\bar{y}_s^2\right] \quad \text{for } s > t$$

Note that this is equivalent to

$$Var_t^{CB}\left(y_{s|s}\right) \leq Var_t^{CB}\left(y_s\right) \text{ and } Cov_t^{CB}\left(\bar{y}_{s|s}, \bar{y}_s\right) \leq Var_t^{CB}\left(y_s\right)$$

since $E_t^{CB}\bar{y}_{s|s} = E_t^{CB}\bar{y}_s$ for $s > t$ so that

$$\begin{aligned}
Cov_t^{CB}\left(\bar{y}_{s|s}, \bar{y}_s\right) &= E_t^{CB}\left[\bar{y}_{s|s}\bar{y}_s\right] - \left(E_{t_0}^{CB}\bar{y}_s\right)^2 \\
Var_t^{CB}\left(y_{s|s}\right) &= E_t^{CB}\left[\bar{y}_{s|s}^2\right] - \left(E_t^{CB}\bar{y}_s\right)^2 \\
Var_t^{CB}\left(y_s\right) &= E_t^{CB}\left[\bar{y}_s^2\right] - \left(E_t^{CB}\bar{y}_s\right)^2
\end{aligned}$$

Then, another set of equivalent conditions is

$$Var_t^{CB}\left(y_{s|s}\right) \leq Var_t^{CB}\left(y_s\right) \text{ and } Corr_t^{CB}\left(\bar{y}_{s|s}, \bar{y}_s\right) = \frac{Cov_t^{CB}\left(\bar{y}_{s|s}, \bar{y}_s\right)}{\sqrt{Var_t^{CB}\left(y_s\right)Var_t^{CB}\left(y_{s|s}\right)}} \leq 1$$

since this gives

$$Cov_t^{CB}\left(\bar{y}_{s|s}, \bar{y}_s\right) \leq \sqrt{Var_t^{CB}\left(y_s\right)Var_t^{CB}\left(y_{s|s}\right)} \leq Var_t^{CB}\left(y_s\right)$$

D.5 Proposition 5

Here, I consider the case where the central bank directly communicates d_t to private agents prior to observing i_t. Then, agents infer $\bar{y}_t$ upon observing i_t. In equilibrium, since agents know beliefs will be correct with $d_{t|t} = d_t$ and $\bar{y}_{t|t} = \bar{y}_t$. However, a key feature of this setup is that the interest rate retains its signaling effect on $\bar{y}_{t|t}$ since from the policymaker's point of view, beliefs are the following function of i_t.

$$\bar{y}_{t|t} = \frac{1}{f_{\bar{y}} + f_{\bar{y},b}}\left(i_t - \left(f_d - f_{d,b}\right)d_t\right)$$

Thus, the policymaker's choice has a marginal impact of $\frac{d\bar{y}_{t|t}}{di_t} = \frac{1}{f_{\bar{y}}+f_{\bar{y},b}}$ on beliefs.

Denoting this case with superscript ^{d}, (33) shows that the inflation-output tradeoff is at its steepest possible value

$$\mathcal{R}^d = \frac{\kappa}{1-\beta\rho_{\bar{y}}}$$

with the following equilibrium outcomes under the optimal discretionary interest rate policy after taking into account that beliefs are correct in equilibrium

$$\pi_t^d = \frac{\kappa\left(1-\beta\rho_{\bar{y}}\right)}{\left(1-\beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa}\bar{y}_t \text{ and } \tilde{y}_t^d - \bar{y}_t = -\frac{\varepsilon\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa}\bar{y}_t$$

Then, the associated welfare loss terms are

$$l_t^d \equiv \left(\tilde{y}_t^d - \bar{y}_t\right)^2 + \frac{\varepsilon}{\kappa}\left(\pi_t^d\right)^2 = \frac{\varepsilon\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa}\bar{y}_t^2$$

$$E_t^{CB}\mathcal{L}_{t+1}^d \equiv E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\frac{1}{2}\left(\left(\tilde{y}_s^d - \bar{y}_s\right)^2 + \frac{\varepsilon}{\kappa}\left(\pi_s^d\right)^2\right) = \frac{1}{2}\frac{\varepsilon\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{y}_s^2\right]$$

Compared to the case of full communication, communicating only d_t is strictly preferable for any realizations of the current shocks.

$$l_t^d - l_t^{PI} = \frac{\varepsilon\kappa}{2}\left(\frac{1}{\left(1-\beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} - \frac{1+\varepsilon\kappa}{\left(1-\beta\rho_{\bar{y}} + \varepsilon\kappa\right)^2}\right)\bar{y}_t^2 \leq 0$$

$$E_t^{CB}\left(\mathcal{L}_{t+1}^d - \mathcal{L}_{t+1}^{PI}\right) = \frac{\varepsilon\kappa}{2}\left(\frac{1}{\left(1-\beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} - \frac{1+\varepsilon\kappa}{\left(1-\beta\rho_{\bar{y}} + \varepsilon\kappa\right)^2}\right)\frac{\rho_{\bar{y}}^2\bar{y}_t^2 + \frac{1}{1-\beta}\sigma_{\bar{y}}^2}{1-\beta\rho_{\bar{y}}^2} \leq 0$$

Both the current period welfare loss and expected future loss are lower in the case of communicating only d_t since

$$\beta\rho_{\bar{y}} \geq 0 \text{ and } \varepsilon\kappa \geq 0 \Rightarrow \frac{1}{\left(1-\beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} \leq \frac{1+\varepsilon\kappa}{\left(1-\beta\rho_{\bar{y}} + \varepsilon\kappa\right)^2}$$

On the other hand, when the case of communicating only d_t is compared to the no additional communication case, neither case produces unambiguously lower losses for either current period or expected future welfare.

$$\begin{aligned} l_t^d - l_t = {} & \frac{\varepsilon}{2}\left(\frac{\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} - \frac{\mathcal{R}^2\varepsilon + \kappa}{\left(1-\beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)^2}\right)\bar{y}_t^2 \\ & - \frac{\varepsilon\left(\mathcal{R}^2\varepsilon + \kappa\right)}{\left(1-\beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)^2}\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}_{\bar{\pi}}\varepsilon}\left(\frac{1}{2}\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\bar{y}_{t|t} - \bar{y}_t\right)^2 + \left(\bar{y}_{t|t} - \bar{y}_t\right)\bar{y}_t\right) \end{aligned}$$

$$\begin{aligned} E_t^{CB}\left(\mathcal{L}_{t+1}^d - \mathcal{L}_{t+1}\right) = {} & \frac{\varepsilon}{2}\left(\frac{\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} - \frac{\mathcal{R}^2\varepsilon + \kappa}{\left(1-\beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)^2}\right)\frac{\rho_{\bar{y}}^2\bar{y}_t^2 + \frac{1}{1-\beta}\sigma_{\bar{y}}^2}{1-\beta\rho_{\bar{y}}^2} \\ & + \frac{\varepsilon}{2}\frac{1}{1-\beta}\frac{\mathcal{R}^2\varepsilon + \kappa}{\left(1-\beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)^2}\frac{\left(2\left(1+\mathcal{R}\varepsilon\right) - \beta\rho_{\bar{y}}\right)\beta\rho_{\bar{y}}\sigma_d^2\sigma_{\bar{y}}^2}{\left(1+\mathcal{R}\varepsilon\right)^2\sigma_d^2 + \sigma_{\bar{y}}^2} \end{aligned}$$

The first term in each of these expressions is negative and reflects the benefit of maximizing the interest rate's effect on inflation expectations, thereby achieving the largest possible reduction in the stabilization bias through the signaling channel. To see that it's always negative, note the following

$$\begin{aligned} & \frac{\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} = \frac{\mathcal{R}^2\varepsilon + \kappa}{\left(1-\beta\rho_{\bar{y}} + \mathcal{R}_{\bar{\pi}}\varepsilon\right)^2} \quad \text{for } \mathcal{R} = \frac{\kappa}{1-\beta\rho_{\bar{y}}} \\ \text{while } & \frac{d}{d\mathcal{R}}\frac{\mathcal{R}^2\varepsilon + \kappa}{\left(1-\beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)^2} = -2\varepsilon\frac{\kappa - \left(1-\beta\rho_{\bar{y}}\right)\mathcal{R}}{\left(1-\beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)^3} \leq 0 \quad \text{for } \mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{y}}}\right] \\ \text{so that } & \frac{\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} \leq \frac{\mathcal{R}^2\varepsilon + \kappa}{\left(1-\beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)^2} \quad \text{for } \mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{y}}}\right] \end{aligned}$$

The second term in $E_t^{CB}\left(\mathcal{L}_{t+1}^d - \mathcal{L}_{t+1}\right)$ is positive since $2\left(1+\mathcal{R}\varepsilon\right) - \beta\rho_{\bar{y}} \geq 1 + 2\mathcal{R}\varepsilon \geq 0$. This reflects the loss of the benefit of decoupling the comovement in agents' beliefs about the output target and its true value. Thus, whether this type of partial communication is beneficial for expected future welfare losses is ambiguous for general parameter values. Meanwhile, the second term in $l_t^d - l_t$ can always be positive for large enough negative realizations of $\left(\bar{y}_{t|t} - \bar{y}_t\right)\bar{y}_t$ so this difference stays ambiguous even for a fixed set of parameter values.

The following can be shown for special parameterizations:

- As $\sigma_d^2 \to 0$ while $\sigma_{\bar{y}}^2$ stays positive, $\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}}$. As the demand shock becomes more negligible, so does the effect of communicating its true value. Even without any additional communication, the interest rate's signaling effect on inflation expectations is already high so the further reduction in the stabilization bias from communicating d_t disappears. Furthermore, as $\sigma_d^2 \to 0$, private agents' forecast errors regarding the output target become negligible and their beliefs $\bar{y}_{t|t}$ approach the true $\bar{y}_t$ so the benefit of reducing their comovement by not directly communicating also disappears.

$$\lim_{\sigma_d^2 \to 0} E_t^{CB}\mathcal{L}_{t+1} \to E_t^{CB}\mathcal{L}_{t+1}^d$$
$$\lim_{\sigma_d^2 \to 0} l_t \to l_t^d \quad \text{if } \epsilon_{d,t} = 0$$

  Here, the benefit of not communicating the true value of $\bar{y}_t$ remains so that

$$\lim_{\sigma_{\bar{y}}^2 \to 0} E_t^{CB}\mathcal{L}_{t+1} < E_t^{CB}\mathcal{L}_{t+1}^{PI} \quad \text{and} \quad \lim_{\sigma_d^2 \to 0} l_t < l_t^{PI}$$

- As $\sigma_{\bar{y}}^2 \to 0$ while σ_d^2 stays positive, $\mathcal{R} \to \kappa$. In this case, the inflation-output tradeoff disappears entirely and the economy approaches one in which the flexible price equilibrium is always efficient and is achievable regardless of the information setting.

$$\lim_{\sigma_{\bar{y}}^2 \to 0} E_t^{CB}\mathcal{L}_{t+1} \to E_t^{CB}\mathcal{L}_{t+1}^d = E_t^{CB}\mathcal{L}_{t+1}^{PI} \quad \text{if } \bar{y}_t = 0$$
$$\lim_{\sigma_{\bar{y}}^2 \to 0} l_t \to l_t^d = l_t^{PI} \quad \text{if } \epsilon_{\bar{y},t} = \bar{y}_t = 0$$

- If $\beta\rho_{\bar{y}} = 0$, then the inflation-output tradeoff is no longer affected by private agents' beliefs since inflation is driven purely by current marginal costs. Then, the information setting again becomes irrelevant.

$$E_t^{CB}\mathcal{L}_{t+1} = E_t^{CB}\mathcal{L}_{t+1}^d = E_t^{CB}\mathcal{L}_{t+1}^{PI} \quad \text{if } \beta\rho_{\bar{y}} = 0$$
$$l_t = l_t^d = l_t^{PI} \quad \text{if } \beta\rho_{\bar{y}} = 0$$

D.6 Proposition 6

I repeat the equilibrium conditions here for convenience

$$\tilde{y}_t = \tilde{y}_{t+1|t} - \frac{1}{\sigma}\left(i_t - \pi_{t+1|t}\right) + d_t - d_{t+1|t}$$
$$\pi_t = \beta\pi_{t+1|t} + \kappa\tilde{y}_t$$

The optimal discretionary interest rate policy under perfect information implements $\tilde{y}_t^{PI} = -\varepsilon\left(\pi_t^{PI} - \bar{\pi}_t\right)$ which yields the solution

$$\begin{bmatrix} \tilde{y}_t^{PI} \\ \pi_t^{PI} - \bar{\pi}_t \end{bmatrix} = \begin{bmatrix} \varepsilon \\ -1 \end{bmatrix} \frac{1-\beta\rho_{\bar{\pi}}}{1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa}\bar{\pi}_t$$

The optimal discretionary interest rate policy under imperfect information implements $\tilde{y}_t = -\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}\left(\pi_t - \bar{\pi}_t\right)$ which yields the following solution

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t - \bar{\pi}_t \end{bmatrix} = \begin{bmatrix} -\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa} \\ 1 \end{bmatrix} \frac{1}{1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon}\left(\frac{\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}_{\bar{\pi}}\varepsilon}\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right) - \left(1-\beta\rho_{\bar{\pi}}\right)\bar{\pi}_t\right)$$

with equilibrium interest rate behavior given by

$$i_t = \underbrace{\sigma d_t - \sigma\rho_d d_{t|t}}_{r_t^n} \underbrace{-\sigma\frac{\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}}{1+\mathcal{R}_{\bar{\pi}}\varepsilon}}_{f_{\bar{\pi}}^*(\mathcal{R}_{\bar{\pi}})}\bar{\pi}_t + \underbrace{\sigma\left(\frac{1}{1+\mathcal{R}_{\bar{\pi}}\varepsilon} - \frac{1}{\Omega_{\bar{\pi}}}\frac{1}{1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon}\right)\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}}_{f_{\bar{\pi},b}^*(\mathcal{R}_{\bar{\pi}})}\bar{\pi}_{t|t}$$

$$\text{where } \Omega_{\bar{\pi}} \equiv \frac{1}{\left(1-\rho_{\bar{\pi}}\right)\left(1-\beta\rho_{\bar{\pi}}\right) - \frac{\kappa}{\sigma}\rho_{\bar{\pi}}}$$

Following steps from the proof of Proposition 2 yields the following equilibrium condition for $\mathcal{R}_{\bar{\pi}}$

$$\begin{aligned} \mathcal{R}_{\bar{\pi}} &= \frac{\frac{\partial\pi_t}{\partial i_t} + \frac{\partial\pi_t}{\partial d_{t|t}}\frac{dd_{t|t}}{di_t} + \frac{\partial\pi_t}{\partial\bar{\pi}_{t|t}}\frac{d\bar{\pi}_{t|t}}{di_t}}{\frac{\partial\tilde{y}_t}{\partial i_t} + \frac{\partial\tilde{y}_t}{\partial d_{t|t}}\frac{dd_{t|t}}{di_t} + \frac{\partial\tilde{y}_t}{\partial\bar{\pi}_{t|t}}\frac{d\bar{\pi}_{t|t}}{di_t}} = \frac{\frac{\partial\pi_t}{\partial i_t} + \frac{\partial\pi_t}{\partial\bar{\pi}_{t|t}}\frac{d\bar{\pi}_{t|t}}{di_t}}{\frac{\partial\tilde{y}_t}{\partial i_t} + \frac{\partial\tilde{y}_t}{\partial\bar{\pi}_{t|t}}\frac{d\bar{\pi}_{t|t}}{di_t}} \\ &= \kappa\frac{\frac{\beta\rho_{\bar{\pi}}}{(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon)(1+\mathcal{R}_{\bar{\pi}}\varepsilon)}\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}K_{\bar{\pi}} - \frac{1}{\sigma}}{\frac{-\beta\rho_{\bar{\pi}}\mathcal{R}_{\bar{\pi}}\varepsilon}{(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon)(1+\mathcal{R}_{\bar{\pi}}\varepsilon)}\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}K_{\bar{\pi}} - \frac{1}{\sigma}} \quad \text{where } K_{\bar{\pi}} = -\frac{1}{\sigma}\frac{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}}{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} + \left(\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}\right)^2} \end{aligned} \tag{40}$$

The same limiting cases hold as in the baseline setup

$$\begin{aligned} \frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} &= 0 \Rightarrow K_{\bar{\pi}} = -\frac{1}{\sigma}\frac{1+\mathcal{R}_{\bar{\pi}}\varepsilon}{\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}} \Rightarrow \mathcal{R}_{\bar{\pi}} = \frac{\kappa}{1-\beta\rho_{\bar{\pi}}} \\ \frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} &\to \infty \Rightarrow K_{\bar{\pi}} \to 0 \Rightarrow \mathcal{R}_{\bar{\pi}} \to \kappa \end{aligned}$$

Rearranging (40) shows that $\mathcal{R}_{\bar{\pi}}$ must satisfy

$$\mathcal{R}_{\bar{\pi}}\left(1-\beta\rho_{\bar{\pi}}\right) - \kappa = \left(\kappa - \mathcal{R}_{\bar{\pi}}\right)\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)\frac{1+\mathcal{R}_{\bar{\pi}}\varepsilon}{\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}}\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2}$$

Then, limiting attention to solutions where $\mathcal{R}_{\bar{\pi}} \geq 0$ shows that $\mathcal{R}_{\bar{\pi}} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{\pi}}}\right]$ since $\mathcal{R}_{\bar{\pi}} \in [0,\kappa)$ produces a negative LHS and positive RHS while $\mathcal{R}_{\bar{\pi}} > \frac{\kappa}{1-\beta\rho_{\bar{\pi}}}$ produces a positive LHS and negative RHS.

The equilibrium belief error is

$$\bar{\pi}_{t|t} - \bar{\pi}_t = \left(K_{\bar{\pi}}f_{\bar{\pi}}^*\left(\mathcal{R}_{\bar{\pi}}\right) - 1\right)\epsilon_{\bar{\pi},t} + K_{\bar{\pi}}\sigma\epsilon_{d,t} = -\frac{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2}}{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} + \left(\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}\right)^2}\epsilon_{\bar{\pi},t} - \frac{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}}{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} + \left(\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}\right)^2}\epsilon_{d,t}$$

which gives

$$E_t^{CB}\left[\left(\bar{\pi}_{s|s}-\bar{\pi}_s\right)^2\right]=\frac{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\sigma_d^2}{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2}+\left(\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}\right)^2}>0 \quad \text{for } s>t$$

$$E_t^{CB}\left[\left(\bar{\pi}_{s|s}-\bar{\pi}_s\right)\bar{\pi}_s\right]=-\frac{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\sigma_d^2}{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2}+\left(\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}\right)^2}<0 \quad \text{for } s>t$$

Thus, in equilibrium

$$l_t^{PI}\equiv\frac{1}{2}\left[\left(\tilde{y}_t^{PI}\right)^2+\frac{\varepsilon}{\kappa}\left(\pi_t^{PI}-\bar{\pi}_t\right)^2\right]=\frac{1}{2}\frac{\frac{\varepsilon}{\kappa}\left(1+\varepsilon\kappa\right)\left(1-\beta\rho_{\bar{\pi}}\right)^2}{\left(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa\right)^2}\bar{\pi}_t^2$$

$$l_t\equiv\frac{1}{2}\left[\tilde{y}_t^2+\frac{\varepsilon}{\kappa}\left(\pi_t-\bar{\pi}_t\right)^2\right]=\frac{1}{2}\frac{\frac{\varepsilon}{\kappa}\left(1+\mathcal{R}_{\bar{\pi}}^2\frac{\varepsilon}{\kappa}\right)}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}\left(\frac{\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}_{\bar{\pi}}\varepsilon}\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right)-\left(1-\beta\rho_{\bar{\pi}}\right)\bar{\pi}_t\right)^2$$

$$\begin{aligned}
E_t^{CB}\mathcal{L}_{t+1}^{PI}&\equiv E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\frac{1}{2}\left(\left(\tilde{y}_s^{PI}\right)^2+\frac{\varepsilon}{\kappa}\left(\pi_s^{PI}-\bar{\pi}_s\right)^2\right)\\
&=\frac{\varepsilon\kappa}{2}\left(1+\varepsilon\kappa\right)\left(\frac{1-\beta\rho_{\bar{\pi}}}{\kappa\left(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa\right)}\right)^2\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right]\\
E_t^{CB}\mathcal{L}_{t+1}&\equiv E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\frac{1}{2}\left(\tilde{y}_s^2+\frac{\varepsilon}{\kappa}\left(\pi_s-\bar{\pi}_s\right)^2\right)\\
&=\frac{\varepsilon}{2}\frac{\mathcal{R}_{\bar{\pi}}^2\varepsilon+\kappa}{\kappa^2\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\left(\frac{\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}_{\bar{\pi}}\varepsilon}\left(\bar{\pi}_{s|s}-\bar{\pi}_s\right)-\left(1-\beta\rho_{\bar{\pi}}\right)\bar{\pi}_s\right)^2\right]\\
&=\frac{\varepsilon}{2}\frac{\mathcal{R}_{\bar{\pi}}^2\varepsilon+\kappa}{\kappa^2\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}\left\{\left(1-\beta\rho_{\bar{\pi}}\right)^2\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right]\right.\\
&\left.+\frac{1}{2}\frac{1}{1-\beta}\frac{\left[2\left(1-\beta\rho_{\bar{\pi}}\right)\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)+\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}\right]\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}\sigma_d^2}{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2}+\left(\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}\right)^2}\right\}
\end{aligned}$$

The difference in the expected future welfare loss is then

$$\begin{aligned}
E_t^{CB}\left[\mathcal{L}_{t+1}-\mathcal{L}_{t+1}^{PI}\right]&=\frac{\varepsilon}{2}\left(\frac{1-\beta\rho_{\bar{\pi}}}{\kappa}\right)^2\left(\frac{\mathcal{R}_{\bar{\pi}}^2\varepsilon+\kappa}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}-\frac{\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa\right)^2}\right)\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right]\\
&+\frac{\varepsilon}{2}\frac{1}{1-\beta}\frac{\mathcal{R}_{\bar{\pi}}^2\varepsilon+\kappa}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}\frac{\left[2\left(1-\beta\rho_{\bar{\pi}}\right)\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)+\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}\right]\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}\sigma_d^2}{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2}+\left(\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}\right)^2}
\end{aligned}$$

The proof of Proposition 4 showed that the first term is negative since $\mathcal{R}_{\bar{\pi}}\in\left[\kappa,\frac{\kappa}{1-\beta\rho_{\bar{\pi}}}\right]$. The second term is clearly positive for positive $\mathcal{R}_{\bar{\pi}}$. Thus, the implications of full communication for expected future welfare will depend on the parameterization. Unlike the case with an output target, output fluctuations and deviations of inflation from target will actually be smaller when the inflation target $\bar{\pi}_{t|t}$ moves with true inflation $\bar{\pi}_t$. However, no direct communication comes with a benefit of disciplining discretionary interest rate policy so the net effect is ambiguous.

The difference in the current period loss is

$$l_t - l_t^{PI} = \frac{1}{2}\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2\left(\frac{1+\mathcal{R}_{\bar{\pi}}^2\frac{\varepsilon}{\kappa}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2} - \frac{1+\varepsilon\kappa}{\left(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa\right)^2}\right)\bar{\pi}_t^2$$
$$+\frac{\frac{\varepsilon}{\kappa}\left(1+\mathcal{R}_{\bar{\pi}}^2\frac{\varepsilon}{\kappa}\right)}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}\frac{\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}_{\bar{\pi}}\varepsilon}\left(\frac{1}{2}\frac{\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}_{\bar{\pi}}\varepsilon}\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right)^2 - \left(1-\beta\rho_{\bar{\pi}}\right)\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right)\bar{\pi}_t\right)$$

Again, the first term is negative, but the second term may be positive and larger than the first term depend on the realizations of shocks even for a given set of parameter values.

Now, I turn to the case where the central bank directly communicates d_t to private agents prior to observing i_t. Then, agents infer $\bar{\pi}_t$ upon observing i_t. In equilibrium, since agents know beliefs will be correct with $d_{t|t} = d_t$ and $\bar{\pi}_{t|t} = \bar{\pi}_t$. However, from the policymaker's point of view, beliefs follow

$$\bar{\pi}_{t|t} = \frac{1}{f_{\bar{\pi}} + f_{\bar{\pi},b}}\left(i_t - \left(f_d + f_{d,b}\right)d_t\right)$$

Thus, the policymaker's choice still has a marginal impact of $\frac{d\bar{\pi}_{t|t}}{di_t} = \frac{1}{f_{\bar{\pi}}+f_{\bar{\pi},b}}$ on beliefs.

Denoting this case with superscript ^{d}, the inflation-output tradeoff is at its steepest possible value with

$$\mathcal{R}_{\bar{\pi}}^d = \frac{\kappa}{1-\beta\rho_{\bar{\pi}}}$$

with the following equilibrium outcomes under the optimal discretionary interest rate policy after taking into account that beliefs are correct in equilibrium

$$\tilde{y}_t^d = \frac{\varepsilon\left(1-\beta\rho_{\bar{\pi}}\right)}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}\bar{\pi}_t \quad \text{and} \quad \pi_t^d - \bar{\pi}_t = -\frac{\left(1-\beta\rho_{\bar{\pi}}\right)^2}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}\bar{\pi}_t$$

Then, the associated welfare loss terms are

$$l_t^d \equiv \frac{1}{2}\left[\left(\tilde{y}_t^d\right)^2 + \frac{\varepsilon}{\kappa}\left(\pi_t^d - \bar{\pi}_t\right)^2\right] = \frac{1}{2}\frac{\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}\bar{\pi}_t^2$$

$$E_t^{CB}\mathcal{L}_{t+1}^d \equiv E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\frac{1}{2}\left(\left(\tilde{y}_s^d\right)^2 + \frac{\varepsilon}{\kappa}\left(\pi_s^d - \bar{\pi}_s\right)^2\right) = \frac{1}{2}\frac{\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right]$$

Compared to the case of full communication, communicating only d_t is strictly preferable for any realizations of the current shocks since

$$l_t^d - l_t^{PI} = \frac{1}{2}\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2\left(\frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa} - \frac{1+\varepsilon\kappa}{\left(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa\right)^2}\right)\bar{\pi}_t^2 \leq 0$$
$$E_t^{CB}\left(\mathcal{L}_{t+1}^d - \mathcal{L}_{t+1}^{PI}\right) = \frac{1}{2}\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2\left(\frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa} - \frac{1+\varepsilon\kappa}{\left(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa\right)^2}\right)\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right] \leq 0$$
$$\text{since } \beta\rho_{\bar{\pi}} \geq 0 \text{ and } \varepsilon\kappa \geq 0 \Rightarrow \frac{1}{\left(1-\beta\rho_{\bar{y}}\right)^2+\varepsilon\kappa} \leq \frac{1+\varepsilon\kappa}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}$$

Compared to the baseline no direct communication case,

$$
\begin{aligned}
l_t^d - l_t = & \frac{1}{2}\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2\left(\frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}-\frac{\left(1+\mathcal{R}_{\bar{\pi}}^2\frac{\varepsilon}{\kappa}\right)}{\left(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa\right)^2}\right)\bar{\pi}_t^2 \\
& -\frac{\frac{\varepsilon}{\kappa}\left(1+\mathcal{R}_{\bar{\pi}}^2\frac{\varepsilon}{\kappa}\right)}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}\frac{\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}_{\bar{\pi}}\varepsilon}\left(\frac{1}{2}\frac{\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}_{\bar{\pi}}\varepsilon}\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right)^2-\left(1-\beta\rho_{\bar{\pi}}\right)\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right)\bar{\pi}_t\right)
\end{aligned}
$$

$$
\begin{aligned}
E_t^{CB}\left(\mathcal{L}_{t+1}^d-\mathcal{L}_{t+1}\right) = & \frac{1}{2}\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2\left(\frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}-\frac{1+\mathcal{R}_{\bar{\pi}}^2\frac{\varepsilon}{\kappa}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}\right)\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right] \\
& -\frac{1}{2}\frac{1}{1-\beta}\frac{\frac{\varepsilon}{\kappa}\left(1+\mathcal{R}_{\bar{\pi}}^2\frac{\varepsilon}{\kappa}\right)}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}\frac{\left[2\left(1-\beta\rho_{\bar{\pi}}\right)\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)+\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}\right]\mathcal{R}_{\bar{\pi}}\varepsilon\beta\rho_{\bar{\pi}}\sigma_d^2}{\left(1+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2}+\left(\mathcal{R}_{\bar{\pi}}\frac{\varepsilon}{\kappa}\right)^2}
\end{aligned}
$$

$l_t^d - l_t$ depends on realizations of shocks and is positive for a large enough positive $\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right)\bar{\pi}_t$. For general parameter values, both terms in $E_t^{CB}\left(\mathcal{L}_{t+1}^d-\mathcal{L}_{t+1}\right)$ are negative since the second term is clearly negative while

$$
\begin{aligned}
& \frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}=\frac{1+\mathcal{R}_{\bar{\pi}}^2\frac{\varepsilon}{\kappa}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}\quad\text{for } \mathcal{R}_{\bar{\pi}}=\frac{\kappa}{1-\beta\rho_{\bar{\pi}}} \\
\text{and } & \frac{\partial}{\partial\mathcal{R}_{\bar{\pi}}}\frac{1+\mathcal{R}_{\bar{\pi}}^2\frac{\varepsilon}{\kappa}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}=-2\frac{\varepsilon}{\kappa}\frac{\kappa-\left(1-\beta\rho_{\bar{\pi}}\right)\mathcal{R}_{\bar{\pi}}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^3}\leq 0\quad\text{for}\quad\mathcal{R}_{\bar{\pi}}\in\left[\kappa,\frac{\kappa}{1-\beta\rho_{\bar{\pi}}}\right] \\
\Rightarrow & \frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}\leq\frac{1+\mathcal{R}_{\bar{\pi}}^2\frac{\varepsilon}{\kappa}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}_{\bar{\pi}}\varepsilon\right)^2}\quad\text{for } \mathcal{R}_{\bar{\pi}}\in\left[\kappa,\frac{\kappa}{1-\beta\rho_{\bar{\pi}}}\right]
\end{aligned}
$$

D.7 Proposition 7

For convenience, I reproduce the policymaker's welfare loss function and model equilibrium conditions here.

$$
\mathcal{L}_{t_0}=E_{t_0}^{CB}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\tilde{y}_t^2+\frac{\varepsilon}{\kappa}\pi_t^2\right)
$$

$$
\tilde{y}_t=\tilde{y}_{t+1|t}-\frac{1}{\sigma}\left[i_t-\pi_{t+1|t}-\sigma\left(d_t-d_{t+1|t}\right)\right] \tag{41}
$$

$$
\pi_t=\beta\pi_{t+1|t}+\kappa\tilde{y}_t+v_t \tag{42}
$$

$$
\text{where}\quad d_t=\rho_d d_{t-1}+\epsilon_{d,t}\quad\text{and}\quad v_t=\rho_v v_{t-1}+\epsilon_{v,t}
$$

For the optimal policy problem, the shocks are assumed to come from independent Gaussian white noise processes with constant variances σ_d^2 and σ_v^2, respectively. The information sets are:

$$
\mathcal{I}_t=\left\{i_t,d^{t-1},v^{t-1}\right\}\quad\text{and}\quad\mathcal{I}_t^{CB}=\left\{i_t,d^t,v^t\right\}=\left\{d_t,v_t\right\}\cup\mathcal{I}_t
$$

The policymaker has an initial supposition that private agents believe the current interest rate behavior to be described by

$$
i_t=f_d d_t+f_{d,b}d_{t|t}+f_v v_t+f_{v,b}v_{t|t}
$$

Then, beliefs mirror the baseline case and are the following function of lagged states and i_t

$$d_{t|t} = \frac{f_v + f_{v,b}}{1 + K_v f_{v,b} + K_d f_{d,b}} \left(K_v \rho_d d_{t-1} - K_d \rho_v v_{t-1}\right) + \frac{K_d}{1 + K_v f_{v,b} + K_d f_{d,b}} i_t$$

$$v_{t|t} = \frac{f_d + f_{d,b}}{1 + K_v f_{v,b} + K_d f_{d,b}} \left(K_d \rho_v v_{t-1} - K_v \rho_d d_{t-1}\right) + \frac{K_v}{1 + K_v f_{v,b} + K_d f_{d,b}} i_t$$

$$\text{where } K_d = \frac{f_d \frac{\sigma_d^2}{\sigma_v^2}}{f_d^2 \frac{\sigma_d^2}{\sigma_v^2} + f_v^2} \quad \text{and} \quad K_v = \frac{f_v}{f_d^2 \frac{\sigma_d^2}{\sigma_v^2} + f_v^2}$$

Following the same process as in the proof of Proposition 2 gives the optimality condition

$$\tilde{y}_t - \bar{y}_t = -\mathcal{R}_v \frac{\varepsilon}{\kappa} \pi_t \quad \text{where } \mathcal{R}_v = \frac{\frac{\partial \pi_t}{\partial i_t} + \frac{\partial \pi_t}{\partial d_{t|t}} \frac{dd_{t|t}}{di_t} + \frac{\partial \pi_t}{\partial v_{t|t}} \frac{dv_{t|t}}{di_t}}{\frac{\partial \tilde{y}_t}{\partial i_t} + \frac{\partial \tilde{y}_t}{\partial d_{t|t}} \frac{dd_{t|t}}{di_t} + \frac{\partial \tilde{y}_t}{\partial v_{t|t}} \frac{dv_{t|t}}{di_t}}$$

Solving for $\tilde{y}_t$ using this optimality condition and substituting this into the inflation condition gives

$$\pi_t = \beta \pi_{t+1|t} - \mathcal{R}_v \varepsilon \pi_t + \kappa \bar{y}_t + v_t$$

By restricting attention to nonnegative values of $\mathcal{R}_v$, I can iterate this forward while using the fact that $v_{t+h|t} = \rho_v^h v_{t|t}$ to get a solution for π_t in terms of $\left\{v_t, v_{t|t}\right\}$. Substituting that expression for π_t back into the optimality condition gives the solution for $\tilde{y}_t$ in terms of the same state variables

$$\begin{aligned}
\pi_t &= \frac{1}{1 + \mathcal{R}_v \varepsilon} v_t + \frac{\beta \rho_v}{\left(1 - \beta \rho_v + \mathcal{R}_v \varepsilon\right)\left(1 + \mathcal{R}_v \varepsilon\right)} v_{t|t} \\
\tilde{y}_t &= -\frac{\mathcal{R}_v \frac{\varepsilon}{\kappa}}{1 + \mathcal{R}_v \varepsilon} v_t - \frac{\mathcal{R}_v \frac{\varepsilon}{\kappa} \beta \rho_v}{\left(1 - \beta \rho_v + \mathcal{R}_v \varepsilon\right)\left(1 + \mathcal{R}_v \varepsilon\right)} v_{t|t}
\end{aligned}$$

This immediately reveals that

$$\frac{\partial \pi_t}{\partial d_{t|t}} = \frac{\partial \tilde{y}_t}{\partial d_{t|t}} = 0, \; \frac{\partial \pi_t}{\partial v_{t|t}} = \frac{\beta \rho_v}{\left(1 - \beta \rho_v + \mathcal{R}_v \varepsilon\right)\left(1 + \mathcal{R}_v \varepsilon\right)}, \text{ and } \frac{\partial \tilde{y}_t}{\partial v_{t|t}} = -\frac{\mathcal{R}_v \frac{\varepsilon}{\kappa} \beta \rho_v}{\left(1 - \beta \rho_v + \mathcal{R}_v \varepsilon\right)\left(1 + \mathcal{R}_v \varepsilon\right)}$$

In addition, the solutions for π_t and $\tilde{y}_t$ can be used along with (1) to back out the implied nominal interest rate in terms of $\left\{d_t, d_{t|t}, v_t, v_{t|t}\right\}$

$$\begin{aligned}
i_t &= \sigma\left(d_t - d_{t+1|t}\right) + \pi_{t+1|t} + \sigma\left(\tilde{y}_{t+1|t} - \tilde{y}_t\right) \\
&= \underbrace{\sigma d_t - \sigma \rho_d d_{t|t}}_{r_t^n} + \underbrace{\sigma \frac{\mathcal{R}_v \frac{\varepsilon}{\kappa}}{1 + \mathcal{R}_v \varepsilon}}_{f_v^*(\mathcal{R}_v)} v_t + \underbrace{\sigma \left(\frac{\frac{1}{\sigma} \rho_v + \mathcal{R}_v \frac{\varepsilon}{\kappa}\left(1 - \rho_v\right)}{1 - \beta \rho_v + \mathcal{R}_v \varepsilon} - \frac{\mathcal{R}_v \frac{\varepsilon}{\kappa}}{1 + \mathcal{R}_v \varepsilon}\right)}_{f_{v,b}^*(\mathcal{R}_v)} v_{t|t}
\end{aligned}$$

Using these optimal response coefficients along with the expressions for $\frac{\partial \pi_t}{\partial v_{t|t}}$ and $\frac{\partial \tilde{y}_t}{\partial v_{t|t}}$ gives the equilibrium condition for $\mathcal{R}_v$

$$\mathcal{R}_v = \kappa \frac{\frac{\frac{\beta \rho_v}{\kappa}}{\left(1 - \beta \rho_v + \mathcal{R}_v \varepsilon\right)\left(1 + \mathcal{R}_v \varepsilon\right)} K_v - \frac{1}{\sigma}}{-\frac{\mathcal{R}_v \frac{\varepsilon}{\kappa} \beta \rho_v}{\left(1 - \beta \rho_v + \mathcal{R}_v \varepsilon\right)\left(1 + \mathcal{R}_v \varepsilon\right)} K_v - \frac{1}{\sigma}} \quad \text{where } K_v = \frac{1}{\sigma} \frac{\mathcal{R}_v \frac{\varepsilon}{\kappa}\left(1 + \mathcal{R}_v \varepsilon\right)}{\left(1 + \mathcal{R}_v \varepsilon\right)^2 \frac{\sigma_d^2}{\sigma_v^2} + \left(\mathcal{R}_v \frac{\varepsilon}{\kappa}\right)^2} \tag{43}$$

Again, when $\beta \rho_v = 0$, the terms involving K_v drop out and $\mathcal{R}_v = \kappa$.

This condition can be rearranged into a fourth-order polynomial

$$\begin{aligned} 0 &= \mathcal{R}_v\left\{(1-\beta\rho_v+\mathcal{R}_v\varepsilon)\left[(1+\mathcal{R}_v\varepsilon)^2\frac{\sigma_d^2}{\sigma_v^2}+\left(\mathcal{R}_v\frac{\varepsilon}{\kappa}\right)^2\right]+\left(\mathcal{R}_v\frac{\varepsilon}{\kappa}\right)^2\beta\rho_v\right\} \\ &\quad -\kappa\left\{(1-\beta\rho_v+\mathcal{R}_v\varepsilon)\left[(1+\mathcal{R}_v\varepsilon)^2\frac{\sigma_d^2}{\sigma_v^2}+\left(\mathcal{R}_v\frac{\varepsilon}{\kappa}\right)^2\right]-\frac{\beta\rho_v}{\kappa}\mathcal{R}_v\frac{\varepsilon}{\kappa}\right\}\end{aligned}$$

Thus, there are up to four distinct equilibrium values for $\mathcal{R}_v$. For $\frac{\sigma_d^2}{\sigma_v^2}>0$, the coefficient on the $\mathcal{R}_v^4$ term is $\varepsilon^3\left(\frac{\sigma_d^2}{\sigma_v^2}+\frac{1}{\kappa^2}\right)>0$ and the term that's constant in $\mathcal{R}_v$ is $-\kappa\left(1-\beta\rho_v\right)\frac{\sigma_d^2}{\sigma_v^2}<0$. Then, Descartes' rule of signs says that there must always be at least one positive root for any values of the other parameters.

Note that rearranging (43) shows that $\mathcal{R}_v \leq \kappa$ if attention is limited to solutions where $\mathcal{R}_v$ is real and nonnegative.

$$\kappa-\mathcal{R}_v=\frac{\beta\rho_v\mathcal{R}_v\frac{\varepsilon}{\kappa}}{\left(\mathcal{R}_v\frac{\varepsilon}{\kappa}\right)^2+(1-\beta\rho_v+\mathcal{R}_v\varepsilon)\left(1+\mathcal{R}_v\varepsilon\right)\frac{\sigma_d^2}{\sigma_v^2}}\geq 0 \;\text{ if }\; \mathcal{R}_v\geq 0$$

Now, I look at the cases given by the limits of $\frac{\sigma_d^2}{\sigma_v^2}$.

- When $\frac{\sigma_d^2}{\sigma_v^2}\to\infty$, $K_v\to 0$ so that (43) gives $\mathcal{R}_v=\kappa$ as the unique solution. The policymaker's problem under perfect information in this setting is

$$\min_{i_t}\frac{1}{2}\left(\tilde{y}_t{}^2+\frac{\varepsilon}{\kappa}\pi_t^2\right)$$

  subject to (41) and (42) with $d_{t|t}=d_t$ and $v_{t|t}=v_t$ being exogenous to the policymaker's choice of i_t. Then, it's clear that the optimality condition is the same as the one given in the proposition with $\mathcal{R}_v=\kappa$.

- When $\frac{\sigma_d^2}{\sigma_v^2}\to 0$, (43) becomes the following quadratic equation

$$\mathcal{R}_v=\kappa\left(1-\frac{\beta\rho_v}{\mathcal{R}_v\varepsilon}\right)\quad\text{since } K_v\to\frac{\kappa}{\sigma}\frac{1+\mathcal{R}_v\varepsilon}{\mathcal{R}_v\varepsilon}$$

  If $\varepsilon\kappa<4\beta\rho_v$, then no real roots exist. Otherwise, there are two real positive roots with the larger root being

$$\frac{\varepsilon\kappa+\sqrt{\left(\varepsilon\kappa\right)^2-4\varepsilon\kappa\beta\rho_v}}{2\varepsilon}<\kappa$$

D.8 Proposition 8

I now add to the baseline case a cost-push shock that private agents are perfectly informed about (i.e., $\mathcal{I}_t=\left\{i^t,v^t,d^{t-1},\bar{y}^{t-1}\right\}$) so that the equilibrium conditions become

$$\begin{aligned}\tilde{y}_t &= \tilde{y}_{t+1|t}-\frac{1}{\sigma}\left(i_t-\pi_{t+1|t}\right)+d_t-d_{t+1|t} \\ \pi_t &= \beta\pi_{t+1|t}+\kappa\tilde{y}_t+v_t\end{aligned}$$

Conjecturing a solution that's linear in the expanded set of state variables $\{d_t, d_{t|t}, \bar{y}_t, \bar{y}_{t|t}, v_t\}$ results in expectations of future outcomes of the form

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} + \mathbf{M}_v \rho_v v_t$$

Private agents now suppose that the equilibrium interest rate can be described by

$$i_t = f_d d_t + f_{\bar{y}} \bar{y}_t + f_v v_t + f_{d,b} d_{t|t} + f_{\bar{y},b} \bar{y}_{t|t}$$

Beliefs are derived using the same procedure as Section 2.3 which results in

$$\begin{aligned} d_{t|t} &= \frac{f_{\bar{y}} + f_{\bar{y},b}}{1 + K_{\bar{y}} f_{\bar{y},b} + K_d f_{d,b}} \left(K_{\bar{y}} \rho_d d_{t-1} - K_d \rho_{\bar{y}} \bar{y}_{t-1} \right) + \frac{K_d}{1 + K_{\bar{y}} f_{\bar{y},b} + K_d f_{d,b}} \left(i_t - f_v v_t \right) \\ \bar{y}_{t|t} &= \frac{f_d + f_{d,b}}{1 + K_{\bar{y}} f_{\bar{y},b} + K_d f_{d,b}} \left(K_d \rho_{\bar{y}} \bar{y}_{t-1} - K_{\bar{y}} \rho_d d_{t-1} \right) + \frac{K_{\bar{y}}}{1 + K_{\bar{y}} f_{\bar{y},b} + K_d f_{d,b}} \left(i_t - f_v v_t \right) \end{aligned}$$

where $K_d = \frac{f_d \frac{\sigma_d^2}{\sigma_{\bar{y}}^2}}{f_d^2 \frac{\sigma_d^2}{\sigma_{\bar{y}}^2} + f_{\bar{y}}^2}$ and $K_{\bar{y}} = \frac{f_{\bar{y}}}{f_d^2 \frac{\sigma_d^2}{\sigma_{\bar{y}}^2} + f_{\bar{y}}^2}$ as before and, along with the believed policy coefficients, are again taken as given by the discretionary policymaker.

Following the same steps as the proof of Proposition 2, I use the form of expectations and beliefs to write the output gap deviation and inflation in terms of the exogenous states and i_t so that the discretionary policy problem becomes

$$\min_{i_t} \frac{1}{2} \left((\tilde{y}_t - \bar{y}_t)^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right)$$

$$\text{subject to} \quad \begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \frac{\mathbf{\Psi} \begin{bmatrix} K_{\bar{y}} (f_{\bar{y}} + f_{\bar{y},b}) & -K_d (f_{\bar{y}} + f_{\bar{y},b}) \\ -K_{\bar{y}} (f_d + f_{d,b}) & K_d (f_d + f_{d,b}) \end{bmatrix}}{1 + K_{\bar{y}} f_{\bar{y},b} + K_d f_{d,b}} \begin{bmatrix} \rho_d d_{t-1} \\ \rho_{\bar{y}} \bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ \kappa & 0 \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix}$$

$$+ \left(\begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \mathbf{M}_v \rho_v + \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \frac{\mathbf{\Psi} \begin{bmatrix} K_d \\ K_{\bar{y}} \end{bmatrix} f_v}{1 + K_{\bar{y}} f_{\bar{y},b} + K_d f_{d,b}} \right) v_t + \begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} i_t$$

$$\text{where } \mathbf{\Psi} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \mathbf{M} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} - \begin{bmatrix} \rho_d & 0 \\ \kappa \rho_d & 0 \end{bmatrix}$$

$$\begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} \equiv \frac{\mathbf{\Psi} \begin{bmatrix} K_d \\ K_{\bar{y}} \end{bmatrix}}{1 + K_{\bar{y}} f_{\bar{y},b} + K_d f_{d,b}} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix}$$

Then, clearly, the optimality condition is again

$$\tilde{y}_t - \bar{y}_t = -\mathcal{R} \frac{\varepsilon}{\kappa} \pi_t \quad \text{with } \mathcal{R} = \frac{H_{\pi,i}}{H_{\tilde{y},i}}$$

Substituting this into the equilibrium conditions and solving again for the endogenous variables as I did in the proof of Proposition 2 gives

$$\pi_t = \frac{\kappa}{1+\mathcal{R}\varepsilon}\bar{y}_t + \frac{\kappa}{1+\mathcal{R}\varepsilon}\frac{\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon}\bar{y}_{t|t} + \frac{1}{1-\beta\rho_v+\mathcal{R}\varepsilon}v_t$$

$$\tilde{y}_t = \frac{1}{1+\mathcal{R}\varepsilon}\bar{y}_t - \frac{\mathcal{R}\varepsilon}{1+\mathcal{R}\varepsilon}\frac{\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\frac{H_{i,2}}{H_{i,1}}\varepsilon}\bar{y}_{t|t} - \frac{\mathcal{R}\frac{\varepsilon}{\kappa}}{1-\beta\rho_v+\mathcal{R}\varepsilon}v_t$$

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1-\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon} \\ 0 & \frac{\kappa}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon} \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} + \underbrace{\begin{bmatrix} -\frac{\mathcal{R}\frac{\varepsilon}{\kappa}}{1-\beta\rho_v+\mathcal{R}\varepsilon} \\ \frac{1}{1-\beta\rho_v+\mathcal{R}\varepsilon} \end{bmatrix}}_{\mathbf{M}_v} \rho_v v_t$$

Then, this implies that the interest rate can be written in terms of $\left\{d_t, d_{t|t}, \bar{y}, \bar{y}_{t|t}, v_t\right\}$

$$\begin{aligned} i_t^* &= \sigma\left(d_t - d_{t+1|t}\right) + \pi_{t+1|t} + \sigma\left(\tilde{y}_{t+1|t} - \tilde{y}_t\right) \\ &= \underbrace{\sigma\left(d_t - \rho_d d_{t|t}\right)}_{r_t^n} \underbrace{-\sigma\frac{1}{1+\mathcal{R}\varepsilon}}_{f_{\bar{y}}^*(\mathcal{R})}\bar{y}_t + \underbrace{\sigma\left(\frac{1}{1+\mathcal{R}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon}\right)}_{f_{\bar{y},b}^*(\mathcal{R})}\bar{y}_{t|t} + \underbrace{\sigma\frac{\frac{1}{\sigma}\rho_v + \mathcal{R}\frac{\varepsilon}{\kappa}(1-\rho_v)}{1-\beta\rho_v+\mathcal{R}\varepsilon}}_{f_v^*(\mathcal{R})}v_t \end{aligned}$$

It's clear that the equilibrium conditions between $\left\{\mathbf{M}, K_{\bar{y}}, f_{\bar{y}}^*, f_{\bar{y},b}^*, \mathcal{R}\right\}$ are the same here as in the previous case without the additional cost push shock and so the equilibrium value(s) of $\mathcal{R}$ are also the same.

In the perfect information case, conjecturing a solution that's linear in state variables $\{d_t, \bar{y}_t, v_t\}$ results in expectations of future outcomes of the form

$$\begin{bmatrix} \tilde{y}_{t+1|t}^{PI} \\ \pi_{t+1|t}^{PI} \end{bmatrix} = \mathbf{M}\begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix}\begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} + \mathbf{M}_v\rho_v v_t$$

Then, the output gap deviation and inflation written in terms of exogenous variables along with the interest rate is

$$\begin{bmatrix} \tilde{y}_t^{PI} - \bar{y}_t \\ \pi_t^{PI} \end{bmatrix} = \left(\mathbf{\Psi} + \begin{bmatrix} 1 & -1 \\ \kappa & 0 \end{bmatrix}\right)\begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} + \left(\begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix}\mathbf{M}_v\rho_v + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)v_t - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} i_t$$

Thus, the discretionary policy problem is equivalent to minimizing the current period loss subject to this condition. Then the perfect information discretionary policy optimality condition and equilibrium conditions (including the interest rate behavior) are again the same as the imperfect information case with κ in place of $\mathcal{R}$.

In the limit where $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to 0$, it's still the case that $\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}}$ since $K_{\bar{y}} \to -\frac{1+\mathcal{R}\varepsilon}{\sigma}$. However, I will now show that this is not equivalent to commitment to a rule of the form

$$i_t = r_t^n + f_{\bar{y}}^c\bar{y}_t + f_{\bar{y},b}^c\bar{y}_{t|t} + f_v^c v_t$$

The belief $\bar{y}_{t|t}$ in the limit where $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to 0$ is again given by

$$\bar{y}_{t|t} = \bar{y}_t + \frac{\sigma}{f_{\bar{y}}^c}\epsilon_{d,t}$$

Following the same steps given in Section B to obtain a solution under a given linear interest rate rule provides me with the solution

$$\left[\begin{array}{c}\tilde{y}_t-\bar{y}_t\\ \pi_t\end{array}\right]=-\left[\begin{array}{c}\frac{\Omega_{\bar{y}}}{\sigma}\left(1-\beta\rho_{\bar{y}}\right)\left(f^c_{\bar{y}}+f^c_{\bar{y},b}\right)+1\\ \frac{\kappa\Omega_{\bar{y}}}{\sigma}\left(f^c_{\bar{y}}+f^c_{\bar{y},b}\right)\end{array}\right]\bar{y}_t-\left[\begin{array}{c}\Omega_{\bar{y}}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}}+\frac{\kappa}{\sigma}\right)\left(1+\frac{f^c_{\bar{y},b}}{f^c_{\bar{y}}}\right)+\frac{f^c_{\bar{y},b}}{f^c_{\bar{y}}}\\ \kappa\Omega_{\bar{y}}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}}+\frac{\kappa}{\sigma}+\beta\right)\left(1+\frac{f^c_{\bar{y},b}}{f^c_{\bar{y}}}\right)+\kappa\frac{f^c_{\bar{y},b}}{f^c_{\bar{y}}}\end{array}\right]\epsilon_{d,t}$$
$$+\left[\begin{array}{c}\frac{\frac{1}{\sigma}\rho_v-\frac{1}{\sigma}f^c_v(1-\beta\rho_v)}{(1-\rho_v)(1-\beta\rho_v)-\frac{\kappa}{\sigma}\rho_v}\\ \frac{1-\rho_v-\frac{\kappa}{\sigma}f^c_v}{(1-\rho_v)(1-\beta\rho_v)-\frac{\kappa}{\sigma}\rho_v}\end{array}\right]v_t$$

Then, the optimality conditions for $f^c_{\bar{y}}$ and $f^c_{\bar{y},b}$ are the same as in the proof of Proposition 2

$$0=E^{CB}_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\left(\tilde{y}_t-\bar{y}_t\right)\left(1-\beta\rho_{\bar{y}}\right)+\varepsilon\pi_t\right)\left[-\frac{1}{\sigma}\bar{y}_t+\frac{f^c_{\bar{y},b}}{\left(f^c_{\bar{y}}\right)^2}\epsilon_{d,t}\right]$$
$$\text{and } 0=E^{CB}_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\left(\tilde{y}_t-\bar{y}_t\right)\left(1-\beta\rho_{\bar{y}}\right)+\varepsilon\pi_t\right)\left[-\frac{1}{\sigma}\bar{y}_t-\frac{1}{f^c_{\bar{y}}}\epsilon_{d,t}\right]$$

The new optimality condition for f^c_v is

$$0=\frac{\partial}{\partial f^c_v}E^{CB}_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\left(\tilde{y}_t-\bar{y}_t\right)^2+\frac{\varepsilon}{\kappa}\pi_t^2\right)\Rightarrow 0=E^{CB}_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\left(\tilde{y}_t-\bar{y}_t\right)+\frac{\varepsilon}{\kappa}\pi_t\frac{\kappa}{1-\beta\rho_v}\right)v_t$$

Using the equilibrium solutions for $\tilde{y}_t-\bar{y}_t$ and π_t and evaluating expectations from an ex-ante unconditional perspective gives the following set of equations that satisfy all three optimality conditions and determines the optimal policy rule coefficients

$$0=\left(\frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f^{*,c}_{\bar{y}}+f^{*,c}_{\bar{y},b}\right)+1\right)\left(1-\beta\rho_{\bar{y}}\right)+\varepsilon\frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f^{*,c}_{\bar{y}}+f^{*,c}_{\bar{y},b}\right)$$
$$0=\Omega_{\bar{y}}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}}+\frac{\kappa}{\sigma}\right)\left(1+\frac{f^{*,c}_{\bar{y},b}}{f^{*,c}_{\bar{y}}}\right)+\frac{f^{*,c}_{\bar{y},b}}{f^{*,c}_{\bar{y}}}+\varepsilon\left(\kappa\Omega_{\bar{y}}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}}+\frac{\kappa}{\sigma}+\beta\right)\left(1+\frac{f^{*,c}_{\bar{y},b}}{f^{*,c}_{\bar{y}}}\right)+\kappa\frac{f^{*,c}_{\bar{y},b}}{f^{*,c}_{\bar{y}}}\right)$$
$$0=\frac{\frac{1}{\sigma}\rho_v-\frac{1}{\sigma}f^{*,c}_v\left(1-\beta\rho_v\right)}{\left(1-\rho_v\right)\left(1-\beta\rho_v\right)-\frac{\kappa}{\sigma}\rho_v}+\varepsilon\frac{1-\rho_v-\frac{\kappa}{\sigma}f^{*,c}_v}{\left(1-\rho_v\right)\left(1-\beta\rho_v\right)-\frac{\kappa}{\sigma}\rho_v}$$

The resulting solutions are

$$f^{*,c}_{\bar{y}}=-\sigma\frac{1}{1+\frac{\varepsilon\kappa}{1-\beta\rho_{\bar{y}}}}$$
$$f^{*,c}_{\bar{y},b}=\sigma\left(\frac{1}{1+\frac{\varepsilon\kappa}{1-\beta\rho_{\bar{y}}}}-\frac{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)-\frac{\kappa}{\sigma}\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\frac{\varepsilon\kappa}{1-\beta\rho_{\bar{y}}}}\right)$$
$$f^{*,c}_v=\sigma\frac{\frac{1}{\sigma}\rho_v+\frac{\varepsilon}{1-\beta\rho_v}\left(1-\rho_v\right)}{1-\beta\rho_v+\frac{\varepsilon\kappa}{1-\beta\rho_v}}$$

Then, it's clear that

$$f^{*,c}_v=f^*_v\left(\frac{\kappa}{1-\beta\rho_v}\right)\neq f^*_v\left(\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right)=\sigma\frac{\frac{1}{\sigma}\rho_v+\frac{\varepsilon}{1-\beta\rho_{\bar{y}}}\left(1-\rho_v\right)}{1-\beta\rho_v+\frac{\varepsilon\kappa}{1-\beta\rho_{\bar{y}}}}$$

and

$$f_v^{*,c} = f_v^*\left(\frac{\kappa}{1-\beta\rho_v}\right) < f_v^*\left(\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right) \quad \text{if} \quad \rho_v < \rho_{\bar{y}}$$

$$\text{since} \quad f_v^{*\prime}(\mathcal{R}) = \sigma\frac{\varepsilon}{\kappa}\frac{(1-\beta\rho_v)(1-\rho_v) - \frac{\kappa}{\sigma}\rho_v}{\left[1-\beta\rho_v+\mathcal{R}\varepsilon\right]^2} > 0 \quad \text{when} \quad \rho_v \in [0,\bar{\rho})$$

D.9 Proposition 9

In the case that lagged observations are not seen perfectly, beliefs are now given by a Kalman filter. To solve for these beliefs, recall that the latent states and the interest rate signal are perceived by the private agents to be of the form

$$\begin{aligned}
d_t &= \rho_d d_{t-1} + \epsilon_{d,t}, \quad \epsilon_{d,t} \sim N\left(0,\sigma_d^2\right) \\
\bar{y}_t &= \rho_{\bar{y}}\bar{y}_{t-1} + \epsilon_{\bar{y},t}, \quad \epsilon_{\bar{y},t} \sim N\left(0,\sigma_{\bar{y}}^2\right) \\
i_t &= f_d d_t + f_{\bar{y}}\bar{y}_t + f_{d,b}d_{t|t} + f_{\bar{y},b}\bar{y}_{t|t}
\end{aligned}$$

The circularity of the signal can again be resolved by conjecturing a belief structure and then writing the problem in expectational errors defined as $x_t^{surp} \equiv x_t - x_{t|t-1}$. The conjecture I use is

$$\begin{aligned}
\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} &= \begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} + \begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix}\left(i_t - f_d d_{t|t-1} - f_{d,b}d_{t|t} - f_{\bar{y}}\bar{y}_{t|t-1} - f_{\bar{y},b}\bar{y}_{t|t}\right) \\
&= \begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} + \begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix}[f_d \quad f_{\bar{y}}]\left(\begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} - \begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix}\right) \quad \text{in equilibrium} \\
\text{where} \begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} &= \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix}\begin{bmatrix} d_{t-1|t-1} \\ \bar{y}_{t-1|t-1} \end{bmatrix}
\end{aligned}$$

Thus, the expectational errors can be written in state-space form as

$$\begin{aligned}
\begin{bmatrix} d_t^{err} \\ \bar{y}_t^{err} \end{bmatrix} &= \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix}\begin{bmatrix} 1-\hat{K}_d f_d & -\hat{K}_d f_d \\ -\hat{K}_{\bar{y}} f_d & -\hat{K}_{\bar{y}} f_d \end{bmatrix}\begin{bmatrix} d_{t-1}^{err} \\ \bar{y}_{t-1}^{err} \end{bmatrix} + \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \end{bmatrix} \\
i_t^{surp} &= \left(1+\hat{K}_d f_{d,b} + \hat{K}_{\bar{y}} f_{\bar{y},b}\right)[f_d \quad f_{\bar{y}}]\begin{bmatrix} d_t^{err} \\ \bar{y}_t^{err} \end{bmatrix}
\end{aligned}$$

In this case, the steady-state Kalman filter gives

$$\begin{aligned}
\begin{bmatrix} d_{t|t}^{err} \\ \bar{y}_{t|t}^{err} \end{bmatrix} &= \begin{bmatrix} d_{t|t-1}^{err} \\ \bar{y}_{t|t-1}^{err} \end{bmatrix} + \tilde{K}\left(i_t^{surp} - i_{t|t-1}^{surp}\right) = \tilde{K}\left(1+\hat{K}_d f_{d,b} + \hat{K}_{\bar{y}} f_{\bar{y},b}\right)[f_d \quad f_{\bar{y}}]\begin{bmatrix} d_t^{err} \\ \bar{y}_t^{err} \end{bmatrix} \\
\text{where } \tilde{K} &= \tilde{P}\begin{bmatrix} f_d \\ f_{\bar{y}} \end{bmatrix}\left(\left(1+\hat{K}_d f_{d,b} + \hat{K}_{\bar{y}} f_{\bar{y},b}\right)\begin{bmatrix} f_d & f_{\bar{y}} \end{bmatrix}\tilde{P}\begin{bmatrix} f_d \\ f_{\bar{y}} \end{bmatrix}\right)^{-1} \\
\tilde{P} &= \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix}\left(\tilde{P} - \tilde{P}\begin{bmatrix} f_d \\ f_{\bar{y}} \end{bmatrix}\left(\begin{bmatrix} f_d & f_{\bar{y}} \end{bmatrix}\tilde{P}\begin{bmatrix} f_d \\ f_{\bar{y}} \end{bmatrix}\right)^{-1}\begin{bmatrix} f_d & f_{\bar{y}} \end{bmatrix}\tilde{P}\right)\begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} + \begin{bmatrix} \sigma_d^2 & 0 \\ 0 & \sigma_{\bar{y}}^2 \end{bmatrix}
\end{aligned}$$

This fulfills our original conjecture with

$$\begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix} = \tilde{P} \begin{bmatrix} f_d \\ f_{\bar{y}} \end{bmatrix}$$

and the property that $f_d \hat{K}_d + f_{\bar{y}} \hat{K}_{\bar{y}} = 1$ is maintained.

Beliefs as a function of past beliefs and i_t are

$$\begin{aligned}
d_{t|t} &= \frac{f_{\bar{y}} + f_{\bar{y},b}}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} \left(\hat{K}_{\bar{y}} d_{t|t-1} - \hat{K}_d \bar{y}_{t|t-1} \right) + \frac{\hat{K}_d}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} i_t && \text{(44a)} \\
\bar{y}_{t|t} &= \frac{f_d + f_{d,b}}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} \left(\hat{K}_d \bar{y}_{t|t-1} - \hat{K}_{\bar{y}} d_{t|t-1} \right) + \frac{\hat{K}_{\bar{y}}}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} i_t && \text{(44b)} \\
\begin{bmatrix} d_{t+1|t} \\ \bar{y}_{t+1|t} \end{bmatrix} &= \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}
\end{aligned}$$

Then, I follow the same steps as the proof of Proposition 2 and use the linear form of expectations

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

to write $[\tilde{y}_t - \bar{y}_t \quad \pi_t]'$ as a linear function of prior beliefs, exogenous states, and i_t

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \frac{\mathbf{\Psi} \begin{bmatrix} \hat{K}_{\bar{y}} (f_{\bar{y}} + f_{\bar{y},b}) & -\hat{K}_d (f_{\bar{y}} + f_{\bar{y},b}) \\ -\hat{K}_{\bar{y}} (f_d + f_{d,b}) & \hat{K}_d (f_d + f_{d,b}) \end{bmatrix}}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} \begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ \kappa & 0 \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} + \begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} i_t \tag{45}$$

$$\text{where } \mathbf{\Psi} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \mathbf{M} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} - \begin{bmatrix} \rho_d & 0 \\ \kappa \rho_d & 0 \end{bmatrix}$$

$$\text{and } \begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} \equiv \frac{\mathbf{\Psi} \begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix}}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix}$$

Now, the discretionary policymaker's problem can be written as the following Bellman recursion where his choice today now has an effect on the expected future welfare loss since today's beliefs become the prior for period $t+1$ beliefs

$$\begin{aligned}
V\left(d_t, \bar{y}_t, d_{t|t-1}, \bar{y}_{t|t-1}\right) = \min_{\tilde{y}_t, \pi_t, i_t, d_{t+1|t}, \bar{y}_{t+1|t}} & \left\{ \frac{1}{2} \left((\tilde{y}_t - \bar{y}_t)^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right) + \beta E_t^{CB} V\left(d_{t+1}, \bar{y}_{t+1}, d_{t+1|t}, \bar{y}_{t+1|t}\right) \right\} \\
& \text{subject to (44) and (45)}
\end{aligned}$$

Then, the FOC and envelope condition combine to give the optimality condition

$$\begin{aligned}
\tilde{y}_t - \bar{y}_t + \frac{H_{\pi,i}}{H_{\tilde{y},i}} \frac{\varepsilon}{\kappa} \pi_t = & -\frac{\beta}{H_{\tilde{y},i}} \frac{1}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} \left(\frac{\partial \tilde{y}_{t+1}}{\partial d_{t+1|t}} \rho_d \hat{K}_d + \frac{\partial \tilde{y}_{t+1}}{\partial \bar{y}_{t+1|t}} \rho_{\bar{y}} \hat{K}_{\bar{y}} \right) E_t^{CB} \left[\tilde{y}_{t+1} - \bar{y}_{t+1} \right] && \text{(46a)} \\
& -\frac{\beta}{H_{\tilde{y},i}} \frac{1}{1 + \hat{K}_{\bar{y}} f_{\bar{y},b} + \hat{K}_d f_{d,b}} \left(\frac{\partial \pi_{t+1}}{\partial d_{t+1|t}} \rho_d \hat{K}_d + \frac{\partial \pi_{t+1}}{\partial \bar{y}_{t+1|t}} \rho_{\bar{y}} \hat{K}_{\bar{y}} \right) \frac{\varepsilon}{\kappa} E_t^{CB} \pi_{t+1} && \text{(46b)}
\end{aligned}$$

Matching coefficients gives the same equilibrium value for $\mathbf{M}$ as a function of the interest rate coefficients as the case derived in Appendix B where agents could see lagged beliefs.

To prove that an interest rate of the form

$$i_t = r_t^n + f_{\bar{y}}\bar{y}_t + f_{\bar{y},b}\bar{y}_{t|t}$$

cannot satisfy this optimality condition, I use these supposed policy coefficients and show that the optimality condition, (46), is violated. Substituting these policy coefficients into (45) gives an equilibrium where $\tilde{y}_t - \bar{y}_t$ and π_t are linear in $\left\{\bar{y}_t, \bar{y}_{t|t}\right\}$

$$\begin{aligned}
\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} &= -\begin{bmatrix} \frac{1}{\sigma}\rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma}\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{1}{\sigma}f_{\bar{y},b} \\ \frac{\kappa}{\sigma}\rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{\kappa}{\sigma}f_{\bar{y},b} \end{bmatrix}\bar{y}_{t|t} + \begin{bmatrix} -1 - \frac{1}{\sigma}f_{\bar{y}} \\ -\frac{\kappa}{\sigma}f_{\bar{y}} \end{bmatrix}\bar{y}_t \\
\text{where } \mathbf{M} &= -\begin{bmatrix} 0 & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1 - \beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right) \\ 0 & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) \end{bmatrix} \\
\text{and } \begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} &= -\frac{1}{1 + \hat{K}_{\bar{y}}f_{\bar{y},b} - \hat{K}_d\sigma\rho_d}\left(\begin{bmatrix} \frac{1}{\sigma}\rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma}\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{1}{\sigma}f_{\bar{y},b} \\ \frac{\kappa}{\sigma}\rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{\kappa}{\sigma}f_{\bar{y},b} \end{bmatrix}\hat{K}_{\bar{y}} + \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix}\right)
\end{aligned}$$

Then, this gives

$$\begin{aligned}
\frac{d\tilde{y}_{t+1}}{dd_{t+1|t}}\rho_d\hat{K}_d + \frac{d\tilde{y}_{t+1}}{d\bar{y}_{t+1|t}}\rho_{\bar{y}}\hat{K}_{\bar{y}} &= \frac{\partial\tilde{y}_{t+1}}{\partial\bar{y}_{t+1|t+1}}\left(\frac{d\bar{y}_{t+1|t+1}}{dd_{t+1|t}}\rho_d\hat{K}_d + \frac{d\bar{y}_{t+1|t+1}}{d\bar{y}_{t+1|t}}\rho_{\bar{y}}\hat{K}_{\bar{y}}\right) \\
&= \frac{\partial\tilde{y}_t}{\partial\bar{y}_{t|t}}\frac{\hat{K}_{\bar{y}}\hat{K}_d\sigma\left(1 - \rho_d\right)}{1 + \hat{K}_{\bar{y}}f_{\bar{y},b} + \hat{K}_d f_{d,b}}\left(\rho_{\bar{y}} - \rho_d\right)
\end{aligned}$$

and similarly for π_{t+1}. This simplifies (46) to

$$\begin{aligned}
\tilde{y}_t - \bar{y}_t + \frac{H_{\pi,i}}{H_{\tilde{y},i}}\frac{\varepsilon}{\kappa}\pi_t &= \frac{\beta}{H_{\tilde{y},i}}\frac{\hat{K}_{\bar{y}}\hat{K}_d\left(\rho_{\bar{y}} - \rho_d\right)\sigma\left(1 - \rho_d\right)}{\left[1 + \hat{K}_{\bar{y}}f_{\bar{y},b} - \hat{K}_d\sigma\rho_d\right]^2}\frac{\partial\tilde{y}_t}{\partial\bar{y}_{t|t}}E_t^{CB}\left[\tilde{y}_{t+1} - \bar{y}_{t+1} + \frac{\partial\pi_t/\partial\bar{y}_{t|t}}{\partial\tilde{y}_t/\partial\bar{y}_{t|t}}\frac{\varepsilon}{\kappa}\pi_{t+1}\right] \\
\text{where } \frac{H_{\pi,i}}{H_{\tilde{y},i}} &= \kappa\frac{\left[\rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + f_{\bar{y},b}\right]\hat{K}_{\bar{y}} + 1}{\left[\rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma}\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + f_{\bar{y},b}\right]\hat{K}_{\bar{y}} + 1} \\
\frac{\partial\pi_t/\partial\bar{y}_{t|t}}{\partial\tilde{y}_t/\partial\bar{y}_{t|t}} &= \kappa\frac{\rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + f_{\bar{y},b}}{\rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma}\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + f_{\bar{y},b}}
\end{aligned}$$

Then, in general, the LHS of this condition is linear in $\left\{\bar{y}_t, \bar{y}_{t|t}\right\}$ while the RHS is linear in $\left\{\bar{y}_t, \bar{y}_{t|t}, d_t - d_{t|t}\right\}$ through $E_t^{CB}\bar{y}_{t+1|t+1}$ since

$$\begin{aligned}
\bar{y}_{t+1|t+1} &= \rho_{\bar{y}}\bar{y}_{t|t} + \hat{K}_{\bar{y}}\left(\sigma\left(d_{t+1} - \rho_d d_{t|t}\right) + f_{\bar{y}}\left(\bar{y}_{t+1} - \rho_{\bar{y}}\bar{y}_{t|t}\right)\right) \\
\Rightarrow E_t^{CB}\bar{y}_{t+1|t+1} &= \rho_{\bar{y}}\bar{y}_{t|t} + \hat{K}_{\bar{y}}\left(\sigma\rho_d\left(d_t - d_{t|t}\right) + f_{\bar{y}}\rho_{\bar{y}}\left(\bar{y}_t - \bar{y}_{t|t}\right)\right)
\end{aligned}$$

Since the coefficients on these variables are not collinear functions of $\{f_{\bar{y}}, f_{\bar{y},b}\}$, it will in general be impossible to find values of $\{f_{\bar{y}}, f_{\bar{y},b}\}$ that satisfy this condition. Thus, the optimality condition cannot be satisfied by $i_t = r_t^n + f_{\bar{y}}\bar{y}_t + f_{\bar{y},b}\bar{y}_{t|t}$ for general parameter values.

D.9.1 Corollary 3

In the special case where $\hat{K}_{\bar{y}}\hat{K}_d\left(\rho_{\bar{y}}-\rho_d\right)=0$, (46) does hold with $f_d=\sigma$ and $f_{d,b}=-\sigma\rho_d$ and the condition collapses to the same as the case where agents could see true states with a lag.

$$\tilde{y}_t-\bar{y}_t=-\frac{H_{\pi,i}}{H_{\tilde{y},i}}\frac{\varepsilon}{\kappa}\pi_t$$

Substituting this into the equilibrium conditions shows that the interest rate rule features the same responses to $\left\{\bar{y}_t,\bar{y}_{t|t}\right\}$ as in the case where agents could see lagged fundamentals. The condition $\hat{K}_{\bar{y}}\hat{K}_d\left(\rho_{\bar{y}}-\rho_d\right)=0$ captures the case where the current policy choice no longer affects future outcomes since it no longer affects the future belief $\bar{y}_{t+1|t+1}$. This can be broken down into the following subcases:

1. $\hat{K}_{\bar{y}}=0$ ($\Leftrightarrow \hat{K}_d=\frac{1}{f_d}$): In this case, equilibrium beliefs are given by

$$\bar{y}_{t|t}=\rho_{\bar{y}}\bar{y}_{t-1|t-1}\quad\text{and}\quad d_{t|t}=\frac{1}{f_d+f_{d,b}}\left(i_t-\left(f_{\bar{y}}+f_{\bar{y},b}\right)\rho_{\bar{y}}\bar{y}_{t-1|t-1}\right)$$

   Then, the interest rate only affects the current belief $d_{t|t}$ and not future beliefs.

2. $\hat{K}_d=0$ ($\Leftrightarrow \hat{K}_{\bar{y}}=\frac{1}{f_{\bar{y}}}$): In this case, equilibrium beliefs are given by

$$d_{t|t}=\rho_d d_{t-1|t-1}\quad\text{and}\quad \bar{y}_{t|t}=\frac{1}{f_{\bar{y}}+f_{\bar{y},b}}\left(i_t-\left(f_d+f_{d,b}\right)\rho_d d_{t-1|t-1}\right)$$

   Again, the interest rate only affects the current belief $\bar{y}_{t|t}$ and not future beliefs.

3. $\rho_d=\rho_{\bar{y}}=\rho$: Note that beliefs are a discounted sum of past interest rate news

$$\begin{aligned}
\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t}\end{bmatrix} &= \begin{bmatrix}\rho_d & 0\\ 0 & \rho_{\bar{y}}\end{bmatrix}\begin{bmatrix} d_{t-1|t-1}\\ \bar{y}_{t-1|t-1}\end{bmatrix}+\frac{\begin{bmatrix}\hat{K}_d\\ \hat{K}_{\bar{y}}\end{bmatrix}}{1+\hat{K}_{\bar{y}}f_{\bar{y},b}+\hat{K}_d f_{d,b}}\left(i_t-i_{t|t-1}\right)\\
d_{t|t} &= \frac{\hat{K}_d}{1+\hat{K}_{\bar{y}}f_{\bar{y},b}+\hat{K}_d f_{d,b}}\sum_{j=0}^{\infty}\rho_d^j\left(i_{t-j}-i_{t-j|t-j-1}\right)\\
\bar{y}_{t|t} &= \frac{\hat{K}_{\bar{y}}}{1+\hat{K}_{\bar{y}}f_{\bar{y},b}+\hat{K}_d f_{d,b}}\sum_{j=0}^{\infty}\rho_{\bar{y}}^j\left(i_{t-j}-i_{t-j|t-j-1}\right)
\end{aligned}$$

   When the autocorrelations are equal, the interest rate forecast is a function of only the current interest rate

$$i_{t+1|t}=\rho\left(f_d d_{t|t}+f_{\bar{y}}\bar{y}_{t|t}+f_{d,b}d_{t|t}+f_{\bar{y},b}\bar{y}_{t|t}\right)=\rho i_t\quad\text{since } f_d d_{t|t}+f_{\bar{y}}\bar{y}_{t|t}=f_d d_t+f_{\bar{y}}\bar{y}_t$$

   Then, beliefs collapse to a function of just today's interest rate in equilibrium

$$\begin{aligned}
d_{t|t} &= \frac{\hat{K}_d}{1+\hat{K}_{\bar{y}}f_{\bar{y},b}+\hat{K}_d f_{d,b}}\sum_{j=0}^{\infty}\rho^j\left(i_{t-j}-\rho i_{t-j-1}\right)=\frac{\hat{K}_d}{1+\hat{K}_{\bar{y}}f_{\bar{y},b}+\hat{K}_d f_{d,b}}i_t\\
\bar{y}_{t|t} &= \frac{\hat{K}_{\bar{y}}}{1+\hat{K}_{\bar{y}}f_{\bar{y},b}+\hat{K}_d f_{d,b}}\sum_{j=0}^{\infty}\rho^j\left(i_{t-j}-\rho i_{t-j-1}\right)=\frac{\hat{K}_{\bar{y}}}{1+\hat{K}_{\bar{y}}f_{\bar{y},b}+\hat{K}_d f_{d,b}}i_t
\end{aligned}$$

Thus, the optimal policy problem is again one where the interest rate only affects current beliefs.

In the special case where $\rho_{\bar{y}} = 0$, imposing $f_d = \sigma$ and $f_{d,b} = -\sigma\rho_d$ simplifies the optimality condition to

$$\tilde{y}_t - \bar{y}_t + \varepsilon\pi_t = -\frac{\beta}{H_{\tilde{y},i}} \frac{\hat{K}_{\bar{y}}\hat{K}_d\rho_d\sigma\left(1-\rho_d\right)}{\left[1+\hat{K}_{\bar{y}}f_{\bar{y},b}-\hat{K}_d\sigma\rho_d\right]^2} \frac{\partial\tilde{y}_t}{\partial\bar{y}_{t|t}} E_t^{CB}\left[\tilde{y}_{t+1}-\bar{y}_{t+1}+\varepsilon\pi_{t+1}\right]$$

This clearly holds if the central bank maintains $\tilde{y}_t - \bar{y}_t = -\varepsilon\pi_t \ \forall t$ which is the same optimality condition as the perfect information case and is consistent with the initial supposition of $f_d = \sigma$ and $f_{d,b} = -\sigma\rho_d$.

In the special case where $\rho_d = 0$, equilibrium beliefs are given by

$$\bar{y}_{t+1|t+1} = \rho_{\bar{y}}\bar{y}_{t|t} + \hat{K}_{\bar{y}}\left(f_d d_{t+1} + f_{\bar{y}}\left(\bar{y}_{t+1} - \rho_{\bar{y}}\bar{y}_{t|t}\right)\right) \Rightarrow E_t^{CB}\bar{y}_{t+1|t+1} = \rho_{\bar{y}}\bar{y}_{t|t} + \hat{K}_{\bar{y}}f_{\bar{y}}\rho_{\bar{y}}\left(\bar{y}_t - \bar{y}_{t|t}\right)$$

and the RHS is now only a function of $\bar{y}_t$ and $\bar{y}_{t|t}$. Then, it's verified that the optimality condition holds with $f_d = \sigma$, $f_{d,b} = -\sigma\rho_d$. In general, the coefficients $f_{\bar{y}}$ and $f_{\bar{y},b}$ will differ from the case where lags can be seen since the coefficients in that case only set the LHS to zero.

E Empirical relationship from structural model

In this section, I show that giving private agents an additional signal about π_t and using a special parameterization where $\rho_d = \rho_{\bar{y}} = \rho$ allows the structural model to produce the same key regression equation as the reduced-form empirical model. In fact, it can be shown that this parameterization admits a VAR(1) representation of the structural model (derivations available upon request). I continue to assume that $\rho \in [0, \bar{\rho})$ and that there's a given interest rate rule

$$i_t = f_d d_t + f_{d,b}d_{t|t} + f_{\bar{y}}\bar{y}_t + f_{\bar{y},b}\bar{y}_{t|t}, \ \text{ where } f_{\bar{y}} < 0,\ f_{\bar{y}} + f_{\bar{y},b} < 0,\ f_d > 0,\ f_d + f_{d,b} > 0$$

Using the solution in B, the solution for the output gap and inflation under an interest rate of this form is

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sigma}\Omega\left(1-\beta\rho\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho\right)\right) - \left(1-\frac{1}{\sigma}f_d\right) \\ -\frac{\kappa}{\sigma}\Omega\left(f_d + f_{d,b} - \sigma\left(1-\rho\right)\right) - \kappa\left(1-\frac{1}{\sigma}f_d\right) \end{bmatrix} d_{t|t}$$
$$+ \begin{bmatrix} -\frac{1}{\sigma}\Omega\left(1-\beta\rho\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{1}{\sigma}f_{\bar{y}} \\ -\frac{\kappa}{\sigma}\Omega\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{\kappa}{\sigma}f_{\bar{y}} \end{bmatrix}\bar{y}_{t|t} + \begin{bmatrix} 1-\frac{1}{\sigma}f_d & -\frac{1}{\sigma}f_{\bar{y}} \\ \kappa\left(1-\frac{1}{\sigma}f_d\right) & -\frac{\kappa}{\sigma}f_{\bar{y}} \end{bmatrix}\begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix}$$
$$\text{where } \Omega_d = \Omega_{\bar{y}} = \frac{1}{\left(1-\rho\right)\left(1-\beta\rho\right) - \frac{\kappa}{\sigma}\rho}$$

Imagine now that agents receive another signal which is

$$s_t = \pi_t + \epsilon_{s,t} = \Gamma_d d_t + \Gamma_{\bar{y}}\bar{y}_t + \Gamma_{d,b}d_{t|t} + \Gamma_{\bar{y},b}\bar{y}_{t|t} + \epsilon_{s,t},\ \epsilon_{s,t} \sim N\left(0, \sigma_{s,t-1}^2\right)$$

where the Γ's are the coefficients in the solution for π_t. Then, the private agents' belief formation problem can be written in state-space form as

$$\begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} = \rho \begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \end{bmatrix}, \quad \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \end{bmatrix} \sim N\left(0, \mathbf{\Sigma}_{d,\bar{y},t-1}\right) \text{ where } \mathbf{\Sigma}_{d,\bar{y},t-1} \equiv \begin{bmatrix} \sigma^2_{d,t-1} & 0 \\ 0 & \sigma^2_{\bar{y},t-1} \end{bmatrix}$$
$$\begin{bmatrix} i_t \\ s_t \end{bmatrix} = \begin{bmatrix} f_d & f_{\bar{y}} \\ \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \epsilon_{s,t}$$

I follow the procedure of Svensson and Woodford (2003) to deal with the circularity involved with signals i_t and s_t depending on beliefs. I conjecture a form of beliefs and then write the system in innovations. The conjecture is

$$\begin{aligned}
\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} &= \rho \begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} + \mathbf{K}_t \left(\begin{bmatrix} i_t \\ s_t \end{bmatrix} - \begin{bmatrix} f_d & f_{\bar{y}} \\ \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} \rho \begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} - \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} \right) \\
&= \rho \begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} + \mathbf{K}_t \begin{bmatrix} f_d & f_{\bar{y}} & 0 \\ \Gamma_d & \Gamma_{\bar{y}} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \\ \epsilon_{s,t} \end{bmatrix}
\end{aligned}$$

Then, writing the system in expectational errors defined as $x_t^{err} \equiv x_t - E\left[x_t | \mathcal{I}_t \setminus \{i_t, s_t\}\right]$ yields

$$\begin{bmatrix} d_t^{err} \\ \bar{y}_t^{err} \end{bmatrix} \equiv \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \end{bmatrix}$$
$$\begin{bmatrix} i_t^{surp} \\ s_t^{surp} \end{bmatrix} = \left(\mathbf{I} + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \mathbf{K}_t \right) \left(\begin{bmatrix} f_d & f_{\bar{y}} \\ \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_t^{err} \\ \bar{y}_t^{err} \end{bmatrix} + \begin{bmatrix} 0 \\ \epsilon_{s,t} \end{bmatrix} \right)$$

Then, beliefs are

$$\begin{aligned}
\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} &= \rho \begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} d_{t|t}^{err} \\ \bar{y}_{t|t}^{err} \end{bmatrix} \\
\text{where } \begin{bmatrix} d_{t|t}^{err} \\ \bar{y}_{t|t}^{err} \end{bmatrix} &= E\left[\begin{bmatrix} d_t^{err} \\ \bar{y}_t^{err} \end{bmatrix} | \mathcal{I}_t \setminus \{i_t, s_t\}, i_t^{surp}, s_t^{surp} \right] \\
&= \mathbf{\Sigma}_{d,\bar{y},t-1} \begin{bmatrix} f_d & \Gamma_d \\ f_{\bar{y}} & \Gamma_{\bar{y}} \end{bmatrix} \left(\mathbf{I} + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \mathbf{K}_t \right)' \mathbf{\Sigma}_{i,s,t}^{-1} \begin{bmatrix} i_t^{surp} \\ s_t^{surp} \end{bmatrix} \\
\mathbf{\Sigma}_{i,s,t} &\equiv \left(\mathbf{I} + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \mathbf{K}_t \right) \left(\begin{bmatrix} f_d & f_{\bar{y}} \\ \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} \mathbf{\Sigma}_{d,\bar{y},t-1} \begin{bmatrix} f_d & \Gamma_d \\ f_{\bar{y}} & \Gamma_{\bar{y}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \sigma^2_{s,t-1} \end{bmatrix} \right) \\
&\quad \times \left(\mathbf{I} + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \mathbf{K}_t \right)'
\end{aligned}$$

Since $\begin{bmatrix} \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} = \begin{bmatrix} \kappa\left(1-\frac{1}{\sigma}f_d\right) & -\frac{\kappa}{\sigma}f_{\bar{y}} \end{bmatrix}$, this matches the conjecture above with

$$\mathbf{K}_t \equiv \begin{bmatrix} K^i_{d,t} & K^s_{d,t} \\ K^i_{\bar{y},t} & K^s_{\bar{y},t} \end{bmatrix} = \mathbf{\Sigma}_{d,\bar{y},t-1} \begin{bmatrix} f_d & \Gamma_d \\ f_{\bar{y}} & \Gamma_{\bar{y}} \end{bmatrix} \left(\begin{bmatrix} f_d & f_{\bar{y}} \\ \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} \mathbf{\Sigma}_{d,\bar{y},t} \begin{bmatrix} f_d & \Gamma_d \\ f_{\bar{y}} & \Gamma_{\bar{y}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \sigma^2_{s,t-1} \end{bmatrix} \right)^{-1}$$

$$= \frac{\begin{bmatrix} \left(\frac{\kappa^2}{\sigma} f_{\bar{y}}^2 \sigma^2_{\bar{y},t-1} + f_d \sigma^2_{s,t-1}\right)\sigma^2_{d,t-1} & \kappa f_{\bar{y}}^2 \sigma^2_{d,t-1}\sigma^2_{\bar{y},t-1} \\ \left(\kappa^2\left(1-\frac{1}{\sigma}f_d\right)\sigma^2_{d,t-1} + \sigma^2_{s,t-1}\right) f_{\bar{y}}\sigma^2_{\bar{y},t-1} & -\kappa f_{\bar{y}} f_d \sigma^2_{d,t-1}\sigma^2_{\bar{y},t-1} \end{bmatrix}}{\left(f_d^2\sigma^2_{d,t-1} + f_{\bar{y}}^2\sigma^2_{\bar{y},t-1}\right)\sigma^2_{s,t-1} + \kappa^2 f_{\bar{y}}^2 \sigma^2_{d,t-1}\sigma^2_{\bar{y},t-1}}$$

Using the fact that $f_{\bar{y}} < 0 < f_d$, I obtain the following properties for fixed interest rate rule coefficients

$$K^i_{\bar{y},t} < 0 < K^i_{d,t}, K^s_{d,t}, K^s_{\bar{y},t}, \quad f_d K^i_{d,t} + f_{\bar{y}} K^i_{\bar{y},t} = 1, \quad f_d K^s_{d,t} + f_{\bar{y}} K^s_{\bar{y},t} = 0$$

Then, I can write forecast revisions and the lagged nowcast error as

$$\begin{aligned}
\pi_{t|t} - \pi_{t|t-1} &= \begin{bmatrix} \Gamma_d + \Gamma_{d,b} & \Gamma_{\bar{y}} + \Gamma_{\bar{y},b} \end{bmatrix} \left(\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} - \rho \begin{bmatrix} d_{t-1|t-1} \\ \bar{y}_{t-1|t-1} \end{bmatrix} \right) \\
&= -\frac{\kappa}{\sigma}\Omega \begin{bmatrix} f_d + f_{d,b} - \sigma(1-\rho) & f_{\bar{y}} + f_{\bar{y},b} \end{bmatrix} \rho \left(\begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} - \begin{bmatrix} d_{t-1|t-1} \\ \bar{y}_{t-1|t-1} \end{bmatrix} \right) \\
&\quad - \frac{\kappa}{\sigma}\Omega \begin{bmatrix} f_d + f_{d,b} - \sigma(1-\rho) & f_{\bar{y}} + f_{\bar{y},b} \end{bmatrix} \mathbf{K}_t \begin{bmatrix} f_d & f_{\bar{y}} & 0 \\ \Gamma_d & \Gamma_{\bar{y}} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \\ \epsilon_{s,t} \end{bmatrix} \\
\pi_{t-1} - \pi_{t-1|t-1} &= \begin{bmatrix} \kappa\left(1-\frac{1}{\sigma}f_d\right) & -\frac{\kappa}{\sigma}f_{\bar{y}} \end{bmatrix} \left(\begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} - \begin{bmatrix} d_{t-1|t-1} \\ \bar{y}_{t-1|t-1} \end{bmatrix} \right) \\
\pi_{t+h|t} - \pi_{t+h|t-1} &= -\frac{\kappa}{\sigma}\Omega \begin{bmatrix} f_d + f_{d,b} - \sigma(1-\rho) & f_{\bar{y}} + f_{\bar{y},b} \end{bmatrix} \left(\begin{bmatrix} d_{t+h|t} \\ \bar{y}_{t+h|t} \end{bmatrix} - \begin{bmatrix} d_{t+h|t-1} \\ \bar{y}_{t+h|t-1} \end{bmatrix} \right) = \rho^h\left(\pi_{t|t} - \pi_{t|t-1}\right)
\end{aligned}$$

where

$$\begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} - \begin{bmatrix} d_{t-1|t-1} \\ \bar{y}_{t-1|t-1} \end{bmatrix} = \begin{bmatrix} \frac{f_{\bar{y}}}{f_d} \\ -1 \end{bmatrix} \begin{bmatrix} K^i_{\bar{y},t-1} f_d + K^s_{\bar{y},t-1}\kappa\left(1-\frac{1}{\sigma}f_d\right) & \left(K^i_{\bar{y},t-1} - \frac{\kappa}{\sigma}K^s_{\bar{y},t-1}\right) f_{\bar{y}} - 1 & K^s_{\bar{y},t-1} \end{bmatrix} \begin{bmatrix} \epsilon_{d,t-1} \\ \epsilon_{\bar{y},t-1} \\ \epsilon_{s,t-1} \end{bmatrix}$$

$$\begin{bmatrix} f_d & f_{\bar{y}} & 0 \\ \Gamma_d & \Gamma_{\bar{y}} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \\ \epsilon_{s,t} \end{bmatrix} = \left(I + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \mathbf{K}_t \right)^{-1} \begin{bmatrix} i^{surp}_t \\ s^{surp}_t \end{bmatrix}$$

This allows me to write forecast revisions as linear in the lagged nowcast error, the interest rate surprise, and other inflation news.

$$\begin{aligned}
\pi_{t|t}-\pi_{t|t-1} &= -\frac{\rho\Omega}{\sigma}\left[f_{d,b}-\frac{f_d}{f_{\bar{y}}}f_{\bar{y},b}-\sigma\left(1-\rho\right)\right]\left(\pi_{t-1}-\pi_{t-1|t-1}\right)\\
&\quad-\frac{\kappa\Omega}{\sigma}\left[\begin{array}{cc} f_d+f_{d,b}-\sigma\left(1-\rho\right) & f_{\bar{y}}+f_{\bar{y},b}\end{array}\right]\mathbf{K}_t\left(I+\left[\begin{array}{cc} f_{d,b} & f_{\bar{y},b}\\ \Gamma_{d,b} & \Gamma_{\bar{y},b}\end{array}\right]\mathbf{K}_t\right)^{-1}\left[\begin{array}{c} i_t^{surp}\\ s_t^{surp}\end{array}\right]\\
\text{where } i_t^{surp} &= i_t-E\left[i_t|\mathcal{I}_t\setminus\{i_t,s_t\}\right]\\
s_t^{surp} &= \pi_t-\pi_{t|t-1}+\frac{\rho\Omega}{\sigma}\left[f_{d,b}-\frac{f_d}{f_{\bar{y}}}f_{\bar{y},b}-\sigma\left(1-\rho\right)\right]\left(\pi_{t-1}-\pi_{t-1|t-1}\right)+\epsilon_{s,t}
\end{aligned}$$

Further algebraic manipulation yields a relationship of the same form given by the above empirical model

$$\begin{aligned}
\pi_{t+h|t}-\pi_{t+h|t-1} &= \rho^h\varsigma_t^i\left(i_t-E\left[i_t|\mathcal{I}_t\setminus\{i_t,s_t\}\right]\right)+\rho^h\varsigma_t^s\left(\pi_t-\pi_{t|t-1}\right)\\
&\quad+\rho^{h+1}\varsigma^{NE}\left(1-\varsigma_t^s\right)\left(\pi_{t-1}-\pi_{t-1|t-1}\right)+\rho^h\varsigma_t^s\epsilon_{s,t}\\
\text{where } \varsigma^{NE} &= -\frac{\Omega}{\sigma}\left[\sigma\left(1-\rho\right)-f_{d,b}+\frac{f_d}{f_{\bar{y}}}f_{\bar{y},b}\right]\quad\text{does not depend on variances}
\end{aligned}$$

When I additionally assume that $f_d<\sigma$ and $f_d+f_{d,b}\leq\sigma\left(1-\rho\right)$, this is sufficient (but not always necessary) to obtain the following properties:

1. ς_t^i may be positive, $\varsigma_t^s\geq 0$, $\varsigma^{NE}\geq 0$
2. ς_t^i increases with $\sigma^2_{s,t-1}$ for $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ large enough, ς_t^i decreases with $\sigma^2_{\bar{y},t-1}$ and increases with $\sigma^2_{d,t-1}$
3. ς_t^s decreases with $\sigma^2_{s,t-1}$, ς_t^s increases with $\sigma^2_{\bar{y},t-1}$ and $\sigma^2_{d,t-1}$

F Empirical robustness checks

Table 7: Baseline effect of federal funds rate surprises on inflation forecasts controlling for news about real output growth

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	0.233	0.234	0.285**	0.133
	[1.14]	[1.61]	[2.25]	[1.24]
$\pi_t - \overline{\pi_{t\|t-1}}$	0.095**	0.019	0.029	0.033
	[2.31]	[0.80]	[1.31]	[1.46]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}}$	0.210***	0.150***	0.073***	0.099***
	[3.41]	[3.84]	[3.01]	[3.54]
$y_t - \overline{y_{t\|t-1}}$	0.002	0.003	0.010	0.013
	[0.07]	[0.25]	[1.01]	[1.19]
$y_{t-1} - \overline{y_{t-1\|t-1}}$	0.028	0.009	0.006	0.003
	[0.97]	[0.47]	[0.44]	[0.20]
Adjusted R^2	0.324	0.265	0.200	0.215
N	88	88	88	88

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from the GNP to GDP deflator making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

Table 8: Effect of federal funds rate surprises on inflation forecasts controlling for news about real output growth with a high vs low prior uncertainty interaction

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	−0.070	0.035	0.066	0.102
	[−0.27]	[0.22]	[0.57]	[0.83]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.689**	0.484**	0.667***	0.123
	[2.13]	[2.12]	[3.44]	[0.48]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.054	−0.022	−0.007	0.027
	[0.75]	[−0.50]	[−0.21]	[0.71]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.114**	0.037	0.046*	0.039
	[2.03]	[1.14]	[1.69]	[1.48]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.267***	0.205***	0.103***	0.115***
	[3.35]	[4.07]	[3.12]	[2.84]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.138**	0.063*	0.056*	0.079*
	[2.47]	[1.69]	[1.77]	[1.84]
$y_t - \overline{y_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	−0.004	0.013	0.006	0.007
	[−0.14]	[0.65]	[0.50]	[0.45]
$y_t - \overline{y_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	−0.005	−0.014	0.019	0.021*
	[−0.18]	[−0.80]	[1.63]	[1.80]
$y_{t-1} - \overline{y_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.065	0.013	0.010	0.007
	[1.36]	[0.46]	[0.55]	[0.32]
$y_{t-1} - \overline{y_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	−0.001	−0.001	0.008	0.004
	[−0.03]	[−0.07]	[0.39]	[0.21]
$\overline{Std^{\pi}_{t-1}}$ high	0.152*	0.084	0.094**	0.034
	[1.83]	[1.65]	[2.28]	[0.70]
Adjusted R^2	0.340	0.297	0.265	0.171
N	88	88	88	88
P-value of F-test of difference in $i_t - \overline{i_{t\|t-1}}$ coef	0.070	0.111	0.010	0.943

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from the GNP to GDP deflator making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

Table 9: Baseline effect of federal funds rate surprises on inflation forecasts controlling for news about unemployment

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	0.208	0.197	0.210*	0.130
	[1.02]	[1.36]	[1.79]	[1.19]
$\pi_t - \overline{\pi_{t\|t-1}}$	0.090**	0.012	0.015	0.023
	[2.01]	[0.43]	[0.65]	[1.00]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}}$	0.198***	0.148***	0.075***	0.099***
	[3.48]	[4.07]	[3.54]	[3.73]
$U_t - \overline{U_{t\|t-1}}$	−0.084	−0.072	−0.093	−0.078
	[−0.36]	[−0.50]	[−1.12]	[−0.81]
$U_{t-1} - \overline{U_{t-1\|t-1}}$	−0.196	−0.117	−0.281*	−0.030
	[−0.62]	[−0.56]	[−1.70]	[−0.18]
Adjusted R^2	0.324	0.277	0.272	0.213
N	88	88	88	88

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from the GNP to GDP deflator making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

Table 10: Effect of federal funds rate surprises on inflation forecasts controlling for news about unemployment with a high vs low prior uncertainty interaction

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	−0.093	0.031	0.047	0.119
	[−0.42]	[0.23]	[0.47]	[1.01]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.846***	0.435**	0.548***	0.069
	[2.75]	[2.22]	[2.79]	[0.23]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.051	−0.032	−0.012	0.023
	[0.75]	[−0.83]	[−0.38]	[0.60]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.131***	0.047*	0.029	0.021
	[3.08]	[1.80]	[1.16]	[0.76]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.224***	0.195***	0.095***	0.110***
	[4.89]	[6.30]	[4.11]	[3.33]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.115**	0.052	0.035	0.067*
	[2.38]	[1.48]	[1.38]	[1.77]
$U_t - \overline{U_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	−0.514**	−0.325**	−0.206**	−0.116
	[−2.05]	[−2.37]	[−2.09]	[−0.78]
$U_t - \overline{U_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.357**	0.224**	0.033	−0.042
	[2.37]	[2.18]	[0.43]	[−0.42]
$U_{t-1} - \overline{U_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	−0.095	0.166	−0.014	0.079
	[−0.19]	[0.58]	[−0.06]	[0.31]
$U_{t-1} - \overline{U_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	−0.461	−0.482*	−0.458**	−0.172
	[−1.27]	[−1.91]	[−2.30]	[−0.69]
$\overline{Std^{\pi}_{t-1}}$ high	0.183**	0.092*	0.091**	0.023
	[2.23]	[1.82]	[2.43]	[0.46]
Adjusted R^2	0.407	0.353	0.327	0.170
N	88	88	88	88
P-value of F-test of difference in $i_t - \overline{i_{t\|t-1}}$ coef	0.015	0.097	0.026	0.880

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from the GNP to GDP deflator making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

G Effect of interest rate surprises on output forecasts

In this section, I repeat the exercises in Section 7.3 for real output forecasts. Romer and Romer (2000) finds that the Federal Reserve also possesses an information advantage in forecasting real output relative to the SPF though the evidence seems to be weaker than that for inflation forecasts (Sims (2003) confirms this difference as well). Nevertheless, it may be possible that a signaling effect of interest rate surprises also exists for real output.

All the variables used in these exercises are constructed in the same way as those corresponding to the above inflation measures. Table 11 shows that the prior uncertainty measure for output exhibits slightly stronger, but still small, correlations with macroeconomic variables and other measures of uncertainty than the prior uncertainty measure for inflation. The contemporaneous correlation between $\overline{Std_t^{\pi}}$ and $\overline{Std_t^{y}}$ is .55.

Table 11: Correlations between $\overline{Std_t^{y}}$ and macro variables

x	x_{t-1}	x_t	x_{t+1}
Macro Variables			
Inflation	−0.12	−0.05	−0.19**
Real GNP/GDP growth	−0.22**	−0.05	−0.01
Uncertainty Measures			
Google econ uncertainty index	0.28**	0.22**	0.12
Stock volatility	0.12	0.02	−0.09
Policy uncertainty index	0.17*	0.13	−0.04

Notes: These correlations are computed with the longest samples available in the data. The sample sizes vary between 110 and 124 quarters. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

Table 12 shows the baseline effect of surprise interest rate tightening on real output forecast revisions. The coefficients are large and positive for shorter forecast horizons but turn negative at the farthest forecast horizon. Nakamura and Steinsson (2013) also find a positive effect of interest rate surprises on real output forecasts from the *Blue Chip Economic Indicators* survey that is larger and more statistically significant for shorter horizons.

Table 12: Baseline effect of federal funds rate surprises on output forecasts

Dependent variable: $\overline{y_{t+h\|t}} - \overline{y_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	1.245*	0.763	0.014	−0.314**
	[1.94]	[1.40]	[0.07]	[−2.11]
$y_t - \overline{y_{t\|t-1}}$	0.205***	0.115***	0.060**	0.027
	[4.21]	[2.92]	[2.07]	[1.25]
$y_{t-1} - \overline{y_{t-1\|t-1}}$	0.204***	0.096**	0.030	0.002
	[3.73]	[2.40]	[1.47]	[0.14]
Adjusted R^2	0.468	0.315	0.097	0.027
N	89	89	89	89

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from real GNP to real GDP making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

Table 13 estimates the same equation with the addition of interactions with a variable indicating whether $\overline{Std^y_{t-1}}$ is below or above its median. Compared to the baseline results, the coefficients on interest rates surprises in periods of high uncertainty are much larger except for the farthest horizon. However, unlike the estimates for inflation, the differences are not statistically significant. Moreover, the interactions on the news captured by the lagged forecast goes in the direction predicted by the model while the interactions for nowcast errors do not.

Table 13: Effect of federal funds rate surprises on output forecasts with a high vs low prior uncertainty interaction

Dependent variable: $\overline{y_{t+h\|t}} - \overline{y_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^y_{t-1}}$ low	1.022*	0.252	−0.140	−0.321**
	[1.98]	[0.54]	[−0.63]	[−2.25]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^y_{t-1}}$ high	2.058	1.921*	0.309	−0.338
	[1.21]	[1.69]	[0.70]	[−0.86]
$y_t - \overline{y_{t\|t-1}} \times \overline{Std^y_{t-1}}$ low	0.249***	0.129**	0.068	0.041
	[3.81]	[2.22]	[1.63]	[1.30]
$y_t - \overline{y_{t\|t-1}} \times \overline{Std^y_{t-1}}$ high	0.123**	0.059	0.039	0.009
	[2.04]	[1.54]	[1.01]	[0.38]
$y_{t-1} - \overline{y_{t-1\|t-1}} \times \overline{Std^y_{t-1}}$ low	0.220***	0.150***	0.043	−0.005
	[3.36]	[3.02]	[1.48]	[−0.23]
$y_{t-1} - \overline{y_{t-1\|t-1}} \times \overline{Std^y_{t-1}}$ high	0.174**	0.044	0.016	0.003
	[2.24]	[0.87]	[0.55]	[0.13]
$\overline{Std^y_{t-1}}$ high	−0.078	0.109	0.077	0.056
	[−0.46]	[0.90]	[0.77]	[0.90]
Adjusted R^2	0.468	0.337	0.067	0.005
N	89	89	89	89
P-value of F-test of difference in $i_t - \overline{i_{t\|t-1}}$ coef	0.562	0.178	0.368	0.967

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from real GNP to real GDP making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

Table 14 shows that similar results can be obtained from an estimation with a continuous interaction with prior uncertainty. Again, I standardize the prior uncertainty measure to have zero mean and standard deviation of one. The point estimates on the interaction between interest rate surprises and prior uncertainty are all positive as predicted by the model, but are not statistically significant at standard levels. One possible explanation for the evidence being weaker here is the above-mentioned fact that the Federal Reserve's information advantage is less strong for output than it is for inflation. Another explanation is that real output growth is not characterized as well by an AR(1) process as inflation is. This could imply that there are omitted variables in the above regressions. This issue will be addressed in future work.

Table 14: Effect of federal funds rate surprises on output forecasts with a continuous prior uncertainty interaction

Dependent variable: $\overline{y_{t+h\|t}} - \overline{y_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	1.266*	0.864*	0.026	−0.297*
	[1.77]	[1.68]	[0.12]	[−1.79]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^y_{t-1}}$	0.166	0.809	0.325	0.201
	[0.21]	[1.17]	[1.64]	[1.27]
$y_t - \overline{y_{t\|t-1}}$	0.199***	0.104**	0.054*	0.025
	[3.94]	[2.60]	[1.77]	[1.12]
$y_t - \overline{y_{t\|t-1}} \times \overline{Std^y_{t-1}}$	−0.033	−0.019	−0.012	−0.016
	[−0.58]	[−0.48]	[−0.36]	[−0.72]
$y_{t-1} - \overline{y_{t-1\|t-1}}$	0.197***	0.091**	0.025	−0.002
	[3.39]	[2.36]	[1.38]	[−0.10]
$y_{t-1} - \overline{y_{t-1\|t-1}} \times \overline{Std^y_{t-1}}$	−0.022	−0.077***	−0.044**	−0.010
	[−0.51]	[−2.70]	[−2.16]	[−0.65]
$\overline{Std^y_{t-1}}$	0.033	0.146**	0.108***	0.060*
	[0.39]	[2.63]	[2.80]	[1.87]
Adjusted R^2	0.446	0.340	0.126	0.023
N	89	89	89	89

Notes: $\overline{Std^y_{t-1}}$ is standardized to have zero mean and standard deviation of one. The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from real GNP to real GDP making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.