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Modified numerals and polarity sensitivity: Between O(nly)$_{\text{DA}}$ and E(ven)$_{\sigma\text{A}}$

Introduction. Superlative-modified numerals (SMNs) exhibit many interesting patterns. Two notable sets of patterns are patterns related to scalar implicatures and ignorance; for lack of space I will not discuss them here, though they will be kept in mind. Two further sets of patterns have to do with polarity sensitivity, and these are the focus of this talk: **(Polarity sensitivity 1)** SMNs are bad in downward-entailing (DE) environments, e.g., the scope of negation, (1a) except for *Strawson*-DE environments, e.g., the first argument of a conditional(/universal), (1b). **(Polarity sensitivity 2)** And their acceptability in the first argument of a conditional(/universal) varies with the lexical/conventional/grammatical polarity of the SMN, (2a), of the predicates in the first/second argument, (2b), and of the first/second argument itself, (2c).

(1) a. Jo didn't solve # at least 3 problems.
  b. If Jo solved ✓at least 3 problems, she passed.

(2) a. If Jo solved # at most 3 problems, she passed.
  b. If Jo solved at least 3 problems, she # failed.
  c. If Jo # didn't solve at least 3 problems, she passed.

Existing literature and this talk. The literature on SMNs has discussed scalarity and ignorance at length, but polarity sensitivity has been mostly neglected. Only a handful of theories make any suggestion at all about polarity sensitivity 1 [1–3], and only one engages with polarity sensitivity 2 also [2]. This latter theory suggests that these two types of polarity sensitivity go back to two different lexical meanings of SMNs, one plain meaning which crashes in DE environments, and one evaluative meaning, which doesn't crash in DE environments, thrives in conditionals/universals, and is only sensitive to whether the continuation is pragmatically positive. While it seems to get at the basic distribution, this theory however fails to explain why SMphrases, even when intended very evaluatively, are still bad under negation (a limitation already acknowledged by the authors), and also doesn't quite capture the complexity of polarity sensitivity 2. The goal of this talk is to offer a new unified account of (scalar implicatures, ignorance, and) polarity sensitivity 1 and 2 in terms of alternatives and exhaustification.

Proposal. Building on the existing extent-based approaches to adjectives [4] and alternative-based approaches to numerals [3, 5–9] and polarity sensitive items [10, 11], I argue that:

⋆ With a certain new decomposition of SMNs (among which *much/little* = positive/negative extent indicators), SMNs make reference in their truth conditions not just to a numeral, i.e., a scalar element, but also to a set of degrees based on the numeral, i.e., a domain, (3a), and thus naturally give rise to both scalar alternatives (σA), (3b), and subdomain alternatives (DA), (3c).

(3) At most/least n people quit.

  a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overbrace{[\![\text{much/little}]\!]\,(n)}^{\{\ldots,n-1,n\}/\{n,n+1,\ldots\}}$ (truth conditions)
  b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(m) \mid m \in S\}$ (σA)
  c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset [\![\text{much/little}]\!]\,(n)\}$ (DA)

For a complete analysis of SMNs, all the σA and DA thus generated must be used, and the DA in pre-exhaustified form. For the data at hand, however, it suffices to consider all the σA but just the plain singleton DA. For example, *at least 3* with abbreviated truth conditions $\max \in \{3, 4, \ldots\}$ will have $\sigma\text{A} = \{\ldots, \max \in \{2, 3, \ldots\}, \max \in \{4, 5, \ldots\}, \ldots\}$ (essentially, classical, Horn-style σA) and $\text{DA} = \{\max \in \{3\}, \max \in \{4\}, \max \in \{5\}, \ldots\}$.

⋆ These alternatives are *all* factored in *by default* via silent exhaustivity operators, (a) O(nly) and (b) E(ven). Given a proposition p and a set of alternatives to p, C, (a) O(nly)$_C(p)$ asserts p and negates its alternatives in C that are not entailed by p [10], where C can be its σA or DA, and (b) E(ven)$_C(p)$ imposes a presupposition that p is less likely/more noteworthy than all its alternatives in C [11], where C can be its σA.

⋆ By considering the action of these exhaustivity operators, we can make sense of our empirical patterns. In particular, O gives us (scalarity, ignorance, and) polarity sensitivity 1, and E gives us polarity sensitivity 2, as shown below.

Capturing polarity sensitivity 1. Consider O_{DA}(Jo didn't solve at least 3 problems). O_{DA} asserts its prejacent ($\neg\max \in \{3, 4, \dots\}$) and negates those of its DA (DA $= \{\neg\max \in \{3\}, \neg\max \in \{4\}, \dots\}$) that are not entailed by it. In this case, however, there are no DA that are not entailed, so O_{DA} is vacuous. I propose, following similar suggestions for other items in [10], that the DA of SMNs come with a requirement that their use must lead to a properly stronger meaning (PS) – that is, they must be exhaustified via O^{PS} – and that if that requirement is not satisfied the result is thrown out. This captures (1a). The fact that O^{PS}_{DA} across a Strawson-DE operator is however successful can be captured by assuming that O^{PS}_{DA} takes into account the positive presuppositional component of such operators [12]. This derives the felicity of (1b). For reasons of space I omit the illustration.

Capturing polarity sensitivity 2. Consider $E_{\sigma A}$(If Jo solved at least 3 problems, she passed). $E_{\sigma A}$imposes a presupposition that its prejacent is less likely than all its σA. Here we need to clarify two points. First, which σA are we talking about? The literature [11] discusses cases where the scalar element is an end-of-scale item, but our SMNs are typically not end-of-scale, so their σA-set contains both weaker and stronger σA. I propose that, while O pitches an SMN prejacent up against those of its DA or σA that it *does not* entail, E pitches it up against those of its σA that it *does* entail. For example, in an UE environment, *at least 3* is pitched up against {…, *at least 2*} and *at most 3* against {*at most 4*, …}, while in a DE environment *at least 3* is pitched up against {*at least 4*, …} and *at most 3* against {…, *at most 2*}. Second, how is likelihood assessed? A natural assumption is that 'least likely' aligns with 'logically strongest' ($\prec_{\rightarrow}$ 'is logically less likely'). However, if the SMN prejacent is always compared to the σA that it entails, the presupposition of E will always be trivially satisfied, so we won't be able to derive our contrasts. I propose, following similar suggestions for other items in [11], that the numeral in the prejacent and in the σA are all in fact used by E in an exact sense, obtainable via $O_{\sigma A}$. On this meaning, they are no longer monotonic, so likelihood can no longer be assessed based on logical strength and defaults to being assessed based on contextual assumptions ($\prec_c$ 'is contextually less likely'). This captures all of (1b)-(2c): (1b) is fine because the scalar presupposition imposed by E fits with common assumptions about how the world works, (4), and (2a)-(2c) are bad because it does not, (5) (for reasons of space shows only (2a)).

(4) $E_{\sigma A}$(If $O_{\sigma A}$(Jo solved at least 3 problems), she passed) (captures (1b))

$$\underbrace{O_{\sigma A}(\text{solve at least 3 problems})}_{\text{solve exactly 3 problems}} \rightarrow \text{pass} \prec_c \underbrace{O_{\sigma A}(\text{solve at least 4 problems})}_{\text{solve exactly 4 problems}} \rightarrow \text{pass} \qquad (✓)$$

(5) a. $E_{\sigma A}$(If $O_{\sigma A}$(Jo solved at most 3 problems), she passed) (captures (2a))

$$\underbrace{O_{\sigma A}(\text{solve at most 3 problems})}_{\text{solve exactly 3 problems}} \rightarrow \text{pass} \prec_c \underbrace{O_{\sigma A}(\text{solve at most 2 problems})}_{\text{solve exactly 2 problems}} \rightarrow \text{pass} \qquad (\#)$$

Conclusion and outlook. SMNs activate both scalar and subdomain alternatives. These are factored into meaning via the silent exhaustivity operators O(nly) and E(ven). O operates on the non-entailed (scalar and) subdomain alternatives to yield (scalar implicatures, ignorance, and) polarity sensitivity 1. E operates on the entailed scalar alternatives to yield polarity sensitivity 2. Overall, SMNs emerge as items that want *all* their alternatives to contribute to their strengthening, and recruit both O and E to achieve that. This view of SMNs makes further welcome empirical predictions, though also raises interesting conceptual questions, as will be discussed.

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