An extent-based GQT-style unified implicature account of bare and modified numerals

3 · more/less than 3 · at most/least 3

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Outline

The classic GQT theory of BNs, CMs, and SMs

An extent based GQT-style unified implicature account of BNs, CMs, and SMs

Additional results

Conclusion
Generalized Quantifier Theory
[Barwise and Cooper, 1981]

(1) \( [\text{every}] = \lambda P . \lambda Q . P \subseteq Q \)
(2) \( [\text{no}] = \lambda P . \lambda Q . P \cap Q = \emptyset \)
(3) \( [a] = \lambda P . \lambda Q . P \cap Q \neq \emptyset \)
(4) \( [\text{three}] = \lambda P . \lambda Q . |P \cap Q| \geq 3 \)
(5) \( [\text{more than three}] = \lambda P . \lambda Q . |P \cap Q| > 3 \)
(6) \( [\text{less than three}] = \lambda P . \lambda Q . |P \cap Q| < 3 \)
(7) \( [\text{at least three}] = \lambda P . \lambda Q . |P \cap Q| \geq 3 \)
(8) \( [\text{at most three}] = \lambda P . \lambda Q . |P \cap Q| \leq 3 \)
(9) \( [\text{exactly three}] = \lambda P . \lambda Q . |P \cap Q| = 3 \)
(10) \( [\text{between three and five}] = \lambda P . \lambda Q . 3 \leq |P \cap Q| \leq 5 \)
Features and bugs

- Uniformity of DPs
- Uniformity of natural language determiners

- Uniformity of bare (BNs, *three*), comparative-modified (CMs, *more/less than three*), and superlative-modified numerals (SMs, *at least/most three*)

Challenged with data pointing to non-uniformity!
Challenges led to theories very different from GQT.

Where exactly does GQT fail?
We will assess it w.r.t. four major yardsticks:

| entailments | scalar implicatures | ignorance | accept in DE env |
Entailments


(11) a. Alice has 3 / more than 3 / at least 3 diamonds.
    b. ¬ The number of diamonds that Alice has is 2 or less / 3 or less / 2 or less.
    c. Alice has 3 / more than 3 / at least 3 diamonds, # if not less.

(12) a. Alice has less than 3 / at most 3 diamonds.
    b. ¬ The number of diamonds that Alice has is 3 or more / 4 or more.
    c. Alice has less than 3 / at most 3 diamonds, # if not more.
The upper bound of BNs as a scalar implicature

[Horn, 1972, Spector, 2013]

(13) a. Alice has 3 diamonds.
   b. ¬ The number of diamonds that Alice has is 4 or more.
   c. Alice has 3 diamonds, if not more.

★ 3 P Q ambiguous between ‘at least 3 P Q’ and ‘exactly 3 P Q’
★ One way to get this is to say that 3 P Q entails ‘at least 3 P Q’,
   derives ‘not at least 4 P Q’ via scalar implicature.
★ Predicted scalar alternatives of BNs, CMs, and SMs:

(14) a. ScalAlts(3 P Q)
    = {…, 2 P Q, 4 P Q, …}
   b. ScalAlts(more/less than 3 P Q)
    = {…, more/less than 2 P Q, more/less than 4 P Q, …}
   c. ScalAlts(at most/least 3 P Q)
    = {…, at most/least 2 P Q, at most/least 4 P Q, …}
Scalar implicatures


★ Unembedded:

(15) Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds.
⇝ ¬ Alice has 4 / *more than 4 / *less than 2 / *at most 2 / *at least 4 diamonds.
(Total predicted meaning: She has exactly 3 / exactly 4 / exactly 2 / exactly 3 / exactly 3 diamonds.)

★ In the scope of a universal operator:

(16) Alice is required to have 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds.
⇝ ¬ Alice is required to have 4 / more than 4 / less than 2 / at most 2 / at least 4 diamonds.
Scalar implicatures

[Mayr, 2013]

★ In the antecedent of a conditional:

(17) If Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds she wins.

¬ If Alice has 2 / more than 2 / less than 4 / at most 4 / at least 2 diamonds she wins.

★ In the scope of negation:

(18) Alice doesn’t have 3 / more than 3 / less than 3 / *at most 3 / *at least 3 diamonds.

¬ Alice doesn’t have *2 / *more than 2 / *less than 4 / *at most 4 / *at least 2 diamonds.

(Total predicted meaning: She has exactly 2 / exactly 3 / exactly 3 / exactly 4 / exactly 2 diamonds.)
Scalar implicatures
[Cummins et al., 2012]

★ Unembedded, coarse granularity scale:

(19) (example from [Spector, 2014, 42])

Context: Grades are attributed on the basis of the number of problems solved. People who solve between 1 and 5 problems get a C. People who solve more than 5 problems but fewer than 9 problems get a B, and people who solve 9 problems or more get an A.

John solved more than 5 problems. Peter solved more than 9.
\[ \sim \rightarrow \neg \text{John solved more than 9.} \]
Ignorance

★ Unembedded:

(20) Alice has 3 diamonds.
   (*→ The speaker is not sure whether Alice has 3 or 4 or . . .)

(21) Alice has more than 3 / less than 3 diamonds.
   (→ The speaker is not sure whether Alice has 4 or 5 or . . . / 2
   or 1 or . . .)

(22) Alice has at least 3 / at most 3 diamonds.
   *(→ The speaker is not sure whether Alice has 3 or 4 or . . . / 3
   or 2 or . . .)
In the scope of a universal operator:

(23) Alice is required to have 3 diamonds.

\( \forall \) The speaker is not sure whether Alice is required to have 3 or 4 or …

(24) Alice is required to have more than 3 / less than 3 / at most 3 / at least 3 diamonds.

(\( \forall \) The speaker is not sure whether Alice is required to have 4 or 5 or … / 2 or 1 or … / 3 or 2 or … / 3 or 4 or ….)
Ignorance

★ In the scope of negation:

(25) Alice doesn’t have 3 diamonds.

\( \not\setminus \) The speaker is not sure whether Alice doesn’t have 3 or 4 or …

(26) Alice doesn’t have more than 3 / less than 3 diamonds.

(\( \not\Rightarrow \) The speaker is not sure whether Alice has 3 or 2 or … / 3 or 4 or …)
Acceptability in DE environments

★ In the scope of negation:

(27) Alice doesn’t have 3 / more than 3 / less than 3 diamonds.
    → Alice has 2 or less / 3 or less / 3 or more diamonds. ✓

(28) Alice doesn’t have *at least three / *at most three diamonds.
    → Alice has 2 or less / 4 or more diamonds. ✗

★ In the antecedent of a conditional:

(29) If Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds, she wins.

★ In the restriction of a universal:

(30) Everyone who has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds wins.
What is GQT missing?

<table>
<thead>
<tr>
<th>entailments</th>
<th>scalar implicatures</th>
<th>ignorance</th>
<th>accept in DE env</th>
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<tbody>
<tr>
<td>✓</td>
<td>✓ + ?</td>
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Sketch of the solution:

- Keep the GQT way of getting entailments.
- Keep scalar implicatures.
- Add domain alternatives [Kennedy, 2015, Spector, 2015].
- Make the domain alternatives of SMs obligatory [Spector, 2015].
- Try to derive rather than stipulate the number, type, and status of the alternatives in each case.
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### Proposal: Truth conditions and presupposition

<table>
<thead>
<tr>
<th>the numeral</th>
<th>[Link, 1983, Buccola and Spector, 2016]</th>
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<tbody>
<tr>
<td>([\text{three}] = 3)</td>
<td>(<a href="3">\text{isCard}</a> = \lambda x .</td>
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<table>
<thead>
<tr>
<th>much/little</th>
<th>[Seuren, 1984, Kennedy, 1997]</th>
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<tr>
<td>(<a href="n">\text{much}</a> = \lambda d . d \leq n)</td>
<td>(<a href="n">\text{little}</a> = \lambda d . d \geq n)</td>
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<tr>
<td>((\exists (n P))(Q) = 1 \text{ iff } \exists x[</td>
<td>x</td>
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<tr>
<td>(<a href="n">\text{sup(much/little)}</a>(P)(Q) = 1 \text{ iff }</td>
<td>P \cap Q</td>
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<table>
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<tr>
<th>the presupposition of [sup]</th>
<th>[Hackl, 2009, Gajewski, 2010]</th>
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<td>(</td>
<td><a href="n">\text{much/little}</a></td>
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Entailments

(31) 3 P Q:
\[ \exists x [ |x| = 3 \land P(x) \land Q(x)] \Rightarrow |P \cap Q| \geq 3 \] (l.b.)

(32) more than 3 P Q:
\[ |P \cap Q| \in [\text{much}] (3) \iff |P \cap Q| \in \{4, 5, \ldots\} \] (l.b.)

(33) less than 3 P Q:
\[ |P \cap Q| \in [\text{little}] (3) \iff |P \cap Q| \in \{\ldots, 0, 1, 2\} \] (u.b.)

(34) at most 3 P Q:
\[ |P \cap Q| \in [\text{much}] (3) \iff |P \cap Q| \in \{\ldots, 0, 1, 2, 3\} \] (u.b.)

(35) at least 3 P Q:
\[ |P \cap Q| \in [\text{little}] (3) \iff |P \cap Q| \in \{3, 4, \ldots\} \] (l.b.)
Proposal: Alternatives

**Scalar alternatives** can be obtained by replacing \( n \) in the numeral argument with its scalar alternatives (other numerals)

BNs: \( \{\exists x[|x| = m \land P(x) \land Q(x)] : m \in S\} \)

CMs: \( \{|P \cap Q| \in \langle\text{much}/\text{little}\rangle(m) : m \in S\} \)

SMs: \( \{|P \cap Q| \in \langle\text{much}/\text{little}\rangle(m) : m \in S\} \)

**Domain alternatives** can be obtained by replacing the whole numeral argument with its subsets

BNs: NA  

CMs: \( \{|P \cap Q| \in A : A \subseteq \langle\text{much}/\text{little}\rangle(n)\} \)

SMs: \( \{|P \cap Q| \in A : A \subseteq \langle\text{much}/\text{little}\rangle(n)\} \)  

active by presup!
Scalar alternatives

\[
\text{ScalAlts}(3 \, P \, Q) \\
= \text{ScalAlts}(\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]) \\
= \{\ldots, \exists x[|x| = 2 \wedge P(x) \wedge Q(x)], \exists x[|x| = 4 \wedge P(x) \wedge Q(x)], \ldots\} \\
= \{\ldots, 2 \, P \, Q, 4 \, P \, Q, \ldots\}
\]

\[
\text{ScalAlts}(\text{more/less than 3} \, P \, Q) \\
= \text{ScalAlts}(|P \cap Q| \in \llbracket\text{much/little}\rrbracket (3)) \\
= \{\ldots, |P \cap Q| \in \llbracket\text{much/little}\rrbracket (2), |P \cap Q| \in \llbracket\text{much/little}\rrbracket (4), \ldots\} \\
= \{\ldots, \text{more/less than 2} \, P \, Q, \text{more/less than 4} \, P \, Q, \ldots\}
\]

\[
\text{ScalAlts}(\text{at most/least 3} \, P \, Q) \\
= \text{ScalAlts}(|P \cap Q| \in \llbracket\text{much/little}\rrbracket (3)) \\
= \{\ldots, |P \cap Q| \in \llbracket\text{much/little}\rrbracket (2), |P \cap Q| \in \llbracket\text{much/little}\rrbracket (4), \ldots\} \\
= \{\ldots, \text{at most/least 2} \, P \, Q, \text{at most/least 4} \, P \, Q, \ldots\}
\]
Subdomain alternatives

\text{SubDomAlts(3 P Q)}: \text{NA}

\text{SubDomAlts(more/less than 3 P Q)}
= \text{SubDomAlts(}|P \cap Q| \in \llbracket \text{much/little} \rrbracket (3))
= \text{SubDomAlts(}|P \cap Q| \in \{4, 5, \ldots \}/\{0, 1, 2\})
= \{|P \cap Q| \in \{4\}, |P \cap Q| \in \{4, 7, \ldots \} \}/\{|P \cap Q| \in \{0\},
|P \cap Q| \in \{0, 1\}, \ldots \}

\text{SubDomAlts(at most/least 3 P Q)}
= \text{SubDomAlts(}|P \cap Q| \in \llbracket \text{much/little} \rrbracket (3))
= \text{SubDomAlts(}|P \cap Q| \in \{0, 1, 2, 3\}/\{3, 4, \ldots \})
= \{|P \cap Q| \in \{0\}, |P \cap Q| \in \{1, 3\}, \ldots \} \)/\{|P \cap Q| \in \{3\},
|P \cap Q| \in \{4, 8\}, \ldots \}

\text{active by presup!}
Proposal: Implicature calculation system

[Chierchia, 2013]

\[ O \] to exhaustify the scalar alternatives of BNs, CMs, and SMs

\[(36) \ [O_{ALT}(\phi)]^{g, w} = [\phi]^{g, w} \land \forall p \in [\phi]^{ALT} [p \rightarrow \lambda w'. [\phi]^{g, w'} \subseteq p] \]

\[ O^{PS} \] to exhaustify the subdomain alternatives of CMs and SMs

\[(37) O_{ALT}^{PS}(\phi) \text{ is defined iff } O_{ALT}^{S}(\phi) \subset \phi. \]
Whenever defined, \( O_{ALT}^{PS}(\phi) = O_{ALT}^{S}(\phi) \),
where
a. \( O_{ALT}^{S}(\phi_w) = \phi_w \land \forall p \in ALT [\pi(p)_w \rightarrow \pi(\lambda w. \phi_w) \subseteq \pi(p)] \),
where
(i) \( \pi(q) = \alpha q \land \pi q \).

\[ \Box \text{ last resort, silent, matrix-level, universal doxastic modal} \]
Implicatures from scalar alternatives
considering only alternatives that do not lead to the problematic ‘exactly’ meanings

★ Unembedded:

(38) $O_{\text{ScalAlts}}$ (Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds.)
\[ \sim \neg \text{Alice has 4 / more than 5 / less than 1 / at most 1 / at least 5 diamonds.} \]

★ In the scope of a universal operator:

(39) $O_{\text{ScalAlts}}$ (Alice is required to have 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds.)
\[ \sim \neg \text{Alice is required to have 4 / more than 4 / less than 2 / at most 2 / at least 4 diamonds.} \]
Implicatures from scalar alternatives
considering only alternatives that do not lead to the problematic ‘exactly’ meanings

★ In the antecedent of a conditional:

(40) $O_{ScalAlts}$ (If Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds she wins.)
\[ \sim \neg \text{If Alice has 2 / more than 2 / less than 4 / at most 4 / at least 2 diamonds she wins.} \]

★ In the scope of negation:

(41) $O_{ScalAlts}$ (Alice doesn’t have 3 / more than 3 / less than 3 / *at most 3 / *at least 3 diamonds.)
\[ \sim \neg \text{Alice doesn’t have 1 / more than 1 / less than 5 / at most 5 / at least 1 diamonds.} \]
Implicatures from subdomain alternatives

* Unembedded:

(42) Alice has more/less than 3 / at most/least 3 diamonds.

a. $|P \cap Q| \in D$

b. $O_{\text{SubDomAlts}}^{PS} (|P \cap Q| \in D) = |P \cap Q| \in D \land \neg (|P \cap Q| \in A) \land \neg (|P \cap Q| \in B) \ldots$, for all $A, B, \ldots \subset D$, $= \bot$

c. $O_{\text{SubDomAlts}}^{PS} \Box (|P \cap Q| \in D) = \Box |P \cap Q| \in D \land \neg \Box (|P \cap Q| \in A) \land \neg \Box (|P \cap Q| \in B) \ldots$, for all $A, B, \ldots \subset D$

* Ignorance optional for CMs, obligatory for SMs.
Implicatures from subdomain alternatives

clash with ‘exactly’-inducing implicature from scalar alternatives!

(43) Alice has more than 2 / at least 3 diamonds.

\[ O^\text{PS}_{\text{SubDomAlts}} \Box O^\text{ScalAlts} \left( |P \cap Q| \in \{3, 4, \ldots \} \right) \]

\[ = \Box O^\text{ScalAlts} \left( |P \cap Q| \in \{3, 4, \ldots \} \right) \land \neg \Box (|P \cap Q| \in \{3\}) \land \neg \Box (|P \cap Q| \in \{4, 7\}) \land \neg \ldots \]

\[ = \Box (|P \cap Q| \in \{3\}) \land \neg \Box (|P \cap Q| \in \{3\}) \land \neg \Box (|P \cap Q| \in \{4, 7\}) \land \neg \ldots \]

\[ = \bot \]

* Prune offending SubDomAlts? That would violate \( O^\text{PS}_{\text{SubDomAlts}} \), so no. ✗
* Prune offending ScalAlt? ✔
Implicatures from subdomain alternatives

* In the scope of a universal operator:

(44) Alice is required to have more/less than 3 / at most/least 3 diamonds.

a. $\Box(|P \cap Q| \in D)$

b. $\Box O^{PS}_{SubDomAlts} (|P \cap Q| \in D)$

c. $O^{PS}_{SubDomAlts} (\Box(|P \cap Q| \in D))$

d. $O^{PS}_{SubDomAlts} (|P \cap \Box Q| \in D)$

e. $O^{PS}_{SubDomAlts} (\Box(|P \cap \Box Q| \in D))$

* Ignorance optional for both CMs and SMs.
Implicatures from subdomain alternatives

★ In the scope of negation:

(45) Alice doesn’t have more/less than 3 / *at most/least 3 diamonds.

a. \( \neg (|P \cap Q| \in D) \)

b. \( \neg O_{SubDomAlts}^{PS} (|P \cap Q| \in D) \)

c. \( O_{SubDomAlts}^{PS} \neg (|P \cap Q| \in D) \)

\[ = \neg (|P \cap Q| \in D) \]

d. \( O_{SubDomAlts}^{PS} \Box \neg (|P \cap Q| \in D) \)

\[ = \Box \neg (|P \cap Q| \in D) \]

★ No ignorance implicatures sanctioned formally.
Acceptability in DE environments

★ In the scope of negation:

(46) Alice doesn’t have more/less than three / *at most/least three diamonds.

a. \( \neg (|P \cap Q| \in D) \)  no \( O^{PS}_{SubDomAlts} \) !

b. \( \neg O^{PS}_{SubDomAlts} (|P \cap Q| \in D) \) contradiction!

c. \( O^{PS}_{SubDomAlts} \neg (|P \cap Q| \in D) \) no proper strengthening!

d. \( O^{PS}_{SubDomAlts} \Box \neg (|P \cap Q| \in D) \) no proper strengthening!

★ CMs can be parsed as in (a). No parsing option for SMs.
Acceptability in DE environments

In the antecedent of a conditional / restriction of a universal:

(47) Everyone who has more/less than 3 / at most/least 3 diamonds wins.

\[ \forall x[\# \text{ di } x \text{ has } \in D \rightarrow \ldots] \land \exists x[\# \text{ of di } x \text{ has } \in D] \]

\[ \Downarrow \]

\[ \forall x[\# \text{ di } x \text{ has } \in D' \rightarrow \ldots] \land \exists x[\# \text{ of di } x \text{ has } \in D'] \]

SubDomAlts not entailed, so they must be false.

However, negating them leads to contradiction.

We can rescue the parse with \[\Box\].

Ignorance implicatures about the presupposition: The speaker is sure that here is someone such that the \# of diamonds they have is in D, but not sure about any subsets of D.
## Taking stock

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Lexical entries for the numeral, *much*/little*, [comp], and [sup] that

★ give us the right truth conditions and a natural way to derive the number, type, and status of the alternatives in each case;

★ link up naturally to meanings elsewhere;

★ ensure that the resulting bare or modified numeral DPs will pose no further compositional challenges, as they are generalized quantifiers.
Predicative uses

(48) The three / more/less than three / at most/least three NP
(49) We are three / more/less than three / at most/least three.
(50) Plant a tree every three houses.
(51) If two relatives of mine die, I’ll be rich.

* Use [Partee, 1987]’s BE to typeshift the generalized quantifier meanings into predicative meanings:

(52) $[\text{BE}] = \lambda Q_{\langle at,t \rangle} \cdot \lambda x_\alpha \cdot Q(\lambda y_\alpha \cdot y = x)$

(53) $[\text{BE}] ([\text{at most three students}])$

$= [\lambda Q_{\langle et,t \rangle} \cdot \lambda x_e \cdot Q(\lambda y_e \cdot y = x)](\lambda Q_{\langle e,t \rangle} \cdot |P \cap Q| \in [\text{much}] (3))$

$= \lambda x_e \cdot [\lambda Q_{\langle e,t \rangle} \cdot |P \cap Q| \in [\text{much}] (3)](\lambda y_e \cdot y = x)$

$= \lambda x_e \cdot |P \cap \lambda y_e \cdot y = x| \in [\text{much}] (3)$
Constituent structure

\[ \exists x [ |x| = 3 \land P(x) \land Q(x)] \]

[Diagram]

- DP
- VP
- Q
- D
- #P
- \( \emptyset \exists \)
- \( \lambda x. |x| = 3 \land P(x) \)
- NumP
- [isCard] (3)
- #'
- P
- #
- NP
- SG/PL
- P
Constituent structure

\[
\lambda Q. \ [\text{comp/sup}] ([\text{much/little}]) (n)(P)(Q) \quad Q
\]

\[
\text{ModP}
\]

- \text{Mod}
- \text{NumP}
  - \text{n}
- \text{[comp]/[sup]}
- \text{much/little}

\[
\#P \quad \#' \quad P
\]

\[
\# \quad \text{SG/PL} \quad \text{NP}
\]

\[
\# \quad P
\]
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★ A unified account of bare and modified numerals that builds conservatively on the original GQT account.

★ Derives their patterns w.r.t. entailments, scalar implicatures, ignorance, and acceptability in DE environments from their morphological pieces.

★ The account is more comprehensive, has better empirical coverage, and is less stipulative than previous accounts.
Generalized quantifiers and natural language.

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Modified numerals and maximality.

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Superlative quantifiers and meta-speech acts.

Raising and resolving issues with scalar modifiers.
References II


References III


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References V


