Ignorance and anti-negativity in the grammar: 
\[ or/some \; NP_{SG} \]
and 
comparative-/superlative-modified (CMNs/SMNs) numerals

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NELS 50 @ MIT
ignorance
polarity sensitivity
\textit{or/some } \textit{NP}^i_{SG}

ignorance
polarity sensitivity
\textit{CMNs/SMNs}

\underline{shockingly similar!}

\textbf{Why?}

\textbf{A UNIFIED APPROACH.}*

*Using alternatives and exhaustification.
Ignorance and polarity sensitivity: \textit{or/some NP}_{SG}

(1) Jo called Alice or Bob / some student\_\{Alice,Bob\} · (truth conditions: $\parallel$)
(2) (Who did Jo call?) Jo called Alice or Bob / some student. (ignorance: $\parallel$)
(3) Jo called Alice. So, she called \# Alice, Bob, or Cindy / √some student. (pos certainty: $\parallel$)
(4) Jo called \# Alice, Bob, or Cindy / √some student, but not Alice. (neg certainty: $\parallel$)
(5) If Jo called √Alice or Bob / √some student, she won. (if > __: $\parallel$)
(6) Everyone who called √Alice or Bob / √some student won. (every > __: $\parallel$)
(7) Jo didn’t call √Alice or Bob / \# some student.

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<tr>
<th>compatibility with certainty</th>
<th>no</th>
<th>yes</th>
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<td>anti-negativity</td>
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<td>yes</td>
<td>some NP_SG</td>
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Ignorance and polarity sensitivity: CMNs/SMNs

(8) Jo called less than 2 people / at most 1 person.  \hspace{2cm} \text{(truth conditions: } \parallel \text{)}

(9) (How many did Jo call?) Jo called less than 2 people / at most 1 person.  \hspace{2cm} \text{(ignorance: } \parallel \text{)}

(10) Jo called 2 people. Therefore, she called \checkmark less than 3 / # at most 2.  \hspace{2cm} \text{(pos certainty: } \not\parallel \text{)}

(11) Jo called \checkmark less than 3 / # at most 2 people, but not 1.  \hspace{2cm} \text{(neg certainty: } \not\parallel \text{)}

(12) If Jo called \checkmark less than 2 people / \checkmark at most 1 person, she won.  \hspace{2cm} \text{\textit{(if} } \checkmark \text{\textit{: } } \parallel \text{)}

(13) Everyone who called \checkmark less than 2 people / \checkmark at most 1 person won.  \hspace{2cm} \text{\textit{(every} } \checkmark \text{\textit{: } } \parallel \text{)}

(14) Jo didn’t call \checkmark less than 2 people / # at most 1 person.  \hspace{2cm} \text{\textit{(not} } \checkmark \text{\textit{: } } \not\parallel \text{)}

\begin{center}
\begin{tabular}{ll}
\hline
compatibility with certainty & no & yes \\
anti-negativity & no & \text{CMNs} \\
& yes & \text{SMNs} \\
\hline
\end{tabular}
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Existing literature

- ignorance and anti-negativity in French disjunctions *soit ... soit/ou*: [Spector, 2014, Nicolae, 2017] An item like *or* that cannot prune its DA-set only has this option.
- experimental evidence that both CMNs and SMNs can give rise to ignorance: [Westera and Brasoveanu, 2014, Cremers et al., 2017, Nouwen et al., 2018]
- experimental evidence that CMNs are compatible with positive certainty but SMNs are not [Geurts and Nouwen, 2007, Geurts et al., 2010, Cummins and Katsos, 2010, Nouwen et al., 2018]
- experimental evidence of *not-if-every* patterns for CMNs and SMNs: [Mihoc and Davidson, 2017]
- the empirical similarity between SMNs and disjunction with respect to ignorance: [Büring, 2008, Kennedy, 2015]
- the empirical similarity between SMNs and some French disjunctions w.r.t. both ignorance and polarity sensitivity [Spector, 2014, Spector, 2015]
Existing literature

- disjunction
- epistemic indefinites
- polarity sensitive items
- modified numerals
Today’s talk

<table>
<thead>
<tr>
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Goal and plan

Goals:

★ Figure out an account for ignorance and polarity sensitivity in \textit{or/some NP}_{SG}.  
★ Identify the shape of a general theory of ignorance and polarity sensitivity.

There are many approaches to ignorance and polarity sensitivity. The only \textit{unified} approaches use alternatives and exhaustification. The only approach with explicit concern for variation: [Chierchia, 2013].

Plan:

★ We will use [Chierchia, 2013] for reference throughout.
Assumptions: Truth conditions
Contain reference to both a domain and a scalar element.

(15) Jo called $a$, $b$ or ...  \hspace{3cm} (16) Jo called some student.
\hspace{1cm} a. $\exists x \in \{a, b, \ldots\} \, [C(j, x)]$  \hspace{2cm} a. $\exists x \in \text{[student]} \, [C(j, x)]$
\hspace{1cm} \text{(assertion)} \hspace{2cm} \text{(assertion)}

* If the domains coincide, this captures (truth conditions: $\parallel$).
Assumptions: Alternatives
Generated by replacing the domain with its subsets and the scalar element with its scalemates.

(17) Jo called $a$, $b$ or ...

a. $\exists x \in \{a, b, \ldots\}[C(j, x)]$ (assertion)
b. $\{\exists x \in D'[C(j, x)] \mid D' \subset \{a, b, \ldots\}\}$ (DA)
c. $\{\forall x \in \{a, b, \ldots\}[C(j, x)]\}$ (σA)
d. $\{\forall x \in D'[C(j, x)] \mid D' \subset \{a, b, \ldots\}\}$ (DσA)

(18) Jo called some student.

a. $\exists x \in \text{[student]}[C(j, x)]$ (assertion)
b. $\{\exists x \in D'[C(j, x)] \mid D' \subset \text{[student]}\}$ (DA)
c. $\{\forall x \in \text{[student]}[C(j, x)]\}$ (σA)
d. $\{\forall x \in D'[C(j, x)] \mid D' \subset \text{[student]}\}$ (DσA)
Assumptions: Exhaustification
A silent exhaustivity operator O negates the non-entailed pre-exhaustified subdomain alternatives and scalar alternatives.

\[ [O_C(p)]^{g,w} = [p]^{g,w} \land \forall q \in [p]^C \left( [q]^{g,w} \rightarrow \lambda w'. [p]^{g,w'} \subseteq q \right) \]

E.g., \( O_{DA}(a \lor b) = (a \lor b) \land \neg a \land \neg b, = \bot \)  \hspace{1cm} (G-trivial)
E.g., \( O_{\sigma A}(a \lor b) = (a \lor b) \land \neg (a \land b) \)  \hspace{1cm} (\sim \land \not\text{every})

\* For or/some \( NP_{SG} \), \( O_{DA} \) is actually \( O_{ExhDA} \): the DA must be used in a pre-exhaustified form, obtained by exhaustifying each fully grown DA relative to other DA of the same size:
E.g., \( O_{ExhDA}(a \lor b) = (a \lor b) \land \neg O(a) \land \neg O(b), = (a \lor b) \land (a \rightarrow b) \land (b \rightarrow a), = (a \land b) \)

\* For or/some \( NP_{SG} \), both the ExhDA and the \( \sigma A \) are used by default, e.g., via \( O_{ExhDA+\sigma A} \).
E.g., \( O_{ExhDA+\sigma A}(a \lor b) = (a \lor b) \land \neg O(a) \land \neg O(b) \land \neg(a \land b), = \bot \)
Jo called Alice or Bob / some student \{Alice, Bob\}.

* Why is this grammatical, and how does it give rise to ignorance?
* Ignorance is a silent modal effect.
* Let’s look at some sentences with modals …
Jo may call Alice or Bob / some student \{Alice, Bob\}.

\[O_{ExhDA+\sigma A} \xrightarrow{\Diamond (a \lor b)} \Diamond (a \lor b) \land \Diamond a \land \Diamond b \land \neg \Diamond (a \land b)\]

Free Choice
Jo must call Alice or Bob / some student\{Alice, Bob\}.

\[ O_{\text{ExhDA+σA}} \]

Free Choice

\[ □(a ∨ b) \]

\[ □(a ∨ b) ∧ ¬□a ∧ ¬□b ∧ ¬□(a ∧ b) \]
Jo called Alice or Bob / some student \{Alice, Bob\}.

\[ (a \lor b) \]

\[ \Box_S (a \lor b) \land \neg \Box_S a \land \neg \Box_S b \land \neg \Box_S (a \land b) \]

epistemic Free Choice = ignorance

★ This captures (ignorance: ||)
★ But the result is total ignorance. How do we capture compatibility with partial ignorance?
★ Assumption: Partial variation effects come from pruning the DA-set down to a natural subset.
★ Let’s study exhaustification relative to SgDA, NonSgDA.
Jo called # Alice, Bob, or Cindy / ✓ some student\{Alice, Bob, or Cindy\}, but not Alice.

\[ (a \lor b \lor c) \rightarrow O_{ExhSgDA+\sigma A} \rightarrow O_{ExhSgDA+\sigma A} \]

\[ \Box_S \neg a \land \neg \Box_S b \land \neg \Box_S c \]

\text{partial ignorance w/ neg certainty}

* Assumption: To accommodate context, some \(NP_{SG}\) can prune its DA-set down to just \(SgDA\).
* This captures (neg certainty: \(\not\parallel\)).
Jo called Alice.
So, she called # Alice, Bob, or Cindy / ✓ some student \{Alice, Bob, Cindy\}.

\[
(a \lor b \lor c) \quad O_{E\text{xhNonSgDA}+\sigmaA} \quad \Box_S a \land \neg \square_S / \square_S \neg b \land \neg \square_S / \square_S \neg c
\]

partial ignorance w/ pos certainty

* Assumption: To accommodate context, some \(NP_{SG}\) can prune its DA-set down to just NonSgDA.
* This captures (pos certainty: \(\parallel\)).
Note on scalar implicatures

★ Quite generally, the ExhDA-implicatures are also compatible with no ignorance.
★ However, as we saw, the $\sigma A$-implicatures prevent that.
★ Yet:

(20) Jo called Alice or Bob / some student$_{\{Alice,Bob\}}$. In fact, she called both / every student.

★ Assumption: To accommodate context, or/some NP$_{SG}$ can both prune their $\sigma A$. 
Jo didn’t call ✓Alice or Bob / # some student\{Alice, Bob\}.

\[ \neg(a \vee b) \rightarrow O_{\text{ExhDA}+\sigma A} \rightarrow O_{\text{ExhDA}}: \text{no proper strengthening} \]

- Assumption: some \( NP_{SG} \) doesn’t tolerate a use of its ExhDA that doesn’t lead to PS.
- This captures (not > __: \( \| \)).
If Jo called Alice or Bob / some student\{Alice, Bob\}, she won. Everyone who called Alice or Bob / some student\{Alice, Bob\} won.

⋆ Assumption: Exhaustification proceeds relative to presupposition-enriched content.

\[\forall v[(a \lor b)v \rightarrow W_v] \land \exists v[(a \lor b)v] \quad \text{O}_{\text{ExhDA}+\sigma A} \]

⋆ This captures \(\text{if/every} > ___: ||\).
Summary

★ Figure out an account for ignorance and polarity sensitivity in or/some \( N_{P_{SG}} \).

★ Identify the shape of a general theory of ignorance and polarity sensitivity.
Comparison to previous literature

Comparison to [Spector, 2014, Nicolae, 2017]’s solutions for French PPI disjunctions:

⋆ similarity in the general use of alternatives-and-exhaustification, but
⋆ differences in the formal assumptions and solution for ignorance and polarity sensitivity
   — consequences for $or/some \text{NP}_{SG}$

Comparison to [Chierchia, 2013]’s solution for variation among epistemic indefinites:

⋆ similarity in all the crucial pieces, but
⋆ differences in some of the details related to pre-exhaustification and pruning
⋆ revisions towards unification that wouldn’t affect the present analysis include:
   — the O used to generate ExhDA is actually $O_{IE-DA}$
   — pre-exhaustification of NonSgDA is actually relative to both NonSgDA and SgDA
Goals and plan

Goals:

★ Figure out an account for ignorance and polarity sensitivity in CMNs/SMNs.
★ Consider consequences for a general theory of bare and modified numerals.
Existing literature: Truth conditions

Contain reference only to a scalar element.

(21) $n$ people quit.
   a. $\exists x [ |x| = n \land P(x) \land Q(x)]$  (assertion)

(22) More/less than $n$ people quit.
   a. $\max(\lambda d. \exists x [ |x| = d \land P(x) \land Q(x)]) > / < n$  (assertion)

(23) At most/least $n$ people quit.
   a. $\max(\lambda d. \exists x [ |x| = d \land P(x) \land Q(x)]) \leq / \geq n$  (assertion)
Assumptions: Truth conditions
As before, contain reference to both a domain and a scalar element.

(24) \( n \) people quit.
   a. \( \exists x[|x| = n \wedge P(x) \wedge Q(x)] \) (assertion)

(adapting [Kennedy, 1997] to degrees)

(25) \([\text{much}] = \lambda n . \lambda d . d \leq n \)  
    e.g., \([\text{much}] (3) = \lambda d . d \leq 3 \)

(26) \([\text{little}] = \lambda n . \lambda d . d \geq n \)  
    e.g., \([\text{little}] (3) = \lambda d . d \geq 3 \)

(27) More/less than \( n \) people quit.
   a. \( \max(\lambda d . \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\text{much/little}] (n) \) (assertion)

(28) At most/least \( n \) people quit.
   a. \( \max(\lambda d . \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\text{much/little}] (n) \) (assertion)

* If the domains coincide, this captures (truth conditions: \parallel\).
Assumptions: Alternatives

As before, generated by replacing the domain with its subsets and the scalar element with its scalemates.

(29) \( n \) people quit.
   a. \( \exists x \left[ |x| = n \land P(x) \land Q(x) \right] \) \hspace{2cm} (assertion)
   b. \( \neg \) \hspace{2cm} (no DA)
   c. \( \{ \exists x \left[ |x| = m \land P(x) \land Q(x) \right] \mid m \in S \} \) \hspace{2cm} (\( \sigma A \))

(30) More/less than \( n \) people quit.
   a. \( \max(\lambda d. \exists x \left[ |x| = d \land P(x) \land Q(x) \right]) \in \text{[much/little]}(n) \) \hspace{2cm} (assertion)
   b. \( \{ \max(\lambda d. \exists x \left[ |x| = d \land P(x) \land Q(x) \right]) \in D' \mid D' \subset \text{[much/little]}(n) \} \) \hspace{2cm} (DA)
   c. \( \{ \max(\lambda d. \exists x \left[ |x| = d \land P(x) \land Q(x) \right]) \in \text{[much/little]}(m) \mid m \in S \} \) \hspace{2cm} (\( \sigma A \))

(31) At most/least \( n \) people quit.
   a. \( \max(\lambda d. \exists x \left[ |x| = d \land P(x) \land Q(x) \right]) \in \text{[much/little]}(n) \) \hspace{2cm} (assertion)
   b. \( \{ \max(\lambda d. \exists x \left[ |x| = d \land P(x) \land Q(x) \right]) \in D' \mid D' \subset \text{[much/little]}(n) \} \) \hspace{2cm} (DA)
   c. \( \{ \max(\lambda d. \exists x \left[ |x| = d \land P(x) \land Q(x) \right]) \in \text{[much/little]}(m) \mid m \in S \} \) \hspace{2cm} (\( \sigma A \))
Assumptions: Exhaustification

As before, O negates the non-entailed pre-exhaustified subdomain alternatives and scalar alternatives.
Scalar implicatures – reasons to rehabilitate them

★ Conceptual generality: All our items entail one bound and implicate another.

★ Makes good empirical predictions in general, and in particular for (35) (indirect SI).

(32) Jo called 3 people / more than 3 / at least 3 people.
   \[\sim \rightarrow \neg \text{Jo called 4 / more than } #4 \checkmark 5 / \text{at least } #4 \checkmark 5 \text{ people.}\]

(33) Jo is required to call 3 / more than 3 / at least 3 people.
   \[\sim \rightarrow \neg \text{Jo is required to call 4 / more than } \checkmark 4 / \text{at least } 4 \text{ people.}\]

(34) Jo didn’t call 3 people / more than 3 / at least 3 people.
   \[\sim \rightarrow \neg \text{Jo didn’t call } #2 \checkmark 1 / \text{more than } #2 \checkmark 1 / \text{at least } #2 \checkmark 1 \text{ people.}\]

(35) If Jo called 3 / more than 3 / at least 3 people, she won.
   \[\sim \rightarrow \neg \text{If Jo called } \checkmark 2 / \text{more than } \checkmark 2 / \text{at least } \checkmark 2, \text{ she won.}\]

★ The bad predictions disappear once we dig deeper.
Jo called less than 2 / at most 1 people.

Why is this grammatical, and how does it give rise to ignorance?

Ignorance is a silent modal effect.

Let’s look at some sentences with modals …
Jo may call less than 2 / at most 1 people.

\[ O_{\text{ExhDA+} \sigma A} \]

\[ \Diamond (0 \lor 1) \land \Diamond 0 \land \Diamond 1 \Rightarrow \Box \emptyset \]

Free Choice

\[ \Diamond (0 \lor 1) \]

* Assumption: CMNs/SMNs can prune their \( \sigma A \) simply to avoid a clash with the ExhDA.

* Justification: \( \sigma A \)-implicatures play a different role for CMNs/SMNs than for or/some NP_{SG}. 
Jo must call less than 2 / at most 1 people.
Jo called less than 2 \(/
\) at most 1 people.

\[ (0 \lor 1) \quad \rightarrow \quad O_{\text{ExhDA+}} \sigma_A \quad \rightarrow \quad S (0 \lor 1) \land \neg S 0 \land \neg S 1 \land \neg S 0 \]

epistemic Free Choice = ignorance

⋆ This captures (ignorance: \( \parallel \)).
⋆ But the result is total ignorance. How do we get compatibility with certainty?
⋆ As before …
Jo called ✓less than 3 / # at most 2 people, but not 1.

Assumption: To accommodate context, CMNs can prune their DA-set to just SgDA.

This captures (neg certainty: $\not\Box$).
Jo called 2 people. Therefore, she called \( \surd \) less than 3 / \# at most 2.

- **Assumption**: To accommodate context, CMNs can prune their DA-set to just NonSgDA.
- **This captures** (pos certainty: \( \| \)).
Ignorance and strong scalar implicatures

(36) Jo called less than 3 / at most 2 people.

\[ O_{\text{ExhDA}}(\Box_S O_{\sigma A}(0 \lor 1 \lor 2)) \]

\[ \neg O\Box_S 0 \land \neg O\Box_S 1 \land \neg O\Box_S 2 \land \neg O\Box_S (0 \lor 1) \land \neg O\Box_S (1 \lor 2) \land \neg O\Box_S (0 \lor 2) \]

\[ = (a) \land (b) \]

\[ \Box_S((0 \lor 1 \lor 2) \land \neg(0 \lor 1)) \land \neg\Box_S 0 \land \neg\Box_S 1 \land \neg\Box_S 2 \]

\[ = \Box_S 2 \]

\[ \perp \]

* Assumption: CMNs/SMNs can prune their \( \sigma A \) simply to avoid a clash with the ExhDA.
* Justification: \( \sigma A \)-implicatures play a different role for CMNs/SMNs than for or/some \( \text{NP}_{SG} \).
* The above can \( \not\sim \not\sim \) not 0.
Jo didn’t call ✓ less than 2 / # at most 1 people.

\( \neg(0 \lor 1) \rightarrow O_{ExhDA+\sigma A} \rightarrow O_{ExhDA}: \text{no proper strengthening} \)

★ Assumption: SMNs don’t tolerate a use of their ExhDA that doesn’t lead to PS.
★ This captures \( \text{not} > \_\_ \_ : \| \).
If Jo called ✓Alice, Bob, or Cindy / ✓some student, she won. Everyone who called ✓Alice, Bob, or Cindy / ✓some student won.

★ Assumption: Exhaustification proceeds relative to presupposition-enriched content.

★ This captures (if/every > __: ||).
Negation and strong scalar implicatures

* Assumption: The \( \sigma_A \) of, e.g., 3 under negation are \{\ldots, \neg 2, \neg 4, \ldots \} but also \{\ldots, 2, 4, \ldots \}.
* Negating all the non-entailed \( \sigma_A \) leads to \( \bot \).
* With last resort insertion of \( \Box_S \), it leads to ignorance:

(37) Jo didn’t call three / more than two / # at least three people. \( \not\leftrightarrow \) ‘exactly 2’
\[
O_{\sigma_A \Box_S \neg (3 \lor 4 \lor \ldots )}
\]
\[\neg \Box_S \neg (3 \lor 4 \lor \ldots ) \land \neg \Box_S \neg (1 \lor \ldots ) \land \ldots \] (traditional \( \sigma_A \))
\[\neg \Box_S (2 \lor \ldots ) \land \neg \Box_S (1 \lor \ldots ) \land \ldots \] (new \( \sigma_A \), obtained by deleting \( \neg )\)

‘In all the worlds compatible with what the speaker believes the relevant number is not three or more but the speaker is not sure which one of the remaining numbers (0 or 1 or 2) it is.’
\[\not\rightarrow \Diamond_S 0 \land \Diamond_S 1 \land \Diamond_S 2 \]
Summary

* Figure out an account for ignorance and polarity sensitivity in CMNs/SMNs. ✓

* Consider consequences for a general theory of bare and modified numerals. ✓
Comparison to the existing alternatives(-and-exhaustification) solutions

* conceptual advantages:
  - more compositional truth conditions
  - more general alternative generation mechanism
  - more general implicature calculation
  - more general approach to ignorance, polarity sensitivity, and scalar implicatures

* empirical advantages:
  - better captures ignorance/other modal/quantificational effects in CMNs vs. SMNs
  - better captures polarity sensitivity in SMNs
  - better captures scalar implicatures in CMNs and SMNs
  - better captures general similarity to disjunction/indefinites
Conclusion: Why are or/some $NP_{SG}$ and CMNs/SMNs so similar?

\[ \begin{align*}
\text{parameters} \quad & \quad \downarrow \\
D, \sigma \quad & \quad \rightarrow \text{output} \\
O \rightarrow \quad & \quad \downarrow \\
\text{DA, } \sigma A \quad & \quad \text{variation}
\end{align*} \]
Further patterns of immediate interest:
- embedding under other DE operators and/or combinations thereof
- sensitivity to other types of polarity

Predictions for the range of empirical variation:
- * or with anti-negativity: French *soit ... soit* or *ou*
- some $NP_{SG}$ incompatible with certainty and with no anti-negativity: *irgendein*
- * or compatible with partial ignorance:
- CMNs like SMNs, SMNs like CMNs:

Predictions for the nature of ungrammaticality:
- How do violations of no DA-pruning and proper strengthening compare to logical contradiction, cancelation of scalar implicatures, or logical redundancy?
Acknowledgments:

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Thank you!
More/less than three / at most/least three people quit

= 1 iff \( \max(\lambda d . \exists x[|x| = d \land \text{people}(x) \land \text{quit}(x)]) \in \text{[much/little]} (3)/\text{[much/little]} (3) \)

**Figure:** The syntax and semantics of CMNs and SMNs.


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