The alternatives of bare and modified numerals

3 (BNS) (ScalAlts)
more/less than 3 (CMNs) (ScalAlts), (SubDomAlts)
at most/least 3 (SMNs) (ScalAlts), SubDomAlts

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3, more/less than 3, and at least/most 3 differ w.r.t. (at least)
- entailments,
- scalar implicatures,
- ignorance, and
- acceptability in downward-entailing environments.

Many theories have been proposed to capture these differences.

Lately a move towards alternative-based theories.

Promising results, but also empirical and conceptual issues.

I will propose a theory that overcomes these issues.
Outline

Empirical patterns, existing proposals, issues
  Entailments
  Scalar implicatures
  Ignorance
  Acceptability in DE environments

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Appendix
Entailments

★ *3 / more than 3 / at least 3* carry lower-bounding entailments.

(1) a. Alice has 3 diamonds.
   b. \(\sim\) not 2 or less
   c. Alice has 3 diamonds, \(\#\) if not less.

★ *less than 3 / at most 3* carry upper-bounding entailments.

(2) a. Alice has less than 3 diamonds.
   b. \(\sim\) not 3 or more
   c. Alice has less than 3 diamonds, \(\#\)if not more.

★ Existing proposals: Multiple possible solutions, typically not compositional down to the smallest pieces.
★ *We want* one that gets these entailments with ease and also minimally uncovers the uniform contribution of the numeral, *much/little*, or [−er]/[at -est] in producing these entailments.
 Scalar implicatures I

★ BNs also carry upper-bounding scalar implicatures. [Horn, 1972]

(3) a. Alice has 3 diamonds.
   b. $\sim \rightarrow$ not 4 yields ‘exactly 3’ meaning ✓
   c. Alice has 3 diamonds, if not 4.

★ CMNs and SMNs don’t seem to. [Krifka, 1999]

(4) a. Alice has more than 3 diamonds.
   b. $\not\rightarrow$ not more than 4 yields ‘exactly 4’ meaning ✗

★ Existing proposals: No scalar implicatures for CMNs and SMNs.
Scalar implicatures II

- But in certain contexts all give rise to scalar implicatures!

(5)  a. If you have at least 3 diamonds, you win.
  b. $\neg\neg$ not if at least 2

- And in some none do:

(6)  a. Alice doesn’t have 3 diamonds.
  b. $\neg\neg$ not not 2 yields ‘exactly 2’ meaning $\nabla$

- We want scalar implicatures for all!
- We need a separate mechanism to rule out certain implicatures.
Scalar implicatures III

★ With coarser granularity, CMNs and SMNs can give rise scalar implicatures too. [Spector, 2014, Cummins et al., 2012, Enguehard, 2018]

(7) Grades are given based on the number of problems solved. People who solve more than 5 problems but fewer than 9 problems get a B, and people who solve 9 problems or more get an A.
   a. John solved more than 5 problems.
   b. \( \sim \gg \) not more than 9 (he gets a B) example from [Spector, 2014]

★ That is true of BNs in the problem cases also.

(8) a. Alice doesn’t have 3 diamonds.
   b. \( \not\sim \not\gg \) not not 1 (she does have some)
SMNs give rise to strong speaker ignorance inferences.

(9) I have 3 / more than 2 / ??at least 3 children.

Existing proposals: e.g., [Büring, 2008, Kennedy, 2015, Spector, 2015]
- SMNs are underlyingly disjunctive (*at least 3 = exactly 3 or more than 3*) and have domain alternatives (the individual disjuncts).
- Ignorance inferences are implicatures from these alternatives.
- Nothing of this sort is assumed / derived for CMNs.
CMNs give rise to ignorance inferences too! [Cremers et al., 2017]

(10) [A:] How many diamonds does Alice have? [B:] More than 3.

Unlike BNs and like SMNs, CMNs are compatible with ignorance:

(11) I don’t know how many diamonds Alice has, but she has # 3 / more than 3 / at least 3.

Unlike CMNs, SMNs are incompatible with exact knowledge. [Nouwen, 2015]

(12) There were exactly 62 mistakes in the manuscript, so that’s more than 50 / # at least 50.

We want ignorance implicatures for CMNs too!
We want ignorance to be weaker for CMNs than for SMNs.
SMNs are bad under negation.


(13) Alice doesn’t have *at least three / *at most three diamonds.
→ Alice has 2 or less / 4 or more diamonds.

Existing proposals: The domain alternatives of SMNs are obligatory and must lead to a stronger meaning, but that cannot happen in a DE environment like negation.

[Spector, 2015]
SMNs are okay in the antecedent of a conditional or the restriction of a universal!


(14) If Alice has at least 3 diamonds, she wins.

(15) Everyone who has at least 3 diamonds wins.

We want a solution that can distinguish between various types of DE environments!
Summary and preview of proposal

⋆ BNs, CMNs, and SMNs are non-uniform w.r.t.
   Entailments
   Scalar implicatures
   Ignorance
   Acceptability in DE environments

⋆ The existing alternative-based proposals are promising, but still:
  - they take into evidence an incomplete dataset;
  - they make non-uniform stipulations about the alternatives;
  - they fail to capture all the patterns we saw.

⋆ In this talk:
  - I take into evidence a revised and extended dataset;
  - I derive the alternatives of BNs, CMNs, and SMNs in a uniform
    way from their truth conditions;
  - I show how, with certain general assumptions about
    implicature calculation, we get all the patterns we saw.
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Proposal: Truth conditions and presupposition

<table>
<thead>
<tr>
<th>the numeral</th>
<th>[Link, 1983, Buccola and Spector, 2016]</th>
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<tbody>
<tr>
<td>([n] = n)</td>
<td>[\text{is}_{\text{Card}}(n) = \lambda x.</td>
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<tr>
<th>much/little</th>
<th>[Seuren, 1984, Kennedy, 1997]</th>
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<tr>
<td>(<a href="n">\text{much}</a> = \lambda d. d \leq n)</td>
<td>(<a href="n">\text{little}</a> = \lambda d. d \geq n)</td>
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<tr>
<td>((\exists (n P))(Q) = 1 \text{ iff } \exists x[</td>
<td>x</td>
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<tr>
<td>(<a href="%5Ctext%7Bmuch%7D/%5Ctext%7Blittle%7D">\text{at-sup}</a>(n)(P)(Q) = 1 \text{ iff }</td>
<td>P \cap Q</td>
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<tr>
<th>the presupposition of at-sup</th>
<th>[Hackl, 2009, Gajewski, 2010]</th>
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<td>(</td>
<td><a href="n">\text{much}/\text{little}</a></td>
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Entailments

(16) $3 \, P \, Q$:
\[ \exists x [ |x| = 3 \land P(x) \land Q(x)] \]
(l.b.)

(17) more than $3 \, P \, Q$:
\[ |P \cap Q| \in \{ \text{much} \} (3) \iff |P \cap Q| \in \{4, 5, \ldots \} \]
(l.b.)

(18) less than $3 \, P \, Q$:
\[ |P \cap Q| \in \{ \text{little} \} (3) \iff |P \cap Q| \in \{\ldots, 0, 1, 2\} \]
(u.b.)

(19) at most $3 \, P \, Q$:
\[ |P \cap Q| \in \{ \text{much} \} (3) \iff |P \cap Q| \in \{\ldots, 0, 1, 2, 3\} \]
(u.b.)

(20) at least $3 \, P \, Q$:
\[ |P \cap Q| \in \{ \text{little} \} (3) \iff |P \cap Q| \in \{3, 4, \ldots \} \]
(l.b.)
Proposal: Alternatives

Scalar alternatives: Replace the \( n \)-domain with an \( m \)-domain.

- BNs: \( \exists x [ |x| = m \land P(x) \land Q(x)] : m \in S \) 
- CMs: \( |P \cap Q| \in \overline{[\text{much/little}]} (m) : m \in S \) 
- SMs: \( |P \cap Q| \in \overline{[\text{much/little}]} (m) : m \in S \) 

Subdomain alternatives: Replace the \( n \)-domain with its subsets.

- BNs: NA (the numeral argument is just a degree) 
- CMs: \( |P \cap Q| \in A : A \subseteq \overline{[\text{much/little}]} (n) \) 
- SMs: \( |P \cap Q| \in A : A \subseteq \overline{[\text{much/little}]} (n) \)  
  active by presup! 
  obligatory exhaustification relative to SubDomAlts
Examples

(21) BNs: 3 $P \land Q$
   a. Truth conditions: $\exists x[|x| = 3 \land P(x) \land Q(x)]$
   b. ScalAlts: $\{\ldots, \exists x[|x| = 2 \ldots], \exists x[|x| = 4 \ldots, \ldots \}$
   c. SubDomAlts: NA

(22) CMNs: e.g., more than 3 $P \land Q$
   a. Truth conditions: $|P \cap Q| \in \underline{\text{[much]}} (3)$
   b. ScalAlts: $\{\ldots, |P \cap Q| \in \underline{\text{[much]}} (2), |P \cap Q| \in \underline{\text{[much]}} (4), \ldots \}$
   c. SubDomAlts: $\{|P \cap Q| \in A : A \subseteq \underline{\text{[much]}} (3)\}$

(23) SMNs: e.g., at least 3 $P \land Q$
   a. Truth conditions: $|P \cap Q| \in \underline{\text{[little]}} (3)$
   b. ScalAlts: $\{\ldots, |P \cap Q| \in \underline{\text{[little]}} (2), |P \cap Q| \in \underline{\text{[little]}} (4), \ldots \}$
   c. SubDomAlts: $\{|P \cap Q| \in A : A \subseteq \underline{\text{[little]}} (3)\}$
Proposal: Implicature calculation system [Chierchia, 2013]

\( O \) to exhaustify the scalar alternatives of BNs, CMNs, and SMNs

\[(24) \quad [O_{ALT}(p)] = p \land \forall q \in ALT \ [q \rightarrow p \subseteq q] \]

\( O^{PS} \) to exhaustify the subdomain alternatives of CMNs and SMNs

\star \quad A \text{ version of } O \text{ that}
- takes into account presuppositions:

\[(25) \quad [O_{ALT}^{S}(p)] = \pi(p) \land \forall q \in ALT \ [\pi(q) \rightarrow \pi(p) \subseteq \pi(q)], \]
- requires a properly stronger result:

\[(26) \quad [O_{ALT}^{PS}(p)] \text{ is defined iff } O_{ALT}^{S}(p) \subset p. \]
Whenever defined, \( [O_{ALT}^{PS}(p)] = [O_{ALT}^{S}(p)]. \)

\[\quad \square \quad \text{last resort, silent, matrix-level, universal doxastic modal} \]
Implicatures from ScalAlts: Scalar implicatures

(27) Alice has 3 diamonds.
   a. $O_{\text{ScalAlts}} \left( \exists x [ |x| = 3 \land P(x) \land Q(x)] \land \right)$
   $= \exists x [ |x| = 3 \land P(x) \land Q(x)] \land$
   $\neg \exists x [ |x| = 4 \land P(x) \land Q(x)] \land \ldots$  

(28) Alice has at least 3 diamonds.
   a. $O_{\text{ScalAlts}} \left( |P \cap Q| \in \llbracket \text{little} \rrbracket (3) \right)$
   $= |P \cap Q| \in \llbracket \text{little} \rrbracket (3) \land$
   $\neg |P \cap Q| \in \llbracket \text{little} \rrbracket (5) \land \ldots$  

(29) If Alice has more than 3 diamonds, she wins.
   a. $O_{\text{ScalAlts}} \left( [|P \cap Q|] \in \llbracket \text{much} \rrbracket (3) \rightarrow \text{win} \right)$
   $= [|P \cap Q|] \in \llbracket \text{much} \rrbracket (3) \rightarrow \text{win} \right) \land$
   $\neg [|P \cap Q|] \in \llbracket \text{much} \rrbracket (2) \rightarrow \text{win} \right) \land \ldots$  

★ And so on. We can derive all the attested scalar implicatures.
★ Scalar implicatures may be restricted by granularity.
★ In unembeded contexts this effect is compounded by ignorance.
(30) Alice has more/less than 3 / at most/least 3 diamonds.

a. \( O_{SubDomAlts}^{PS} (|P \cap Q| \in D) \)
   \[
   = |P \cap Q| \in D \land \\
   \neg |P \cap Q| \in A \land \\
   \neg |P \cap Q| \in B \land \ldots, \text{for all } A, B, \ldots \subset D, = \bot
   \]
   contradiction \( \times \)

b. \( O_{SubDomAlts}^{PS} (\square|P \cap Q| \in D) \)
   \[
   = \square|P \cap Q| \in D \land \\
   \neg \square|P \cap Q| \in A \land \\
   \neg \square|P \cap Q| \in B \land \ldots, \text{for all } A, B, \ldots \subset D
   \]
   ignorance \( \checkmark \)

\* The only consistent \( O_{SubDomAlts}^{PS} \) parse yields ignorance.
\* SMNs can only have an \( O_{SubDomAlts}^{PS} \) parse, so *(ignorance)*
\* CMNs can also have a parse without \( O_{SubDomAlts}^{PS} \), so (ignorance).
Scalar implicatures vs. ignorance implicatures

(31) Alice has more than 2 / at least 3 diamonds.

\[ O_{\text{SubDomAlts}}^{PS} \Box O_{\text{ScalAlts}} (|P \cap Q| \in \{3, 4, \ldots\}) \]
\[ = \Box O_{\text{ScalAlts}} (|P \cap Q| \in \{3, 4, \ldots\}) \land \]
\[ \neg \Box (|P \cap Q| \in \{3\}) \land \]
\[ \neg \Box (|P \cap Q| \in \{4, 7\}) \land \ldots \]

\[ = \Box(|P \cap Q| \in \{3, 4, \ldots\}) \land \neg |P \cap Q| \in \{4, \ldots\}) \land \]
\[ \neg \Box (|P \cap Q| \in \{3\}) \land \]
\[ \neg \Box (|P \cap Q| \in \{4, 7\}) \land \ldots = \bot \]

\[ \text{contradiction} \times \]

★ Prune offending SubDomAlts? That would violate \( O_{\text{SubDomAlts}}^{PS} \). \( \times \)
★ Prune offending ScalAlt? \( \checkmark \)
Implicatures from SubDomAlts: Negation

(32) Alice doesn’t have more/less than three / *at most/least three diamonds.

a. $\neg O_{SubDomAlts}^{PS} (|P \cap Q| \in D)$
   contradiction ✗

b. $O_{SubDomAlts}^{PS} (\neg |P \cap Q| \in D)$
   PS violated ✗

c. $O_{SubDomAlts}^{PS} (\square \neg |P \cap Q| \in D)$
   PS violated ✗

★ All $O_{SubDomAlts}^{PS}$ parses fail.
★ SMNs cannot have a non-$O_{SubDomAlts}^{PS}$ parse, so bad.
★ CMNs can be parsed without $O_{SubDomAlts}^{PS}$, so okay.
**Implicatures from SubDomAlts: AntCond/RestUniv**

(33) $O_{SubDomAlts}^{PS}$ (Everyone who has at least 3 diamonds wins.)

Prejacent: $\forall x[\# \text{ di } x \text{ has } \in D \to \ldots] \land \exists x[\# \text{ of di } x \text{ has } \in D]$  
$\downarrow$

SubDomAlt: $\forall x[\# \text{ di } x \text{ has } \in D' \to \ldots] \land \exists x[\# \text{ of di } x \text{ has } \in D']$

- SubDomAlts not entailed, so they must be false.
- However, negating them leads to contradiction.
- We can rescue the parse with $\Box$:

(34) $\Box \exists x[\# \text{ of di } x \text{ has } \in D] \land \neg \Box \exists x[\# \text{ of di } x \text{ has } \in D']$

PS satisfied

- Thus there is a consistent $O_{SubDomAlts}^{PS}$ parse for SMNs, which is why they are felicitous in this type of DE environments.
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The existential implicature of *at most*  

(35) LeBron scored at most 20 points (and it’s even possible that he didn’t score any points at all).

\[ O_{ScalAlts} (|P \cap Q| \in \text{[much]} (20)) \]

\[ = |P \cap Q| \in \text{[much]} (20) \land \]

\[ \neg |P \cap Q| \in \text{[much]} (18) \land \]

\[ \neg |P \cap Q| \in \text{[much]} (17) \land \]

\[ \ldots \]

\[ \neg |P \cap Q| \in \text{[much]} (0) \]

**existential implicature ✓**

★ Lower-bounding inference is a scalar implicature, which is why it is defeasible.

★ The same can be observed for *less than*.

★ Both follow if we assume CMNs and SMNs have scalar alternatives.
The ‘not possible more’ reading of \textit{at most} under \textdagger

(36) a. You are allowed to drink at most one beer.

b. $O_{\text{ScalAlts}}(O_{\text{SubDomAlts}}^{PS} (\Diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (1)))$

c. Prejacent: $O_{\text{ExhSubDomAlts}}^{PS} (\Diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (1))$

d. ScalAlts: $O_{\text{ExhSubDomAlts}}^{PS} (\Diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (m))$

e. Outcome:

\[
O_{\text{ScalAlts}} (O_{\text{ExhSubDomAlts}}^{PS} (\Diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (1))) \\
= O_{\text{ExhSubDomAlts}}^{PS} (\Diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (1)) \land \\
\neg O_{\text{SubDomAlts}}^{PS} (\Diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (2)) \\
= \Diamond |P \cap Q| \in \{0\} \land \Diamond |P \cap Q| \in \{1\} \land \\
\neg (\Diamond |P \cap Q| \in \{0\} \land \Diamond |P \cap Q| \in \{1\} \land \Diamond |P \cap Q| \in \{2\})
\]

\[\quad \quad \text{not possible more ✓}\]

\[\star \text{ This follows from a system where } O_{\text{SubDomAlts}} \text{ can apply to } \]
\[\quad \text{pre-exhaustified alternatives, where } O_{\text{SubDomAlts}} \text{ and } O_{\text{ScalAlts}} \text{ can be} \]
\[\quad \text{manipulated separately, and where } O_{\text{SubDomAlts}} \text{ can be part of the} \]
\[\quad \text{prejacent and the alternatives operated on by } O_{\text{ScalAlts}} .\]
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Conclusion

- A unified account of bare, comparative-modified, and superlative-modified numerals that
  - captures more patterns than previous accounts; and
  - derives them from
    - truth conditions and alternatives obtained in a uniform way from the morphological pieces of BNs, CMNs, and SMNs, and
    - general implicature calculation mechanisms, using general recipes for deriving scalar implicatures, ignorance effects, polarity sensitivity, or free choice behavior.
Open issues

★ How does superlativity in SMNs (at-sup) connect to superlativity in adjectives (sup)?

★ Why Proper Strengthening?
(At present it is a stipulation. We could replace it with a ban on vacuous exhaustification but I think in the general case that might be too strong. It however seems to be a necessary general assumption for items with a positive polarity behavior such as SMNs - [Spector, 2014, Nicolae, 2017]. Parametric choice? Is there any evidence of SMNs that are not PPIs?)
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(37) 3 people quit.

\[ \exists x [\mid x \mid = 3 \land [\text{people}] (x) \land [\text{quit}] (x)] \]

\[ \lambda Q . \exists x [\mid x \mid = 3 \land [\text{people}] (x) \land Q(x)] \]

\[ \exists \lambda x . [\mid x \mid = 3 \land [\text{people}] (x)] \]

\[ [\text{isCard}] ([3]) \]

people
(38) More/less than 3 people quit.

\[ \lambda D_{d,dt} \cdot \lambda d_d \cdot \lambda P_{e,t} \cdot \lambda Q_{e,t} \cdot |P \cap Q| \in D(n) \]
(39) At most/least 3 people quit.

\[ |[\text{people}] \cap [\text{quit}]| \in [\text{much/little}] \]
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