The alternatives of bare and modified numerals

3 (BNS)	(ScalAlts)
more/less than 3 (CMNs)	(ScalAlts), (SubDomAlts)
at most/least 3 (SMNs)	(ScalAlts), SubDomAlts

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Preview

- *3*, *more/less than 3*, and *at least/most 3* differ w.r.t. (at least)

  - entailments,
  - scalar implicatures,
  - ignorance, and
  - acceptability in downward-entailing environments.

- Many theories have been proposed to capture these differences.

- Lately a move towards alternative-based theories.

- Promising results, but also empirical and conceptual issues.

- I will propose a theory that overcomes these issues.

Outline

Entailments

[Horn, 1972, van Benthem, 1986]

⋆ *3 / more than 3 / at least 3* carry lower-bounding entailments.

(1) a. Alice has 3 diamonds.
b. ⇝ not 2 or less
c. Alice has 3 diamonds, # if not less.

⋆ *less than 3* / *at most 3* carry upper-bounding entailments.

(2) a. Alice has less than 3 diamonds.
b. ⇝ not 3 or more
c. Alice has less than 3 diamonds, #if not more.

⋆ Existing proposals: Multiple possible solutions, typically not compositional down to the smallest pieces.
⋆ We want one that gets these entailments with ease and also minimally uncovers the uniform contribution of the numeral, *much*/*little*, or [-er]/[at -est] in producing these entailments.

Scalar implicatures I

- BNs also carry upper-bounding scalar implicatures. [Horn, 1972]

(3) a. Alice has 3 diamonds.
 b. ⇝ not 4 — yields 'exactly 3' meaning ✓
 c. Alice has 3 diamonds, if not 4.

- CMNs and SMNs don't seem to. [Krifka, 1999]

(4) a. Alice has more than 3 diamonds.
 b. ⇝̸ not more than 4 — yields 'exactly 4' meaning ✗

- Existing proposals: No scalar implicatures for CMNs and SMNs.

Scalar implicatures II

- ⋆ But in certain contexts all give rise to scalar implicatures!

(5) a. If you have at least 3 diamonds, you win.
    b. ⇝ not if at least 2

- ⋆ And in some none do:

(6) a. Alice doesn't have 3 diamonds.
    b. ⇝̸ not not 2 yields 'exactly 2' meaning ✗

- ⋆ We want scalar implicatures for all!
- ⋆ We need a separate mechanism to rule out certain implicatures.

Scalar implicatures III

⋆ With coarser granularity, CMNs and SMNs can give rise scalar implicatures too. [Spector, 2014, Cummins et al., 2012, Enguehard, 2018]

(7) *Grades are given based on the number of problems solved. People who solve more than 5 problems but fewer than 9 problems get a B, and people who solve 9 problems or more get an A.*

a. John solved more than 5 problems.
b. $\rightsquigarrow$ not more than 9 (he gets a B) example from [Spector, 2014]

⋆ That is true of BNs in the problem cases also.

(8) a. Alice doesn't have 3 diamonds.
b. $\not\rightsquigarrow$ not not 1 (she does have some)

Ignorance I

- ⋆ SMNs give rise to strong speaker ignorance inferences.

(9) I have 3 / more than 2 / ??at least 3 children.

- ⋆ Existing proposals: e.g., [Büring, 2008, Kennedy, 2015, Spector, 2015]
  - SMNs are underlyingly disjunctive (*at least 3* = exactly 3 or more than 3) and have domain alternatives (the individual disjuncts).
  - Ignorance inferences are implicatures from these alternatives.
  - Nothing of this sort is assumed / derived for CMNs.

Ignorance II

- ⋆ CMNs give rise to ignorance inferences too! [Cremers et al., 2017]

(10) [A:] How many diamonds does Alice have? [B:] More than 3.

- ⋆ Unlike BNs and like SMNs, CMNs are compatible with ignorance:

(11) I don't know how many diamonds Alice has, but she has # 3 / more than 3 / at least 3.

- ⋆ Unlike CMNs, SMNs are incompatible with exact knowledge. [Nouwen, 2015]

(12) There were exactly 62 mistakes in the manuscript, so that's more than 50 / # at least 50.

- ⋆ We want ignorance implicatures for CMNs too!
- ⋆ We want ignorance to be weaker for CMNs than for SMNs.

Acceptability in DE environments I

⋆ SMNs are bad under negation.

[Nilsen, 2007, Geurts and Nouwen, 2007, Cohen and Krifka, 2014, Spector, 2015]

(13) Alice doesn't have *at least three / *at most three diamonds.
→ Alice has 2 or less / 4 or more diamonds. ✗

⋆ Existing proposals: The domain alternatives of SMNs are obligatory and must lead to a stronger meaning, but that cannot happen in a DE environment like negation. [Spector, 2015]

Acceptability in DE environments II

⋆ SMNs are okay in the antecedent of a conditional or the restriction of a universal!

[Geurts and Nouwen, 2007, Cohen and Krifka, 2014, Spector, 2015]

(14) If Alice has at least 3 diamonds, she wins.

(15) Everyone who has at least 3 diamonds wins.

⋆ We want a solution that can distinguish between various types of DE environments!

Summary and preview of proposal

- ⋆ BNs, CMNs, and SMNs are non-uniform w.r.t.
  - Entailments
  - Scalar implicatures
  - Ignorance
  - Acceptability in DE environments

- ⋆ The existing alternative-based proposals are promising, but still:
  - they take into evidence an incomplete dataset;
  - they make non-uniform stipulations about the alternatives;
  - they fail to capture all the patterns we saw.

- ⋆ In this talk:
  - I take into evidence a revised and extended dataset;
  - I derive the alternatives of BNs, CMNs, and SMNs in a uniform way from their truth conditions;
  - I show how, with certain general assumptions about implicature calculation, we get all the patterns we saw.

Outline

Proposal: Truth conditions and presupposition

the numeral	[Link, 1983, Buccola and Spector, 2016]

$[\![\text{n}]\!] = n$ $\quad\quad$ $[\![\text{is}_{\text{Card}}]\!](n) = \lambda x_e \,.\, |x| = n$

much/little	[Seuren, 1984, Kennedy, 1997]

$[\![\text{much}]\!](n) = \lambda d \,.\, d \leq n$ $\quad\quad$ $[\![\text{little}]\!](n) = \lambda d \,.\, d \geq n$

truth conditions (t)	[Krifka, 1999, Von Stechow, 2005, Heim, 2007, Hackl, 2009]

$$
\begin{aligned}
(\exists\ (\text{n P}))(\text{Q}) &= 1 \text{ iff } \exists x[|x| = \underline{n \wedge P(x) \wedge Q(x)]}\\
[\text{comp}](\text{much/little})(\text{n})(\text{P})(\text{Q}) &= 1 \text{ iff } |P \cap Q| \in [\![\text{much/little}]\!](n)\\
[\text{at-sup}](\text{much/little})(\text{n})(\text{P})(\text{Q}) &= 1 \text{ iff } |P \cap Q| \in [\![\text{much/little}]\!](n)
\end{aligned}
$$

the presupposition of at-sup	[Hackl, 2009, Gajewski, 2010]

$$|[\![\text{much/little}]\!](n)| \geq 2$$

✓ Entailments

(16) *3 P Q:*

$\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$ (l.b.)

(17) *more than 3 P Q:*

$|P \cap Q| \in \overline{[\![\text{much}]\!]\ (3)} \Leftrightarrow |P \cap Q| \in \{4, 5, \dots\}$ (l.b.)

(18) *less than 3 P Q:*

$|P \cap Q| \in \overline{[\![\text{little}]\!]\ (3)} \Leftrightarrow |P \cap Q| \in \{\dots, 0, 1, 2\}$ (u.b.)

(19) *at most 3 P Q:*

$|P \cap Q| \in [\![\text{much}]\!]\ (3) \Leftrightarrow |P \cap Q| \in \{\dots, 0, 1, 2, 3\}$ (u.b.)

(20) *at least 3 P Q*:

$|P \cap Q| \in [\![\text{little}]\!]\ (3) \Leftrightarrow |P \cap Q| \in \{3, 4, \dots\}$ (l.b.)

Proposal: Alternatives

Scalar alternatives: Replace the n-domain with an m-domain.

BNs: $\{\exists x[|x| = m \wedge P(x) \wedge Q(x)] : m \in S\}$

CMs: $\{|P \cap Q| \in \overline{[\![\text{much/little}]\!](m)} : m \in S\}$

SMs: $\{|P \cap Q| \in [\![\text{much/little}]\!](m) : m \in S\}$

Subdomain alternatives: Replace the n-domain with its subsets.

BNs: NA (the numeral argument is just a degree)

CMs: $\{|P \cap Q| \in A : A \subseteq \overline{[\![\text{much/little}]\!](n)}\}$

SMs: $\{|P \cap Q| \in A : A \subseteq [\![\text{much/little}]\!](n)\}$

active by presup!

obligatory exhaustification relative to SubDomAlts

Examples

(21) BNs: *3 P Q*

a. Truth conditions: $\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$

b. ScalAlts: $\{\ldots, \exists x[|x| = 2\ldots], \exists x[|x| = 4\ldots, \ldots\}$

c. SubDomAlts: NA

(22) CMNs: e.g., *more than 3 P Q*

a. Truth conditions: $|P \cap Q| \in \overline{[\![\text{much}]\!](3)}$

b. ScalAlts: $\{\ldots, |P \cap Q| \in \overline{[\![\text{much}]\!](2)}, |P \cap Q| \in \overline{[\![\text{much}]\!](4)},$
$\ldots\}$

c. SubDomAlts: $\{|P \cap Q| \in A : A \subseteq \overline{[\![\text{much}]\!](3)}\}$

(23) SMNs: e.g., *at least 3 P Q*

a. Truth conditions: $|P \cap Q| \in [\![\text{little}]\!](3)$

b. ScalAlts: $\{\ldots, |P \cap Q| \in [\![\text{little}]\!](2), |P \cap Q| \in [\![\text{little}]\!](4), \ldots\}$

c. SubDomAlts: $\{|P \cap Q| \in A : A \subseteq [\![\text{little}]\!](3)\}$ active!

Proposal: Implicature calculation system [Chierchia, 2013]

O	to exhaustify the scalar alternatives of BNs, CMNs, and SMNs

(24) $[\![O_{ALT}(p)]\!] = p \wedge \forall q \in ALT\ [q \rightarrow p \subseteq q]$

O^{PS}	to exhaustify the subdomain alternatives of CMNs and SMNs

⋆ A version of O that

- takes into account presuppositions:

(25) $[\![O^{S}_{ALT}(p)]\!] = \pi(p) \wedge \forall q \in ALT\ [\pi(q) \rightarrow \pi(p) \subseteq \pi(q)]$,

- requires a properly stronger result:

(26) $[\![O^{PS}_{ALT}(p)]\!]$ is defined iff $O^{S}_{ALT}(p) \subset p$.
Whenever defined, $[\![O^{PS}_{ALT}(p)]\!] = [\![O^{S}_{ALT}(p)]\!]$.

$\boxdot$	last resort, silent, matrix-level, universal doxastic modal

Implicatures from ScalAlts: Scalar implicatures ✓

(27) Alice has 3 diamonds.

a. $O_{ScalAlts}\ (\exists x[|x| = 3 \wedge P(x) \wedge Q(x)] \wedge)$
$= \exists x[|x| = 3 \wedge P(x) \wedge Q(x)] \wedge$
$\neg\ \exists x[|x| = 4 \wedge P(x) \wedge Q(x)] \wedge \ldots$ — not 4 ✓

(28) Alice has at least 3 diamonds.

a. $O_{ScalAlts}\ (|P \cap Q|) \in [\![\text{little}]\!]\ (3))$
$= |P \cap Q| \in [\![\text{little}]\!]\ (3) \wedge$
$\neg\ |P \cap Q| \in [\![\text{little}]\!]\ (5) \wedge \ldots$ — not at least 5 ✓

(29) If Alice has more than 3 diamonds, she wins.

a. $O_{ScalAlts}\ ([|P \cap Q|) \in \overline{[\![\text{much}]\!]\ (3))} \rightarrow \text{win}]$
$= [|P \cap Q| \in \overline{[\![\text{much}]\!]\ (3)} \rightarrow \text{win}] \wedge$
$\neg\ [|P \cap Q| \in \overline{[\![\text{much}]\!]\ (2)} \rightarrow \text{win}] \wedge \ldots$ — not if more than 2 ✓

- ⋆ And so on. We can derive all the attested scalar implicatures.
- ⋆ Scalar implicatures may be restricted by granularity.
- ⋆ In unembeded contexts this effect is compounded by ignorance.

Implicatures from SubDomAlts: Ignorance ✓

(30) Alice has more/less than 3 / at most/least 3 diamonds.

a. $O^{PS}_{SubDomAlts}\ (|P \cap Q|) \in D)$
$= |P \cap Q| \in D\ \wedge$
$\neg\ |P \cap Q| \in A\ \wedge$
$\neg\ |P \cap Q| \in B\ \wedge \ldots$, for all $A, B, \cdots \subset D$, $= \bot$

contradiction ✗

b. $O^{PS}_{SubDomAlts}\ (\boxdot|P \cap Q| \in D)$
$= \boxdot|P \cap Q| \in D\ \wedge$
$\neg\ \boxdot\ |P \cap Q| \in A\ \wedge$
$\neg\ \boxdot\ |P \cap Q| \in B\ \wedge \ldots$, for all $A, B, \cdots \subset D$

ignorance ✓

- ⋆ The only consistent $O^{PS}_{SubDomAlts}$ parse yields ignorance.
- ⋆ SMNs can only have an $O^{PS}_{SubDomAlts}$ parse, so *(ignorance)
- ⋆ CMNs can also have a parse without $O^{PS}_{SubDomAlts}$, so (ignorance).

Scalar implicatures vs. ignorance implicatures

(31) Alice has more than 2 / at least 3 diamonds.

$$O^{PS}_{SubDomAlts} \boxdot O_{ScalAlts} (|P \cap Q|) \in \{3, 4, \dots\})$$
$$= \boxdot O_{ScalAlts} (|P \cap Q| \in \{3, 4, \dots\}) \wedge$$
$$\neg \boxdot (|P \cap Q| \in \{3\}) \wedge$$
$$\neg \boxdot (|P \cap Q| \in \{4, 7\}) \wedge \dots$$

$$= \boxdot(|P \cap Q| \in \{3, 4, \dots\} \wedge \neg|P \cap Q| \in \{4, \dots\}) \wedge$$
$$\neg \boxdot (|P \cap Q| \in \{3\}) \wedge$$
$$\neg \boxdot (|P \cap Q| \in \{4, 7\}) \wedge \dots$$

$$= \underline{\boxdot(|P \cap Q| \in \{3\})} \wedge \underline{\neg \boxdot (|P \cap Q| \in \{3\})} \wedge$$
$$\neg \boxdot (|P \cap Q| \in \{4, 7\}) \wedge \dots = \bot$$

contradiction ✗

- ⋆ Prune offending SubDomAlts? That would violate $O^{\boxed{PS}}_{SubDomAlts}$. ✗
- ⋆ Prune offending ScalAlt? ✓

Implicatures from SubDomAlts: Negation ✓

(32) Alice doesn't have more/less than three / *at most/least three diamonds.

a. $\neg O^{PS}_{SubDomAlts}\ (|P \cap Q| \in D)$ — contradiction ✗

b. $O^{PS}_{SubDomAlts}\ (\neg|P \cap Q| \in D)$ — PS violated ✗

c. $O^{PS}_{SubDomAlts}\ (\boxdot\neg|P \cap Q| \in D)$ — PS violated ✗

- ⋆ All $O^{PS}_{SubDomAlts}$ parses fail.
- ⋆ SMNs cannot have a non-$O^{PS}_{SubDomAlts}$ parse, so bad.
- ⋆ CMNs can be parsed without $O^{PS}_{SubDomAlts}$, so okay.

Implicatures from SubDomAlts: AntCond/RestUniv ✓

(33) $O^{PS}_{SubDomAlts}$ (Everyone who has at least 3 diamonds wins.)

$$\begin{array}{llcl} \text{Prejacent:} & \forall x[\#\text{ di } x \text{ has } \in D \rightarrow \ldots] & \wedge & \exists x[\#\text{ of di } x \text{ has } \in D] \\ & \Downarrow & & \Uparrow \\ \text{SubDomAlt:} & \forall x[\#\text{ di } x \text{ has } \in D' \rightarrow \ldots] & \wedge & \exists x[\#\text{ of di } x \text{ has } \in D'] \end{array}$$

- ⋆ SubDomAlts not entailed, so they must be false.
- ⋆ However, negating them leads to contradiction.
- ⋆ We can rescue the parse with $\boxdot$:

(34) $\boxdot \exists x[\#\text{ of di } x \text{ has } \in D] \wedge \neg \boxdot \exists x[\#\text{ of di } x \text{ has } \in D']$

PS satisfied ✓

⋆ Thus there *is* a consistent $O^{PS}_{SubDomAlts}$ parse for SMNs, which is why they are felicitous in this type of DE environments.

Outline

Empirical patterns, existing proposals, issues
- Entailments
- Scalar implicatures
- Ignorance
- Acceptability in DE environments

Proposal

Bonus results

Conclusion and open issues

Appendix

The existential implicature of *at most* [Alrenga, 2016]

(35) LeBron scored at most 20 points (and it's even possible that he didn't score any points at all).

$$O_{ScalAlts}\ (|P \cap Q| \in [\![\text{much}]\!]\ (20))$$
$$= |P \cap Q| \in [\![\text{much}]\!]\ (20) \wedge$$
$$\neg\ |P \cap Q| \in [\![\text{much}]\!]\ (18) \wedge$$
$$\neg\ |P \cap Q| \in [\![\text{much}]\!]\ (17) \wedge$$
$$\ldots$$
$$\neg\ |P \cap Q| \in [\![\text{much}]\!]\ (0)$$

existential implicature ✓

⋆ Lower-bounding inference is a scalar implicature, which is why it is defeasible.

⋆ The same can be observed for *less than*.

⋆ Both follow if we assume CMNs and SMNs have scalar alternatives.

The ‘not possible more’ reading of *at most* under $\Diamond$

(36) a. You are allowed to drink at most one beer.

b. $O_{ScalAlts}\ (O^{PS}_{SubDomAlts}\ (\Diamond|P \cap Q| \in [\![\text{much}]\!]\ (1)))$

c. Prejacent: $O^{PS}_{ExhSubDomAlts}\ (\Diamond|P \cap Q| \in [\![\text{much}]\!]\ (1))$

d. ScalAlts: $O^{PS}_{ExhSubDomAlts}\ (\Diamond|P \cap Q| \in [\![\text{much}]\!]\ (m))$

e. Outcome:

$O_{ScalAlts}\ (O^{PS}_{ExhSubDomAlts}\ (\Diamond|P \cap Q| \in [\![\text{much}]\!]\ (1)))$
$= O^{PS}_{ExhSubDomAlts}\ (\Diamond|P \cap Q| \in [\![\text{much}]\!]\ (1)) \wedge$
$\neg\ O^{PS}_{SubDomAlts}\ (\Diamond|P \cap Q| \in [\![\text{much}]\!]\ (2))$
$= \Diamond|P \cap Q| \in \{0\} \wedge \Diamond|P \cap Q| \in \{1\} \wedge$
$\neg(\Diamond|P \cap Q| \in \{0\} \wedge \Diamond|P \cap Q| \in \{1\} \wedge \Diamond|P \cap Q| \in \{2\})$

not possible more ✓

⋆ This follows from a system where $O_{SubDomAlts}$ can apply to pre-exhaustified alternatives, where $O_{SubDomAlts}$ and $O_{ScalAlts}$ can be manipulated separately, and where $O_{SubDomAlts}$ can be part of the prejacent and the alternatives operated on by $O_{ScalAlts}$.

Outline

Conclusion

⋆ A unified account of bare, comparative-modified, and superlative-modified numerals that

- captures more patterns than previous accounts; and

- derives them from

    - truth conditions and alternatives obtained in a uniform way from the morphological pieces of BNs, CMNs, and SMNs, and

    - general implicature calculation mechanisms, using general recipes for deriving scalar implicatures, ignorance effects, polarity sensitivity, or free choice behavior.

Open issues

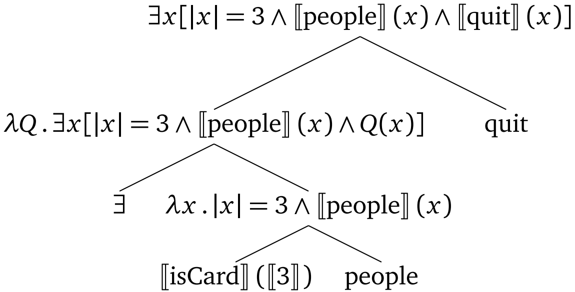
- ⋆ How does superlativity in SMNs (at-sup) connect to superlativity in adjectives (sup)?

- ⋆ Why Proper Strengthening?
(At present it is a stipulation. We could replace it with a ban on vacuous exhaustification but I think in the general case that might be too strong. It however seems to be a necessary general assumption for items with a positive polarity behavior such as SMNs - [Spector, 2014, Nicolae, 2017]. Parametric choice? Is there any evidence of SMNs that are not PPIs?)

Outline

Truth conditions, BNs

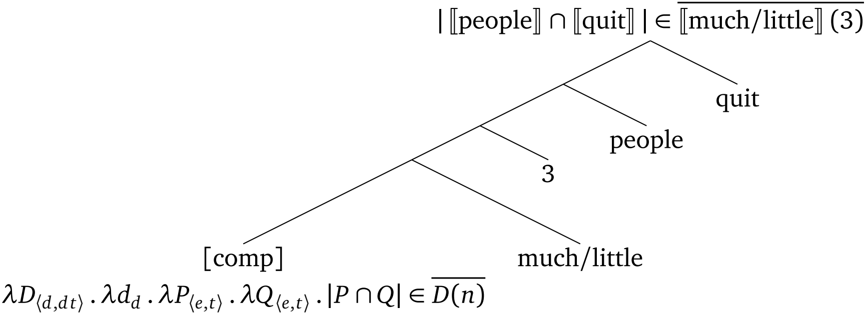
(37) 3 people quit.



Truth conditions, CMNs

back

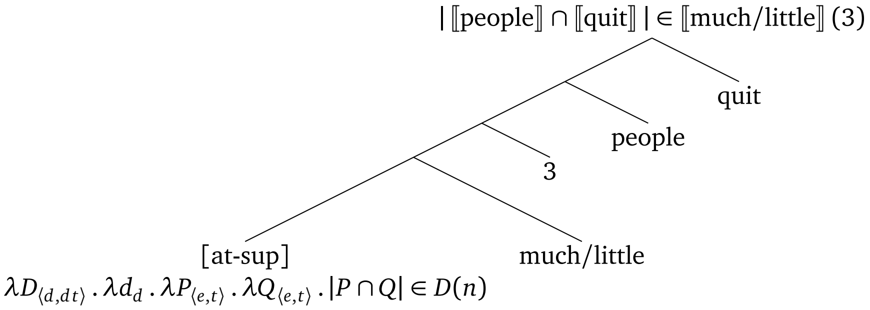
(38) More/less than 3 people quit.



Truth conditions, SMNs

back

(39) At most/least 3 people quit.



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