Online Appendix to: "Axiomatization and measurement of Quasi-hyperbolic Discounting"

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1 Sample Selection

As discussed before our initial sample consists of two groups of subjects. Group "M" has 639 subjects that answered the "Money" questionnaire. Group "IC" has 640 subjects that answered the "Ice-cream" questionnaire. Inside each group, we associate subjects with an Internet Protocol address (IP) and we verify that there is no IP repetition inside the group. Consequently, we do not allow for a single IP address to answer the same questionnaire more than once.

Remark 1. We do allow for the same IP address to answer both questionnaires. Our sample contains 548 IPs in this situation. Note that this within-subject information could be useful for partially identifying the joint and conditional distributions of money/ice-cream preference parameters. For instance, let β_i^X denote the short-run discount factor for a good X related question. We could try to understand whether or not β_i^M and β_i^{IC} are independent; this is:

$$\mu\Big\{i \in \mathcal{P} \mid \beta_i^M \leq \beta_1, \quad \beta_i^{IC} \leq \beta_2\Big\} = (\neq)\, \mu\Big\{i \in \mathcal{P} \mid \beta_i^M \leq \beta_1\Big\}\mu\Big\{i \in \mathcal{P} \mid \beta_i^{IC} \leq \beta_2\Big\}$$

We left these (and other related) questions for future research and we focus on inference concerning the distribution of (β_i^X, δ_i^X) for a fixed good X.

1.1 Monotonicity and Understanding

We checked the subjects' understanding of the instructions by asking two simple questions, see Figure 1. Table 1 summarizes our sample selection based on the two questions reported in Figure 1:

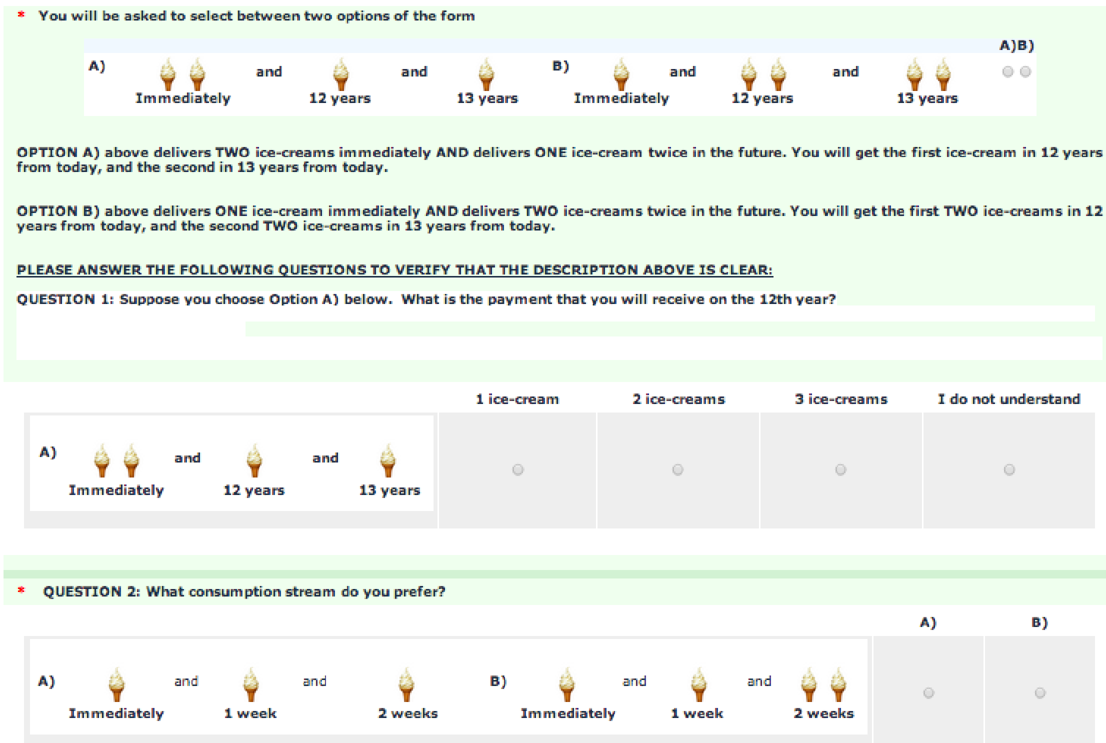


Figure 1: Initial Checks

	Unique IPs	Survive u Check	Survive m Check	Survive m-u Check
Money	639	608	526	506 (79.1%)
IC	640	611	526	507 (79.2%)

NOTE: "m" stands for "monotonicity" and "u" stands for "understanding"

Table 1: Sample Selection based on m-u check

1.2 Consistency

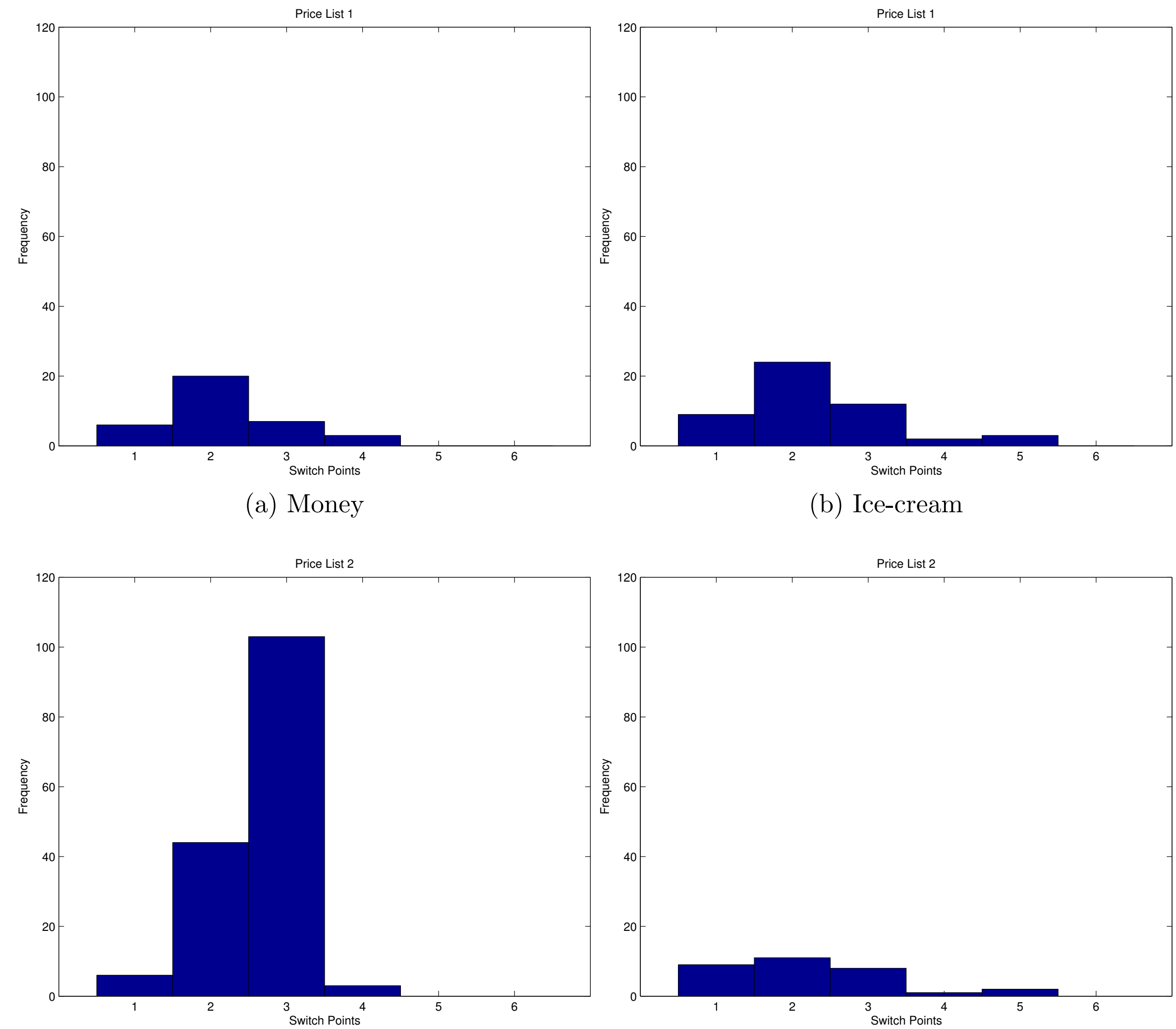
After selecting subjects that survive the monotonicity and understanding check, we further refine the sample by considering agents that are *consistent* with the quasi-hyperbolic model. A necessary condition for an agent to admit a quasi hyperbolic representation is the existence of at most one switch point (from patient to impatient prospect) per price list. Thus, we discard all subjects that violate this condition.

	Sample after m-u check	Inconsistent (L1)	Inconsistent (L2)	Consistent
Money	506	36	156	336
IC	507	50	31	444

Table 2: Sample Selection based on consistency check

We also summarize the type of inconsistency observed in each price list. The histograms in Figure 2 report the frequency of switching points. Note that agents with a single switch point moving from an impatient reward in question j to a patient reward in $j + h$ are also inconsistent.

A quick observation concerning the behavior of inconsistent subjects. It seems reasonable to ask whether subjects with inconsistent answers in price list 1 also have inconsistent answers in price list 2. One way to get a simple statistic to summarize this dependence is as follows. Consider first the money questionnaire. For price list 1, we create a vector of dummy variables d_1 (of dimension 506) with value of 1 if the agent is inconsistent, and zero otherwise. The dummy variable d_2 is defined analogously. We then look at the sample correlation between these two random vectors. The correlation equals 1 if and only if the subjects that are inconsistent in the price list 1 are also inconsistent in price list 2. Likewise, the correlation equals 0 if and only if there is no overlap. For the money questionnaire, we found a correlation of .1815; for the ice-cream questionnaire the correlation is .4126.



(a) Money (b) Ice-cream

(c) Money (d) Ice-cream

Figure 2: Distribution of Swtich Points for Inconsistent Subjects

2 Response Times

Given the online nature of our pilot experiment and the lack of incentives, a concern is that subjects click at random, or always choose the same answer (for example always choose A) in order to save time and move quickly to another task. We find little support of this story in the data. I this section, we describe the data collected on response times and the statistical test we implemented to show that the population's upper and lower bounds are statistically independent of response times.

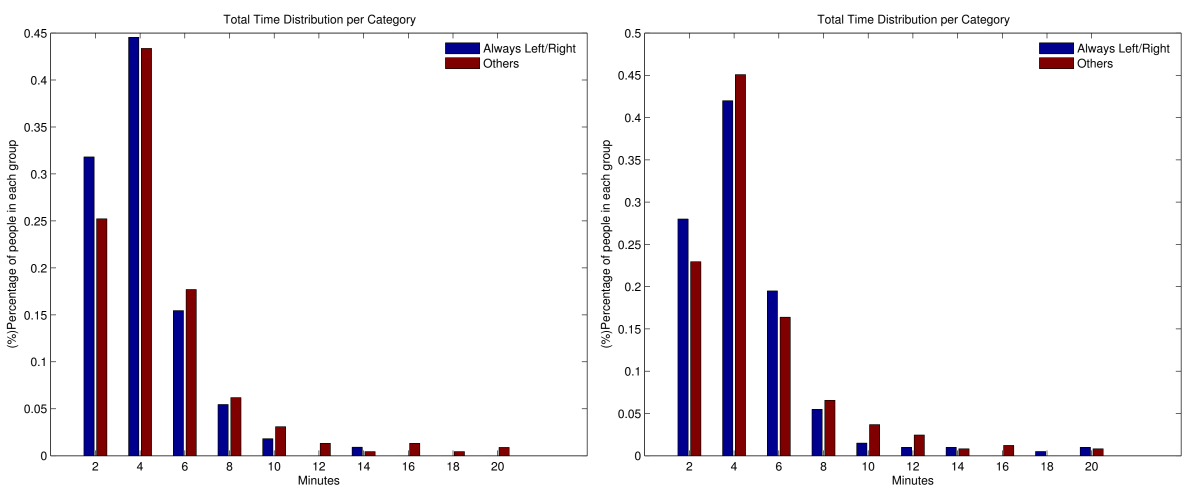
2.1 Data on Response Times

For each price list 1-2 in the M-IC questionnaires, we collected three variables measuring subjects' response times:

1. r_i : The total time spent in questionnaire M (IC), measured as total number of seconds that each subject spent in completing the two price lists.

2. $r_{i,1}$: The time spent in price list 1 of questionnaire M (IC), measured as total number of seconds that each subject spent in completing the first price list of of questionnaire M (IC).

3. $r_{i,2}$: The time spent in price list 2 of questionnaire M (IC), measured as total number of seconds that each subject spent in completing the second price list of of questionnaire M (IC).



(a) Money (b) Ice-cream

Figure 3: Conditional Distributions of Total Response Time

Figure 3 compares the total response times r_i for consistent subjects that always select Option A ($s_{i,1} = s_{i,2} = 8$) or Option B ($s_{i,1} = s_{i,2} = 1$) against consistent subjects with other behavior.

The distributions in Figure 3 are conditional distributions of response times for certain values of $(s_{i,1}, s_{i,2})$:

$$r_i \,\Big|\, (s_{i,1} = s_{1,2} = 1 \text{ or } s_{i,1} = s_{1,2} = 8)$$

and

$$r_i \,\Big|\, (s_{i,1} = s_{1,2} = 1 \text{ or } s_{i,1} = s_{1,2} = 8)^c$$

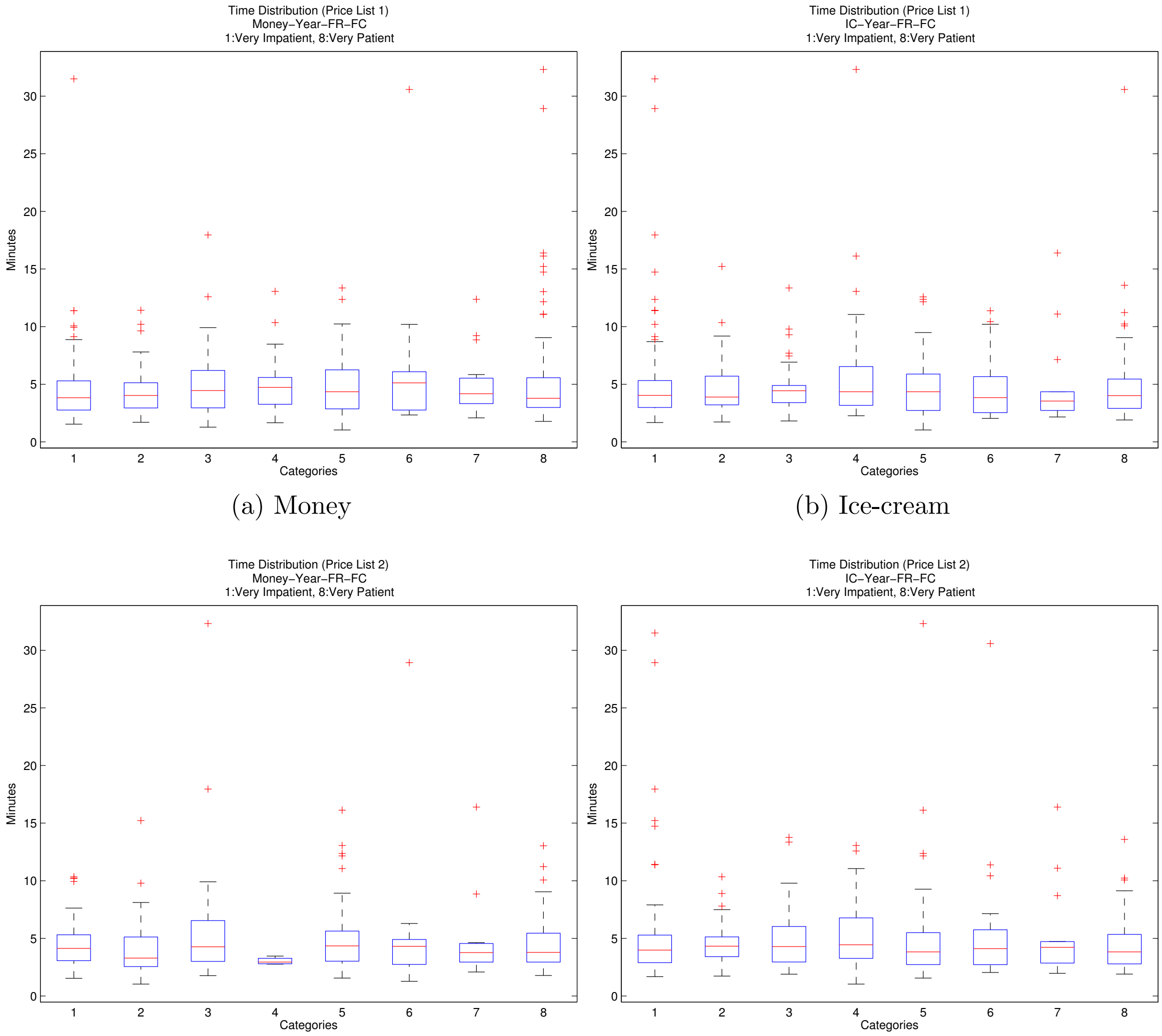
Figure 4 below reports the conditional distributions of response times given $s_{i,1} = k$ (first row) and given $s_{i,2} = k$ (second row).

Both graphs suggest that the response time r_i is independent of both $s_{i,1}$ and $s_{i,2}$. We test this statistical hypothesis using the *distance covariance* statistic of **?**. The distance covariance statistic compares the weighted difference between the sample analog of the characteristic function of $(s_{i,1}, s_{i,2}, r_i)$ against the product of the characteristic functions of $(s_{i,1}, s_{i,2})$ and r_i, see **?**, pp. 6-7. Under the null hypothesis of independence, the (properly scaled) distance covariance between $(s_{i,1}, s_{i,2}, r_i)$ and r_i converges in distribution to a weighted sum of chi-squared random variables and a 5%-level conservative critical value is given 1.96^2; see Theorem 5, 6 in **?**. The scaled *distance covariance* statistic is 1.03 for the M questionnaire and .8510 for IC. In both cases, the conservative 5% critical value is 1.96^2. Thus, we cannot reject the null hypothesis that the distributions of $(s_{i,1}, s_{i,2})$ and r_i are independent.[1] The *distance correlation* (normalized to be in [0,1]) is .1027 for the Money questionnaire and .0750 for the Ice-cream. The distance correlation is zero in the population if and only the random vectors are independent.

The population lower and upper bounds in our design are functions of the switch points $(s_{i,1}, s_{i,2})$. If $(s_{i,1}, s_{i,2})$ and r_i are independent then:

$$\mu\{i \in \mathcal{P} \mid s_{i,1} \leq k, s_{i,2} = j + 1, r_i > r\}/\mu\{i \in \mathcal{P} \mid r_i > r\}$$

[1]The distance covariance statistic was computed using the matlab file distcorr.m available here: `http://www.mathworks.com/matlabcentral/fileexchange/39905-distance-correlation/content/distcorr.m`



(a) Money (b) Ice-cream

(c) Money (d) Ice-cream

Figure 4: Conditional Distributions of Response Time by Switch Point Category

equals

$$\mu\{i \in \mathcal{P} \mid s_{i,1} \leq k, s_{i,2} = j+1\}$$

for all c. We conclude by saying that there is no statistical evidence suggesting that the lower and upper bounds will change if we condition on response times.

3 Worker's Qualifications

In terms of qualifications, we divide the subjects in our sample into "Masters" (MA) and "Non-Masters with qualifications" (NMAQ). AMT defines Masters as an "elite groups of workers who have demonstrated accuracy on specific types of HITs on the Mechanical Turk marketplace". Workers achieve a Masters distinction by consistently completing HITs of a certain type with a high degree of accuracy.

For non masters, AMT allows the users to require different degrees of *qualifications*. A qualification represents a worker's skill, ability or reputation. The "NonMasters with qualifications" subjects in our sample are workers with 95% of approved prior tasks and at minimum 5000 approved prior tasks.

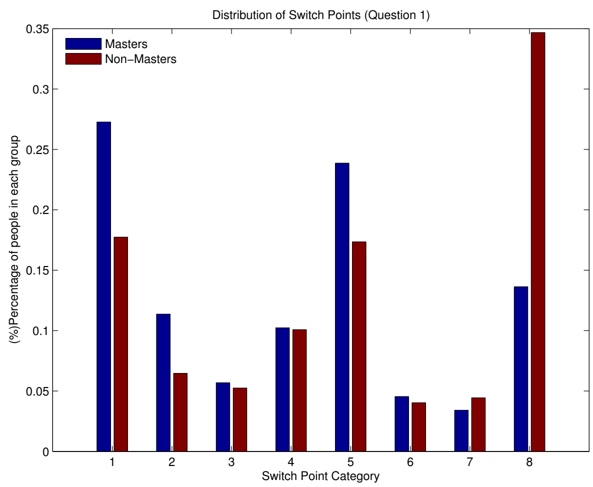
In this section, we analyze the number of MA and NMAQ in our sample. We also discuss the dependence of switch points to this categorization of workers. In particular, we reject the null hypothesis that the distribution of switch points in the population is independent of the MA/NMAQ category dummy. Finally, we report lower and upper bounds for MA and NMAQ.

3.1 MA and NMAQ in our sample

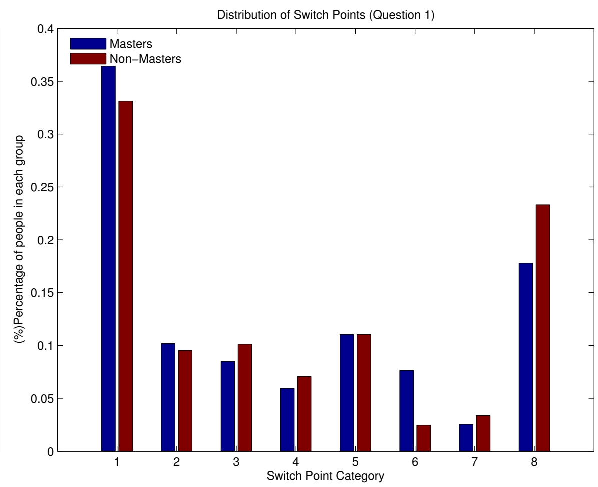
	Sample m-u-check	MA m-u sample	Consistent MA	Consistent NMAQ
Money	506	144	88	248
IC	507	142	118	326

Table 3: Sample Selection based on consistency check

Table 3 reports the number of MA and NMAQ workers in our sample. We observe a larger share of NMAQ (the ratio is almost 3:1) in both the Money and Ice-cream treatments. Figure 5 presents the conditional distribution of switch points in price list 1 for the money and ice-cream treatments. Panel a) of this figure suggests that the distribution of switch points in question 1 is not independent of the MA/NMAQ dummy variable (d_i^{MA}): the conditional probability of never switching (category 8) is larger for non-masters. Interestingly, Panel b) suggests that the conditional distribution of switch points in price list 1 for the ice-cream treatment does not vary in the MA/NMAQ groups. At the end of this section we will provide statistical tests for the null hypothesis of independence for the random variables $s_{i,1}$ and d_i^{MA}. We will also report the estimated bounds for $G(\beta)$ for the MA and NMAQ.



(a) Money



(b) Ice-cream

Figure 5: Conditional Distributions of Switch Point

Figure 6 presents the conditional distribution of switch points in price list 2 for the money and ice-cream treatments. The graphs suggest that the conditional distributions of $s_{i,1}|d_i^{\mathrm{MA}} = 1$ and $s_{i,1}|d_i^{\mathrm{MA}} = 0$ are similar.

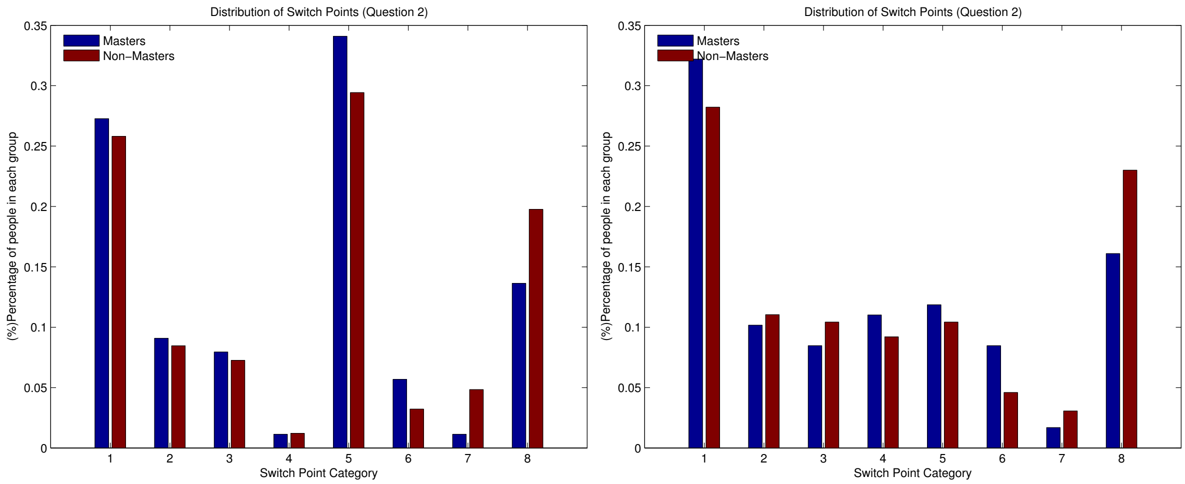
We now consider the tests for three different null hypothesis (each of them tested in the money and ice-cream treatment separately).

1. $\mathbf{H}_0^1 : s_{i,1}$ is independent of d_i^{MA} vs. $\mathbf{H}_0^1 : s_{i,1}$ is not independent of d_i^{MA}

2. $\mathbf{H}_0^2 : s_{i,2}$ is independent of d_i^{MA} vs. $\mathbf{H}_0^1 : s_{i,2}$ is not independent of d_i^{MA}

3. $\mathbf{H}_0^3 : (s_{i,1}, s_{i,2})$ is independent of d_i^{MA} vs. $\mathbf{H}_0^1 : (s_{i,1}, s_{i,2})$ is not independent of d_i^{MA}

Once again, we test these statistical hypotheses using the *distance covariance* statistic of **?** discussed in Appendix 2. The following table reports the (properly scaled) distance covariance



(a) Money (b) Ice-cream

Figure 6: Conditional Distributions of Switch Point

statistic (see pp. 6-7 in **?**). The null hypothesis of independence is rejected at the 5% asymptotic level if the scaled distance covariance statistic is larger than $1.96^2 = 3.84$. We also report the distance correlation (normalized to be in $[0, 1]$) as a measure of dependence.

	$\mathbf{H}_0^1$	$\mathbf{H}_0^2$	$\mathbf{H}_0^3$
Money			
Distance Correlation	0.197	0.063	0.169
Distance Covariance statistic	9.308	0.958	5.403
IC			
Distance Correlation	0.428	0.055	0.054
Distance Covariance statistic	0.601	0.992	0.864

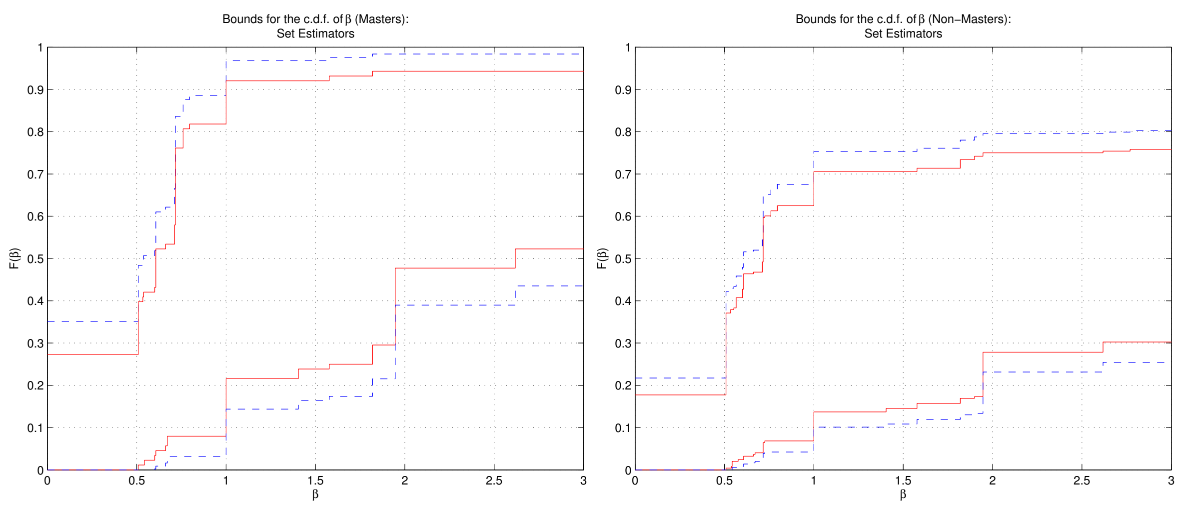
Table 4: Distance Correlation and Distance Covariance statistics

For the money treatment, $\mathbf{H}_0^1$ is rejected at the 5% asymptotic level as the distance covariance statistic is larger than 3.84. Therefore, we reject the null hypothesis that $s_{i,1}$ is independent of d_i^{MA}. For the same treatment, we cannot reject the null hypothesis $\mathbf{H}_0^2$. The latter suggests that the populations bounds for $F(\delta)$ do not depend on whether we condition on MA/NMAQ.

For the IC treatment, we cannot reject $\mathbf{H}_0^1$ at the 5% asymptotic level as the distance covariance statistic is no larger than 3.84. Likewise, the null hypotheses $\mathbf{H}_0^2$, $\mathbf{H}_0^3$.

3.2 Lower and upper bounds for MA/NMAQ

Since $\mathbf{H}_0^2$ is rejected for both the money and the ice-cream treatments, the population bounds for $F(\delta|d_i^{\text{MA}})$ should not depend on whether we condition on MA or NMAQ. The same is true for the bounds for $G(\beta|d_i^{\text{MA}})$ in the ice-cream treatment. However, for the money treatment we could expect the bounds for $G(\beta|d_i^{\text{MA}})$ to depend on the values of d_i^{MA}. Figure 7 and 8 present the results.

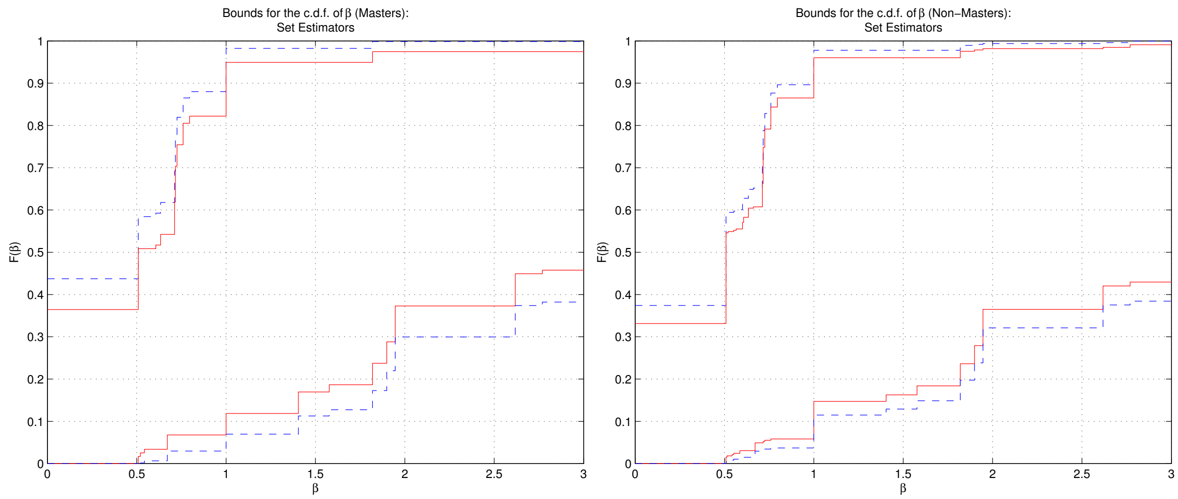


(a) MA (b) NMAQ

Figure 7: Conditional Lower and Upper Bounds for $G(\beta|d_i^{\text{MA}})$: Money

4 Joint Distributions

The previous section focused on the partial identification of the marginal distributions of β_i and δ_i for two different rewards. We know discuss the findings concerning the joint distributions of $(\beta_i,\, \delta_i)$ and $(\beta_i^M, \beta_i^{IC})$.



(a) MA (b) NMAQ

Figure 8: Conditional Lower and Upper Bounds for $G(\beta|d_i^{\text{MA}})$: Ice-Cream

4.1 Joint distribution of (β_i, δ_i)

Our experimental design allows us to partially identify the joint distribution of (β_i, δ_i). For instance, note that:

$$\mu\{i \in \mathcal{P} \mid \beta_i \leq 1 \text{ and } \delta_i \leq \delta^*(j)\} \geq \sum_{k=1}^{j} \mu\{i \in \mathcal{P} \mid s_{i,1} < k \text{ and } s_{i,2} = k\}$$

and

$$\mu\{i \in \mathcal{P} \mid \beta_i \leq 1 \text{ and } \delta_i \leq \delta^*(j)\} \leq \sum_{k=1}^{j} \mu\{i \in \mathcal{P} \mid s_{i,1} \leq k+1 \text{ and } s_{i,2} = k\}$$

One question we could ask concerning the joint distribution of β_i and δ_i is whether the time preference parameters are independent in the population. Although we are not aware of statistical tests for the independence of two random variables whose joint (and marginals) c.d.f's are partially identified, we present a simple analysis that can shed some light on the issue.

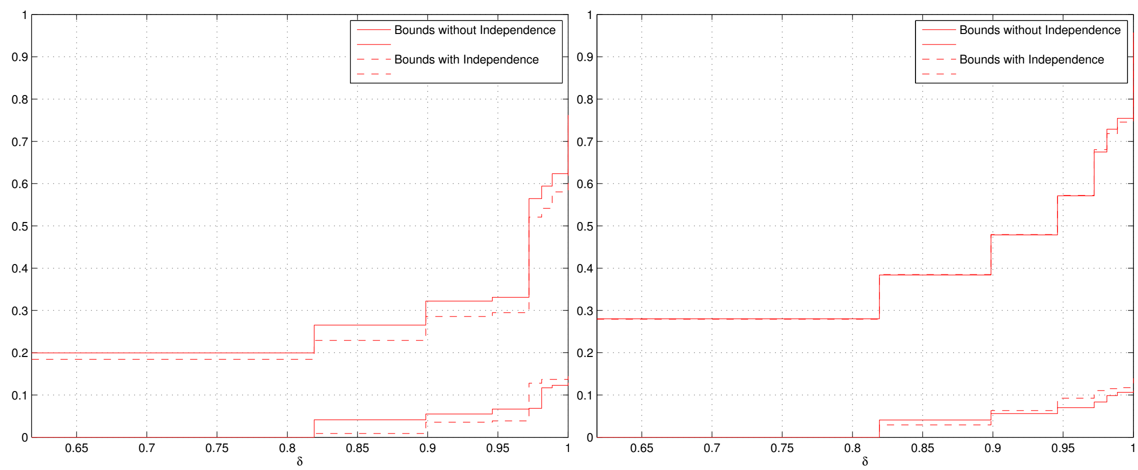
Note that under the assumption of independence, the following upper and lower bounds obtain:

$$\begin{aligned} \mu\{i \in \mathcal{P} \mid \beta_i \leq 1 \text{ and } \delta_i \leq \delta^*(j)\} &= G(1)F(\delta^*(j)) \\ &\geq \Big(\sum_{j=1}^{8} \mu\{i \in \mathcal{P} \mid s_{i,1} < j \text{ and } s_{i,2} = j\}\Big)\underline{F}(\delta^*(j)) \end{aligned}$$

and

$$\begin{aligned} \mu\{i \in \mathcal{P} \mid \beta_i \leq 1 \text{ and } \delta_i \leq \delta^*(j)\} &= G(1)F(\delta^*(j)) \\ &\leq \Big(\sum_{j=1}^{8} \mu\{i \in \mathcal{P} \mid s_{i,1} \leq j+1 \text{ and } s_{i,2} = j\}\Big)\overline{F}(\delta^*(j)) \end{aligned}$$

Figure 9 presents upper and lower bounds for $\mu\{i \in \mathcal{P} \mid \beta_i \leq 1 \text{ and } \delta_i \leq \delta\}$ as a function of $\delta \in [.6, 1]$. The figure suggests that regardless of statistical significance, the difference that arises from the independence assumption does not seem to be very important, at least when the joint c.d.f. is evaluated at $\beta = 1$.



(a) Money (b) Ice-cream

Figure 9: Lower and Upper bounds for $\mu\{i \mid \beta_i \leq 1 \text{ and } \delta_i \leq \delta\}$

4.2 Joint distribution of preference parameters for different primary rewards

As mentioned before, there are 548 participants that answered both the money and ice-cream questionnaire. Out of those, there are 437 that survive the 'm-u' check and 273 that survive the 'm-u-c' check. In principle, one could use the information concerning the switch points in the 4 price lists (2 price lists per questionnaire) to bound probability statements concerning preference parameters for different rewards; for example, the probability of the event:

$$\{i \mid 0 \leq \beta_i^M \leq 1 \text{ and } 0 \leq \beta_i^{IC} \leq 1\}$$

The probability of this event cannot be bounded directly using the results concerning the marginals β_i^M and β_i^{IC}, as these distributions need not be independent. Thus, the first question that we ask is whether there is dependence between the vectors $(s_{i,1}^M, s_{i,2}^M)$ and $(s_{i,1}^{IC}, s_{i,2}^{IC})$. A simple statistic to report is the correlation matrix between the 4 random vectors $(s_{i,1}^M, s_{i,2}^M, s_{i,1}^{IC}, s_{i,2}^{IC})$:

$$\begin{pmatrix} 1 & .7072 & .4734 & 0.4150 \\ 0.7072 & 1 & 0.5914 & 0.5267 \\ 0.4734 & 0.5914 & 1 & 0.8847 \\ 0.4150 & 0.5267 & 0.8847 & 1 \end{pmatrix}$$

The matrix above suggests that there is dependence between switch points within the same questionnaire, but also across questionnaires. If this dependence is also present in the switch points associated to more complicated annuities (such as those required by our annuity compensation axiom), then we could expect the preference parameters across different primary rewards to exhibit dependence as well. Thus, we test the null hypothesis:

$$\mathbf{H}_0 : (s_{i,1}^M, s_{i,2}^M) \text{ is independent of } (s_{i,1}^{IC}, s_{i,2}^{IC})$$

against the alternative that the random vectors are not independent. The distance correlation of **?** is .5608 and their test statistic for independence is 31.9643. Since the 5% asymptotically valid critical value is 3.84, the null hypothesis of independence is rejected.

Finally, we report the share of agents (out of those that solved both questions) for which $s^M_{i,1} < s^M_{i,2}$ and $s^{IC}_{i,1} < s^{IC}_{i,2}$. This is, we report the share of agents that exhibit behavior consistent with present bias in both questionnaires. The share of agents that exhibit present bias is 12.82% in the money questionnaire and 11.36% in the IC questionnaire. The share of agents that exhibit present bias in both questionnaires is only 1.47%.