Subjective Beliefs and Ex Ante Trade

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Question

S – states of nature

$\mathbb{R}^S$ – acts with monetary payoffs: financial assets, bets

n agents

Endowed with constant acts

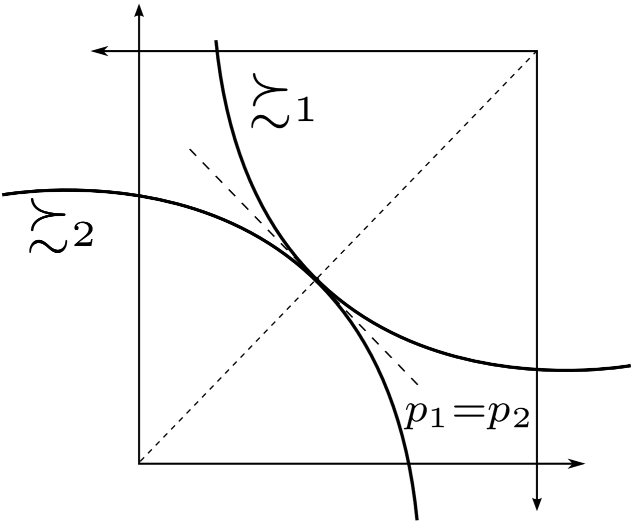
Question: Will they bet / trade assets?

$\succsim$ = *Expected Utility*

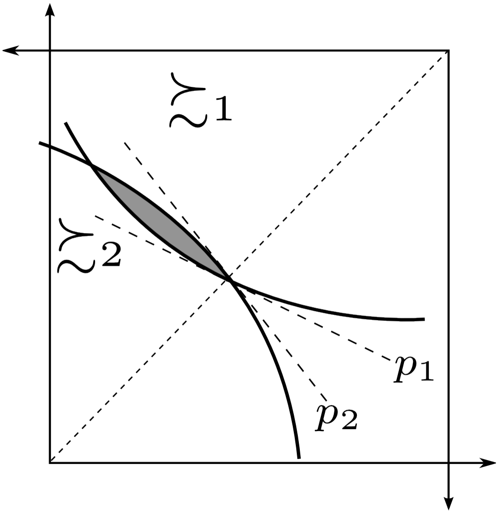
Fact: Assume u_i are strictly concave. Then

$$\text{No Pareto optimal trade} \Updownarrow p_1 = p_2 = \ldots = p_n.$$

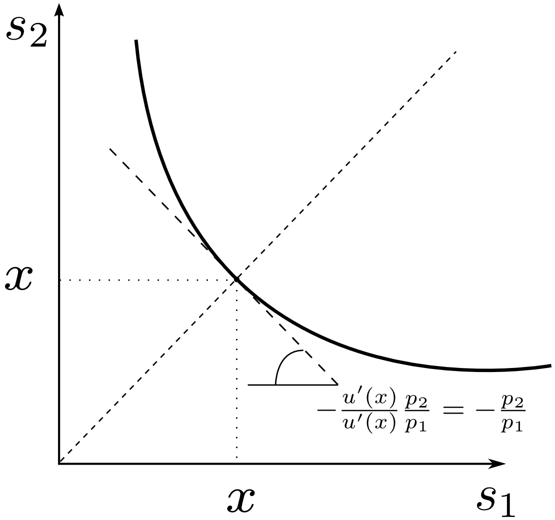
$\succsim$ = *Expected Utility*



$\succsim$ = *Expected Utility*



EU: p $\sim$ *supporting hyperplane*



Ambiguity

Elsberg paradox

Urn 1: 50 black, 50 white balls

Urn 2: unknown proportion of black and white balls

Indifferent to betting on black and white from Urn 1

Indifferent to betting on black and white from Urn 2

But prefer betting on Urn 1 than Urn 2.

Consequences

Urn 1: WIG20

Urn 2: Nikkei

Indifferent to bets on Urn 1

Indifferent to bets on Urn 2

But prefer betting on Urn 1 than Urn 2: *Home bias*

Consequences

Equity premium puzzle: to justify the discrepancy between prices of stocks and risk-free assets need to assume absurd risk aversion.

Ambiguity

This cannot be justified by a probabilistic model of choice

Risk and Ambiguity (Knight, Keynes)

Maxmin expected utility (MEU)—**?**

Ambiguity—set P of probabilities

$V(f) = \min_{p \in P} \mathbb{E}_p \, u(f)$

Ambiguity

This cannot be justified by a probabilistic model of choice

Risk and Ambiguity (Knight, Keynes)

Maxmin expected utility (MEU)—**?**

Ambiguity—set P of probabilities

$V(f) = \min_{p \in P} \mathbb{E}_p \, u(f)$

↑

pessimism

(ambiguity aversion)

$\succsim = MEU$

?, *Econometrica* 2000.

Theorem: Assume u_i are strictly concave. Then

$$\text{No Pareto optimal trade}$$
$$\Updownarrow$$
$$P_1 \cap P_2 \cap \ldots \cap P_n = \emptyset$$

$\succsim = MEU$

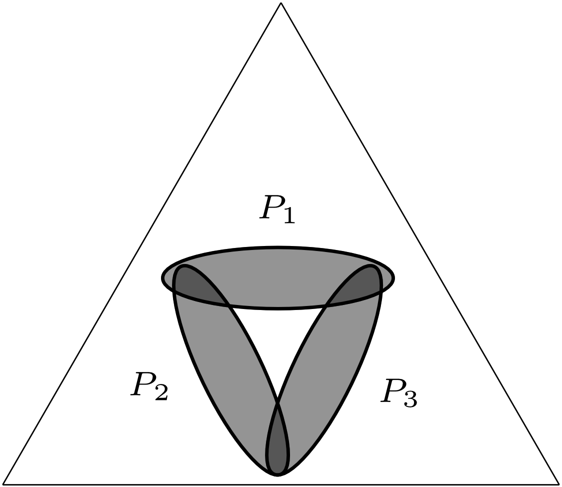
No Pareto optimal trade

$$\Updownarrow$$

$$\bigcap_{i=1}^{n} P_i = \emptyset$$

Beliefs don't have to be identical. Just overlapping.
Sharing one probability is enough.

$$\succsim \; = MEU$$



This paper

What happens beyond MEU?

Smooth (**?**; Nau; Ergin and Gul;
Seo; Halevy and Ozdenoren; Segal)

Variational (**?**)

Confidence (**?**)

This paper

Idea: Don't solve for each model separately:

1. Solve the problem for a general class of preferences

2. Plug in for special cases.

This paper

Idea: Don't solve for each model separately:

1. Solve the problem for a general class of preferences

  — Study a general notion of "beliefs"
  — Apply results from GE to the abstract problem

2. Plug in for special cases.

This paper

Idea: Don't solve for each model separately:

1. Solve the problem for a general class of preferences

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2. Plug in for special cases. ←

This paper

Why this route?

1. Identify forces behind betting independent of representation.
2. Useful if new models come up.
3. Heterogeneity.

Beliefs in general

Notation

S – states (finite)

$\mathcal{F}$ – acts ($=\mathbb{R}^S$)

$x \in \mathbb{R} \xrightarrow[\text{notation}]{\text{abuse}} x \in \mathbb{R}^S$ – constant acts

$\succsim$ – preference of an agent

Convex Preferences

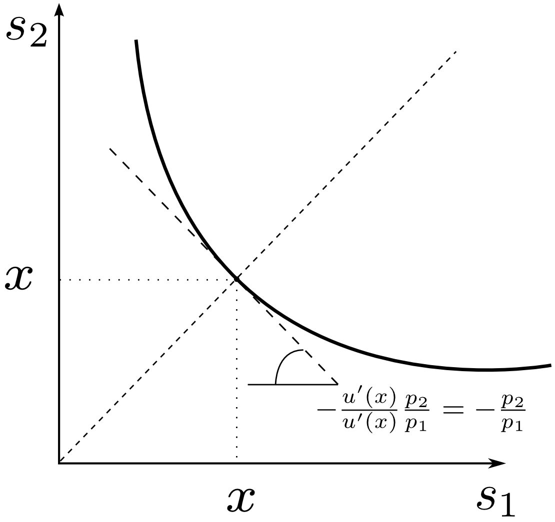
Preference
The relation $\succsim$ is complete and transitive.

Continuity
For all $f \in \mathcal{F}$, the sets $\{g \in \mathcal{F} | g \succsim f\}$ and $\{g \in \mathcal{F} | f \succsim g\}$ are closed.

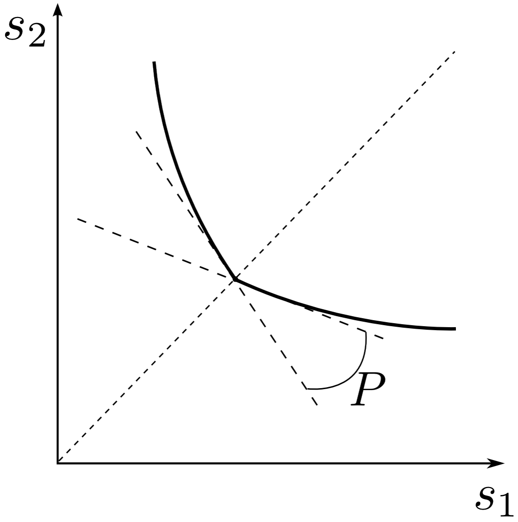
Monotonicity
For all $f, g \in \mathcal{F}$, if $f(s) > g(s)$ for all $s \in S$, then $f \succ g$.

Convexity
For all $f \in \mathcal{F}$, the set $\{g \in \mathcal{F} | g \succsim f\}$ is convex.

EU: p $\sim$ *supporting hyperplane*



MEU: P $\sim$ *set of supporting hyperplanes*



General definition of "subjective beliefs"

Yaari (JET, 69) proposed using the supporting hyperplane
in situations when representation is absent.

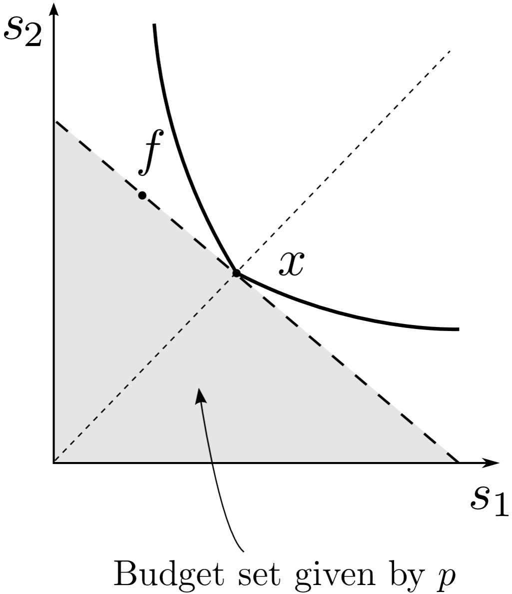
Calls it "subjective probability"

We call the set "subjective beliefs" and denote it by $\boldsymbol{\pi}$.

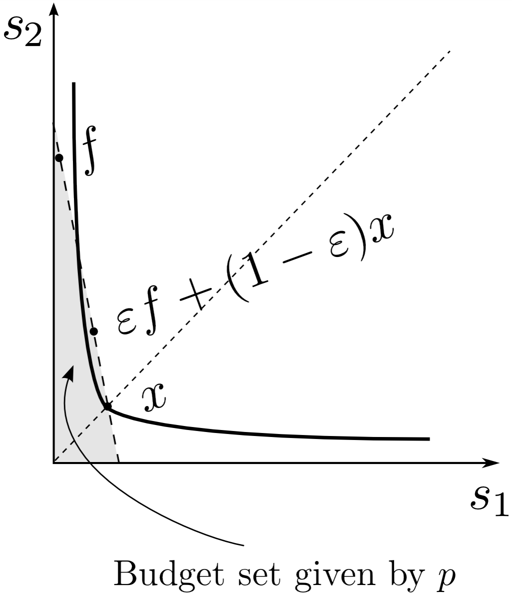
Standard representations

	functional form	subjective beliefs
EU	$\mathbb{E}_p\, u(f)$	$\{p\}$
MEU	$\min_{p\in P}\mathbb{E}_p\, u(f)$	P
variational	$\min_{p\in\Delta}\mathbb{E}_p\, u(f) + c(p)$	$\{p \mid c(p) = 0\}$
confidence	$\min_{p\in\Delta}\mathbb{E}_p\, u(f)\cdot\frac{1}{\varphi(p)}$	$\{p \mid \varphi(p) = 1\}$
smooth	$\int_\Delta \phi(\mathbb{E}_p\, u(f))\, \mathrm{d}\mu(p)$	$\{\int_\Delta p\, \mathrm{d}\mu(p)\}$

Unwillingness to trade



Willingness to trade



Properties of "subjective beliefs"

$$\boldsymbol{\pi}(x)$$

$$\shortparallel$$

$$\left\{p \in \Delta \mid \mathbb{E}_p f \geq x \text{ for all } f \succsim x\right\}$$

$$\shortparallel$$

$$\left\{p \in \Delta \mid \mathbb{E}_p f > x \text{ for all } f \succ x\right\}$$

$$\shortparallel$$

$$\left\{p \in \Delta \mid f \succsim x \text{ for all } \mathbb{E}_p f = x\right\}$$

$$\shortparallel$$

$$\bigcap \left\{P \subseteq \Delta \text{ cnvx, cpct } \mid \forall_{p \in P} \left(\mathbb{E}_p f > x\right) \Rightarrow \exists_\varepsilon \left(\varepsilon f + (1-\varepsilon)x \succ x\right)\right\}$$

Betting

Additional Axioms

Strong Monotonicity
For all $f \neq g$, if $f \geq g$, then $f \succ g$.

Strict Convexity
For all $f \neq g$ and $\alpha \in (0,1)$, if $f \succsim g$, then $\alpha f + (1-\alpha)g \succ g$.

Additional Axioms

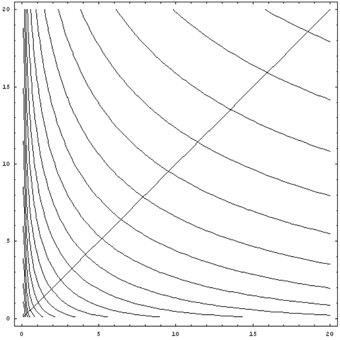
Constant Beliefs (Weak Translation Invariance)
For all acts z and all constant acts x, x'

$$\underset{\varepsilon>0}{\exists}\; x + \varepsilon z \succsim x \implies \underset{\varepsilon>0}{\exists}\; x' + \varepsilon z \succsim x'$$

In the presence of other axioms, this means that

$$\boldsymbol{\pi}(x) = \boldsymbol{\pi}(x') \quad \text{for all constant acts } x, x'$$

$\neg$ *Weak Translation Invariance*



$$\frac{1}{2}\log(f(s_1)) + \frac{1}{2}\sqrt{f(s_1)}$$

Main Theorem

Theorem. If $\{\succsim_i\}_{i=1}^n$ satisfy our axioms, then the following statements are equivalent:

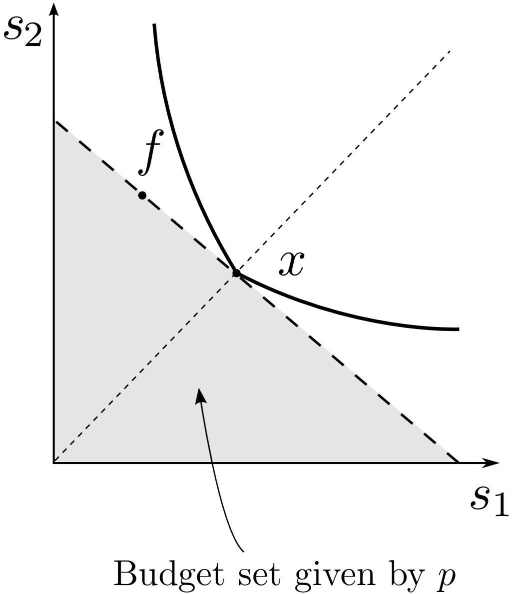
(i) There exists an interior full insurance Pareto optimal allocation.

(ii) Any Pareto optimal allocation is a full insurance allocation.

(iii) Every full insurance allocation is Pareto optimal.

(iv) $\bigcap_{i=1}^n \boldsymbol{\pi}_i \neq \emptyset$.

Unwillingness to trade



Theorem If $\succsim_i$ satisfy *Axioms* then the following statements are equivalent:

(i) There exists a full insurance Pareto optimal allocation.

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