Efficient Allocations under Ambiguity

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Goal

Understand risk sharing among agents with ambiguity averse preferences

30 balls Red
60 balls Green or Blue

Ambiguity

	R	G	B
r^+	1	0	0

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Understand risk sharing among agents with ambiguity averse preferences

Setup and notation

S — states of the world (finite)

$\Delta(S)$ — all probabilities on S

two agents exchange economy, one shot ex ante trade

$f : S \to \mathbb{R}_+$ — allocation of agent 1

$g : S \to \mathbb{R}_+$ — allocation of agent 2

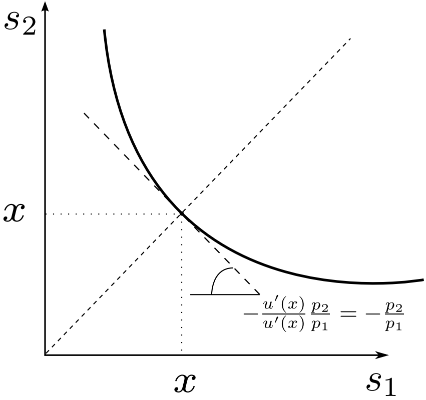
Question 1: Full Insurance

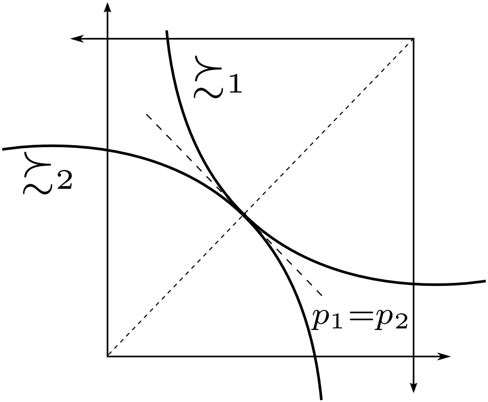
Full Insurance

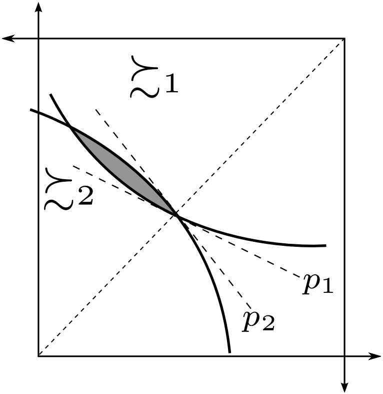
Theorem

agents have strictly risk averse EU
the aggregate endowment is risk-free
common beliefs

$\Longrightarrow$ all PO allocations are risk-free


s_2
x
$-\frac{u'(x)}{u'(x)}\frac{p_2}{p_1} = -\frac{p_2}{p_1}$
x
s_1


$\succsim_1$
$\succsim_2$
$p_1 = p_2$



$\succsim_1$
$\succsim_2$
p_1
p_2

Question 2: Conditional Full Insurance

Conditional Full Insurance

Theorem

agents have strictly risk averse EU
the aggregate endowment is $\mathcal{G}$-measurable
$\mathcal{G}$-concordant beliefs

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$p(\cdot \mid G) = q(\cdot \mid G)$ for all $G \in \mathcal{G}$

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$\implies$ all PO allocations are $\mathcal{G}$-measurable

$p(\cdot \mid G) = q(\cdot \mid G)$ for all $G \in \mathcal{G}$

$\mathbb{E}_p[f|\mathcal{G}] = \mathbb{E}_q[f|\mathcal{G}]$ for all f

Proof

$$\frac{u'(f(s_1))}{u'(f(s_2))}\frac{p(s_1)}{p(s_2)} = \frac{v'(g(s_1))}{v'(g(s_2))}\frac{q(s_1)}{q(s_2)}$$

Proof

$$\frac{u'(f(s_1))}{u'(f(s_2))} = \frac{v'(g(s_1))}{v'(g(s_2))}$$

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If $f(s_1) > f(s_2)$ then $g(s_1) > g(s_2)$, but that can't be since $f(s_1) + g(s_1) = f(s_2) + g(s_2)$

Question 3: Comonotonicity

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$$[f(s_1) - f(s_2)][g(s_1) - g(s_2)] \geq 0$$

Comonotonicity

Theorem

agents have strictly risk averse EU
common probability beliefs

$\Longrightarrow$ all PO allocations are comonotone

Proof

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Concordant not enough, because I need this to hold for any two states, so boils down to $p = q$

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If $f(s_1) > f(s_2)$ then $g(s_1) > g(s_2)$

Question:

What is the analogue of these results for ambiguity averse $\succsim$?

Main Characters

1. *Expected utility (EU)* : $U(f) = \mathbb{E}_p u(f)$

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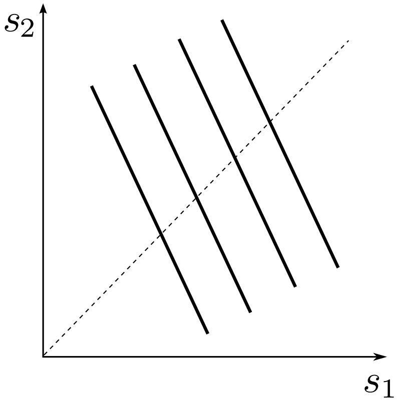
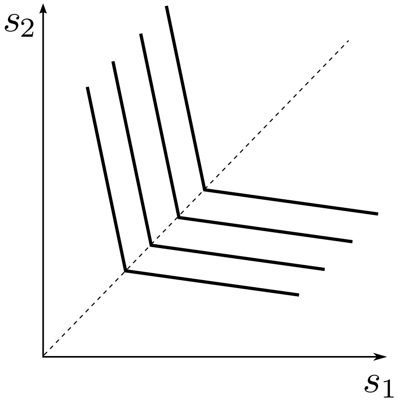
3. *General* $\succsim$: strictly convex, monotone, continuous

This gives us freedom to play with the risk-neutral probabilities without bending the utility too much

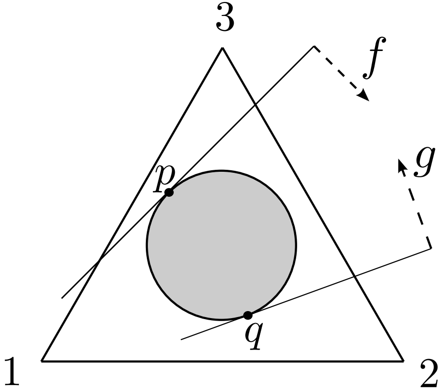
EU



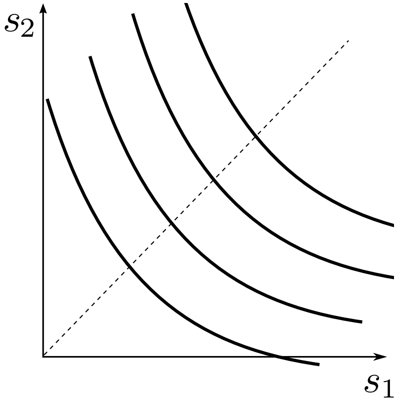
MEU



MEU dual space



Variational



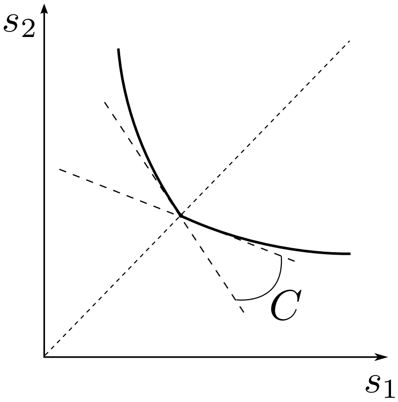
Full Insurance for Ambiguity averse $\succsim$

What is the analogue of the common beliefs condition?

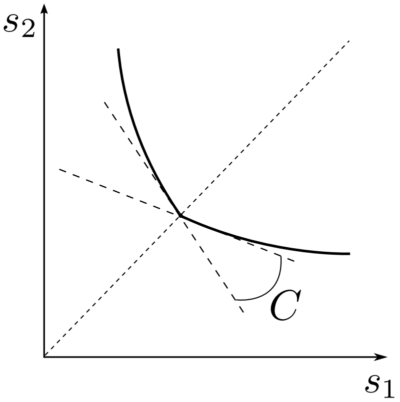
Full Insurance for Ambiguity averse $\succsim$

Billot, Chateauneuf, Gilboa, and Tallon (2000)
Rigotti, Shannon, and Strzalecki (2008)

Beliefs



Beliefs

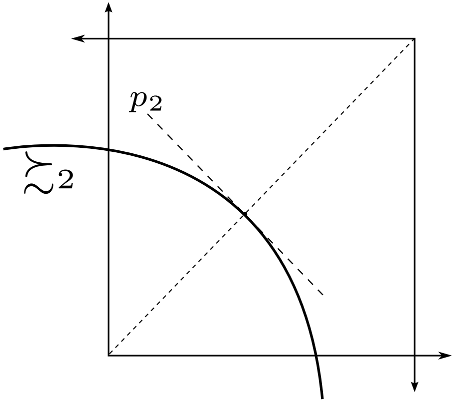


$p \in \Delta(S)$ is a **subjective belief** at f if $\mathbb{E}_p(h) \geq \mathbb{E}_p(f)$ for all $h \succsim f$

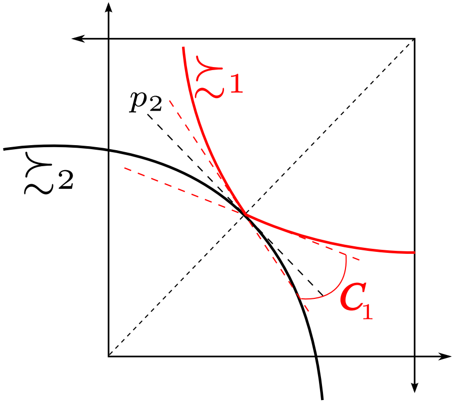
Full Insurance

agents have strictly convex preferences
the aggregate endowment is risk-free
shared beliefs

$\Longrightarrow$ all PO allocations are risk-free



p_2
$\succsim_2$



$\succsim_1$
p_2
$\succsim_2$
C_1

Conditions on Beliefs

	EU	$\succsim$
Full Insurance	same beliefs	shared beliefs

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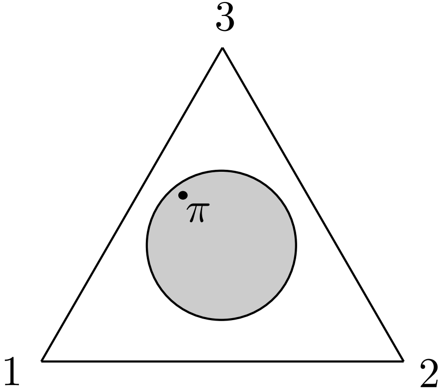
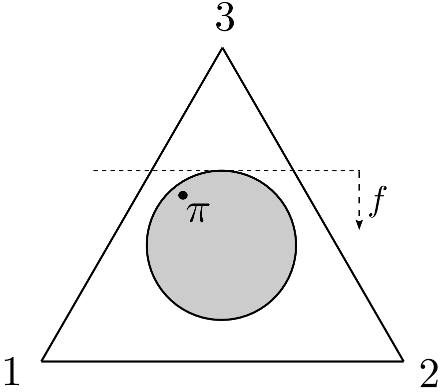
	EU	$\succsim$
Full Insurance	same beliefs	shared beliefs
Conditional Full Insurance	concordant beliefs	?

Conditional Full Insurance



Conditional Full Insurance



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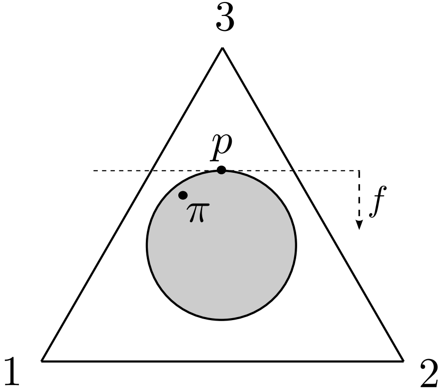

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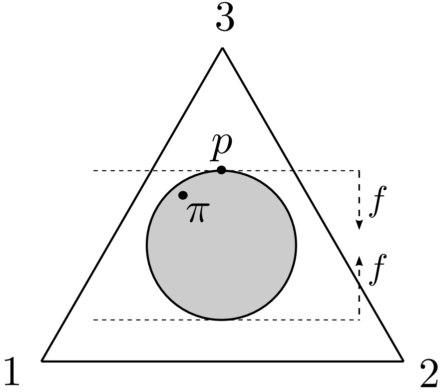

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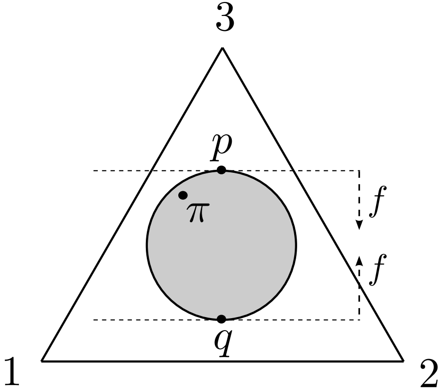

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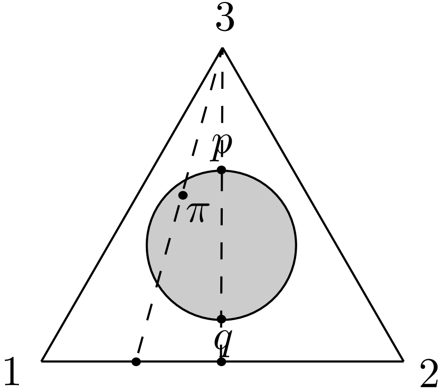

Conditional Full Insurance

The problem is that MRS_{12} depends on what is going on in state 3

(Sure thing principle violated)

Conditional Full Insurance

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Can show: p is a consistent $\mathcal{G}$-conditional belief iff $\mathbb{E}_p[h|\mathcal{G}] \succsim h$ for all h

Or: p is a consistent $\mathcal{G}$-conditional belief iff $f \succsim f + \epsilon$ for every ϵ with $\mathbb{E}_p[\epsilon|\mathcal{G}] = 0$

When does this happen?

MEU with concave utility and set of priors C

q is a consistent $\mathcal{G}$-conditional belief iff $p^q_{\mathcal{G}} \in C$ for every $p \in C$

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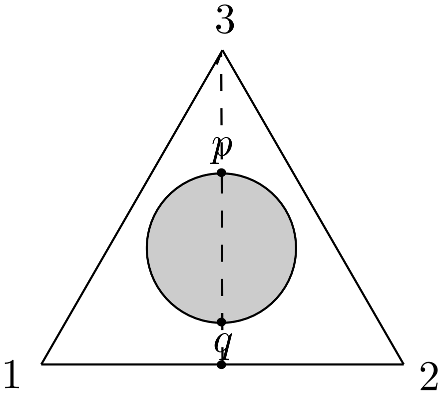
$p^q_{\mathcal{G}} =$ conditionals from q, marginals from p

When does this happen?



Examples

Constraint preferences: $C^{q,\epsilon} = \{p \in \Delta(S) \mid R(p \parallel q) \leq \epsilon\}$

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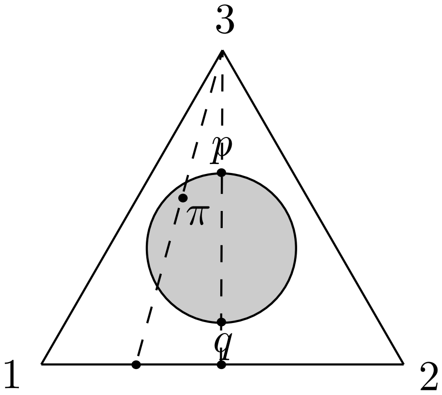
Conditional Full Insurance



Comonotonicity

Theorem

agents have strictly convex preferences
the aggregate endowment is $\mathcal{G}$-measurable
shared consistent $\mathcal{H}$-conditional beliefs for any $\mathcal{H}$ coarser than $\mathcal{G}$

$\Longrightarrow$ all PO allocations are comonotone

Other papers

Chateauneuf, Dana, and Tallon (2000)

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Chateauneuf, Dana, and Tallon (2000)

de Castro and Chateauneuf (2009)

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Chateauneuf, Dana, and Tallon (2000)

de Castro and Chateauneuf (2009)

Kajii and Ui (2009); Martins da Rocha (forthcoming)