Behavioral Competitive Equilibrium and Extreme Prices

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behavioral optimization

- *behavioral optimization* restricts agents' ability by imposing additional constraints on their maximization problem

- the model captures cognitive limitations

- examples:
  - limited attention
  - inability to formulate complex plans
  - bounded memory

common feature

agents cannot react to all changes in the economic fundamentals

- limited information processing capacity: cannot adjust to every observable change in economic fundamentals
- complexity constraints: cannot plan for every possible realization of economic circumstances
- bounded memory: cannot react to every variation in the history

crude consumption plans

- agents cannot perfectly tailor action to the state of the economy
- so they do the same thing in different but similar situations
- group the states into coarse events
- choose the same consumption in each state of an event

alternative formulations of crudeness

- crudeness in net trades / savings / finances
- actions = fraction of endowment
- costly flexibility

Key: not too crude to balance budget

important feature

the agents can:

- allocate their attention optimally
- form optimal categories
- form optimal memories

more generally, we want this choice to respond to incentives

optimality is the cleanest way of getting that

role of prices in the economy

there is nobody in the economy who understands everything

agents will endogeneously specialize—“division of attention”

the markets have to allocate not only goods, but also attention

prices need to “shout” at our inattentive agents to make them focus on things that in equilibrium someone needs to notice

it doesn’t make sense to focus on rare events if attention is costly

prices will exaggerate rare events

Model

$N = \{1, \dots, n\}$ – states; $\pi \in \Delta(N)$ – belief

$c : N \to \mathbb{R}_+$ – consumption plan; $\mathcal{C} = \mathbb{R}^n_+$ – consumption set

$B \subseteq \mathcal{C}$ – budget set

optimal choice:

$$\Upsilon(B) := \arg\max_{c \in B} \sum_{i=1}^{n} u(c_i)\pi_i$$

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agent constrained to *crude plans*: $c \in \mathcal{C}_k$ iff $|\{c(i) : i \in N\}| \leq k$

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behavioral optimization:

$$\Upsilon_k(B) := \arg\max_{c \in B \cap \mathcal{C}_k} \sum_{i=1}^{n} u(c_i)\pi_i$$

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interpretations:

- limited attention: plans in $\mathcal{C}_k$ convey less information than s
- limited memory: states/histories lumped together
- complexity: plans in $\mathcal{C}_k$ are simpler

behavioral optimization

$$\max_{c \in B \cap \mathcal{C}_k} \sum_{i=1}^{n} u(c_i)\pi_i$$

can be written as

$$\max_{P \in \mathcal{P}_k} \max_{\substack{c \in B \\ c \text{ is } P\text{-meas.}}} \sum_{i=1}^{n} u(c_i)\pi_i$$

where the partition P of S belongs to $\mathcal{P}_k$ iff $|P| \leq k$.

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optimal attention/memory allocation; optimal categorization

related literature

- Rubinstein (86), Neyman (87), Ben Porath (90, 93), Dow (91) finite automata in game theory and search models
- Wilson (04), Kocer (10) motivate crudeness constraints as a memory restriction
- Jehiel (05), Eyster and Piccione (11) reasoning by analogies/crude inference
- Sims (03), Woodford (12) model rational inattention by imposing constraints similar to crudeness; motivated by information processing capacity from information theory
- Grossman, and Laroque (90), Gabaix and Laibson (02)
- Gabaix (11, 12a, 12b, 12c, . . .)

Economy

static model

- continuum of identical households, each has the same stochastic endowment
- CRRA utility
- $N = \{1, 2, \ldots, n\}$ is a finite state space with n elements
- $\pi_1, \ldots, \pi_n$ – probabilities of states
- $s_i > 0$ – endowment in state i
- $s_i \leq s_j$ if $i < j$

prices

p_i – price in state $i \in N$

$p = (p_1, \ldots, p_n)$ – vector of prices

$\sum_{i=1}^{n} p_i = 1$ prices are normalized to sum to 1

$\frac{p_i}{\pi_i}$ – price in state i per unit of probability (price kernel)

budget and household choice

budget:

$$B(p) = \{c \in \mathbb{R}^n_+ : \sum_{i \in N} p_i \cdot (c_i - s_i) \leq 0\}$$

household's choice:

$$\max U(c)$$
$$\text{subject to } c \in B(p) \cap \mathcal{C}_k$$

allocations...

- the set of crude consumption plans is not convex
- market clearing (feasibility) requires that (identical) households choose distinct consumption plans
- an allocation is a probability on $\mathcal{C}_k$; it specifies the fraction of households that choose c
- $\Delta(\mathcal{C}_k)$ is the set of probabilities on $\mathcal{C}_k$ with finite support.

$\dots allocations$

economy $E = (u, k, s)$

Definition : an *allocation* is an element of $\Delta(\mathcal{C}_k)$.

the allocation $\mu \in \Delta(\mathcal{C}_k)$ is *feasible* for E if for all $i \in N$

$$\sum_{c \in \mathcal{C}_k} c_i \mu(c) \leq s_i$$

behavioral competitive equilibrium

Definition an allocation μ and a price p constitute a *BCE* for $E = (u, k, s)$ if μ is feasible and if every $c \in \mathrm{supp}\,(\mu)$ maximizes utility in $B(p) \cap \mathcal{C}_k$.

BCE exists and is monotone

- the consumption plan c is *measurable* if $s_i = s_j$ implies $c_i = c_j$.
- the consumption plan c is *monotone* if $s_i > s_j$ implies $c_i \geq c_j$.
- the allocation μ is *measurable/monotone* if every $c \in \operatorname{supp}(\mu)$ is monotone.
- the price is *monotone* if $s_i > s_j$ implies $\frac{p_i}{\pi_i} \leq \frac{p_j}{\pi_j}$.

Theorem:

(1) there exists a BCE (μ, p) for E

(2) if μ is a BCE allocation then μ is measurable and monotone

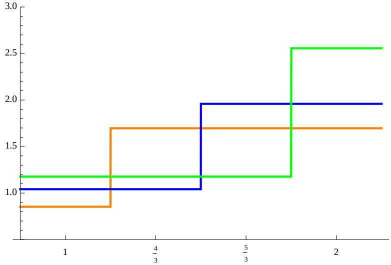
(3) if p is a BCE price in a pure endowment economy then p is monotone

example

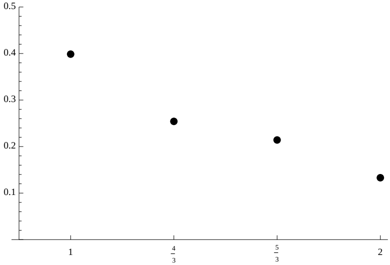
- four equally likely states
- $s_i \in \left\{1, \frac{4}{3}, \frac{5}{3}, 2\right\}$
- 2-crude plans: $k = 2$

example: log utility

$$u(x) = \ln x$$



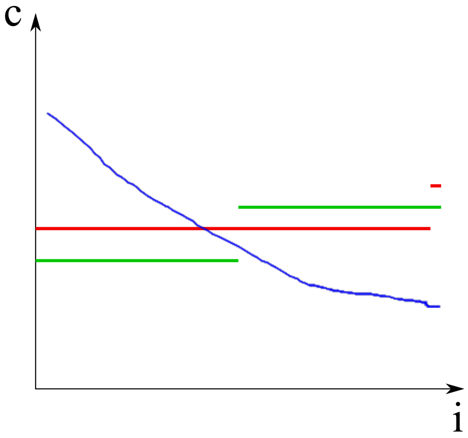
equilibrium consumption plans



equilibrium prices

Prices

idea: taking risks has to be rewarded in eqm.



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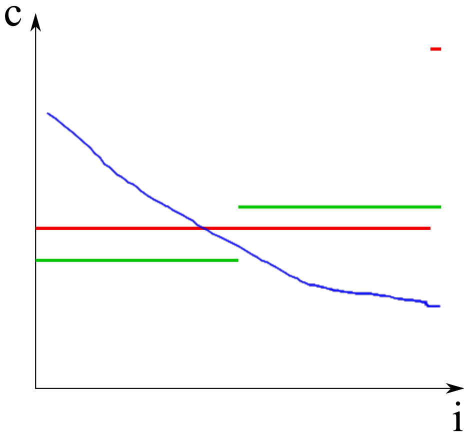

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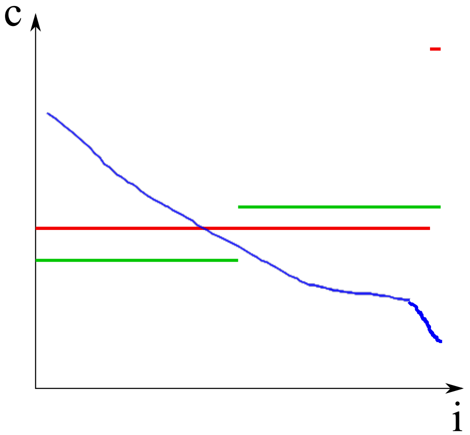


prices as $n \to \infty$

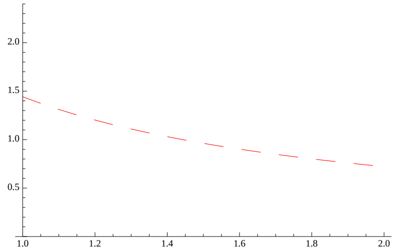
- we will examine a sequence of pure endowment economies that converge to an economy with a continuous endowment distribution

- compare the limit equilibrium prices in a BCE to the limit equilibrium prices of a standard economy without the crudeness constraint

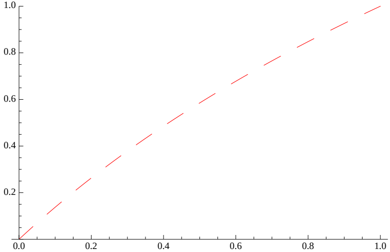
- along the sequence k and u stay fixed: $E^n = (u, k, s^n)$ is the $n-$th entry in the sequence

- s^n converges in distribution to a random variable with a continuous, strictly positive density on the interval $[a, b]$ where $0 < a < b$

standard equilibrium prices



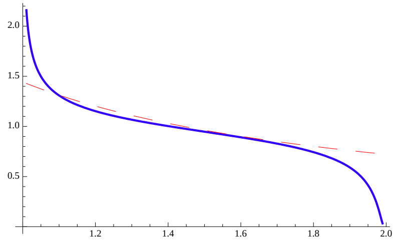
price density p



cumulative price P

$$u(z) = \log z,\ s \sim U[1,2]$$

BCE prices



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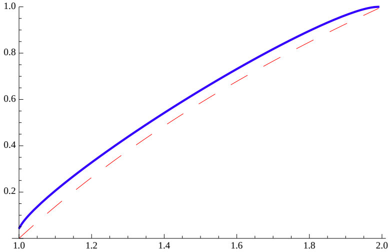

cumulative price P

$$u(z) = \log z,\ s \sim U[1,2],\ k = 2$$

extreme prices

Let $P = P(0) + \int_0^x p(r)dr$ be a limit BCE price of an almost continuous sequence.

(i) P has *heavy high tails* if $P(0) > 0$ and *heavy low tails* if $P(x) = 1$ for some $x < 1$.

(ii) P has *extreme highs* if $\lim_{x \to 0} p(x) = \infty$ and *extreme lows* if $\lim_{x \to 1} p(x) = 0$.

main result

Theorem Let P be a limit price of an almost continuous sequence $E^n = (u, k, s^n)$.

(1) If $\rho < 1$ then P has heavy high tails and extreme lows.

(2) If $\rho = 1$ then P has heavy high tails, extreme lows and extreme highs.

(3) If $\rho > 1$ then P has heavy high tails, heavy low tails, extreme highs and extreme lows.

economic implications of the result

Theorem

- there exists a sequence of consumption plans c^n such that
  - the expected consumption under $c^n \to 0$
  - the expenditure on $c^n \to +\infty$

- there exists a sequence of consumption plans d^n such that
  - the expected consumption under $d^n \to +\infty$
  - the expenditure on $d^n \to 0$

safe haven premium

- a (risk free) bond delivers one unit of consumption in every state
- an almost risk free bond delivers one unit of consumption in every state except in the most expensive $\epsilon-$fraction of states
- safe haven premium: limit price difference between these two assets as $n \to \infty$

Theorem the safe haven premium stays bounded away from zero for all ϵ

straightforward implication of heavy low tails of the equilibrium price

conclusion

- (Lucas tree) model with crude consumption plans
- limited attention/costly contemplation interpretations
- to capture agents' attention, equilibrium prices are extremely volatile at high or low realizations of the endowment
- in equilibrium optimal attention allocation is "easy"
- model is easy to compute (diff. eqns.)

Thank you