ON THE WELFARE SIGNIFICANCE OF NATIONAL PRODUCT IN A DYNAMIC ECONOMY *

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I. INTRODUCTION

Repeatedly and unabashedly the national income statistician calculates a number he calls net national product by adding in the value of the nation's net investment to its consumption. What does this single figure measure? The usual welfare interpretation of index numbers can perhaps be used to excuse combining apples and oranges, but it falls short of providing an adequate justification for NNP. Economic activity has as its ultimate end consumption, not capital formation. The most complete inference that can be drawn from such a tenet is that investment must be treated as an intermediate good in a multiperiod system whose final products are the various consumptions of different years. As Samuelson has convincingly argued,[1] the rigorous search for a meaningful welfare concept leads to a rejection of all current income concepts and ends up with something closer to a "wealth-like magnitude," such as the present discounted value of future consumption.

Actually, it is not really a question of choosing between a conventional but inappropriate current income concept and an impractical but correct wealth-like magnitude, because in principle they are merely different sides of the same coin. As I hope to show in this paper, the welfare justification of net national product is just the idea that in theory it is a proxy for the present discounted value of future consumption.

II. A FORMULATION OF THE PROBLEM

We abstract heroically in more ways than one.

First of all, for simplicity it is assumed that (in effect) there is just one composite consumption good. It might be calculated as

* For very useful discussions on this subject and for encouraging me to write up my results, I would like to thank Professor Paul A. Samuelson.

1. See P. A. Samuelson, "The Evaluation of 'Social Income': Capital Formation and Wealth," in *The Theory of Capital*, Proceedings of an IEA Conference, Lutz and Hague, eds. (New York: St. Martin's Press, 1961).

an index number with given price weights, or as a multiple of some fixed basket of goods, or more generally as any cardinal utility function. The important thing is that the consumption level in period t can be unambiguously registered by the single number $C(t)$. Purging consumption of the index number problem will allow us to focus more sharply on the general meaning and significance of combining it with investment.

The notion of a capital good used in this paper is meant to be quite a bit more general than the usual equipment, structures, and inventories. Strictly speaking, pools of exhaustible natural resources ought to qualify as capital, and so should states of knowledge resulting from learning or research activities.[2] For convenience, all noncapital contributions to production are treated as fixed over time. In the case of a growing labor force, this assumption would always be satisfied as long as everything were calculated on a per-capita basis.[3] Note that in effect we are making the extreme abstraction that *all* sources of economic growth have been identified and attributed to one or another form of capital, broadly defined.

Suppose that altogether there are n capital goods. The stock of capital of type $i(1 \leqslant i \leqslant n)$ in existence at time t is denoted $K_i(t)$ and its net investment flow is $I_i(t) \equiv dK_i/dt$.[4] From what has previously been said, the production possibilities set at time t can be expressed in the form $S(K(t))$. The consumption-investment pair (C,I) is producible at time t if and only if

$$(1) \qquad (C,I) \in S(K(t)).$$

Let p_i represent the price of investment good i relative to a consumption price of unity. A real net national product function (with consumption as numeraire) could be defined as follows:

$$(2) \qquad Y(K,p) \equiv \max_{(C,I)\in S(K)} [C+pI].$$

While we could get by on much weaker assumptions, it will be postulated here that $\partial Y/\partial K_i$ and $\partial Y/\partial p_i$ exist for all K and $p \geqq 0$.

A *feasible* trajectory $\{C(t),K(t)\}$ is one satisfying for all $t \geqslant 0$

$$(3) \qquad \left(C(t), \frac{dK}{dt}(t)\right) \in S(K(t)),$$

$$(4) \qquad K(0) = K_0,$$

2. Included here would be so-called "human capital." Investment in learning or research is of course most easily calculated on the cost side, since it obviates the necessity for evaluating increases in the semi-imaginary "stock of knowledge."

3. A similar comment applies to nonattributable ("atmospheric") technical change that is purely labor-augmenting; in such a case everything should be calculated per unit of "effective" or "augmented" labor.

4. As usual, $K \equiv (K_i)$.

where K_0 is the original endowment of capital that is available at starting time $t=0$.

Generally speaking, there are an infinite number of feasible trajectories. We can narrow them down to a single family by presuming a competitive-like economy with a fixed own interest rate on the consumption good equal to r. A *competitive* trajectory $\{C^*(t), K^*(t)\}$ with rate of return r is any feasible trajectory for which there exists a set of investment prices $\{p(t)\}$ such that for all $t \geqslant 0$:

$$(5) \qquad Y(K^*(t), p(t)) = C^*(t) + p(t)\frac{dK^*}{dt}(t)$$

$$(6) \qquad \left.\frac{\partial Y}{\partial K_i}\right|_* = rp_i(t) - \frac{dp_i}{dt}(t) \qquad i=1, \ldots, n.$$

Equation (5) just states that what is actually produced by the economy at any time maximizes its income — in other words, relative prices are equal to marginal rates of transformation. Condition (6) is the well-known intertemporal efficiency condition of a competitive capital market with perfect foresight.[5]

Actually, equations (5) and (6) are necessary Pontryagin-type conditions [6] for any solution of the optimal control problem: [7]

5. If I buy an extra unit of capital good i, it will cost me p_i, whereas I can use it to produce $\partial Y/\partial K_i$ worth of output and sell it for $p_i + dp_i/dt$ one period later. In a competitive capital market we must therefore have

$$p_i = \frac{\dfrac{\partial Y}{\partial K_i} + p_i + \dfrac{dp_i}{dt}}{1+r},$$

which reduces to equation (6). For a more rigorous explanation, see R. Dorfman, P. A. Samuelson, and R. M. Solow, *Linear Programming and Economic Analysis* (New York: McGraw-Hill, 1958), p. 318. The * symbol in equation (6) means that the derivative is evaluated (at time t) along a competitive trajectory.

6. See, for example, L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mischenko, *The Mathematical Theory of Optimal Processes* (New York and London: Interscience Publishers, 1962); or K. J. Arrow, "Applications of Control Theory to Economic Growth," in *Mathematics of the Decision Sciences*, Part 2, Dantzig and Veinott, eds. (Providence: American Mathematical Society, 1968), pp. 85–119.

7. Sometimes it is argued that the maximand of an optimal growth problem *should* have the form (7) (once it has been determined that $C(t)$ is the appropriate measure of satisfaction at time t) because it is reasonable to require that the future look the same from any initial time point; see, for example, R. H. Strotz, "Myopia and Inconsistency in Dynamic Utility Maximization," *Review of Economic Studies*, XXIII (1956), 165–80. Note that it is trivial to force any technology formally into the nominally time-independent form (8) merely by introducing a new artificial "capital good" K_{n+1}, which is a proxy for time and satisfies the differential equation,

$$\frac{dK_{n+1}}{dt} = 1;$$

the difficulty with such a trick from a strictly *economic* point of view is that it is not clear where the investment,

(7) maximize $\int_0^{\infty} C(t)e^{-rt}dt$

(8) subject to $\left(C(t), \frac{dK}{dt}(t)\right) \epsilon S(K(t))$

(9) $K(0)=K_0$.

What we have been calling net national product is just the Hamiltonian for a general optimization problem of the form (7)–(9). Therefore, an alternative but equivalent approach to the one taken in this paper would be to ask what the Hamiltonian along an optimal growth path measures.

III. What Is Net National Product?

Even granted that consumption is the ultimate end of economic activity, the national income statistician's practice of adding in investment goods to the value of consumption by weighting them with prices measuring their marginal rates of transformation might still be defended as a measure of the economy's power to consume at a constant rate. After all, a standard welfare interpretation of NNP is that it is the largest permanently maintainable value of consumption. If all investment were convertible into consumption at the given price-transformation rates, the maximum attainable level of consumption that could be maintained forever without running down capital stocks would appear to be NNP as conventionally measured by $C^*+p\frac{dK^*}{dt}$.

Unfortunately, such reasoning is insufficient because *marginal* transformation rates cannot in general be used to change *nonmarginal* amounts of investment into consumption. For this reason, the *consumption* level $C^*(t)+p(t)\frac{dK^*}{dt}(t)$ is undoubtedly not attainable at time t. Such a situation is depicted in Figure I for case $n=1$. The economy is located at point A on the production possibilities frontier $B'B$. Real net national product $C^*+p\frac{dK^*}{dt}$ is geometrically represented as OC'. But OC' is a strictly *hypothetical* consumption level at the present time, since the largest permanently maintainable level of consumption that can *actually* be obtained is

$$p_{n+1}\frac{dK_{n+1}}{dt}=p_{n+1},$$

(which measures the value of time per se) shows up in the national income accounting of any real economy.

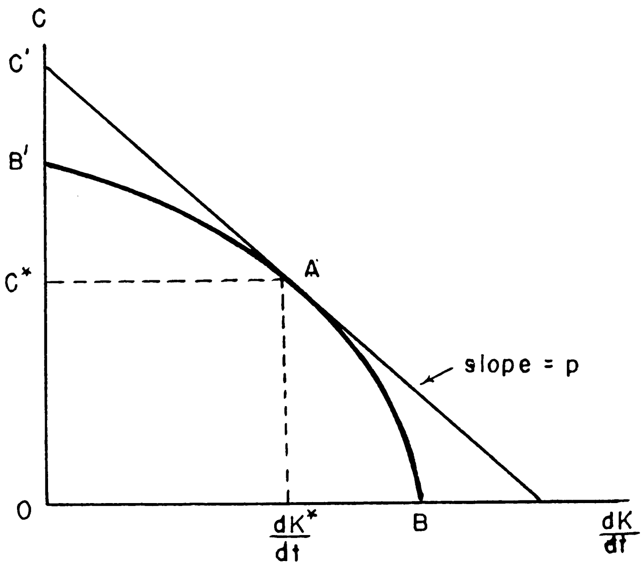

FIGURE I

OB'. The two are equal only if the production possibilities frontier is a linear surface.

All this notwithstanding, it turns out that the maximum welfare actually attainable from time t on along a competitive trajectory,

$$\int_t^{\infty} C^*(s)\, e^{-r(s-t)} ds,$$

is exactly the same as what *would* be obtained from the *hypothetical* constant consumption level $C^*(t)+p(t)dK^*/dt$. In this sense the naive interpretation of the current power to consume at a constant rate idea gives the right answer, although for the wrong reason. Net national product is what might be called the *stationary equivalent* of future consumption, and this is its primary welfare interpretation.[8]

IV. Proof of the Main Proposition

We want to show that along a competitive trajectory

$$\int_t^{\infty} \left[C^*(t) + p(t)\frac{dK^*}{dt}(t) \right] e^{-r(s-t)} ds = \int_t^{\infty} C^*(s)\, e^{-r(s-t)} ds,$$

8. If there is nonattributable or "atmospheric" technical change, national product will be less than the stationary equivalent of consumption by a term measuring the present discounted value of the pure effect of time alone on increasing output.

or that

$$(10) \qquad Y^*(t) = r \int_t^\infty C^*(s) e^{-r(s-t)} ds,$$

where

$$(11) \qquad Y^*(t) \equiv Y(K^*(t), p(t)) = C^*(t) + p(t)\frac{dK^*}{dt}(t).$$

This is easily shown as follows. First differentiate Y^* totally with respect to time:

$$(12) \qquad \frac{dY^*}{dt}(t) = \sum_1^n \left.\frac{\partial Y}{\partial K_i}\right|_* \frac{dK^*_i}{dt}(t) + \sum_1^n \left.\frac{\partial Y}{\partial p_i}\right|_* \frac{dp_i}{dt}(t).$$

From (2), (5), and the theory of cost functions,[9]

$$(13) \qquad \left.\frac{\partial Y}{\partial p_i}\right|_* = \frac{dK^*_i}{dt}(t).$$

Substituting from (6) and (13) into (12) and canceling out the term

$$\sum_1^n \frac{dp_i}{dt}(t)\frac{dK^*_i}{dt}(t)$$

yields

$$\frac{dY^*}{dt}(t) = r \sum_1^n p_i(t)\frac{dK^*_i}{dt}(t),$$

which by (11) is equivalent to

$$(14) \qquad \frac{dY^*}{dt}(\mathrm{t}) = r(Y^*(t) - C^*(t)).$$

It can easily be verified that the solution to the differential equation (14) is given by (10), the proposition to be proved.

V. Unanticipated Technological Change: an Application

So far we have in effect been using the law of large numbers to abstract away the sporadic or stochastic nature of technical change. Instead, it has been attributed in regular fashion (ultimately) to capital accumulation of one form or another (including educational investment in human capital and research investment in increased stocks of knowledge). Now we shall go to the opposite extreme in asking what happens if an unanticipated invention or innovation is suddenly and unexpectedly discovered.[10] Such "windfall" technical

9. See, for example, D. McFadden, "An Econometric Approach to Production Theory," unpublished manuscript, Ch. 1.

10. Alternatively, we could consider the situation where a seemingly foreseen invention or innovation failed to materialize after performing the necessary research that was expected to uncover it.

change can be expressed analytically as an enlargement of the production possibilities set from $S(K)$ to $S'(K)$, where

(15) $S'(K) \supseteq S(K)$.

For notational convenience and without loss of generality, this unexpected change is presumed to occur at time zero. In a perfect-foresight, competitive economy, a new competitive trajectory $\{C'^*(t), K'^*(t)\}$ will come into being with a new set of prices $p'(t)$ obeying conditions just like (2)–(6), the only substantive difference being that $S'(K)$ now replaces $S(K)$. From (15) and the sufficiency [11] of conditions (2)–(6) for describing a solution to (7)–(9), it follows that

$$\int_0^\infty C'^*(t)e^{-rt}dt > \int_0^\infty C^*(t)e^{-rt}dt.$$

This means, along with (10), (11), that

$$C'^*(0) + p'(0)\frac{dK'^*}{dt}(0) =$$

$$= r\int_0^\infty C'^*(t)e^{-rt}dt > r\int_0^\infty C^*(t)e^{-rt}dt$$

$$= C^*(0) + p(0)\frac{dK^*}{dt}(0).$$

In other words, the welfare effect on present discounted consumption of an unexpected windfall gain in production possibilities should exactly and immediately be capitalized by a sudden rise in net national product.[12] This is true even if the technological discovery is of the embodied kind that cannot increase *current* productive capacity but will be helpful in adding to production possibilities only at some time in the future, after the necessary capital has been accumulated. From Figure I it is apparent that the only way current NNP ($= OC'$) can rise is if the economy moves along BB' from A towards B. Thus, at the time of its discovery, unanticipated capital-embodied technical change should (other things being equal) immediately result in less consumption and more investment.

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11. Actually, for sufficiency to hold, we would need something additional like convexity of S and fulfillment of a transversality condition. See, e.g., Arrow, *op. cit.*; or M. L. Weitzman, "Duality Theory for Infinite Horizon Convex Models," *Management Science*, XIX (March 1973), 783–89.

12. Naturally a windfall *decrease* in production possibilities (like the discovery that petroleum reserves are less than expected) leads to a *decline* in current NNP, which reflects the extent of the loss.