A MODEL OF THE DEMAND FOR MONEY BY FIRMS: COMMENT

MARTIN WEITZMAN

Recent macroeconomic explanations of the demand for cash have tended to focus on the so-called "indirect utility" approach. One of the latest models to embody this viewpoint is that of Miller and Orr,¹ hereafter referred to as the M-O model. Although earlier models isolated certain key factors in the analysis of household cash inventory decisions, they generally failed to capture the essence of the business habitat, a void which Miller and Orr have endeavored to fill.

In this note an attempt is made to clarify and extend certain parts of the M-O analysis to cover an important but neglected aspect of the business situation which Miller and Orr seek to quantify. Roughly speaking, it is found that the M-O model, like many inventory models, is "robust" (i.e., general results are unaltered) with respect to the assumptions to be modified.

The policy structure of the M-O model is characterized by two parameters, $h$ and $z$. Cash balances are allowed to wander freely until they reach either the lower bound, zero, or the upper bound, $h$, at which times a portfolio transfer is undertaken to restore cash balances to a level of $z$. The lower bound is set at zero because transfers are regarded as taking place instantaneously.

Miller and Orr postulate that the fixed cost of transference from bonds to cash, $\beta$, is equal to the fixed cost of transference from cash to bonds, $\gamma$. I find this assumption implausible. The prospect of running out of cash forces a firm to appropriate managerial attention on a scale of priority far higher than would be the case for the problem of converting excess cash into bonds. Consequently, it is felt, the total opportunity loss (the sum of direct plus indirect costs) of running out of cash altogether will typically exceed the total opportunity loss of converting cash to bonds.

The return point is prescribed by Miller and Orr to be independent of whether the last transaction was a purchase or a sale. Such a policy, they state, is simpler and more "natural" than one involving different return points after a purchase and a sale, because fixed transfer costs are assumed to be identical in both directions.

Actually, whether or not transfer costs are equal, it suffices to consider only a single return point because an optimal policy allowing for double return points is a single return point strategy. This is so because the entire process is Markovian. The system "has no memory" and it matters only what state the system is in, not how it got there. Having made the fixed payment, we are free to choose initial conditions (the point of return). The optimal choice depends only on the future, and not on the past in any way — it is thus identical from either transfer. The optimal point of return will in general depend on $\beta$ and $\gamma$, but it will always be one point rather than two.$^2$

The path is now clear for investigating the effects of differing transfer costs. The following symbols are used:

- $\beta =$ fixed cost of a transfer from bonds to cash
- $\gamma =$ fixed cost of a transfer from cash to bonds
- $\nu =$ rate of interest earned on the portfolio
- $T =$ length of the planning period
- $E(N_1) =$ expected number of portfolio transfers from bonds to cash during the planning period of length $T$
- $E(N_2) =$ expected number of portfolio transfers from cash to bonds during the planning period of length $T$
- $E(N) =$ expected total number of portfolio transfers during a period of length $T$
- $E(M) =$ the average cash balance.

The expected cost per day of managing the firm's cash balances over a horizon of $T$ days can be expressed as

$$E(c) = \beta \frac{E(N_1)}{T} + \delta \frac{E(N_2)}{T} + \nu E(M). \tag{1}$$

From now on, we assume zero-drift in the underlying random walk. With the general case it is much more difficult to obtain analytically tractable results, and they depend on a larger number of parameters.

The resulting steady state distribution is triangular with base $h$ and mode $z$. It follows that the relative occupancy rates of the reflecting barriers at zero and $h$ are given by the relative slopes of the two sides of the triangle. The mean of such a distribution is $(h + z)/3$.

$^2$ Optimality of a single return point strategy under the conditions assumed is also indicated by Fama and Eppen in an as yet unpublished paper entitled "Solutions for Cash Balance and Simple Dynamic Portfolio Problems."

$^3$ Miller and Orr, op. cit., p. 422.
THE DEMAND FOR MONEY BY FIRMS

Since the area of the triangle is one, its height must be $2/h$. It follows that

$$\frac{E(N_1)}{E(N_2)} = \frac{2/h}{h-z} = \frac{h-z}{z}. \tag{2}$$

Using the fact that

$$\frac{E(N)}{T} = \frac{E(N_1) + E(N_2)}{T}$$

is distributed as $1/D(z,h)$, where $D(z,h) = z(h-z)$; $^4$

(1) and (2) combine to form

$$E(c) = \beta \frac{\gamma}{hz} + \frac{\gamma(h+z)}{(h-z)h} + \frac{\nu(h+z)}{3} \tag{3}$$

(for notational convenience, we have chosen units so that $m^2t = 1.5$

Define $w = h - z$, substitute in (3), and fulfill the necessary conditions

$$\frac{2E(c)}{2z} = \frac{-\beta}{z^2(w+z)} - \frac{\beta}{z(w+z)^2} \frac{-\gamma}{w(w+z)^2} + \frac{2\nu}{3} = 0 \tag{4}$$

$$\frac{2E(c)}{2w} = \frac{-\gamma}{w^2(w+z)} - \frac{\gamma}{w(w+z)^2} - \frac{\beta}{z(w+z)^2} + \frac{\nu}{3} = 0. \tag{5}$$

Let $w = kz$, where $k$ is a constant to be determined.

Substituting in (4) and (5), canceling terms, and rewriting, we derive the following cubic equation for $k$:

$$\beta k^3 = 3 \gamma k + 2 \gamma.$$

Note that $k$ is invariant to $\nu$ and the ratio of $\beta$ to $\gamma$. Changes in $\nu$ and proportionate changes in $\beta$ and $\gamma$ serve only to shrink or dilate the system as a whole, and leave unaffected the ratio of $z$ to $h$.

If $\gamma = 0$, then $k = 0$ ($w = 0$, $z = h$), in accordance with the intuitive notion that the optimal policy under a zero charge for conversion of cash to bonds requires us to convert every extra unit of cash beyond $h$ into bonds, going right back to level $h$.

Similar comments apply for the case $\beta = 0$, where no cash is held (the cubic equation actually yields a negative value of $h$ in this case, and a boundary solution is the relevant one).

Our greatest interest centers on analyzing less extreme ratios of $\beta$ to $\gamma$. The following four are examples:

4. Ibid., p. 421.
5. Ibid.
values of $\beta$ and $\gamma$  |  solution in $k$  |  solution for $z$ in terms of $h$
---|---|---
$\beta = 44 \gamma$  |  $k = \frac{1}{2}$  |  $z = \frac{2}{3} h$
$\beta = 5 \gamma$  |  $k = 1$  |  $z = \frac{1}{2} h$
$\beta = \gamma$  |  $k = 2$  |  $z = \frac{1}{2} h$
$\beta = \frac{11}{27} \gamma$  |  $k = 3$  |  $z = \frac{1}{4} h$

The results speak for themselves. To push $z$ up from $\frac{1}{3}$ to $\frac{1}{2}$ of $h$ requires that $\beta = 5 \gamma$, an extremely high ratio. The interesting M-O result that $z < \frac{1}{2} h$ holds true unless $\beta$ is over five times as great as $\gamma$.

Yale University