

# The “ratchet principle” and performance incentives

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*The use of current performance as a partial basis for setting future targets is an almost universal feature of economic planning. This “ratchet principle,” as it is sometimes called, creates a dynamic incentive problem for the enterprise. Higher rewards from better current performance must be weighed against the future assignment of more ambitious targets. In this paper I formulate the problem of the enterprise as a multiperiod stochastic optimization model incorporating an explicit feedback mechanism for target setting. I show that an optimal solution is easily characterized, and that the incentive effects of the ratchet principle can be fully analyzed in simple economic terms.*

## 1. Introduction

■ Understanding how incentive systems work is an important task of economic theory. To date, most analyses of reward structures have been essentially static (Weitzman, 1976 and references cited there). For some situations this is not a serious limitation, but, certain important incentive issues have an inherently dynamic character that cannot even be formulated, let alone analyzed, in a timeless framework (Yunker, 1973; Weitzman, 1976; Snowberger, 1977).

Consider the “standard reward system.” Let  $y$  be a performance indicator for the enterprise. In most settings,  $y$  will symbolize output, but profits, cost, or productivity might be the appropriate performance measure in some contexts. Let the target, goal, or quota be denoted  $q$ . In a standard reward system the variable component of an enterprise’s bonus is typically proportional to the difference between  $y$  and  $q$ ; where the relationship is more complicated, proportionality is still a good approximation for most analytical purposes.

There are two basic incentive problems associated with a standard reward system: one is static and the other is dynamic. The immediate difficulty is essentially a *static* problem of misrepresentation which has to do with bluffing or gaming in hopes of influencing the plan while it is being formulated. The worker or manager will typically try to convince his superiors that  $y$  is likely to be small, thereby entitling him to a lower  $q$  and a bonus that is easier to attain. To focus sharply on the *dynamic* incentive problem, the present paper abstracts from the static misrepresentation issue by not allowing the planners to base quotas on any message other than previous actual output.

The *dynamic* incentive problem, on which this paper concentrates, arises from the well-known tendency of planners to use current performance as a criterion in determining future goals. This tendency has sometimes been called

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the “ratchet principle” of economic planning, because current performance acts like a notched gear wheel in fixing the point of departure for next period’s target.<sup>1</sup> Operation of the ratchet principle is widespread in planning or regulatory contexts ranging from the determination of piecework standards for individual workers to fixing budgets or output quotas for large bureaucracies. In such situations, agents face a dynamic tradeoff between present rewards from better current performance and future losses from the assignment of higher targets.

Realistic treatment of the ratchet principle necessitates a multiperiod stochastic statement of the enterprise’s problem, which at first glance appears to be extremely complicated. One of the principal aims of this paper is to show that under a reasonable formulation, the enterprise’s dynamic problem can be easily solved and given a neat economic interpretation. In this formulation, the effect of the ratchet principle on economic performance is simple to state and analyze.

## 2. The model

■ The economic unit whose behavior we shall be studying is called an “enterprise.” This term is employed in a broad sense because, depending on the context, the unit might be an individual worker, an intermediate sized department, or a giant sector. The enterprise operates in a planned environment where it and the planners interact; the environment might be a multidivisional private firm, a government or quasi-public organization, or a nationalized branch of the economy.

Let  $t = 1, 2, 3, \dots$  index the plan period. Enterprise performance during any period will typically be affected by the plan target for that period and will in turn influence the formation of next period’s target.

The planning period discount rate is denoted  $r$ . That is, next period’s gains are transformed into this period’s by the factor  $1/(1 + r)$ . If  $\rho$  is the instantaneous force of interest and  $l$  is the length of the plan period,

$$\frac{1}{1 + r} = e^{-\rho l},$$

or

$$r = e^{\rho l} - 1. \quad (1)$$

Thus, the size of  $r$  depends on the length of the review lag  $l$  and the interest rate  $\rho$ .

The variable  $y_t$  will symbolize performance of the enterprise in period  $t$ . It is perhaps easiest to think of inputs being exogenously determined and let  $y_t$  denote output; and, for convenience, this will be our primary interpretation. As noted earlier, though, profits or productivity could also be accommodated as measures of performance. In budgeting contexts it may be more appropriate to envision a fixed task given in period  $t$  and have  $y_t$  be a negative output representing the funds needed to accomplish the task.

Let  $\epsilon_t$  be a random variable, known at time  $t$  but uncertain before, which characterizes cost or technological conditions of the enterprise during period  $t$ . For ease of exposition, it will be assumed the  $\{\epsilon_t\}$  are independently distributed.

Given conditions described by  $\epsilon_t$ , if the enterprise chooses to perform at level  $y_t$  in period  $t$ , it incurs net disutility, loss, or cost  $C_t(y_t; \epsilon_t)$  exclusive of

<sup>1</sup> The term “ratchet principle” was coined by Berliner. For descriptions see Berliner (1957, pp. 78–80), Bergson (1964, pp. 75–76), Zielinski (1973, p. 122), Berliner (1976, pp. 408–409).

any bonus payments received. The cost of performance is typically time-dependent because the means available for meeting plan assignments, treated here as exogenously predetermined, may differ from period to period. For example, a growing enterprise will frequently have an ever increasing capacity to utilize. It is postulated that for all  $\epsilon_t$ ,

$$C_t'' \geq 0, \quad (2)$$

which ensures that second-order conditions are always met.<sup>2</sup>

We denote the performance target in period  $t$  as  $q_t$ . We assume that the bonus received by the enterprise is proportional to the difference between actual performance and the plan target:

$$b(y_t - q_t),$$

where  $b$  is a bonus coefficient. In reality bonus systems tend to be more complex, but the present formulation is a first-order approximation that fairly represents many situations.

If  $q_t$  were exogenously fixed for all  $t$ , the enterprise in period  $t$  would seek  $y_t$  to maximize the total gain<sup>3</sup>

$$b(y_t - q_t) - C_t(y; \epsilon_t).$$

It would choose the performance level  $\tilde{y}_t$  satisfying

$$C_t'(\tilde{y}_t; \epsilon_t) = b.$$

A more realistic scenario (and the point of this paper) is to have  $q_t$  determined by some version of the ratchet principle. The specific form postulated here is:

$$q_t - q_{t-1} = \delta_t + \lambda_t(y_{t-1} - q_{t-1}). \quad (3)$$

The independent increment  $\delta_t$  represents how much the target would be changed in period  $t$  if last period's target were exactly met. For every notch that last period's performance exceeded last period's target, this period's target will be pushed up by an additional  $\lambda_t$  notches. The adjustment coefficient  $\lambda_t$  is treated as a behavioral parameter of the planners that quantifies the strength of the ratchet principle.

An instructive way of rewriting (3) is

$$q_t = \lambda_t y_{t-1} + (1 - \lambda_t)q_{t-1} + \delta_t.$$

This period's target is a weighted average of last period's performance and last period's target, plus an independent increment. The weight on last period's performance is the adjustment coefficient  $\lambda_t$ .

The firm views the elements in the target-setting process,  $\delta_t$  and  $\lambda_t$ , as independently distributed random variables whose realized values are *not* known at time  $t$  (unlike  $\epsilon_t$ ). The mean value of the adjustment coefficient is assumed to be the same in each period, denoted

<sup>2</sup> We employ the notation,

$$C_t' = \frac{\partial C_t(y_t; \epsilon_t)}{\partial y_t}, \quad C_t'' = \frac{\partial^2 C_t(y_t; \epsilon_t)}{\partial y_t^2}.$$

<sup>3</sup> We are implicitly assuming that the one-period gain can be written as bonus income minus a disutility-of-effort term which is independent of income.

$$\lambda = E\lambda_t. \quad (4)$$

This is a fair representation of an environment where the adjustment coefficient is uncertain, but without systematic bias or trend. It is possible to incorporate into the model some more general forms of uncertainty without altering the main results<sup>4</sup>, but the notation would become too unwieldy.

With  $\{\delta_t\}$ ,  $\{\lambda_t\}$ ,  $\{\epsilon_t\}$  independently distributed, under the informational constraints, and the given target-setting procedure, all relevant statistical history at time  $t$  is summarized for the enterprise by the state variables  $q_t$  and  $\epsilon_t$ . A decision rule,

$$y_t(q_t, \epsilon_t),$$

expresses the performance level at time  $t$  as a function of the assigned target  $q_t$  and the situation of the enterprise  $\epsilon_t$ . The set of decision rules  $\{y_t(q_t, \epsilon_t)\}$ , with one rule for each  $t$ , results in an expected value to the enterprise of

$$V(\{y_t(q_t, \epsilon_t)\}) = E \sum_{t=1}^{\infty} [b(y_t(q_t, \epsilon_t) - q_t) - C_t(y_t(q_t, \epsilon_t); \epsilon_t)] \left( \frac{1}{1+r} \right)^t, \quad (5)$$

where

$$q_t = (1 - \lambda_t)q_{t-1} + \lambda_t y_{t-1} + \delta_t, \quad (6)$$

$$q_0 = \bar{q}_0, \quad y_0 = \bar{y}_0 \text{ (initial conditions)}. \quad (7)$$

The expectation operator  $E$  in (5) is taken over the random variables  $\{\delta_t\}$ ,  $\{\lambda_t\}$ , and  $\{\epsilon_t\}$ .

Given the passive target-setting behavior of the planners, the problem of the enterprise is to maximize expected present discounted value<sup>5</sup>, or to find a set of *optimal* decision rules  $\{y_t^*(q_t, \epsilon_t)\}$  satisfying

$$V(\{y_t^*(q_t, \epsilon_t)\}) = \max_{\{y_t(q_t, \epsilon_t)\}} V(\{y_t(q_t, \epsilon_t)\}). \quad (8)$$

This problem is representative of a class of models which attempt to characterize optimal behavior in the presence of a regulatory lag.

We assume that the problem (5)–(8) is well defined and that an optimal solution exists. The issue of existence is not of interest in its own right; and, in any event, it is not difficult to specify a set of sufficient conditions for (5)–(8) to be a meaningful problem.<sup>6</sup>

### 3. The ratchet effect

■ At first glance, it might appear that problem (5)–(8) is difficult or impossible to solve analytically. In fact, an exceedingly straightforward solution is avail-

<sup>4</sup> For example, if the random variables  $\{\delta_t\}$  or  $\{\epsilon_t\}$  are not independently distributed, the formulation of the problem becomes more cumbersome, but the results do not change at all. However, if the  $\{\lambda_t\}$  were not independently distributed with identical mean, the form of an optimal policy would be greatly complicated, although it would have properties analogous to those of the solution developed here.

<sup>5</sup> If we were to consider expected present discounted utility, the general case would not yield a simple solution concept. It seems reasonable as a first approximation to put aside the higher-order considerations involved in treating a curved utility function.

<sup>6</sup> As is frequently (but not always) the case, sufficient conditions for this problem would be complicated, somewhat arbitrary, and more or less devoid of economic content.

able. The following theorem, which is proved in the Appendix, is the basic result of the present paper:

*Theorem 1:*  $y_t^*$  is the optimal performance level in period  $t$  if and only if it satisfies

$$C'_t(y_t^*; \epsilon_t) = \frac{b}{1 + \frac{\lambda}{r}}. \quad (9)$$

Note the extreme simplicity of an optimal policy. Rule (9) is completely myopic:  $y_t^*$  depends only on the parameters,  $b$  and  $\lambda/r$ , and the current cost function.

As a specific case, suppose costs are deterministic and time-invariant so that

$$C_t(y_t; \epsilon_t) = C(y_t).$$

Then, the optimal strategy is always to perform at the constant level  $y^*$  satisfying

$$C'(y^*) = \frac{b}{1 + \frac{\lambda}{r}}.$$

Perhaps it is easiest to think of  $\{y_t^*\}$  as the performance levels that would be elicited if the same hypothetical "ratchet price"

$$p = \frac{b}{1 + \frac{\lambda}{r}} \quad (10)$$

were offered for each period's output. If the enterprise were to

$$\underset{y_t}{\text{maximize}} \quad py_t - C_t(y_t; \epsilon_t)$$

by setting the marginal cost of output equal to the ratchet price, it would automatically attain the optimal solution  $y_t^*$ . The entire effect of the ratchet principle can be thought of as transmitted through the ratchet price. The higher the ratchet price, the higher the optimal output in each period.

The ratchet price is essentially the bonus coefficient  $b$  adjusted by a term in  $\lambda/r$  which captures the effect of the ratchet on future plan quotas and enterprise bonuses. Note that with  $\lambda > 0$ , the ratchet price  $p$  is lower than the bonus coefficient  $b$ . The ratchet effect diminishes performance in each period.

Comparative statics are easily performed;  $p$  and hence  $y_t^*$  are lower as  $b$  is lower, as  $\lambda$  is higher, or as  $r$  is lower. The ratchet effect varies directly with the adjustment coefficient, as of course it should. There is also a stronger ratchet effect as  $r$  is smaller. From (1), shorter review lags or lower interest rates will cause the enterprise to weigh more strongly the adverse effects of overzealous present performance on raising future targets.

It is instructive to look at extreme values of  $\lambda$  and  $r$ . There is no ratchet effect—that is,  $p \rightarrow b$ —as either  $\lambda \rightarrow 0$  or  $r \rightarrow \infty$ . There is a maximal ratchet effect equivalent to a zero price of output,  $p \rightarrow 0$ , as either  $\lambda \rightarrow \infty$  or  $r \rightarrow 0$ . Such extreme results accord well with economic intuition.

The aim of this paper has been to investigate the ratchet principle's effects on enterprise performance. Although the model is a gross oversimplification of reality, it captures the main ingredients of the dynamic incentive problem, and

it does allow a sharp quantification of the basic tradeoffs involved in the ratchet effect. The possibility of explicitly constructing an optimal solution makes the problem analyzed here a natural preliminary to more general formulations. In addition, the present model may be a reasonable description of particular planning or regulatory situations.

## Appendix

### Proof of the optimal policy

■ Consider any decision rule  $\{y_t(q_t, \epsilon_t)\}$ . For notational convenience, drop the explicit dependence on  $q_t$  and  $\epsilon_t$ , and simply write  $y_t$  to stand for  $y_t(q_t, \epsilon_t)$ .

It is tedious but not difficult to verify that the solution of (6), (7) is

$$q_s = q_0 \prod_{i=1}^s (1 - \lambda_i) + \sum_{t=0}^{s-1} (\lambda_{t+1} y_t + \delta_{t+1}) \left( \prod_{i=t+2}^s (1 - \lambda_i) \right). \quad (\text{A1})$$

Note from independence of the random variables and (4) that:

$$E q_0 \prod_{i=1}^s (1 - \lambda_i) = q_0 (1 - \lambda)^s, \quad (\text{A2})$$

$$E \lambda_{t+1} y_t \left( \prod_{i=t+2}^s (1 - \lambda_i) \right) = \lambda (1 - \lambda)^{s-1-t} E y_t, \quad \text{and} \quad (\text{A3})$$

$$E \delta_{t+1} \left( \prod_{i=t+2}^s (1 - \lambda_i) \right) = (1 - \lambda)^{s-1-t} E \delta_{t+1}. \quad (\text{A4})$$

Using (A1)–(A4), equation (5) (the expected value of the decision rule) can be expressed as

$$V = E \sum_{s=1}^{\infty} [b(y_s - q_0(1 - \lambda)^s) - \sum_{t=0}^{s-1} (\lambda y_t + \delta_{t+1})(1 - \lambda)^{s-1-t}) - C_s(y_s; \epsilon_s)] \left( \frac{1}{1+r} \right)^s. \quad (\text{A5})$$

Changing the order of summation, (A5) can be rewritten as

$$V = E \sum_{t=1}^{\infty} [b(y_t - q_0(1 - \lambda)^t) - C_t(y_t; \epsilon_t)] \left( \frac{1}{1+r} \right)^t - E \sum_{t=0}^{\infty} b(\lambda y_t + \delta_{t+1}) \sum_{s=t+1}^{\infty} (1 - \lambda)^{s-1-t} \left( \frac{1}{1+r} \right)^s. \quad (\text{A6})$$

Using the fact that

$$\sum_{s=t+1}^{\infty} (1 - \lambda)^{s-1-t} \left( \frac{1}{1+r} \right)^s = \frac{1}{\lambda + r} \left( \frac{1}{1+r} \right)^t,$$

rewrite (A6) as

$$V = E \sum_{t=1}^{\infty} \left[ \frac{b}{1 + \frac{\lambda}{r}} y_t - C_t(y_t; \epsilon_t) \right] \left( \frac{1}{1+r} \right)^t - K, \quad (\text{A7})$$

where

$$K \equiv by_0 \left( \frac{\lambda}{\lambda + r} \right) + \sum_{t=1}^{\infty} bq_0 \left( \frac{1 - \lambda}{1 + r} \right)^t + E \sum_{t=0}^{\infty} b\delta_{t+1} \left( \frac{1}{\lambda + r} \right) \left( \frac{1}{1 + r} \right)^t. \quad (\text{A8})$$

From (A8),  $K$  is a constant independent of  $\{y_t\}$ . The variable part of (A7) is additively separable across periods in functions of  $y_t$ . Hence, (A7) will be maximized if and only if in each period  $t$ ,  $y_t$  is selected to maximize

$$\frac{b}{1 + \frac{\lambda}{r}} y_t - C_t(y_t; \epsilon_t)$$

Note that the optimal value  $y_t^*$  does not depend on  $q_t$ .

Given the second-order condition (2) and no constraint on the domain of  $y_t$ , the optimal value  $y_t^*$  must be an interior solution satisfying (9).

This concludes our proof of the form of an optimal policy.

## References

- BERGSON, A. *The Economics of Soviet Planning*. New Haven: Yale University Press, 1964.
- BERLINER, J.S. *Factory and Manager in the Soviet Union*. Cambridge: Harvard University Press, 1957.
- . *The Innovation Decision in Soviet Industry*. Cambridge: M.I.T. Press, 1976.
- SNOWBERGER, V. "The New Soviet Incentive Model: Comment." *Bell Journal of Economics*, Vol. 8, No. 2 (Autumn 1977), pp. 591–600.
- WEITZMAN, M.L. "The New Soviet Incentive Model." *Bell Journal of Economics*, Vol. 7, No. 1 (Spring 1976), pp. 251–257.
- YUNKER, J.A. "A Dynamic Optimization Model of the Soviet Enterprise." *Economics of Planning*, Vol. 13, Nos. 1–2 (1973), pp. 33–51.
- ZIELINSKI, J.G. *Economic Reforms in Polish Industry*. Oxford: Oxford University Press, 1973.