Potentially large equilibrium climate sensitivity tail uncertainty[☆]

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Abstract

Equilibrium climate sensitivity (ECS), the link between concentrations of greenhouse gases in the atmosphere and eventual global average temperatures, has been persistently and perhaps deeply uncertain. Its 'likely' range has been approximately between 1.5 and 4.5 degrees Centigrade for almost 40 years (Wagner and Weitzman, 2015). Moreover, Roe and Baker (2007), Weitzman (2009), and others have argued that its right-hand tail may be long, 'fat' even. Enter Cox et al. (2018), who use an 'emergent constraint' approach to characterize the probability distribution of ECS as having a central or best estimate of 2.8°C with a 66% confidence interval of 2.2-3.4 °C. This implies, by their calculations, that the probability of ECS exceeding 4.5°C is less than 1%. They characterize such kind of result as "renewing hope that we may yet be able to avoid global warming exceeding 2[°C]". We

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share the desire for less uncertainty around ECS (Weitzman, 2011; Wagner and Weitzman, 2015). However, we are afraid that the upper-tail emergent constraint on ECS is largely a function of the assumed normal error terms in the regression analysis. We do not attempt to evaluate Cox et al. (2018)'s physical modeling (aside from the normality assumption), leaving that task to physical scientists. We take Cox et al. (2018)'s 66% confidence interval as given and explore the implications of applying alternative probability distributions. We find, for example, that moving from a normal to a log-normal distribution, while giving identical probabilities for being in the 2.2-3.4°C range, increases the probability of exceeding 4.5°C by over five times. Using instead a fat-tailed Pareto distribution, an admittedly extreme case, increases the probability by over forty times.

Keywords: climate change, climate sensitivity, fat tails

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1. Introduction

Our methodology is straightforward. We simply wish to show that Cox et al. (2018)'s hopeful upper-tail result is, at least in part, a consequence of the probability density function (PDF) 'chosen' by them. In fact, Cox et al. (2018)'s methodology involves establishing bounds for equilibrium climate sensitivity (ECS) based on identifying a statistic that is highly correlated with ECS in global climate models. Their chosen statistic is the ratio of the standard deviation of temperature divided by a measure of the autocorrelation in temperatures across time. Cox et al. (2018) shows that this statistic in climate models is highly correlated with ECS in those models. Based on calibrating the statistic on the historic record with a least-squares regression, they then constrain ECS to lie within 2.2-3.4 °C with a 66% probability.[1]

We make no judgment on the appropriateness of this 'emergent constraint' on ECS other than to argue that Cox et al. (2018)'s least-squares linear regression immediately leads to normal error terms and that this partially accounts for their optimistic conclusions.[2] We do not re-analyze Cox et al. (2018)'s underlying data and time-series econometric assumptions. We instead proxy for such different formulations by simple exercises that examine some consequences of alternative probability distributions.

2. Analysis

Let x stand for ECS and let $f_\theta(x)$ represent a PDF of family θ. We consider three families of two-parameter PDFs: Normal ($\theta = N$), Pareto ($\theta = P$), Lognormal ($\theta = L$). For each such family, we fix the two free parameters by an appropriate condition characterizing the central estimate and by simply imposing, as if given, Cox et al. (2018)'s condition that 66% of the probability lies within the interval [2.2, 3.4]. Mathematically, this Cox

[1]We surmise that Cox et al. (2018) present the 66% confidence interval in a nod to the Intergovernmental Panel on Climate Change (IPPC)'s convention of presenting the 66% "likely" interval (Mastrandrea et al., 2011).

[2]In fact, a closer climate-econometric examination of Cox et al. (2018)'s proposed metric may well reveal deeper issues linked to its relatively small sample and the assumed AR(1) structure, which is likely inappropriate for discrete annual temperature data. See, e.g., Bruns et al. (2018).

et al. (2018) 66% condition is represented for each θ by the equation

$$\int_{2.2}^{3.4} f_\theta(x)\,dx = 0.66. \tag{1}$$

After calibration, we calculate for each θ the probability that ECS exceeds 4.5°C, denoted as $Prob(S_\theta > 4.5)$. This upper-tail behavior is our object of greatest interest here, as it is in much of climate science. Mathematically,

$$Prob(S_\theta > 4.5) \equiv \int_{4.5}^{\infty} f_\theta(x)\,dx. \tag{2}$$

A thin-tailed PDF $f(x)$ approaches zero exponentially ($f(x) \propto \exp(-\lambda x)$ for some $\lambda > 0$) or faster as $x \to \infty$. A fat-tailed PDF $f(x)$ approaches zero polynomially ($f(x) \propto x^{-k}$ for some $k > 0$) or slower as $x \to \infty$. (The ratio of a fat-tailed PDF divided by a thin-tailed PDF therefore approaches infinity as $x \to \infty$.) An intermediate-tailed PDF has a tail which goes to zero slower than exponentially but faster than polynomially.

A prototype thin-tailed PDF is the Normal:

$$f_N(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right). \tag{3}$$

Cox et al. (2018), whose underlying PDF is effectively Normal, characterize the central or best estimate of climate sensitivity to be 2.8 degrees Celsius. We interpret this as signifying here that $\mu = 2.8$ in (3). The standard deviation σ in (3) is then determined by condition (1) for $\theta = N$, and turns out to be $\sigma = 0.629$. For these two parameter values, we calculate $Prob(S_N > 4.5) = 0.34\%$, confirming Cox et al. (2018)'s calculation of "the probability of ECS exceeding 4.5 degrees Celsius to less than 1 per cent".

Because the normal PDF is symmetric, mean, mode, and median are identical. When the PDF is right-skewed, mode < median < mean, and we have to choose which of these three measures of central tendency should represent a ‘best estimate of 2.8°C’. For the purpose of this set of numerical exercises we choose the median, which is in between the mode and the mean. This particular measure of central tendency has the intuitively appealing and readily visualizable characterization that half the probability is above the median while the other half is below the median.

Fat-tailed polynomial (alternatively power-law) distributions are used to characterize many physical phenomena, such as earthquakes, hurricanes, volcanic eruptions, floods, meteorite sizes, etc. (Sornette, 2013). ECS, too, is typically assumed to follow a considerably skewed distribution, with many estimates seeming to show a thick if not outright fat right tail (Roe and Baker, 2007; Weitzman, 2009). A candidate for the prototype two-parameter fat-tailed PDF is the Type I Pareto:

$$f_P(x) = \frac{a\, b^a}{x^{a+1}}, \tag{4}$$

for $x \geq b$, while $f_P(x) = 0$ elsewhere. The positive parameter b represents the minimum possible value of x, while the positive parameter a is known as the so-called tail index (smaller values of a correspond to relatively fatter tails). We do not take seriously the full Pareto PDF for approximating the distribution of ECS. We simply take it to proxy for a fat upper tail. Parameters b and a can be simultaneously fixed or calibrated by setting the median of $f_P(x)$ equal to 2.8 and by imposing the condition (1) for $\theta = P$. The result of this particular curve-fitting exercise is $b = 2.164$ and $a = 2.69$.[3]

[3]Note that since $b = 2.164$, the Pareto PDF excludes any ECS values below 2.164, clearly an extreme case.

With these two parameter values, and for what it is worth without thinking deeply, we mechanically calculate $Prob(S_P > 4.5) = 13.95\%$.

In our opinion, the Normal and Pareto distributions represent two extreme poles in upper-tail behavior. To use the Normal here is to choose an extremely thin upper-tailed PDF. To use the Pareto here is to choose an extremely fat upper-tailed PDF. This leads us directly to consider fitting an intermediate-tailed PDF. The Lognormal distribution is of the form

$$f_L(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right), \tag{5}$$

where x is constrained to be non-negative. A convenient property of the Lognormal PDF is that its tail goes to zero slower than exponentially but faster than polynomially, making it intermediate between a thin-tailed and a fat-tailed PDF. In this sense the Lognormal represents a lower bound on $Prob(S > 4.5)$ for fat-tailed power-law distributions.

The median of the Lognormal PDF (6) is $\exp(\mu)$, where $\mu = 2.8$ here. The appropriate value of σ that appears in the Lognormal PDF (6) is then fixed or calibrated by condition (1) for $\theta = L$, and turns out to be $\sigma = 3.04$. For these two parameter values, we calculate $Prob(S_L > 4.5) = 1.82\%$.

Note that, perhaps by coincidence, this is very close to the geometric mean of the comparable thin-tailed Normal and the fat-tailed Pareto probabilities: $\sqrt{0.34 \cdot 13.95}\% = 2.18\%$.

Let us also re-emphasize briefly that all these calibrations are purely illustrative. Instead of taking the entire range between 2.2 and 3.4°C as the 66% interval (equation 1), imagine that we took the interval between the median and the upper bound to be 33%:

$$\int_{2.8}^{3.4} f_\theta(x)\,dx = 0.33. \tag{6}$$

That formulation makes no different for the Normal PDF. It makes a clear difference for both the Pareto and Lognormal PDFs. In the former case, $Prob(S_P > 4.5) = 3.58\%$ instead of 13.95%; in the latter case, $Prob(S_L > 4.5) = 0.99\%$ instead of 1.82%. The magnitudes are very different, the spirit of the story remains the same.

3. Conclusion

This is the end of our story. What is its moral? Tail behavior is often postulated rather than empirically derived, because oftentimes it is statistically very difficult, and sometimes even impossible, to estimate the probabilities of extreme values when there are so few extreme values of rare tail-events in the existing data. This is overwhelmingly true for estimates of ECS tail-probability distributions. Pending other climate-econometric challenges, Cox et al. (2018) may have found a useful new way of measuring the 'best estimate' of ECS. In doing so, however, they have effectively assumed something close to a Normal distribution around the best estimate. While this analysis may be used to justify statements around the 'best estimate' of ECS, it does not justify statements concerning its tail behavior and, in particular, cannot rule out the fat tails that characterize many physical processes.

Here we demonstrate that $Prob(S > 4.5)$ can vary enormously, depending on what tail behavior the underlying PDF is representing. Taking Cox et al. (2018)'s 66%-interval as given, the intermediate-tailed Lognormal PDF, has

$Prob(S_L > 4.5) = 1.82\%$. This is a probability over five times higher than what we impute to be the Cox et al. (2018) estimate of $Prob(S_N > 4.5) = 0.34\%$. However, this five-times probability result represents a lower bound for fat tails and could be made an order of magnitude higher by considering tail behavior that is fatter than the Lognormal. Sadly, the spirit of these exercises does not give much sustenance to the hope that extremely high values of ECS are exceedingly rare.

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