MEMORANDUM

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TO: File

FROM: Bill Hogan

SUBJ: DC Optimal Power Flow Model, V. 3.0 through V. 3.06

This note summarizes the assumptions and formulation behind the GAMS implementation of version 3.0 of the DC optimal power flow model suitable for testing illustrative examples of electric network power flows and prices. The discussion covers the development of the DC load flow model in terms of a nonlinear program that yields the loads, flows, and bus prices for an optimal solution that is also a market equilibrium under the appropriate competitive conditions.

DC LOAD FLOW MODEL FOR MWs

This summary of the "DC Load" model follows almost exactly the development in Schweppe et al. with minor modifications suitable for use in an optimization model.[1] Let:

z_{Pijk} = Real power flowing from bus i to bus j along line k. Then

1. $$z_{Pijk} = G_k[V_i^2 - V_iV_j\cos(\delta_i - \delta_j)] + \Omega_k V_iV_j\sin(\delta_i - \delta_j),$$

where,

2. $$G_k = R_k/(R_k^2 + X_k^2),$$

and

3. $$\Omega_k = X_k/(R_k^2 + X_k^2).$$

The line resistance is R_k and the reactance is X_k. Here δ_j = Voltage phase angle at bus j, and

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[1] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R.E. Bohn, Spot Pricing of Electricity, Kluwer Academic Publishers, Norwell, MA, 1988, Appendices A and D. The "DC Load" flow refers to the real power half of the nonlinear AC load flow model. Under the maintained assumptions there is a weak link between the reactive power and real power halves of the full problem. And the real power flow equations have the same general form as the direct current flow equations in a purely resistive network; hence the name "DC Load Flow."

V_i = Voltage magnitude at bus i.

Losses are given by

4. $$L_{Pk} = z_{Pijk} + z_{Pjik} \quad ,$$

Hence,

5. $$L_{Pk} = G_k[V_i^2 + V_j^2 - 2V_iV_j\cos(\delta_i - \delta_j)] \quad .$$

Now if $\delta_i - \delta_j$ is small, then

6. $$\cos(\delta_i - \delta_j) \approx 1 - (\delta_i - \delta_j)^2/2 \quad .$$

Assuming that this is a per unit system[2] where $V_i \approx V_j \approx 1$, we have,

7. $$L_{Pk} \approx G_k(\delta_i - \delta_j)^2 \quad .$$

As an approximation to the flows we can also assume $\delta_i - \delta_j$ is small and use the approximations

8. $$\cos(\delta_i - \delta_j) \approx 1$$

and

9. $$\sin(\delta_i - \delta_j) \approx (\delta_i - \delta_j) \ ,$$

in which case with $V_i \approx V_j \approx 1$ we have[3]

10. $$z_{Pijk} \approx \Omega_k(\delta_i - \delta_j) \ .$$

Then combined with the line loss approximation we have,

11. $$L_{Pk} = G_k z_{Pk}^2/\Omega_k^2 \quad ,$$

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[2] Per unit refers to the common scaling of the power flow equations to a reference value for two of the three factors of voltage, current and power. The third reference value is determined by the two others in order to satisfy the power flow equations. See A. R. Bergen, Power Systems Analysis, Prentice Hall, Englewood Cliffs, New Jersey, 1986, chapter 5.

[3] This approximation also requires that $G_k << \Omega_k$, which is appropriate for transmission lines. Otherwise we need to attend to the percentage differences in the voltages.

where in (11) for the losses approximation we have adopted the notation

12. $z_{Pijk} \approx -z_{Pjik} \approx z_{Pk}$.

Let

n_B = Number of buses,

n_L = Number of lines,

A = n_L x n_B Network incidence matrix with elements of 0, 1, -1 corresponding to the network interconnections. If link k originates at bus i and terminates at bus j, then $a_{ki} = 1 = -a_{kj}$.

y_P = $g - d = n_B$ Vector of real bus injections, generation minus demand,

z_P = n_L Vector of line real power flows,

δ = n_B Vector of voltage angles relative to the swing bus where by definition for the swing bus, $\delta_{swing}=0$,

Ω = Diagonal matrix of the elements Ω_k ,

G = Diagonal matrix of the elements G_k ,

R = Diagonal matrix of the elements G_k/Ω_k^2 .[4]

Then, by conservation of flow in and out of a bus

13. $y_P = A^t z_P$,

and by (10)

14. $z_P = \Omega A \delta$.

Hence,

[4] Note that if $X_k >> R_k$ then $G_k/\Omega_k^2 \approx R_k$ and the matrix R becomes the diagonal matrix of resistances. This approximation is the one developed in Schweppe et al., but the alternative applied here seems a better approximation without any loss of simplicity.

15. $$y_P = A^t \Omega A \delta \ .$$

Then from (11), the approximation of losses is

16. $$L_P = z_P{}^t R z_P \ .$$

This formulation provides the foundation needed for the optimization model. Schweppe et al. carry the approximation forward to the corresponding matrix solutions for losses and prices as a function of the net loads. This last step requires an explicit matrix inversion which is implicit in the optimization solution.

OPTIMAL POWER FLOW MODEL

With the above simplifications, the optimal power flow model can be expressed as a nonlinear optimization problem. Suppose that we have

B = Benefit function defined on demand d, typically calculated as the area under the bus demand curves;

C = Cost function defined on generation g, typically calculated as the area under the bus supply curves;

z_{min}, z_{max} = The lower and upper bounds on the real power line flows;

U_{int}, W_{int} = Matrices defining the line flows and bus loads included in an interface nomogram limit;

U_{min},U_{max}= The lower and upper limits on the interface constraints.

Then the optimal power flow problem can be defined as:

$$\underset{d,g,y,z_p,\delta}{\text{Max}} \quad B(d) - C(g) \qquad \text{(Net Benefits)}$$

subject to:

17. $$g - d = y \quad , \qquad \text{(Net Input Balance)}$$

18. $$y = A^t z \quad , \qquad \text{(Network Balance)}$$

19. $$z_P = \Omega A \delta \ , \qquad \text{(Line Flows)}$$

20. $\delta_{swing} = 0$, (Swing Bus Angle)

21. $z_{min} \leq z_P \leq z_{max}$, (Line Thermal Limits)

22. $U_{min} \leq U_{int} z_P - W_{int} y \leq U_{max}$. (Interface Limits)

This model consists of linear constraints and a nonlinear objective function. If the cost function includes only the area under the supply curve, then model gives the optimal dispatch ignoring losses.

The GAMS implementation in version 3.06 assumes linear supply and demand curves that are independent across buses. Because there can be more than one supply or demand at a bus, with upper and lower limits, it is possible to create piecewise linear curves.

In any event, the optimal dual variables for the network balance constraints (17) yield the optimal generation and load prices at the buses.

The original Schweppe formulation assumed zero losses for the approximation of the flows, but calculated prices using a variant of (7) to determine the marginal losses. In this formulation, if there is no limit to generation at the swing bus and losses are included in the cost function as an additional load at the swing bus, with $L_P = z_P^t R z_P$, then the model solves the DC load model and attempts to minimize for all costs including losses. The losses would not affect the flows, but the prices would be the Schweppe prices. In the event that the swing bus has limited generation (i.e., a finite price elasticity), then the choice of the swing bus could affect the dispatch and the prices.

An alternative loss formulation appears first in version 3.06. Here the line flows are modeled as in (19). However, equation (18) for bus loads is modified to include one-half the losses for every line flowing in or out of the bus, taken from (11). This makes the constraints non-linear, which appears to have no effect on the ability of the GAMS-MINOS implementation to find optimal solutions for the test problems. This formulation with losses affects the flows and the dispatch, with the nodal balance including losses at each bus. However, this formulation makes the prices and the dispatch independent of the choice of the swing bus, even in the with losses formulation.

As always, the decomposition of prices into generation, losses and congestion depends on the choice of the swing bus, which is used as the reference price bus. Before version 3.06, the price decomposition exploited the special linear structure of the Schweppe assumptions. The nonlinear formulation with losses at each bus requires a more general approach. In order to calculate the decomposition, version 3.06 solves a related problem in order to determine the price of generation and losses. In essence, we fix generation and demand at the optimal solution everywhere but the swing bus, drop constraint equations in (21) and (22), and set the objective function price at the reference bus price for load and generation at the swing bus, which is the only slack bus. The price elsewhere is set at a high number. This problem has only one feasible dispatch, and the corresponding dual solution gives the prices for generation and losses only. Subtracting these from the original bus prices gives the cost of congestion relative to the reference price at the swing bus.

GAMS IMPLEMENTATION

The GAMS implementation consists of several generic files for controlling the model input and output:

In the GAMS386 directory:

GAMS:	The GAMS386 system located in an appropriate directory to allow execution from another directory.

In the local directory:

DCMAIN.GMS:	The main program that implements the model, calls the MINOS solver, and prepares the output reports.
DCGAMS.BAT	A batch file that copies the case data to an input file, calls GAMS386, retrieves the output to be saved in the case output file, and displays the results on the screen using any convenient text editor.
case.MOD	A GAMS style file that includes all the tables and so on for the particular "case" being run. The user changes the tables and parameters in this file to define the MODel and associated input assumptions.
case.PUT	The GAMS outPUT file that contains the results for the "case" being run.

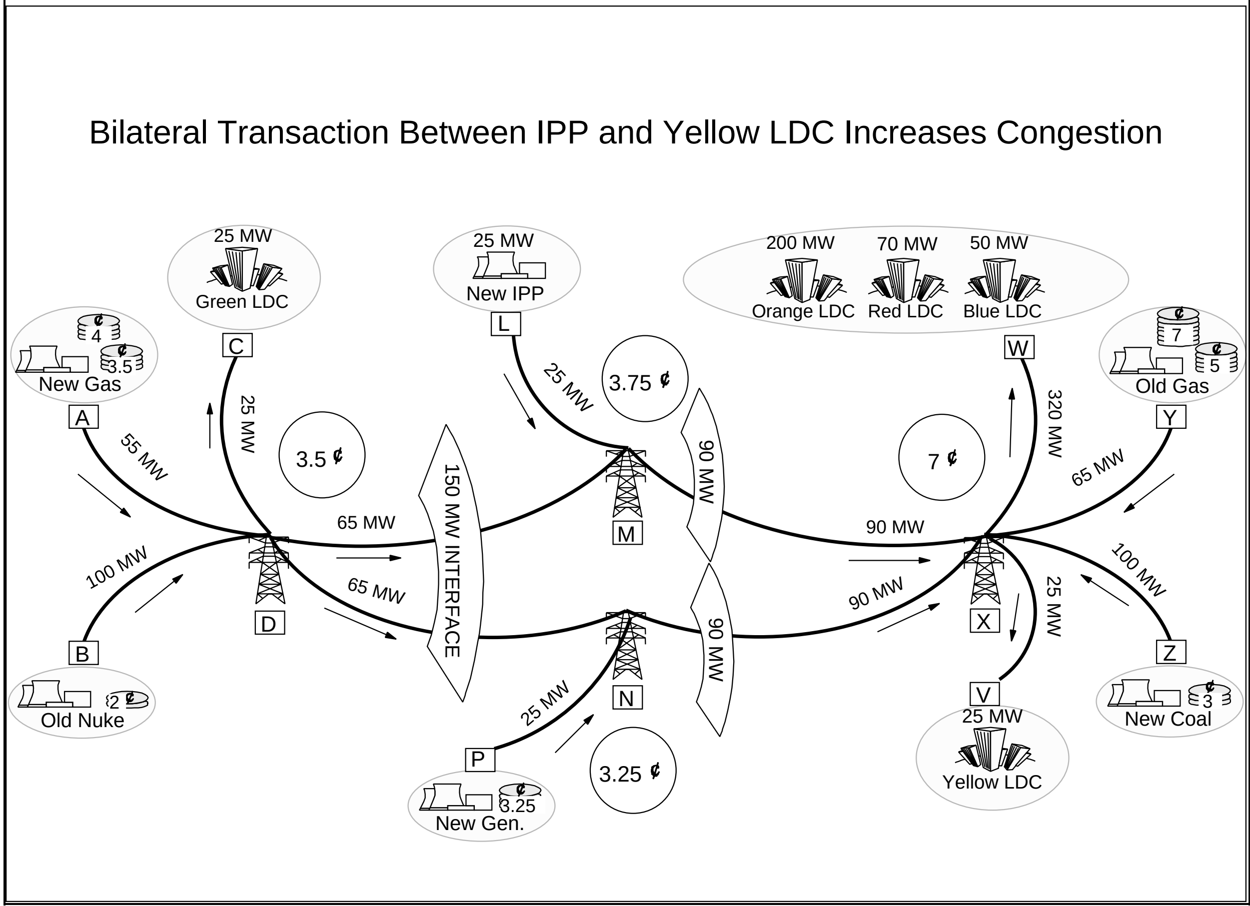
Once the data have been defined in the appropriate model file "case.MOD", the command to execute the run is:

```
C: > DCGAMS  case
```

EXAMPLE INPUT AND OUTPUT FILES

The network shown in Figure 1 provides an example of the application of the DC optimal power flow model.

Figure 1



Here there is one interface constraint plus thermal limits on two individual lines. The interface constraint involves only lines but no buses, and in any event is not binding in this case. The IPP at bus "L" has bid in a must run plant at 25 MW, having arranged a corresponding sale to the Yellow distribution company at bus "V". Were it not for the IPP sale, more power could be taken from the inexpensive generators at bus "P" and at bus "A". However, because of the effects of loop flow, these plants are constrained at less than their maximum output, and there are different prices applicable at buses "D", "M", "N", and "X".

The following is the input file DEMO.MOD for this network:

```
$Title -- DC Optimal Dispatch Model for Demonstration Network

$OFFSYMXREF
option sysout = on;
$onmulti
* allows multiple redefinition of tables


SCALAR
     Swing      Select the index of the swing bus / 1 /
     WITHLOSS   Set to one for losses and zero for no losses / 0 /
;

SETS
* The following parameters must be set to describe the network
* Bus names should be a maximum of 8 characters
     P   Players / 1*8 /
     N   Buses
         /
         A,B,C,D,L,P,M,N,V,W,X,Y,Z
         /
*  Index: A-1,B-2,C-3,D-4,L-5,P-6,M-7,N-8,V-9,W-10,X-11,Y-12,Z-13

     M   Links / 1*13 /
     G   Generators / 1*8 /
     L   Loads / 1*3 /
     INT Interface Constraints /1*1/
     IntZRows Number of links included in interface constraints /1*2/
     IntYRows Number of buses included in interface constraints /1*1/

* The column data are fixed and set up the tables below

     GenCols Number of Columns in GenData   / 1*6 /
     LoadCols Number of Columns in LoadData / 1*6 /
     LinkCols Number of Columns in LinkData / 1*6 /
     IntZCols Number of Columns in IntZData / 1*3 /
     IntYCols Number of Columns in IntYData / 1*3 /
     IntUCols Number of Columns in IntUData / 1*2 /
;


Table LinkData(M,LinkCols)
* Column 1:  From bus index
* Column 2:  To bus index
* Column 3:  Resistance
* Column 4:  Reactance
* Column 5:  Minimum flow
* Column 6:  Maximum flow

    1    2       3       4      5       6
1   1    4 0.00005 0.00025  -1000    1000
2   2    4 0.00005 0.00025  -1000    1000
3   3    4 0.00005 0.00025  -1000    1000
4   4    7 0.00005 0.00025  -1000    1000
5   4    8 0.00005 0.00025  -1000    1000
6   7   11 0.00005 0.00025    -90      90
7   8   11 0.00005 0.00025    -90      90
8  12   11 0.00005 0.00025  -1000    1000
9  11   10 0.00005 0.00025  -1000    1000
```

```
10 13   11 0.00005 0.00025  -1000    1000
11  5    7 0.00005 0.00025  -1000    1000
12 11    9 0.00005 0.00025  -1000    1000
13  6    8 0.00005 0.00025  -1000    1000
;

Table GenData(G,GenCols)
* Column 1:  Price at which supply is zero at a given bus.
* Column 2:  Inverse of slope of supply curve (dQ/dP) at a bus. ENTER INF
*            FOR HORIZONTAL AND ZERO FOR VERTICAL SUPPLY CURVES.
* Column 3:  Minimum supply at a given bus.
* Column 4:  Maximum supply at a given bus.
* Column 5:  Index to bus number.
* Column 6:  Index of player owning generation.

        1          2          3     4    5    6
1      3.5       inf          0   100    1    1
2        4       inf          0   100    1    2
3        2       inf          0   100    2    3
4        5       inf          0    50   12    4
5        7       inf          0    50   12    5
6        3       inf          0   100   13    6
7        0       inf         25    25    5    7
8     3.25       inf          0   250    6    8
;

Table LoadData(L,LoadCols)
* Column 1:  Price at which demand is zero at a given bus. ANY NUMBER CAN
*            BE ENTERED FOR VERTICAL DEMAND CURVES.
* Column 2:  Inverse of slope of demand curve (dQ/dP) at a bus.
*            ENTER -INF FOR HORIZONTAL AND ZERO FOR VERTICAL DEMAND CURVES.
* Column 3:  Minimum load at a given bus. MIN LOAD SHOULD EQUAL MAX LOAD FOR
*            VERTICAL DEMAND CURVES.
* Column 4:  Maximum load at a given bus.
* Column 5:  Index to bus number.
* Column 6:  Index of player owning load.

           1       2      3          4      5    6
1        257    -0.1      0         95      9    1
2       3207    -0.1      0        390     10    2
3        100    -inf     25         25      3    2
;


Table IntZData(IntZRows,IntZCols)
* Column 1:  Index of interface constraint.
* Column 2:  Index of link included in interface constraint.
* Column 3:  Coefficient for link in interface constraint.

        1          2          3
1       1          4          1
2       1          5          1
;

Table IntYData(IntYRows,IntYCols)
* Column 1:  Index of interface constraint.
* Column 2:  Index of bus included in interface constraint.
* Column 3:  Coefficient for bus in interface constraint.

        1          2          3
1       1          1          0
;
```

```
Table IntUData(INT,IntUCols)
* Column 1:  Lower bound on the interface constraint.
* Column 2:  Upper bound on the interface constraint.

          1         2
1    -2000       150
;

* With these tables, the model links to the main GAMS program
* for implementing the DC optimal power flow computation.

$include dcmain.gms
```

The command for executing this input file is:

DCGAMS DEMO

With this command, the GAMS-MINOS system finds the optimal dispatch and produces the output file DEMO.PUT:

```
DCMODEL V3.06: I:\GAMS225\HOGLIB\DCMODL3\DEMO.GMS ON 09/06/98 AT 15:48:10

Note:  This simplified DC model is designed to illustrate constrained network
       dispatch and pricing.  However, it does not account for the spinning
       reserve or contingency constraints needed for a full analysis of a
       security constrained dispatch and the true (lower) system capacity.


Dispatch Ignoring Losses
Buses                Gen          Load        Net Input      Price
    1 A           55.00000      0.00000     55.00000        3.50000
    2 B          100.00000      0.00000    100.00000        3.50000
    3 C            0.00000     25.00000    -25.00000        3.50000
    4 D            0.00000      0.00000      0.00000        3.50000
    5 L           25.00000      0.00000     25.00000        3.75000
    6 P           25.00000      0.00000     25.00000        3.25000
    7 M            0.00000      0.00000      0.00000        3.75000
    8 N            0.00000      0.00000      0.00000        3.25000
    9 V            0.00000     25.00000    -25.00000        7.00000
   10 W            0.00000    320.00000   -320.00000        7.00000
   11 X            0.00000      0.00000      0.00000        7.00000
   12 Y           65.00000      0.00000     65.00000        7.00000
   13 Z          100.00000      0.00000    100.00000        7.00000


Price Decomposition Relative to Swing Bus at A
Buses                Gen&Loss     Congestion   Price
    1 A            3.50000      0.00000      3.50000
    2 B            3.50000      0.00000      3.50000
    3 C            3.50000      0.00000      3.50000
    4 D            3.50000      0.00000      3.50000
    5 L            3.50000      0.25000      3.75000
    6 P            3.50000     -0.25000      3.25000
    7 M            3.50000      0.25000      3.75000
    8 N            3.50000     -0.25000      3.25000
    9 V            3.50000      3.50000      7.00000
   10 W            3.50000      3.50000      7.00000
   11 X            3.50000      3.50000      7.00000
```

12	Y	3.50000	3.50000	7.00000
13	Z	3.50000	3.50000	7.00000

Lines	From	To	Min	Zp	Max	ZloMult	ZupMult
1	A	D	-1000.00	55.00000	1000.00	0.000	0.000
2	B	D	-1000.00	100.00000	1000.00	0.000	0.000
3	C	D	-1000.00	-25.00000	1000.00	0.000	0.000
4	D	M	-1000.00	65.00000	1000.00	0.000	0.000
5	D	N	-1000.00	65.00000	1000.00	0.000	0.000
6	M	X	-90.00	90.00000	90.00	0.000	3.000
7	N	X	-90.00	90.00000	90.00	0.000	4.000
8	Y	X	-1000.00	65.00000	1000.00	0.000	0.000
9	X	W	-1000.00	320.00000	1000.00	0.000	0.000
10	Z	X	-1000.00	100.00000	1000.00	0.000	0.000
11	L	M	-1000.00	25.00000	1000.00	0.000	0.000
12	X	V	-1000.00	25.00000	1000.00	0.000	0.000
13	P	N	-1000.00	25.00000	1000.00	0.000	0.000

	Interface	Lower	Upper
Trans. Interface Mult,	1	0.00000	0.00000

Individual Generator Output:

Generator	Bus	Index	Output
1	A	1	54.00000
2	A	1	0.00000
3	B	2	0.00000
4	Y	12	0.00000
5	Y	12	0.00000
6	Z	13	0.00000
7	L	5	25.00000
8	P	6	0.00000

Total Generation Cost, $ 11287.50000

Total Generation, MW 370.00000

Total Load Value, $ 5200400.00000

Total Load, MW 370.00000

Total Losses, MW 0.00000

Total Surplus, $ 5189112.50000

Total Generation Pmts, $ 18724.99986

Total Load Pmts, $ 25024.99971

Total Rents, $ 6299.99985

```
* LoadData:
* Column 1:  Price at which demand is zero at a given bus.
* Column 2:  Inverse of slope of demand curve (dQ/dP) at a bus.
* Column 3:  Minimum load at a given bus.
* Column 4:  Maximum load at a given bus.
* Column 5:  Index to bus number.
* Column 6:  Index of player owning load.
```

	1	2	3	4	5	6
1	257.00	-0.10	0.00	95.00	9	1
2	3207.00	-0.10	0.00	390.00	10	2
3	100.00	-INF	25.00	25.00	3	2

```
* GenData:
* Column 1:  Price at which supply is zero at a given bus.
* Column 2:  Inverse of slope of supply curve (dQ/dP) at a bus.
* Column 3:  Minimum supply at a given bus.
* Column 4:  Maximum supply at a given bus.
* Column 5:  Index to bus number.
* Column 6:  Index of player owning generation.
              1       2       3       4       5       6
1          3.50    +INF    0.00  100.00       1       1
2          4.00    +INF    0.00  100.00       1       2
3          2.00    +INF    0.00  100.00       2       3
4          5.00    +INF    0.00   50.00      12       4
5          7.00    +INF    0.00   50.00      12       5
6          3.00    +INF    0.00  100.00      13       6
7          0.00    +INF   25.00   25.00       5       7
8          3.25    +INF    0.00  250.00       6       8
```

```
* LinkData:
* Column 1:  From bus index
* Column 2:  To bus index
* Column 3:  Resistance
* Column 4:  Reactance
* Column 5:  Minimum flow
* Column 6:  Maximum flow
           1    2        3        4         5         6
1          1    4  0.00005  0.00025   -1000.0    1000.0
2          2    4  0.00005  0.00025   -1000.0    1000.0
3          3    4  0.00005  0.00025   -1000.0    1000.0
4          4    7  0.00005  0.00025   -1000.0    1000.0
5          4    8  0.00005  0.00025   -1000.0    1000.0
6          7   11  0.00005  0.00025     -90.0      90.0
7          8   11  0.00005  0.00025     -90.0      90.0
8         12   11  0.00005  0.00025   -1000.0    1000.0
9         11   10  0.00005  0.00025   -1000.0    1000.0
10        13   11  0.00005  0.00025   -1000.0    1000.0
11         5    7  0.00005  0.00025   -1000.0    1000.0
12        11    9  0.00005  0.00025   -1000.0    1000.0
13         6    8  0.00005  0.00025   -1000.0    1000.0
```

```
* IntZData:
* Column 1:  Index of interface constraint.
* Column 2:  Index of link included in interface constraint.
* Column 3:  Coefficient for link in interface constraint.
           1    2        3
    1      1    4  1.00000
    2      1    5  1.00000
```

```
* IntYData:
* Column 1:  Index of interface constraint.
* Column 2:  Index of bus included in interface constraint.
* Column 3:  Coefficient for bus in interface constraint.
           1    2        3
    1      1    1  0.00000
```

```
* IntUData:
* Column 1:  Lower bound on the interface constraint.
* Column 2:  Upper bound on the interface constraint.
```

	1	2
1	-2000.00000	150.00000

PLAYER 1

Generation

Bus	Price C/KWH	MWH	Revenue	Avg. Cost C/KWH	Total Cost	Quasi-Rent
A	3.50	54.00	1890	3.50	1890	0
		54.00	1890		1890	0

Load

Bus	Price C/KWH	MWH	Cost
V	7.00	25.00	1750
		25.00	1750

PLAYER 2

Generation

Bus	Price C/KWH	MWH	Revenue	Avg. Cost C/KWH	Total Cost	Quasi-Rent
A	3.50	0.00	0	4.00	0	0
		0.00	0		0	0

Load

Bus	Price C/KWH	MWH	Cost
W	7.00	320.00	22400
C	3.50	25.00	875
		345.00	23275

PLAYER 3

Generation

Bus	Price C/KWH	MWH	Revenue	Avg. Cost C/KWH	Total Cost	Quasi-Rent
B	3.50	0.00	0	2.00	0	0
		0.00	0		0	0

Load

Bus	Price C/KWH	MWH	Cost
		0.00	0

PLAYER 4

Generation

Bus	Price C/KWH	MWH	Revenue	Avg. Cost C/KWH	Total Cost	Quasi-Rent
Y	7.00	0.00	0	5.00	0	0
		0.00	0		0	0

Load

Bus	Price C/KWH	MWH	Cost
		0.00	0

PLAYER 5

Generation

Bus	Price C/KWH	MWH	Revenue	Avg. Cost C/KWH	Total Cost	Quasi-Rent
Y	7.00	0.00	0	7.00	0	0
		0.00	0		0	0

Load

Bus	Price C/KWH	MWH	Cost
		0.00	0

PLAYER 6

Generation

Bus	Price C/KWH	MWH	Revenue	Avg. Cost C/KWH	Total Cost	Quasi-Rent
Z	7.00	0.00	0	3.00	0	0
		0.00	0		0	0

Load

Bus	Price C/KWH	MWH	Cost
		0.00	0

PLAYER 7

Generation

Bus	Price C/KWH	MWH	Revenue	Avg. Cost C/KWH	Total Cost	Quasi-Rent
L	3.75	25.00	938	0.00	0	938
		25.00	938		0	938

Load

Bus	Price C/KWH	MWH	Cost
		0.00	0

PLAYER 8

Generation

Bus	Price C/KWH	MWH	Revenue	Avg. Cost C/KWH	Total Cost	Quasi-Rent
P	3.25	0.00	0	3.25	0	0
		0.00	0		0	0

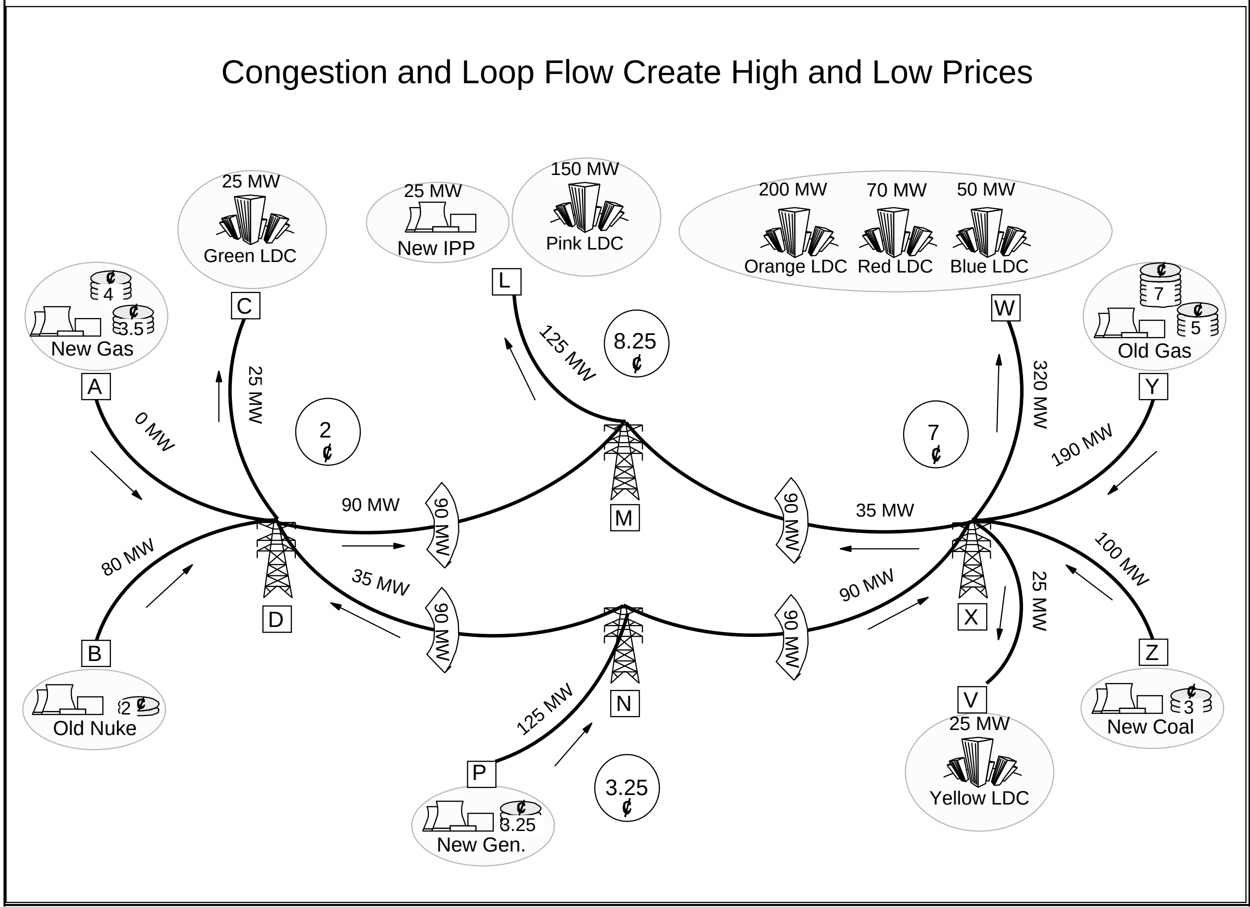
Load

Bus	Price C/KWH	MWH	Cost
		0.00	0

FURTHER EXAMPLES

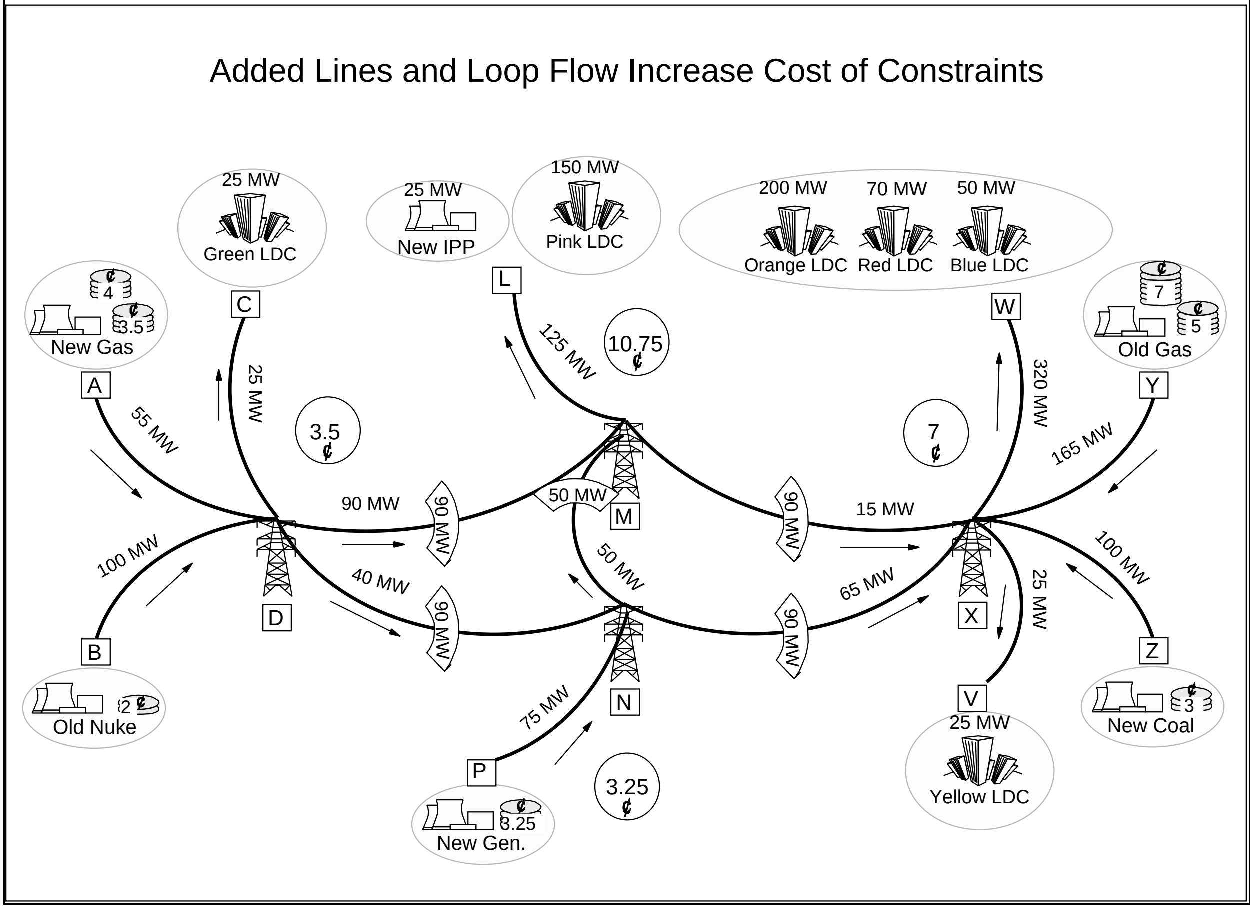
The example network can be modified to illustrate a range of price effects induced by the interaction of loop flow and constraints. In Figure 2, every line in the main loop is constrained by a thermal limit of 90 MW. With these constraints, an added load of 150 MW at bus "L" alters the flows for the market equilibrium. In this case, the combined effect of the increased load and the constraints leads to a price of 8.25¢ per kWh at bus "L". This price is higher than the 7¢ marginal running cost of the old gas plant at bus "Y", the most expensive plant in the system. Apparently the load at bus "L" causes an expensive rearrangement of the dispatch throughout the network.

Figure 2



In Figure 3, a new line has been added to the network, connecting bus "N" to bus "M". This line is assumed to have a thermal limit of 50 MW. The new line adds to the capability of the network in that the new pattern of generation lowers the overall cost of satisfying the same load. The total cost in the case in Figure 2 is $20,962.50, while the corresponding cost in the case of Figure 3 is $19,912.50. However, although the average cost of power generation has

Figure 3



fallen, the marginal cost of power has increased at bus "L", where the price is now 10.75¢ per kWh.

A further extension of this example network in Figure 4 adds a new bus "O" between bus "M" and bus "N", and lowers the thermal limit to 30 MW between bus "O" and bus "M". In addition, bus "O" has a small load of 15 MW. In this case, the total cost of meeting the higher load is $20,900, again less than in Figure 2. The cost is higher than in Figure 3, due in part to the tighter thermal limit on the line between bus "O" and bus "M".[5] The increased load of 15 MW at bus "O" actually lowers the total cost of the dispatch, as reflected in the negative price. Each additional MW of load at bus "O" changes the flows to allow a new dispatch that lowers the overall cost of meeting the remaining load. Even further, with the 30 MW limit and no load at bus "O", there is no feasible dispatch.

Hence, as summarized in these figures, it is possible to have a network where marginal

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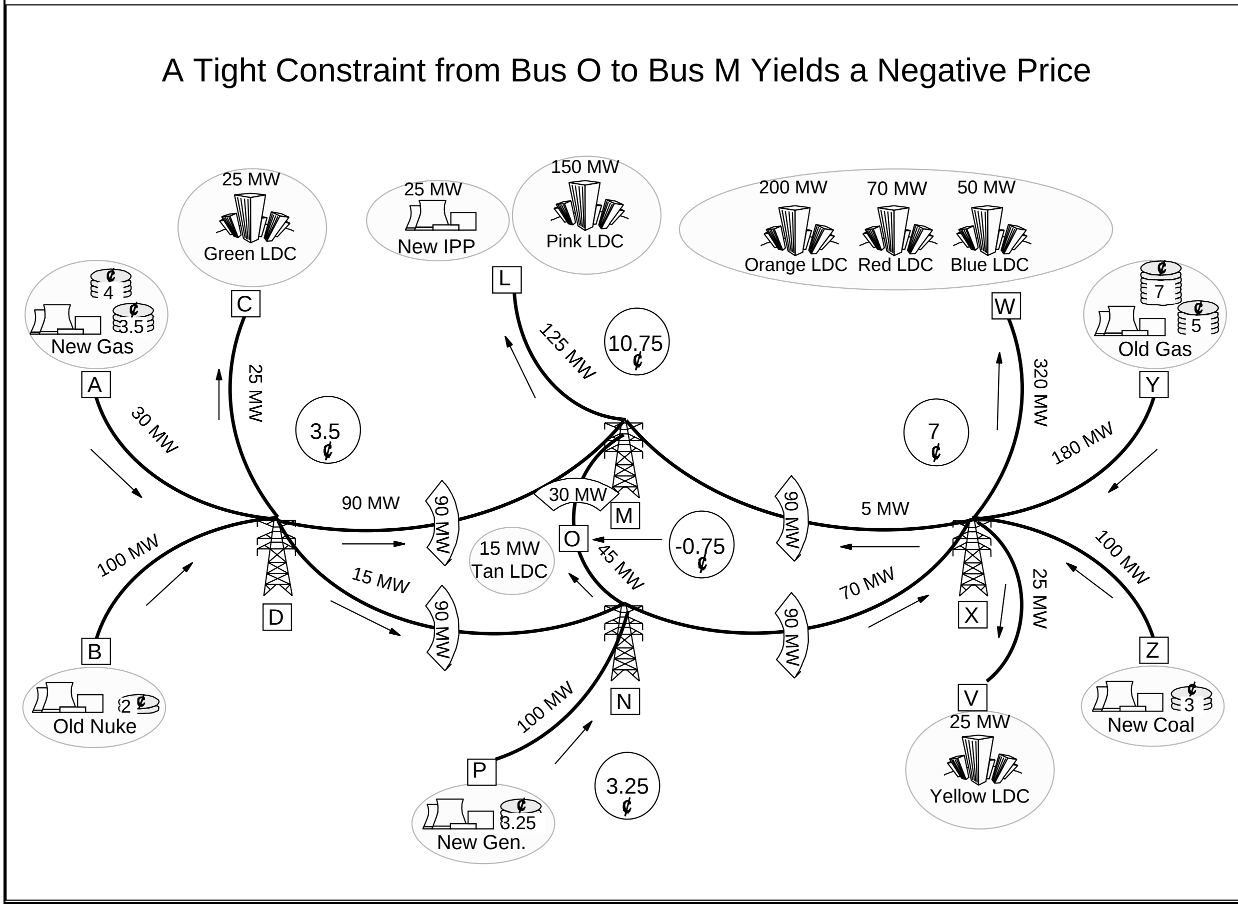
[5] For simplicity, all lines have the same impedance. Hence, introducing bus "O" converts the single line between "M" and "N" into two lines, and doubles the impedance.

Figure 4



costs at some locations that are higher than the variable cost of the most expensive plant and lower than the variable cost of the least expensive plant in operation. In principle, it is even possible to have negative prices in a least-cost dispatch and market equilibrium.