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Quantitative flow analysis of swimming dynamics with coherent Lagrangian vortices

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Undulatory swimmers flex their bodies to displace water, and in turn, the flow feeds back into the dynamics of the swimmer. At moderate Reynolds number, the resulting flow structures are characterized by unsteady separation and alternating vortices in the wake. We use the flow field from simulations of a two-dimensional, incompressible viscous flow of an undulatory, self-propelled swimmer and detect the coherent Lagrangian vortices in the wake to dissect the driving momentum transfer mechanisms. The detected material vortex boundary encloses a Lagrangian control volume that serves to track back the vortex fluid and record its circulation and momentum history. We consider two swimming modes: the C-start escape and steady anguilliform swimming. The backward advection of the coherent Lagrangian vortices elucidates the geometry of the vorticity field and allows for monitoring the gain and decay of circulation and momentum transfer in the flow field. For steady swimming, momentum oscillations of the fish can largely be attributed to the momentum exchange with the vortex fluid. For the C-start, an additionally defined jet fluid region turns out to balance the high momentum change of the fish during the rapid start. © 2015 AIP Publishing LLC.

Lagrangian coherent structures (LCSs) characterize fluid transported coherently inside a vortex and can help elucidate the governing mechanisms of momentum transfer in a flow field. We use the dynamics of the LCS to characterize the flow field and propulsion mechanism of an anguilliform swimmer employing different swimming modes.

I. INTRODUCTION

The way swimmers take advantage of the surrounding liquid environment is subtle and its governing mechanisms remain elusive to a large extent. Fishes propel themselves by deforming their bodies to transfer momentum to the surrounding fluid. Both for fast starts and steady-state swimming, their body strokes correspond to sequences of acceleration and deceleration. The continuously alternating rates of momentum transfer, both globally in the entire flow field and locally along the body surface, make it difficult to separate thrust and drag forces.1,2 This complicates our understanding of the hydrodynamic mechanisms underlying self-propulsion in swimming.

Understanding the momentum transfer between the swimmer and the flow structures in its wake holds the promise of elucidating the fundamental physical mechanisms governing fish swimming.2,3 In order to extract the information as to when and where the swimmer accelerates its surrounding fluid, we rely on the detection and tracking of LCSs. LCSs are persistent material objects in the flow that characterize the fluid deformation over a certain period of time.9,10 LCSs are composed of finite-time Lagrangian trajectories of fluid particles, obtained from the application of concepts of dynamical systems theory to Lagrangian fluid motion. Haller9 reviews various types of LCS and shows a number of applications to geophysical flows.

In the context of fish propulsion, LCSs have been used to reveal the flow topology of several flapping propulsion devices using experimental and numerical data. Peng et al.16 construct a boundary of an isolated vortex pair by combining attracting and repelling hyperbolic LCSs, based on finite-time Lyapunov exponents (FTLE), in order to estimate propulsive forces. In a follow-up article, Peng and Dabiri17 study a numerical simulation of a self-propelled flexible plate and compute hyperbolic FTLE-based LCSs. They define an upstream wake and use the area enclosed by the LCSs to determine an unsteady mass flow rate past the swimmer. Green et al.7 use experimental flow data of a pitching panel, mimicking a caudal fish fin. The transition of the wake topology for increasing Strouhal number is shown by means of FTLE-based hyperbolic LCS. All three studies detect LCSs as FTLE ridges, while by now, a precise definition of a hyperbolic LCS is available that returns a coherent material line.8

In the first two studies, the authors attempt to track fluid enclosed by FTLE ridges. For the tracking of a coherent fluid region, this approach seems problematic, since hyperbolic structures stretch exponentially under the flow and are therefore unable to define a closed vortex boundary that evolves coherently under the flow. Moreover, FTLE ridges do not
form closed loops, and rather spiral into coherent vortices, incorrectly suggesting the lack of a coherent core in these cases. Even if closed curves are constructed from FTLE ridges computed in forward and backward time direction, the ridge curves neither close up, and the curves have to be closed manually.\textsuperscript{16} As an additional drawback, the material flux across an FTLE ridges is generally non-zero.\textsuperscript{8,9} If consecutive snapshots of FTLE ridges are used to define an evolving boundary, fluid is flowing into and out of the tracked fluid region, so that the Lagrangian property is lost.

In contrast to FTLE ridges that only roughly identify hyperbolic structures in the flow, our novel method\textsuperscript{11} detects elliptic LCS, denoted by coherent Lagrangian vortices. The resulting closed material line has a zero material flux across it by construction. This is a key property for the quantification of momentum exchange between the engulfed fluid and the swimming body. The obtained material vortex boundary behaves coherently in the sense that—unlike generic material lines in a time-dependent flow—it does not stretch or form filaments over the time-period used in its identification. Hence, the coherent vortex boundary encloses fluid that stays inside the vortex, coherently separated from the ambient fluid. The vortex fluid region serves as a Lagrangian control volume that allows us to track vorticity and linear momentum history to the swimming dynamics and monitor the vortex formation process. We relate the evolution of a fluid trajectory is given by
\begin{equation}
\dot{x} = v(x, t).
\end{equation}
A trajectory is denoted \(x(t; x_0, t_0)\), with the initial position \(x_0\) at time \(t_0\). The flow map is
\begin{equation}
F^t_t(x_0) := x(t; t_0, x_0),
\end{equation}
which maps initial positions to current positions at time \(t\). The finite time \(T = t - t_0\) is predefined and plays the role of the required coherence time for the coherent vortex boundary. The Cauchy–Green strain tensor
\begin{equation}
C^t_t(x_0) = [\nabla F^t_t(x_0)]^T \nabla F^t_t(x_0)
\end{equation}
measures Lagrangian strain along trajectories and has eigenvalues and eigenvectors,
\begin{equation}
C^t_t \xi_i = \lambda_i \xi_i, \quad 0 < \lambda_1 \leq \lambda_2, \quad i = 1, 2.
\end{equation}
As derived by Haller and Beron-Vera,\textsuperscript{11} \(\lambda\)-lines are trajectories of one of the two vector fields.
The relative tolerance of the employed Matlab fluid onto the body is given as conditions on the body surface. In this case, the force of the unit density in an infinite 2D fluid, with no-slip boundary conditions to the variational principle of stationary averaged Lagrangian strain.

\[ \lambda \text{ is a parameter close to one, namely, the stretching factor for the coherent vortex boundaries. To detect coherent vortex boundaries, we vary } \lambda \text{ in the range [0.95, 1.05], i.e., we allow for maximally 5\% stretching or shrinking of the curve. For each } \lambda \text{-value, we seek closed orbits in the vector fields } \eta^\lambda, \text{ and define the outermost such closed orbit as the Lagrangian vortex boundary. A detailed description of the individual steps of the algorithm along with descriptive examples are given by Onu et al.}^{15} \text{ The associated MATLAB toolbox LCS Tool is available at https://github.com/jeixav/LCS-Tool.} \]

The numerical advection of the detected vortex boundary lines in the simulated flow field has to be accurate. Therefore, the relative tolerance of the employed Matlab ode45 function has been set to \(10^{-8}\) and additional points are added to the vortex polygons when the distance between neighbor points exceeds a threshold. Linear interpolation in time and space is sufficient, since the numerical data set has a high spatial and temporal resolution. The critical location for advection is the fish boundary: in case of insufficient accuracy, advected material lines may cross the fish boundary.

For the plots and diagrams, vorticity is computed from the velocity field with finite differences. The circulation \(\Gamma\) of the vortex fluid is obtained as \(\Gamma = A \langle \omega \rangle_A\), where \(A\) is the area and \(\langle \omega \rangle_A\) is the mean vorticity of the enclosed fluid region. Linear momentum is obtained as \(p = \rho A \langle \mathbf{u} \rangle_A\), where \(\langle \mathbf{u} \rangle_A\) is the mean velocity, and density \(\rho = 1\).

We consider a self-propelled body of constant area and unit density in an infinite 2D fluid, with no-slip boundary conditions on the body surface. In this case, the force of the fluid onto the body is given as

\[ \mathbf{F}^H = m \ddot{\mathbf{x}} + \frac{d}{dt} \int_{V_\infty(t)} \mathbf{u} \, dV - \oint_{S_\infty} p n \, dS, \]

where \(V_\infty\) is the region of the domain occupied by fluid, \(S_\infty\) is the bounding surface of the domain at infinity, and \(n\) is the normal vector to that surface.\(^{14,19}\) The left-hand side of the equation is obtained by differentiating the time history of the center-of-mass velocity of the swimmer. On the right hand side, the pressure integral cannot be calculated numerically with the current method. However, we found that neglecting the pressure term and restricting the volume integral to the computational domain only introduces a discrepancy in the momentum balance of \(\sim 10\%\) or smaller. This supports our approach of investigating to what extent the momentum change of the swimmer can be attributed to that of the coherent structures in the swimmer’s wake.

### III. RESULTS AND DISCUSSION

**A. C-start**

For the C-start maneuver, coherent vortices are detected at time \(t_0 = 1.25\) with an integration time of \(T = 0.5\). Figure 1 shows the detected vortex boundaries at the beginning and end of the coherent phase at times \(t_0\) and \(t_0 + T\). As guaranteed by the theory of coherent Lagrangian vortices, no filamentation or material break-away occurs along the advected curves over this period (inspect the dashed-dotted curves). The time \(t_0\) corresponds to the time of the second tail stroke cycle when the second vortex pair (#4 and #5) is shed. The first vortex pair (#2 and #3), generated when the C-shape is straightened, is shed at \(t \approx 0.65\). With this choice of \(t_0\) and \(T\), the first and the second vortex pairs are included in the analysis, but the major acceleration of the swimmer is due to the first vortex pair. Vortex #1 is generated during the preparatory stroke when the fish bends from a straight line to the C-shape. Its momentum contribution is small, rather opposing propulsion.

In order to obtain time series of vorticity and linear momentum of the vortex fluid, the vortex boundaries are advected with the flow over the whole time interval of the C-start maneuver, \(t = [0.0, 1.77]\). Figure 2 gives an overview of the temporal evolution of the material fluid regions during vortex formation. The fluid, finally forming the vortices, is initially stretched out to an elongated shape. Interestingly, fluid from both sides of the fish constitutes the vortex fluid, while vorticity with the same sign as in each final vortex is located only on one side. In the presented sequence, this effect can be best observed for vortex #4.

In the first shown snapshot at \(t = 0.3\), a pocket of fluid (black, #6) is enclosed by the deformed vortex boundaries of the first vortex pair (#2 and #3), corresponding to the white enclosed fluid region in Fig. 6 in Gazzola et al.\(^3\) This fluid region constitutes the jet fluid that is expelled backwards inside the first vortex pair. It corresponds to the propulsive

**FIG. 1.** Coherent vortex boundaries at \(t_0 = 1.25\) (solid) with vorticity field and advected curves at \(t_0 + T = 1.75\) (dashed-dotted). A short piece of the fish tail can be seen at the upper left corner. The trajectory of the fish’s center of mass is drawn (solid, gray).
“Jet 2” in the experimental study with bluegill sunfish by Tytell and Lauder. We define the boundary of the jet fluid at time $t = 0$ when the fluid is at rest. At this time, the bounding curves of vortex regions #2 and #3 are almost indistinguishably close, and the jet boundary is constructed by concatenating the corresponding segments of these two boundaries. The two segments are connected where they are closest. Although the jet fluid region does not transform into a coherent vortex, we add it to the five coherent vortex regions we consider, since it experiences a high acceleration during the start and is therefore a crucial part of the momentum balance.

The evolution of the vortex circulation $C(t)$ is shown in Fig. 3. Initially, the fluid is at rest and all vortices have zero circulation. The circulation increases slowly in the fish’s boundary layer and rises abruptly right before the vortex is shed at the tail. This process is emphasized by Fig. 3(a), in which peak circulation corresponds to the shedding of the fluid region from the fish tail. The peak is followed by a slow decay in the wake due to viscosity. As an example, we show the decay of vortex region #2, and find that it fits well with the circulation decay of the viscous Lamb–Oseen vortex model

$$C(t) = C_0 [1 - e^{-\pi(r-t_0)^2}],$$

where $C_0$ is the peak circulation, $t_0$ is the time of vortex shedding, and $x = r^2/(4\nu)$ is an inverse time constant, depending on the vortex radius $r$ and the kinematic viscosity $\nu$ of the fluid. The Lamb–Oseen vortex is a circular two-dimensional vortex model with a Gaussian vorticity profile. The diffusive outflow of vorticity through a bounding material curve leads to the above decay of circulation with time. The peak circulation is similar for the coherent vortices #2, #3, and #4, with values $C_0 \sim 3 \times 10^{-3}$. The circulation of vortex region #5 results smaller, since only a small core part of the vortex is detected as coherent in the prescribed time window, cf. Fig. 1. The preparatory vortex (#1) is also weaker with a peak circulation of $C_0 \sim 1 \times 10^{-3}$.

The gain of linear momentum of the fish in the accelerating phase of the C-start is closely linked to the generated coherent vortices. However, the primary contribution to the forward acceleration comes from the expelled jet of fluid. Figure 4 shows the temporal evolution of forward momentum for the six discrete fluid regions. The forward direction of the fish motion is defined by the trajectory of the center of mass of the fish over the interval $[0, 0.47]$ (see trajectory in Fig. 1) and corresponds to an angle of $\sim 45^\circ$ to the vertical. Linear momentum of the fluid regions is projected onto the opposite direction to ease a direct comparison with the fish momentum.

The sum of forward linear momentum of all fluid regions (Fig. 4(a), black) is dominated by the momentum of the jet of fluid (magenta). Vortices #2 (cyan) and #3 (green) also contribute, but the forward linear momentum they carry is five times smaller. During the C-start, the fish gains its final momentum before $t = 0.5$, mainly during a steep rise of the momentum curve at $t = 0.3$, see Fig. 4(b) (red, solid). We note that both the fish’s momentum and the (negative) momentum of the fluid regions (black) exhibit this steep rise approximately at $t = 0.3$. Consequently, the force impulse accelerating the fish is provided by the reaction force of the ejected fluid regions, in particular, by the jet fluid. The momentum of the fluid regions rises to values twice as high as
the fish’s momentum. This imbalance could be explained by
the added mass of the fish or the vortices\textsuperscript{2} that we do not
consider in the detected fluid regions. In the lateral direction,
the momentum of the fish and the momentum of the fluid
regions are not in balance, indicating that additional parts of
the fluid have to be taken into account for a complete mo-
mentum balance of fluid and body.

In summary, the detection of coherent vortices in the C-
start serves to define discrete Lagrangian fluid regions rele-
vant for propulsion and to track their dynamics. Backward
advection of these regions identifies a further jet region
around which Lagrangian vortex boundaries accumulate
beyond their coherence time.

B. Steady swimming

In the case of steady swimming, during the first cycle, a
quarter-period sinusoidal ramp is employed, and for each
swimming cycle afterwards, the swimmer’s body deforma-
tion corresponds to anguilliform swimming.\textsuperscript{12} We consider
the swimming mode of the fish as steady after an initial
acceleration phase of eight cycles, corresponding to \( t = 8.0 \).
After this period, the momentum of the center of mass of the
fish is constant, apart from periodic body stroke-induced
oscillations (cf. gray curve in Fig. 7(c)).

Coherent vortices are detected in the wake of the steady
swimmer, one cycle after being shed from the tail, with an
integration time also of one cycle, i.e., each coherent vortex
has an individual initial time \( t_0 = t_{\text{shed}} + 1.0 \) and \( T = T_c = 1.0 \).
The time \( t_{\text{shed}} \) is determined by visual inspection. We detect
three consecutive vortices at \( t_0 = [13.0, 13.5, 14.0] \) and advect
the resulting boundary curves over the whole time interval of
steady swimming [8.5 15.0].

The temporal evolution of the coherent vortex bounda-
ries is shown in Fig. 5. Initially, the vortex fluid is stretched
into an elongated shape with two larger patches on both sides of
the swimmer. Similar lobes appear in Fig. 7 of Peng
and Dabiri,\textsuperscript{17} in FTLE-based hyperbolic LCS. This suggests
that vortex fluid is embedded in the hyperbolic structures at
this location. It is surprising that the fluid patch on the side with opposite vorticity sign compared to the final vortex has a significant size, such that a large volume of fluid with opposite sign vorticity is entrained into the final vortex during the shedding phase at the tail. The deformed fluid regions have qualitatively the same shape at \( t = 0.3 \), apart from translation and mirroring with respect to the horizontal axis, as expected for the periodic steady motion. Yet, a careful inspection reveals different details. This is because the detected circular vortex regions, at their individual \( t_0 \), have slightly different sizes, which may be related to an imperfect periodic flow. These small differences are propagated through the non-linear advection to earlier times.

Both fluid patches on either side of the fish are connected with a thin filament of negligible area. The filaments of the three vortices are arranged side by side, overlap with the boundary layer of the swimmer’s skin, and exhibit a broader band of fluid in front of the swimmer. When the fish passes through the vortex regions, the contained fluid gains vorticity, and finally rolls up to a coherent vortex when shed at the tail. Figure 6 shows the circulation \( \Gamma(t) \) of three detected vortices. The circulation is practically zero when the fluid regions are initially at rest far upstream of the fish. Then, the circulation rises as the fish proceeds, until the vortex is shed, and viscous decay starts. Due to the periodic motion, the curves show the same behavior, as expected. Since they are associated with three consecutive vortices, all three curves are shifted in time by \( \Delta t = 0.5 \). The second vortex region (red) has a negative circulation, indicating the opposite sense of rotation compared to the first and third vortex region. The first local maximum of circulation, for example, for the red curve at \( t = 10.0 \), corresponds to the vorticity generated in a small vortex behind the head (see arrow in Fig. 5).

Concerning the linear momentum balance, the steady swimming case differs from the accelerating C-start case. During steady swimming, the momentum of the fish, averaged over a flapping period, is constant, and therefore, the net momentum exchange with the fluid is null. However, oscillations in the forward momentum of the fish (in x-direction) exist due to the intermittent propulsion by tail strokes. These sinusoidal oscillations can be seen in Fig. 7(a) represented by the gray curve, the linear momentum of the fish. The curve is shifted by an arbitrary \( p_x \)-offset to ease visual comparison with the momentum of the vortex fluid.

The constant component of the linear momentum of the fish during steady swimming was gained during the initial acceleration phase prior to our analysis period. Therefore, we focus on the oscillations in the fish’s forward momentum and relate them to the momentum of the coherent vortices. Fig. 7(a) shows the momentum time series of the fluid regions associated with the detected coherent vortices (thin black, red, and blue). Color-coded bars indicate the time when the vortex region goes past the head and when the vortex is shed at the tail. The momentum history of an individual fluid patch can be divided into three phases depending on the location of the fluid: upstream, alongside, or downstream of the fish.

In the first phase, starting from zero, the individual fluid region gains momentum in the swimming direction of the fish. This corresponds to the fluid being pushed forward due to the approaching swimmer, a part of the added mass of the body, which can be associated with a drag acting on the swimmer. In the second phase, after contact with the body, the fluid is accelerated backwards until it reaches zero momentum again, right before being shed by the tail. This phase is associated to thrust production. Note that the shedding times of the three vortices, \( t_{shed} = [12.0, 12.5, 13.0] \), agree...
frequency is not present in the momentum history of an individual vortex fluid region. The individual momentum curves have lower frequencies of typically $f_p \sim 4/3$. The oscillations occur when alternating drag and thrust regions along the fish propagate with the fish through the vortex material patches (Fig. 7(b)). Only if the momentum time series of several vortices is superposed (Fig. 7(a), bold black curve), the dominant frequency in the total momentum matches the frequency of the fish momentum. The dominant frequency of $f_p = 2$ can be seen in the bold black total momentum curve approximately over the range $[10.25, 12.50]$.

To strengthen this observation, we construct a total momentum time series of the vortex fluid, by summing up a periodically shifted individual momentum curve. Figure 7(c) shows the individual momentum curve (blue), the copies shifted by 0.5 in time (faint blue), and the summed up total momentum (black). While it is not surprising that the resulting total momentum has a dominant period of 0.5 (by construction), it is noteworthy that the amplitude of the constructed total momentum is very similar to the amplitude of the oscillations of the fish momentum. Consequently, it seems that the oscillations in the fish momentum are balanced by the superimposed momentum time series of individual vortex regions.

Our results show that, by following the momentum history of a coherent vortex fluid region as a material entity, drag and thrust can be spatially and temporally separated to some extent. According to the momentum history, depicted in Fig. 7(a), an individual vortex fluid region is first accelerated in swimming direction upstream of the fish, acting as drag, and later accelerated in backward direction aside the fish, acting as thrust. Yet, the backward acceleration aside the fish is not monotonic. Regions of thrust and drag alternate along the undulating body (Fig. 7(b)). Thrust (drag) regions are located close to concave (convex) regions of the fish boundary. This is consistent with the C-start, where the thrust-generating accelerated fluid is located within the concave C-shaped body during the preparatory stroke. Since the vortex fluid is distributed along both sides of the fish body, it both covers drag and thrust regions when the fish passes. However, the net thrust effect can then be predominantly attributed to the fluid located in the concave sections of the fish’s flexing body.

We wish to stress that in our analysis, we only considered a discrete set of coherent vortices, while non-negligible contributions of other parts of the fluid may have been overlooked. In particular, the jet fluid in close contact with the fish in-between the lobes of the vortex fluid in Fig. 5(a) could be relevant. Yet, the fact that the momentum of the vortex fluid alone can largely explain the momentum fluctuations of the fish suggests that complementary jet fluid regions predominantly carry lateral momentum that is of minor importance for propulsion.

IV. CONCLUSION

In this study, we quantify the contribution of discrete coherent Lagrangian vortices to the momentum balance of two different modes of swimming propulsion, impulsive C-start escape, and steady anguilliform swimming. For the
C-start, Gazzola et al.\textsuperscript{5} report that the fluid region trapped inside the C-shape consists of a vortex pair and a low vorticity jet region. They find that bending the body into a C-shape is an optimal strategy for fast escapes, as it allows the fish to trap and accelerate the largest amount of fluid. We quantify the momentum transfer to the trapped fluid and show that the main contribution comes from the jet fluid region, initially enclosed by vortex fluid and finally expelled as a jet. For steady swimming, the amplitude and frequency of momentum oscillations of the fish are matched when the momentum records of individual vortex regions are periodically superposed. For an individual vortex region, a phase of drag production along the fish. Consistent with the C-start, thrust regions aside the fish are located where the body is concave suggesting a similar propulsion mechanism. The geometry of the deformed vortex fluid upstream of the swimmer, revealed by our method, can be of further interest when the swimmer interacts with vorticity from upstream obstacles, like a stationary bluff body\textsuperscript{13} or other fish in fish schools.\textsuperscript{4}

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