On the Use of Complementary Encoding Techniques to Improve MR Imaging

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Overview

• Part I: Overview
  ◦ Fourier Encoding (MRI Foundation)
  ◦ Temporal Encoding (UNFOLD)
  ◦ Spatial Encoding (pMRI: SENSE, GRAPPA)
  ◦ Review Strengths and Weaknesses

• Part II: Regularization in SENSE
  ◦ LSQR-Hybrid Algorithm for efficient solutions
  ◦ Accelerated partial-Fourier imaging

• Part III: Useful Combinations
  ◦ GRAPPA+SENSE together
  ◦ Real-Time Self-Referenced pMRI
  ◦ Cardiac CINE via UNFOLD/GRAPPA/SPACE RIP

• Part IV: EPI Artifact Correction
  ◦ Review of the problem
  ◦ A spatial + temporal encoding solution
Basics of MRI: Fourier Imaging

MRI uses magnetic field gradients and RF signals to encode field-of-view

Encoding typically corresponds to a Fourier transform (data is acquired in $k$–space domain)

$k$–space is sampled line-by-line

Reduce number of lines $\leftrightarrow$ Reduce acquisition time
Fast MR Imaging: Keyhole Sampling

One idea: acquire just the low frequency content of the image

Produces images with low resolution.
Fast MR Imaging: Sub-sampling

An Alternative: sub-sampling to increase imaging efficiency.

Sub-sampling in k-space produces spatial domain aliasing. Images can be recovered using complementary encoding.

- Temporal encoding, by varying the sampling pattern
- Spatial encoding, using multiple receiver coils
Temporal Encoding

A shift in the sampling grid alters the point spread function

Odd Frames

Even Frames

Point Spread Functions

UNFOLD uses variations in sampling time to encode aliasing artifacts.

UNFOLD Processing

Alternating the sampling grid will cause artifact “flicker”

With appropriate time domain filtering

the artifact can be removed.
Multiple receiver coils were first introduced to improve signal-to-noise.

A secondary effect is that it provides a *spatial domain encoding*. Each receiver coil sees the FOV from a different perspective.

This encoding complements traditional Fourier encoding.
Sub-sampling combined with spatial domain encoding yields the basic parallel imaging problem.

Each coil acquires an aliased image ... weighted by the coil sensitivity ... at each pixel that contributes

\[ s_l(x, y) = W_l(x, y)\rho(x, y) + W_l(x, y + N/2)\rho(x, y + N/2) \]
Cartesian SENSE Formulated

Combining the equations for all coils, we have

\[
\begin{bmatrix}
    s_1(x, y) \\
    s_2(x, y) \\
    s_3(x, y) \\
    s_4(x, y)
\end{bmatrix}
= \begin{bmatrix}
    W_1(x, y) & W_1(x, y + N/2) \\
    W_2(x, y) & W_2(x, y + N/2) \\
    W_3(x, y) & W_3(x, y + N/2) \\
    W_4(x, y) & W_4(x, y + N/2)
\end{bmatrix}
\begin{bmatrix}
    \rho(x, y) \\
    \rho(x, y + N/2)
\end{bmatrix}
\]

or, in a compact matrix/vector form

\[ s = P\rho \]

Solving the SENSE Problem

Recast as a minimization problem

$$\hat{\rho} = \arg\min_{\rho} \left\{ f(\rho) = \| P \rho - s \|^2_2 \right\}$$

Adding regularization terms allows one to balance between noise reduction and artifact suppression.

$$\min_{\rho} \left\{ \| s - P \rho \|^2_2 + \ldots \right\}$$

All SENSE-related methods (including SPACE RIP, Generalized SENSE, Generalized GRAPPA) can be described this way.
GRAPPA generalizes so-called k-space methods:

- coil-by-coil reconstruction,
- flexible kernel size,
- multiple auto-calibration signal lines acquired.

\[ s^{(l)}(j) = \sum_{m=1}^{L} \sum_{b=0}^{3} \sum_{c=-2}^{2} s_m(c + k_x, (2b - 3) + k_y) n(m, b, c, l) \]

\[ \text{GRAPPA generalizes so-called k-space methods:} \]

- coil-by-coil reconstruction,
- flexible kernel size,
- multiple auto-calibration signal lines acquired.

GRAPPA Model

While not explicit in the reconstruction model, GRAPPA does rely on spatial encoding via multiple coils.

Consider the encoding model for the *the full data* in each coil

\[
\begin{bmatrix}
    s_l^{(a)} \\
    s_l^{(u)}
\end{bmatrix} = \begin{bmatrix}
    G_a \\
    G_u
\end{bmatrix} \kappa.
\]

This gives two equations, acquired lines: \( s_l^{(a)} = G_a \kappa \)

un-acquired lines: \( s_l^{(u)} = G_u \kappa \)

Eliminating \( \kappa \) yields

\[
s_l^{(u)} = G_u (G_a^\dagger s_l^{(a)}).
\]

GRAPPA Visualized

\[ s_l^{(u)} = (G_u G_a^\dagger) s_l^{(a)} \]

The GRAPPA algorithm seeks to estimate a composite encoding matrix, \((G_u G_a^\dagger)\) to combine acquired coil data, \(s_l^{(a)}\), and form unacquired coil data, \(s_l^{(u)}\).

GRAPPA is efficient (low number of parameters) because of the particular structure of the composite encoding matrix.
## Comparison of SENSE and GRAPPA

<table>
<thead>
<tr>
<th></th>
<th><strong>Strengths</strong></th>
<th><strong>Weaknesses</strong></th>
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<tbody>
<tr>
<td><strong>SENSE</strong></td>
<td>- optimal in least-squares sense</td>
<td>requires <em>good</em> coil sensitivity estimates</td>
</tr>
<tr>
<td></td>
<td>- easy to constrain the reconstruction (e.g. regularization)</td>
<td></td>
</tr>
<tr>
<td><strong>GRAPPA</strong></td>
<td>- <em>does not</em> require coil sensitivity estimates</td>
<td>sub-optimal solution in least-squares sense</td>
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SENSE (and related methods) and GRAPPA use separate models to reconstruct an image.

*One can combine them* and build on each of their strengths.
Part II

Regularization in SENSE
SENSE with Damped Least Squares

SENSE pMRI problems can benefit from a single (scalar) regularization parameter

$$\min_{\rho} \left\{ \| P \rho - s \|_2^2 + \lambda \| \rho \|_2^2 \right\}$$

This problem can be solved quickly using *LSQR-Hybrid*

- solved in a projected space
- solutions for multiple $\lambda$ can be found very efficiently
- “best” image selected automatically via balance between error and solution norm. (e.g. L-curve, Discrepancy)

LSQR Algorithm

For the moment, assume that $\lambda$ is fixed and let $\rho_0 = 0$.

LSQR produces iterates $\rho_k$ such that

$$
\rho_k = \arg \min_{\rho \in K_k} \left\| \begin{bmatrix} P \\ \lambda I \end{bmatrix} \rho - \begin{bmatrix} s \\ 0 \end{bmatrix} \right\|.
$$

The solution, $\rho_k$, exists within a $k$-dimensional Krylov subspace:

$$
K_k = \text{span}\{ P^H s, (P^H P)P^H s, \ldots, (P^H P)^{k-1} P^H s \}.
$$
LSQR-Hybrid

Multiple regularized solutions can be found quickly because the Krylov subspace is independent of $\lambda$.

That is,

$$\text{span}\{ P^H s, (P^H P) P^H s, \ldots, (P^H P)^{k-1} P^H s \}$$

describes the same space as

$$\text{span}\{ P^H s, (P^H P + \lambda^2 I) P^H s, \ldots, (P^H P + \lambda^2 I)^{k-1} P^H s \}.$$  

- Points on L-curve can be computed in $\mathcal{O}\{N \lambda\}$ flops.
- Recurrence relationships can update all $\rho_{k}^{(\lambda)}$ solutions in $\mathcal{O}\{NN \lambda\}$ flops.

LSQR-Hybrid Example

Acceleration factor = 3,
Variable density sub-sampling,
8-channel FSE data acquired on a 1.5T GE Signa EXCITE scanner.

\[ \lambda = 1.0000 \times 10^{-4} \]
LSQR-Hybrid Example

Acceleration factor = 3, Variable density sub-sampling, 8-channel FSE data acquired on a 1.5T GE Signa EXCITE scanner.

\[ \lambda = 6.2506 \times 10^3 \]
LSQR-Hybrid Example

Acceleration factor = 3, Variable density sub-sampling, 8-channel FSE data acquired on a 1.5T GE Signa EXCITE scanner.

\[ \lambda = 1.4563 \times 10^4 \]
LSQR-Hybrid Example

Acceleration factor = 3, Variable density sub-sampling, 8-channel FSE data acquired on a 1.5T GE Signa EXCITE scanner.

\[ \lambda = 7.9060 \times 10^4 \]
### LSQR-Hybrid Recon Time

<table>
<thead>
<tr>
<th>Number of λ solutions:</th>
<th>LSQR recon time (sec):</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.51</td>
</tr>
<tr>
<td>5</td>
<td>4.46</td>
</tr>
<tr>
<td>10</td>
<td>4.63</td>
</tr>
<tr>
<td>15</td>
<td>4.73</td>
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<tr>
<td>20</td>
<td>5.12</td>
</tr>
<tr>
<td>25</td>
<td>5.32</td>
</tr>
<tr>
<td>30</td>
<td>5.47</td>
</tr>
<tr>
<td>40</td>
<td>5.81</td>
</tr>
<tr>
<td>50</td>
<td>5.83</td>
</tr>
</tbody>
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C-code implementation,
1 core of 2.16 GHz Centrino, 2GB RAM.
256 x 256 image, R = 3.2x, variable density subsampling

http://ncigt.org/pages/Research_Projects/ImagingCoreToolbox/Imaging_Toolkit
Partial-Fourier Problem Formulation

In partial-Fourier imaging, only half-plane of k-space is acquired, and solution constrained to be ’nearly’ real.

We use a two-parameter minimization, to constrain real and imaginary components separately.

$$\min_{\rho} \left\{ \| P\rho - s \|^2 + \lambda_{re} \| \Re \{\rho\} \|^2 + \lambda_{im} \| \Im \{\rho\} \|^2 \right\}$$

Downside: large regularization parameter search space
Reformulating the partial-Fourier problem

To maintain scalar-lambda modification of Krylov subspace, we recast the 2-parameter partial-Fourier problem as

\[
\begin{bmatrix}
\Re\{s\} \\
\Im\{s\} \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}
\Re\{P\} & -\Im\{P\} \\
\Im\{P\} & \Re\{P\} \\
\lambda_{re} \begin{bmatrix}
I \\
0
\end{bmatrix} \\
0 & \lambda_{im}/\lambda_{re} I
\end{bmatrix}
\begin{bmatrix}
\Re\{\rho\} \\
\Im\{\rho\}
\end{bmatrix}
\]

- hold the ratio \( c = \lambda_{im}/\lambda_{re} \) constant
- iterate over \( \lambda_{re} \)
- repeat for multiple values of \( c \).

LSQR-Hybrid with a regularization operator

Define \( L = \begin{bmatrix} I & 0 \\ 0 & cI \end{bmatrix} \), and \( c = \lambda_{im}/\lambda_{re} \), and solve

\[
\min_{\rho} \{ \|P \rho - s\|_2^2 + \lambda_{re}^2 \|L \rho\|_2^2 \}
\]

A change of variables, \( y = L \rho \), and \( \rho = L^{-1} y \), gives

\[
\min_{y} \{ \|P L^{-1} y - s\|_2^2 + \lambda_{re}^2 \|y\|_2^2 \}
\]

Problem is now compatible with LSQR-Hybrid

Constraint: \( L \) must be invertible. (e.g. non-zero \( \lambda \)'s)
Partial-Fourier pMRI Results

Highly efficient computations along lines of constant $c$

Contour plot of residual error, $\| P \rho - s \|_2^2$, 
2-D MRI with partial-Fourier Results

R=4x. (>2x $k_y$ acceleration + partial-Fourier)
16 conjugate-symmetric lines (past $k_y = 0$)
2-D MRI with partial-Fourier Results

R=4x. (>2x $k_y$ acceleration + partial-Fourier)
8 conjugate-symmetric lines (past $k_y = 0$)
2-D MRI with partial-Fourier Results

R=4x. (>2x $k_y$ acceleration + partial-Fourier)
1 conjugate-symmetric line (past $k_y = 0$)
3-D MRI with partial-Fourier Results

3D FSE Data acquired at 3T, 8x total acceleration, 8 channel head coil unaccelerated acquisition time $\sim$ 28 min
accelerated acquisition time $\sim$ 4 min
Part III

Useful Combinations
GRAPPA-enhanced self-referenced SENSE

Strategy: use GRAPPA to improve coil sensitivity estimates in self-referenced SENSE methods.

Acquisition pattern subdivided into three regions

Use GRAPPA in light gray outer central region, estimate coil sensitivities using all gray regions, then use SENSE (LSQR-Hybrid) reconstruction on original data set.

GEYSER + SPACE RIP

3.2x acceleration, exponentially weighted sampling distribution, 128-x-128 image size, 4 ACS lines, 8-channel cardiac coil, 3T scanner.

SPACE RIP (4 ACS lines)  GRAPPA alone (4 ACS lines)  GEYSER + SPACE RIP (20 ACS lines)
GEYSER + LSQR-Hybrid in Real-Time MRI

Short movie showing real-time imaging capabilities

Data acquired at 3T, TR/TE: 3.4ms/1.9ms,
128x128 image size,
34 phase-lines acquired (3.76x).
Completely self-referenced.
Image acquisition time: 163ms (6 frames/sec)
Image reconstruction time: 100ms (8-core CPU, 4GB RAM)

Credit: Renxin Chu
UNFOLD Cardiac Imaging Example

UNFOLD uses temporal variations in sampling to encode aliasing artifacts.

Complements parallel imaging techniques.

Separate temporal-frequency bands can be processed independently.

UNFOLD Cardiac Imaging Images

Best results with GRAPPA and SPACE RIP combined

Part IV

Useful Combinations: Nyquist Ghost Correction
Echo Planar Imaging

EPI is a fast imaging method that acquires multiple k-space lines after each RF excitation.

Challenge: Data sampled on positive readout (+RO) and negative readout (-RO) gradients is often misaligned, introducing artifacts known as *Nyquist ghosts.*
EPI Nyquist Ghost Correction Review

Many methods have been proposed to correct them:

- use pre-scan data to estimate the shift (sensitive to scanner drift)
- acquire additional k-space lines w/ image data (extends echo train, yielding greater distortion)
- interleave data (loss of temporal resolution)
- self-referenced phase alignment of +RO and -RO data (not robust)
- use parallel imaging (the focus here)
Ghost Correction via Temporal Encoding

Temporal Encoding: alternate readout polarity on alternate frames

Odd Frames

Even Frames

Ghost correction options:

- interleave adjacent data
- filter temporal series with UNFOLD
Ghost Correction via Interleaves (PLACE)

- Interleaved data may be corrupted by scanner instability.
- (+) Phase-alignment and addition can remove any residual ghosts.
- (-) Signal cancellation, with loss of SNR and temporal variations.

Benefit to pMRI

*Ghost-free data is critical for good pMRI reconstructions*

Data: $R=3.2x$ variable density EPI, 8-channel coil, 1.5T
Recon: Double-pass GRAPPA (1st: 6 ACS; 2nd: 20 ACS)

<table>
<thead>
<tr>
<th>Ghost Correction</th>
<th>GRAPPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) pre-scan data</td>
<td>self referenced</td>
</tr>
<tr>
<td>(b) pre-scan data</td>
<td>20 ACS reference frame</td>
</tr>
<tr>
<td>(c) interleaved data</td>
<td>self referenced</td>
</tr>
</tbody>
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Ghost Elimination via Spatial/Temporal Enc.

- Temporal encoding: alternate polarity of readout gradients
- Spatial encoding: employ multiple channels and parallel MR imaging
Comparison of Ghost Correction Methods

Using both temporal encoding and pMRI, robust and exceptionally good Nyquist ghost correction can be obtained.
EPI-GESTE summary

Ghost Elimination via Spatial and Temporal Encoding:

Parallel MRI calibration
- two temporally encoded frames are used
- interleaved data forms two images, +RO and -RO
- coherent summation of +RO and -RO image is ghost free
- ghost removal improves pMRI coefficient calibration

Image reconstruction
- recon +RO and -RO data separately w/ pMRI coefficients
- adding images coherent cancels residual pMRI artifacts

Advantages
- no loss of temporal resolution
- self-referenced - no pre-scan data or longer ETL
- exceptionally good ghost suppression

GESTE + 3D EPI

Data acquired as planes of kx-ky, at kz locations

Standard Ghost Correction  
EPI–GESTE Ghost Correction

Credit: O. Afacan
Measuring CBF with ASL MRI

Arterial Spin Labeling (ASL) MRI
• for cerebral blood flow (CBF) measurement
• non-invasive and repeatable

How does ASL work?

Spin Preparation

Image Acquisition

Difference

Credit: H. Tan, R. Kraft
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Arterial Spin Labeling (ASL) MRI
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How does ASL work?

Spin Preparation → Image Acquisition → Difference → 1 average

Credit: H. Tan, R. Kraft
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• for cerebral blood flow (CBF) measurement
• non-invasive and repeatable

How does ASL work?

Spin Preparation ➔ wait ➔ Image Acquisition

60 averages

Credit: H. Tan, R. Kraft
GESTE vs Other GC methods in ASL

Fluctuations in ghost suppression can cause false brightening in the perfusion image.
Summary

MR Imaging can benefit from spatial and temporal encoding.

Spatial and Temporal encoding are complementary, and reduce the amount data needed to create an image.

Combining methods is often beneficial:
- artifacts visible in one method can be compensated with another.

Leveraging complementary encoding to generate ‘redundant’ data can be used to remove artifacts.
- also useful in: motion artifact correction,
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