# Crash Risk in Currency Markets

# — Appendix —

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The objective of this appendix is to present and derive the theoretical propositions of the paper regarding the model-implied prices of carry trade returns, interest rates, currency options, and risk-reversals. The appendix is self-contained, and thus reproduces some material presented in the paper. It is organized as follows. Section 1 recalls the laws of motion of the SDFs in both countries and the key model assumptions. Section 2 presents the Black and Scholes (1973) formula applied to currency options. Section 3 restates the discrete-time Girsanov's lemma in order to prove a key lemma that will be used for the derivations of all the proofs. Section 4 presents the propositions and their proofs. Section 5 reports simulation results. Finally, Section 6 presents our data set and reports additional estimates and pricing errors.

# 1 Model Assumptions

Let us first summarize the notation used in the main text, starting with the two SDFs.

**Pricing Kernels** In the home country, the log SDF evolves as:

$$\log M_{t,t+\tau} = -g\tau + \varepsilon\sqrt{\tau} - \frac{1}{2}var(\varepsilon)\tau + \left\{ \begin{array}{ll} 0 & \text{if there is no disaster at time } t+\tau \\ \log(J) & \text{if there is a disaster at time } t+\tau \end{array} \right\}.$$

Likewise, the log of SDF in the foreign country evolves as:

$$\log M^{\star}_{t,t+\tau} = -g^{\star}\tau + \varepsilon^{\star}\sqrt{\tau} - \frac{1}{2}var(\varepsilon^{*})\tau + \left\{ \begin{array}{ll} 0 & \text{if there is no disaster at time } t+\tau \\ \log(J^{\star}) & \text{if there is a disaster at time } t+\tau \end{array} \right\}.$$

The parameters $g$ and $g^{*}$ are drift parameters which are constant between t and t+$\tau$. The random variables $\varepsilon$ and $\varepsilon^{\star}$ are jointly normally distributed with mean 0, variance $\sigma$ and $\sigma^{*}$ and correlation $\rho$. The probability of a disaster between $t$ and $t+\tau$ is given by $p\tau$. $J$ and $J^{*}$, which measure the magnitudes of the disaster, are independent of the process driving the realization of a disaster. The variables $\varepsilon$ and $\varepsilon^{\star}$ are independent of random variables $J$ and $J^{\star}$, and of the process driving the realization of a disaster.

**Exchange Rates** In a complete markets economy, the change in the nominal exchange rate is given by the ratio of the SDFs:

$$\frac{S_{t+\tau}}{S_t} = \frac{M^{\star}_{t,t+\tau}}{M_{t,t+\tau}}.$$

**Disaster Uncertainty** We introduce within-month uncertainty about the realization of $J$ and $J^{\star}$ by considering that home and foreign disaster size within a month are two-value random variable define by:

$$J(\eta) = \bar{J} \cdot (1 + \eta\sigma_J),\, J^{*}(\eta^{*}) = \bar{J}^{*} \cdot (1 + \eta^{*}\sigma_{J^{*}}) \tag{1}$$

where $\eta, \eta^{*}$ are i.i.d. variables equal to 1 and $-1$ with equal probability.

**Risk Exposure** We define: $\tilde{J} = \frac{p\bar{J}}{1-p\tau}$ and $\tilde{J}^{*} = \frac{p\bar{J}^{*}}{1-p\tau}$. The disaster exposure and the Gaussian exposure as defined by:

$$\Pi_D = pE(J - J^*) = \tilde{J} - \tilde{J^*} + O(\tau),$$
$$\Pi_G = cov(\epsilon, \epsilon - \epsilon^*) = \sigma^2 - \sigma\sigma^*\rho.$$

**Conditional Moments** Conditional on no disaster, the expected SDFs are:

$$\begin{aligned} E^{ND} M_{t,t+\tau} &= e^{-g\tau}, \\ E^{ND} M^{\star}_{t,t+\tau} &= e^{-g^*\tau}. \end{aligned}$$

where superscript $ND$ indicates that expectations are taken conditional on no disaster.
The expected ratio of the SDFs conditional on no disaster is:

$$\begin{aligned} E^{ND}\frac{M^*}{M} &= E^{ND} e^{\left(-g^*\tau + \epsilon^*\sqrt{\tau} - \frac{1}{2}\sigma^{*2}\tau\right) - \left(-g\tau + \epsilon\sqrt{\tau} - \frac{1}{2}\sigma^2\tau\right)} \\ &= e^{\left(g - g^* + \frac{1}{2}(\sigma^2 - \sigma^{*2})\right)\tau} E^{ND} e^{(\epsilon^* - \epsilon)\sqrt{\tau}} \\ &= e^{\left(g - g^* + \frac{1}{2}(\sigma^2 - \sigma^{*2})\right)\tau} e^{\frac{1}{2}\left(\sigma^2 + \sigma^{*2} - 2\rho\sigma\sigma^*\right)\tau} \\ &= e^{\left(g - g^* + \Pi_G\right)\tau} \end{aligned}$$

where we used the definition of the Gaussian risk exposure ($\Pi_G = \sigma^2 - \sigma\sigma^*\rho$).
We denote by $\sigma_h^2$ the volatility of the change in exchange rate conditional on no disaster:

$$var^{ND}(\log M_{t,t+\tau} - \log M^{\star}_{t,t+\tau}) = var(\epsilon - \epsilon^*)\tau = \sigma_h^2\tau.$$

In all that follows, we drop the time subscripts for notational simplicity.

# 2 The Black and Scholes (1973) formula

$V^P_{BS}(S, K, \sigma, r, r^{\star}, \tau)$ and $V^C_{BS}(S, K, \sigma, r, r^{\star}, \tau)$ denote the Black and Scholes prices for a put and a call, respectively, when the spot exchange rate is $S$, the strike is $K$, the exchange rate volatility is $\sigma$, the time to maturity is $\tau$, the home interest rate is $r$, and the foreign interest rate is $r^{\star}$. The

Black and Scholes prices of a call and a put are given by:

$$
\begin{aligned}
V_{BS}^{C}(S,K,\sigma,r,r^{\star},\tau) &= Se^{-r^{\star}\tau}\mathbb{N}(d_{+}) - Ke^{-r\tau}\mathbb{N}(d_{-}), && (2)\\
V_{BS}^{P}(S,K,\sigma,r,r^{\star},\tau) &= Ke^{-r\tau}\mathbb{N}(-d_{-}) - Se^{-r^{\star}\tau}\mathbb{N}(-d_{+}), && (3)\\
d_{+} &= \frac{\log(S/K) + (r - r^{\star} + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, && (4)\\
d_{-} &= d_{+} - \sigma\sqrt{\tau}, && (5)
\end{aligned}
$$

where $\mathbb{N}$ is the cumulative standard normal distribution function. The Black–Scholes formula has a simple scaling property with respect to the time to maturity $\tau$ and the interest rates $r$ and $r^{\star}$:

$$V_{BS}^{P}(S,K,\sigma,r,r^{\star},\tau) = V_{BS}^{P}(Se^{-r^{\star}\tau}, Ke^{-r\tau}, \sigma\sqrt{\tau}, 0, 0, 1).$$

For notational convenience, we will omit the arguments 0 and 1 and simply write the value of a generic put as $V_{BS}^{P}(S,K,\sigma)$.

Options are quoted in terms of their implied volatility, the volatility parameter which has to be plugged into the Black–Scholes prices in order to retrieve the market option prices. In the case of currency options quotes, it is common to refer to the spot delta of a Black–Scholes option instead of its strike. The spot delta of an option is its first derivative with respect to the underlying asset S. We will refer to a spot delta as a delta for simplicity. In the case of Black–Scholes prices, the delta is given by:

$$
\begin{aligned}
\Delta_{BS}^{C}(S,K,\sigma,r,r^{\star},\tau) &= +e^{-r^{*}\tau}\mathbb{N}(+d_{+}),\\
\Delta_{BS}^{P}(S,K,\sigma,r,r^{\star},\tau) &= -e^{-r^{*}\tau}\mathbb{N}(-d_{-})
\end{aligned}
$$

Let $(\Delta^{C}, \Delta^{P})$ be the values for the Black–Scholes call delta and a put delta respectively. Let $(\sigma_{\Delta^C}, \sigma_{\Delta^P})$ be the corresponding implied volatility. The strikes $(K_{\Delta^C}, K_{\Delta^P})$ can be obtained by inverting the formula for the Black–Scholes delta:

$$K_{\Delta^C} = Se^{-\mathbb{N}^{-1}\left(+e^{r^{*}\tau}\Delta^{P}\right)\sigma_{\Delta^C}\sqrt{\tau} + \left(r - r^{*} + 1/2\sigma^{2}_{\Delta^C}\right)\tau} \tag{6}$$

$$K_{\Delta^P} = Se^{+\mathbb{N}^{-1}\left(-e^{r^{*}\tau}\Delta^{P}\right)\sigma_{\Delta^P}\sqrt{\tau} + \left(r - r^{*} + 1/2\sigma^{2}_{\Delta^P}\right)\tau} \tag{7}$$

# 3 Some Useful Lemmas

We start with a well-known Lemma, whose proof we provide for completeness.

**Lemma 1.** *(Discrete-time Girsanov's lemma) Suppose that* $(x, y)$ *are jointly Gaussian distributed random variables under probability measure* $P$*. Consider the measure* $Q$ *such that* $dQ/dP = \exp(x - E[x] - \mathrm{var}(x)/2)$*. Then, under* $Q$*,* $y$ *is Gaussian, with distribution*

$$y \sim^Q \mathcal{N}(E[y] + \mathrm{cov}(x, y), \mathrm{var}(y)), \tag{8}$$

*where* $E[y], \mathrm{cov}(x, y), \mathrm{var}(y)$ *are calculated under* $P$*.*

*Proof.* We calculate the characteristic function of $y$. For a purely imaginary number $k$, $E^Q\left[e^{ky}\right]$ is given by

$$E\left[e^{x-E[x]-\sigma_x^2/2}e^{ky}\right] = \exp\left(kE[y] + \frac{k^2\sigma_y^2}{2} + kcov(x, y)\right) = \exp\left(k(E[y] + \mathrm{cov}(x, y)) + \frac{k^2\sigma_y^2}{2}\right).$$

That is indeed the characteristic function of distribution (8). $\square$

**Lemma 2.** *For* $\ln X$*,* $\ln Y$ *jointly Gaussian distributed,*

$$\begin{aligned} E\left[(X-Y)^+\right] &= V_{BS}^C\left(E[X], E[Y], var(\log X - \log Y)^{1/2}\right) \\ &= V_{BS}^P\left(E[Y], E[X], var(\log X - \log Y)^{1/2}\right), \end{aligned}$$

*where* $V_{BS}^C(S_0, K, \sigma)$ *and* $V_{BS}^P(S_0, K, \sigma)$ *are the Black and Scholes call and put prices with home and foreign interest rates equal to 0, and horizon equal to* $1$*.*

*Proof.* Let us write $X = E[X]e^{x-var(x)/2}$ and $Y = E[Y]e^{y-var(y)/2}$, where $(x, y)$ are jointly Gaussian distributed with mean 0 and respective variance $var(\log X)$ and $var(\log Y)$ under probability measure P. We define the probability measure Q by: $dQ/dP = exp(x - E[x] - var(x)/2) = \exp(x - var(x)/2)$ (as $E[x] = 0$). Using Lemma 1:

$$\begin{aligned} E\left[(X-Y)^+\right] &= E\left[\left(E[X]e^{x-var(x)/2} - E[Y]e^{y-var(y)/2}\right)^+\right] \\ &= E\left[e^{x-var(x)/2}(E[X] - E[Y]e^z)^+\right] \\ &= E^Q\left[(E[X] - E[Y]e^z)^+\right], \end{aligned}$$

where $z = y - var(y)/2 - x + var(x)/2$. Applying Lemma 1 implies that the variable $z$ is distributed as: $z \sim^Q \mathcal{N}\left(E^Q[z], var(y-x)\right)$, where:

$$\begin{aligned} E^Q[z] &= -var(y)/2 + var(x)/2 + cov(x, y - x), \\ &= -var(y-x)/2, \end{aligned}$$

As a result, the variable $z$ is distributed as:

$$z \sim^{Q} \mathcal{N}\left(-var\left(y-x\right)/2, var\left(y-x\right)\right).$$

Let us define the variable $u$ as:

$$u = (z - E^{Q}(z))/var(y-x)^{1/2}.$$

Thus the variable $u$ is gaussian, with mean zero and variance 1: $u \sim^{Q} \mathcal{N}(0,1)$. We can define $z$ as a function of $u$:

$$\begin{aligned} u &= (z + var(y-x)/2)/var^{1/2}(y-x) \\ z &= u.var^{1/2}(y-x) - var(y-x)/2 \\ e^{z} &= e^{u.var^{1/2}(y-x) - var(y-x)/2} \end{aligned}$$

Therefore,

$$E\left[(X-Y)^{+}\right] = E^{Q}\left[\left(E\left[X\right] - E\left[Y\right]e^{u.var^{1/2}(y-x)-var(y-x)/2}\right)^{+}\right]$$

The Black and Scholes call and put prices are also given by:

$$\begin{aligned} V_{BS}^{P}(S,K,\sigma) &= E^{Q}\left[\left(K - Se^{\sigma u - \sigma^2/2}\right)^{+}\right], \\ V_{BS}^{C}(S,K,\sigma) &= E^{Q}\left[\left(Se^{\sigma u - \sigma^2/2} - K\right)^{+}\right], \end{aligned}$$

where $u$ is a normal variable with mean 0 and variance 1 under probability measure Q. Using the previous result then implies:

$$E\left[(X-Y)^{+}\right] = V_{BS}^{P}\left(E\left[Y\right], E\left[X\right], var\left(\log X - \log Y\right)^{1/2}\right).$$

A similar reasoning implies that:

$$E\left[(X-Y)^{+}\right] = V_{BS}^{C}\left(E\left[X\right], E\left[Y\right], var\left(\log X - \log Y\right)^{1/2}\right).$$

□

**Lemma 3.** *For* $\log X$, $\log Y, \log Z$ *jointly Gaussian distributed,*

$$\begin{aligned}\operatorname{cov}\left(Z,(X-Y)^{+}\right) &= V_{BS}^{C}\left(E\left[ZX\right], E\left[ZY\right], \operatorname{var}\left(\log X-\log Y\right)^{1/2}\right)\\ &- E\left[Z\right] V_{BS}^{C}\left(E\left[X\right], E\left[Y\right], \operatorname{var}\left(\log X-\log Y\right)^{1/2}\right)\\ &= V_{BS}^{P}\left(E\left[ZY\right], E\left[ZX\right], \operatorname{var}\left(\log X-\log Y\right)^{1/2}\right)\\ &- E\left[Z\right] V_{BS}^{P}\left(E\left[Y\right], E\left[X\right], \operatorname{var}\left(\log X-\log Y\right)^{1/2}\right).\end{aligned}$$

*Proof.* The proof is a straightforward application of the previous Lemma. □

# 4 Propositions and Proofs.

## 4.1 Interest rates and currency excess returns

Let us now turn to carry trade returns. Consider the case of an investment in foreign currency funded by borrowing in domestic currency. The symmetric case — borrowing in foreign currency to invest in domestic currency — would yield the opposite of the result. The trade has return $X$ in domestic currency, and does not require any investment:

$$X = e^{r^{*}\tau}\frac{S_{t+\tau}}{S_t} - e^{r\tau}$$

Let $X^e$ be the annualized expected value of the carry trade return conditional on no disaster:

$$X^e = \frac{E^{ND}X}{\tau}$$

**Proposition 1.** *Recall that* $\tilde{J} = \frac{p\bar{J}}{1-p\tau}$ *and* $\tilde{J}^{*} = \frac{p\bar{J}^{*}}{1-p\tau}$. *In the limit of small time intervals* ($\tau \to 0$), *the interest rate* $r$ *in the home country is approximately equal to:*

$$r = g - pE\big(J-1\big) + O(\tau)$$

*In the limit of small time intervals* ($\tau \to 0$), *the carry trade expected returns (conditional on no disasters) are approximately equal to:*

$$X^e = \Pi_D + \Pi_G + O(\tau)$$

*where:*

$$\Pi_D = pE(J - J^*) = \tilde{J} - \tilde{J}^* + O(\tau)$$
$$\Pi_G = cov(\epsilon, \epsilon - \epsilon^*)$$

*Proof.* Let us first focus on interest rates. The home interest rate $r$ is determined by the Euler equation:

$$E\left[Me^{r\tau}\right] = 1$$

The Euler equation implies:

$$e^{-r\tau} = E\left[M\right] = e^{-g\tau}\Big(1 + p\tau E\big(J - 1\big)\Big) \tag{9}$$

Taking logs gives:

$$r\tau = g\tau - \log\Big(1 + p\tau E\big(J - 1\big)\Big)$$

A Taylor expansion yields:

$$\log\Big(1 + p\tau E\big(J - 1\big)\Big) = p\tau E\big(J - 1\big) + O(\tau^2)$$

The interest rate is thus:

$$r = g - pE\big(J - 1\big) + O(\tau)$$

Now let's turn to the carry trade excess return. The Euler equation for the unhedged carry trade excess return X is:

$$E\left[MX\right] = 0,$$

Recall that the change in the nominal exchange rate is given by the ratio of the SDFs:

$$\frac{S_{t+\tau}}{S_t} = \frac{M^\star}{M}$$

Therefore, the currency excess return is equal to:

$$X = e^{r^*\tau}\frac{M^*}{M} - e^{r\tau}$$

The Euler equation can be decomposed in two terms, corresponding to the expected gaussian and disaster states:

$$\begin{aligned}
E\left[MX\right] &= (1-p\tau)E^{ND}\left[MX\right] + p\tau E^{D}\left[MX\right], \\
&= (1-p\tau)E^{ND}[M]E^{ND}[X] + (1-p\tau)cov^{ND}(M,X) \\
&+ p\tau E^{D}\left[MX\right], \\
&= 0,
\end{aligned}$$

where superscripts $ND$ and $D$ denote moments conditional on no disasters and disasters respectively. The expected currency excess returns conditional on no disasters is thus:

$$\begin{aligned}
E^{ND}[X] &= \frac{-p\tau E^{D}[MX] - (1-p\tau)cov^{ND}(M,X)}{(1-p\tau)E^{ND}[M]}, \\
&= -\frac{p}{(1-p\tau)}e^{g\tau}\tau E^{D}[MX] - e^{g\tau}cov^{ND}(M,X). \qquad (10)
\end{aligned}$$

The first term in Equation (10) yields:

$$-\frac{p}{(1-p\tau)}\tau e^{g\tau}E^{D}[MX] = \frac{p}{(1-p\tau)}\tau e^{g\tau}\left(E[J]e^{(r-g)\tau} - E[J^*]e^{(r^*-g^*)\tau}\right)$$

which leads to:

$$-\frac{p}{(1-p\tau)}\tau e^{g\tau}E^{D}[MX] = \Pi_D\tau + O(\tau^2)$$

The second term in Equation (10) is given by:

$$-e^{g\tau}cov^{ND}(M,X) = -e^{g\tau}e^{r^*\tau}cov^{ND}(M,\frac{M^*}{M}),$$

The covariance between the home SDF and the ratio of SDFs is:

$$cov^{ND}\Big(M, \frac{M^*}{M}\Big) = E^{ND}M^* - E^{ND}\frac{M^*}{M}E^{ND}M$$
$$= e^{-g^*\tau} - e^{-g^*\tau}e^{\Pi_G\tau}$$
$$= e^{-g^*\tau}\Big(-\Pi_G\tau + O(\tau^2)\Big)$$

The second term in Equation (10) is finally:

$$-e^{g\tau}cov^{ND}(M, X) = e^{(g-g^*+r^*)\tau}\Big(\Pi_G\tau + O(\tau^2)\Big) = \Pi_G\tau + O(\tau^2)$$

Collecting terms of Equation (10), the approximation at the order $\tau$ of the expected return, conditional on no-disaster is given by:

$$X^e = \frac{E^{ND}[X]}{\tau} = \Pi_D + \Pi_G + O(\tau)$$

Substituting in the definition of the Gaussian and the disaster risk exposure gives therefore:

$$X^e = pE(J - J^*) + cov(\epsilon, \epsilon - \epsilon^*) + O(\tau)$$

□

## 4.2 Option prices

In all that follows, options prices and strikes are normalized by the spot. The main results are derived in the case of a put option but can easily be generalized to a call option.

Recall that $\tilde{J} = \frac{p\bar{J}}{1-p\tau}$ and $\tilde{J}^* = \frac{p\bar{J}^*}{1-p\tau}$. We introduce within-month uncertainty about the realization of $J$ and $J^\star$ by considering that home and foreign disaster size within a month are two-value random variable define by:

$$J(\eta) = \bar{J}\cdot(1 + \eta\sigma_J)\,, J^*(\eta^*) = \bar{J}^*\cdot(1 + \eta^*\sigma_{J^*}) \tag{11}$$

where $\eta, \eta^*$ are i.i.d. variables equal to 1 and $-1$ with equal probability. The price of a put option is immediate to derive:

**Proposition 2.** *In a model in which disaster sizes vary according to the expressions in (11) , the put option price is given by*

$$P_{t,t+\tau}(K,\tilde{J},\tilde{J}^{*},\sigma_J,\sigma_{J^*},\sigma_h) = E\Big[P^{ND}_{t,t+\tau}\Big(K,\tilde{J}\cdot(1+\eta\sigma_J)\,,\tilde{J}^{*}\,(1+\eta^{*}\sigma_{J^*})\,,\sigma_h\Big)$$
$$+\,P^{D}_{t,t+\tau}\Big(K,\tilde{J}\cdot(1+\eta\sigma_J)\,,\tilde{J}^{*}\,(1+\eta^{*}\sigma_{J^*})\,,\sigma_h\Big)\Big]$$

*where*

$$P^{ND}(K) = V^{P}_{BS}\Big(\frac{e^{-r^{*}\tau}}{1+\tilde{J}^{*}\tau},K\frac{e^{-r\tau}}{1+\tilde{J}\tau},\sigma_h\sqrt{\tau}\Big)$$
$$P^{D}(K) = \tau V^{P}_{BS}\Big(\frac{e^{-r^{*}\tau}\tilde{J}^{*}}{1+\tilde{J}^{*}\tau},K\frac{e^{-r\tau}\tilde{J}}{1+\tilde{J}\tau},\sigma_h\sqrt{\tau}\Big)$$

*where the strike is $K$, the exchange rate volatility conditional on no disaster is $\sigma_h$, the time to maturity is $\tau$, the home interest rate is $r$, the foreign interest rate is $r^{\star}$, and the expectation is taken over $\eta,\eta^{*}$, which are i.i.d. Bernoulli variables with values in $\{-1,1\}$.*

*Proof.* First let's assume that the disaster sizes $(J,J^{*})$ *are constant* between t and $t+\tau$. Using the model expression for the exchange rate movement, we obtain the following put option price:

$$P(K) = E\Big(KM-M^{*}\Big)^{+}$$

Decomposing the expectation into its non-disaster and disaster components gives:

$$P(K) = (1-p\tau)E^{ND}\Big(KM-M^{*}\Big)^{+} + p\tau E^{D}\Big(KM-M^{*}\Big)^{+}$$

The non-disaster part can be expressed in terms of Black–Scholes formula using Lemma (2):

$$P^{ND}(K) = (1-p\tau)E^{ND}\Big(KM-M^{*}\Big)^{+}$$
$$= (1-p\tau)V^{P}_{BS}\Big(e^{-g^{*}\tau},Ke^{-g\tau},\sigma_h\sqrt{\tau}\Big)$$

Plugging the equation for the interest rate given in equation (9) gives:

$$P^{ND}(K) = V_{BS}^{P}\Big(e^{-r^*\tau}\frac{1-p\tau}{1+p\tau(J^*-1)}, Ke^{-r\tau}\frac{1-p\tau}{1+p\tau(J-1)}, \sigma_h\sqrt{\tau}\Big)$$

Using the definition of $\tilde{J}$ and $\tilde{J}^*$ implies:

$$P^{ND}(K) = V_{BS}^{P}\Big(\frac{e^{-r^*\tau}}{1+\tilde{J}^*\tau}, K\frac{e^{-r\tau}}{1+\tilde{J}\tau}, \sigma_h\sqrt{\tau}\Big)$$

When $J, J^*$ are constant between t and $t+\tau$, the SDFs conditional on a disaster are also log-normally distributed and so we can apply Lemma 2 to compute $P^D(K)$:

$$\begin{aligned}
P^D(K) &= p\tau E^D\Big(KM - M^*\Big)^+ \\
&= \tau V_{BS}^{P}\Big(e^{-g^*\tau}pJ^*, Ke^{-g\tau}pJ, \sigma_h\sqrt{\tau}\Big) \\
&= \tau V_{BS}^{P}\Big(e^{-r^*\tau}\frac{pJ^*}{1+p\tau(J^*-1)}, Ke^{-r\tau}\frac{pJ}{1+p\tau(J-1)}, \sigma_h\sqrt{\tau}\Big) \\
&= \tau V_{BS}^{P}\Big(\frac{e^{-r^*\tau}\tilde{J}^*}{1+\tilde{J}^*\tau}, K\frac{e^{-r\tau}\tilde{J}}{1+\tilde{J}\tau}, \sigma_h\sqrt{\tau}\Big)
\end{aligned}$$

Finally given the expressions for $J, J^*$ given in equations (11), we obtain the expression for the put price by taking expectation over $(\eta, \eta^*)$. We obtain:

$$\begin{aligned}
P_{t,t+\tau}(K, \tilde{J}, \tilde{J}^*, \sigma_J, \sigma_{J^*}, \sigma_h) = E\Big[&P_{t,t+\tau}^{ND}\Big(K, \tilde{J}\cdot(1+\eta\sigma_J), \tilde{J}^*(1+\eta^*\sigma_{J^*}), \sigma_h\Big) \\
&+ P_{t,t+\tau}^{D}\Big(K, \tilde{J}\cdot(1+\eta\sigma_J), \tilde{J}^*(1+\eta^*\sigma_{J^*}), \sigma_h\Big)\Big]
\end{aligned}$$

□

From now on, we keep the assumption that the disaster sizes $(J, J^*)$ *are constant* between t and $t+\tau$. The following lemma identifies some conditions under which we can simplify the non-disaster component of a put price $P^{ND}$. Similar results can be obtained for a call price.

**Lemma 4.** *We assume that the disaster sizes* $(J, J^*)$ are constant *between* $t$ *and* $t+\tau$*. Given a Black–Scholes delta* $\Delta$*, let* $\sigma_\Delta$ *be the option implied volatility and* $K_\Delta$ *be the corresponding strike.*

*Let* $\mathbb{N}()$ *be the cumulative standard normal distribution and let* $n()$ *be the standard normal distribution. We define* $\alpha = -\phi\mathbb{N}^{-1}\big(\phi\Delta e^{r^*\tau}\big)$ *where* $\phi = 1$ *for a call delta and* $\phi = -1$ *for a put delta.*

*In the limit of small time intervals (*$\tau \to 0$*), the non–disaster component of a put price with a strike* $K_\Delta$ *can be approximated by:*

$$
\begin{aligned}
P^{ND}(K_\Delta) \;&=\; \Big(n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_h + \alpha\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\Big)\sqrt{\tau} \\
&+\; \Big(\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(1/2\big(1+\alpha^2\big)\sigma_\Delta^2 - \Pi_D\Big) + 1/2\alpha n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\sigma_h\Big)\tau + O(\tau\sqrt{\tau})
\end{aligned}
$$

*where the exchange rate volatility conditional on no disaster is* $\sigma_h$ *and* $\Pi_D$ *is the disaster exposure.*

*Proof.* Recall that the strike price given a Black-Scholes delta $\Delta$ is given by equation (6) and (7):

$$
K_\Delta = e^{-\phi\mathbb{N}^{-1}\left(\phi e^{r^*\tau}\Delta\right)\sigma_\Delta\sqrt{\tau} + \left(r - r^* + 1/2\sigma_\Delta^2\right)\tau}
$$

where $\phi = 1$ for a call delta and $\phi = -1$ for a put delta. Let's define:

$$
\begin{aligned}
\alpha &= -\phi\mathbb{N}^{-1}(\phi e^{r^*\tau}\Delta) \\
\beta &= \frac{1/2\sigma_\Delta^2 - \Pi_D}{\sigma_h^2} \\
\gamma &= \Big(1/2\big(1+\alpha^2\big)\sigma_\Delta^2 - \Pi_D\Big)
\end{aligned}
$$

Using these notations, the non-disaster component of the option price can be written :

$$
\begin{aligned}
P^{ND}(K_\Delta) &= (1 - p\tau)V^P_{BS}(e^{-g^*\tau}, K_\Delta e^{-g\tau}, \sigma_h\sqrt{\tau}) \\
&= e^{-g^*\tau}(1 - p\tau)\Big(K_\Delta e^{-(g-g^*)\tau}\mathbb{N}(-d_-) - \mathbb{N}(-d_+)\Big)
\end{aligned}
$$

where:

$$
\begin{aligned}
-d_{\pm} &= \frac{\log(K_\Delta) + (g^* - g \mp 1/2\sigma_h^2)\tau}{\sigma_h\sqrt{\tau}} \\
&= \frac{-\phi\mathbb{N}^{-1}\big(\phi e^{r^*\tau}\Delta\big)\sigma_\Delta\sqrt{\tau} + \big(r - r^* + 1/2\sigma_\Delta^2 + g^* - g \mp 1/2\sigma_h^2\big)\tau}{\sigma_h\sqrt{\tau}} \\
&= \alpha\frac{\sigma_\Delta}{\sigma_h} + \Big(\beta \mp 1/2\Big)\sigma_h\sqrt{\tau} + O(\tau\sqrt{\tau})
\end{aligned}
$$

When $\tau$ is small, the Taylor expansion of $\mathbb{N}(-d_\pm)$ gives :

$$
\mathbb{N}(-d_\pm) = \mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big) + n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(\beta \mp 1/2\Big)\sigma_h\sqrt{\tau} + 1/2n'\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(\beta \mp 1/2\Big)^2\sigma_h^2\tau + O(\tau\sqrt{\tau})
$$

Similarly, the Taylor expansion of $K_\Delta e^{-(g-g^*)\tau}$ gives :

$$
\begin{aligned}
K_\Delta e^{-(g-g^*)\tau} &= e^{-\phi\mathbb{N}^{-1}\left(\phi e^{r^*\tau}\Delta\right)\sigma_\Delta\sqrt{\tau} + \left(r - r^* - (g-g^*) + 1/2\sigma_\Delta^2\right)\tau} \\
&= 1 + \alpha\sigma_\Delta\sqrt{\tau} + \gamma\tau + O(\tau^2)
\end{aligned}
$$

Combining the terms, we get:

$$
\begin{aligned}
P^{ND}(K_\Delta) = (1 - g^*\tau - p\tau)\Bigg[&\Big(1 + \alpha\sigma_\Delta\sqrt{\tau} + \gamma\tau\Big). \\
&\Bigg(\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big) + n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)(\beta + 1/2)\sigma_h\sqrt{\tau} + 1/2n'\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(\beta + 1/2\Big)^2\sigma_h^2\tau\Bigg) \\
&- \Bigg(\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big) + n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)(\beta - 1/2)\sigma_h\sqrt{\tau} + 1/2n'\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(\beta - 1/2\Big)^2\sigma_h^2\tau\Bigg)\Bigg] + O(\tau\sqrt{\tau}) \\
&= n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_h\sqrt{\tau} + n'\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\beta\sigma_h^2\tau + \alpha\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\sqrt{\tau} \\
&+ \mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\gamma\tau + \alpha n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)(\beta + 1/2)\sigma_\Delta\sigma_h\tau + O(\tau\sqrt{\tau})
\end{aligned}
$$

By plugging the expression for $\gamma$, and using $n'(x) = -xn(x)$ we get:

$$P^{ND}(K_\Delta) = \Big(n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_h + \alpha\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\Big)\sqrt{\tau}$$
$$+ \Big(\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(1/2(1+\alpha^2)\sigma_\Delta^2 - \Pi_D\Big) + 1/2\alpha n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\sigma_h\Big)\tau + O(\tau\sqrt{\tau})$$

□

The following lemma identifies some conditions under which we can simplify the disaster component of a put price $P^D$. Similar results can be obtained for a call price.

**Lemma 5.** *We assume that the disaster sizes* $(J, J^*)$ are constant *between* $t$ *and* $t+\tau$*. Given a Black–Scholes delta* $\Delta$*, let* $\sigma_\Delta$ *be the option implied volatility and* $K_\Delta$ *be the corresponding strike.*

*In the limit of small time intervals* ($\tau \to 0$)*, the disaster component of a put price with a strike* $K_\Delta$ *can be approximated by:*

$$P^D(K_\Delta) = \begin{cases} (K_\Delta pJ - pJ^*)\tau + O(\tau^2) & \text{if } J > J^*; \\ O(\tau^2) & \text{if } J < J^*. \end{cases}$$

*Proof.* When $J, J^*$ are constant between t and $t+\tau$, we get from proposition (2):

$$P^D(K_\Delta) = K_\Delta pJe^{-g\tau}\mathbb{N}(-d_-)\tau - pJ^*e^{-g^*\tau}\mathbb{N}(-d_+)\tau$$

where:

$$-d_\pm = \frac{\log\big((JK_\Delta)/J^*\big) - (g - g^* \pm \frac{1}{2}\sigma_h^2)\tau}{\sigma_h\sqrt{\tau}}$$
$$= \frac{\log\big(J/J^*\big)}{\sigma_h\sqrt{\tau}} + O(1)$$

We want to approximate the cumulative normal terms $\mathbb{N}(-d_\pm)$ by zero or one with sufficient precision. We apply Chebychev's inequality and derive the following bounds:

$$\begin{cases} 1 - \frac{1}{2d_\pm^2} < \mathbb{N}(-d_\pm) < 1 & \text{if } d_\pm < 0; \\ 0 < \mathbb{N}(-d_\pm) < \frac{1}{2d_\pm^2} & \text{if } d_\pm > 0. \end{cases}$$

The Taylor expansion of $\frac{1}{2d_{\pm}^2}$ gives :

$$\frac{1}{2d_{\pm}^2} = O(\tau)$$

Notice from the Taylor expansion of $-d_{\pm}$ that when $\tau$ is small enough we have:

$$\text{sign}(d_{\pm}) = \text{sign}(J^* - J)$$

By plugging the above approximation for $\mathbb{N}(-d_{\pm})$ into the expression for $P^D(K_\Delta)$, we obtain:

$$P^D(K_\Delta) = \begin{cases} (K_\Delta pJ - pJ^*)\tau + O(\tau^2) & \text{if } J > J^*; \\ O(\tau^2) & \text{if } J < J^*. \end{cases}$$

□

The following lemma identifies some conditions under which we can simplify the implied volatility $\sigma_\Delta$ for a given Black–Scholes delta $\Delta$.

**Lemma 6.** *We assume that the disaster sizes* $(J, J^*)$ are constant *between* $t$ *and* $t+\tau$*. For a given Black–Scholes delta* $\Delta$*, let* $\sigma_\Delta$ *be the corresponding implied implied volatility.*

*In the limit of small time intervals* ($\tau \to 0$)*, the implied volatility can be approximated by:*

$$\sigma_\Delta = \begin{cases} \sigma_h + \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ^* - pJ\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)}\sqrt{\tau} + O(\tau) & \text{if } \Delta > 0; \\ \\ \sigma_h + \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ - pJ^*\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)}\sqrt{\tau} + O(\tau) & \text{if } \Delta < 0. \end{cases}$$

*where* $\mathbb{N}()$ *is the cumulative standard normal distribution and n() the standard normal distribution.*

*Proof.* By definition, the implied volatility $\sigma_\Delta$ verifies:

$$P^{ND}(K_\Delta) + P^D(K_\Delta) = V_{BS}^P\left(e^{-r^*\tau}, K_\Delta e^{-r\tau}, \sigma_\Delta\sqrt{\tau}\right) \tag{12}$$

We guess that the Taylor expansion of the implied volatility with respect to the maturity has the following form :

$$\sigma_\Delta = \sigma_h + A\sqrt{\tau} + O(\tau) \tag{13}$$

and we want to find the value for A. We proceed in 3 steps:

1. Recall that when $\alpha = -\phi \mathbb{N}^{-1}(\phi e^{r^*\tau}\Delta)$, with $\phi = 1$ for a call delta and $\phi = -1$ for a put delta, the approximation for the non-disaster component of the put price is given by lemma (4):

$$\begin{aligned} P^{ND}(K_\Delta) = & \Big(n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_h + \alpha\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\Big)\sqrt{\tau} \\ & + \Big(\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(1/2\big(1+\alpha^2\big)\sigma_\Delta^2 - \Pi_D\Big) + 1/2\alpha n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\sigma_h\Big)\tau + O(\tau\sqrt{\tau}) \end{aligned}$$

Plugging the guess from equation (13) into this expression for $P^{ND}$ gives:

$$\begin{aligned} P^{ND}(K_\Delta) = & \Big(n\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big)\sigma_h + \alpha\mathbb{N}\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big)\big(\sigma_h + A\sqrt{\tau}\big)\Big)\sqrt{\tau} \\ & + \Big(\mathbb{N}\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big)\Big(1/2\big(1+\alpha^2\big)\big(\sigma_h + A\sqrt{\tau}\big)^2 - \Pi_D\Big) \\ & + 1/2\alpha n\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big)\big(\sigma_h + A\sqrt{\tau}\big)\sigma_h\Big)\tau + O(\tau\sqrt{\tau}) \\ = & \Big(n\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big) + \alpha\mathbb{N}\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big)\Big)\sigma_h\sqrt{\tau} \\ & + \Big(\mathbb{N}\big(\alpha\big)\Big(1/2\big(1+\alpha^2\big)\sigma_h^2 - \Pi_D\Big) + 1/2\alpha n\big(\alpha\big)\sigma_h^2 + \alpha\mathbb{N}\big(\alpha\big)A\Big)\tau + O(\tau\sqrt{\tau}) \end{aligned}$$

A Taylor expansion of the standard normal terms gives :

$$
\begin{aligned}
P^{ND}(K_\Delta) = &\Big(n\big(\alpha\big)\sigma_h + n'\big(\alpha\big)\alpha A\sqrt{\tau} + \alpha\mathbb{N}\big(\alpha\big)\sigma_h + \alpha^2 n\big(\alpha\big)A\sqrt{\tau}\Big)\sqrt{\tau} \\
&+ \Bigg(\mathbb{N}\big(\alpha\big)\Big(1/2(1+\alpha^2)\sigma_h^2 - \Pi_D\Big) + 1/2\alpha n\big(\alpha\big)\sigma_h^2 + \alpha\mathbb{N}\big(\alpha\big)A\Bigg)\tau + O(\tau\sqrt{\tau}) \\
\\
= &\Big(n\big(\alpha\big) + \alpha\mathbb{N}\big(\alpha\big)\Big)\sigma_h\sqrt{\tau} \\
&+ \Bigg(\mathbb{N}\big(\alpha\big)\Big(1/2(1+\alpha^2)\sigma_h^2 - \Pi_D\Big) + 1/2\alpha n\big(\alpha\big)\sigma_h^2 + \alpha\mathbb{N}\big(\alpha\big)A\Bigg)\tau + O(\tau\sqrt{\tau})
\end{aligned}
$$

2. Now, let's write the non-disaster component of the put price $P^{ND}(K_\Delta)$ as:

$$
P^{ND}(K_\Delta) = P^{ND}(K_\Delta, p, g, g^*, \sigma_h)
$$

So $P^{ND}(K_\Delta)$ is formally equivalent to the right–hand side of equation (12) when $p = 0$, $g = r$, $g^* = r^*$ and $\sigma_h = \sigma_\Delta$:

$$
P^{ND}(K_\Delta, p = 0, g = r, g^* = r^*, \sigma_h = \sigma_\Delta) = V_{BS}^P\Big(e^{-r^*\tau}, K_\Delta e^{-r\tau}, \sigma_\Delta\sqrt{\tau}\Big)
$$

So we can apply lemma (4) to compute the Taylor expansion for the right–hand side of equation (12):

$$
\begin{aligned}
V_{BS}^P\Big(e^{-r^*\tau}, K_\Delta e^{-r\tau}, \sigma_\Delta\sqrt{\tau}\Big) = &\Big(n\big(\alpha\big) + \alpha\mathbb{N}\big(\alpha\big)\Big)\sigma_\Delta\sqrt{\tau} \\
&+ 1/2\Big(\mathbb{N}\big(\alpha\big)(1+\alpha^2) + \alpha n\big(\alpha\big)\Big)\sigma_\Delta^2\tau + O(\tau\sqrt{\tau})
\end{aligned}
$$

Plugging the expression for $\sigma_\Delta$ in equation (13) gives :

$$
\begin{aligned}
V_{BS}^{P}\Big(e^{-r^*\tau}, K_\Delta e^{-r\tau}, \sigma_\Delta\sqrt{\tau}\Big) = &\Big(n\Big(\alpha\Big) + \alpha\mathbb{N}\Big(\alpha\Big)\Big)\sigma_h\sqrt{\tau} \\
&+ \Big(n\Big(\alpha\Big) + \alpha\mathbb{N}\Big(\alpha\Big)\Big)A\tau \\
&+ 1/2\Big(\mathbb{N}\Big(\alpha\Big)(1+\alpha^2) + \alpha n\Big(\alpha\Big)\Big)\sigma_h^2\tau + O(\tau\sqrt{\tau})
\end{aligned}
$$

3. Finally, when $\tau$ is small enough we can simplify the disaster component of the put price using lemma (5):

$$
P^D(K_\Delta) = \big(pJ - pJ^*\big)^+\tau + O(\tau\sqrt{\tau})
$$

Plugging these results into equation (12) gives:

$$
-\mathbb{N}\big(\alpha\big)\big(pJ - pJ^*\big) + \big(pJ - pJ^*\big)^+ = n\big(\alpha\big)A
$$

So the unknown coefficient A is given by:

$$
A = \frac{\big(pJ - pJ^*\big)^+ - \mathbb{N}\big(\alpha\big)\big(pJ - pJ^*\big)}{n\big(\alpha\big)}
$$

Recall the expression for $\alpha$:

$$
\begin{aligned}
\alpha &= -\phi\mathbb{N}^{-1}(\phi e^{r^*\tau}\Delta) \\
&= -\phi\mathbb{N}^{-1}(\phi\Delta) + O(\tau)
\end{aligned}
$$

So we can simplify the expression for A depending on the type of option delta ($\phi = 1$ for a call delta and - 1 for a put delta):

$$A = \begin{cases} \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ^* - pJ\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)} + O(\tau) & \text{if } \phi = 1; \\ \\ \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ - pJ^*\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)} + O(\tau) & \text{if } \phi = -1. \end{cases}$$

Recall that by construction a call delta is positive and a put delta is negative. So by plugging the value of A into equation (13) gives the expression for the implied volatility:

$$\sigma_\Delta = \begin{cases} \sigma_h + \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ^* - pJ\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)} \sqrt{\tau} + O(\tau) & \text{if } \Delta > 0; \\ \\ \sigma_h + \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ - pJ^*\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)} \sqrt{\tau} + O(\tau) & \text{if } \Delta < 0. \end{cases}$$

□

### 4.3 Hedged currency excess returns

In all that follows, we assume that the foreign currency is the investment currency.

Let $\Delta^P$ be a Black–Scholes put delta, i.e. $\Delta^P < 0$ and let $K_{\Delta^P}$ be the corresponding strike. The return $X(K_{\Delta^P})$ to the hedged carry trade is the payoff of the following zero-investment trade: borrow one unit of the home currency at interest rate $r$ ; use the proceeds to buy $\lambda^P(K_{\Delta^P})$ puts with strike $K_{\Delta^P}$, protecting against a depreciation in the foreign currency below $K_{\Delta^P}$; and invest the remainder $\left(1 - \lambda^P(K_{\Delta^P})P(K_{\Delta^P})\right)$ in the foreign currency at interest rate $r^\star$. So the hedged return is given by:

$$X(K_{\Delta^P}) = \left(1 - \lambda^P(K_{\Delta^P})P(K_{\Delta^P})\right)e^{r^\star\tau}\frac{S_{t+\tau}}{S_t} + \lambda^P\left(K_{\Delta^P}\right)\left(K_{\Delta^P} - \frac{S_{t+\tau}}{S_t}\right)^+ - e^{r\tau},$$

where the hedge ratio $\lambda^P(K_{\Delta^P})$ is given by:

$$\lambda^P(K_{\Delta^P}) = \frac{e^{r^*\tau}}{1 + e^{r^*\tau}P(K_{\Delta^P})}$$

To summarize the notation: $X$ denotes the carry trade return and $X^e$ is its annualized expected value conditional on no disaster; $X(K_{\Delta^P})$ denotes the hedged carry trade return with strike $K_{\Delta^P}$; $P\left(K_{\Delta^P}\right)$ is the home currency price of a put yielding $\left(K_{\Delta^P} - S_{t+\tau}/S_t\right)^+$ in the home currency;

$X^e(K_{\Delta^P})$ is the annualized expected value of the hedged carry trade return conditional on no disaster and $E^{ND}$ denotes expectations under the assumption of no disaster:

$$X^e(K_{\Delta^P}) = \frac{E^{ND} X\left(K_{\Delta^P}\right)}{\tau}.$$

The following proposition offers a closed form formula for the hedged returns.

**Proposition 3.** *We assume that the disaster sizes* $(J, J^*)$ are constant *between* $t$ *and* $t+\tau$ *with* $J > J^*$. *Let* $\Delta^P$ *be a Black-Scholes put delta i.e.* $\Delta^P < 0$, *and let* $K_{\Delta^P}$ *be the corresponding strike. We define:*

$$\begin{aligned}
\beta &= n\big(\mathbb{N}^{-1}(-\Delta^P)\big) - \mathbb{N}^{-1}(-\Delta^P)(1+\Delta^P) \\
\gamma &= \big(1+\Delta^P\big)\Delta^P \mathbb{N}^{-1}(-\Delta^P) - \big(2+\Delta^P\big) n\big(\mathbb{N}^{-1}(-\Delta^P)\big)
\end{aligned}$$

*where* $\mathbb{N}()$ *is the cumulative standard normal distribution and* $n()$ *the standard normal distribution.*

*In the limit of small time intervals* ($\tau \to 0$)*, the hedged carry trade expected return (conditional on no disasters) can be approximated by:*

$$X^e(K_{\Delta^P}) = \big(1+\Delta^P\big)\Pi_G + \left(\beta\left(pJ + \frac{\Pi_D \Pi_G}{\sigma_h^2}\right) + \gamma \Pi_G\right)\sigma_h\sqrt{\tau} + O(\tau)$$

*where* $\Pi_G$ *is the Gaussian exposure,* $\sigma_h$ *is the exchange rate volatility conditional on no disaster and* $\Pi_D$ *is the disaster exposure.*

*Proof.* Let's write the hedged return as a function of the unhedged return X:

$$X(K_{\Delta^P}) = X - \lambda^P(K_{\Delta^P})P(K_{\Delta^P})e^{r^*\tau}\frac{M^*}{M} + \lambda^P(K_{\Delta^P})\left(K_{\Delta^P} - \frac{M^*}{M}\right)^+, \tag{14}$$

Like in proposition (1), we can write the Euler equation for the hedged return as the sum of a disaster component $\Pi_{HD}$ and a non–disaster component $\Pi_{HG}$:

$$\frac{E^{ND} X(K_{\Delta^P})}{\tau} = \Pi_{HD} + \Pi_{HG}$$

where:

$$\Pi_{HD} = -\frac{pe^{g\tau}}{1-p\tau} E^D MX(K_{\Delta^P})$$

and:

$$\Pi_{HG} = -\frac{e^{g\tau}}{\tau} cov^{ND}\Big(M, X(K_{\Delta^P})\Big)$$

We then proceed in 3 steps:

1. First let's replace the expression for the hedged return given by equation (14) in $\Pi_{HD}$.

   From proposition (1), the first term in $\Pi_{HD}$ can be approximated by:

$$-\frac{pe^{g\tau}}{1-p\tau} E^D MX = (pJ - pJ^*) + O(\tau)$$

   Recall from lemma (4) and (5) that $P(K_{\Delta^P})$ is of order $O(\sqrt{\tau})$. So the second term in $\Pi_{HD}$ can be approximated by:

$$\frac{pe^{g\tau}}{1-p\tau} \lambda^P(K_{\Delta^P}) P(K_{\Delta^P}) e^{r^*\tau} E^D M^* = \lambda^P(K_{\Delta^P}) P(K_{\Delta^P}) pJ^* + O(\tau)$$

   Recall that $K_{\Delta^P}$ is of order $O(\sqrt{\tau})$. So when $J > J^*$ and $\tau$ is small enough the third term in $\Pi_{HD}$ can be approximated using lemma (5) by:

$$-\frac{pe^{g\tau}}{1-p\tau} \lambda^P(K_{\Delta^P}) E^D \big(K_{\Delta^P} M - M^*\big)^+ = -\lambda^P(K_{\Delta^P}) \big(K_{\Delta^P} pJ - pJ^*\big) + O(\tau)$$

   Summing up the components of $\Pi_{HD}$, we get:

$$\begin{aligned} \Pi_{HD} &= (pJ - pJ^*) + \lambda^P(K_{\Delta^P}) P(K_{\Delta^P}) pJ^* - \lambda^P(K_{\Delta^P}) \big(K_{\Delta^P} pJ - pJ^*\big), \\ &= pJ\Big(1 - K_{\Delta^P} \lambda^P(K_{\Delta^P})\Big) - pJ^*\Big(1 - \lambda^P(K_{\Delta^P})\big(1 + P(K_{\Delta^P})\big)\Big) + O(\tau) \end{aligned}$$

   The hedge ratio can be approximated by:

$$\lambda^P(K_{\Delta^P}) = \frac{1}{1 + P(K_{\Delta^P})} + O(\tau)$$

So the expression for $\Pi_{HD}$ can be simplified to:

$$\Pi_{HD} = \Big(1 + P(K_{\Delta^P}) - K_{\Delta^P}\Big).pJ + O(\tau)$$

Let's use the notation: $\alpha = \mathbb{N}^{-1}(-\Delta^P)$. Lemma (4) and (5) give:

$$P(K_{\Delta^P}) = \Big(n(\alpha) + \alpha\mathbb{N}(\alpha)\Big)\sigma_h\sqrt{\tau} + O(\tau)$$

Recall that given a Black-Scholes put delta $\Delta^P$ the strike $K_{\Delta^P}$ is given by equation (7):

$$K_{\Delta^P} = e^{\mathbb{N}^{-1}\left(-e^{r^*\tau}\Delta^P\right)\sigma_{\Delta^P}\sqrt{\tau}+\left(r-r^*+1/2\sigma^2_{\Delta^P}\right)\tau}$$

which can be simplified to:

$$K_{\Delta^P} = 1 + \alpha\sigma_h\sqrt{\tau} + O(\tau)$$

Let's define $\beta = n\big(\mathbb{N}^{-1}(-\Delta^P)\big) - \mathbb{N}^{-1}(-\Delta^P)\big(1 + \Delta^P\big)$. We can then $\Pi_{HD}$:

$$\begin{aligned}\Pi_{HD} &= \Big(n(\alpha) + \alpha\mathbb{N}(\alpha) - \alpha\Big).pJ\sigma_h\sqrt{\tau} + O(\tau)\\ &= \beta.pJ.\sigma_h.\sqrt{\tau} + O(\tau)\end{aligned}$$

2. Now let's replace the expression for the hedged return given by equation (14) in $\Pi_{HG}$.

   From proposition (1) the first term in $\Pi_{HG}$ can be approximated by:

$$-\frac{e^{g\tau}}{\tau}cov^{ND}(M, X) = \Pi_G + O(\tau)$$

   In proposition (1) we derived:

$$cov^{ND}\Big(M, \frac{M^*}{M}\Big) = -\Pi_G\tau + O(\tau^2)$$

So the second term in $\Pi_{HG}$ can be approximated by:

$$\frac{e^{g\tau}}{\tau}\lambda^P(K_{\Delta^P})P(K_{\Delta^P})e^{r^*\tau}cov^{ND}\left(M,\frac{M^*}{M}\right)=-\lambda^P(K_{\Delta^P})P(K_{\Delta^P})\Pi_G+O(\tau)$$

Recall that the ratio of the SDFs is given by:

$$E^{ND}\frac{M^*}{M}=e^{\left(g-g^*+\Pi_G\right)\tau}$$

So using lemma 3 we have:

$$cov^{ND}\left(M,\left(K_{\Delta^P}-\frac{M^*}{M}\right)^+\right)=V^P_{BS}\left(e^{-g^*\tau},K_{\Delta^P}e^{-g\tau},\sigma_h\sqrt{\tau}\right)-V^P_{BS}\left(e^{(-g^*+\Pi_G)\tau}K_{\Delta^P}e^{-g\tau},\sigma_h\sqrt{\tau}\right)$$

Recall that a put delta is the first derivative of the put price with respect to the spot. So the third term in $\Pi_{HG}$ can be approximated by:

$$\frac{-e^{g\tau}\lambda^P(K_{\Delta^P})}{\tau}cov^{ND}\left(M,\left(K_{\Delta^P}-\frac{M^*}{M}\right)^+\right)=\ \lambda^P(K_{\Delta^P})\Delta^P_{BS}\left(e^{(g-g^*)\tau},K_{\Delta^P},\sigma_h\sqrt{\tau}\right)\Pi_G+O(\tau)$$

Summing up the components of $\Pi_{HG}$ gives:

$$\Pi_{HG}=\left(1+\lambda^P(K_{\Delta^P})\left(\Delta^P_{BS}\left(e^{(g-g^*)\tau},K_{\Delta^P},\sigma_h\sqrt{\tau}\right)-P(K_{\Delta^P})\right)\right)\Pi_G+O(\tau)$$

which can be simplified using the approximation for $\lambda^P(K_{\Delta^P})$:

$$\Pi_{HG}=\left(1+\Delta^P_{BS}\left(e^{(g-g^*)\tau},K_{\Delta^P},\sigma_h\sqrt{\tau}\right)\right).\left(1-P(K_{\Delta^P})\right)\Pi_G+O(\tau)$$

Recall the expression for a Black–Scholes put delta:

$$\Delta^P_{BS}\big(e^{(g-g^*)\tau}, K_{\Delta^P}, \sigma_h\sqrt{\tau}\big) = -\mathbb{N}\big(-d_-\big)$$

where:

$$-d_- = \frac{\log(K_{\Delta^P}) + (g^* - g + 1/2\sigma_h^2)\tau}{\sigma_h\sqrt{\tau}}$$

and the strike $K_{\Delta^P}$ is given by equation (7):

$$K_{\Delta^P} = e^{\mathbb{N}^{-1}\left(-e^{r^*\tau}\Delta^P\right)\sigma_{\Delta^P}\sqrt{\tau}+\left(r-r^*+1/2\sigma^2_{\Delta^P}\right)\tau}$$

When $J > J^*$, the implied volatility $\sigma_{\Delta^P}$ given by lemma (6) is:

$$\sigma_{\Delta^P} = \sigma_h + \frac{\big(1+\Delta^P\big)\Pi_D}{n\big(\mathbb{N}^{-1}(-\Delta^P)\big)}\sqrt{\tau} + O(\tau)$$

So when $\tau$ is small, the Taylor expansion of $-d_-$ is:

$$\begin{aligned} -d_- &= \frac{\mathbb{N}^{-1}\big(-e^{r^*\tau}\Delta\big)\sigma_\Delta\sqrt{\tau} + \big(r - r^* + 1/2\sigma_\Delta^2 + g^* - g + 1/2\sigma_h^2\big)\tau}{\sigma_h\sqrt{\tau}} \\ &= \alpha + \left(\sigma_h - \frac{\Pi_D}{n(\alpha)\sigma_h}\beta\right)\sqrt{\tau} + O(\tau) \end{aligned}$$

where $\alpha = \mathbb{N}^{-1}(-\Delta^P)$ and $\beta = n\big(\mathbb{N}^{-1}(-\Delta^P)\big) - \mathbb{N}^{-1}(-\Delta^P)(1+\Delta^P)$. So the Taylor expansion of $-\mathbb{N}()$ around $\alpha$ gives :

$$\Delta^P_{BS}\big(e^{(g-g^*)\tau}, K_{\Delta^P}, \sigma_h\sqrt{\tau}\big) = \Delta^P - \left(n(\alpha)\sigma_h - \beta\frac{\Pi_D}{\sigma_h}\right)\sqrt{\tau} + O(\tau)$$

Plugging this expression and the approximation for $P(K_\Delta)$ given by lemma (4) and (5) back into $\Pi_{HG}$ gives :

$$\Pi_{HG} = \left(\left(1+\Delta^P\right) - \left(n(\alpha)\sigma_h - \beta\frac{\Pi_D}{\sigma_h}\right)\sqrt{\tau}\right).\left(1 - \left(n(\alpha) + \alpha\mathbb{N}(\alpha)\right)\sigma_h\sqrt{\tau}\right)\Pi_G + O(\tau)$$

$$= \left(1+\Delta^P\right)\Pi_G - \left(n(\alpha)\sigma_h - \beta\frac{\Pi_D}{\sigma_h} + (1+\Delta^P)\left(n(\alpha) - \Delta^P\alpha\right)\sigma_h\right)\Pi_G\sqrt{\tau} + O(\tau)$$

$$= (1+\Delta^P)\Pi_G + \left(\beta\frac{\Pi_D}{\sigma_h} + \gamma\sigma_h\right)\Pi_G\sqrt{\tau} + O(\tau)$$

where $\gamma = \left(1+\Delta^P\right)\Delta^P\mathbb{N}^{-1}(-\Delta^P) - \left(2+\Delta^P\right)n\left(\mathbb{N}^{-1}(-\Delta^P)\right)$.

3. Finally, we combine the terms to compute $X^e\left(K_{\Delta^P}\right)$:

$$X^e(K_{\Delta^P}) = \left(1+\Delta^P\right)\Pi_G + \left(\beta\left(pJ + \frac{\Pi_D\Pi_G}{\sigma_h^2}\right) + \gamma\Pi_G\right)\sigma_h\sqrt{\tau} + O(\tau)$$

□

## 4.4 Risk-reversals

Given a $\Delta > 0$ we can consider the corresponding Black–Scholes put delta : $\Delta^P = -\Delta$ and the Black–Scholes call delta $\Delta^C = \Delta$. A risk-reversal is defined as the difference between the implied volatility at the Black–Scholes put delta and the implied volatility at the Black–Scholes call delta:

$$RR_\Delta = \sigma_{-\Delta} - \sigma_\Delta$$

**Proposition 4.** *Given $\Delta > 0$, when there is no disaster risk:*

$$RR_\Delta = \sigma_{-\Delta} - \sigma_\Delta = 0$$

*Proof.* The result follows by taking $p = 0$ in proposition (2):

$$\sigma_\Delta = \sigma_{-\Delta} = \sigma_h$$

So:

$$RR_\Delta = 0$$

$\square$

In the presence of disaster risk the following proposition identifies conditions under which we can simplify the expression for the risk–reversal.

**Proposition 5.** *We assume that the disaster sizes* $(J, J^*)$ are constant *between* $t$ *and* $t+\tau$*. Given a Black–Scholes delta* $\Delta > 0$*, the risk–reversal* $\sigma_{-\Delta} - \sigma_\Delta$ *can be approximated in the limit of small time intervals (*$\tau \to 0$*) by:*

$$RR_\Delta = \sigma_\Delta - \sigma_{-\Delta} = \frac{1-2\Delta}{n\big(\mathbb{N}^{-1}(\Delta)\big)}\Pi_D\sqrt{\tau} + O(\tau)$$

*where* $\mathbb{N}()$ *is the cumulative standard normal distribution, n() the standard normal distribution and* $\Pi_D$ *is the disaster risk exposure.*

*Proof.* From lemma (6), we have:

$$\sigma_\Delta = \begin{cases} \sigma_h + \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ^* - pJ\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)}\sqrt{\tau} + O(\tau) & \text{if } \Delta > 0; \\ \\ \sigma_h + \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ - pJ^*\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)}\sqrt{\tau} + O(\tau) & \text{if } \Delta < 0. \end{cases}$$

Notice that $\left(pJ - pJ^*\right)^+ - \left(pJ^* - pJ\right)^+ = \left(pJ - pJ^*\right)$. So given $\Delta > 0$, we obtain:

$$\begin{aligned} RR_\Delta &= \sigma_{-\Delta} - \sigma_\Delta \\ &= \frac{-\Delta\left(pJ - pJ^*\right) + \left(pJ - pJ^*\right)^+}{n\left(\mathbb{N}^{-1}(\Delta)\right)}\sqrt{\tau} - \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ^* - pJ\right)^+}{n\left(\mathbb{N}^{-1}(\Delta)\right)}\sqrt{\tau} + O(\tau) \\ &= \frac{1-2\Delta}{n\left(\mathbb{N}^{-1}(\Delta)\right)}\Pi_D\sqrt{\tau} + O(\tau) \end{aligned}$$

$\square$

# 5 Simulation

Propositions (1), (3) and (5) are derived in the limit of small time intervals. We check their validity for one-day and one-month horizons by simulating a calibrated version of the model.

Table 1 reports the parameter values used in the calibration, while Table 2 reports simulation results.

[Table 1 about here.]

[Table 2 about here.]

We verify that the higher order term in Proposition 3 remains positive given reasonable values for the parameter estimates. Notice that the higher order term is the sum of two positive terms $\left(pJ + \frac{\pi^D \pi^G}{\sigma_h^2}\right)$ and $\pi^G$ which are multiplied by 2 coefficients $\beta$ and $\gamma$ respectively. In our simulation $\left(pJ + \frac{\pi^D \pi^G}{\sigma_h^2}\right)$ is equal to 34% and $\pi^G$ is equal to 3% and in each of the subsample estimations we considered $pJ$ is roughly one order of magnitude larger than $\pi^G$. At 10 delta, $\beta$ is equal to 1.3 and $\gamma$ is equal to -0.2. At 25 delta, $\beta$ is equal to 0.8 and $\gamma$ is equal to -0.4. At 50 delta, $\beta$ is equal to 0.4 and $\gamma$ is equal to -0.6. These results supports the claim that the higher order term is positive given a large range of values for the parameters of the model.

# 6 Data and Additional Estimation Results

## 6.1 Data

Our dataset contains spot exchange rates, one-month forward exchange rates and the U.S. LIBOR interest rate obtained from Datastream, as well as one-month implied volatilities obtained from JP Morgan, for the period 1/1996 to 08/2014. For each country, the spot and forward exchange rates are expressed in U.S. dollars per unit of foreign currency. The foreign interest rates are computed by using covered interest rate parity :

$$F = Se^{(r_{us} - r^*)\tau}.$$

Using the JP Morgan volatility datasets, we express all the implied volatilities as options on spot exchange rates for which the foreign currency is the base currency and the dollar is the quote currency[1]. The convention in the forex market is to quote these implied volatilities by using the value of a Black and Scholes (1973) delta. With one-month maturity options (or any option with maturity less than one year), if the country is a G7 country the convention is to use a Black–Scholes spot delta. For instance, we call $\sigma_{10C}$ ($\sigma_{10P}$) is the implied volatility at 10 delta call (put). It is the implied volatility given that the option Black–Scholes spot delta is equal to 0.1 (-0.1). We can then retrieve the strikes $K_{10C}$ and $K_{10P}$ by using equation (6) and (7).

When the country is not a G7 country, then the spot delta is replaced by the forward delta in the calculation of $K_{10C}$ and $K_{10P}$:

$$\Delta_{BS,F}(\phi) = \phi \mathbb{N}(\phi d_+),$$

where $\phi = 1$ for a delta call and $\phi = -1$ for a delta put.

The at-the-money strike is a special case. With one-month maturity options (or any option with maturity less than 10 years), if the country is a G7 country except Japan, the at-the-money strike is the strike which cancels the spot delta of a straddle (i.e., the sum of a put and a call):

$$\Delta_{BS}^{C} + \Delta_{BS}^{P} = 0$$

This implies:

$$K_{atm} = S e^{(r_{us} - r^* + \frac{1}{2}\sigma_{\text{atm}}^2)\tau}.$$

## 6.2 Pricing Errors

Figure 1 compares the distribution of pricing errors and the distribution of bid ask spreads.

[Figure 1 about here.]

Figure 2 compares the average implied volatilities at different strike in the model and in the data at the country level.

[Figure 2 about here.]

[1]Prior to 2012 the JP Morgan volatility dataset was referring to currency pairs for which the quote currency was the US dollar. Since 2012 they switched to the market convention where the only the Australian Dollar, the Euro, the British Pound and the New-Zealand Dollar were quoted using the US Dollar as the quote currency. For more details on the quotation of currency options, see Wystup (2007).

## 6.3 Additional Estimates

Table (3) is similar to table 2 in the main paper, except that the sample period excludes fall 2008.

[Table 3 about here.]

Figure 3 reports the time-series estimates for the disaster risk exposure at the country level.

[Figure 3 about here.]

Finally, Figure 4 reports similar findings as in Figure 2 in the text. In Figure 4, however, the average estimated disaster risk exposure is estimated over the period leading to the crisis (from May 2008 to August 2008) and compared to the cumulative percentage change in exchange rate for each country during the crisis (from September 2008 to January 2009), while in Figure 2 in the text, both disaster risk and changes in exchange rates are estimated during the crisis.

[Figure 4 about here.]

## 6.4 Asset Pricing

Table 4 reports asset pricing results obtained with two risk factors: the average excess returns of a U.S. investor on currency markets (denoted $RX$) and the risk-reversals at 25-delta on S&P500 index options (denoted $RR$). The test assets are the six portfolios of Lustig, Roussanov and Verdelhan (2011).

[Table 4 about here.]

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Table 1: Simulation Parameters

| Parameter | Symbol | Value |
|---|---|---|
| Disaster probability | $p$ | 3.60% |
| Disaster size (domestic) | $J$ | 7.50 |
| Disaster size (foreign) | $J^{\star}$ | 6.67 |
| SDF drift (domestic) | $g$ | 26.17% |
| SDF drift (foreign) | $g^{\star}$ | 26.23% |
| Volatility of gaussian shocks (domestic) | $\sigma$ | 82.94% |
| Volatility of gaussian shocks (foreign) | $\sigma^{\star}$ | 80.00% |
| Correlation of gaussian shocks | $\rho$ | 99.15% |

*Notes: This table shows the parameters used in the simulation. The disaster probability is taken from Barro (2006). The domestic and foreign disaster sizes ($J$ and $J^{\star}$) as well as the domestic and foreign drifts ($g$ and $g^{\star}$) of the pricing kernel come from the estimation results for the high interest currency portfolio during the period 1/1996–12/2011 excluding fall 2008. In this estimation $J$ and $J^{\star}$ are assumed to be constant within each month (i.e. $\sigma_J = \sigma_J^{*} = 0$). The domestic and foreign volatility ($\sigma$ and $\sigma^{\star}$) of the Gaussian shocks, as well as their correlation ($\rho$), are calibrated to match a Gaussian risk exposure of 3% and a volatility of the bilateral exchange rate equal to 10% to match their counterparts on the high interest currency portfolio, as well as a maximum Sharpe Ratio equal to 80%.*

Table 2: Simulation Results

| | One-Month Horizon | | One-Day Horizon | |
|---|---|---|---|---|
| | Model | Approximation | Model | Approximation |
| | | Excess Returns | | |
| Unhedged Returns | 6.20 | 6.00 | 6.02 | 6.00 |
| Hedged Returns at 10 delta | 4.27 | | 3.06 | |
| Order 0 | | 2.70 | | 2.70 |
| Order $\sqrt{\tau}$ | | 4.13 | | 2.96 |
| Hedged Returns at 25 delta | 3.22 | | 2.53 | |
| Order 0 | | 2.25 | | 2.25 |
| Order $\sqrt{\tau}$ | | 3.11 | | 2.41 |
| Hedged Returns at-the-money | 1.99 | | 1.66 | |
| Order 0 | | 1.50 | | 1.50 |
| Order $\sqrt{\tau}$ | | 1.88 | | 1.57 |
| | | Risk Reversals | | |
| Risk-Reversals at 10 delta | 2.39 | 3.95 | 0.58 | 0.72 |
| Risk-Reversals at 25 delta | 0.88 | 1.36 | 0.23 | 0.25 |

*Notes: This table compares the simulation results obtained by running a Monte-Carlo simulation on the model quantities of interest for a one-month and a one-day horizon to the closed form formula that derived in the paper. This simulation uses the parameters described in Table 1. The results are expressed in percentage points. The excess returns are annualized (multiplied by 12). The risk reversal is computed as the difference between the volatility of an out-of-the-money put and an out-of-the-money call.*

Table 3: Exchanges Rate Changes, Risk-Reversals, and Currency Excess Returns

| Portfolios | 1 | 2 | 3 |
|---|---|---|---|
| | | Panel I: Exchange Rates | |
| Mean | 0.39 | 1.32 | 2.45 |
| | [1.63] | [1.72] | [2.05] |
| | | Panel II: Interest Rates | |
| Mean | −1.91 | 0.14 | 2.31 |
| | [0.39] | [0.30] | [0.30] |
| | | Panel III: Risk-Reversals 10 Delta | |
| Mean | −0.39 | 0.67 | 1.30 |
| | [0.25] | [0.20] | [0.28] |
| | | Panel IV: Risk-Reversals 25 Delta | |
| Mean | −0.19 | 0.38 | 0.71 |
| | [0.14] | [0.12] | [0.15] |
| | | Panel V: Excess Returns | |
| Mean | −1.03 | 1.91 | 5.38 |
| | [1.80] | [1.85] | [2.24] |
| Sharpe Ratio | −0.13 | 0.24 | 0.55 |

*Notes:* This table reports portfolio average changes in exchange rates, interest rates, risk-reversals, as well as average currency excess returns. Countries are sorted by the level of foreign interest rates and allocated into three portfolios, which are rebalanced every month. The first portfolio contains the lowest interest rate currencies while the last portfolio contains the highest interest rate currencies. The table reports the mean excess return and its standard error, along with the corresponding Sharpe ratio for excess returns. The mean and standard deviations for the exchange rates, the interest rates, and the excess returns are annualized (multiplied respectively by 12 and $\sqrt{12}$). The Sharpe ratio corresponds to the ratio of the annualized mean to the annualized standard deviation. The standard errors, reported between brackets, are obtained by bootstrapping both the time-series using a block bootstrap and the cross-section of countries. The block sizes are 10 months. Data are monthly, from J.P. Morgan. The sample period is January 1996 to August 2014 excluding the fall of 2008.

Table 4: Asset Pricing with Risk Reversals

| Panel I: Risk Prices | | | | | | | |
|---|---|---|---|---|---|---|---|
| | $\lambda_{RX}$ | $\lambda_{RR}$ | $b_{RX}$ | $b_{RR}$ | $R^2$ | $RMSE$ | $\chi^2$ |
| $GMM_1$ | 1.03 | −4.66 | −1.16 | −60.09 | 90.80 | 0.75 | |
| | [5.23] | [2.96] | [0.86] | [37.38] | | | 85.85 |
| $GMM_2$ | 2.20 | −4.76 | −0.95 | −60.97 | 67.82 | 1.40 | |
| | [3.85] | [2.48] | [0.77] | [31.62] | | | 87.62 |
| $FMB$ | 1.03 | −4.66 | −1.15 | −59.77 | 90.83 | 0.75 | |
| | [1.63] | [1.18] | [0.47] | [15.23] | | | 46.83 |
| | (1.64) | (2.27) | (0.73) | (29.28) | | | 91.82 |

| Panel II: Factor Betas | | | | |
|---|---|---|---|---|
| $Portf.$ | $\alpha_0^j$ | $\beta_{RX}^j$ | $\beta_{RR}^j$ | $R^2$ |
| 1 | −8.74 | 0.91 | 0.87 | 66.76 |
| | [2.24] | [0.07] | [0.38] | |
| 2 | −3.04 | 0.86 | 0.14 | 69.88 |
| | [2.24] | [0.06] | [0.34] | |
| 3 | −1.78 | 0.93 | 0.18 | 79.60 |
| | [1.90] | [0.05] | [0.31] | |
| 4 | 0.99 | 0.94 | −0.08 | 78.75 |
| | [1.94] | [0.05] | [0.30] | |
| 5 | 2.48 | 1.12 | −0.15 | 77.56 |
| | [2.95] | [0.07] | [0.47] | |
| 6 | 10.09 | 1.24 | −0.96 | 68.01 |
| | [3.45] | [0.08] | [0.59] | |

*Notes:* Panel I reports results from GMM and Fama-McBeth asset pricing procedures. The market prices of risk $\lambda$, the adjusted $R^2$, the square-root of mean-squared errors $RMSE$ and the $p$-values of $\chi^2$ tests on pricing errors are reported in percentage points. The log pricing kernel is here: $m_{t+1}^{US} = 1 - b_{RX}RX_{t+1} - b_{RR}RR_{t+1}$, where $b$ denotes the vector of factor loadings. Excess returns used as test assets and risk factors do not take into account bid-ask spreads. All excess returns are multiplied by 12 (annualized). Shanken (1992)-corrected standard errors are reported in parentheses. We do not include a constant in the second step of the FMB procedure. Panel II reports OLS estimates of the factor betas. $R^2$s and $p$-values are reported in percentage points. The alphas are annualized and in percentage points. The standard errors in brackets are Newey and West (1987) standard errors computed with the optimal number of lags according to Andrews (1991). Note that risk reversals are not excess returns. As a result, constants in the time-series regressions reported in the second panel do not have to be zero. (Each constant $\alpha_0^j$ is equal to $\beta^j(\lambda - E(f)$, where $\lambda$ denotes the vector of risk prices and $E(f)$ the mean of the risk factors. When the factor is an excess return, then the Euler equation implies that $\alpha_0^j = 0$). The test assets are the six currency portfolios of Lustig et al. (2011). Countries are sorted on the basis of their interest rates. The first portfolio corresponds to low interest rate countries while the last portfolio corresponds to high interest rate countries. Data are monthly, from the Datastream and CRSP databases. The sample period is 2/1996–12/2011.

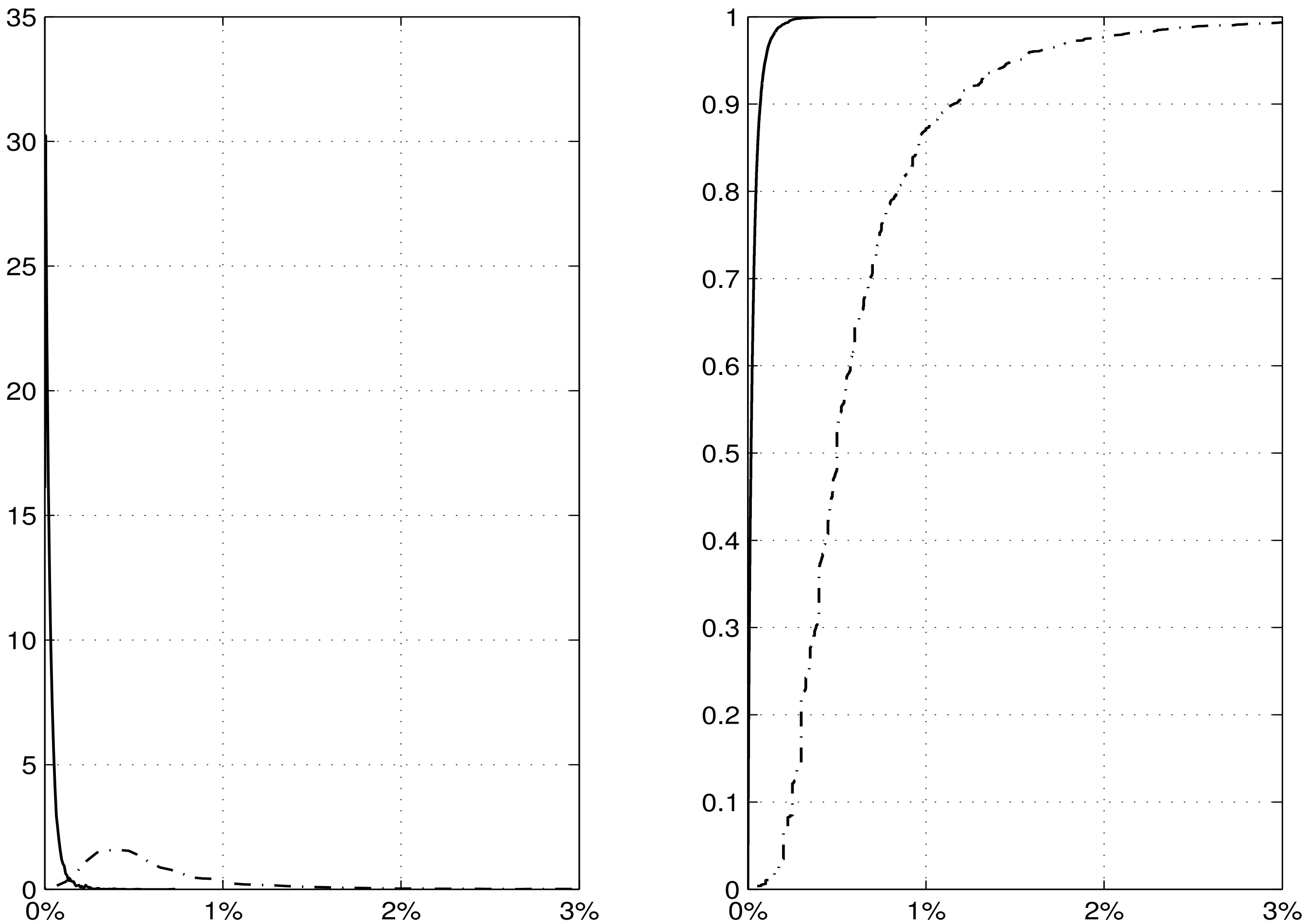


Figure 1: Distribution of Pricing Errors and Bid-Ask Spreads

This figure presents the empirical distribution (left panel) and cumulative distribution for the pricing errors (full line) and the bid-ask spreads (dotted line). Pricing errors are computed as the absolute difference in implied volatility between the model and the data. Spot and forward exchange rates are from Datastream, currency options are from JP Morgan, bid-ask spreads are from Bloomberg. Data are monthly. The sample period for the pricing error (bid-ask spreads) is 1/1996 – 08/2014 (09/2004 – 08/2014).

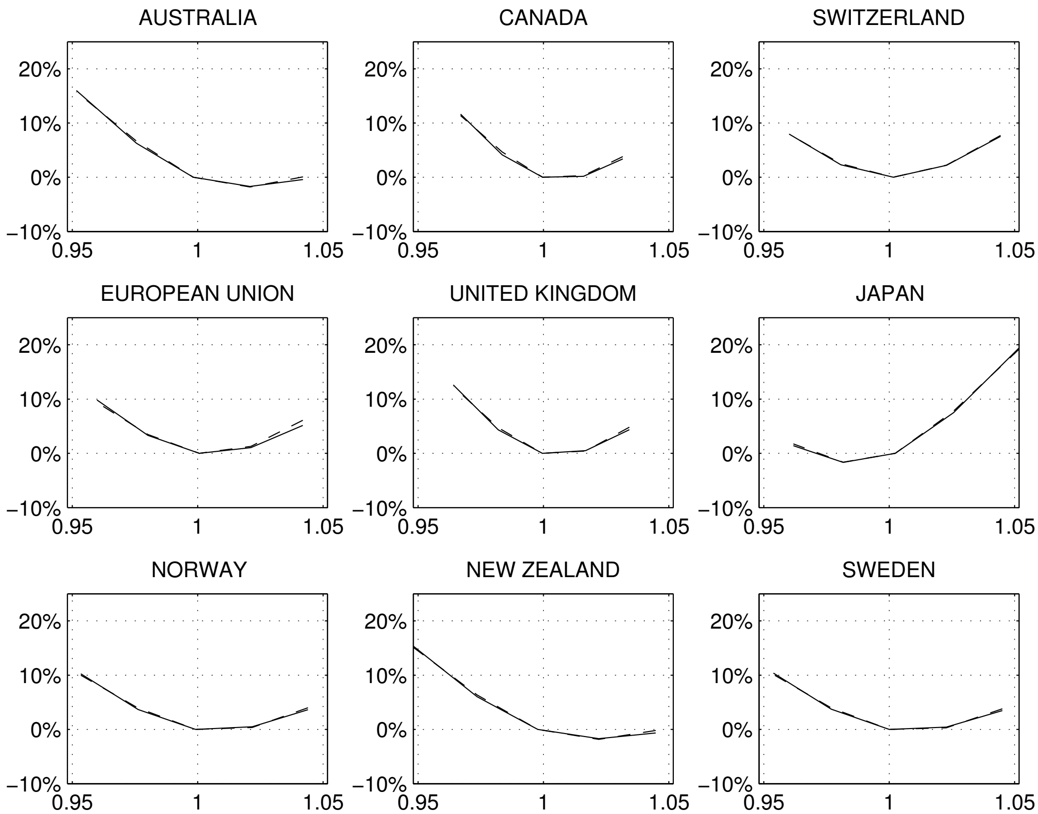


Figure 2: Average Currency Option Smiles in the Model and in the Data

This figure presents the average quoted implied volatilities in the data (dotted line) and in the model (full line) as a function of their strikes. To maintain comparability across currencies and periods, the implied volatilities at different strikes are scaled by the average implied volatility of at-the-money options. The quoted strikes are normalized by the spot exchange rate. Spot and forward exchange rates are from Datastream, while currency options are from J.P. Morgan. Data are monthly. The sample period is 1/1996 – 08/2014.

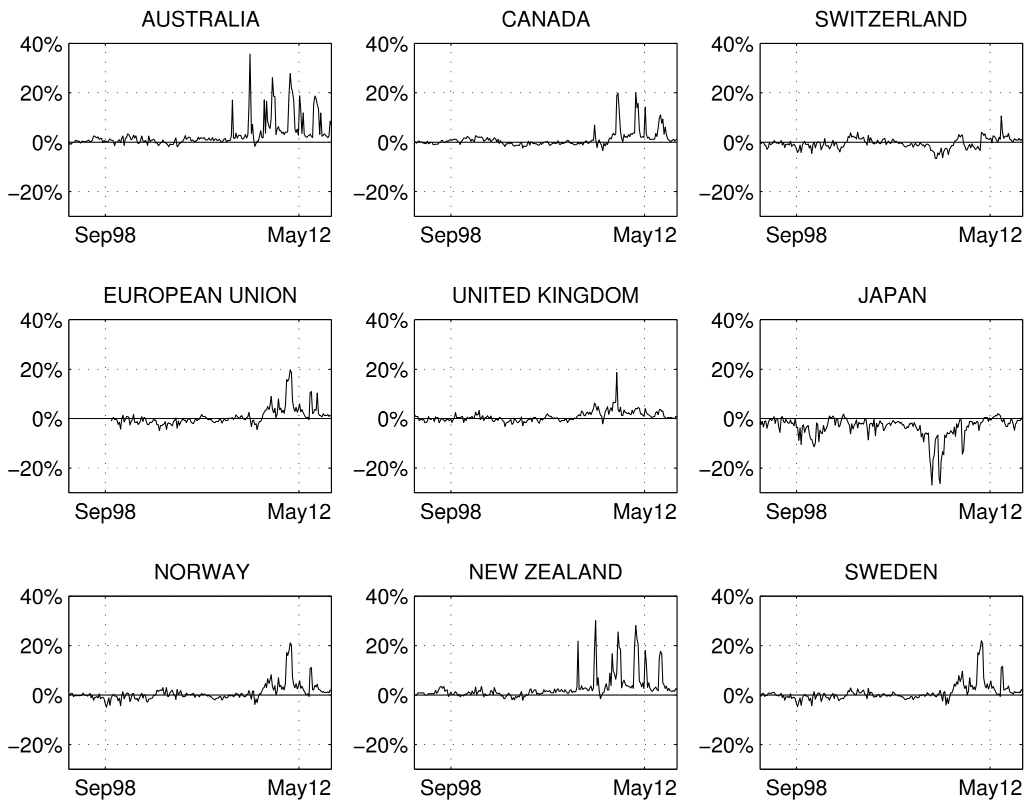


Figure 3: Country-level Estimates of Disaster Risk Exposure

This figure shows time-series estimates for the disaster risk exposure for each country. Spot and forward exchange rates are from Datastream, currency options are from JP Morgan. Data are monthly. The sample period is 1/1996–08/2014.

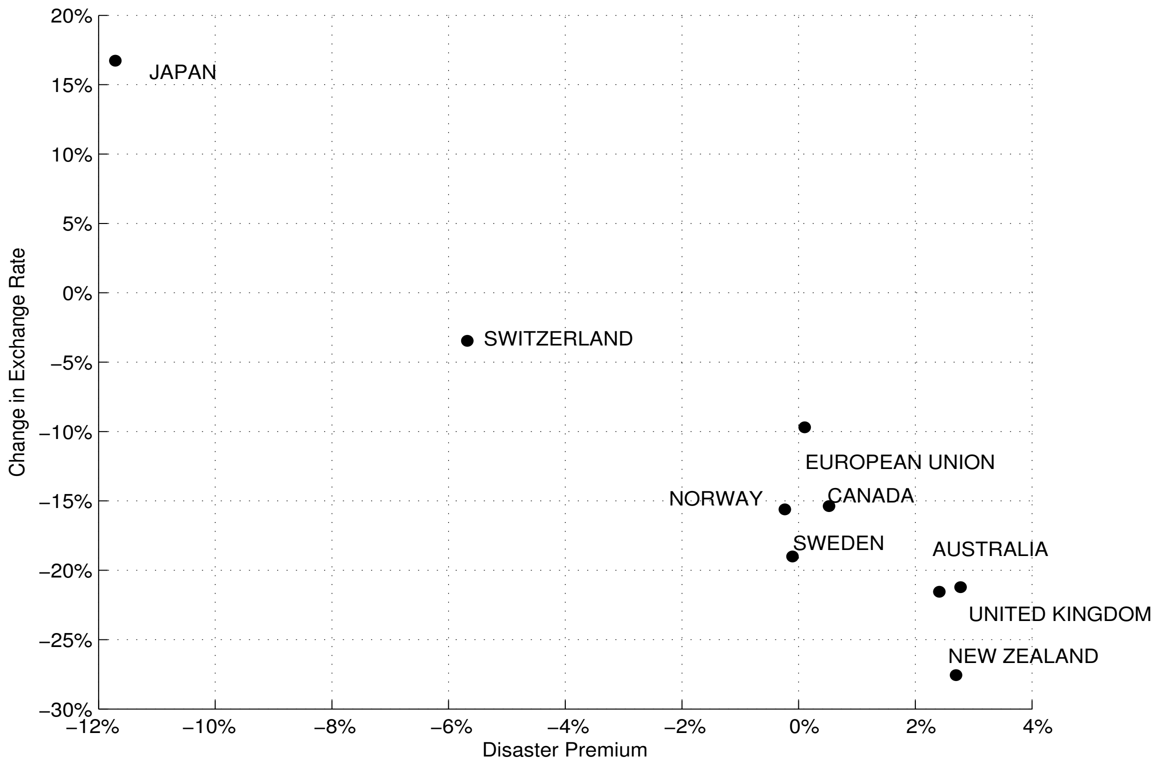


Figure 4: Disaster Risk Exposures and Changes in Exchange Rates During the Crisis

This figure reports the average estimated disaster risk exposure leading to the crisis (from May 2008 to August 2008) and the cumulative percentage change in exchange rate for each country during the crisis (from September 2008 to January 2009). Spot and forward exchange rates are from Datastream, currency options are from JP Morgan. Data are monthly.