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C A Tractable Model Exhibiting Comovement

A Model Setup

To think quantitatively about comovement, we specify the model of Section I. We use a CES production function for one final good.[42] Thus, we can solve the structure of this economy in closed form, and quantify its comovement. Our model differs in a series of small ways from Long and Plosser (1983); Horvath (1998, 2000); Shea (2002); Foerster, Sarte, and Watson (2011); and Carvalho (2010).[43] Its main virtue is that it is solvable in closed form, so that the mechanisms are fairly transparent.

There is an aggregate good and n intermediary goods. Unit i uses L_i, K_i, X_i of labor, capital, and the aggregate good to produce Q_i goods i:

$$Q_i = \kappa A_i \left(L_i^{\alpha} K_i^{1-\alpha}\right)^b X_i^{1-b} \tag{18}$$

with $\kappa = 1/\left(b^b (1-b)^{1-b}\right)$, and $1-b$ is the share of intermediate inputs, so that b will be the ratio of value added to sales, both at the level of the unit and of the economy.[44] GDP is production net of the intermediate inputs, the X_i's:

$$Y = (\sum_i Q_i^{1/\psi})^{\psi} - \sum_i X_i \tag{19}$$

with $\psi > 1$. The transformation from the goods Q_i to the final good $(\sum_i Q_i^{1/\psi})^{\psi}$ and the intermediary inputs is made by a competitive fringe of firms.

[42] In the CES world that we parameterize (with positive elasticity of substitution), a positive TFP shock increases a sector's size. This is not necessarily a good thing. When a sector is very new (say electronic gadgets), the size of that sector grows as the sector becomes more productive (that is, as more products are invented). However, perhaps in a long run sense, some sectors shrink as they become very productive (e.g., agriculture). Following the macro tradition, we eschew here a calibrated modeling of this heterogeneity in the link between productivity and size.

[43] Long and Plosser (1983) impose a Cobb-Douglas structure, with zero idiosyncratic movement in the sales per employee and value of sales.

[44] This uniformity of the sales/value added ratio is only an approximation of reality. We conjecture that this is purely for convenience, and that the economics would go through with non-uniform S_{it}/Y_{it}, although the algebra would be harder to handle.

The representative agent's utility function is $U = Y - L^{1+1/\phi}$. Capital can be rented at a rate r. Thus, the social planner's problem is $\max_{\{K_i, L_i, X_i\}} Y - L^{1+1/\phi} - rK$ subject to $\sum K_i = K; \sum L_i = L$. We assume that the prices equal marginal cost. This could be caused by competition or by an input subsidy equal to ψ for the intermediary firms.

The model gives:

$$\text{GDP} : Y = \Lambda L^{\alpha} K^{1-\alpha} \tag{20}$$

$$\text{TFP} : \Lambda = (\sum_i A_i^{1/(\psi-1)})^{(\psi-1)/b} \tag{21}$$

$$\frac{\text{Sales}_i}{\text{GDP}} : \frac{p_i Q_i}{Y} = \frac{1}{b} \left(\frac{A_i}{\Lambda^b} \right)^{1/(\psi-1)}. \tag{22}$$

B Comovement in Output

To study comovement, we assume that we start from a steady-state equilibrium, and study the one-shot response of our economy to shocks.

Models such as (18) always deliver a sales/employees ratio that is independent of the unit's productivity. The reason for this almost surely counterfactual prediction is that labor is assumed to be costlessly adjustable. To capture the realistic case of costly labor adjustment, we assume that a fraction $1 - \nu$ of labor is a quasi-fixed factor, in the sense of Oi (1962). Technically, we represent $L_i = L_{V,i}^{\nu} L_{F,i}^{1-\nu}$, where $L_{V,i}$ and $L_{F,i}$ are respectively the variable part of labor and the quasi-fixed part of labor. After a small shock, only $L_{V,i}$ adjusts. The disutility of labor remains $L^{1+1/\phi}$, where $L = L_V^{\nu} L_F^{1-\nu}$ is aggregate labor. We assume that capital and intermediary inputs are flexible. The online appendix relaxes that assumption.

One can now study the effect of a productivity shock $\widehat{A}_i$ to each unit i. We call $S_i = p_i Q_i$ the sales of unit i. The next proposition, whose proof is in section D (which contains a generalization to the case where all factors have finite elasticity), describes how the economy reacts to microeconomic shocks.

Proposition 2 *(Aggregate factor emerging from microeconomic shocks) Suppose that each unit i receives a productivity shock $\widehat{A}_i$. Macroeconomic variables change according to:*

$$\textit{GDP and TFP}: \ \widehat{Y} = \frac{1+\phi}{\alpha} \widehat{\Lambda}, \qquad \widehat{\Lambda} = \sum \frac{S_i}{Y} \widehat{A}_i = \sum \frac{\textit{Sales}_i}{\textit{GDP}} \widehat{A}_i \tag{23}$$

$$\textit{Employment and Wage}: \widehat{L} = \frac{\phi}{1+\phi} \widehat{Y}, \qquad \widehat{w} = \frac{1}{1+\phi} \widehat{Y}. \tag{24}$$

Microeconomic-level variables change according to

$$\textit{Value of sales and value added}: \widehat{S}_i = \widehat{Y}_i = \beta \widehat{A}_i + \overline{\Phi}\widehat{Y} \tag{25}$$

$$\textit{Production}: \widehat{Q}_i = \psi\beta \widehat{A}_i + (1-\psi\Phi)\,\widehat{Y} \tag{26}$$

$$\textit{Price}: \widehat{p}_i = -(\psi-1)\,\beta\widehat{A}_i + (\psi-1)\,\beta\Phi\widehat{Y} \tag{27}$$

$$\textit{Employment}: \widehat{L}_i = \nu\beta\widehat{A}_i + \left(\frac{\phi}{1+\phi} - \nu\Phi\right)\widehat{Y} \tag{28}$$

$$\textit{Use of intermediary inputs and capital}: \widehat{X}_i = \widehat{K}_i = \widehat{S}_i \tag{29}$$

$$\beta = \frac{1}{\psi - 1 + b\alpha\,(1-\nu)}, \qquad \Phi = \frac{\beta b\alpha}{1+\phi}, \qquad \overline{\Phi} = 1 - \Phi. \tag{30}$$

In other terms, this economy exhibits a common factor $\widehat{Y}$ (equations 25-29) which is itself nothing but a sum of idiosyncratic shocks (equation 23).

The new results are the sectoral-level changes, in equations 25-29. The economy behaves like in a one-factor model with an "aggregate shock," the GDP factor $\widehat{Y}$. Again, this factor stems from a multitude of idiosyncratic shocks (equation 23).[45] It causes all microeconomic-level quantities to comove. Economically, when sector i has a positive shock, it makes the aggregate economy more productive, and affects the other sectors in three different ways. First, other sectors can use more intermediary inputs produced by sector i, thereby increasing their production. Second, sector i demands more inputs from the other firms (equation 25), which leads their production to increase. Third, given that sector i commands a large share of output, it will use more of the inputs of the economy, which tends to reduce the other sectors' outputs. The net effect depends on the magnitudes of the elasticities[46]. For instance, when ψ is higher, so that all goods are less substitutable, the loadings of the above sales, production, and employment on the GDP shock increase (and, indeed, more positive in the calibration). This makes sense: as goods are less substitutable, a productivity shock in one area makes the economy wish to consume more of all goods.

We calibrate the model using conventional parameters to the extent possible,[47] with parameters summarized in Table 5. Using the decomposition (14), the ratio $f^{GDP} \equiv N/\sigma_Y^2$

[45] In this economy, firms are affected by this GDP factor, but this GDP factor is entirely made of firm-level shocks. That makes identification very difficult. It's an instance of the "reflection problem" identified by Manski (1993). It has no general solution, but some of the partial techniques generated by Manski's impulse might be useful to identify causality in the type of macroeconomic equilibrium described by Proposition 2.

[46] See Baumol (1967) for an early analysis of models with more than 1 sector.

[47] For instance, $\psi = 1.2$ may be a typical estimate. As Chad Jones (2011, p.23, who uses $\psi = 1.5$) states, this parameter is not "especially well pinned down in the literature." By taking $\psi = 1.2$, we are rather conservative about the importance of comovement, as a higher ψ (like Jones's) would generate even more comovement.

Table 5: Model Calibration

Calibrated Values	
Labor share	$\alpha = 2/3$
One minus share of intermediate inputs	$b = 1/2$
Elasticity of labor supply	$\varphi = 2$
Product differentiation parameter	$\psi = 1.2$
Share of labor that is variable in the short run	$\nu = 1/2$
Fraction of mismeasured temporary labor utilization	$\theta = 0.43$
Resulting Values	
Micro-level productivity multiplier	$\beta = 2.7$
Aggregate productivity multiplier	$\mu = 4.5$
Elasticitity Φ	$\Phi = 0.3$
Fraction of GDP variance attributed to comovement	$f^{GDP} = 0.90$
Fraction of employment variance attributed to comovement	$f^{Labor} = 0.95$
Fraction of measured TFP variance attributed to comovement	$f^{TFP,m} = 0.59$

Notes: The first part of the table shows the postulated values. The second part shows the resulting values for a few quantities. Because of linkages, the fraction of variances attributed to comovement is non-zero, although all primitive shocks are assumed to be idiosyncratic.

captures how much of GDP variance is due to comovement.

Proposition 3 *(Magnitude of the comovement in output and employment) Call f^{GDP} (resp. f^{Labor}) the fraction of GDP (resp. employment) variance attributed to comovement in a variance-accounting sense. An exact value is given in equation (52) of the appendix. If most shocks are idiosyncratic at the micro level ($\beta^2\sigma_A^2 \gg \overline{\Phi}^2\sigma_Y^2$), we have*

$$f^{GDP} = 1 - \left(\frac{b\beta}{\mu}\right)^2, \qquad f^{Labor} = 1 - \left(\nu\left(1 + \frac{1}{\phi}\right)\frac{b\beta}{\mu}\right)^2, \tag{31}$$

and we have (15) with $c_D \simeq b^2\beta^2$ and $c_N \simeq \mu^2 - b^2\beta^2$. However, economically, all the shocks are primitively idiosyncratic.

Of course, as μ (defined in equation 7) increases, so does the fraction attributed to comovement. Using our calibration, we find $f^{GDP} = 0.90$ and $f^{Labor} = 0.95$. This is to say that even though primitive shocks are purely idiosyncratic in our model, linkages create such a large comovement that, in a volatility-accounting sense, 90% are mechanically attributed to comovement. This measure is congruent with the empirical findings of Shea (2002), who reports estimates of f^{GDP} in the range of 80-85% and $f^{Labor} = 0.95$. It might also explain why Foerster, Sarte, and Watson (2011) find that a large part of fluctuations comes from the non-diagonal part, N_t.

C Comovement in Measured TFP

The data show a positive correlation (the average pairwise correlation is 2.3 percent) in measured TFP innovations across sectors. The previous model generates positive comovement from independent TFP shocks. Hence, if there is perfect measurement of TFP, it will generate no comovement of TFP. Against this background, we interpret the data in the following way. We say that a fraction θ of the change in the effective number of hours is not measured. For instance, a secretary will work harder when there is much work to do, and will work less intensely when there is less work. Still, the total number of hours that are counted is the same, say 40 hours per week. In that case, $\theta = 1$. If she works some overtime, so that some of her extra efforts appear in the labor-supply statistics, then $\theta < 1$. For simplicity, we assume that only labor is mismeasured (the same argument would go through if more factors were mismeasured). The measured number of hours is:

$$\widehat{L}_i^m = (1-\theta)\,\widehat{L}_i,$$

where the superscript m denotes the $m$easured quantities. Measured TFP growth in sector i is:

$$\begin{aligned}\widehat{A}_i^m &= \widehat{Q}_i - b\alpha\widehat{L}_i^m - b(1-\alpha)\,\widehat{K}_i - (1-b)\,\widehat{X}_i \\ &= \left[\widehat{Q}_i - b\alpha\widehat{L}_i - b(1-\alpha)\,\widehat{K}_i - (1-b)\,\widehat{X}\right] + \theta b\alpha\left(\widehat{L}_i - \widehat{L}_i^m\right).\end{aligned}$$

Hence:

$$\widehat{A}_i^m = \widehat{A}_i + \theta b\alpha\widehat{L}_i. \tag{32}$$

In other terms, the measured TFP is the true TFP, plus the increase in effective labor $\widehat{L}_i$ times labor share in output-cum-intermediary-inputs $b\alpha$ times the mismeasurement factor θ.

In this benchmark economy, the comovement in true productivity growth $\widehat{A}_i$ is 0. However, there will be some comovement in measured productivity growth, as all sectors tend to increase factor utilization (in a partially unmeasured way) during booms. The following proposition quantifies this.

Proposition 4 *(Magnitude of the comovement in measured TFP) Measured TFP follows the factor structure:*

$$\widehat{A}_i^m = c_A \widehat{A}_i + c_Y \widehat{Y}, \tag{33}$$

where $c_A \equiv 1+\theta b \alpha \nu \beta$ *and* $c_Y \equiv \theta b \alpha \left(\frac{\phi}{1+\phi} - \nu \Phi \right)$*. Hence, if there is mismeasurement, measured TFP covaries. Call* $f^{TFP,m}$ *the fraction of measured TFP variance attributed to comovement in a variance-accounting sense. If most shocks are idiosyncratic at the micro level, we have*

$$f^{TFP,m} = 1 - \left(1 + \frac{c_Y \mu}{c_A b} \right)^{-2}. \tag{34}$$

Note that if there is no mismeasurement ($\theta = 0$)*,* $c_Y = 0$ *and* $f^{TFP,m} = 0$*: there is no comovement in TFP.*

Empirically, we measure $f^{TFP,m} = 0.59$ in US data. Solving for θ in equation 34, this corresponds to $\theta = 43$ *percent* of the variable-labor input being undermeasured. It says that if from trough to peak measured hours go up by 5.7 *percent*, effort goes up 4.3 *percent*.[48] The corresponding value of σ_F^m / σ_F is simply $c_A = 1.19$. So, mismeasurement of inputs affects a lot the apparent comovement between sectors (as it is the cause of comovement and the productivity multiplier is large), but only relatively little the measurement of sectoral-level productivity (idiosyncratic factors generally dominate aggregate factors at the microeconomic level). In our model, all primitive shocks come from idiosyncratic microeconomic shocks, but there is comovement in output because of production linkages. In other terms, there is positive comovement in measured TFP because statistical agencies do not control well for unmeasured increases in labor inputs, i.e., "effort" or "utilization."

We wish to conclude that our model simply illustrates important quantitative features of an economy with comovement. We suspect that the highlighted features will survive with other sources of comovement, e.g., a financial accelerator or expectations.

D Derivations

Proof of Proposition 2 We found it useful to state a general proposition with an arbitrary number of fixed and variable factors. We call $\mathcal{F}$ the primitive factors (e.g., labor and capital) and $\mathcal{F} \cup X$ the set of all factors – the primitive factor and the intermediary inputs. Consider the microeconomic production function:

$$Q_i = \kappa A_i \left(\prod_f F_{if}^{\alpha_f} \right)^b X_i^{1-b}$$

[48] Adding the possibility of mismeasurement in capital utilization would decrease θ.

with $\kappa = 1/\left(b^b (1-b)^{1-b}\right)$, where sector i produces a quantity Q_i using F_{if} of factor $f \in \mathcal{F}$ and X_i intermediary inputs. This can be rewritten:

$$Q_i = \kappa A_i \prod_{f \in \mathcal{F} \cup X} F_{if}^{\gamma_f}, \tag{35}$$

where we define $\gamma_f = b\alpha_f$ for f a primitive factor, and $\gamma_f = (1-b)$ for f the intermediary input. Using this notation, the intermediary inputs in sector i are $X_i = F_{iX}$.

For instance, in the economy studied in Section B there are three factors,

$$(F_f)_{f=1\ldots3} = (\text{Labor,Capital,Intermediary inputs}) = (L, K, X),$$

and their weights are $\left(\gamma_f\right)_{f=1\ldots3} = (\alpha b, (1-\alpha) b, 1-b)$. We will call it the "3-factor economy."

Each factor F_f has a cost $C_f F_f^{1/\xi_f}$ for a constant f. In the the 3-factor economy, $1/\xi_1 = 1 + 1/\phi$, i.e., $\xi_1 = \phi/(1+\phi)$. On the other hand, as the cost of the intermediary good X is linear in X, $\xi_3 = C_3 = 1$. If capital is elastic in the short run, $\xi_2 = 1$ and $C_2 = r$; if it is completely inelastic, $\xi_2 = 0$ in all results below.

GDP is output net of intermediary inputs: $Y = H - X = H - \sum_i F_{iX}$, with

$$H = \left(\sum_i Q_i^{1/\psi}\right)^{\psi}. \tag{36}$$

GDP is the solution of the planner's problem:

$$\max_{F_{if}} H - \sum_f C_f F_f^{1/\xi_f} \text{ such that for all } f, \sum_i F_{if} \le F_f.$$

Finally, as in the body of the paper, a fraction ν_f of a factor f is flexible in the short run.

We start with a general proposition.

Proposition 5 *(General case) The static equilibrium is described by* $Y = \Lambda \prod_{f \in \mathcal{F}} F_f^{\alpha_f}$ *with (21)-(22). Furthermore, suppose that each unit* i *receives a productivity shock* $\widehat{A}_i$*. Macroeconomic variables change according to:*

$$\text{TFP}: \ \widehat{\Lambda} = \sum_i \frac{S_i}{Y} \widehat{A}_i = \sum_i \frac{\text{Sales}_i}{\text{GDP}} \widehat{A}_i \tag{37}$$

$$\text{GDP}: \ \widehat{Y} = \frac{1}{1 - \sum_f \alpha_f \xi_f} \widehat{\Lambda} \tag{38}$$

$$\text{Employment of factor } f: \ \widehat{F}_f = \xi_f \widehat{Y} \tag{39}$$

$$\text{Wage of factor } f: \widehat{w}_f = \left(1 - \xi_f\right) \widehat{Y}. \tag{40}$$

Microeconomic-level variables change according to:

$$\text{\textit{Value of sales}}: \ \widehat{S}_i = \beta \widehat{A}_i + \overline{\Phi} \widehat{Y} \tag{41}$$

$$\text{\textit{Production}}: \widehat{Q}_i = \psi \beta \widehat{A}_i + (1 - \psi \Phi) \widehat{Y} \tag{42}$$

$$\text{\textit{Price}}: \widehat{p}_i = -(\psi - 1) \beta \widehat{A}_i + (\psi - 1) \beta \Phi \widehat{Y} \tag{43}$$

$$\text{\textit{Employment of factor } } f: \widehat{F}_{if} = \nu_f \beta \widehat{A}_i + \left(\xi_f - \nu_f \Phi\right) \widehat{Y} \tag{44}$$

$$\text{\textit{Use of intermediary input}}: \widehat{X}_i = \widehat{S}_i \tag{45}$$

with

$$\begin{aligned}\beta &= \frac{1}{\psi - \sum_f \gamma_f \nu_f} = \frac{1}{\psi - (1-b) - b \sum_f \alpha_f \nu_f} \\ &= \frac{1}{\psi - \sum_{f \in \mathcal{F} \cup X} \text{\textit{Share of factor} } f \times \text{\textit{Flexibility ratio of factor} } f}\end{aligned}$$

and

$$\begin{aligned}\Phi &= \beta \left(1 - \sum_f \gamma_f \xi_f\right) = \beta b \left(1 - \sum_f \alpha_f \xi_f\right) \\ &= \beta \left(1 - \sum_{f \in \mathcal{F} \cup X} \text{\textit{Share of factor} } f \times \text{\textit{Adjusted supply elasticity of factor} } f\right),\end{aligned}$$

where $f \in \mathcal{F} \cup X$ denotes the primitive factors (labor, capital) and also the intermediary input X.

Proof of Proposition 5 *Step 1. Frictionless equilibrium.* The price of unit i is $p_i = \frac{\partial H}{\partial Q_i}$, hence by (36): $S_i/H = p_i Q_i/H = Q_i \frac{\partial H}{\partial Q_i}/H$ and

$$\frac{S_i}{H} = \left(\frac{Q_i}{H}\right)^{1/\psi}. \tag{46}$$

Because H is homogenous of degree 1, $H = \sum \frac{\partial H}{\partial Q_i} Q_i = \sum S_i$. H is the sum of sales in the economy.

Unit i solves $\max_{F_{if}} p_i Q_i - \sum_f w_f F_{if}$, which gives $F_{if} = S_i \gamma_f / w_f \propto S_i$. We use $\propto$ to mean that the variables are proportional, up to a factor that does not depend on i. So, $S_i^{\psi} \propto Q_i \propto A_i S_i$ by (18) and, hence, $S_i \propto A_i^{1/(\psi-1)}$. Calling $B = \sum_i A_i^{1/(\psi-1)}$ and using the adding-up constraint $\sum F_{if} = F_f$, we find the constant of proportionality: $F_{if} = F_f A_i^{1/(\psi-1)}/B$. Plugging this in (36), we obtain $H = \kappa B^{\psi-1} \prod_{f \in \mathcal{F} \cup X} F_f^{\gamma_f}$. Now, we solve for X: $\max_X Y = H - X$, i.e., $\max_X \kappa B^{\psi-1}\left(\prod_{f\in\mathcal{F}} F_f^{\alpha_f}\right)^b X^{1-b} - X$. The solution yields $X=(1-b)\,H$, $Y=H-X=B^{(\psi-1)/b}\prod_{f\in\mathcal{F}} F_f^{\alpha_f}$, i.e.,

$$Y=\Lambda\prod_{f\in\mathcal{F}} F_f^{\alpha_f}, \qquad \Lambda=B^{(\psi-1)/b}, \tag{47}$$

as announced in the statement of Proposition 5. In the 3-factor economy, we obtain (20). Also, $Y=H-X=bH$.

Step 2. Changes, assuming $\nu_f=1$. To keep the proof streamlined, we first consider the case $\nu_f=1$, i.e., the case with no frictions in the adjustment of labor, and with the possibility that $\sum_f \gamma_f$ is not 1. TFP growth comes from (21), and is also Hulten's formula. $Y=bH$ gives $\widehat{Y}=\widehat{H}$. The optimal use of factor f maximizes $Y-\sum C_f F_f^{1/\xi}$, for constants C_f. Hence it follows that $\gamma_f Y/F_f = C_f F_f^{1/\xi_f-1}$, $F_f=Y^{\xi_f}$ times a constant, and

$$\widehat{F_f}=\xi_f\widehat{Y}. \tag{48}$$

Equation 47 implies that:

$$\widehat{Y}=\widehat{\Lambda}+\sum_f \alpha_f\widehat{F}_f=\widehat{\Lambda}+\left(\sum_f\alpha_f\xi_f\right)\widehat{Y}$$

and $\widehat{Y}=\frac{\widehat{\Lambda}}{1-\sum_f\alpha_f\xi_f}$. The wage is $w_f=\frac{1}{\xi_f}L^{1/\xi_f-1}$, so: $\widehat{w}_f=\left(\frac{1}{\xi_f}-1\right)\widehat{F}_f$, hence

$$\widehat{w}_f=\left(1-\xi_f\right)\widehat{Y}. \tag{49}$$

It is convenient that one can solve for changes in the macroeconomic variables without revisiting the sectors' decision problems.

We now turn to the unit-level changes. Optimization of the demand for labor gives $w_f F_{if}=\gamma_f S_i$, so that

$$\widehat{F}_{if}=\widehat{S}_i-\widehat{w_f}=\widehat{S}_i-\left(1-\xi_f\right)\widehat{Y}.$$

We have, from (18),

$$\begin{aligned}\widehat{Q}_i&=\widehat{A}_i+\sum_f\gamma_f\widehat{F}_{if}=\widehat{A}_i+\sum_f\gamma_f\left(\widehat{S}_i-\left(1-\xi_f\right)\widehat{Y}\right)\\&=\widehat{A}_i+\left(\sum_f\gamma_f\right)\widehat{S}_i-\sum_f\gamma_f\left(1-\xi_f\right)\widehat{Y}.\end{aligned}$$

Equation 46 gives $\widehat{Q}_i=\psi\widehat{S}_i+(1-\psi)\,\widehat{H}$, and using $\widehat{Y}=\widehat{H}$,

$$\psi\widehat{S}_i+(1-\psi)\,\widehat{Y}=\widehat{Q}_i=\widehat{A}_i+\left(\sum_f\gamma_f\right)\widehat{S}_i-\sum_f\gamma_f\left(1-\xi_f\right)\widehat{Y},$$

which gives

$$\widehat{S}_i = \frac{\widehat{A}_i + \left[\psi - 1 - \sum_f \gamma_f \left(1 - \xi_f\right)\right] \widehat{Y}}{\psi - \sum_f \gamma_f} = \frac{\widehat{A}_i}{\psi - \sum_f \gamma_f} + \left(1 - \frac{1 - \sum_f \gamma_f \xi_f}{\psi - \sum_f \gamma_f}\right) \widehat{Y}$$
$$= \beta \widehat{A}_i + \overline{\Phi} \widehat{Y},$$

where

$$\beta = \frac{1}{\psi - \sum_f \gamma_f}, \qquad \Phi = \beta \left(1 - \sum_f \gamma_f \xi_f\right),$$

and, as $\widehat{Q}_i = \psi \widehat{S}_i + (1 - \psi) \widehat{Y}$, we obtain the announced expressions for $\widehat{S}_i$ and $\widehat{Q}_i$. $\widehat{F}_{if}$ comes from $\widehat{F}_{if} = \widehat{S}_i - \widehat{w}_f$. S_i was defined as $S_i = p_i Q_i$, which gives $\widehat{p}_i = \widehat{S}_i - \widehat{Q}_i$.

Step 3. With general $\nu_f \in [0,1]$. After the changes $\widehat{A}_i$, only $L_{V,i}$ can adjust. The planner optimizes the variable part of labor supply: $\max_{L_{Vf}} A \prod_f \left(F_{Vf}^{\nu_f} F_{Ff}^{1-\nu_f}\right)^{\gamma_f} - \left(F_{Vf}^{\nu_f} F_{Ff}^{1-\nu_f}\right)^{1/\xi_f}$. Note that this is isomorphic to optimizing the total labor supply, defining $F_f = F_{Vf}^{\nu_f} F_{Ff}^{1-\nu_f}$. Hence, we have (48) and (49).

For the unit-level variables, one replaces $\left(\alpha_f, \gamma_f, \xi_f\right)$ by $\left(\alpha'_f, \gamma'_f, \xi'_f\right) = \left(\alpha_f \nu_f, \gamma_f \nu_f, \xi_f / \nu_f\right)$, which delivers

$$\beta = \frac{1}{\psi - \sum_f \gamma_f \nu_f}, \qquad \Phi = \beta \left(1 - \sum_f \gamma_f \xi_f\right).$$

Remember that (25) holds. Then, the expression for employment stemming from the optimization of labor demand becomes:

$$\begin{aligned}
\widehat{F}_{V,i} &= \nu_f \left[\widehat{S}_i - \left(1 - \frac{\xi_f}{\nu_f}\right) \widehat{Y}\right] = \nu_f \left[\beta \widehat{A}_i + (1 - \Phi) \widehat{Y} - \left(1 - \frac{\xi_f}{\nu_f}\right) \widehat{Y}\right] \\
&= \nu_f \left[\beta \widehat{A}_i + \left(\frac{\xi_f}{\nu_f} - \Phi\right) \widehat{Y}\right] = \nu_f \beta \widehat{A}_i + \left(\xi_f - \nu_f \Phi\right) \widehat{Y}.
\end{aligned}$$

This concludes the proof of Proposition 5.

Proof of Proposition 2 We apply the results from Proposition 5. We particularize them to the case of flexible capital ($\xi_2 = 1, \nu_2 = 1$) and flexible intermediary inputs ($\xi_3 = 1, \nu_3 = 1$), whereas labor is less flexible ($\xi_1 = \xi, \nu_1 = \nu$ can be less than 1). Then, using $\xi = \phi / (1 + \phi)$,

$$\widehat{Y} = \frac{1}{1 - (1 - \alpha) - \alpha \xi_1} \widehat{\Lambda} = \frac{1}{\alpha (1 - \xi)} \widehat{\Lambda} = \frac{1 + \phi}{\alpha} \widehat{\Lambda}$$
$$\widehat{L} = \xi \widehat{Y} = \frac{\phi}{1 + \phi} \widehat{Y}, \qquad \widehat{w} = (1 - \xi) \widehat{Y} = \frac{\widehat{Y}}{1 + \phi}.$$

Also:

$$\beta^{-1} = \psi - \sum_f \gamma_f \nu_f = \psi - b\alpha\nu - b(1-\alpha) - (1-b) = \psi - 1 + b\alpha(1-\nu)$$

$$\Phi = \beta\left(1 - \sum_f \gamma_f \xi_f\right) = \beta(1 - b\alpha\xi - b(1-\alpha) - (1-b)) = \beta b\alpha(1-\xi) = \beta b\alpha \frac{1}{1+\phi}.$$

Proof of Proposition 3 In this model, value added is proportional to sales, $Y_i = bS_i$ (this comes from the first-order condition with respect to X_i), so that:

$$\begin{aligned} D &\equiv \sum_i \left(\frac{Y_i}{Y}\right)^2 var\left(\widehat{Y}_i\right) = \sum_i b^2 \left(\frac{S_i}{Y}\right)^2 var\left(\widehat{S}_i\right) \\ &= b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 var\left(\beta\widehat{A}_i + \overline{\Phi}\widehat{Y}\right) \end{aligned} \quad (50)$$

and $f = N/\sigma_Y^2 = 1 - D/\sigma_Y^2$. Consider first the case where most shocks at the microeconomic level are idiosyncratic, i.e., $\beta^2\sigma_A^2 \gg \overline{\Phi}^2\sigma_Y^2$. Then,

$$f^{GDP} \simeq 1 - \frac{b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 \beta^2\sigma_A^2}{\sigma_Y^2} = 1 - \frac{b^2\beta^2\sigma_F^2}{\mu^2\sigma_F^2} = 1 - \frac{b^2\beta^2}{\mu^2}, \quad (51)$$

$c_D \simeq b^2\beta^2$, and $c_N \simeq \mu^2 - b^2\beta^2$.

In the general case, (23) implies $cov\left(\widehat{A}_i, \widehat{Y}\right) = \frac{S_i}{Y}\mu\sigma_A^2$, so (50) gives:

$$\begin{aligned} f^{GDP} &= 1 - \frac{b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 \left[\beta^2\sigma_A^2 + \overline{\Phi}^2\mu^2\sigma_F^2 + 2\beta\overline{\Phi}\left(\frac{S_i}{Y}\right)\mu\sigma_A^2\right]}{\mu^2\sigma_F^2} \\ &= 1 - \frac{b^2\beta^2}{\mu^2} - \overline{\Phi}^2 b^2 \left(\sum_i \left(\frac{S_i}{Y}\right)^2\right) - \frac{2b^2\beta\overline{\Phi}\sigma_A^2}{\mu\sigma_F^2} \sum_i \left(\frac{S_i}{Y}\right)^3 \end{aligned}$$

$$\therefore f^{GDP} = 1 - \frac{b^2\beta^2}{\mu^2} - \overline{\Phi}^2 b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 - \frac{2b^2\beta\overline{\Phi}}{\mu} \frac{\sum_i \left(\frac{S_i}{Y}\right)^3}{\sum_i \left(\frac{S_i}{Y}\right)^2}. \quad (52)$$

We verify numerically that the approximation (51) is quite good. Likewise, the comovement in labor follows, using (24), (28), and $L_i/L = bS_i/Y$:

$$\begin{aligned} 1 - f^{Labor} &= \frac{\sum_i var\left(\widehat{L}_i\right)(L_i/L)^2}{var\left(\widehat{L}\right)} \simeq \frac{\sum_i (\nu\beta)^2 \sigma_A^2 (bS_i/Y)^2}{\left(\frac{\phi}{1+\phi}\right)^2 \sigma_Y^2} \\ &= \left(\frac{\nu\beta(1+\phi)}{\phi}\right)^2 \frac{b^2\sigma_F^2}{\mu^2\sigma_F^2} = \left(\frac{\nu(1+\phi)}{\phi}\frac{b\beta}{\mu}\right)^2. \end{aligned}$$

Proof of Proposition 4 Equation 33 comes from (28) and (32). The measured change in productivity is (using $\sum_i S_i/Y = 1/b$)

$$\widehat{\Lambda}^m = \sum_i \frac{S_i}{Y}\widehat{A}_i^m = \sum_i \frac{S_i}{Y}\left(c_A\widehat{A}_i + c_Y\widehat{Y}\right) = c_A\widehat{\Lambda} + \frac{c_Y}{b}\widehat{Y} = \left(c_A + \frac{c_Y}{b}\mu\right)\widehat{\Lambda},$$

so the volatility of measured TFP, $\sigma_F^{Full,m} = Var\left(\widehat{\Lambda}^m\right)^{1/2}$, is $\sigma_F^{Full,m} = \left(c_A + \frac{c_Y}{b}\mu\right)\sigma_F$. On the other hand, the measured productivity using only the diagonal terms is $\sigma_F^m = c_A\sigma_F$ by (33). Hence, we obtain:

$$\frac{\sigma_F^{Full,m}}{\sigma_F^m} = 1 + \frac{c_Y\mu}{c_A b}. \tag{53}$$

Finally, we yield $f^{TFP,m} = 1 - \left(\frac{\sigma_F^{Full,m}}{\sigma_F^m}\right)^{-2} = 1 - \left(1 + \frac{c_Y\mu}{c_A b}\right)^{-2}$.

E Some Additional Empirical Results

A Construction of the International Data

Because of data limitations, to construct the fundamental volatility in country c, $\sigma_{Fct} = \sqrt{\sum_{i=1}^{n}\left(\frac{S_{ict}}{Y_{ct}}\right)^2\sigma_i^2}$, we use the sectoral volatility σ_i that we have computed for sector i in the US.

To implement this, we have contacted Dale Jorgenson and Mun Ho, who kindly provided us with a bridge between the sectoral classification used by Dale Jorgenson and Associates (DJA henceforth; our main source of data) and the sectoral definitions in the NACE classification system used by the EUKLEMS project (where we have obtained the data for France, Germany, Japan, and the UK).

The match performs best for the UK, since this is the country with the most disaggregated data in the EUKLEMS database. Thus, for most sectors we have a one-to-one match. In these cases we simply impute the TFP volatility for the UK sector from the corresponding sector in the DJA data set. There are three sectors for which the UK data are more aggregated than the original DJA sectors: "Hotels and Restaurants," "Other Inland Transport," and "Legal, Technical, and Advertising Sectors." For these sectors we first compute TFP volatility in the corresponding DJA sectors, and then take a simple average. For example, in the DJA data we observe the sectors "Hotels" and "Eating and Drinking" both of which correspond to the NACE category "Hotels and Restaurants." We then compute TFP for the DJA sectors, and take a simple average to obtain the corresponding TFP volatility of "Hotels and Restaurants."

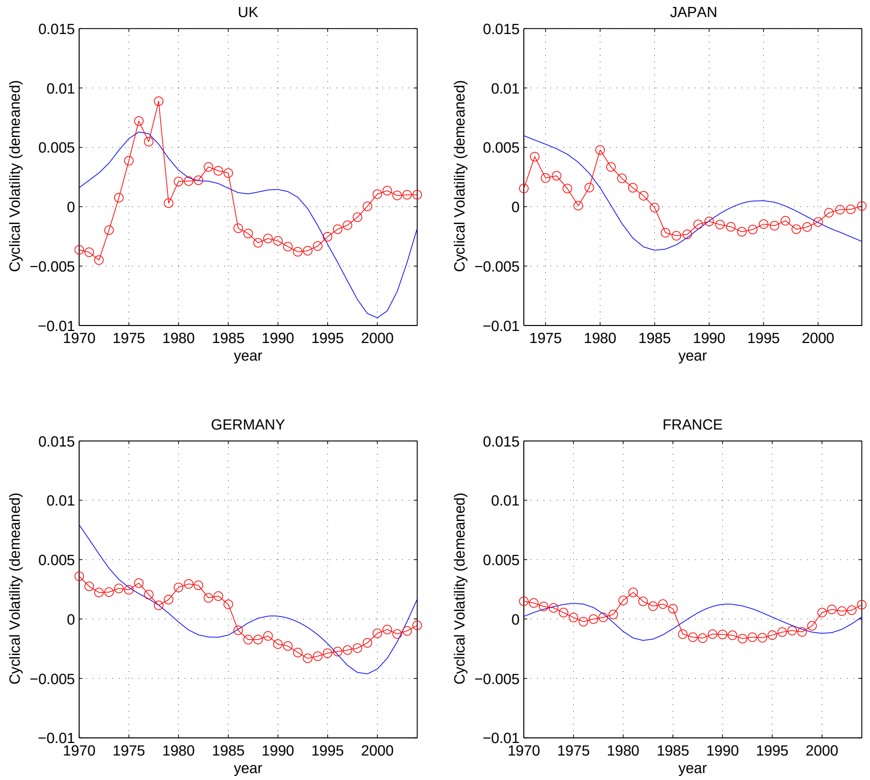
For two other sectors – "Recycling" and "Water Supply" – the match is not possible, since there is no corresponding sector in the DJA data. In this case we have chosen to apply the average sectoral-level TFP volatility.

For Japan, Germany, and France we have followed the exact same procedure, the only difference being that in these countries the original EUKLEMS data are more aggregated (Japan has 6 sectors that are more aggregated than the original DJA data, Germany 11, and France 16).

B Alternative International Evidence (Industry Volatility σ_i Varying by Country)

Here we present an alternative to the computations in the international evidence presented above. The key difference is that while above we imputed σ_i^2 from the US Jorgenson data, here we use EUKLEMS σ_i^2, with variation at the country and industry levels.

The problem we face here is that we do not have σ_i^2 for every sector, but rather for aggregates of sectors (since we only have sector-specific price data at this higher level of disaggregation). For example, for the UK we only have TFP growth data for the aggregate sector "Electrical and Optical Equipment," but we use more detailed data covering "Office, Accounting, and Computing machinery," "Insulated Wire," "Other Electrical Machinery and Apparatus Nec," "Electronic Valves and Tubes," "Telecommunication Equipment," "Radio and Television Receivers," "Scientific Instruments," "Other Instruments," and all subsectors of the aggregate sector for which we have data. Thus, for each of these more disaggregated sectors, we opt to assign σ_i^2 for which we do have TFP data of the more aggregated sector. The graph summarizes the evolution of fundamental volatility and aggregate volatility when we consider this method:



The long-term trends in volatility are broadly consistent both with the alternative method presented above and with the trends currently in the paper. The panel regressions (with and without) fixed effects are also similar.

	σ_{ct}	σ_{ct}
$\widehat{\beta}$	3.401 (6.45;0.527)	3.411 (4.51;0.757)
α_t	No	Yes
Observations	172	172

We can again use these estimates for a quantitative accounting of the role of fundamental volatility. Thus, as above, looking at the US and computing average aggregate volatility, σ_{Yct}, over the subperiods 1970-1984 and 1985-2000 implies a decline in business-cycle volatility of 0.96 percentage points. Across these two periods, the decline of our fundamental-volatility measure in the US is of 0.16 percentage points. Using the new panel estimate in this section (with country and time fixed effects), this implies a decline of $0.16 \times 3.411 = 0.55$ percentage points or 57 percent of the observed low-frequency decline in aggregate volatility.

C Technological Diversification Patterns

Recall that, in the construction of our fundamental-volatility measure, the only time-varying element that we allow for is the sum of squared Domar weights, $H_t^D = \sum_{i=1}^{n} (S_{it}/Y_t)^2$, where S_{it} is sector i nominal gross output in year t and Y_t gives the total (nominal) value added for the private-sector economy in year t. While Domar weights do not sum to one – as gross output at the sector level exceeds sectoral value added by the amount of intermediate inputs consumed by that sector – this measure is akin to the more common Herfindahl indices of concentration. In particular, looking at the cross-sectional (uncentered) second moment to characterize dispersion/concentration in technology loadings is still valid. The graph below shows the evolution of this measure for the US.

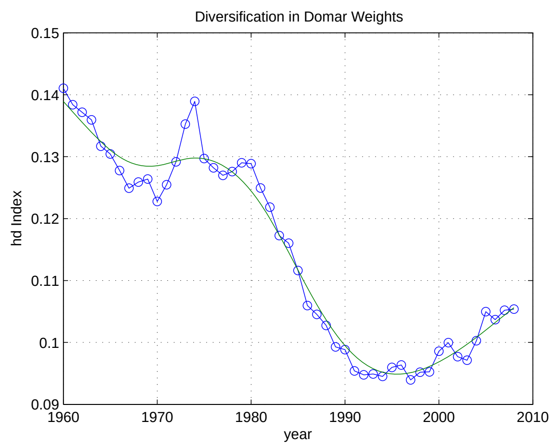


Figure 8: Evolution of H_t^D, 1960-2008

From the peak in 1960 to the trough in 1997, there is a 33% drop in the H_t^D measure. These dynamics are key to explaining the evolution of our fundamental-volatility measure. As such, it is important to perform a number of robustness checks and confirm that the same pattern obtains: i) in more disaggregated data and across different classification systems, ii) for different dispersion/concentration measures, and iii) for value-added shares.

C.1 More Disaggregated Data

First, we look at the raw BLS data underlying much of the construction of Jorgenson's data set. These data are both defined at a more disaggregated level and according to different classification systems. Namely, we source two different vintages of BLS Inter-industry Relationships data (i.e., input-output data). The first is based on SIC classifications (a mixture of two- and three-digit SIC sectors) running from 1977 to 1995 for a total of 185 sectors. The second is the latest vintage produced by the BLS, based on the newer NAICS classification system, and runs from 1993 to 2008 (for a total of 200 sectors).

Based on these data, we calculate the implied Herfindahl-like measure. Note that, given the difference in underlying classification systems, the levels are not comparable, neither among themselves nor with the ones reported above. Nevertheless, the dynamics seem to be in broad agreement with the ones above: a fall in technological concentration in the late 1970s and early 1980s and, if anything, a stronger reversal of this pattern from the late 1990s onwards.

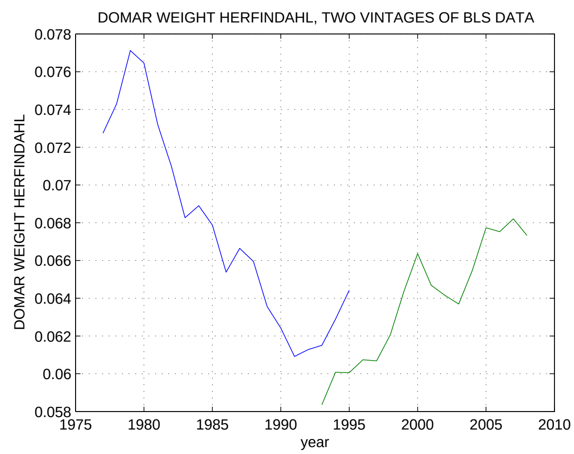


Figure 9: Herfindahl Measure for Domar Weights Computed from Two Vintages of Raw BLS Data (1977-1995 SIC Data; 1993-2008 NAICS Data)

C.2 Checking Against Other Dispersion Measures

Although they are directly related to the story of this paper, Herfindahl indices of concentration are not the only dispersion measure. However, looking at alternative measures of concentration, such as the Gini index, the broad story is unchanged: there is a decline in concentration up until the early 1990s, followed by re-concentration from the mid to late 1990s onwards, the latter now showing up much more strongly. These patterns are also robust to other measures of dispersion: cross-sectional standard deviation, coefficient of variation, max-min spread, and max-median spread.

An alternative is to look into diversification patters in sectoral value-added shares when the corresponding Herfindahl in year t, H_t^{VA}, is defined as $H_t^{VA} = \sum_{i=1}^{n} (Y_{it}/Y_t)^2$, where Y_{it} denotes nominal value added in sector i in year t. Figure 11 depicts the evolution of this

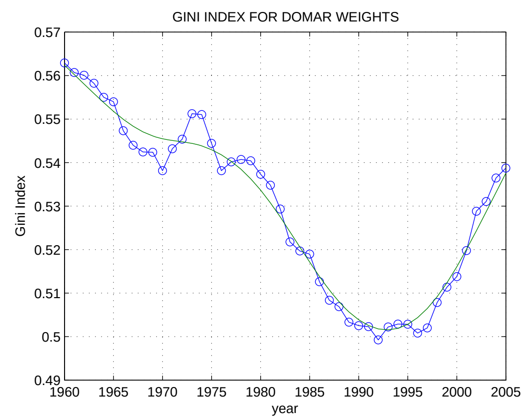


Figure 10: Gini Index for Sectoral Domar Weights

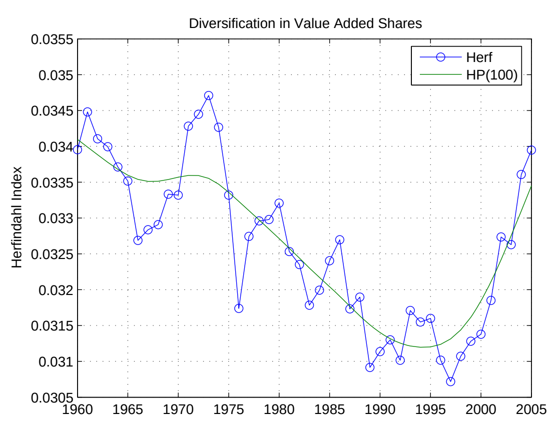


Figure 11: Herfindahl Index of Sectoral Concentration from Value-added Shares Data from 1960-2005.

index and the corresponding HP-filtered series.

Again, looking at value-added shares leads to the same U-shaped pattern. We also see that, quantitatively, the peak-to-trough movement in the value-added Herfindahl is smaller. This is as it should be: manufacturing technologies are relatively more intermediate-input-intensive. As such, the gradual move away from manufacturing and into services implies that the Domar-weights-based measure of concentration fell relatively more than the value-added one: not only has economic activity relied more on what were initially small – in a value-added sense – sectors, but these latter sectors have relatively lower ratios of gross output to value added.[49]

D Time-varying Elasticity of Labor Supply

Jaimovich and Siu (JS, 2009) find that changes in the composition of the labor force account for part of the movement in GDP volatility across the G7 countries. As the young have a more elastic labor supply, the aggregate labor-supply elasticity should be increasing in the fraction of the labor force that is between 15 and 29 years old – which is the JS_t variable.

In terms of our model in (6)-(7), this corresponds to having a time-varying Frisch elasticity of labor supply ϕ_t, i.e.,

$$\sigma_{Yt} = \frac{1+\phi_t}{\alpha}\sigma_{Ft}. \tag{54}$$

The JS composition of the work force effect is in the term $\phi(t) = A + B \cdot JS_t$, while the effect we focus on in this paper is the σ_{Ft} term. Put differently, the JS variable is about the amplification of primitive shocks, while our variable is about the primitive shocks themselves. To investigate (54), we run the panel regression:

$$\sigma_{Yct} = \alpha_c + \alpha_t + \beta\sigma_{Fct} + \gamma JS_{ct} + \varepsilon_{ct}. \tag{55}$$

That is, we return to the cross-country exercise above, this time including the JS measure of the labor-force share of the "volatile age group" in each of our economies, JS_{ct}. The inclusion of this measure shortens our sample somewhat, as it extends only to 1999 (and begins only in 1979 for the case of the UK). Finally, we run the regression with or without time fixed-effects.

Table 6 reports the results. Both coefficients β and γ are positive and significant in all specifications. We conclude that the JS labor-supply elasticity and fundamental volatility are both relevant to explain the cross-country evolution of business cycle volatility.

[49] A way to confirm this is to regress the average growth rate of each sector's value-added share on the average weight of intermediate-input purchases in sectoral gross output (average over the entire sample period 1960-2005). We obtain a negative and significant slope coefficient.

Table 6: Fundamental Volatility and Jaimovic-Siu Variable

	(1)	(2)
$\widehat{\beta}$	1.261 (3.25;0.388)	2.440 (3.30;0.740)
$\widehat{\gamma}$	0.054 (4.97;0.011)	0.060 (5.51;0.011)
α_t	No	Yes
Observations	134	134

Notes: We run the regression $\sigma_{Yct} = \alpha_c + \alpha_t + \beta\sigma_{Fct} + \gamma JS_{ct} + \varepsilon_{ct}$, where σ_{Yct} is the country volatility using a rolling-window measure, σ_{Fct} is the fundamental volatility of the country, JS_{ct} is the Jaimovich and Siu (2009) measure of the labor-force share of the "volatile age group," α_c a country fixed effect, and α_t a time fixed effect. t-statistics and Newey-West standard errors (2 lags) in parentheses.

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