Online Appendix to "International Liquidity and Exchange Rate Dynamics"

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A.1 Further Details for the Main Body of the Paper

A.1.A A More Abstract Version of the Market Structure

It may be useful to have a more abstract presentation of the basic model. We focus on the US side, as the Japanese side is entirely symmetric.

For generality, we present the monetary model, and then show how the real model can be viewed as a special case of it. We call

$$c = (C_H, C_F, C_{NT}, M, 0, 0),$$

the vector of consumptions of C_H US tradables, C_F Japanese tradables, C_{NT} US non-tradables, and a quantity M of US money, respectively. The last 2 slots in vector c (set at 0) are the consumption of Japanese non-tradables, and Japanese money: they are zero for the US consumer. Likewise, the Japanese consumer has consumption:

$$c^* = (C_H^*, C_F^*, 0, 0, C_{NT}^*, M^*).$$

The Japanese household consumes C_H^* US tradables, C_F^* Japanese tradables, 0 US non-tradables, 0 US money, C_{NT}^* Japanese non-tradables, and M^* Japanese money.

The US production vector is

$$y = (y_H, 0, y_{NT}, M^s, 0, 0).$$

This shows that the US produces y_H US tradables, 0 Japanese tradables, y_{NT} US non-tradables, and 0 Japanese non-tradables and money. Here M^s is the money supply given by the government to the household. Japanese production is similarly

$$y^* = (0, y_F^*, 0, 0, y_{NT}^*, M^{s*}).$$

The vector of prices in the US is

$$p = (p_H, p_F, p_{NT}, 1, 0, 0).$$

Utility is $u(c_t, \phi_t)$, where ϕ_t is a taste shock. In the paper, $\phi_t = (a_t, \iota_t, \chi_t, \omega_t, 0, 0)$, so that in the utility function is

$$u(c_t, \phi_t) = \sum_{i=1}^{6} \phi_{it} \ln c_{it}, \text{ for } t = 0, \ldots, T.$$

Consumptions are non-negative, $c_{it} \geq 0$ for all i, t.

The fourth and sixth components of the above vectors correspond to money. In the real model they are set to 0. Then, in this real model, the numéraire is the non-tradable good, so that $p_{NT} = 1$.

We call $\Theta_t = \left(\Theta_t^{US}, \Theta_t^J\right)$ the holding by the US of US bonds and Japanese bonds, $P_t = (1, e_t)$ the price of bonds in dollars.

The US consumers' problem is:

$$\text{(A.1)} \qquad \max_{\left(c_t, \Theta_t^{US}\right)_{t \leq T}} \mathbb{E} \sum_{t=0}^{T} \beta^t u(c_t, \phi_t),$$

s.t.

$$p_t \cdot (y_t - c_t) + P_t \cdot D_t \Theta_{t-1} + \pi_t^F = P_t \cdot \Theta_t, \text{ for } t = 0, ..., T, \tag{A.2}$$

and

$$\Theta_T = 0. \tag{A.3}$$

Here $D_t = diag\,(R, R^*)$ is the diagonal matrix expressing the gross rate of return of bonds in each currency and π_t^F is a profit rebated by financiers. The left-hand side of (A.2) is the households' financial wealth (in dollars) after period t. US firms are fully owned by US households. Because the economy is fully competitive, they make no profit. The entire production comes as labor income, whose value is $p_t \cdot y_t$. The budget constraint is the terminal asset holdings should be 0, which is expressed by (A.3). Finally, as is usual, c_t and Θ_t^{US} are adapted process, i.e. they depend only on information available at date t.

In the above maximization problems, US consumers choose optimally their consumption vector c_t and their dollar bond holdings Θ_t^{US}. However, they do not choose their holding of Japanese bonds Θ_t^J optimally. In the basic model we preclude such holdings and set $\Theta_t^J = 0$. In the extended model, we allow for such holdings and study simple and intuitive cases: for instance, at time 0 the holdings of Japanese bonds can be an endowment $\Theta_{-1}^J = D^J$ (or Japanese debt denoted in Yen). Alternatively, they could be a liquidity (noise trader) shock $\Theta_0^J = -f$, or we could have f be a function of observables, but not the exchange rate directly, e.g. $f = b + c\,(R - R^*)$ for a carry-trader. We do not focus on the foundations for each type of demand, but actually take the demands as exogenously specified. Possible microfoundations for these demands range from rational models of portfolio delegation where the interest rate is an observable variable that is known, in equilibrium, to load on the sources of risk of the model (see Section III.A), to models of "reaching for yield" (Hanson and Stein, 2014), or to the "boundedly rational" households who focus on the interest rate when investing without considering future exchange rate changes or covariance with marginal utility (as in Gabaix (2014)).

To summarize, while all goods are frictionlessly traded within a period (with the non-tradable goods being traded only within a country), asset markets are restricted: only US and Japanese bonds are traded (rather than a full set of Arrow-Debreu securities).

The *goods market clearing condition* is:

$$y_t + y_t^* = c_t + c_t^* \text{ at all dates } t \le T. \tag{A.4}$$

Firms produce and repatriate their sales at every period. They have net asset flows,

$$\Theta_t^{firms} = p_{Ht}^* c_{Ht}^* \,(e_t, -1)\,,$$
$$\Theta_t^{firms,*} = p_{Ft} c_{Ft} \left(-1, \frac{1}{e_t}\right).$$

The first equation expresses the asset flows of US exporters: in Japan, they have sales of $p_{Ht}^* c_{Ht}^*$ Yen in Japan market; they repatriate those yens (hence a flow of $-p_{Ht}^* c_{Ht}^*$ in Yen), to buy dollars (hence a flow of $p_{Ht}^* c_{Ht}^* e_t$ dollars).

For instance, in the model with the log specification,

$$\Theta_t^{firms} = p_{Ht}^* c_{Ht}^* \,(e_t, -1) = m_t^* \xi_t \,(e_t, -1)\,,$$
$$\Theta_t^{firms,*} = p_{Ft} c_{Ft} \left(-1, \frac{1}{e_t}\right) = m_t \iota_t \left(-1, \frac{1}{e_t}\right),$$

so that $\Theta_t^{firms} + \Theta_t^{firms,*} = (m_t^* \xi_t e_t - m_t \iota_t)\left(1, -\frac{1}{e_t}\right)$. The real model is similar, replacing m_t and m_t^* by 1.

The gross demand by financiers is $Q_t\,(1, -1/e_t)$. Each period the financiers sell the previous period position, so that their net demand is:

$$Q_t\left(1,-1/e_t\right)-D_tQ_{t-1}\left(1,-1/e_{t-1}\right)=\left(1-D_t\mathcal{L}\right)Q_t\left(1,-1/e_t\right), \tag{A.5}$$

where $\mathcal{L}$ is the lag operator, $\mathcal{L}X_t = X_{t-1}$.

Financiers choose Q_t optimally, given the frictions, as in the main body of the paper and we do not restate their problem here for brevity. In the last period, holdings are 0, i.e. $Q_T = 0$.

The *asset market clearing condition* is that the net demand for bonds is 0

$$\Theta_t^{firms}+\Theta_t^{firms,*}+\left(1-D_t\mathcal{L}\right)\left(\Theta_t+\Theta_t^*+Q_t\left(1,-1/e_t\right)\right)=0. \tag{A.6}$$

For instance, for consumers, $(1-D_t\mathcal{L})\,\Theta_t$ is the increased asset demand by the agent. To gain some intuition, the first coordinate of equation (A.6), evaluated at time $t=0$, in the case where $\Theta_t=\Theta_t^*=0$, gives equation (25) of the paper:

$$\xi_0 e_0 m_0^* - \iota_0 m_0 + Q_0 = 0;$$

and in the real case (corresponding to $m_0 = m_0^* = 1$), we obtain the basic equation (13) of the paper:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0.$$

We now state formally the definition of equilibrium in the case of flexible prices. Recall that we assume the law of one price in goods market to hold such that:

$$p_{Ht}^* = p_{Ht}/e_t, p_{Ft}^* = p_{Ft}/e_t. \tag{A.7}$$

Definition A competitive equilibrium consists of allocations $\left(c_t, c_t^*, \Theta_t, \Theta_t^*, \Theta_t^{firms}, \Theta_t^{firms,*}, Q_t\right)$, prices p_t, p_t^*, exchange rate e_t, for $t=0,\ldots,T$ such that the US consumers optimize their utility function (A.1) under the above constraints (A.2-A.3), Japanese consumers optimize similarly, goods markets clear (A.4), and asset markets clear (A.6), and the law of one price (A.7) holds.

As explained in the paper (Lemma 4), if we use local currency pricing (i.e. change (A.7), and replace the value of p_{Ht}^* and p_{Ft} by other, potentially arbitrary, values), the equilibrium value of the exchange rate does not change (though consumptions do change).

The timing was already stated in the paper, but for completeness we restate it here. At time 0, producers produce, consumers demand and consume, exporters repatriate their sales, financiers take their FX positions, and asset and goods market clear (simultaneously, like in Arrow-Debreu). The potential diversion by the financiers happens at time 0^+, right after time 0 (of course, no diversion happens on the equilibrium path). Then, at time 1 and potentially future periods, the same structure is repeated (with no financiers' position in the last period).

A.1.B Maximization Problem of the Japanese Household

We include there many details excluded from Section II for brevity. The dynamic budget constraint of Japanese households (which holds state by state) is:

$$\sum_{t=0}^{1}\frac{Y_{NT,t}^*+p_{F,t}^*Y_{F,t}+\pi_t^*}{R^{*t}}=\sum_{t=0}^{1}\frac{C_{NT,t}^*+p_{H,t}^*C_{H,t}^*+p_{F,t}^*C_{F,t}^*}{R^{*t}},$$

where π_t^* are the financiers' profits remittances to the Japanese, $\pi_0^*=0$, $\pi_1^*=Q_0(R-R^*e_1/e_0)/e_1$.

The static utility maximization problem of the Japanese household:

$$\max_{C_{NT,t}^*,C_{H,t}^*,C_{F,t}^*}\quad \chi_t^*\ln C_{NT,t}^*+\xi_t\ln C_{H,t}^*+a_t^*\ln C_{F,t}^*+\lambda_t^*\left(CE_t^*-C_{NT,t}^*-p_{H,t}^*C_{H,t}^*-p_{F,t}^*C_{F,t}^*\right),$$

where CE_t^* is aggregate consumption expenditure of the Japanese household, λ_t^* is the associated Lagrange multiplier, p_H^* is the Yen price in Japan of US tradables, and p_F^* is the Yen price in Japan of Japanese tradables. Standard optimality conditions imply:

$$C^*_{NT,t} = \frac{\chi^*_t}{\lambda^*_t}; \qquad p^*_{H,t}C^*_{H,t} = \frac{\xi_t}{\lambda^*_t}; \qquad p^*_{F,t}C^*_{F,t} = \frac{a^*_t}{\lambda^*_t}.$$

Our assumption that $Y^*_{NT,t} = \chi^*_t$, combined with the market clearing condition for Japanese non-tradables $Y^*_{NT,t} = C^*_{NT,t}$, implies that in equilibrium $\lambda^*_t = 1$. We obtain:

$$p^*_{H,t}C^*_{H,t} = \xi_t; \qquad p^*_{F,t}C^*_{F,t} = a^*_t.$$

A.1.C The Euler Equation when there are Several Goods

We state the general Euler equation when there are several goods.

With utility $u^t(\mathsf{C}_t) + \beta u^{t+1}(\mathsf{C}_{t+1})$, where C_t is the vector of goods consumed (for instance, $\mathsf{C}_t = (C_{NT,t}, C_{H,t}, C_{F,t})$ in our setup), if the consumer is at his optimum, we have:

Lemma A.1. *When there are several goods, the Euler equation is:*

$$1 = \mathbb{E}_t \left[\beta R \frac{u^{t+1}_{c_{j,t+1}} / p_{j,t+1}}{u^t_{c_{i,t}} / p_{i,t}} \right] \text{ for all } i, j. \tag{A.8}$$

This should be understood in "nominal" terms, i.e. the return R is in units of the (potentially arbitrary) numéraire.

Proof. It is a variant on the usual one: the consumer can consume $d\varepsilon$ fewer dollars' worth (assuming that the "dollar" is the local unit of account) of good i at time t (hence, consume $dc_{i,t} = -\frac{d\varepsilon}{p_{i,t}}$), invest them at rate R, and consume the proceeds, i.e. $Rd\varepsilon$ more dollars of good j at time $t+1$ (hence, consume $dc_{j,t+1} = \frac{Rd\varepsilon}{p_{j,t+1}}$). The total utility change is:

$$dU = u^t_{c_{i,t}} dc_{i,t} + \beta \mathbb{E}_t u^{t+1}_{c_{j,t+1}} dc_{j,t+1} = \mathbb{E}_t \left(-u^t_{c_{i,t}} / p_{i,t} + \beta R u^{t+1}_{c_{j,t+1}} / p_{j,t+1} \right) d\varepsilon.$$

At the margin, the consumer should be indifferent, so $dU = 0$, hence (A.8). □

Applying this to our setup, with $i = j = NT$, with $p_{NT,t} = 1$ and $u^t_{c_{NT,t}} = \frac{\chi_t}{C_{NT,t}} = 1$ for $t = 0, 1$, we obtain: $1 = \mathbb{E}\left[\beta R \frac{1/1}{1/1}\right]$, hence $R = 1/\beta$.

A.1.D Price Indices, Nominal and Real Exchange Rates

We explore here the relationship between the nominal and the real CPI-based exchange rate in our framework. The real exchange rate can be defined as the ratio of two broad price levels, one in each country, expressed in the same numéraire. It is most common to use consumer price indices (CPI) adjusted by the nominal exchange rate, in which case one has: $\mathcal{E} \equiv \frac{P^* e}{P}$. Notice that a fall in $\mathcal{E}$ is a US Dollar real appreciation.

Consider the nominal version of the basic Gamma model in Section IV. Standard calculations reported below imply that the real CPI-based exchange rate is:

$$\mathcal{E} = \tilde{\theta} \frac{(p^*_H)^{\xi'} (p^*_F)^{a^{*'}} (p^*_{NT})^{\chi^{*'}}}{(p_H)^{a'} (p_F)^{\iota'} (p_{NT})^{\chi'}} e_t, \tag{A.9}$$

where $\tilde{\theta}$ is a function of exogenous shocks and primed variables are normalized by θ. The above equation is the most general formulation of the relationship between the CPI-RER and the nominal exchange rate in the Gamma model.

Let us first derive the price indices $\{P, P^*\}$. The US price index P is defined as the minimum cost, in units of the numéraire (money), of obtaining one unit of the consumption basket:

$$C_t \equiv \left[\left(\frac{M_t}{P_t} \right)^{\omega_t} (C_{NT,t})^{\chi_t} (C_{H,t})^{a_t} (C_{F,t})^{\iota_t} \right]^{\frac{1}{\theta_t}}.$$

Let us define a "primed" variable as being normalized by the sum of the preference coefficients θ_t; so that, for example, $\chi_t^{'} \equiv \frac{\chi_t}{\theta_t}$. Substituting the optimal demand for goods (see the first order conditions at the beginning of Section IV) in the consumption basket formula we have:

$$1 = \left(\omega^{'} P \right)^{\omega^{'}} \left(a^{'} \frac{P}{p_H} \right)^{a^{'}} \left(\iota^{'} \frac{P}{p_F} \right)^{\iota^{'}} \left(\chi^{'} \frac{P}{p_{NT}} \right)^{\chi^{'}}.$$

Hence:

$$P = (p_H)^{a'} (p_F)^{\iota'} (p_{NT})^{\chi'} \left[(\omega_t^{'})^{-\omega_t^{'}} (\iota_t^{'})^{-\iota_t^{'}} (a_t^{'})^{-a_t^{'}} (\chi_t^{'})^{-\chi_t^{'}} \right].$$

The part in square brackets is a residual and not so interesting. Similarly for Japan, we have:

$$P^* = (p_H^*)^{\xi'} (p_F^*)^{a^{*'}} (p_{NT}^*)^{\chi^{*'}} \left[(\omega_t^{*'})^{-\omega_t^{*'}} (\xi_t^{'})^{-\xi_t^{'}} (a_t^{*'})^{-a_t^{*'}} (\chi_t^{*'})^{-\chi_t^{*'}} \right].$$

The CPI-RER in equation (A.9) is then obtained by substituting the price indices above in the definition of the real exchange rate $\mathcal{E} \equiv \frac{P^* e}{P}$. For completeness, we report below the full expression for the function $\tilde{\theta}$ that enters in equation (A.9):

$$\tilde{\theta}_t = \frac{(\omega_t^{*'})^{-\omega_t^{*'}} (\xi_t^{'})^{-\xi_t^{'}} (a_t^{*'})^{-a_t^{*'}} (\chi_t^{*'})^{-\chi_t^{*'}}}{(\omega_t^{'})^{-\omega_t^{'}} (\iota_t^{'})^{-\iota_t^{'}} (a_t^{'})^{-a_t^{'}} (\chi_t^{'})^{-\chi_t^{'}}}.$$

If we impose further assumptions on Equation (A.9), we can derive some useful special cases.

The Basic Gamma Model Assume that $\omega = \omega^* = 0$ and $p_{NT} = p_{NT}^* = 1$ so that there is no money and the numéraire in each economy is the non-tradable good. Recall that in the basic Gamma model of Section II the law of one price holds for tradables, so we have $p_H = p_H^* e$ and $p_F = p_F^* e$. Equation (A.9) then reduces to: $\mathcal{E} = \tilde{\theta}\, (p_H)^{\xi' - a'} (p_F)^{a^{*'} - \iota'} e^{\chi^{*'}}$. This equation describes the relationship between the RER as defined in the basic Gamma model and the CPI-based RER. Notice that the two are close proxies of each other whenever the baskets' shares of tradables are symmetric across countries (i.e. $\xi^{'} \approx a^{'}$ and $a^{*'} \approx \iota^{'}$) and the non-tradable goods are a large fraction of the Japanese overall basket (i.e. $\chi^{*'} \approx 1$).

The Basic Complete Market Model We maintain all the assumptions from the paragraph above on the Basic Gamma model, except that we now assume markets to be complete and frictionless. Recall from Lemma 3 that we then obtain $e_t = \nu$. Hence, the CPI-RER now follows: $\mathcal{E} = \tilde{\theta}\, (p_H)^{\xi' - a'} (p_F)^{a^{*'} - \iota'} \nu^{\chi^{*'}}$. Notice that while the real exchange rate (e) is constant in complete markets in the basic Gamma model, the CPI-RER will in general not be constant as long as the CPI baskets are not symmetric and relative prices of goods move.

A.1.E The Backus and Smith Condition

In the spirit of re-deriving some classic results of international macroeconomics with the Gamma model, let us analyze the Backus and Smith condition (Backus and Smith (1993)). Let us first consider the basic Gamma set-up but with the additional assumption of complete markets as in Lemma 3. Then by equating marginal utility growth in the two countries and converting, via the exchange rate, in the same units, we have: $\frac{P_0 C_0/\theta_0}{P_1 C_1/\theta_1} = \frac{P_0^* C_0^*/\theta_0^*}{P_1^* C_1^*/\theta_1^*}\frac{e_0}{e_1}$. Re-arranging we conclude:

$$\frac{C_0/\theta_0}{C_1/\theta_1} = \frac{C_0^*/\theta_0^*}{C_1^*/\theta_1^*}\frac{\mathcal{E}_0}{\mathcal{E}_1}, \tag{A.10}$$

where the reader should recall the definition $\mathcal{E} = \frac{P^* e}{P}$. This is the Backus and Smith condition in our set-up under complete markets: the perfect risk sharing benchmark equation.

Of course, this condition fails in the basic Gamma model because agents not only cannot trade all Arrow-Debreu claims, but also have to trade with financiers in the presence of limited commitment problems. In our framework (Section II), however, an extended version of this condition holds:

$$\frac{C_0/\theta_0}{C_1/\theta_1} = \frac{C_0^*/\theta_0^*}{C_1^*/\theta_1^*}\frac{\mathcal{E}_0}{\mathcal{E}_1}\frac{e_1}{e_0}. \tag{A.11}$$

The simple derivation of this result is reported below. The above equation is the extended Backus-Smith condition that holds in our Gamma model. Notice that our condition in equation (A.11) differs from the standard Backus-Smith condition in equation (A.10) by the growth rate of the "nominal" exchange rate $\frac{e_1}{e_0}$. Since exchange rates are much more volatile in the data than consumption, this omitted term creates an ample wedge between the complete market and the Gamma version of the Backus-Smith condition.

The condition in equation (A.11) can be verified as follows:

$$\frac{C_0/\theta_0}{C_1/\theta_1} = \frac{C_0^*/\theta_0^*}{C_1^*/\theta_1^*}\frac{\mathcal{E}_0}{\mathcal{E}_1}\frac{e_1}{e_0} \iff \frac{P_0 C_0/\theta_0}{P_1 C_1/\theta_1} = \frac{P_0^* C_0^*/\theta_0^*}{P_1^* C_1^*/\theta_1^*} \iff \frac{1}{1} = \frac{1}{1},$$

where the first equivalence simply makes use of the definition $\mathcal{E} \equiv \frac{P^* e}{P}$, and the second equivalence follows from $P_t C_t = \theta_t$ and $P_t^* C_t^* = \theta_t^*$ for $t = 0, 1$. These latter equalities (we focus here on the US case) can be recovered by substituting the households' demand functions for goods in the static household budget constraint: $P_t C_t = C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t} = \chi_t + a_t + \iota_t = \theta_t$.

A.2 Extensions of the Model

A.2.A Japanese Households and the Carry Trade

In most of the main body of the paper, consumers do not do the carry trade themselves. In this subsection, we extend Proposition 6 by analyzing the case in which Japanese consumers buy a quantity f^* of dollar bonds, financing the purchase by shorting an equivalent amount of Yen bonds. We let this demand take the form:

$$f^* = b\left(R - R^*\right).$$

Recall that Proposition 6 assumes $R < R^*$, so that if $b \geq 0$ the Japanese household demand is a form of carry trade. The flow equations now are:

$$NX_0 + Q + f^* = 0; \qquad NX_1 - R\left(Q + f^*\right) = 0.$$

We summarize the implications for the equilibrium carry trade in the Proposition below.

Proposition A.1. *Assume $\xi_t = 1$ for $t = 0, 1$, $R < R^*$ and that Japanese consumers do the carry trade in amount f^*, the expected return to the carry trade in the Gamma model is:*

$$\overline{R}^c = \Gamma \frac{\frac{R^*}{R}\mathbb{E}\left[\iota_1\right] - \iota_0 + f^*(1 + R^*)}{(R^* + \Gamma)\,\iota_0 + \frac{R^*}{R}\mathbb{E}\left[\iota_1\right] - \Gamma f^*}.$$

Hence the carry trade return is bigger: (i) when R^*/R is higher, (ii) when the funding country is a net foreign creditor, and (iii) when consumers do the carry trade less (f^* increases).

If consumers do the carry trade on too large a scale (f^* too negative), then the carry trade becomes unprofitable, $\overline{R}^c < 0$.

A.2.B Endogenizing the Number of Financiers

In the basic model, there is a fixed quantity of financiers. We now show a possible way to endogenize entry of financiers. This will confirm that the first order results of the paper are unchanged, except that γ is now endogenous.

We call $\Omega = \mathbb{E}_0\left[1 - \frac{e_1}{e_0}\frac{R^*}{R}\right]$ the expected discounted return of currency trading. Suppose that each potential trader has an incentive constraint of the form:

$$V_0 \equiv \Omega q_0 = \mathbb{E}_0\left[\beta\left(R - R^*\frac{e_1}{e_0}\right)\right] q_0 \geq G\frac{q_0^2}{e_0},$$

and we have $G = g\left(var_0\left(e_1\right)\right)^{\alpha}$ for a parameter g. Hence g and G are the agent's γ and Γ. Using $R\beta = 1$, this entails an individual demand:

$$q_0 = \frac{\Omega e_0}{G},$$

and a benefit

$$V_0 = \Omega q_0 = \frac{\Omega^2 e_0}{G} = \frac{\Omega^2 e_0}{g\; var_0\left(e_1\right)^{\alpha}}.$$

In the spirit of Jeanne and Rose (2002), we posit that financiers decide to enter at date -1 (before the values of $\iota_0, \mathbb{E}_0\left[\iota_1\right]$ are realized, hence before the actual expected currency trading return is known). Potential financier i enters if and only if $\mathbb{E}_{-1}\left[V_0\right] \geq \kappa_i$, where κ_i is a (perhaps psychological) cost drawn from a distribution with CDF $F\left(x\right) = P\left(\kappa_i \leq x\right)$. This implies that the mass n of financiers is

$$n = F\left(\mathbb{E}_{-1}\left[\Omega^2 e_0 var_0\left(e_1\right)^{-\alpha}\right]/g\right).$$

The aggregate demand at time 0 is then:

$$Q_0 = nq_0 = n\frac{\Omega e_0}{g\; var_0\left(e_1\right)^{\alpha}}.$$

Hence, we have $Q_0 = \frac{\Omega e_0}{\gamma\; var_0\left(e_1\right)^{\alpha}}$ with

$$\gamma = \frac{g}{n}, \tag{A.12}$$

so that

$$\gamma = \frac{g}{F\left(\mathbb{E}_{-1}\left[\Omega^2 e_0 var_0\left(e_1\right)^{-\alpha}\right]/g\right)}. \tag{A.13}$$

Hence, we have a fixed point determining γ, since e_0 and e_1 depend on γ.

Starting after date 0, the analysis is exactly like in the paper, except that the value of γ is pinned down by considerations at time -1.

For instance, take our baseline case, where $R = R^* = 1$. From Proposition 1, $e_0 = \frac{(1+\Gamma)\iota_0 + \mathbb{E}_0[\iota_1]}{2+\Gamma}$, $\Omega = \frac{\Gamma(\iota_0 - \mathbb{E}_0[\iota_1])}{(1+\Gamma)\iota_0 + \mathbb{E}_0[\iota_1]}$, and γ solves:

$$g = \gamma F\left(\mathbb{E}_{-1}\left[\frac{\Gamma\left(\gamma\right)^2\left(\iota_0 - \mathbb{E}_0\left[\iota_1\right]\right)^2 var_0\left(e_1\right)^{-\alpha}}{\left[\left(1+\Gamma\left(\gamma\right)\right)\iota_0 + \mathbb{E}_0\left[\iota_1\right]\right]\left(2+\Gamma\left(\gamma\right)\right)g}\right]\right) \text{ with } \Gamma\left(\gamma\right) = \gamma\; var_0\left(\iota_1\right)^{\alpha}. \tag{A.14}$$

We note that the government might wish to subsidize entry in the financial sector so to effectively remove the financial constraint. This is a property common to many models of financial imperfections: for example if the financiers had limited capital as in Kiyotaki and Moore (1997); Gertler and Kiyotaki (2010); Brunnermeier and Sannikov (2014); He and Krishnamurthy (2013), the government would want to recapitalize them in many states of the world.[59] Like those papers, we do not consider the optimal subsidy to financiers. One reason for this is that in practice, it is difficult as the government might be facing frictions with the financiers such as moral hazard or adverse selection. For example, the government might want to screen for "smart" FX traders that stabilize FX markets, and not subsidize noise traders, who might actually worsen the situation (they would be creating f, f^* shocks in our model).[60]

A.2.C A "Short-Run" Vs "Long-Run" Analysis

As in undergraduate textbooks, it is handy to have a notion of the "long run". We develop here a way to introduce it in our model. We have periods of unequal length: we say that period 0 is short, but period "1" lasts for a length T. The equilibrium flow equations in the dollar-yen market become:

$$\begin{aligned} \xi_0 e_0 - \iota_0 + Q_0 &= 0, \\ T(\xi_1 e_1 - \iota_1) - RQ_0 &= 0. \end{aligned} \tag{A.15}$$

The reason for the "T" is that the imports and exports will occur over T periods. We assume a zero interest rate "within period 1". This already gives a good notion of the "long run".[61]

Some extra simplicity is obtained by taking the limit $T \to \infty$. The interpretation is that period 1 is "very long" and period 0 is "very short". The flow equation (A.15) can be written: $\xi_1 e_1 - \iota_1 - \frac{RQ_0}{T} = 0$. So in the large T limit we obtain: $\xi_1 e_1 - \iota_1 = 0$. Economically, it means the trades absorbed by the financiers are very small compared to the trades in the goods markets in the long run. We summarize the environment and its solution in the following proposition.[62]

Proposition A.2. *Consider a model with a "long-run" last period. Then, the flow equations become* $\xi_0 e_0 - \iota_0 + Q_0 = 0$ *and* $\xi_1 e_1 - \iota_1 = 0$, *while we still have* $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right]$. *The exchange rates become:*

$$e_0 = \frac{\frac{R^*}{R}\mathbb{E}\left[\frac{\iota_1}{\xi_1}\right] + \Gamma \iota_0}{1 + \Gamma \xi_0}; \qquad e_1 = \frac{\iota_1}{\xi_1}.$$

In this view, the "long run" is determined by fundamentals $e_1 = \frac{\iota_1}{\xi_1}$, while the "short run" is determined both by fundamentals and financial imperfections (Γ) with short-run considerations (ι_0, ξ_0). In the simple case $R = R^* = \xi_t = 1$, we obtain: $e_0 = \frac{\Gamma\iota_0 + \mathbb{E}[\iota_1]}{\Gamma+1}$ and $e_1 = \iota_1$.

Application to the carry trade with three periods. In the 3-period carry trade model of Section III.A, we take period 2 to be the "long run". We assume that in period $t = 1$ financiers only intermediate the new flows; stocks arising from previous flows are held passively by the households (long term investors) until t=2. That allows us to analyze more clearly the dynamic environment. Without the "long-run" period 2, the expressions are less intelligible, but the economics is the same.

---

[59] We thank a referee for remarks along these lines.

[60] With endogenous entry, the FX intervention considered in Section III.B will also ex ante affect entry, similarly to the analysis in Jeanne and Rose (2002). We leave this interesting analysis to future research.

[61] The solution is simply obtained by Proposition 3, setting $\widetilde{\iota}_1 = T\iota_1, \widetilde{\xi}_1 = T\xi_1$.

[62] One derivation is as follows. Take Proposition 3, set $\widetilde{\iota}_1 = T\iota_1, \widetilde{\xi}_1 = T\xi_1$, and take the limit $T \to \infty$.

A.2.D The Fama Regression over Longer Horizons

We take the context of the Fama regresssion in the paper, and now consider the Fama regression over a 2-period horizon:

$$\frac{1}{2}\frac{e_2 - e_0}{e_0} = \alpha + \beta_{UIP,2}\left(R - R^*\right) + \varepsilon_1$$

i.e. regressing (normalized) the 2-period return on the interest rate differential. We assume that Γ_1 is deterministic.

Lemma A.2. *The coefficient* $\beta_{UIP,2} = \frac{1+\Gamma_1/2}{(1+\Gamma_0)(1+\Gamma_1)}$*, while the UIP coefficient in a 1-period regression is* $\beta_{UIP,1} \equiv \beta_{UIP} = \frac{1+\Gamma_1-\Gamma_0}{(1+\Gamma_0)(1+\Gamma_1)}$*, as in the main text.*

Proof We evaluate the derivative at $R = R^* = 1$, and for simplicity take the case Γ_1 deterministic.

$$\begin{aligned}
\beta_{UIP,2} &= \frac{-1}{2}\frac{\partial \mathbb{E}\left[\frac{e_2-e_0}{e_0}\right]}{\partial R^*} = \frac{-1}{2}\frac{\partial}{\partial R^*}\frac{\Gamma_0+1}{\Gamma_0\iota_0 + \mathcal{R}^*\mathbb{E}\left[\frac{\Gamma_1\iota_1+\mathcal{R}^*\iota_2}{\Gamma_1+1}\right]} \\
&= \frac{1}{2}\frac{\Gamma_0+1}{(\Gamma_0+1)^2}\left(\mathbb{E}\left[\frac{\Gamma_1\iota_1+\iota_2}{\Gamma_1+1}\right] + \mathbb{E}\left[\frac{\iota_2}{\Gamma_1+1}\right]\right)] = \frac{1}{2}\frac{1}{\Gamma_0+1}\left(1+\mathbb{E}\left[\frac{1}{\Gamma_1+1}\right]\right) \\
&= \frac{1}{2}\frac{2+\Gamma_1}{(\Gamma_0+1)(1+\Gamma_1)} = \frac{1+\Gamma_1/2}{(1+\Gamma_0)(1+\Gamma_1)}.
\end{aligned}$$

□

Hence, we have $1 \geq \beta_{UIP,2}$ as often found empirically. Furthermore, we have $\beta_{UIP,2} \geq \beta_{UIP,1}$ if and only if $\Gamma_1 \leq 2\Gamma_0$. For instance, suppose that Γ_1 and Γ_0 are drawn from the same distribution. Then, $\mathbb{E}\left[\beta_{UIP,2}\right] \geq \mathbb{E}\left[\beta_{UIP,1}\right]$: this means that as the horizon expands, the coefficient of the Fama regression is closer to 1. This is consistent with the empirical evidence that the Fama regression coefficient is higher, and closer to 1, at long horizons (Chinn and Meredith (2005)).

A.3 Model Extensions: Multi-Country, Multi-Asset Model, and Additional Material on the Variance in the Constraint

We provide below generalizations of the model. In particular, we develop a multi-asset, multi-country model.

A.3.A Verification of the tractability of the model when the variance is in the constraint

In the paper, we propose a formulation of $\Gamma = \gamma var(e_1)^\alpha$. We verify that it leads to a tractable model in the core parts of the paper. In this subsection of the appendix, we check that we also keep a tractable model in a more general model with T periods.

When ξ_t is deterministic, the formulation remains tractable. We obtain each Γ_t in closed form.[63] Let us

[63]However, when ξ_t is stochastic, the formulation is more complex. We obtain a fixed point problem not just in Γ_0 (like in the 2-period model), but in $(\Gamma_{0,...,}\Gamma_{T-1})$.

work out explicitly a 3-period example. We take $\xi_t = R = R^* = 1$ for simplicity. The equations are:

$$
\begin{aligned}
e_0 - \iota_0 + Q_0 &= 0, \\
e_1 - \iota_1 - Q_0 + Q_1 &= 0, \\
e_2 - \iota_2 - Q_1 &= 0, \\
Q_t &= \frac{\mathbb{E}_t\left[e_t - e_{t+1}\right]}{\Gamma_t} \text{for } t = 0, 1, \\
\Gamma_t &= \gamma\, var_t\left(e_{t+1}\right)^{\alpha}.
\end{aligned}
$$

Notice that the model at $t = 1, 2$ is like the basic model with 2 periods, except for the pseudo-import term $\widetilde{\iota}_1 = \iota_1 - Q_1$. Hence, we have $\{e_2\} = \{\iota_2\}$, and

(A.16) $$\Gamma_1 = \gamma \sigma_{\iota_2}^{2\alpha}.$$

This also implies that (by Proposition 3 applied to (e_1, e_2) rather than (e_0, e_1)): $\{e_1\} = \frac{1+\Gamma_1}{2+\Gamma_1}\{\iota_1\}$, which gives:

(A.17) $$\Gamma_0 = \gamma\left(\frac{1+\Gamma_1}{2+\Gamma_1}\sigma_{\iota_1}\right)^{2\alpha},$$

so we endogenize Γ_0. Note that the σ_{ι_1} is, in general, the variance of pseudo-imports, hence it would include the volatility due to financial flows. Notice also that fundamental variance is endogenously amplified by the imperfect financial market: $var(e_1)$ depends positively on Γ_1, that itself depends positively on fundamental variance.

The same idea and procedure applies to an arbitrary number of periods, and indeed to the infinite period model. We could also have correlated innovations in ι_t.

A.3.B A tractable multi-country model

We call e_i^t the exchange rate of country i at date t, with a high e_i^t being an appreciation of country i's currency versus the USD. There is a central country 0, for which we normalize $e_0^t = 1$ at all dates t. As a short hand, we call this country "the US". For $i \neq j$, call $\xi_{ij} < 0$ exports of country i to country j (*minus* the Cobb-Douglas weight), and $x_i = -\xi_{i0} > 0$ exports of country i to country 0. Define the import weight as:

$$\xi_{ii} \equiv -\sum_{j=0,\ldots,n, j\neq i} \xi_{ji} > 0,$$

so that ξ_{ii} equals total imports of country i. Call θ_i the holdings of country i's bonds by financiers, expressed in number of bonds: so, the dollar value of those bond holdings is $q_i \equiv \theta_i e_i^0$.

Hence, the net demand for currency i in the currency i / USD spot market, expressed in dollars, is:

(A.18) $$-\sum_{j\neq 0} \xi_{ij}^0 e_j^0 + x_i^0 + \theta_i e_i^0, = 0,$$

and has to be 0 in market equilibrium. Indeed, at time 0 the country imports a dollar value $\xi_{ii}^0 e_i^0$, creating a negative demand $-\xi_{ii}^0 e_i^0$ for the currency. It also exports a dollar value $-\sum_{j\neq 0} \xi_{ij}^0 e_j^0 + x_i^0$ (recall that $\xi_{ij} < 0$ for $i \neq j$); as those exports are repatriated, they lead to a demand for the currency. Finally, financiers demand a dollar value $\theta_i e_i^0$ of the country's bonds. Using $q_i \equiv \theta_i e_i^0$, equation (A.18) can be rewritten in vector form:

(A.19) $$-\xi^0 e^0 + x^0 + q = 0.$$

The flow equation at time $t = 1$ is (again, net demand for currency i in the dollar-currency i market, expressed in dollars):

$$-\sum_{j\neq 0}\xi^1_{ij}e^1_j + x^1_i - \theta_i e^1_i + \Pi_i = 0 \tag{A.20}$$

where Π_i is the time-1 rebate of financiers profits to country i. In the first equation, imports enter as $-\xi^0_{ii}e^0_i < 0$, creating a net negative demand for currency i, and exports to other countries enter as $-\sum_{j\neq 0,i}\xi^0_{ij}e^0_j > 0$.

Total financiers' profit is: $\Pi \equiv \sum_i \Pi_i = \sum_i \theta_i\left(e^1_i - e^0_i\right)$. We posit the following rule for the rebate Π_i to country i: $\Pi_i = \theta_i\left(e^1_i - e^0_i\right)$. Then, (A.20) becomes: $-\sum_{j\neq 0}\xi^1_{ij}e^1_j + x^1_i - \theta_i e^0_i = 0$, i.e., in vector form:

$$-\xi^1 e^1 + x^1 - q = 0. \tag{A.21}$$

Finally, we will have the generalized demand for assets:

$$q = \Gamma^{-1}\mathbb{E}\left[e^1 - e^0\right], \tag{A.22}$$

where q, e^t are vectors, and Γ is a matrix. We provide a derivation of this demand in section A.3.C. The financiers buy a dollar value q_i of country i's bonds at time 0, and $-\sum_{i=1}^n q_i$ dollar bonds, so that the net time-0 value of their initial position is 0. The correspondence with the basic Gamma model (with only 2 countries) is $q = -Q$, $x_{it} = \iota_{it}$.

We summarize the set-up below.

Lemma A.3. *In the extended n-country model, the basic equations describing the vectors of exchange rates e^t are:*

$$\xi^0 e^0 - x^0 - q = 0, \tag{A.23}$$

$$\xi^1 e^1 - x^1 + q = 0, \tag{A.24}$$

$$\mathbb{E}\left[e^1 - e^0\right] = \Gamma q. \tag{A.25}$$

Those are exactly the equations of the 2−country model (with $Q^{\text{Gamma}} = -q^{\text{here}}$), and $\iota^{\text{Gamma}} = x^{\text{here}}$, but with n countries (so $e^t \in \mathbb{R}^{n-1}$). Hence the solution is the same (using matrices). We assume that ξ^1 is deterministic.

Proposition A.3. *The exchange rates in the n−country model are given by the following vectors:*

$$e^0 = \left(\xi^0 + \xi^1 + \xi^1\Gamma\xi^0\right)^{-1}\left(\left(1 + \xi^1\Gamma\right)x^0 + \mathbb{E}\left[x^1\right]\right), \tag{A.26}$$

$$e^1 = \left(\xi^0 + \xi^1 + \xi^0\Gamma\xi^1\right)^{-1}\left(x^0 + \left(1 + \xi^0\Gamma\right)\mathbb{E}\left[x^1\right]\right) + \left(\xi^1\right)^{-1}\left\{x^1\right\}. \tag{A.27}$$

Hence, the above model has networks of trade in goods, and multi-country asset demand.

A.3.C Derivation of the multi-asset, multi-country demand

We derive the financiers' demand function in a multi-asset case. We start with a general asset case, and then specialize our results to exchange rates.

A.3.C.1 General asset pricing case

Basic case We use notations that are valid in general asset pricing, as this makes the exposition clearer and more general. We suppose that there are assets $a = 1, \ldots, A$, with initial price p^0, and period 1 payoffs p^1 (all in $\mathbb{R}^A$). Suppose that the financiers hold a quantity position $\theta \in \mathbb{R}^A$ of those assets, so that the terminal value is $\theta \cdot p^1$. We want to compute the equilibrium price at time 0.

Let

$$\pi = \mathbb{E}\left[p^1\right] - p^0,$$

denote the expected gain (a vector), and

$$V = var\left(p^1\right),$$

denote the variance-covariance matrix of period 1 payoffs.

Given a matrix G, our demand will generate the relation

$$\pi = G\theta^*, \tag{A.28}$$

This is a generalization to an arbitrary number of assets of the basic demand of Lemma 2, $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right]$. The traditional mean-variance case is $G = \gamma V$. The present machinery yields more general terms: for example, we could have $G = VH'$, for a "twist" matrix H. The mean-variance case is $H = \gamma I_n$, for a risk-aversion scalar γ. The H can, however, represent deviations from that benchmark, e.g. source-dependent risk aversion (if $H = diag\left(\gamma_1, ..., \gamma_A\right)$, we have a "risk aversion" scalar γ_a for source a), or tractability-inducing twists (our main application here). Hence, the machinery we develop here will allow to go beyond the traditional mean-variance setup.

The financiers' profits (in dollars) are: $\theta \cdot \left(p^1 - p^0\right)$, and their expected value is $\theta'\pi$, where $\pi := \mathbb{E}\left[p^1\right] - p^0$. We posit that financiers solve:

$$\max_{\theta \in R^A} \theta'\pi \text{ s.t. } \theta'\pi \geq \theta' S\theta,$$

where S is a symmetric, positive semi-definite matrix. This is a limited commitment constraint: the financiers' outside option is $\theta' S\theta$. Hence, the incentive-compatibility condition is $\theta'\pi \geq \theta' S\theta$. Again, this is a generalization (to an arbitrary number of assets) of the constraint in the paper in Equation (8).

The problem implies:

$$\pi = S\theta^*,$$

where θ^* is the equilibrium θ.[64]

Hence, we would deliver (A.28) if we could posit $S = G$. However, this is not exactly possible, because S must be symmetric, and G is not necessarily symmetric.

We posit that the outside option $\theta' S\theta$ equals:[65]

$$\theta' S\theta \equiv \sum_{i,j} \theta_i^2 \frac{1_{\theta_i^* \neq 0}}{\theta_i^*} G_{ij}\theta_j^*, \tag{A.29}$$

where θ is chosen by the financier under consideration, and θ^* is the equilibrium demand of *other* financiers (in equilibrium, $\theta = \theta^*$). This functional form captures the fact that as the portfolio or balance sheet expands (θ_i high), it is "more complex" and the outside option of the financiers increases. In addition (if say $G = \gamma V$), it captures that high variance assets tighten the constraint more (perhaps again because they are more "complex" to monitor). The non-diagonal terms indicate that "similar" assets (as measured by covariance) matter. Finally, the positions of other financiers matter. Mostly, this assumption is made for convenience. However, it captures the idea (related to Basak and Pavlova (2013)) that the positions of other traders influence the portfolio choice of a given trader. The influence here is mild: when G is diagonal, there is no influence at all.

We will make the assumption that

$$\forall i,\ sign\left(\pi_i^*\right) = sign\left(\theta_i^*\right), \text{ where } \pi^* \equiv G\theta^*. \tag{A.30}$$

This implies that S is a positive semi-definite matrix: for instance, when $\theta_i^* \neq 0$, $\sum_j \frac{1}{\theta_i^*} G_{ij}\theta_j^* \geq 0$. Equation (A.30) means that the sign of the position θ_i^* is equal to the sign of the expected return π_i. This is a mild

[64]The proof is as follows. Set up the Lagrangian $\mathcal{L} = \theta'\pi + \lambda\left(\theta'\pi - \theta' S\theta\right)$. The first-order condition reads $0 = \mathcal{L}_{\theta'} = (1+\lambda)\pi - 2\lambda S\theta$. So, $\pi = \frac{2\lambda}{1+\lambda} S\theta$. Left-multiplying by θ' yields $\theta'\pi = \frac{2\lambda}{1+\lambda}\theta' S\theta$. Since $\theta'\pi \geq \theta' S\theta$, we need $\lambda \geq 1$. Hence, $\pi = S\theta$.

[65]This is, $S_{ij} = 1_{i=j}\frac{1}{\theta_i^*} G_{ij}\theta_j^*$ if $\theta_i^* \neq 0$, $S_{ij} = 0$ if $\theta_i^* = 0$.

assumption that rules out situations where hedging terms are very large.

We summarize the previous results. Recall that we assume (A.30).

Proposition A.4. *(General asset pricing case: foundation for the financiers' demand) With the above set-up, the financiers' equilibrium holdings θ^* satisfy:*

$$\mathbb{E}\left[p^1 - p^0\right] = G\theta^*, \tag{A.31}$$

with G a matrix. When G is invertible, we obtain the demand $\theta^* = G^{-1}\mathbb{E}\left[p^1 - p^0\right]$.

Proof: First, take the case $\theta_i^* \neq 0$. Deriving (A.29) w.r.t. θ_i: $2\,(S\theta)_i = \sum_j \frac{2\theta_i}{\theta_i^*} G_{ij}\theta_j^*$, so that $(S\theta^*)_i = \sum_j G_{ij}\theta_j^* = (G\theta^*)_i$. When $\theta_i^* = 0$, assumption (A.30) implies again $(S\theta^*)_i = \sum_j S_{ij}\theta_j^* = 0 = \pi_i^* = (G\theta^*)_i$.

Thus, $S\theta^* = G\theta^*$. Hence, the set-up induces $\pi = S\theta^* = G\theta^*$. □

Proposition A.5. *Suppose that we can write $G = VH'$, for some matrix H. Then, a riskless portfolio simply offers the riskless US return, and in that sense the model is arbitrage-free.*

Proof: Suppose that you have a riskless, 0-investment portfolio κ: $\kappa' V = 0$. Given $\pi = VH'\theta^*$, we have $\kappa'\pi = \kappa' G\theta^* = \kappa' VH'\theta^* = 0$, i.e. the portfolio has 0 expected return, hence, as it is riskless, the portfolio has 0 return. □

Proposition A.7 offers a stronger statement that the model is arbitrage-free.

A.3.C.2 Extension with derivatives and other redundant assets

The reader may wish to initially skip the following extension. When there are redundant assets (like derivatives), some care needs to be taken when handling indeterminacies (as many portfolios are functionally equivalent). Call Θ the full portfolio, including redundant assets, and P^t the full price vector. We say that assets $a \leq B$ are a basis, and we reduce the portfolio Θ into its "basis-equivalent" portfolio in the basis, $\theta \in \mathbb{R}^B$, with price p^t, defined by:

$$\Theta \cdot P^1 = \theta \cdot p^1, \quad \text{for all states of the world.}$$

For instance, if asset c is redundant and equal to asset a minus asset b ($p_c^1 = p_a^1 - p_b^1$), then $(\theta_a, \theta_b) = (\Theta_a + \Theta_c, \Theta_b - \Theta_c)$.

More generally, partition the full portfolio into basis assets Θ_B and derivative assets Θ_D, $\Theta = (\Theta_B, \Theta_D)$, and similarly partition prices in $P = (p, p_D)$. We sometimes write p_B rather than p when this clarifies matters. As those assets are redundant, there is a matrix Z such that

$$p_D^1 = Zp^1.$$

Then, the basis-equivalent portfolio is $\theta = \Theta_B + Z'\Theta_D$.[66]

Then, we proceed as above, with the "basis-equivalent portfolio". This gives the equilibrium pricing of the basis assets, p_B^0. Then, derivatives are priced by arbitrage:

$$p_D^0 = Zp^0,$$

A.3.C.3 Formulation with a Stochastic Discount Factor

The following section is more advanced, and may be skipped by the reader.

It is often useful to represent pricing via a Stochastic Discount Factor (SDF). Let us see how to do that here. Call $w = P_1'\Theta = p_1'\theta$ the time-1 wealth of the financiers. Recall that we have $\pi = G\theta^*$, with $G = VH'$.

If we had traditional mean-variance preferences, with $\pi = \gamma V\theta^*$, we could use a SDF: $M = 1 - \gamma\{w\}$, for a scalar γ. We want to generalize that idea.

[66] Proof: the payoffs are $\Theta' P^1 = \Theta_B' p^1 + \Theta_D' p_D^1 = \Theta_B' p^1 + \Theta_D' Z p_B^1 = \theta' p_B^1$ with $\theta' = \Theta_B' + \Theta_D' Z$.

As before, we define

$$\{X\} \equiv X - \mathbb{E}[X]$$

to be the innovation to a random variable X.

Recall that we are given B basis assets $a = 1, \ldots, B$ (i.e., $\left(\{p_a^1\}\right)_{a=1,\ldots,B}$ are linearly independent), while assets $a = B+1, ..., A$ are derivatives (e.g. forward contracts), and so their payoffs are spanned by the vector $\left(p_a^1\right)_{a \le B}$.

Next, we choose a linear operator Ψ for the basis assets that maps random variables into random variables.[67] It is characterized by:

$$\Psi\left\{p_a^1\right\} = \sum_b H_{ab}\left\{p_b^1\right\} \text{ for } a = 1, \ldots, B, \text{ and for } b = 1, \ldots, B,$$

or, more compactly:

$$\Psi\left\{p^1\right\} = H\left\{p^1\right\}.$$

This is possible because $\left\{p_1^a\right\}$ are linearly independent. The operator extends to the whole space S of traded assets (including redundant assets).

Proposition A.6. *The pricing is given by the SDF:*

$$M = 1 - \Psi\{w\}, \tag{A.32}$$

where $w = P_1'\Theta = p_1'\theta$ is the time-1 wealth of the financiers.

Proposition A.7. *If the shocks $\left\{p^1\right\}$ are bounded and the norm of matrix H, $\|H\|$, is small enough, then $M > 0$ and the model is arbitrage-free.*

In addition, it shows that the SDF depends linearly on the agents' total terminal wealth w, including their proceeds from positions in derivatives.

Proof. We need to check that this SDF generates: $p^0 = \mathbb{E}\left[Mp^1\right]$. Letting $M = 1 - m$ with $m = \Psi\{w\}$, we need to check that $p^0 = \mathbb{E}\left[p^1 - mp^1\right] = \mathbb{E}\left[p^1\right] - \mathbb{E}\left[mp^1\right]$, i.e. $\pi := \mathbb{E}\left[p^1 - p^0\right] = \mathbb{E}\left[mp^1\right]$. Recall that we have $\pi = G\theta = VH'\theta$.

Hence, we compute:

$$\begin{aligned}
\mathbb{E}\left[mp_a^1\right] &= \mathbb{E}\left[p_a^1\left(\Psi\{w\}\right)\right] \\
&= \mathbb{E}\left[p_a^1 \sum_{b,c} \theta_c H_{cb}\left\{p_b^1\right\}\right] = \sum_{b,c} \mathbb{E}\left[p_a^1\left\{p_b^1\right\}\right] H_{cb}\theta_c \\
&= \sum_{b,c} V_{ab}\left(H'\right)_{bc}\theta_c = \left(VH'\theta\right)_a
\end{aligned}$$

i.e., indeed, $\mathbb{E}\left[mp^1\right] = VH'\theta = \pi$. □

A.3.C.4 Application to the FX case in a multi-country set-up

We now specialize the previous machinery to the FX case. In equilibrium, we will indeed have (with $q = (q_i)_{i=1,\ldots,n}$):

$$q = \Gamma^{-1}\mathbb{E}\left[e^1 - e^0\right], \tag{A.33}$$

[67] Mathematically, call S the space of random payoffs spanned by (linear combinations of) the traded assets, $\left(p_a^1\right)_{a=1,\ldots,B}$. S is a subset of $L^2(\Omega)$, where Ω is the underlying probability space. Ψ: $S \to S$ is an operator from S to S, while H is a $B \times B$ matrix.

and $q_0 = -\sum_{i=1}^{n} q_i$ ensures $\sum_{i=0}^{n} q_i = 0$. We endogenize this demand, with

$$\Gamma = \gamma V^{\alpha}, \tag{A.34}$$

where $V = var\left(e^1\right)$, and $var\left(x\right) = \mathbb{E}\left[xx'\right] - \mathbb{E}\left[x\right]\mathbb{E}\left[x'\right]$ is the variance-covariance matrix of a random vector x. Note that $var\left(e^1\right) = var\left(x^1\right)$ is independent of e^0. Hence, with this endogenous demand, we have a model that depends on variance, is arbitrage free, and (we believe) sensible.

Let us see how the general asset pricing case applies to the FX case. The basis assets are the currencies, with $p^t = e^t$, θ_a is the position in currency a, and $q_a = \theta_a e_a^0$ is the initial dollar value of the position. The position held in dollars is q_0 (and we still have $e_0^t = 1$ as a normalization). We define

$$D = diag\left(e^0\right), \tag{A.35}$$

so that $q = D\theta$. We take the G matrix to be

$$G = \gamma V^{\alpha} D, \tag{A.36}$$

for scalars $\gamma > 0$ and $\alpha \geq 0$. Recall that $V = var\left(e^1\right)$ is a matrix. The reader is encouraged to consider the leading case where $\alpha = 1$. In general, V^{α} is the variance-covariance matrix to the power α: if we write $V = U' \Lambda U$ for U an orthogonal matrix and $\Lambda = diag\left(\lambda_i\right)$ a diagonal matrix, $V^{\alpha} = U' diag\left(\lambda_i^{\alpha}\right) U$.

Proposition A.8. *(FX case: Foundation for the financiers' demand (A.22)) With the above set-up, the financiers' equilibrium holding q satisfies:*

$$\mathbb{E}\left[e^1 - e^0\right] = \Gamma q, \tag{A.37}$$

with

$$\Gamma = \gamma V^{\alpha},$$

where $\gamma > 0$ and $\alpha \geq 0$ are real numbers, and $V = var\left(e^1\right)$ is the variance-covariance matrix of exchange rates. In other terms, when Γ is invertible, we obtain the Gamma demand (A.22), $q = \Gamma^{-1}\mathbb{E}\left[e^1 - e^0\right]$.

Proof. This is a simple correlate of Proposition A.4. This Proposition yields

$$\mathbb{E}\left[p^1 - p^0\right] = G\theta^*,$$

Using $p^t = e^t$, $\Gamma \equiv \gamma V^{\alpha}$, $G \equiv \Gamma D$, $q = D\theta^*$, we obtain

$$\mathbb{E}\left[e^1 - e^0\right] = G\theta^* = \Gamma D\theta^* = \Gamma q.$$

□

It may be useful to check the logic by inspecting what this yields in the Basic Gamma model. There, the outside option of the financiers is given by (A.29) (using $\theta = -q/e_0$, since in the basic Gamma model the dollar value of the yen position is $-q$)

$$\theta' S\theta = \gamma\theta^2 var\left(e_1\right)^{\alpha} e_0 = \gamma var\left(e_1\right)^{\alpha} \frac{q^2}{e_0}.$$

The financiers' maximization problem is thus:

$$\max_{q} V_0 \text{ where } V_0 := \mathbb{E}\left[1 - \frac{e_1}{e_0}\right] q,$$
$$\text{s.t. } V_0 \geq \gamma\, var\left(e_1\right)^{\alpha} \frac{q^2}{e_0},$$

i.e., the divertable fraction is $\gamma var(e_1)^{\alpha}\frac{q}{e_0}$. It is increasing in q and the variance of the trade (a "complexity" effect).

The constraint binds, and we obtain:

$$\mathbb{E}\left[1-\frac{e_1}{e_0}\right]q=\gamma var(e_1)^{\alpha}\frac{q^2}{e_0},$$

or,

$$\mathbb{E}[e_0-e_1]=\gamma var(e_1)^{\alpha}q, \tag{A.38}$$

which confirms the intuitive properties of this derivation.

A.3.C.5 Application to the CIP and UIP trades

Suppose that the assets are: dollar bonds paying at time 1, yen bonds paying at time 1 (so that their payoff is e_1), and yen futures that pay e_1-F at time 1, where F is the futures' price. The payoffs (expressed in dollars) are:

$$P_1=(1,e_1,e_1-F)',$$

and the equilibrium time-0 price is:

$$P_0=(1,e_0,0)',$$

as a futures position requires 0 initial investment.

Suppose that financiers undertake the CIP trade, i.e. they hold a position:

$$\Theta^{CIP}=(e_0,-1,1)',$$

where they are long the dollar, short the yen, and long the future. To review elementary notions in this language, the initial price is $\Theta^{CIP}\cdot P^0=0$. The terminal payoff is $\Theta^{CIP}\cdot P^1=e_0-F$, hence, by no arbitrage, we should have $F=e_0$.

The financiers can also engage in the UIP trade; in the elementary UIP trade they are long 1 dollar, and short the corresponding yen amount:

$$\Theta^{UIP}=\left(1,\frac{-1}{e_0},0\right)'.$$

Assume that financiers' portfolio is composed of C CIP trades, and q UIP trades:

$$\Theta=C\Theta^{CIP}+q\Theta^{UIP}.$$

We expect the risk premia in this economy to come just from the risk currency part (q), not the CIP position (C). Let us verify this.

In terms of the reduced basis, we have

$$\theta^{CIP}=(e_0-F,0)',$$
$$\theta^{UIP}=\left(1,\frac{-1}{e_0}\right)',$$

so that

$$\theta=C\theta^{CIP}+q\theta^{UIP}.$$

Hence, the model confirms that the financiers have 0 exposure to the yen coming from the CIP trade. We then have

$$\mathbb{E}[e_0-e_1]=\Gamma q,$$

with $\Gamma=\gamma var(e_1)^{\alpha}$. The CIP trade, causing no risk, causes no risk premia. We summarize the results in the following lemmas.

Lemma A.4. *If the financiers undertake both CIP and UIP trades, only the net positions coming from the UIP trades induce risk premia.*

Lemma A.5. *Assume that $\alpha \geq 1$, or that $V = var\left(x^1\right)$ is invertible (and $\alpha \geq 0$). Then, in the FX model risk-less portfolios earn zero excess returns. In particular, CIP holds in the model, while UIP does not.*

Proof: Define $W = \gamma V^{\alpha-1} D$, which is well-defined under the lemma's assumptions. Then, we can write $G = \gamma V^{\alpha} D$ as $G = VW$, and apply Proposition A.5. □

A.4 Numerical Generalization of the Model

We include here a generalization of the basic Gamma model of Section II that relaxes some of the assumptions imposed in the main body of the paper for tractability. The generalization of the model in Section II has to be solved numerically. Our main aim is to verify, at least numerically, that all the core forces of the basic model carry through to this more general environment. We provide a brief numerical simulation and stress that this is only a numerical example without any pretense of being a full quantitative assessment. A full quantitative assessment, with its need for further channels and numerical complications, while interesting, is the domain of future research.

Model Equations Since the model is a generalization of the basic one, we do not restate, in the interest of space, the entire structure of the economy. We only note here that the model has infinite horizon, symmetric initial conditions (both countries start with zero bond positions), and report below the system of equations needed to compute the solution.

$$\text{(A.39)} \quad R_{t+1} = \frac{\chi_t / Y_{NT,t}}{\beta_t \mathbb{E}_t[\chi_{t+1}/Y_{NT,t+1}]},$$

$$\text{(A.40)} \quad R^*_{t+1} = \frac{\chi^*_t / Y^*_{NT,t}}{\beta^*_t \mathbb{E}_t[\chi^*_{t+1}/Y^*_{NT,t+1}]},$$

$$\text{(A.41)} \quad Q_t = \frac{1}{\Gamma}\mathbb{E}_t\left[\left(\eta\beta_t \frac{Y_{NT,t}/\chi_t}{Y_{NT,t+1}/\chi_{t+1}} + (1-\eta)\beta^*_t \frac{e_t}{e_{t+1}} \frac{Y^*_{NT,t}/\chi^*_t}{Y^*_{NT,t+1}/\chi^*_{t+1}}\right)\left(e_t R_{t+1} - R^*_{t+1} e_{t+1}\right)\right]$$

$$\text{(A.42)} \quad Q_t = f_t e_t - f^*_t - D_t,$$

$$\text{(A.43)} \quad D_t = D_{t-1} R_t + (\eta Q_{t-1} - e_{t-1} f_{t-1})\left(R_t - R^*_t \frac{e_t}{e_{t-1}}\right) + e_t \frac{\xi_t}{\chi^*_t} Y^*_{NT,t} - \frac{\iota_t}{\chi_t} Y_{NT,t},$$

where η is the share of financiers' profits repatriated to the US, and D are the US net foreign assets. This is a system of five nonlinear stochastic equations in five endogenous unknowns $\{R, R^*, e, Q, D\}$. We solve the system numerically by second order approximation. The exogenous variables evolve according to:

$$\text{(A.44)} \quad \ln \iota_t = (1-\phi_\iota)\ln \iota_{t-1} + \sigma_\iota \varepsilon_{\iota,t}; \qquad \ln \xi_t = (1-\phi_\xi)\ln \xi_{t-1} + \sigma_\xi \varepsilon_{\xi,t},$$

$$\text{(A.45)} \quad f_t = (1-\phi_f) f_{t-1} + \sigma_f \varepsilon_{f,t}; \qquad f^*_t = (1-\phi_f) f^*_{t-1} + \sigma_f \varepsilon_{f^*,t},$$

$$\text{(A.46)} \quad \beta_t = \bar{\beta}\exp(x_t); \qquad \beta^*_t = \bar{\beta}\exp(x^*_t),$$

$$\text{(A.47)} \quad x_t = (1-\phi_x) x_{t-1} + \sigma_x \varepsilon_{x,t}; \qquad x^*_t = (1-\phi_x) x^*_{t-1} + \sigma_x \varepsilon_{x^*,t},$$

where $[\varepsilon_\iota, \varepsilon_\xi, \varepsilon_f, \varepsilon_{f^*}, \varepsilon_x, \varepsilon_{x*}] \sim N(0, I)$. We assume that all other processes, including the endowments, are constant.

The deterministic steady state is characterized by: $\{\bar{e} = 1, \bar{R} = \bar{R}^* = \bar{\beta}^{-1}, \bar{Q} = \bar{D} = \bar{D}^* = 0\}$.[68] In order to provide a numerical example of the solution, we briefly report here the chosen parameter values. We stress that this is not an estimation, but simply a numerical example of the solutions. We set $\bar{\beta} = 0.985$

[68]Note that the deterministic steady state is stationary whenever $\Gamma > 0$, which we always assume here (i.e $\alpha = 0$ from the main text). Similarly the portfolio of the intermediary is determinate via the assumption that households only actively save in domestic currency and via the limited commitment problem of the intermediary.

to imply a steady state annualized interest rate of 6%. We set the share of financiers' payout to households at $\eta = 0.5$, so that it is symmetric across countries. We set all constant output parameters at 1 ($Y_H = Y_F = a = a^* = 1$), except for the value of non-tradables set at 18 ($Y_{NT} = Y^*_{NT} = \chi = \chi^* = 18$), so that they account for 90% of the consumption basket. We set $\Gamma = 0.1$.[69] Finally, we set the shock parameters to: $\phi_\iota = \phi_\xi = 0.018, \sigma_\iota = \sigma_\xi = 0.037, \phi_f = 0.0001, \sigma_f = 0.05, \phi_x = 0.0491, \sigma_x = 0.0073$.

We report in Table A.1 below a short list of simulated moments.[70] For a rough comparison, we also provide data moments focusing on the GBP/USD exchange rate and US net exports.

Table A.1: Numerical Example of Simulated Moments

Moment	*Data*	*Model*
$SD\left(\frac{e_{t+1}}{e_t} - 1\right)$	0.1011	0.1269
$\phi(e_{t+1}, e_t)$	0.2442	0.0831
$\bar{R}^c$	0.0300	0.0408
$SD\left(R^c_t\right)$	0.1011	0.1269
$SD(nx_t)$	0.0335	0.0143
$\phi(nx_t, nx_{t-1})$	0.0705	0.1438
$SD(R_t)$	0.0479	0.0479
$\phi(R_{t+1}, R_t)$	0.1821	0.1824

Data and model-simulated moments. The first column reports the standard deviation ($SD(\frac{e_{t+1}}{e_t} - 1)$) and (one minus) autocorrelation ($\phi(e_{t+1}, e_t)$) of exchange rates, the average carry trade return ($\bar{R}^c$) and its standard deviation ($SD\left(R^c_t\right)$), the standard deviation ($SD(nx_t)$) and (one minus) the autocorrelation coefficient ($\phi(nx_t, nx_{t-1})$) of net exports over GDP for the US, and the standard deviation ($SD(R_t)$) and (one minus) autocorrelation of interest rates ($\phi(R_{t+1}, R_t)$). Data sources: exchange rate moments are for the GBP/USD, the carry trade moments are based on Lettau, Maggiori and Weber (2014) assuming the interest rate differential is 5%, the interest rate moments are based on the yield on the 6-month treasury bill minus a 6-year moving average of the 6-month rate of change of the CPI. All data are quarterly 1975Q1-2012Q2 (150 observations). The reported moments are annualized. Model implied moments are computed by simulating 500,000 periods (and dropping the first 100,000). The carry trade moments are computed selecting periods in the simulation when the interest rate differential is between 4% and 6%.

Finally, we provide a numerical example of classic UIP regressions. The regression specification follows:

$$\Delta \ln(e_{t+1}) = \alpha + \beta_{UIP}[\ln(R_t) - \ln(R^*_t)] + \varepsilon_t.$$

The above regression is the empirical analog to the theoretical results in Section III.A.[71] We find a regression coefficient well below one ($\hat{\beta} = 0.33$), the level implied by UIP. Indeed, on average we strongly reject UIP with an average standard error of 0.19. The regression adjusted R^2 is also low at 0.018. The results are broadly in line with the classic empirical literature on UIP.

[69] We set this conservative value of Γ based on a thought experiment on the aggregate elasticity of the exchange rate to capital flows. We suppose that an inelastic short-term flow to buy the Dollar, where the scale of the flow is comparable to 1 year worth of US exports (*i.e.* $f^* = 1$), would induce the Dollar to appreciate 10%. The numbers are simply illustrative, but are in broad congruence with the experience of Israel and Switzerland during the recent financial crisis. Let us revert to the basic Gamma model. Suppose that period 1 is a "long run" during which inflows have already mean-reverted (so that the model equations are: $e_0 - 1 + f^* + Q = 0$, $e_1 = 1$, $Q = \frac{1}{\Gamma}\left(e_0 - e_1\right)$). Then, we have $e_0 = 1 - \frac{\Gamma}{1+\Gamma} f^*$. Hence, the price impact is $e_0 - 1 = -\frac{\Gamma}{1+\Gamma} \simeq -0.1$. This leads to $\Gamma \simeq 0.1$.

[70] The moments are computed by simulating 500,000 periods with pruning. We drop the first 100,000 observations (burn-in period).

[71] To estimate the regression based on model-produced data, we simulate the model for 500,000 periods, dropping the first 100,000, and then sample at random 10,000 data intervals of length 150. The length is chosen to reflect the data span usually available for the modern period of floating currencies (150 quarters). On each data interval, we estimate the above regression. Finally, we average across the regression output from the 10,000 samples.

A.5 Proofs for the Main Body of the Paper

Proof of Proposition 3 The flow equilibrium conditions in the dollar-yen markets are:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0, \tag{A.48}$$
$$\xi_1 e_1 - \iota_1 - RQ_0 = 0. \tag{A.49}$$

Summing (A.48) and (A.49) gives the intertemporal budget constraint: $R(\xi_0 e_0 - \iota_0) + \xi_1 e_1 - \iota_1 = 0$. From this, we obtain:

$$e_1 = \xi_1^{-1}(R\iota_0 + \iota_1 - R\xi_0 e_0). \tag{A.50}$$

The market clearing in the Dollar / Yen market, $\xi_0 e_0 - \iota_0 + \frac{1}{\Gamma}\mathbb{E}\left[e_0 - \frac{R^*}{R}e_1\right] = 0$, gives:

$$\frac{R^*}{R}\mathbb{E}[e_1] = e_0 + \Gamma(\xi_0 e_0 - \iota_0) = (1+\Gamma\xi_0)e_0 - \Gamma\iota_0. \tag{A.51}$$

Combining (A.50) and (A.51),

$$\mathbb{E}[e_1] = \mathbb{E}\left[\xi_1^{-1}(R\iota_0 + \iota_1)\right] - \mathbb{E}\left[\xi_1^{-1}\right]\xi_0 R e_0 = \frac{R}{R^*}(1+\Gamma\xi_0)e_0 - \frac{R}{R^*}\Gamma\iota_0,$$

i.e.

$$\begin{aligned} e_0 &= \frac{\frac{R}{R^*}\Gamma\iota_0 + \mathbb{E}\left[\xi_1^{-1}(R\iota_0+\iota_1)\right]}{\frac{R}{R^*}(1+\Gamma\xi_0) + \mathbb{E}\left[\xi_1^{-1}\right]\xi_0 R} = \frac{\left(\mathbb{E}\left[R^*\xi_1^{-1}\right]+\Gamma\right)\iota_0 + \mathbb{E}\left[\frac{R^*}{R}\xi_1^{-1}\iota_1\right]}{\left(\mathbb{E}\left[R^*\xi_1^{-1}\right]+\Gamma\right)\xi_0 + 1} \\ &= \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0 + \frac{\iota_1}{R}\right)\right] + \Gamma\iota_0}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma\xi_0}. \end{aligned}$$

We can now calculate e_1. We start from its expected value:

$$\begin{aligned} \frac{R^*}{R}\mathbb{E}[e_1] &= (1+\Gamma\xi_0)e_0 - \Gamma\iota_0 = (1+\Gamma\xi_0)\frac{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\iota_0 + \mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1} - \Gamma\iota_0 \\ &= \frac{\left\{(1+\Gamma\xi_0)\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right) - \Gamma\left[\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1\right]\right\}\iota_0 + (1+\Gamma\xi_0)\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1} \\ &= \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\right]\iota_0 + (1+\Gamma\xi_0)\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1} = \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0+\frac{\iota_1}{R}\right)\right] + \Gamma\xi_0\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0+\frac{\xi_1}{R^*}\right)\right]+\Gamma\xi_0}. \end{aligned}$$

To obtain the time-1 innovation, we observe that $e_1 = \frac{1}{\xi_1}(R\iota_0 + \iota_1 - R\xi_0 e_0)$ implies:

$$\{e_1\} = \left\{\frac{\iota_1}{\xi_1}\right\} + R(\iota_0 - \xi_0 e_0)\left\{\frac{1}{\xi_1}\right\}.$$

As:

$$\iota_0 - \xi_0 e_0 = \iota_0 - \xi_0\frac{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\iota_0 + \mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1} = \frac{\iota_0 - \mathbb{E}\left[\xi_0\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1},$$

we obtain:

$$\{e_1\} = \left\{\frac{\iota_1}{\xi_1}\right\} + R\frac{\iota_0 - \mathbb{E}\left[\xi_0 \frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right] + \Gamma\right)\xi_0 + 1}\left\{\frac{1}{\xi_1}\right\}.$$

We next derive the value of Γ. Notice that we can write the above equation as:

$$\begin{aligned}
\{e_1\} &= \varepsilon + \frac{1}{a+\Gamma}\eta, \\
\varepsilon &\equiv \left\{\frac{\iota_1}{\xi_1}\right\}, \\
\eta &\equiv \left(\iota_0 - \mathbb{E}\left[\xi_0 \frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]\right)\frac{1}{\xi_0}\left\{\frac{1}{\xi_1}\right\}, \\
a &\equiv \mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right]\frac{1}{\xi_0}.
\end{aligned}$$

Then,

$$var\left(e_1\right) = \sigma_\varepsilon^2 + \frac{2\sigma_{\varepsilon\eta}}{a+\Gamma} + \frac{\sigma_\eta^2}{\left(a+\Gamma\right)^2}.$$

Letting $G(\Gamma)$ be

$$G\left(\Gamma\right) \equiv \Gamma - \gamma\left(\sigma_\varepsilon^2 + \frac{2\sigma_{\varepsilon\eta}}{a+\Gamma} + \frac{\sigma_\eta^2}{\left(a+\Gamma\right)^2}\right)^\alpha, \tag{A.52}$$

then Γ is defined as

$$G\left(\Gamma\right) = 0. \tag{A.53}$$

When $\alpha = 0$, we get the basic Gamma model. When $\alpha = 1$, we have a polynomial of degree 3 in Γ. When there is no noise and $\alpha > 0$, $\Gamma = 0$. In general, it is still amenable to computation: there is a unique positive solution of $G\left(\Gamma\right)$ (as $G\left(\Gamma\right)$ is increasing in Γ, and $G\left(0\right) < 0$, $\lim_{\Gamma\to\infty} G\left(\Gamma\right) = \infty$).

Proof of Lemma 3 In the decentralized allocation, the consumer's intra-period consumption, Equation (5), gives the first order conditions:

$$\begin{aligned}
p_{NT}C_{NT} &= \frac{\chi}{\lambda}; & p_{NT}^*C_{NT}^* &= \frac{\chi^*}{\lambda^*}; \\
p_H C_H &= \frac{a}{\lambda}; & \frac{p_H}{e}C_H^* &= \frac{\xi}{\lambda^*}; \\
e p_F^* C_F &= \frac{\iota}{\lambda}; & p_F^* C_F^* &= \frac{a^*}{\lambda^*}.
\end{aligned} \tag{A.54}$$

so that

$$e = \frac{\frac{C_H^*\lambda^*}{\xi}}{\frac{C_H\lambda}{a}}.$$

Suppose that the Negishi weight is ν. The planner maximizes $U + \nu U^*$ subject to the resource constraint; hence, in particular $\max_{C_H + C_H^* \le Y_H} a\ln C_H + \nu\xi\ln C_H^*$, which gives the planner's first order condition $\frac{a}{C_H} = \frac{\nu\xi}{C_H^*}$. Hence, in the first best exchange rate satisfies:

$$e_t^{FB} = \nu\frac{\lambda_t^*}{\lambda_t} = \nu\frac{p_{NT}C_{NT,t}/\chi_t}{p_{NT}^*C_{NT,t}^*/\chi_t^*}.$$

In the basic case of Lemma 3, we have $\lambda_t = \lambda_t^* = 1$, so $e_t^{FB} = \nu$. Note that this is derived under the assumption of identical discount factor $\beta = \beta^*$. □

Proof of Proposition 6

$$\overline{R}^c = \frac{\mathbb{E}\left[\frac{R^*}{R}e_1 - e_0\right]}{e_0} = \frac{-\Gamma Q_0}{e_0}$$
$$= -\Gamma\left(\frac{\iota_0 - e_0}{e_0}\right) = \Gamma\left(1 - \frac{\iota_0}{e_0}\right).$$

Recall that:

$$e_0 = \frac{(R^* + \Gamma)\,\iota_0 + \frac{R^*}{R}\mathbb{E}\left[\iota_1\right]}{R^* + \Gamma + 1},$$

so that we conclude:

$$\overline{R}^c = \Gamma\left(1 - \iota_0 \frac{R^* + \Gamma + 1}{(R^* + \Gamma)\,\iota_0 + \frac{R^*}{R}\mathbb{E}\left[\iota_1\right]}\right).$$

which, rearranged, gives the announced expression.

Derivation of 3-period economy exchange rates We will use the notation:

$$\mathcal{R}^* \equiv \frac{R^*}{R}.$$

Recall that we assume that in period $t = 1$ financiers only intermediate the new flows; stocks arising from previous flows are held passively by the households (long term investors) until t=2. Therefore, from the flow demand equation for $t = 1$, $e_1 - \iota_1 + Q_1 = 0$, and the financiers' demand, $Q_1 = \frac{e_1 - \mathcal{R}^*\mathbb{E}[e_2]}{\Gamma_1}$, we get an expression for e_1:

$$e_1 = \frac{\Gamma_1 \iota_1 + \mathcal{R}^* \mathbb{E}_1\left[e_2\right]}{\Gamma_1 + 1}.$$

The flow demand equation for $t = 2$ gives $e_2 = \iota_2$, so we can rewrite e_1 as:

$$e_1 = \frac{\Gamma_1 \iota_1 + \mathcal{R}^* \mathbb{E}_1\left[\iota_2\right]}{\Gamma_1 + 1}.$$

Similarly for e_0, we have

$$e_0 = \frac{\Gamma_0 \iota_0 + \mathcal{R}^* \mathbb{E}_0\left[e_1\right]}{\Gamma_0 + 1},$$

and we can use our expression for e_1 above to express e_0 as:

$$e_0 = \frac{\Gamma_0 \iota_0 + \mathcal{R}^* \mathbb{E}_0\left[\frac{\Gamma_1 \iota_1 + \mathcal{R}^* \iota_2}{\Gamma_1 + 1}\right]}{\Gamma_0 + 1}. \square$$

Proof of Proposition 7 We have already derived Claim 1. For Claim 2, we can calculate, from the definition of carry trade returns ($R^c \equiv \frac{R^*}{R}\frac{e_1}{e_0} - 1$) and equation (24):

$$\overline{R}^c = (\mathcal{R}^* - 1)\,\Gamma_0 \frac{\overline{\Gamma}_1 + 1 + \mathcal{R}^*}{\overline{\Gamma}_1(\Gamma_0 + \mathcal{R}^*) + \Gamma_0 + (\mathcal{R}^*)^2} > 0.$$

Hence, the expected carry trade return is positive.

For Claim 3, recall that a function $\frac{ax+b}{cx+d}$ is increasing in x iff $\Delta^x \equiv ad - bc > 0$. For Γ_0,

$$\Delta^{\Gamma_0} = \left(1 + \overline{\Gamma}_1 + \mathcal{R}^*\right)\left(\overline{\Gamma}_1 \mathcal{R}^* + (\mathcal{R}^*)^2\right) > 0,$$

which proves $\frac{\partial \overline{R}^c}{\partial \Gamma_0} > 0$.

For $\overline{\Gamma}_1$, the discriminant is

$$\frac{\Delta^{\overline{\Gamma}_1}}{(\mathcal{R}^* - 1)\,\Gamma_0} = \Gamma_0 + (\mathcal{R}^*)^2 - (1 + \mathcal{R}^*)\,(\Gamma_0 + \mathcal{R}^*) = -\mathcal{R}^*\,(1 + \Gamma_0) < 0,$$

so that $\frac{\partial \overline{R}^c}{\partial \overline{\Gamma}_1} < 0$.

Finally, for $\mathcal{R}^*$, we simply compute:

$$\frac{\partial \overline{R}^c}{\partial \mathcal{R}^*} = \frac{\Gamma_0\,(1 + \Gamma_0)\,(1 + \overline{\Gamma}_1)\,(2\mathcal{R}^* + \overline{\Gamma}_1)}{\left(\Gamma_0\,(1 + \overline{\Gamma}_1) + \overline{\Gamma}_1 \mathcal{R}^* + (\mathcal{R}^*)^2\right)^2} > 0. \square$$

Proof of Proposition 8 The regression corresponds to: $\beta_{\text{UIP}} = \frac{-\partial}{\partial R^*}\mathbb{E}\left[\frac{e_1}{e_0} - 1\right]$. For simplicity we calculate this derivative at $R = R^* = \mathbb{E}\iota_t = 1$, and with deterministic $\Gamma_1 = \overline{\Gamma}_1$. Equation (24) yields, for those values but keeping R^* potentially different from 1:

$$e_0 = \frac{\Gamma_0 + R^* \frac{\overline{\Gamma}_1 + R^*}{\overline{\Gamma}_1 + 1}}{\Gamma_0 + 1}; \qquad \mathbb{E}e_1 = \frac{\overline{\Gamma}_1 + R^*}{\overline{\Gamma}_1 + 1}.$$

Calculating $\beta_{\text{UIP}} = \frac{-\partial}{\partial R^*}\mathbb{E}\left[\frac{e_1}{e_0} - 1\right] = \frac{-\partial}{\partial R^*}\frac{\mathbb{E}e_1}{e_0}$ gives:

$$\beta_{\text{UIP}} = \frac{1 + \overline{\Gamma}_1 - \Gamma_0}{(1 + \Gamma_0)\,(1 + \overline{\Gamma}_1)}.$$

Hence, $\beta_{\text{UIP}} \leq \frac{1+\overline{\Gamma}_1}{(1+\Gamma_0)(1+\overline{\Gamma}_1)} = \frac{1}{1+\Gamma_0} < 1$. $\square$

Proof of Proposition 9 Lemma 6 shows that the Yen (strictly) monotonically depreciates as a function of the intervention q^*. Let $e_0(q^*)$ be the exchange rate as a function of the intervention. From Section II.E and the assumption in this proposition that output is demand determined under PCP, we know that:

$$Y_{F,0} = \frac{1 + \frac{1}{e_0(q^*)}}{\overline{p}_F^*} \quad \forall q^* \in [0, \overline{q}^*), \tag{A.55}$$

so that Japanese tradable output increases monotonically as a function of the intervention. We define $\overline{q}^* \equiv \min\{\operatorname{argmax}_{q^*} \; Y_{F,0}(q^*)\}$ as the smallest intervention that achieves full employment. Strict monotonicity of $Y_{F,0}(q^*)$ for all q^* such that $Y_{F,0} < L$ and the fact that $Y_{F,0}$ is bounded above by L guarantee that $\overline{q}^*$ exists and is unique.

The consumption shares are obtained from the household demand functions plus market clearing, so that:

$$C_{H,t} = (1 - s_t^*)L; \qquad C_{F,t} = (1 - s_t^*)Y_{F,t};$$
$$C^*_{H,t} = s_t^* L; \qquad C^*_{F,t} = s_t^* Y_{F,t};$$

where $s_t^* \equiv \frac{e_t}{1+e_t}$. To derive the solution for $C_{F,t}$, recall that the US household demand function is given by $C_{F,t} = \frac{\iota_t}{p_{F,t}}$. At time $t = 0$ we have $\iota_0 = 1$ and $p_{F,0} = \overline{p}^*_F e_0$, and substituting in the output expression in (A.55), we obtain $C_{F,0} = \frac{1}{1+e_0} Y_{F,0}$. At time $t = 1$ we have $p_{F,1} = e_1 p^*_{F,1} = e_1 \frac{a^*_1 + \iota_1 / e_1}{L} = \frac{\iota_1(1+e_1)}{L}$, so that $C_{F,1} = \frac{1}{1+e_1} L$. The rest of the expressions can be derived by analogy. □

Proof of Proposition 12 We first prove a Lemma.

Lemma A.6. *In the setup of Proposition 3, e_0 is increasing in ι_t and R^* and decreasing in ξ_t and R; $\frac{\partial e_0}{\partial \iota_0}$ increases in Γ. In addition, e_0 increases in Γ if and only if the US is a natural net debtor at time 0^+, i.e. $N_{0^+} \equiv \xi_0 e_0 - \iota_0 < 0$.*

Proof: The comparative statics with respect to ι_t, ξ_0, and R are simply by inspection. We report here the less obvious ones:

$$\frac{\partial e_0}{\partial \xi_1} = \frac{\mathbb{E}\left[\frac{e_0\xi_0 - \iota_0 - \frac{\iota_1}{R}}{\xi_1^2}\right]}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} = -\frac{\mathbb{E}\left[\frac{e_1}{R\xi_1}\right]}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} < 0,$$

where we made use of the state-by-state budget constraint $e_0\xi_0 - \iota_0 + \frac{e_1\xi_1 - \iota_1}{R} = 0$. To be very precise, a notation like $\frac{\partial e_0}{\partial \xi_1}$ is the sensitivity of e_0 to a small, deterministic increment to random variable ξ_1.

$$\frac{\partial e_0}{\partial R^*} = \frac{1}{R^{*2}} \frac{e_0 - \Gamma Q}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} = \frac{1}{RR^*} \frac{\mathbb{E}\left[e_1\right]}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma\xi_0}{R^*}} > 0,$$

where we made use of the financiers' demand equation, $\Gamma Q_0 = \mathbb{E}\left[e_0 - \frac{R^*}{R} e_1\right]$, and the flow equation, $\xi_0 e_0 - \iota_0 + Q_0 = 0$.

We also have,

$$\frac{\partial e_0}{\partial \Gamma} = -N_{0^+} \frac{1}{1 + R^*\mathbb{E}\left[\frac{\xi_0}{\xi_1}\right] + \xi_0\Gamma} < 0,$$

where we made use of the definition $N_{0^+} = e_0\xi_0 - \iota_0$. This implies:

$$\frac{\partial^2 e_0}{\partial \Gamma \partial \iota_0} = \frac{1}{\left(R^*\mathbb{E}\left[\frac{\xi_0}{\xi_1}\right] + 1 + \Gamma\xi_0\right)^2} > 0. \square$$

This implies all the points of Proposition 12 with two exceptions. The effects with respect to interest rate changes, both domestic and foreign, hold for f, f^* sufficiently small. Finally, we focus on the impact of f^*. Simple calculations yield:

$$\frac{\partial e_0}{\partial f^*} = -\frac{\Gamma}{R^*\mathbb{E}\left[\frac{\xi_0}{\xi_1}\right] + 1 + \Gamma\xi_0} < 0.$$

We notice that the comparative statics with respect to f are less clear-cut, because f affects the value of $\Gamma\widetilde{\xi}_1$, and hence affects risk-taking. However, we have $\frac{\partial e_0}{\partial f} > 0$ for typical values (e.g. $R = R^* = 1, \widetilde{\xi}_0 = \widetilde{\xi}_1$). □

Appendix References

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