

International Liquidity and Exchange Rate Dynamics

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Imperfect Finance and the Determination of Exchange Rates

"One very important and quite robust insight is that the nominal exchange rate must be viewed as an asset price" Obstfeld and Rogoff (1996)

- Exchange rates are disconnected from traditional macroeconomic fundamentals
- They are instead connected to financial forces: e.g. capital flows and financial conditions
- Demand and Supply of assets in different currencies is central to exchange rate determination
- Financial determination *in imperfect capital markets* is key for welfare analysis: floating exchange rates do not move to absorb real shocks as in Mundellian analysis

Important issues: framework is desirable, but has proven elusive

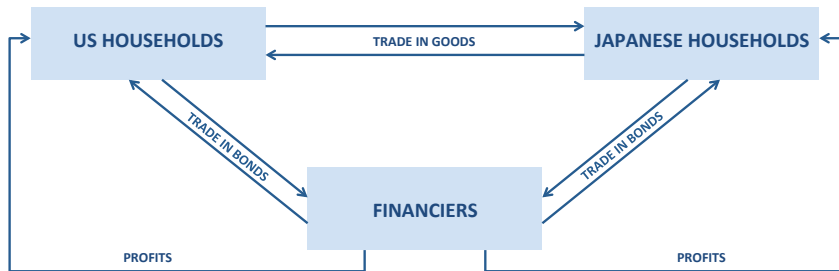
Imperfect Finance and the Determination of Exchange Rates

We provide a basic framework of capital flows and exchange rates:

- Capital flows alter balance sheet of financiers who absorb resulting imbalances
- Financiers' balance sheets and risk bearing capacity determine the required compensation for absorbing unbalanced capital flows
- Such compensation determines both the level and dynamics of exchange rates
- Practical Example:
US investors demand Brazilian Real bonds → Financiers provide these bonds in the short-medium run, Short Real and Long Dollar → To compensate financiers, the Real appreciates on impact and is expected to depreciate relative to the Dollar
- Our framework is a basic theory where a price, *the exchange rate*, has to move to *balance the demand/supply of assets in financial markets*

Building up the Framework

- Real Model: basic exchange rate determination in a financial world



- Real effects of financial determination of exchange rates
 - Welfare and heterodox financial policies
- Monetary Model:
 - Nominal vs real exchange rates
 - Monetary shocks and exchange rate dynamics

Exchange Rate Determination Frameworks

Two important papers in 1976: Dornbusch's "overshooting" model, and Kouri's "portfolio balance" model

- Obstfeld, Rogoff (1995) brought Mundell-Fleming-Dornbusch model into modern macroeconomics
- We provide a modern general equilibrium theory of the financial market forces first sketched by Kouri

Kouri's ideas:

- The demand and supply of assets denominated in different currencies as a determinant of exchange rates
- Key ingredients: domestic and foreign assets are imperfect substitutes, and imperfect capital markets
- Partial equilibrium framework
- Lack of foundations

Basic Model

We present here the simplest model: real model, imperfect capital markets

- Two countries (US, Japan (*)). Two periods ($t = 0, 1$)
- Unit measure of households in each country
- Four goods: 1 non-tradable (NT) and 1 tradable good in each country
- NT are endowments, tradables produced with int. immobile inelastically supplied labor
- NT good is the numéraire in each economy
- Incomplete Markets: two “risk-free” bonds that pay for sure one unit of the domestic numéraire (the NT good) for each economy
- Households borrow/lend in domestic “risk-free” bonds with the financiers
- Financiers absorb resulting imbalances in global capital flows

The Household Problem

US households' consumption/saving decision:

$$\begin{aligned} \max_c \quad & \mathbb{E} [\theta_0 \ln C_0 + \beta \theta_1 \ln C_1] \\ \text{s.t.} \quad & \sum_{t=0}^1 \frac{C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t}}{R^t} \leq \sum_{t=0}^1 \frac{Y_{NT,t} + p_{H,t} Y_{H,t}}{R^t} \end{aligned}$$

where $C_t \equiv [(C_{NT,t})^{\chi_t} (C_{H,t})^{a_t} (C_{F,t})^{l_t}]^{\frac{1}{\theta_t}}$, and $\theta_t = \chi_t + a_t + l_t$

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where $C_t \equiv [(C_{NT,t})^{\chi_t} (C_{H,t})^{a_t} (C_{F,t})^{\iota_t}]^{\frac{1}{\theta_t}}$, and $\theta_t = \chi_t + a_t + \iota_t$

Corresponding Japanese households' problem:

$$\begin{aligned} \max_{c^*} \quad & \mathbb{E} [\theta_0^* \ln C_0^* + \beta^* \theta_1^* \ln C_1^*] \\ \text{s.t.} \quad & \sum_{t=0}^1 \frac{C_{NT,t}^* + p_{H,t}^* C_{H,t}^* + p_{F,t}^* C_{F,t}^*}{R^{*t}} \leq \sum_{t=0}^1 \frac{Y_{NT,t}^* + p_{F,t}^* Y_{F,t}^* + \pi_t}{R^{*t}} \end{aligned}$$

where $C_t^* \equiv [(C_{NT,t}^*)^{\chi_t^*} (C_{H,t}^*)^{\xi_t} (C_{F,t}^*)^{a_t^*}]^{\frac{1}{\theta_t^*}}$; and $\theta_t^* = \chi_t^* + a_t^* + \xi_t$

Net Exports

US households' time t problem:

$$\max_{C_{i,t}} \chi_t \ln C_{NT,t} + a_t \ln C_{H,t} + l_t \ln C_{F,t} - \lambda_t (C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t})$$

Focus on two intra-temporal FOCs with respect to $C_{NT,t}$ and $C_{F,t}$:

$$\frac{\chi_t}{C_{NT,t}} = \lambda_t; \quad \frac{l_t}{C_{F,t}} = \lambda_t p_{F,t}$$

Simplifying assumption: $Y_{NT} = \chi_t \Rightarrow \lambda_t = 1$

Dollar value of US imports: $p_{F,t} C_{F,t} = l_t$

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Dollar value of US imports: $p_{F,t} C_{F,t} = l_t$

Similarly, Yen value of Japanese imports: $p_{H,t}^* C_{H,t} = \xi_t$

So, Dollar value of US exports: $\xi_t e_t$

where e_t is the exchange rate: $e_t \uparrow$ is a Yen appreciation

Dollar value of US **net exports**: $NX_t = \xi_t e_t - l_t$

Interest Rates

US households' inter-temporal optimality condition (Euler Equation):

$$1 = \mathbb{E} \left[\beta R \frac{U'_{1,C_{NT}}}{U'_{0,C_{NT}}} \right] = \mathbb{E} \left[\beta R \frac{\chi_{1/C_{NT,1}}}{\chi_{0/C_{NT,0}}} \right] = \beta R,$$

Recall: simplifying assumption $C_{NT} = Y_{NT} \equiv \chi_t$

Hence: $R = \frac{1}{\beta}$

Likewise: $R^* = \frac{1}{\beta^*}$

Financiers' Asset Demand

- Unit measure of intermediaries, each financier runs one intermediary
- Agents are selected at random. Zero starting capital. Rebate all profits to households
- Trade Dollar and Yen bonds. Balance sheet: $q_0 = -q_{F,0}e_0$
- Financiers maximize expected returns in dollars:

$$V_0 = \mathbb{E} \left[\beta \left(R - R^* \frac{e_1}{e_0} \right) \right] q_0$$

Intermediation Friction: After taking positions, but before uncertainty is realized financiers can divert funds. If financiers divert, creditors recover $\left(1 - \Gamma \left| \frac{q_0}{e_0} \right| \right)$ of their claims $\left| \frac{q_0}{e_0} \right|$:

$$\underbrace{\frac{V_0}{e_0}}_{\text{Intermediary Value in Yen}} \geq \underbrace{\left| \frac{q_0}{e_0} \right|}_{\text{Total Claims}} \underbrace{\Gamma \left| \frac{q_0}{e_0} \right|}_{\text{Diverted Portion}} = \underbrace{\Gamma \left(\frac{q_0}{e_0} \right)^2}_{\text{Total divertable Funds}}$$

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Financiers' Asset Demand: Micro-foundations

Financiers' problem:

$$\max_{q_0} \quad V_0 = \mathbb{E} \left[\beta \left(R - R^* \frac{e_1}{e_0} \right) \right] q_0 \quad \text{s.t.} \quad V_0 \geq \Gamma \frac{q_0^2}{e_0}$$

Optimality \Rightarrow Constraint always binds \Rightarrow Financiers' demand q_0 dollar and $-q_0/e_0$ yen, according to:

$$q_0 = \frac{1}{\Gamma} \mathbb{E} \left[e_0 - \frac{R^*}{R} e_1 \right]$$

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- $\Gamma \uparrow \infty$: no amount of intermediation is possible \Rightarrow financial autarky
- $\Gamma = 0$: any amount of intermediation is possible \Rightarrow Uncovered Interest Parity holds

This Γ demand function is key to the model: Basic Gamma model

With $\Gamma = \gamma \text{Var}(e_1)^\alpha$, and $\alpha > 0 \Rightarrow$ UIP fails, but CIP holds

Simplifying assumption: financiers pay all profits to Japanese households

Equilibrium Exchange Rate

From the three previous equations,

$$\xi_0 e_0 - \iota_0 + Q_0 = 0; \quad \xi_1 e_1 - \iota_1 - RQ_0 = 0.$$

$$Q_0 = \frac{1}{\Gamma} \mathbb{E} \left[e_0 - \frac{R^*}{R} e_1 \right]$$

the equilibrium exchange rate follows (assume $\xi_t = R = R^* = 1$):

$$e_0 = \frac{(1 + \Gamma) \iota_0 + \mathbb{E}[\iota_1]}{2 + \Gamma}; \quad \mathbb{E} \left[\frac{e_0 - e_1}{e_0} \right] = \frac{\Gamma (\iota_0 - \mathbb{E}[\iota_1])}{(1 + \Gamma) \iota_0 + \mathbb{E}[\iota_1]}$$

Derivation

- Financial Autarky ($\Gamma \uparrow \infty$): $e_0 = \iota_0$
- UIP ($\Gamma \downarrow 0$): $e_0 = \mathbb{E}[e_1] = \frac{\iota_0 + \mathbb{E}[\iota_1]}{2}$
- $\Gamma = \gamma \text{var}(\iota_1)^\alpha$

Equilibrium Exchange Rate

Recall, the equilibrium exchange rate:

$$e_0 = \frac{(1 + \Gamma) \iota_0 + \mathbb{E}[\iota_1]}{2 + \Gamma}$$

- US net foreign assets: $N_{0+} = e_0 - \iota_0 = \frac{\mathbb{E}[\iota_1] - \iota_0}{2 + \Gamma}$
- **Proposition:** When there is a financial disruption ($\uparrow \Gamma$), countries that are net external debtors ($N_{0+} < 0$) experience a currency depreciation ($\uparrow e$), while the opposite is true for net-creditor countries. [▶ Derivation](#)
 - Suppose $\iota_0 - \mathbb{E}[\iota_1] > 0$, US runs a trade deficit and borrows in dollars
 - Financiers are long Dollar and short Yen ($Q_0 > 0$)
 - If financial conditions worsen ($\uparrow \Gamma$), the Dollar depreciates ($\uparrow e_0$)
 - Empirical support: Della Corte, Riddiough and Sarno (2013)

Gross Portfolio Flows

- So far households only traded bonds in *domestic* currency
- For simplicity, assume that some Japanese households have a noise demand f^* for Dollar bonds (financed in Yen bonds), then the equilibrium exchange rate follows:

$$e_0 = \frac{(1 + \Gamma) l_0 + \mathbb{E}[l_1] - f^* \Gamma}{2 + \Gamma}$$

► Derivation

- $\frac{\partial e_0}{\partial f^*} = -\frac{\Gamma}{2 + \Gamma}$: if Japanese households demand Dollar bonds ($f^* > 0$), then the Dollar appreciates ($\downarrow e_0$): supply and demand of assets matters!
- This effect is absent both in complete market models or in models that assume UIP. Empirical support: Hau et al. (2010)
- Generalization: any portfolio demand that depends on fundamentals (but not on e directly) is tractable

Flows not just Stocks Matter

- Recall: $R = 1$
- US has an exogenous Dollar-denominated debt toward Japan. D_0 due at time zero, and D_1 due at time one
- Equilibrium exchange rate

$$e_0 = \frac{(1 + \Gamma) \iota_0 + \mathbb{E}[\iota_1]}{2 + \Gamma} + \frac{(1 + \Gamma) D_0 + D_1}{2 + \Gamma}$$

- When finance is imperfect ($\Gamma > 0$):
 - The timing of repayment (a flow) matters, not just the stock of debt ($D_0 + D_1$)
 - The higher Γ the more weight on early repayment

The Exchange Rate Disconnect

Consider two worlds: *Tranquil Times*, and *Distressed Times*

- Tranquil Times and Distressed Times have *identical macro fundamentals*, but...
- Financiers' risk bearing capacity: $\Gamma_D > \Gamma_T$
- Financiers' balance sheet: $Q_{0-}^T = -f < 0$; $Q_{0-}^D = -f - \Delta_f$
- Equilibrium *exchange rates are different*:

$$e_0^T - e_0^D \propto (\Gamma_D - \Gamma_T)[\mathbb{E}[\iota_1] - \iota_0 + 2f] + \Gamma_D(2 + \Gamma_T)\Delta_f$$

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Intuition:

- $\mathbb{E}[\iota_1] - \iota_0 > 0$: fundamental capital flows. US lends in Dollar
- $f > 0$: starting balance sheet. Financiers short Dollar, long Yen

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- $\Delta_f > 0$: increase financiers' imbalance. Shorter Dollar, longer Yen. Dollar appreciates
- $\Gamma_D(2 + \Gamma_T)$: worse financial conditions reinforce the latter effect

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- $\Gamma_D(2 + \Gamma_T)$: worse financial conditions reinforce the latter effect

Empirical Evidence

- Little connection between traditional fundamentals and exchange rates
Meese, Rogoff (1983)
- More evidence that exchange rates are connected to flows in the medium run
 - Adrian, Etula, Groen (2011), Adrian, Etula, Shin (2013): financiers' balance sheet forecast USD FX
 - Hau, Massa, Peress (2010): inflows cause currency appreciation. Clean IV approach
 - Yogo, Hong (2012): CME speculators positions help predict currency returns
 - Froot, Ramodorai (2005): flows are associated with most of the variation in expected currency returns over medium horizons, fundamentals matter only at long horizon

Currency Interventions: Welfare Consequences

Welfare analysis full details are in NBER working paper.

- Simple environment: uncertainty $\mathbb{E}[\iota_1] = 1$, sticky prices $\bar{p}_{H,0}$
- The Japanese government buys q^* dollars and sells $\frac{q^*}{e_0}$ yen at time 0

$$e_0 - \iota_0 - q^* + Q_0 = 0; \quad e_1 - \iota_1 + q^* - Q_0 = 0.$$

- $e_0(q^*) = 1 - \frac{\Gamma}{2+\Gamma} q^*$: Dollar depreciates, creating employment
- Recall: $Y_{H,0}(e_0) = \min\left(\frac{a_0 + e_0 \xi_0}{\bar{p}_{H,0}}, L\right)$
- US Welfare:

$$V(q^*) \equiv \mathbb{E}[U_0 + U_1] = V^{FB} + \ln \frac{Y_{H,0}(e_0(q^*))}{L} + O(q^{*2})$$

- **Proposition** If $\Gamma > 0$ and $Y_{H,0}(q^* = 0) < L$, then welfare $V(q^*)$ is increasing in intervention $q^* \in [0, \bar{q}^*]$, where $e(\bar{q}^*)$ generates full employment
- Note that in this case there are no private incentives to intervene

Rethinking Currency Interventions

Recent implementation on a *massive scale* of policies with similar rationale:

- **Switzerland:** In Sept. 2011 SNB set explicit floor for CHF/EUR at 1.2 and has accumulated CHF 450bn of reserves (80% of GDP)
- **US:** Starting in 2007 Fed provided Dollar liquidity via “unlimited” currency swaps, amounts outstanding reached \$600bn
- **Israel:** Bol has been intervening since 2008, accumulated reserves of 30% of GDP

“I have no doubt that the massive purchases [of foreign exchange] we made between July 2008 and into 2010 [...] had a serious effect on the exchange rate which I think is part of the reason that we succeeded in having a relatively short recession.” **Stanley Fischer** (WSJ 2010)

Summary of other Financial Policies

- **Taxing international finance:**

- The government taxes financiers' profits at rate τ , rebates lump sum
- Then, financiers' demand: $Q_0 = \frac{\mathbb{E}[e_0 - e_1](1 - \tau)}{\Gamma} \equiv \frac{\mathbb{E}[e_0 - e_1]}{\Gamma^{eff}}$
- Policy warning: financiers matter, effect of the tax on ER depends on the sign of Q_0 before the tax

- **Dilemma: joint monetary and FX policy**

- Use FX interventions (q) or monetary policy (m_0)?
- **Proposition:** Suppose that at time zero \bar{p}_H is downwards rigid at a level inconsistent with full-employment, and that at time one it is either:
 - 1 Flexible \Rightarrow use both FX and monetary policy. Rely more on the FX intervention when Γ is higher
 - 2 Rigid \Rightarrow use only monetary policy. Currency intervention reduces welfare

The Carry Trade: Financial risks

- We study carry trade returns:

$$R_1^c \equiv \mathcal{R}^* \frac{e_1}{e_0} - 1, \text{ with } \mathcal{R}^* \equiv \frac{R^*}{R}$$

- (To simplify the math take 3 periods with financial shocks: Γ_1 stochastic, and with “very long” period 2):

$$e_0 = \frac{\Gamma_0 l_0 + \mathcal{R}^* \mathbb{E}_0 \left[\frac{\Gamma_1 l_1 + l_2 \mathcal{R}^*}{\Gamma_1 + 1} \right]}{\Gamma_0 + 1};$$

$$e_1 = \frac{\Gamma_1 l_1 + \mathcal{R}^* \mathbb{E}_1 [l_2]}{\Gamma_1 + 1}; \quad e_2 = l_2.$$

- The carry trade does badly if there is a financial squeeze, i.e. if Γ_1 goes up.: $\frac{\partial R_1^c}{\partial \Gamma_1} < 0$. (Brunnermeier, Nagel and Pedersen (2009)).
- Hence, the carry trade is exposed to “financial shocks”, not simply “fundamental shocks”

The Carry Trade: Expected returns

- Carry trade return:

$$R_1^c \equiv \mathcal{R}^* \frac{e_1}{e_0} - 1, \quad \text{with } \mathcal{R}^* \equiv \frac{R^*}{R}$$

- (To simplify the math take 3 periods with financial shocks: Γ_1 stochastic, and with “very long” period 2):
- Expected carry trade returns:

$$\mathbb{E}[R_1^c] = \frac{(\mathcal{R}^* - 1)\Gamma_0(\bar{\Gamma}_1 + 1 + \mathcal{R}^*)}{\bar{\Gamma}_1(\Gamma_0 + \mathcal{R}^*) + \Gamma_0 + (\mathcal{R}^*)^2}$$

$\frac{\bar{\Gamma}_1 + \mathcal{R}^*}{1 + \bar{\Gamma}_1} \equiv \mathbb{E}_0 \left[\frac{\Gamma_1 + \mathcal{R}^*}{1 + \Gamma_1} \right]$. Expected returns are higher:

- the higher is the interest rate differential \mathcal{R}^* (Lustig, Verdelhan (2007))
- the worse financial conditions are today ($\uparrow \Gamma_0$), the better they are tomorrow ($\downarrow \{\bar{\Gamma}_1, \Gamma_1\}$) (Brunnermeier, Nagel and Pedersen (2009))
- the more one invests in net-external-debtor countries' currencies (Della Corte, Riddiough and Sarno 2013)

The Carry Trade: Fama regression

- Fama (1984) regresses:

$$\frac{e_1 - e_0}{e_0} = \alpha + \beta (R - R^*) + \varepsilon_1$$

- Under UIP, $\beta = 1$. Data shows $\beta < 1$
- In the Gamma model:

$$\beta = \frac{1 + \bar{\Gamma}_1 - \Gamma_0}{(1 + \Gamma_0)(1 + \bar{\Gamma}_1)} < 1$$

- Note $\beta < 0$ iff $\bar{\Gamma}_1 + 1 < \Gamma_0$, i.e. financial conditions are expected to improve

UIP & CIP

Recall $\Gamma_t = \gamma \text{Var}_t(e_{t+1})^\alpha$, take $\alpha > 0$

Proposition All replication trades are satisfied, hence **CIP holds**. Risky trades are affected by the constraint, hence **UIP fails**. Under mild boundedness of the shocks, all arbitrages are satisfied

- Previous propositions are about financial risk bearing capacity
- Constraint amplifies fundamental variance
- Model solution is still analytical

Model extensions: Nominal ER, Portfolio Flows, Financial External Adjustment

The flow equations are now extended to be:

$$0 = m_0^* \xi_0 e_0 - m_0 l_0 + Q_0 + f^* - f e_0 - D^{US} + D^J e_0$$

$$0 = m_1^* \xi_1 e_1 - m_1 l_1 - RQ_0 - Rf^* + R^* f e_1$$

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$$0 = m_1^* \xi_1 e_1 - m_1 \iota_1 - RQ_0$$

- Nominal: m_t and m_t^* are the US and Japanese money supplies

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$$0 = m_1^* \xi_1 e_1 - m_1 \iota_1 - RQ_0$$

- Nominal: m_t and m_t^* are the US and Japanese money supplies
 - Money used domestically; nominal bonds traded internationally
 - Money is the numéraire in each country; P_t is the nominal price level
 - The US household problem:

$$\max_{\frac{M}{P}, C_{NT}, C_H, C_F} \omega_t \ln \frac{M_t}{P_t} + \chi_t \ln C_{NT,t} + a_t \ln C_{H,t} + \iota_t \ln C_{F,t}$$

$$s.t. \quad M_t + p_{NT,t} C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t} \leq CE_t$$

- Optimality $\Rightarrow M_t = \frac{\omega_t}{\lambda_t}$. Let $m_t \equiv \frac{M_t}{\omega_t}$
- Cash-less limit *à la* Woodford (1998):
 $M_t \downarrow 0, \omega_t \downarrow 0, s.t. m_t \rightarrow \text{finite positive}$, a policy variable

Model extensions: Nominal ER, Portfolio Flows, Financial External Adjustment

The flow equations are now extended to be:

$$0 = m_0^* \xi_0 e_0 - m_0 l_0 + Q_0 + f^* - f e_0$$

$$0 = m_1^* \xi_1 e_1 - m_1 l_1 - RQ_0 - Rf^* + R^* f e_1$$

- Nominal: m_t and m_t^* are the US and Japanese money supplies
- Capital flows: f and f^* are the demand for foreign bonds by the US and Japanese households

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- Nominal: m_t and m_t^* are the US and Japanese money supplies
- Capital flows: f and f^* are the demand for foreign bonds by the US and Japanese households
 - Flows that depend on all fundamentals, but not directly on e are tractable
 - E.g. Carry trade flows:

$$f = b + c(R - R^*)$$

$$f^* = d + g(R - R^*)$$

Model extensions: Nominal ER, Portfolio Flows, Financial External Adjustment

The flow equations are now extended to be:

$$0 = m_0^* \xi_0 e_0 - m_0 \iota_0 + Q_0 + f^* - f e_0 - D^{US} + D^J e_0$$

$$0 = m_1^* \xi_1 e_1 - m_1 \iota_1 - RQ_0 - Rf^* + R^* f e_1$$

- Nominal: m_t and m_t^* are the US and Japanese money supplies
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- Nominal: m_t and m_t^* are the US and Japanese money supplies
- Capital flows: f and f^* are the demand for foreign bonds by the US and Japanese households
- Financial adjustment: D^{US} and D^J are the Dollar net liabilities of the US and the Yen net liabilities of Japan
 - D^{US} and D^J are "legacy" positions
 - US net foreign assets: $D^J e_0 - D^{US}$
 - Currency composition of gross asset and liabilities matters, not just net positions (Gourinchas and Rey (2007), Lane and Shambaugh (2010))

Equilibrium Exchange Rate

Proposition The equilibrium exchange rate follows immediately from the Basic Gamma model by defining "pseudo" imports, exports, and risk bearing capacity, such that:

$$e_0 = \frac{\mathbb{E} \left[\frac{\tilde{\iota}_0 + \frac{\tilde{\iota}_1}{R}}{\tilde{\xi}_1} \right] + \frac{\tilde{\Gamma} \tilde{\iota}_0}{R^*}}{\mathbb{E} \left[\frac{\tilde{\xi}_0 + \frac{\tilde{\xi}_1}{R^*}}{\tilde{\xi}_1} \right] + \frac{\tilde{\Gamma} \tilde{\xi}_0}{R^*}}$$

$$\tilde{\iota}_0 \equiv m_0 \iota_0 + D^{US} - f^*;$$

$$\tilde{\xi}_0 \equiv m_0^* \xi_0 + D^J - f;$$

$$\tilde{\iota}_1 \equiv m_1 \iota_1 + Rf^*;$$

$$\tilde{\xi}_1 \equiv m_1^* \xi_1 + R^*f;$$

$$\tilde{\Gamma} \equiv \Gamma / m_0^*$$

Next slide analyzes main properties of this more general economy

Equilibrium Exchange Rate

The Dollar is weaker (i.e., the Yen is stronger):

① Debts and their currency denomination:

The higher the US net external liabilities in dollars are (higher D^{US}); the lower the Japanese net external liabilities in Yen are (lower D^J)

② Demand pressure:

If $\Gamma > 0$, the lower the demand for the Dollar is (lower f^*)

③ Interest rates:

The higher the Japanese real interest rate is; the lower the US real interest rate is.

④ Money supply:

The higher the US present money supply is (high m_0); the lower the Japanese present money supply is (low m_0^*).

⑤ Financial disruption:

The higher Γ is, if the US is running a trade deficit.

Infinite horizon

- **Proposition:** The exchange rate is:

$$e_t = \iota_* + \beta Q_t^- + \mathbb{E}_t \int_t^\infty e^{-\beta(s-t)} [\beta \hat{\iota}_s + \iota_* (r_s^* - r_s) - \beta f_s^*] ds$$

$$\beta = \frac{r + \sqrt{r^2 + 4\Gamma}}{2}$$

where demand shocks are f_s^* .

- Hence, the Yen is stronger if:
 - Japan is a creditor ($Q_t^- > 0$)
 - There is high demand import demand for Japanese goods ($\hat{\iota}_s > 0$)
 - Interest rates are higher in Japan than in the US ($r_s^* - r_s > 0$)
 - Noise traders (or governments) are selling the dollar ($f_s^* < 0$)

Paper also analytically extends to N countries.

Take-Aways

We presented a basic model with:

- Imperfect capital markets: limited risk bearing, supply and demand of assets matters!
- Production: real effects of ER fluctuations, unemployment
- (Potentially) sticky prices: PCP, LCP, incomplete pass-through
- Welfare analysis: monetary policy, heterodox financial policies

Key implications: Exchange rates are a financial phenomenon determined by supply and demand of assets in different currencies. Financiers' balance sheets and risk tolerance are important determinants of ER

Key take-away: Floating exchange rate regimes can be the source of problems. Heterodox policies (interventions, capital controls) make sense when imbalances are big and financial markets distressed

Derivation of Equilibrium Exchange Rate

Recall that we assume $\xi_t = R^* = R = 1$.

Adding the two flow equations, $e_0 - \iota_0 + Q_0 = 0$ and $e_1 - \iota_1 - Q_0 = 0$, and taking expectations gives:

$$\mathbb{E}[e_1] = \iota_0 + \mathbb{E}[\iota_1] - e_0.$$

The financiers' demand simplifies to $Q_0 = \frac{1}{\Gamma} (e_0 - \mathbb{E}[e_1])$ and from the time-0 flow equation, $Q_0 = e_0 - \iota_0$, so we have:

$$\mathbb{E}[e_1] = (1 + \Gamma) e_0 - \Gamma \iota_0$$

Combining the two equations gives the expression for the time-0 exchange rate:

$$e_0 = \frac{(1 + \Gamma)\iota_0 + \mathbb{E}[\iota_1]}{2 + \Gamma}$$

Effect of financial disruptions on the exchange rate

- Recall the equilibrium exchange rate $e_0 = \frac{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma}$ and US net foreign assets $N_{0+} = \frac{\mathbb{E}[\iota_1] - \iota_0}{2+\Gamma}$.
- The derivative of the exchange rate e_0 with respect to financiers' risk bearing capacity Γ is

$$\begin{aligned}\frac{\partial e_0}{\partial \Gamma} &= \frac{(2 + \Gamma)\iota_0 - (1 + \Gamma)\iota_0 - \mathbb{E}[\iota_1]}{(2 + \Gamma)^2} \\ &= \frac{\iota_0 - \mathbb{E}[\iota_1]}{(2 + \Gamma)^2} \\ &= \frac{-N_{0+}}{2 + \Gamma}.\end{aligned}$$

- When the US is a net external debtor ($N_{0+} < 0$), the derivative is positive, so a financial disruption ($\uparrow \Gamma$) causes the Dollar to depreciate ($\uparrow e$).

Gross portfolio flows and exchange rates

- Japanese households have a noise demand f^* for Dollar bonds, funded by an offsetting position $-f^*/e_0$ in Yen bonds.
- The flow equations are now given by

$$\xi_0 e_0 - \iota_0 + Q_0 + f^* = 0, \quad \xi_1 e_1 - \iota_1 - RQ_0 - Rf^* = 0,$$

and the financiers' demand is still given by $Q_0 = \frac{1}{r} \mathbb{E} \left[e_0 - \frac{R^*}{R} e_1 \right]$.

- Assume $\xi_t = R = R^* = 1$ as before, and define $\tilde{\iota}_0 \equiv \iota_0 - f^*$ and $\tilde{\iota}_1 \equiv \iota_1 + f^*$. Then the previous expression for the equilibrium exchange rate holds for the “tilde” economy:

$$\begin{aligned} e_0 &= \frac{(1 + \Gamma)\tilde{\iota}_0 + \mathbb{E}[\tilde{\iota}_1]}{2 + \Gamma} \\ &= \frac{(1 + \Gamma)(\iota_0 - f^*) + \mathbb{E}[\iota_1 + f^*]}{2 + \Gamma} \\ &= \frac{(1 + \Gamma)\iota_0 + \mathbb{E}[\iota_1] - \Gamma f^*}{2 + \Gamma} \end{aligned}$$

Equilibrium Exchange Rate in the Extended Gamma Model

The flow equations,

$$0 = m_0^* \xi_0 e_0 - m_0 \iota_0 + Q_0 + f^* - fe_0 - D^{US} + D^J e_0$$

$$0 = m_1^* \xi_1 e_1 - m_1 \iota_1 - RQ_0 - Rf^* + R^* fe_1$$

and the financiers' demand

$$Q_0 = \frac{m_0^*}{\Gamma} \mathbb{E} \left[e_0 - e_1 \frac{R^*}{R} \right]$$

can be expressed:

$$0 = \tilde{\xi}_0 e_0 - \tilde{\iota}_0 + Q_0, \quad 0 = \tilde{\xi}_1 e_1 - \tilde{\iota}_1 - RQ_0, \quad Q_0 = \frac{1}{\tilde{\Gamma}} \mathbb{E} \left[e_0 - e_1 \frac{R^*}{R} \right],$$

where we define

$$\begin{aligned} \tilde{\iota}_0 &\equiv m_0 \iota_0 + D^{US} - f^*; & \tilde{\xi}_0 &\equiv m_0^* \xi_0 + D^J - f; \\ \tilde{\iota}_1 &\equiv m_1 \iota_1 + Rf^*; & \tilde{\xi}_1 &\equiv m_1^* \xi_1 + R^* f & \tilde{\Gamma} &\equiv \Gamma / m_0^* \end{aligned}$$

See the paper for the derivation of e_0 in the “tilde” economy.