Amendment Voting with Incomplete Preferences:
A Revealed Preference Approach to Robust Identification*

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Abstract

We study the outcome of the amendment voting procedure based on a potentially incomplete preference relation. A decision-maker evaluates candidates in a list and iteratively updates her choice by comparing the status-quo to the next candidate. She favors the status-quo when the two candidates are incomparable according to her underlying preference. Developing a revealed preference approach, we characterize all choice functions that can arise from such a procedure and discuss to what extent the underlying preference can be identified from observed choices.

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1 Introduction

A popular voting method is the "amendment voting procedure", whose description and analysis dates back to Farquharson [11]. According to this procedure, the decision-maker (henceforth DM) is faced sequentially with a finite set of candidates, or agendas. At any time along the list, the DM compares her current choice to the next candidate in line, choosing which candidate to keep based on a complete and asymmetric binary relation that represents her underlying preference. Miller [25] shows that a candidate is the final vote from some list if and only if it belongs to the "top cycle" of the binary relation.[1]

In many real-world situations, the DM's preference need not be *complete*. For example, the preference could be dependent on multiple factors that are difficult to aggregate, as in Dubra et al. [10]. It is also possible that the DM is indifferent between certain pairs of candidates due to "just noticeable differences", in the sense of Luce [21]. These possibilities motivate us to generalize the classic model of amendment voting by allowing the DM to entertain an incomplete preference. We will however restrict attention to transitive preferences, so as to capture the aforementioned types of incompleteness we have in mind.

To define the amendment voting procedure for incomplete preferences, we must specify how the DM chooses between two candidates who are incomparable. Following the findings in the behavioral economics literature (e.g. Kahneman and Tversky [18]), we assume throughout that the DM favors her current choice to the next candidate whenever incomparability arises. In Appendix C, we discuss the alternative model for "recency-biased" agents (who are biased toward the later candidates) and the corresponding results.

We are primarily concerned with the following question: given a choice function from all lists to final votes, is it possible to identify the DM's underlying preference that induces this mapping via the generalized amendment procedure? The answer turns

[1] Given a binary relation P on a finite set S, the top cycle is defined as the set of candidates in S that are directly or indirectly P-preferred to every other candidate.

out to be negative: often times multiple incomplete preferences lead to the same choice function. However, we show that among these possible preference relations, there is a minimal one with respect to set inclusion. This minimal preference admits the "robust inference" interpretation, in the sense that it captures everything the analyst can deduce with certainty from observed votes.

We derive the minimal preference using a novel revealed preference method. Say that a candidate y is revealed-preferred to another candidate x, if in a list that begins with x, either y is the final vote, or changing the position of y affects the final vote. Due to transitivity, either of these conditions implies that y must be preferred to x according to the DM's true preference. Thus our revealed preference is included in any possible underlying preference. The core of our argument, then, is to show that such a revealed preference is rich enough to generate the choice data.

While our result is strongest when choices from *all* lists are observed, the revealed preference approach does extend to settings with less data, yielding out-of-sample predications. We discuss this more general identification and prediction problem after presenting the main results.

The rest of the paper is organized as follows. In Section 2 we formally set up the model of amendment voting with incomplete preferences. Section 3 introduces the revealed preference and shows it is the minimal possible underlying preference. Section 4 extends the analysis to the case of more limited data sets. We review the related literature and conclude in Section 5. Proofs omitted from the main text can be found in the Appendix.

2 The Model

Let S denote a finite set of candidates. A *list* over S is an ordered sequence (or permutation) $\pi = x_1 x_2 \dots x_n$ which enumerates the candidates in S without repetition.[2] Given an asymmetric, transitive binary relation P on S,[3] we say that the DM's choice

[2]Throughout, we will write lists in this way, without commas or brackets, to distinguish them from sets.
[3]Such a binary relation is also called a *strict partial order*, a terminology we will use in what follows.

follows the amendment procedure with preference P if his choice $c(\pi)$ can be defined as follows:

$$c(x_1) = x_1;$$

$$c(x_1x_2\dots x_k) = \begin{cases} x_k, & \text{if } x_k \text{ is } P\text{-preferred to } c(x_1x_2\dots x_{k-1}); \\ c(x_1x_2\dots x_{k-1}), & \text{otherwise.} \end{cases} \quad \forall 2 \leq k \leq n. \tag{2.1}$$

In words, the DM is faced with the candidates $x_1, x_2, \dots, x_n$ in this order. She always keeps one candidate in mind and compares him to the next candidate from the list. We assume that the DM suffers from status-quo bias, so that she replaces the current candidate in mind with the new one if and only if the latter is strictly preferred according to P. We call the candidate $c(x_1x_2\dots x_n)$ the DM's *final vote* from the list $\pi = x_1x_2\dots x_n$.[4]

A pair of simple examples will help illustrate. First consider three candidates x, y, z, and suppose the DM's preference P is such that yPx, zPx, but y, z are incomparable. Her final votes from different lists are $c(xyz) = c(yxz) = c(yzx) = y$, and $c(xzy) = c(zxy) = c(zyx) = z$. We see here that with a transitive but incomplete preference, different lists result in different final votes. This example also has the property that either of the two P-maximal candidates (y and z) is chosen depending on which one occurs earlier in the list. In general however, the final vote depends more subtly on the list. For example, consider four candidates x, y, z, w and a preference P such that yPx, wPz and all other pairs are indifferent. In this case a DM following the amendment procedure chooses the candidate w from the list $zxyw$ despite the candidate y appearing earlier, and she chooses y from the list $xzwy$ despite w appearing earlier.

We provide a plausible story for this last example: imagine that x, y, z, w are four

[4]In a different specification, one might consider a DM who chooses the later candidate whenever direct comparison cannot be made. That variant of the model and its implications are discussed in Appendix C. We remark here that status-quo bias fits our sequential choice setting better, both empirically and theoretically. On the theoretical side, we show below that a status-quo biased DM chooses a P-maximal candidate from any list, thus achieving Pareto-efficiency. This is not true for "recency-biased" decision-makers.

political candidates. Candidates x and y both brag about their expertise in managing the economy, although y has more experience in this regard. On the other hand, candidates z and w campaign about their practical solutions to the country's racial divide, and it is believed that w has a better policy proposal than z. When faced with the candidates z, x, y, w in this order, the DM's attention is immediately drawn to candidate z's claim for a more unified nation. This leads her to underweight the value of economic reforms brought by the next two candidates x and y. But when it comes lastly to candidate w, the DM recognizes his better policy for dealing with the racial problem, and so she votes for w. In a similar way, she would vote for y from the list $xzwy$. Such effects of salience on choice have been more explicitly studied in Bordalo et al. [5] [6].

Given an asymmetric transitive relation P, our first result characterizes the set of final votes as the list varies.

Proposition 1. *Suppose the DM follows the amendment procedure with strict partial order P. Then the candidates that are ever chosen from some list are the P-maximal candidates in S.*

Proof. In one direction, suppose candidate x is P-maximal in S. Then in any list π that begins with x, x will be the final vote due to status-quo bias. Conversely, suppose x is not P-maximal and there exists some candidate y s.t. yPx. Assume for contradiction that x is the final vote in some list π. Observe that according to the amendment procedure, in any list $\pi = x_1 \dots x_n$, $c(x_1 \dots x_j)$ is either equal to or P-preferred to $c(x_1 \dots x_{j-1}), \forall j$. Hence by transitivity, $x = c(x_1 \dots x_n)$ is P-preferred to every $c(x_1 \dots x_{j-1})$. Let us now choose the subscript j so that $y = x_j$. Then by the previous claim, y is preferred to x which is in turn preferred to $c(x_1 \dots x_{j-1})$. By transitivity, this implies $y = c(x_1 \dots x_j)$. But then $x = c(x_1 \dots x_n)$ is preferred to $y = c(x_1 \dots x_j)$, contradicting the asymmetry of P. ■

The above proposition enables the analyst to recover partial information about the

DM's preference from her final votes. Can the analyst learn more? This is the question we turn to in the next section.

3 The Revealed Preference

We begin by formalizing the choice function that we will take as primitive. Let $\Pi(S)$ denote the set of all lists over S. For any preference relation P, the amendment voting rule as defined in (2.1) yields a choice function $c^P : \Pi(S) \to S$, where $c^P(\pi)$ represents the DM's final vote from the list π.

To understand the properties of such a choice function, we will take a revealed preference approach. Given any choice function $c : \Pi(S) \to S$, we define a binary relation P^* such that yP^*x if and only if either of the following two conditions holds:
Condition I: $c(\pi) = y$ for some list π that begins with x;
Condition II: $c(\pi) \neq c(\pi')$ for some list π that begins with x and another list π' that is obtained from π by moving the candidate y to an arbitrary position.

Any choice function c determines a *revealed preference* $P^*(c)$ through this pair of conditions, and we will often simply write P^* when there is no confusion. The next lemma shows that when the choice function arises from the amendment procedure, the revealed preference is part of the true underlying preference.

Proposition 2. *If $c = c^P$ for some strict partial order P, then $P^*(c) \subset P$.*

Proof. It suffices to show that if y is not P-preferred to x, then y is not revealed-preferred to x. From transitivity, we see that Condition I can never be satisfied. In fact, when faced with a list that begins with x, the DM never chooses y at any time along the list. Thus the position of y does not affect the final vote, and Condition II will not be satisfied either. ■

Although Condition I looks easier, it is Condition II that really captures the sequential aspect of choice in our model. To give a simple example, consider four candidates

x, y, z, w and a preference P such that $wPyPx, zPx$ and all other pairs are incomparable. A decision-maker following the amendment procedure chooses w from the list $xyzw$ but z from the list $xzyw$. Only by Condition II (and not Condition I) can the analyst infer that the DM prefers y to x, and this preference is crucial for the previous distinct choices to occur. Furthermore, Condition II provides a simple experimental design to study such sequential choice problems — by varying the position of y in a list that begins with x (and vice versa), one can glean information regarding the preference between these two candidates.

It turns out that the revealed preference so defined contains everything the analyst can identify from observed votes. In other words, if the choice function c arises from the amendment procedure with some transitive preference P, then the revealed preference P^* is also transitive, and it generates the same final votes via amendment voting.

Theorem 1. *Fix a choice function $c : \Pi(S) \to S$ and define P^* as its revealed preference. Then $c = c^P$ for some strict partial order P iff (if and only if) P^* is a strict partial order and $c = c^{P^*}$.*

Proof. Because this theorem is the main result of the paper, we sketch an outline of the proof here. The "if" statement is trivial, and we focus on the "only if". First, we note by Condition II that any candidate z not revealed-preferred to the first candidate x can be moved to later in the list, without affecting the final vote. Repeatedly applying this observation, we may without loss assume that the candidate y immediately after x is P^*-preferred to x. Next, as P^* is included in P, we deduce from yP^*x that yPx. Thus, switching the first two candidates x and y does not change the final vote by a DM with true preference P. This leads us to a different list with the same final vote: the new list starts with y instead of x, while y was the first candidate in the original list revealed-preferred to x. Thus the DM's choice from the original list is as if she carries out the amendment procedure with underlying preference P^*, proving that $c = c^{P^*}$. The proof that P^* is also a strict partial order is more involved, which we relegate to Appendix A. ■

We remark that the theorem not only tells us that the revealed preference is what the analyst can "robustly" infer from observed votes. It also solves the characterization problem of what choice data are consistent with the generalized (status-quo biased) amendment procedure with incomplete preferences. All one has to do is to compute the revealed preference and verify whether the choice function it generates coincides with what is observed. As a corollary of the proof above, we have an alternative method to verify whether a given choice function can arise from amendment voting:

Corollary 1. *Fix a choice function $c : \Pi(S) \to S$ and define P^* as its revealed preference. Then $c = c^P$ for some strict partial order P iff P^* is a strict partial order and $c(xyA) = c(yxA)$ whenever yP^*x.*

In Appendix A, we show that our results extend without change to the case where the true preference P is further restricted to be an interval-order or semi-order.[5] To be specific, we prove that $c = c^P$ for some interval-order/semi-order iff the revealed preference P^* is an interval-order/semi-order and $c = c^{P^*}$.

Is the true preference P necessarily equal to the revealed preference P^* — in other words, is there a unique underlying preference generating observed votes? The answer is negative, for example when there are more than two candidates and P is a complete preference. Despite this non-uniqueness, we have the following structural result regarding the family of preferences that induce the same choice function.

Theorem 2. *Suppose P and Q are strict partial orders such that $c^P = c^Q = c$. Let P^* be the revealed preference derived from c. Then for any strict partial order R satisfying $P^* \subset R \subset P \vee Q$, it holds that $c^R = c$ as well. In particular, $c^{P \wedge Q} = c$.*[6]

Proof. See Appendix B.

[5] An interval-order P is a strict partial order that satisfies $PIP \subset P$, where I is the indifference relation associated with P, and PIP denotes the concatenation of these binary relations. Any such preference admits an "interval representation": $yPx \Leftrightarrow u(y) > u(x) + b(x)$, for some functions $u(\cdot)$ and $b(\cdot)$. If in addition $PPI \subset P$, then one can take $b(x) = 1, \forall x$ and obtain a semi-order. See Beja and Gilboa [4] and Fishburn [12], Section 2.4.

[6] Here, $P \vee Q$ denotes the union of the binary relations, while $P \wedge Q$ denotes their intersection. Note that we do not claim $c^{P \vee Q} = c$, because $P \vee Q$ is not necessarily a strict partial order.

4 Limited Data Sets

We have demonstrated the power of the revealed preference approach when the analyst can observe the final votes from *all* lists. A natural question to pursue next is what to do with less choice data, as is often the case in reality. As shown recently by de Clippel and Rozen [9], extending choices from limited data sets is not always a trivial exercise. This difficulty also appears in our setting, and we are not able to completely characterize which (partial) choice functions can arise from the amendment procedure.[7] Rather, we devote this section to discussing additional properties of the choice function, and how they may be used to make out-of-sample predictions and/or to identify the DM's true preference.

Our analysis builds on the following proposition:

Proposition 3. *Suppose $c = c^P$ for some strict partial order P. These axioms hold:*
(Primacy). *If x is the final vote from a list π, and π' differs from π only in that the position of x is moved earlier. Then x remains the final vote from the list π'.*
(Switching). *If yPx, then in any list where these two candidates are adjacent, switching their positions does not change the final vote.*

Proof. The proofs of these axioms are similar to the proof of Proposition 1, and so they are omitted. ■

The primacy axiom could be directly used to generate out-of-sample votes. In contrast, the switching axiom depends on knowing the underlying preference P. Thus in practice, one should instead apply this axiom with the revealed preference P^*, which is always included in P. Either way, these out-of-sample predictions provide additional information from which one could derive extra revealed preference. The following example illustrates this iterative procedure of eliciting the true preference:

Example 4.1: Let there be 5 candidates x, y, z, u, v. Suppose the analyst observe the final votes from two lists: $c(xyzvu) = u$ and $c(xzyuv) = v$. What can be inferred about

[7] Curiously, the revealed preference method we develop for a recency-biased DM completely solves the identification problem for any data set. See Appendix C for details.

the DM's true preference? Applying the revealed preference method, we could only deduce that v and w are both revealed-preferred to x. Note that Condition II does not yet have any bite. However, we could invoke the primacy axiom and deduce the out-of-sample final vote that $c(xyzuv) = u$. At this stage we recognize the difference between $c(xyzuv)$ and $c(xzyuv)$, allowing us to obtain by Condition II that both candidates y and z are revealed-preferred to x.

So far we have shown that all candidates other than x are preferred to x. But that is not the end of the story. We could apply the switching axiom to $c(xyzvu) = u$ and make another out-of-sample prediction: $c(yxzvu) = u$. This then enables us to conclude from Condition I that u is preferred to y. Similarly we could derive that v is revealed-preferred to z, so that the revealed preference consists of uP^*yP^*x and vP^*zP^*x. Since the observed choices do arise under the amendment procedure with this revealed preference, we have successfully identified the DM's underlying preference.[8] ■

Unfortunately, we are not always guaranteed to end up with a revealed preference P^* that generates the observed final votes. In fact, when the data set is limited, there may not exist a minimal preference relation that explains the data. We conclude this section with such a negative example, leaving further study for future work.

Example 4.2: Consider 7 candidates x, y, z, y_1, z_1, u, v. The observed final votes are $c(xyzy_1z_1vu) = u$ and $c(xzyz_1y_1uv) = v$. One underlying preference that generates these votes consists of $uPyPx$ and $vPzPx$. By similar reasoning, the preference Q with uQy_1Qx and vQz_1Qx also induces the same final votes. Nevertheless, the intersection of these preferences only has v and w preferred to x, which is clearly not sufficient to explain the observations.

[8] Once again, the DM's true preference P might be even richer than P^*. But P^* is as much as we can infer from the given data set.

5 Related Literature

Following the paper of Arrow [2], there has been a large literature relating choice *correspondence* to the underlying preference relation. Notably, Jamison and Lau [17] and Fishburn [13] establish necessary and sufficient conditions for a choice correspondence $c : 2^S \backslash \{\emptyset\} \to S$ to be given by $c(T) = \{P\text{-maximal elements in } T\}$, where P is an interval order or a semi-order. Like these authors, we are also interested in the problem of recovering underlying preference from choice data. But our domain is the space of lists, which differs from these classical papers.

Empirical literature has well documented the order effect on choice. Miller and Krosnick [24] and Krosnick et al. [20] find statistically significant and sometimes large effects of being listed first on the vote shares of major party candidates in the U.S. state and federal elections. The "first-position advantage" these papers highlight is a special case of the status-quo bias captured by our model. On the other hand, Bruine de Bruin [7] reports on order effects in panel decisions in contests such as the World Figure Skating Competition and the Eurovision Song Contest. He finds that the *last* few participants in the contest have an advantage, corresponding to the model with recency bias in Appendix C.

The agenda-rationalizable choice of Apesteguia and Ballester [1], the tournament choice of Horan [16] and the list-rationalizable choice of Yildiz [32] are similar to our model in that the DM in their models also performs pairwise comparisons along a list. However, these papers assume a *fixed but unknown* order in which candidates are evaluated, and they attempt to endogenously derive this order.[9][10] Their choice domain is unordered subsets of candidates, while we consider observations from ordered lists. Furthermore, these papers consider underlying preference relations that are tournaments, which are complete but not necessarily transitive binary relations. We have

[9] The sequentially rationalizable choice model of Manzini and Mariotti [22] is also related and has a similar structure to these papers. The distinction is that their DM sequentially evaluates "rationales" (instead of candidates), or multiple underlying preferences, to eliminate certain candidates.

[10] Caplin and Dean [8] and Masatlioglu and Nakajima [23] consider choice by search, a different kind of dynamic choice procedure. They focus on deriving the search rule, while we take the ordering of candidates as the natural search rule.

however focused on incomplete but transitive relations.

The framework introduced in Rubinstein and Salant [28] is closely related to ours.[11] These authors consider a DM whose underlying preference is a weak order, and who uses the list to resolve indifferences by choosing either the first or the last most-preferred candidate. Choosing the earliest maximal candidate is equivalent to satisficing in the sense of Simon [31], which specifies a set A and dictates choosing the earliest candidate that belongs to A. We note that satisficing is a special case of our model with status-quo bias, when the DM's true preference is such that yPx iff $y \in A$ and $x \notin A$.

Salant [30] presents a more general model of iterative choice from lists, not restricting attention to the amendment procedure.[12] He proves that framing effects generally exist, and he characterizes choice rules that exhibit optimal tradeoff between maximizing utility and minimizing computational complexity. By comparison, our DM exhibits a specific form of framing effect: either the status-quo bias or the recency bias.

The recent paper of Guney [15] is closest to this work. She also considers the amendment procedure, but she works with the larger domain of all lists over all subsets of S. One of her main results is that the primacy axiom together with some other minor conditions characterizes the choice behavior of a status-quo biased DM on this larger domain. In contrast, the primacy axiom is not sufficient in our setting.[13] Another distinction is that in Guney's model, choices from two-candidate lists are observed, so that the DM's preference can be perfectly identified from data. Working with the smaller domain $\Pi(S)$, we have shown that the analyst can robustly infer a revealed preference, which is the minimal underlying preference that explains the data. Our revealed preference method extends to more limited data sets, where the problem of identification and out-of-sample prediction becomes interesting and nontrivial.

[11] The choice with frames model of Rubinstein and Salant [29] is more general. We are studying a specific type of frame, that is, the ordering of candidates.

[12] A further generalization is obtained by considering a list as a special case of a decision tree, for example see Mukherjee [26].

[13] We provide an example here for completeness. There are four candidates x, y, z, w. The DM chooses y if he is the first candidate. Otherwise the DM follows the amendment procedure with the preference $wQyQx$ and zQx. The primacy axiom is satisfied, but these choices are not consistent with amendment voting.

Appendix A: Proof of Theorem 1

For the sake of clarity, the proof is broken down into several claims.

Claim A.1. *Let P^* be the revealed preference derived from the choice function c. If P^* is a strict partial order and $c(xyA) = c(yxA)$ whenever yP^*x, then $c = c^{P^*}$.*

Proof. Fix a list π over S. Let x_1 denote the first candidate in π, and for $k \geq 1$ let x_{k+1} denote the first candidate after x_k in this list that is P^*-preferred to x_k. In this way we can write $\pi = x_1A_1x_2A_2 \dots x_nA_n$, so that $c^{P^*}(\pi) = x_n$. Because any candidate y in A_1 is not P^*-preferred to x_1, moving y to the end of the list does not change the DM's choice by Condition II. Applying this observation to every candidate in A_1, we obtain

$$c(x_1A_1x_2A_2 \dots x_nA_n) = c(x_1x_2A_2 \dots x_nA_nA_1). \tag{A.1}$$

But $x_2P^*x_1$, so by assumption we have

$$c(x_1x_2A_2 \dots x_nA_nA_1) = c(x_2x_1A_2 \dots x_nA_nA_1) = c(x_2A_2 \dots x_nA_nx_1A_1), \tag{A.2}$$

where the second equality follows because $\neg x_1P^*x_2$ and Condition II. From Eq. (A.1) and Eq. (A.2) we obtain $c(x_1A_1x_2A_2 \dots x_nA_n) = c(x_2A_2 \dots x_nA_nx_1A_1)$. Repeating this procedure, we can eventually derive

$$c(x_1A_1x_2A_2 \dots x_nA_n) = c(x_nA_nx_1A_1 \dots x_{n-1}A_{n-1}). \tag{A.3}$$

Let us show that no other candidate is P^*-preferred to x_n: this is because x_n is P^*-preferred to x_k and no candidate in A_k is P^*-preferred to x_k. Hence, by Condition I, the final vote $c(x_nA_nx_1A_1 \dots x_{n-1}A_{n-1})$ can only be x_n. By Eq. (A.3), $c(\pi) = x_n = c^{P^*}(\pi)$ as we desire to show. ■

Claim A.2. *Let $c = c^P$ where P is a strict partial order, and P^* be its revealed preference. Then yP^*x implies yPx and $c(xyA) = c(yxA)$ whenever yP^*x.*

Proof. The first half has been proved in the main text, while the second half follows from $c = c^P$. ■

Claim A.3. *Let $c = c^P$ with P a strict partial order, then P^* is a strict partial order as well.*

Proof. Suppose P is asymmetric and transitive, we show that P^* has the same properties. Asymmetry follows from $P^* \subset P$. To prove transitivity, assume that zP^*y and yP^*x. By $P^* \subset P$, we deduce $zPyPx$. The next lemma suffices to show that zP^*x must hold in this case. ■

Lemma A.4. *Let $c = c^P$ where P is a strict partial order, and P^* be its revealed preference. If zPx but $\neg zP^*x$, then for any y with $\neg yPz$, we have $\neg yP^*x$.*

Proof. Pick any list $\pi = xyAzB$, we will show that $c(\pi) \neq y$ and moving y to any later position does not alter the final vote. This would imply that neither Condition I nor Condition II is satisfied, and so y would not be revealed-preferred to x.

Since z is not revealed-preferred to x, we deduce by Condition II that

$$c(xyAzB) = c(xzyAB). \tag{A.4}$$

Using $c = c^P$, zPx, $\neg yPz$, we further obtain

$$c(xzyAB) = c^P(xzyAB) = c^P(xzABy) = c(xzABy), \tag{A.5}$$

In particular, we see that $c(\pi) \neq y$. Next, we use $\neg zP^*x$ and Condition II to deduce

$$c(xzABy) = c(xAzBy). \tag{A.6}$$

The preceding three equations together imply that $c(xyAzB) = c(xAzBy)$, so that moving y to the end of the list does not affect the final vote. In fact, the same

argument can be applied to move y to any position in the list, completing the proof. ■

Theorem 1 and Corollary 1 now follow from Claims A.1 to A.3. Next we show that the results extend to interval-orders and semi-orders. It suffices to re-prove Claim A.3 for those cases:

Interval-orders: We need to show that $P^*(c)$ is an interval order whenever $c = c^P$ where P is an interval-order. We already know that P^* is a strict partial order. Let I^* be the indifference relation associated with P^*, it remains to check $P^*I^*P^* \subset P^*$. Thus assume wP^*z, zI^*y, yP^*x. Without loss we take these candidates to be distinct, otherwise the result is trivial. Recall from Fn. 5 that P admits an interval representation $[u(x), u(x) + b(x)]$. From wP^*z we have wPz and $u(w) > u(z) + b(z)$. If $u(y) \geq u(w)$, then $u(y) > u(z) + b(z)$ which yields yPz. Thus yPz but $\neg yP^*z$, and $\neg wPy$ because $u(w) \leq u(y)$. From Lemma A.4 we deduce that $\neg wP^*z$, contradicting our assumption.

Thus $u(y) \leq u(w)$, showing that $\neg yPw$. Furthermore, wPx because $u(w) \geq u(y) > u(x) + b(x)$. If $\neg wP^*x$, we would obtain from Lemma A.4 that $\neg yP^*x$, another contradiction. Hence wP^*x must hold, and we have proved $P^*I^*P^* \subset P^*$ so that P^* is an interval-order. ■

Semi-orders: We already know that P^* is an interval-order. To show P^* is in fact a semi-order, we need to check $P^*P^*I^* \subset P^*$. Thus assume wP^*z, zP^*y, yI^*x, where the candidates are distinct. Let $[u(x), u(x) + 1]$ be a representation for P. If $u(x) \geq u(z)$, then from zPy and the interval representation, it holds that xPy. But from $\neg xP^*y$, $\neg zPx$ and Lemma A.4, we would deduce $\neg zP^*y$, a contradiction. So $u(z) > u(x)$. The following lemma suffices to complete the proof.

Lemma A.5. *Let P be a semi-order with interval representation $[u(x), u(x)+1]$. Suppose wP^*z and $u(z) > u(x)$. Then wP^*x also holds.*

Proof. By assumption we have wPz. Thus $u(w) > u(z) + 1 > u(x) + 1$, which implies

wPx. From wP^*z, we know that either Condition I or Condition II is satisfied. If w is the final vote in a list that begins with z, w must be a P-maximal candidate. Since wPx, it follows that w is the final vote in any list that begins with xw. This yields wP^*x as we desire.

Suppose instead that Condition II is satisfied, so that for some A, B we have

$$c(zwAB) \neq c(zAwB). \tag{A.7}$$

Note that the position of w affects the final vote. Define $A_1 = \{y \in A : yPz\}$, where the candidates are ordered in the same way as in A. Also define $B_1 = (A \backslash A_1) \cup (B \backslash \{x\})$, where order is again preserved. Because any candidate $y \in A \backslash A_1$ (and x) is not P-preferred to z or w, its position in the list does not change the final vote. Thus

$$c(zwAB) = c^P(zwAB) = c^P(wzAB) = c^P(wzA_1B_1x) = c^P(xwzA_1B_1) = c(xwzA_1B_1) \tag{A.8}$$

The penultimate equality follows because wPx. Similarly we have

$$c(zAwB) = c^P(zAwB) = c^P(zA_1wB_1x) = c^P(xzA_1wB_1) = c(xzA_1wB_1). \tag{A.9}$$

Here, the penultimate equality follows because by definition, the first element in A_1w is P-preferred to z. It is thus P-preferred to x due to the interval representation and $u(z) > u(x)$.

From the preceding equations, we obtain that

$$c(xwzA_1B_1) = c(zwAB) \neq c(zAwB) = c(xzA_1wB_1). \tag{A.10}$$

Hence by Condition II, we must have wP^*x as desired. ■

Appendix B: Proof of Theorem 2

We will prove the following lemma, which implies the theorem.

Lemma B.1. *Let c be a choice function generated by some strict partial order via the amendment procedure (equivalently, the revealed preference P^* is a strict partial order and $c = c^{P^*}$). Suppose R is a strict partial order containing P^*, such that for any yRx, there exists a strict partial order Q with yQx and $c^Q = c$. Then $c^R = c$ also holds.*

Proof. Fix a list π. Because the binary relation R contains P^*, the amendment procedure with preference R differs from the one with preference P^* only at those times where the new candidate y is R-preferred but not P^*-preferred to the current choice x. We can write π as

$$\pi = A_1 y_1 A_2 y_2 \dots A_m y_m A_{m+1}, \tag{B.1}$$

with $x_1 = c^R(A_1) = c^{P^*}(A_1)$ and y_1 is R-preferred but not P^*-preferred to x_1.[14] For $2 \leq k \leq m+1$, $x_k = c^R(y_{k-1}A_k) = c^{P^*}(y_{k-1}A_k)$; y_k is R-preferred but not P^*-preferred to x_1. Note that x_k could be y_{k-1}.

By construction,

$$c^R(\pi) = x_{m+1} = c^{P^*}(y_m A_{m+1}). \tag{B.2}$$

Because $y_m = c^R(A_1 y_1 \dots A_m y_m)$, it is R-maximal in the partial list $A_1 y_1 \dots A_m y_m$. Since $P^* \subset R$, we deduce that y_m is P^*-maximal in this partial list and so

$$c^{P^*}(y_m A_{m+1}) = c^{P^*}(y_m x_m A_1 y_1 \dots y_{m-1} A_m A_{m+1}), \tag{B.3}$$

where on the RHS x_m need to be removed from the partial list $y_{m-1}A_m$.

Now we invoke the assumption that $c^{P^*} = c^Q$ for some strict partial order Q with

[14] The choice function for the partial list A_1 is given by Eq. (2.1).

$y_m Q x_m$. This tells us that

$$c^{P^*}(y_m x_m A_1 y_1 \dots A_m A_{m+1}) = c^Q(y_m x_m \dots) = c^Q(x_m y_m \dots) = c^{P^*}(x_m y_m A_1 y_1 \dots A_m A_{m+1}). \tag{B.4}$$

But $x_m = c^{P^*}(y_{m-1} A_m)$, so we also have

$$c^{P^*}(x_m y_m A_1 y_1 \dots A_m A_{m+1}) = c^{P^*}(y_{m-1} A_m y_m A_1 y_1 \dots A_{m-1} A_{m+1}). \tag{B.5}$$

From the preceding equations, $c^{P^*}(y_m A_{m+1}) = c^{P^*}(y_{m-1} A_m y_m A_1 y_1 \dots A_{m-1} A_{m+1})$. We have thus successfully inserted $y_{m-1} A_m$ before the candidate y_m without changing the final vote.

Next, we will move $y_{m-2} A_{m-1}$ to the beginning of the list. To do this, we note from $y_{m-1} R x_{m-1}$ that $\neg x_{m-1} P^* y_{m-1}$. Condition II allows us to move $x_{m-1} \in y_{m-2} A_{m-1}$ to immediately after y_{m-1} in the list $y_{m-1} A_m y_m A_1 y_1 \dots A_{m-1} A_{m+1}$. After that we can switch the candidates y_{m-1} and x_{m-1} as in Eq. (B.4). We can then move the remaining candidates in $y_{m-2} A_{m-1}$ to the front, as we did in Eq. (B.5).

Repeating this procedure, we derive that

$$c^{P^*}(y_{m-1} A_m y_m A_1 y_1 \dots A_{m-1} A_{m+1}) = c^{P^*}(A_1 y_1 \dots A_m y_m A_{m+1}) = c^{P^*}(\pi). \tag{B.6}$$

From Eq. (B.2)-(B.6), we conclude $c^R(\pi) = c^{P^*}(\pi)$ as desired. ■

Appendix C: The Model with Recency Bias

In this appendix we consider the variant of our model with recency bias: when the DM cannot compare two candidates according to P, she favors the one who appears more

recently. The DM's choice function is thus defined as follows:

$$c(x_1) = x_1;$$

$$c(x_1x_2\dots x_k) = \begin{cases} c(x_1x_2\dots x_{k-1}), & \text{if } c(x_1x_2\dots x_{k-1}) \text{ is } P\text{-preferred to } x_k; \\ x_k & \text{otherwise.} \end{cases} \quad \forall 2 \le k \le n. \tag{C.1}$$

If P could be any binary relation, then this is equivalent to the original model upon transforming the underlying preference. Specifically, if a choice function c is generated by Eq. (2.1) with true preference P, then it is also generated by Eq. (C.1) with true preference Q, where yQx if and only if $\neg xPy$. However, this equivalence breaks down once we restrict attention to transitive preferences.

To distinguish from the notation for status-quo biased decision-makers, we will use $\bar{c}^P$ (with an upper-bar) to denote the choice function that arises from the recency-biased amendment procedure (C.1). We will develop a different revealed preference by the following condition:

Condition III: $y\bar{P}x$ iff in some list π where y precedes x, y is the final vote.

For a recency-biased DM, the revealed preference $\bar{P}$ is always part of the true preference P. Analogous to Theorem 1, our main result is that P^* can generate the observed choices.

Theorem 3. *Fix a choice function $c : \Pi(S) \to S$ and define $\bar{P}$ by Condition III. Then $c = \bar{c}^P$ for some strict partial order P iff $\bar{P}$ is a strict partial order and $c = \bar{c}^{\bar{P}}$.*

Proof. All proofs in this appendix can be found at the end.

We also have an analogue of Proposition 1 that characterizes which candidates are ever chosen via the recency-biased amendment procedure.

Proposition 4. *Let T be the smallest non-empty subset of S such that any candidate in T is $\bar{P}$-preferred to every candidate in $S \backslash T$.*[15] *Then T is the set of candidates that*

[15] Such subsets are closed under intersection, so there exists a smallest one.

are ever chosen from some list, according to $\bar{c}^{\bar{P}}$.

We also have the following structural result regarding those underlying preferences that generate the same choice function.

Proposition 5. *Given a choice function* c *that arises from recency-biased amendment voting.* $c = \bar{c}^P$ *iff* $\bar{P} \subset P$*, and whenever* $yPx, \neg y\bar{P}x$ *it holds that* $x, y \in S \backslash T$*. As a corollary, there is a unique possible true preference if and only if* $S \backslash T$ *is the empty set or a singleton.*

Our final result shows that a choice function cannot simultaneously exhibit status-quo bias and recency bias except in trivial situations.

Proposition 6. *Suppose* $c = c^P = \bar{c}^Q$ *for strict partial orders* P *and* Q*. Then* $\exists x \in S$ *such that* $c(\pi) = x, \forall \pi$.

Proof of Theorem 3: The "if" statement is trivial, so we focus on the "only if." We state a simple lemma that is useful in the analysis.

Lemma C.1. *Let* $c = \bar{c}^P$ *with a strict partial order* P*. In any list, the recency-biased DM votes for the earliest candidate that is* P*-preferred to every later candidate. As a corollary, if* yPx*, then changing the position of* x *does not affect whether or not* y *is the final vote.*

The first part of the lemma follows from transitivity. For the corollary, we argue that in a list where y is the final vote, moving x does not change the fact that y is P-preferred to every later candidate. Furthermore, any candidate z before y is still P-worse than some later candidate — in case that candidate was x, we could as well take y because yPz by transitivity.

Let us show that the revealed preference $\bar{P}$ is also a strict partial order. Because P is asymmetric and $\bar{P} \subset P$, $\bar{P}$ is also asymmetric. To prove transitivity, we consider three distinct candidates x, y, z with $z\bar{P}y$ and $y\bar{P}x$. By definition, $z\bar{P}y$ implies there

exists a list π with $c(\pi) = z$. Moreover, as $zPyPx$, we can move x to after z in the list π, without changing the fact that z is the final vote. Thus $z\bar{P}x$ as desired.

Next we show that $c(\pi) = \bar{c}^{\bar{P}}(\pi)$. Fix a list π. It is without loss to write

$$\pi = x_1 A_1 x_2 A_2 \dots x_n A_n, \tag{C.2}$$

where x_1 is the first candidate, and for $1 \leq k \leq n-1$, x_{k+1} is the first candidate after x_k such that $\neg x_k \bar{P} x_{k+1}$. By construction, $\bar{c}^{\bar{P}}(\pi) = x_n$. We thus need to show $c(\pi) = x_n$. We note that $c(\pi) \neq x_k (\forall 1 \leq k \leq n-1)$ due to Condition III defining the revealed preference. Furthermore, x_k is $\bar{P}$-preferred to every candidate in A_k, so that by transitivity no candidate in A_k can be $\bar{P}$-preferred to x_{k+1}. This shows $c(\pi) \notin A_k$. Finally, note that x_n is $\bar{P}$-preferred and thus P-preferred to every candidate in A_n. By transitivity, we deduce that the candidate chosen from the partial list $x_1 A_1 x_2 A_2 \dots x_n$ must be P-preferred to x_n and thus to every candidate in A_n. Hence $c(\pi) \notin A_n$, which proves $c(\pi) = x_n$. ■

Remark: with more limited data sets, the first part of this proof fails and we are not guaranteed that $\bar{P}$ defined by Condition III is transitive. However, the transitive closure of $\bar{P}$ is still included in the DM's true preference. Since the second part of the above proof goes through without change, we are able to achieve robust identification of the DM's preference for any given set of observations. This contrasts with the model of a status-quo biased DM, see the discussion in Section 4.

Proof of Proposition 4: We first show that for any list π, $c(\pi) \in T$. Since $c(\pi) = \bar{c}^{\bar{P}}(\pi)$, the earliest candidate in π that belongs to T must be chosen in the partial list up to it, by the definition of T. Simple induction shows that afterwards, the DM only keeps in mind a candidate in T.

Conversely, take $z \in T$, we will construct a list π in which z is the final vote. Define $V_0 = \{z\}$. For $k \geq 1$ define $V_k = \{x \in S \setminus \cup_{j=0}^{k-1} V_j : \exists y \in V_{k-1} \text{ with } \neg x \bar{P} y\}$.[16] Finally let $V = \cup_{k=0}^{\infty} V_k$. By construction, any element in $S \setminus V$ must be $\bar{P}$-preferred to every

[16]The iterative definition corresponds to running a breath-first-search on an auxiliary directed graph.

element in V. But $S \backslash V$ does not contain T because $z \in V$ and $z \in T$. Thus by the minimality of T, we deduce that $S \backslash V$ must be the empty set, so that $V = S$. Now note that $V_0, V_1, \ldots$ are disjoint sets. So we can find a positive integer m such that $V_k = \emptyset$ for $k > m$. Consider the list $\pi = V_m V_{m-1} \ldots V_0$, where the ordering of those candidates in any V_k can be arbitrary. As $V = S$, π is indeeda list over S. Moreover, by our construction, for any candidate $x \neq z$ there exists some candidate y later in the list such that $\neg x \bar{P} y$. This implies that x cannot be the final vote, and we conclude that $c(\pi) = z$. ■

Proof of Proposition 5: Suppose P satisfies the assumptions. Take any list π. Let π_T be the partial list over T that preserves the ordering of these candidates in π. Because any candidate in T is P-preferred to every candidate in $S \backslash T$, it is straightforward to see that $\bar{c}^P(\pi) = \bar{c}^P(\pi_T)$. Similarly $\bar{c}^{\bar{P}}(\pi) = \bar{c}^{\bar{P}}(\pi_T)$. But P and $\bar{P}$ agree on T, so $\bar{c}^P(\pi_T) = \bar{c}^{\bar{P}}(\pi_T)$. It follows that $\bar{c}^P(\pi) = \bar{c}^{\bar{P}}(\pi)$ as desired.

Conversely, suppose $\bar{c}^P = c$. We know that $\bar{P}$ is included in P. Take any pair of candidates x, y with yPx and $\neg y \bar{P} x$. From Lemma C.1 above, y cannot be the final vote in any list, otherwise we can move x to the end of that list and deduce $y \bar{P} x$ from Condition III. Thus by Proposition 4, y does not belong to T. Since yPx, we deduce from the definition of T that $x \notin T$ either. This completes the proof. ■

Proof of Proposition 6: Take any list π and let $c(\pi) = c^P(\pi) = x$. By the primacy axiom, we can move x to the beginning of this list without changing the fact that it is chosen. But $c = \bar{c}^Q$, so x is Q-preferred to every other candidate. Thus $c(\sigma) = \bar{c}^Q(\sigma) = x$ for every list σ. ■

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