Earthquakes Within Earthquakes: Patterns in Rupture Complexity

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Abstract Earthquake source time functions carry information about the complexity of seismic rupture. We explore databases of earthquake source time functions and find that they are composed of distinct peaks that we call subevents. We observe that earthquake complexity, as represented by the number of subevents, grows with earthquake magnitude. Patterns in rupture complexity arise from a scaling between subevent moment and main event moment. These results can be explained by simple 2-D dynamic rupture simulations with self-affine heterogeneity in fault prestress. Applying this to early magnitude estimates, we show that the main event magnitude can be estimated after observing only the first few subevents.

Plain Language Summary Seismograms are measurements of waves from earthquakes. They give us information about what happened on the fault at the place where the earthquake occurred. Seismograms can be difficult to interpret because they are often very complicated. Why? One reason is that the waves change when they travel long distances between the fault and a seismometer. Seismologists correct for this effect by constructing something called a source time function. Source time functions are much easier to understand than raw seismograms. We examine a catalog of source time functions from around the world. We find that large earthquakes are composed of many smaller events that we call subevents. The size of a subevent is related to the size of the main earthquake. One important outcome is that we can predict the final size of an earthquake after observing only the first few subevents.

1. Introduction Whether earthquake complexity reflects underlying organization or randomness is a fundamental issue in earthquake physics and predictability. Recent work has suggested that organization in earthquake complexity permits a sense of determinism whereby observations made in a short period of time at the beginning of an earthquake may be used to infer the overall event size and style (Olson & Allen, 2005; Melgar & Hayes, 2017; Goldberg et al., 2018; Allen & Melgar, 2019). Previous studies have found that early P wave arrivals carry information about the overall earthquake size (Iio, 1995; Beroza & Ellsworth, 1996; Olson & Kanamori, 2003; Olson & Allen, 2005), but such correlations bear large uncertainties that necessitate additional information, such as attenuation parameters, in order to conduct meaningful early magnitude estimation (Wu et al., 2006; Colombelli et al., 2014; Noda & Ellsworth, 2016). The direct P wave arrival, however, represents only a small fraction of the available and potentially deterministic information (Vallée et al., 2017; Vallée & Juhel, 2019).

The seismic signature of earthquake processes is captured in the Source Time Function (STF). Conventional approaches to estimating the STF fall into two categories. The first inverts for a kinematic evolution of slip on the fault from recorded body and surface waves and yields the moment rate function (Hayes, 2017; Ji et al., 2002; Kikuchi & Kanamori, 1991; Ye et al., 2016). Due to data resolution, this approach works best for large earthquakes; published databases have few examples. The second approach makes a point source approximation to the rupture and directly deconvolutes recorded seismic waves with a Green’s function (Mueller, 1985; Tanioka & Ruff, 1997; Vallée & Douet, 2016). This second method assumes that the far-field pulse is proportional to the moment rate function when averaged over stations, is appropriate for small to moderate earthquakes, and results in larger ensembles of STFs.

Here, we take advantage of existing STF catalogs in order to explore earthquake complexity. The main result of this paper is that larger earthquakes are more complex—on average—than smaller ones. We find that...
earthquake complexity grows with earthquake size in a structured manner. To our knowledge, no previous study has found systematic growth in complexity with earthquake size (Boatwright, 1984; Houston et al., 1998; Houston, 2001; Meier et al., 2017; Stork & Ito, 2004). The main reason why our results differ from these previous studies is probably that we use much larger catalogs. As a result, we find that inherent structure in earthquake complexity can be used to infer a likely final earthquake size in the first few seconds of large earthquakes.

2. Methods
2.1. Data
The SCARDEC (Seismic source ChActeristics Retrieved from DEConvolvolving teleseismic body waves, Vallée and Douet (2016), http://scardec.projects.sismo.ipgp.fr/, last accessed 11 November 2018) database contains 3,395 STFs for earthquakes with magnitude from $M_{5.5}$ to $M_{9.0}$ between 1992 and 2017. The SCARDEC functions are constructed from the deconvolution of teleseismic $P$ waves ($P$, $pP$, $sP$, $PP$, and $PcP$) with a theoretical Green’s function that is estimated from a radially symmetric and anelastic Earth. Because the global models of attenuation are better constrained by seismic frequencies lower than 1 Hz, we do not interpret signals that are shorter than 1 s in the following analysis. The United States Geological Survey (USGS) STF database (Hayes, 2017) contains 180 events (as of 12 April 2018) of earthquakes of magnitude greater than 7 during the period 1990–2017. STFs in the USGS database are reconstructed from an inverted kinematic model that follows the algorithm developed by Ji et al. (2002) and that jointly inverts for body and surface waves. This latter data set serves as a validation set to our analysis of the SCARDEC database and has been shown to provide STFs comparable to the SCARDEC database by Meier et al. (2017). Both databases have been extensively studied to extract physical properties of earthquakes such as moment, duration, stress drop, radiated energy, and rupture velocity (Chounet et al., 2018; Chounet & Vallée, 2018; Denolle, 2019; Meier et al., 2017; Melgar & Hayes, 2017; Vallée, 2013; Vallée & Douet, 2016). The sampling rate of the SCARDEC database is $dt = 0.0703$ s and we recalculate the USGS moment rate function to that sampling rate from the finite fault models and the slip rate bases described in Ji et al. (2002).

STFs have a basic structure consisting of an initial acceleration of seismic moment followed by a deceleration. When averaged over many events, STFs have maximum values around 30–50% of their duration (Denolle, 2019; Meier et al., 2017; Melgar & Hayes, 2017). Despite this first-order similarity in shapes, dissimilarity among STFs becomes obvious for large events (as discussed in Meier et al., 2017). In the following, we explore the roughness in the shape of the STF as a metric of earthquake complexity.

2.2. Numerical Simulations
We construct a database of synthetic STFs derived from dynamic rupture simulations. These simulations solve the equations of elastodynamics in two spatial dimensions (antiplane strain) with linear slip weakening friction acting along a one-dimensional fault. This coupled elasticity-friction problem is solved using the spectral boundary integral method (SBIEMLAB, code developed by Jean-Paul Ampuero, http://web.gps.caltech.edu/~ampuero/software.html, last accessed 27 November 2018). Although rupture complexity may arise from heterogeneity in prestress (Huang et al., 2012; Ripperger et al., 2007), geometry (Ando & Kaneko, 2018; Hamling et al., 2017), roughness (Denolle et al., 2011; Fang & Dunham, 2013), or fault strength (Ide & Aochi, 2005), the goal of the paper is not to explore this full parameter space, but rather test whether heterogeneity in prestress explains observed STF characteristics (presented below). The fault prestress is defined as having a spatially uniform mean value plus a spatially varying perturbation that follows a power law power spectral density $P_m(k) \sim k^{-\gamma}$, where $k$ is the wave number and the exponent $\gamma$ is chosen as 0.8. Additional details concerning the numerical simulations are discussed in the supporting information (see Text S3 and Figures S7 and S8).

We produce 600 prestress distributions and run dynamic ruptures by randomly selecting a nucleation location. We only keep the 499 of these where the rupture did not propagate beyond the zone of self-affine prestress. We generate earthquakes with over 2 orders of magnitude difference. For these simulated earthquakes we calculate STFs by integrating the moment rate (per unit fault width) over the fault dimension (see Figure S8). Because the simulated STF contains higher-frequency content than the data set of observed STFs, we smooth the simulated moment rate functions by convolving the simulated STF with a Gaussian of width $\sigma = 2$ s. We also attempted to use a Butterworth-type low-pass filter, but this resulted in nonphysical negative values of the moment rate function.
Figure 1. (top) Observed Source Time Function (STF) from the SCARDEC database from the 14 March 1994 M7.1 Central Mid-Atlantic Ridge Earthquake in black and the fitted Gaussian-built STF in red. (bottom) Modeled STF in moment rate per unit of fault width (black) and the corresponding fitted Gaussian-built STF (red). This illustrates the reliability of our fitting procedure in capturing the shape and moment of STFs. SCARDEC = Seismic source ChActeristics Retrieved from DEConvolvevolving teleseismic body waves.

2.3. Subevent Decomposition
The roughness of both the observed and simulated STFs may be described in several ways. Several studies use the zero crossings of the moment acceleration function (Houston et al., 1998; Houston, 2001; Sato & Mori, 2006), while others use the log-residuals between the STF and a smoothed model (Meier et al., 2017). Here, we decompose the STF as a sum of “subevents” that are Gaussian pulses, similar to the pulse stripping method of Kikuchi and Kanamori (1982) and Zhan et al. (2014). Gaussian subevents were found to best fit the shape of the SCARDEC STFs (see Figures S5 and S6). We perform the subevent decomposition from onset of rupture (time zero) as follows:

1. go forward in time and detect a peak \( S_S \) (local maximum over three points or 0.21 s) at time \( t_S \) that satisfies \( S_S > 0.1 \) max(STF);
2. fit a Gaussian function centered around \( t_S \) to the STF with an amplitude to \( S_S \) and a width \( \sigma \) estimated using a grid search minimizing the root-mean-square residuals (effectively to get a subevent duration and moment) over 11 grid points, or 0.77 s;
3. if \( 4\sigma > 1 \) s, count the detection as subevent and move forward, otherwise go back to step 1;
4. subtract the fitted Gaussian function from the STF;
5. if more time remains in the STF, return to step 1.

We apply the same algorithm to all STFs (simulated and the observed SCARDEC and USGS). Examples of the reconstructed STFs are shown in Figure 1 for both the observed STF and the simulated STF. The choice of a threshold to select the peak based on the maximum amplitude of the STF, here 0.1, is necessary to ignore spurious residuals that are not resolvable by the data (e.g., for shorter duration or lower amplitude signals). Other thresholds resulted in detecting smaller subevents that we interpret as being overfit (Figure S2). This interpretation is based on the low likelihood of resolving small subevents given the depletion of high-frequency energy at teleseismic distances. Regardless of the threshold, the results presented below remained unchanged.
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Figure 2. Number of subevents as a function of main event seismic moment for (a) the observed SCARDEC STFs and (b) the modeled STFs. Dots are the individual earthquakes. Dots are colored according to whether the faulting type parameter $F_M$ defined in Shearer et al. (2006), which is between $-0.5$ and $0.5$ for strike slip (red dots), and between $-1$ and $-0.5$ or $0.5$ to $1$ for dip slip (blue dots). The slopes that best explain the variance in a linear regression of both subsets are shown in colored letters. The growth in complexity is monotonic with earthquake size for the observations, except for the $M_9.0$ 2011 Tohoku earthquake, and the growth is noticeable for the simulations. SCARDEC = Seismic source ChActeristics Retrieved from DEConvolving teleseismic body waves; STF = Source Time Function.

3. Results
3.1. Earthquake Complexity

Using the number of subevents as a metric for earthquake complexity, we find our first main result: earthquake complexity increases with earthquake size (Figure 2a). In general, $M_0 < 4 \times 10^{19}$ (Nm) ($\sim M_W 7$) earthquakes have about 1–4 subevents (mean of 3.08), while the $M_0 > 4 \times 10^{19}$ earthquakes have 4–10 subevents (mean of 5.84). We find similar systematic growth in complexity using the smaller USGS database, as discussed in the supporting information (Figure S1a).

The growth in the number of observed subevents with earthquake magnitude is more pronounced for strike-slip earthquakes with magnitudes greater than $M_W \approx 6.75$ ($M_0 \geq 1.68 \times 10^{19}$ Nm) as the slope is 2.5 subevents/log($M_0$) for lower magnitudes and 3.3 subevents/log($M_0$) for greater magnitude. These earthquakes have a source dimension $\sim 15$ km, a typical seismogenic depth of crustal earthquakes (Figure 2a). Large crustal strike-slip earthquake tend to have subvertical faults limited in width by the seismogenic depth and are known for their complex multifault geometry (Kokoxili 2001 (Klinger et al., 2005; Tocheport et al., 2006), Denali 2002 (Eberhart-Phillips et al., 2003), Sumatra 2012 (Meng et al., 2012; Yue et al., 2012), Kaikoura 2016 (Hamling et al., 2017; Wen et al., 2018)). Offshore strike-slip event complexity may be overestimated, however, by contamination of the direct seismic phases with their reverberation in the water column (Yue et al., 2017). Furthermore, among the strike-slip earthquakes of magnitudes greater than $M_W 6.5$, the 267 crustal events (depth lower than 35 km) have a median number of 5 subevents while the 142 deeper events have a median number of 3 subevents.

Large dip-slip earthquakes tend to exhibit less complexity. Their growth in the number of subevents is clearer for moments of $M_0 \geq 6 \times 10^{20}$ ($M_W \approx 7.8$), where the slope becomes 2.5 subevents/log($M_0$) if we ignore the $M_9.0$ 2011 Tohoku earthquake. The $M_9.0$ 2011 Tohoku earthquake is an outlier and an extreme case in this regard consistent with rupture of a single, large patch (Ide et al., 2011; Shao et al., 2011). Depth has less influence for $M_W > 6.5$ dip-slip earthquakes since both crustal and deep earthquakes have a median number of subevents of 3.

We find a similar pattern of monotonic growth in the number of subevents in our numerical simulations (Figure 2b). There is a close relationship between prestress variations, and rupture velocity is itself a basic aspect of elastodynamic crack propagation (Eshelby, 1969; Freund, 1998; Koizdon & Dunham, 2013).
3.2. Subevent Scaling

Our second result is that subevent moment is correlated with main event total moment (Figure 3a). To our knowledge, this finding has not previously been reported in the literature. Independent of event magnitude, the ratio of the main event moment to subevent moment is between 1 and 100. The lower bound of this distribution arises from the fact that subevents are rarely found to be larger than the main events, which is true by definition, but the approximation of the Gaussian function to the local moment rate function sometimes brings errors. The upper bound of this distribution arises from the choice of threshold to detect subevents and from ignoring the subevents with duration shorter than 1 s ($\sigma \geq 1 \text{ s}$). Of 3,395 STFs, 19 are left out of the analysis with this latter criterion. The scaling remains unchanged by varying the threshold (Figure S2). We see a similar scaling in the USGS data set (Figure S1). Another interesting and supporting aspect is that we did not detect small subevents in the large $M_w \geq 8$ earthquakes: subevents for such earthquakes are either buried in the signal of the largest subevents and undetectable in our decomposition, or they are absent.

The scaling between subevent moment $M_S$ and main event $M_0$ for the observed STF is $\log_{10}(M_S) = 0.79\log_{10}(M_0) + 3.22$ and for the simulated STF is $\log_{10}(M_S) = 0.8\log_{10}(M_0) + 2.30$. Our simulations are 2-D, whereas real earthquakes in nature occur on 3-D faults. For this reason, the two scaling relations are not expected to be identical. We are, however, confident that a scaling slope lower than 1, which would support earthquake self-similarity, is a robust feature for both the observed and simulated STFs. It is likely that complexity in the small STFs is underestimated, as discussed in Houston et al. (1998), because the teleseismic data are less reliable at frequencies greater than 1 Hz. Thus, we may not be able to see the smaller and higher frequency subevents. Furthermore, the use of a symmetric Gaussian function for the pulse shape may not be physical (Tinti et al., 2005) as it is intrinsically smoothed. Similar limitations apply to the simulated STF given the low-pass filtering we apply and given the numerical limitations of the experiment.

3.3. Possible Application to Earthquake Early Warning

The correlation between the subevent and main event moment carries potentially a predictive power in estimating the final earthquake moment given observations of the first few subevents. To test this idea, we extract the STF constructed from the 28 September 2018 $M_w \geq 7.5$ Palu earthquake in Indonesia from the USGS database (https://earthquake.usgs.gov/earthquakes/eventpage/us1000h3p4/finite-fault, last accessed on 19 October 2018; Figure 4). We interpolated the STF with the sampling rate of our previous anal-
Figure 4. (top) United States Geological Survey (USGS) source time function estimate (https://earthquake.usgs.gov/archive/product/finite-fault/us1000h3p4/us/1539813424642/moment_rate.mr, last accessed on 19 October 2018) in black, linearly interpolated to $dt = 0.0703$ s, and our Gaussian-built STF in green line. (bottom) “Real time” moment magnitude estimate throughout the rupture: dark green curve is the magnitude difference from the cumulative seismic moment, black dots are the magnitude estimates from individual subevents fitting (using scaling statistics from Figure S3), red dots are the median of all previous magnitude estimates.

ysis. This event was not used in forming the main scaling relation. We begin scanning the STF at the onset of rupture time and we fit the first subevent at $t = 1.8$ s, which is the time of the first detected peak. At this time we estimate the subevent moment and duration by fitting a Gaussian function (the first half duration and a few samples after its peak), and then apply an algorithm slightly modified from the above described scaling relation to estimate the final event moment (see Text S2).

We find that our algorithm predicts a magnitude of $M_W \approx 7.2$ at $\sim 1.8$ s of rupture time, while only $M_W \approx 6.3$ equivalent of moment magnitude was released, and while the final size of the event is $M_W \approx 7.5$ (Figure 4).

Upon the second subevent detection at $\sim 3.6$ s, which we constrain as being no smaller than 0.25 times the amplitude of the first peak, we calculate an updated moment magnitude estimate and take the mean of both estimates. We iterate as the rupture evolves and refine the magnitude estimates based on the distributions of the previous individual estimates by taking the median value of the peak amplitudes as a threshold of the previous moment magnitude estimates. This allows us to stabilize the magnitude estimates as the variability in subevent size fluctuates during the rupture. An alternative approach to rapid moment estimation based on observing the maximum moment rate (Meier et al., 2017) would require approximately an additional $\sim 10$ s of observation beyond that required for our approach.

Earthquake early warning (EEW) systems have latency due to data limitations (proximity of the sensors to the source), due to network delays, and due to methodological errors and uncertainties in interpreting the data. Furthermore, there is not yet an implementation of real-time source time function inversion, which would require an a priori knowledge of the source location and $P$ wave Green’s function (the latter can be precomputed and efficiently searched in databases). Goldberg et al. (2018) and Minson et al. (2018) indicate that, at best, magnitude estimates are slightly lower but track the evolution of cumulative moment release (green curve in Figure 4b). The advantage of using the scaling provided described in this paper is therefore a speed up, in the Palu example, of the duration of observation required to produce a final magnitude estimate by about 10 s from current methods.

We have performed a similar exercise as in Figure 4 for the entire SCARDEC database and find that we can predict the final magnitude with a bias of $+0.03$ and an uncertainty (standard deviation) of 0.52 in moment magnitude within the first 20% of rupture duration (there is a negative bias before 10%, then an overall slight positive bias, see Figure S3b). We find that the largest subevent is detected within 30–50% of the rupture time; however, there are systematic biases, likely due to a structure in the distribution of the subevent size.
during the rupture that we have not explored. Nevertheless, these biases may be accounted for in a future improvement and application of this scaling.

4. Discussion and Conclusions

Through an analysis of earthquake STFs and their subevents, we make several observations. First, large earthquakes have more subevents than small earthquakes, contradicting the idea of self-similarity in earthquakes (Allmann & Shearer, 2009; Prieto et al., 2004) but agreeing with several previous and local observations (Ellsworth & Beroza, 1998; Sato & Mori, 2006). Second, the subevent moment scales with the main event moment with a power exponent of about 0.8.

We find that simulations with spatially uniform fault parameters fail to produce subevents but that introducing a range of heterogeneity levels in the stress prestress yields similar results. We conclude that we were unable to constrain the strength of fault heterogeneity from the subevent scaling. Instead, we simply make the weaker claim that the heterogeneous prestress distributions we analyze are consistent with the observations.

Because both simple simulations and observations share similar statistical properties of subevent-to-main event moment, we conclude that the dynamics of rupture, with the physics tested in our simulations, exhibits a sense of self-organization such that the complexity of earthquakes is structured; the subevents have neither a fixed nor a random size. We propose that the same variations in fault prestress that create subevents in STFs also give rise to the scaling between subevent moment and main event moment. Main event-subevent scaling then occurs because a favorably prestressed region creates a large subevent that is more likely to rupture through a surrounding unfavorably prestressed region (Eschelby, 1969; Kozdon & Dunham, 2013).

It is in this way that a large subevent is more likely to be part of a large total rupture rather than a small one.

We have shown that the statistical properties of the subevent moment relative to the main event moment may be utilized for early magnitude estimates. Our results do not imply that earthquakes are deterministic in the sense that early nucleation is indicative of the entire evolution of rupture (Beroza & Ellsworth, 1996; Goldberg et al., 2018; Iio, 1995; Meier et al., 2017; Melgar & Hayes, 2017), but rather that we can exploit the patterns in earthquake complexity to statistically infer the future number and size of subevents from the first subevent. Our results are mostly practical for strike-slip events that tend to have shorter duration subevents. Using STFs for early warning is not yet in place as the latency of current EEW systems delay magnitude estimates. Rapid STF estimation for EEW could be possible pending further investigation.

The caveats of this study are mainly the frequency bandwidth limitations for small events. Furthermore, the attenuation models used in SCARDEC and USGS databases are globally averaged, yielding inaccuracies in regions that have abnormally strong or weak seismic attenuation. Our results are nevertheless robust for magnitudes 7 and greater.

References


