Resolvability of linear BackProjection (BP) method for earthquake kinematics

1. *Physical meaning of BP image*
2. *Generalized way to quantify the BP resolution*

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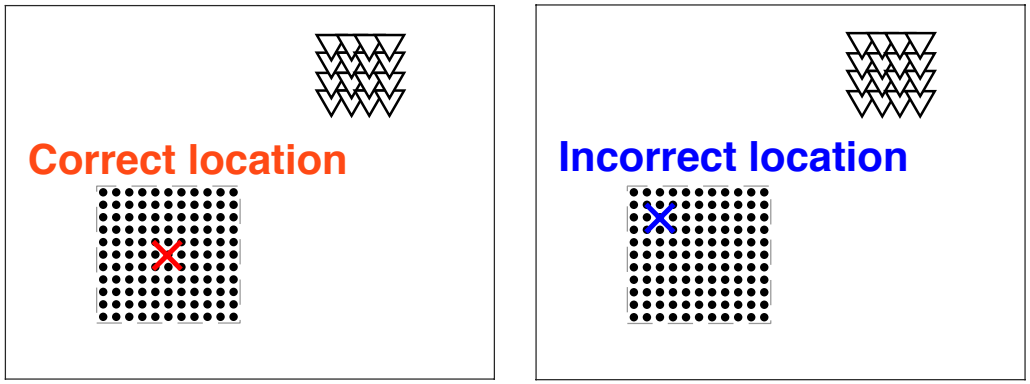
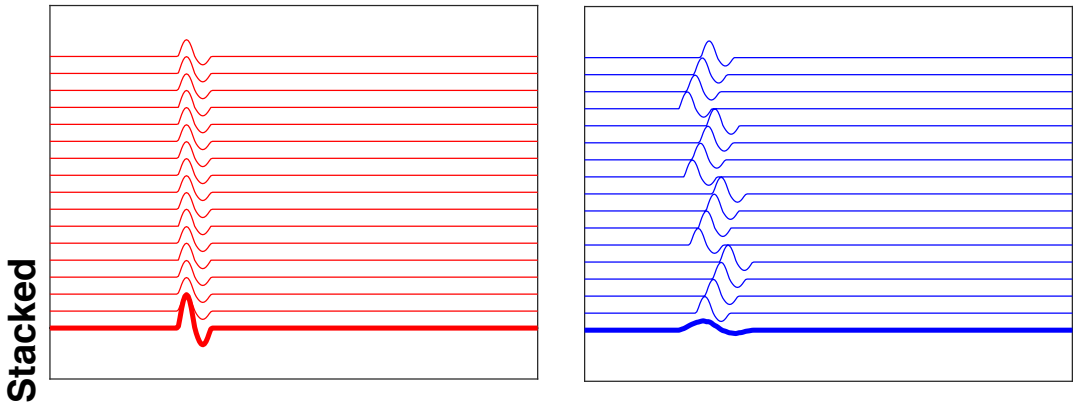
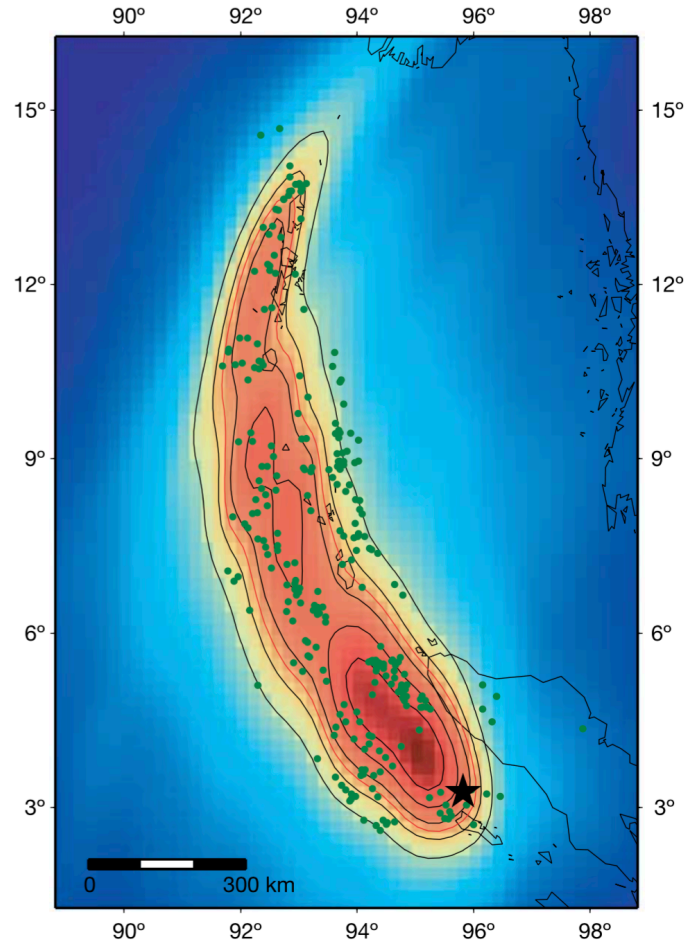
Earthquake Seismology

Backprojection (BP) of seismograms

Two ingredients: Realignment + Stacking







(Ishii et al., 2005)

❖ **Physical meaning of the BP images?**

Theoretical Formulation of linear BP (I)

Displacement seismograms (Representation theorem):

Radiation pattern

$$d_k(t) = \sum_{n=1}^{N} \frac{R_{kn}^P}{4\pi\rho\alpha^3} \frac{\mu\Delta S}{r_{kn}} \dot{u}_n(t - t_{kn})$$

Slip rate function with travel time delay

Geometrical spreading

In matrix form for the entire array:

$$\begin{bmatrix} D_1(\omega) \\ D_2(\omega) \\ \vdots \\ D_K(\omega) \end{bmatrix} = \frac{\mu\Delta S}{4\pi\rho\alpha^3} \begin{bmatrix} \frac{R_{11}^P}{r_{11}} e^{-i\omega t_{11}} & \frac{R_{12}^P}{r_{12}} e^{-i\omega t_{12}} & \cdots & \frac{R_{1N}^P}{r_{1N}} e^{-i\omega t_{1N}} \\ \frac{R_{21}^P}{r_{21}} e^{-i\omega t_{21}} & \frac{R_{22}^P}{r_{22}} e^{-i\omega t_{22}} & \cdots & \frac{R_{2N}^P}{r_{2N}} e^{-i\omega t_{2N}} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{R_{K1}^P}{r_{K1}} e^{-i\omega t_{K1}} & \frac{R_{K2}^P}{r_{K2}} e^{-i\omega t_{K2}} & \cdots & \frac{R_{KN}^P}{r_{KN}} e^{-i\omega t_{KN}} \end{bmatrix}_{K\times N} \begin{bmatrix} \dot{U}_1(\omega) \\ \dot{U}_2(\omega) \\ \vdots \\ \dot{U}_N(\omega) \end{bmatrix}$$

Theoretical Formulation of linear BP (II)

Linear BP is to multiply a phase shift matrix $\tilde{\mathbf{A}}(\omega)$ to the array data $\mathbf{D}(\omega)$:

$$\tilde{\mathbf{A}}(\omega) = \begin{bmatrix} e^{i\omega t_{11}} & e^{i\omega t_{21}} & \cdots & e^{i\omega t_{K1}} \\ e^{i\omega t_{12}} & e^{i\omega t_{22}} & \cdots & e^{i\omega t_{K2}} \\ \vdots & \ddots & \ddots & \vdots \\ e^{i\omega t_{1N}} & e^{i\omega t_{2N}} & \cdots & e^{i\omega t_{KN}} \end{bmatrix}_{N\times K}$$

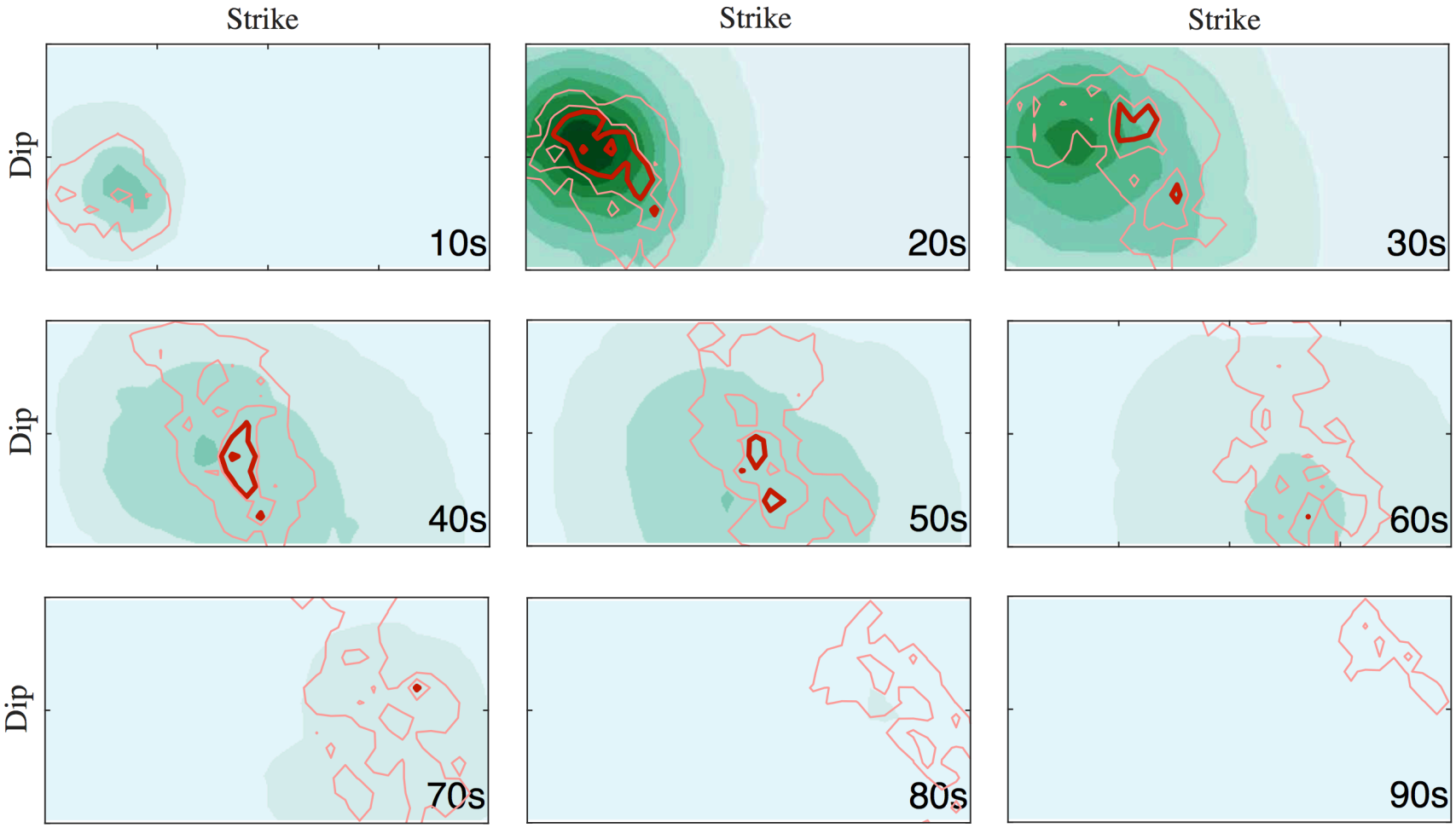
$$\mathbf{D}^{BP}(\omega) = \tilde{\mathbf{A}}(\omega)\mathbf{D}(\omega) = \tilde{\mathbf{A}}(\omega)\mathbf{A}(\omega)\dot{\mathbf{U}}(\omega) = \mathbf{F}(\omega)\dot{\mathbf{U}}(\omega)$$

BP Image = Resolution Matrix × Slip Rate distribution

❖ **Linear BP results ($\mathbf{D}^{BP}(\omega)$) are proportional to the images of slip motion $\dot{\mathbf{U}}(\omega)$, through a resolution matrix $\mathbf{F}(\omega)$**

Validation with kinematic source

Displacement seismogram → slip rate; Velocity seismogram → slip acceleration



Red: 20% and 50% of peak slip rate

Pink: 1% and 10% of peak slip rate

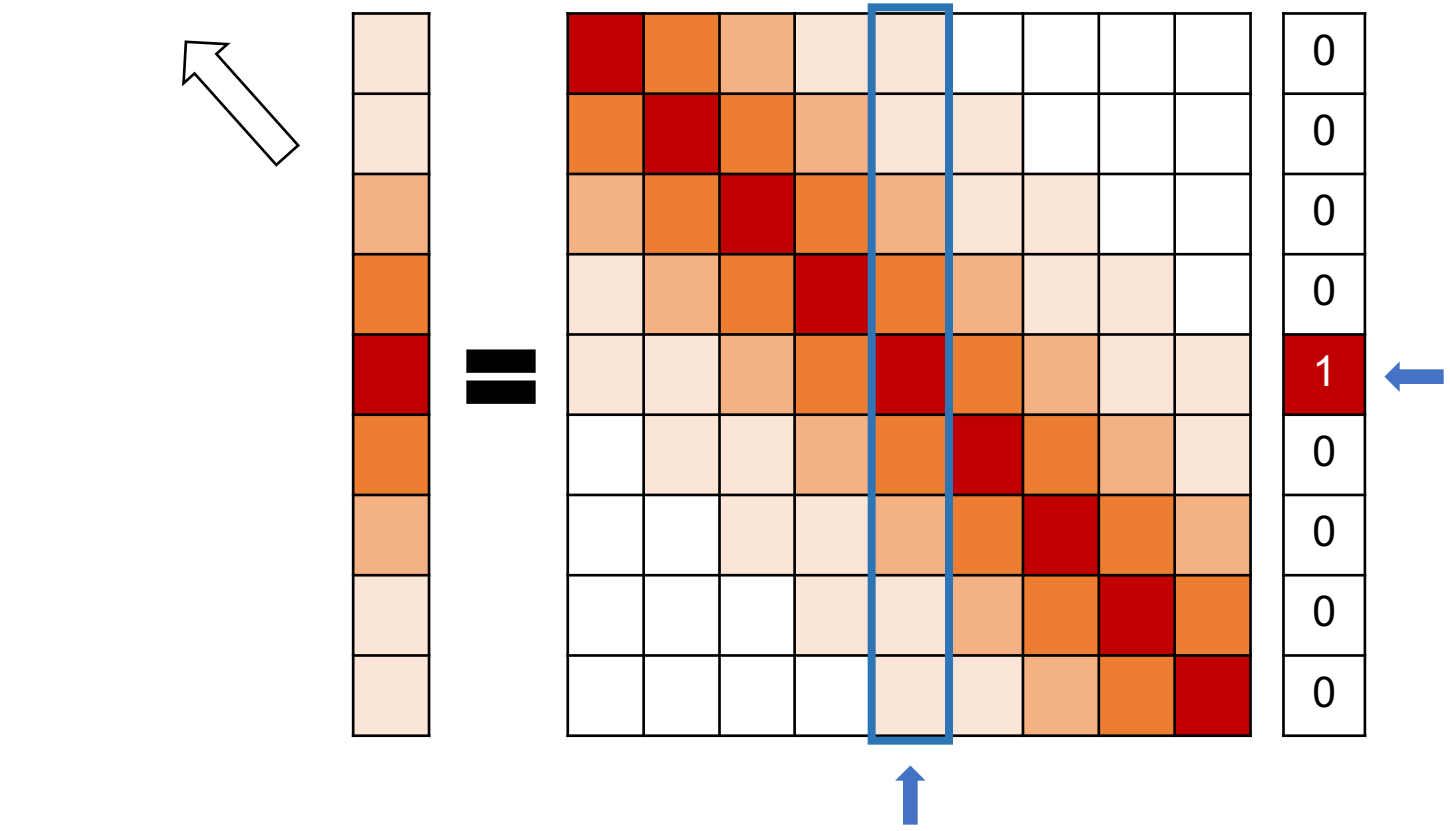
Resolution matrix $\mathbf{F}(\omega)$

Recovered BP $\mathbf{D}^{BP}(\omega)$

$$\mathbf{D}^{BP}(\omega) = \mathbf{F}(\omega)\dot{\mathbf{U}}(\omega)$$

Impulsive source $\dot{\mathbf{U}}(\omega)$

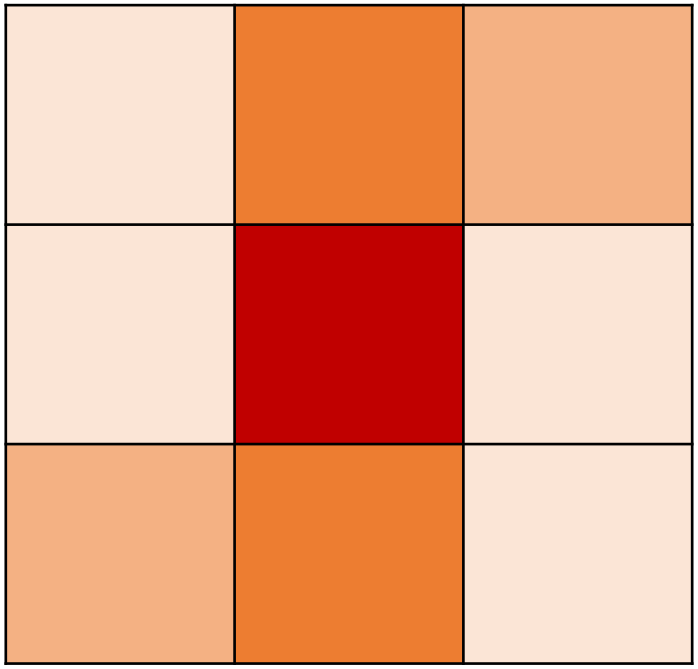
0	0	0
0	1	0
0	0	0



Resolution matrix $\mathbf{F}(\omega)$

$$\mathbf{D}^{BP}(\omega) = \mathbf{F}(\omega)\dot{\mathbf{U}}(\omega)$$

Recovered BP $\mathbf{D}^{BP}(\omega)$
Array Response

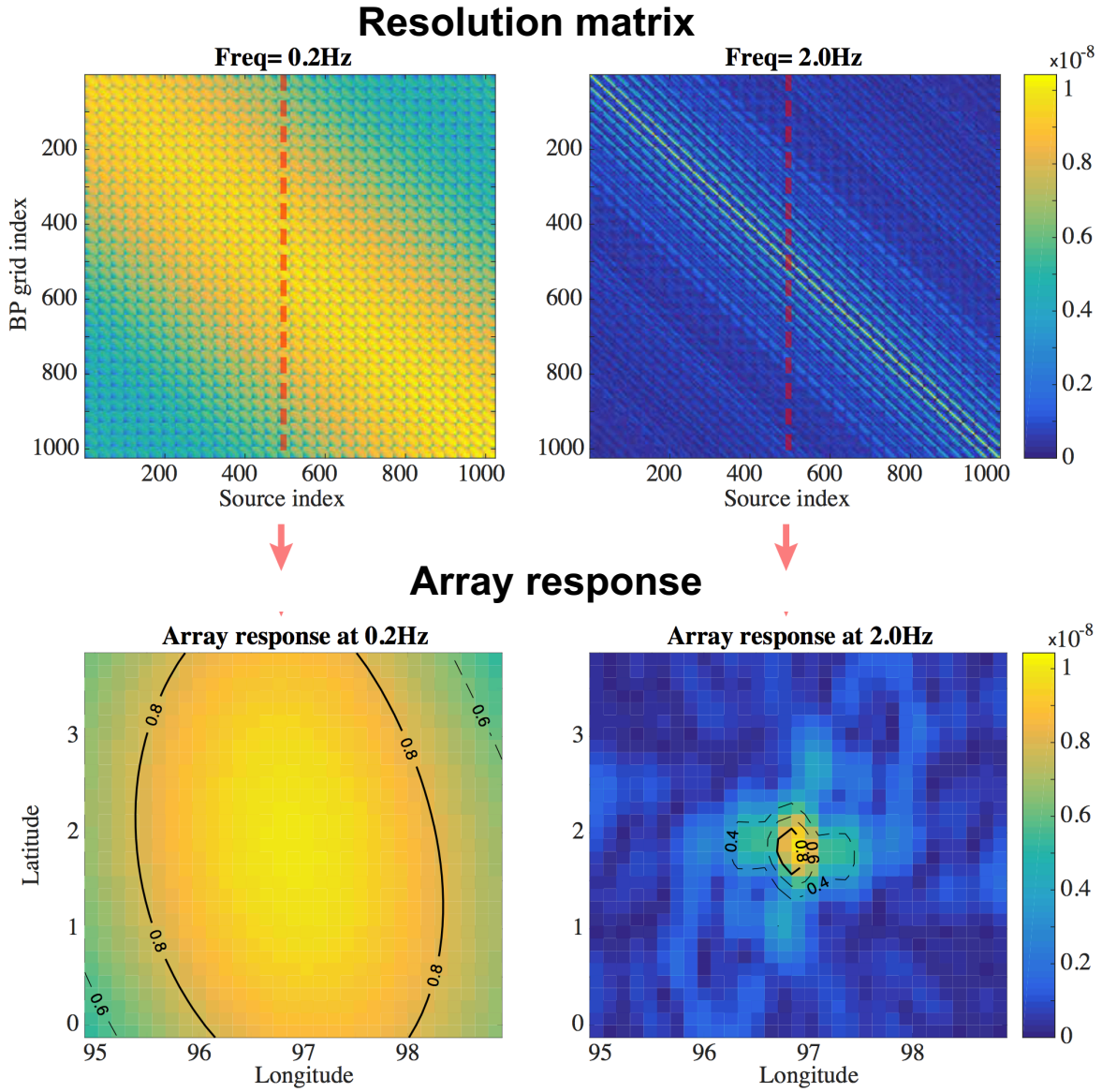


Impulsive source $\dot{\mathbf{U}}(\omega)$

0	0	0
0	1	0
0	0	0

- **Resolution matrix $\mathbf{F}(\omega)$ contains the information of BP spatial resolution**

Quantifying BP resolution using $\mathbf{F}(\omega)$



1. BP Resolvability

$$\epsilon_I(\omega) = |corr2(\mathbf{F}, \mathbf{I})|$$

- 2D image correlation coefficient
- Similarity between F and identity matrix
- Dimensionless parameter varying from 0 – 1

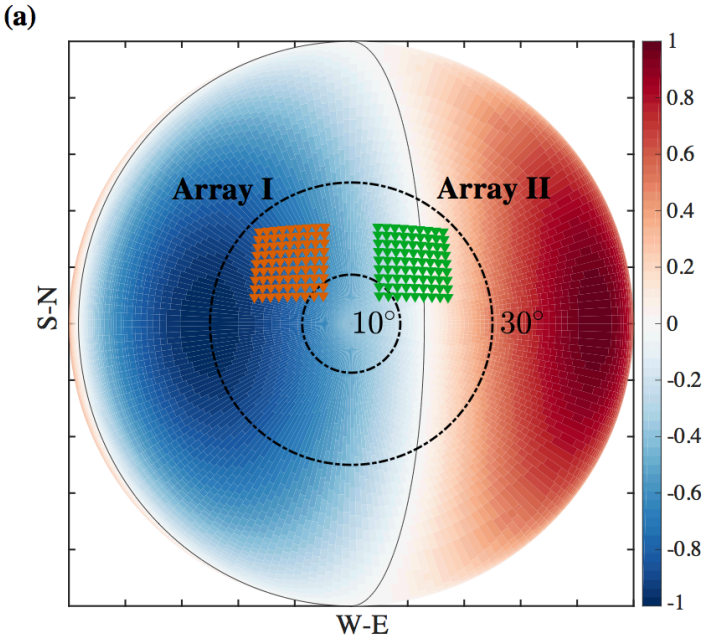
2. Resolvable area

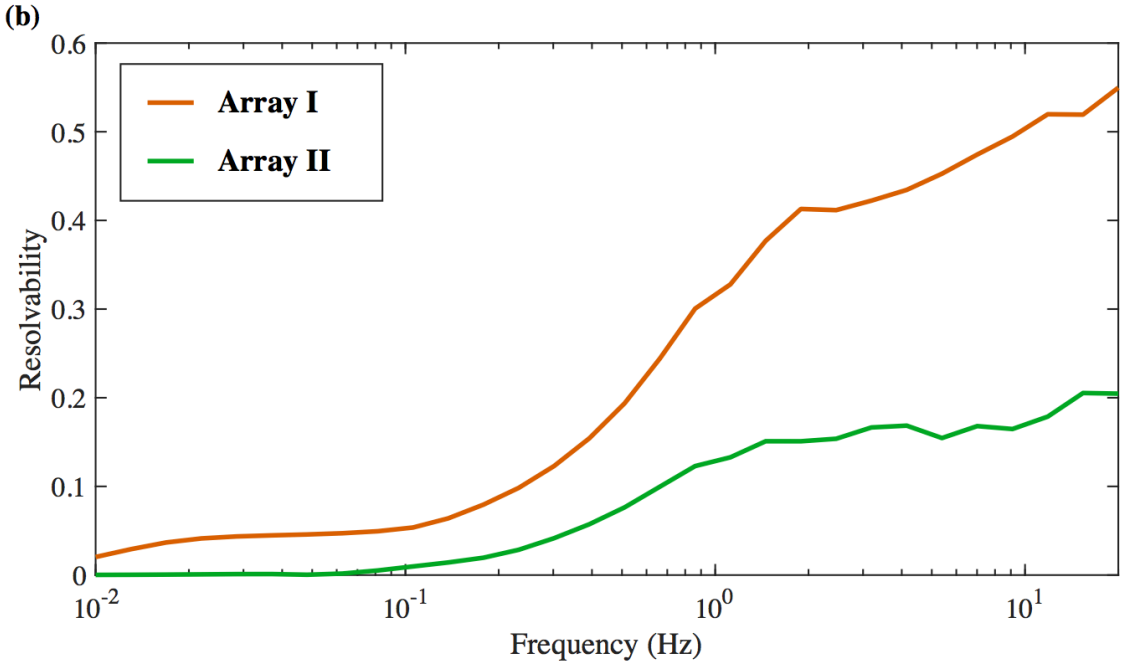
- Area with array response function > 80% of its maximum
- Physical quantity that can be related to frequency

BP resolvability

$$\mathbf{F}(\omega) = \tilde{\mathbf{A}}(\omega)\mathbf{A}(\omega) = \begin{bmatrix} e^{i\omega t_{11}} & e^{i\omega t_{21}} & \cdots & e^{i\omega t_{K1}} \\ e^{i\omega t_{12}} & e^{i\omega t_{22}} & \cdots & e^{i\omega t_{K2}} \\ \vdots & \ddots & \ddots & \vdots \\ e^{i\omega t_{1N}} & e^{i\omega t_{2N}} & \cdots & e^{i\omega t_{KN}} \end{bmatrix} \begin{bmatrix} \frac{R^P_{11}}{r_{11}} e^{-i\omega t_{11}} & \frac{R^P_{12}}{r_{12}} e^{-i\omega t_{12}} & \cdots & \frac{R^P_{1N}}{r_{1N}} e^{-i\omega t_{1N}} \\ \frac{R^P_{21}}{r_{21}} e^{-i\omega t_{21}} & \frac{R^P_{22}}{r_{22}} e^{-i\omega t_{22}} & \cdots & \frac{R^P_{2N}}{r_{2N}} e^{-i\omega t_{2N}} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{R^P_{K1}}{r_{K1}} e^{-i\omega t_{K1}} & \frac{R^P_{K2}}{r_{K2}} e^{-i\omega t_{K2}} & \cdots & \frac{R^P_{KN}}{r_{KN}} e^{-i\omega t_{KN}} \end{bmatrix}$$

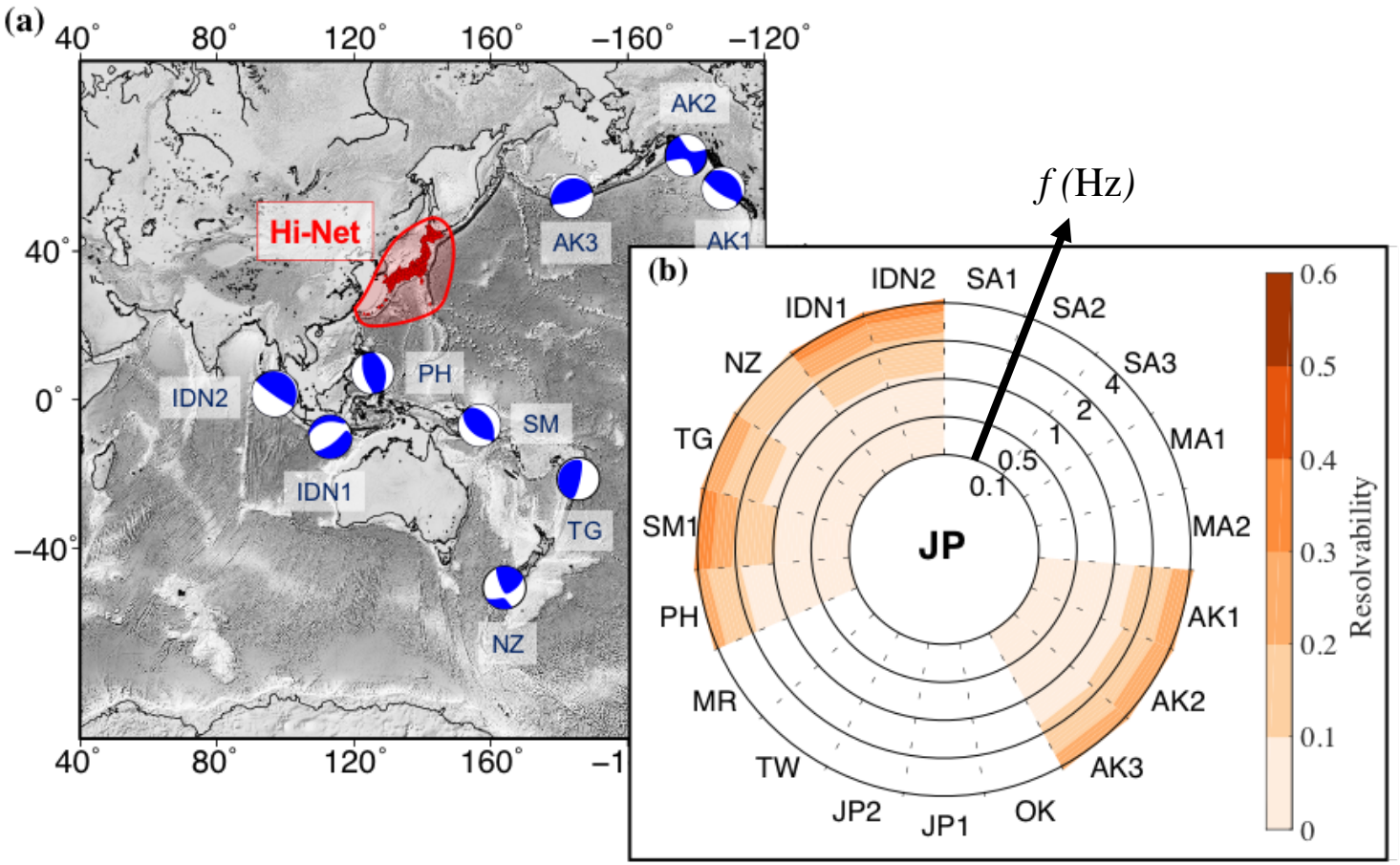
- Focal mechanism (radiation pattern R^P_{KN}), source-array location and velocity model (geometrical spreading r_{KN} and travel time t_{KN}) are required



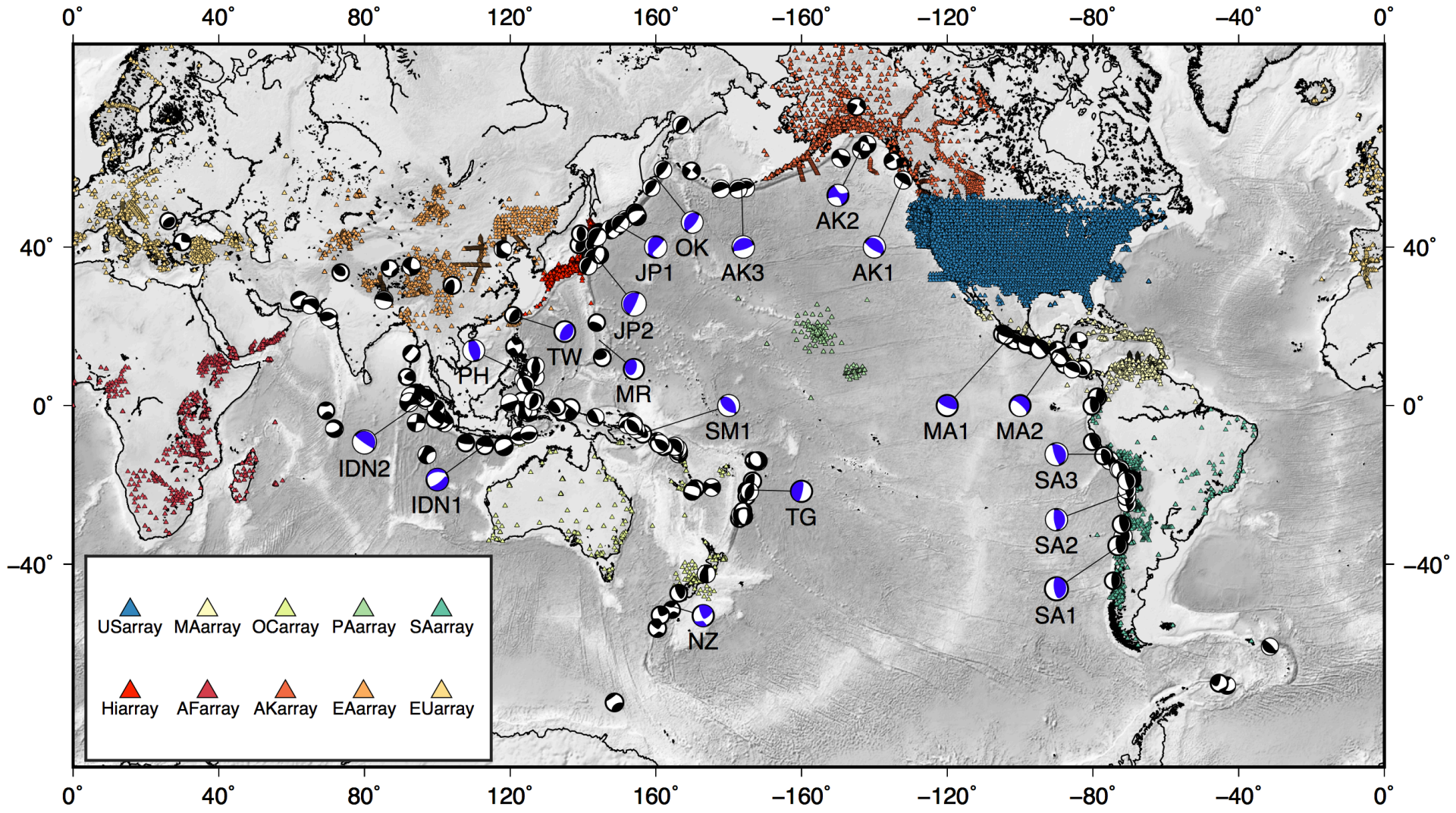


Resolvability for real source and array

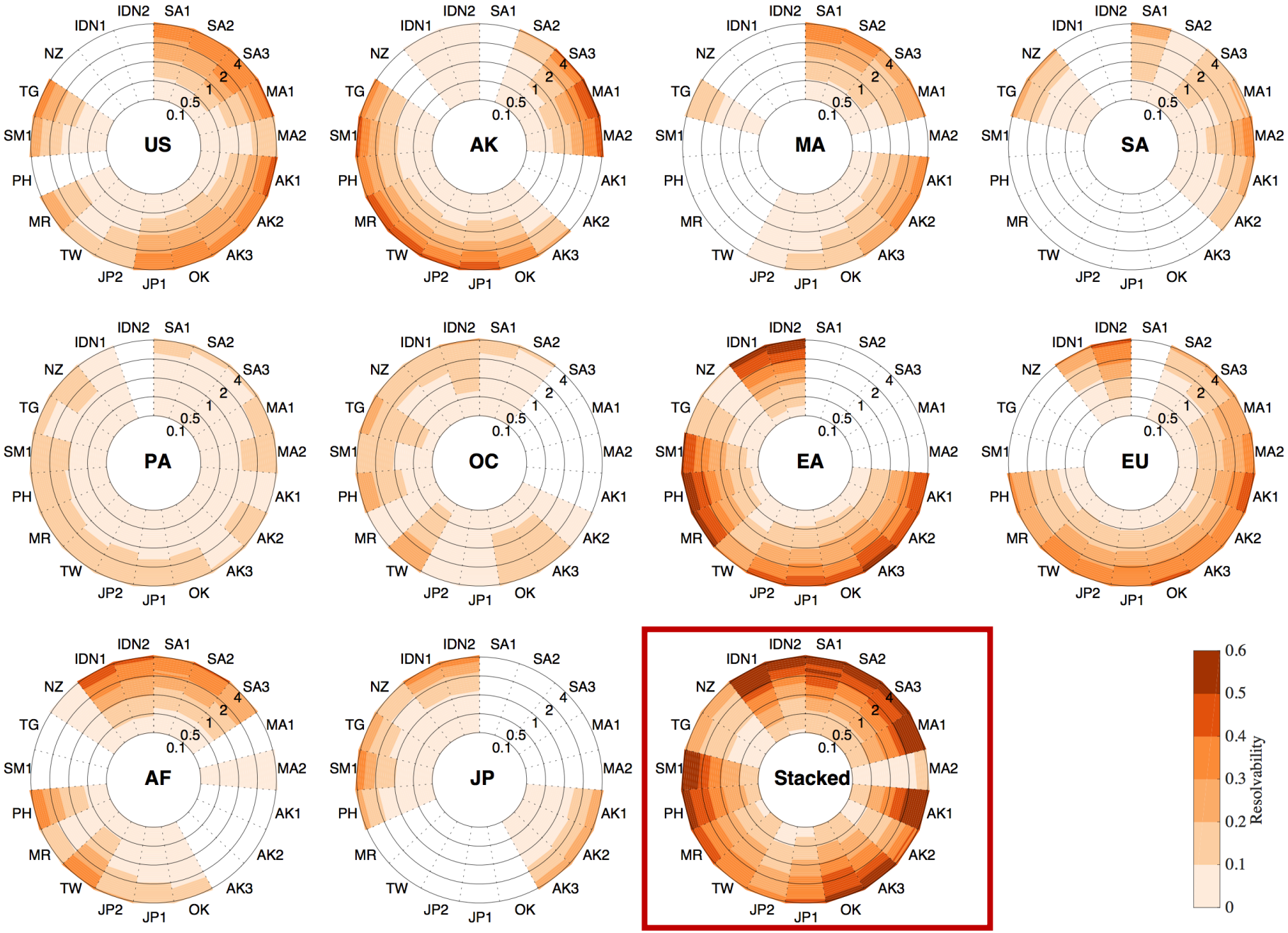
- Radiation pattern R^{P}_{KN} from GCMT historical large events
- Geometrical spreading r_{KN} and travel time t_{KN} from IASP91 velocity model



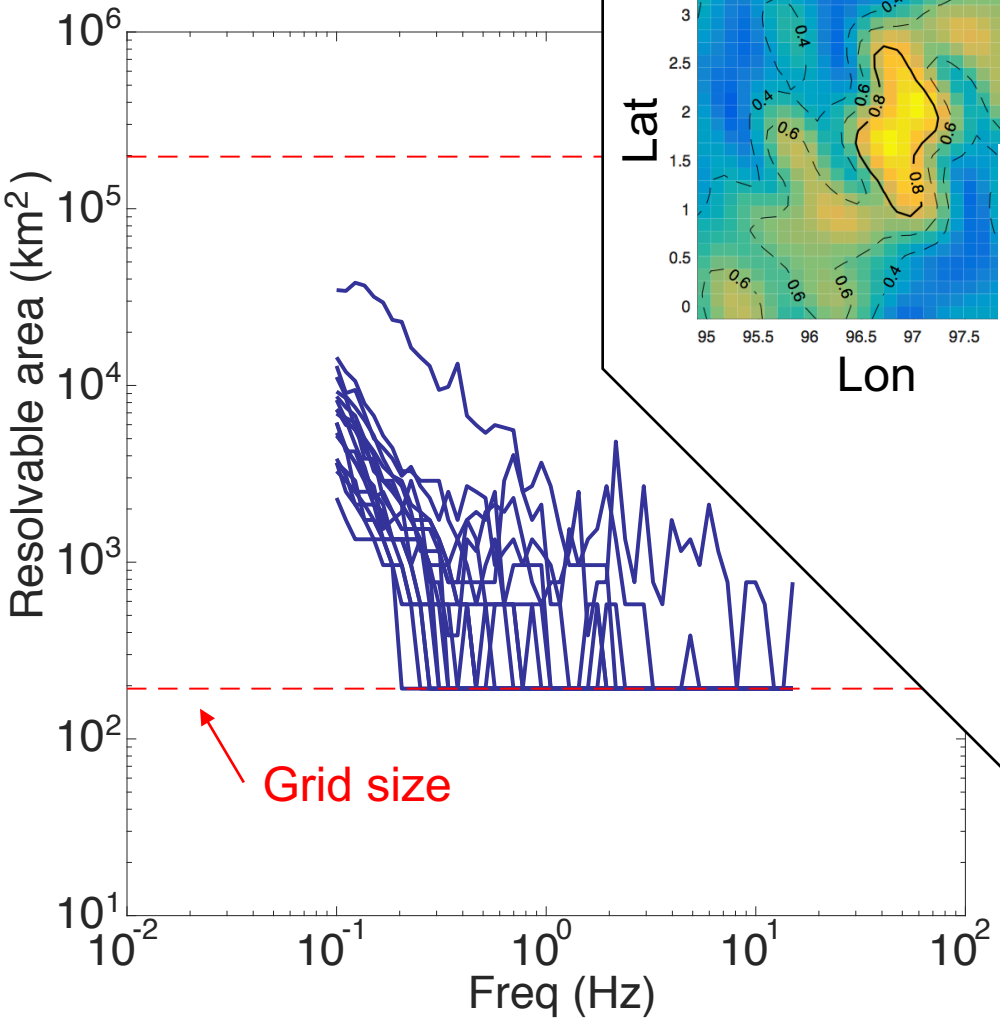
Global BP resolvability

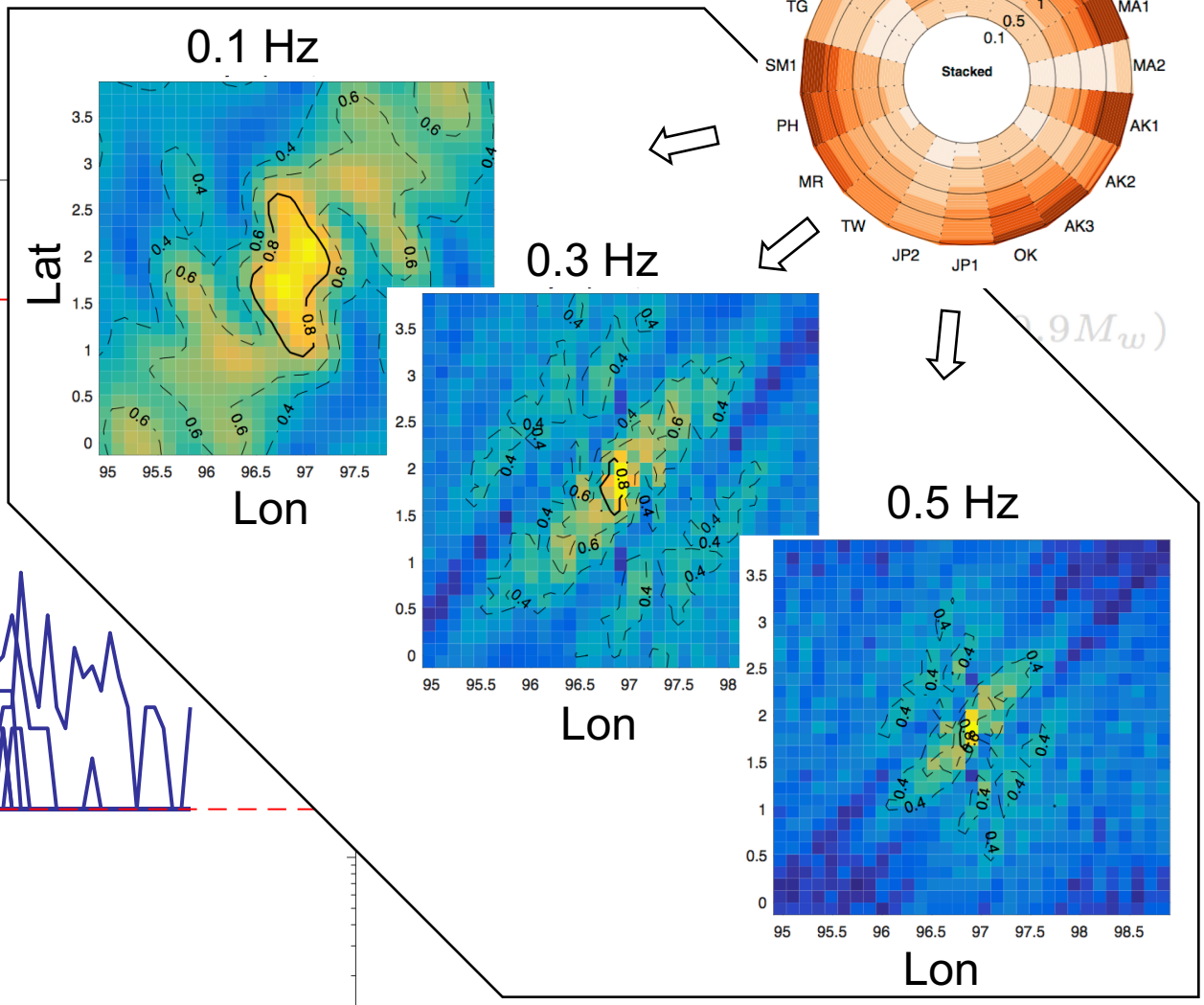


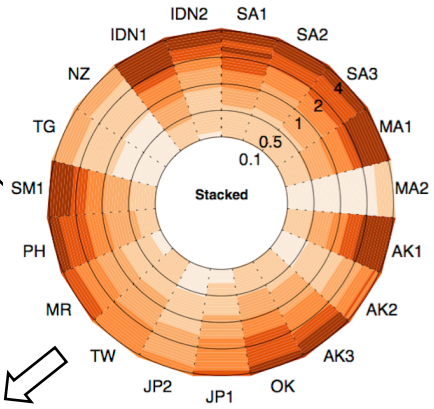
- Array location from IRIS SeismicQuery website (https://ds.iris.edu/SeismiQuery/station.htm) and NIED Hi-net websites (http://www.hinet.bosai.go.jp/). Historical event focal mechanisms from GCMT (http://www.globalcmt.org/)


US
AK
MA
SA
PA
OC
EA
EU
AF
JP
Stacked
IDN2
SA1
SA2
SA3
MA1
MA2
AK1
AK2
AK3
OK
JP1
JP2
TW
MR
PH
SM1
TG
NZ
IDN1
0.1
0.5
1
2
4
Resolvability
0
0.1
0.2
0.3
0.4
0.5
0.6

Frequency resolution of BP

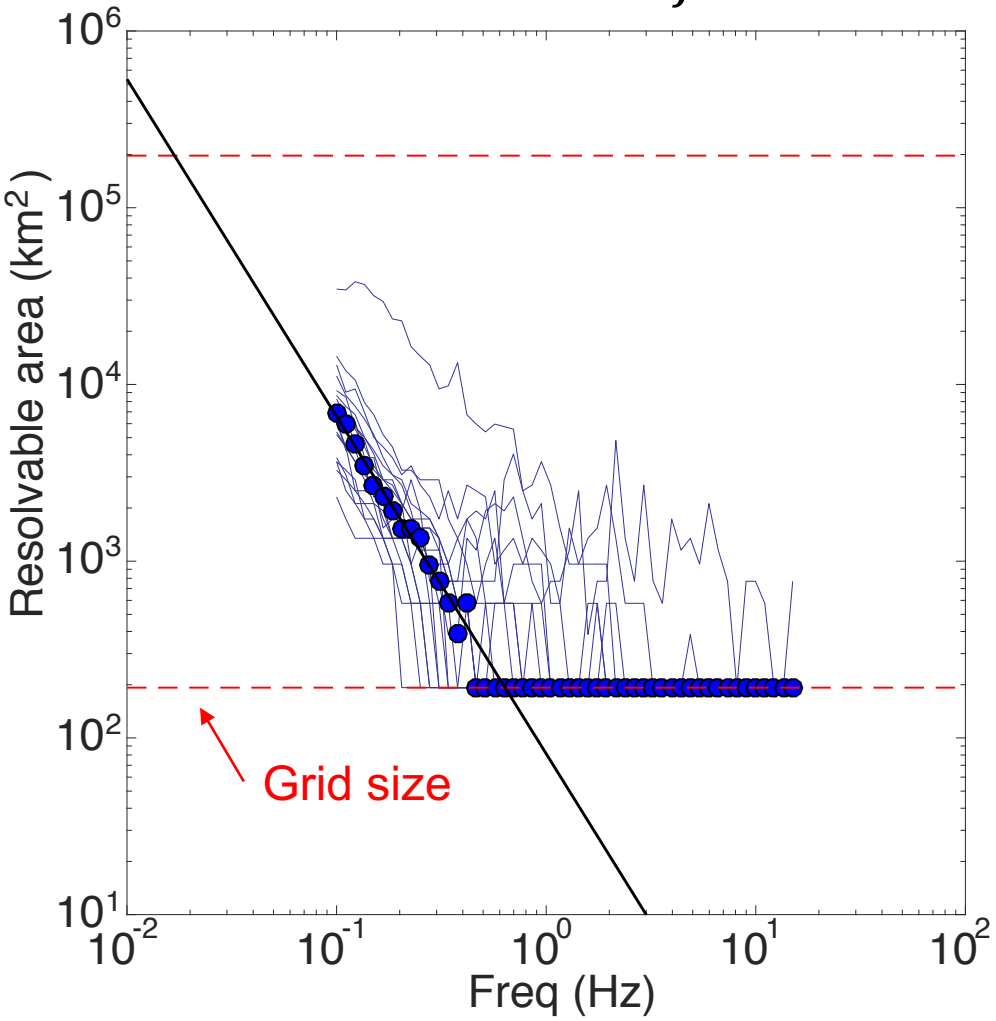






Frequency resolution of BP

$$S^{\mathbf{BP}} \approx 10^{1.91} f^{-1.91} \approx \left(2\frac{V_P}{f}\right)^2 = (2\lambda_P)^2$$



Scaling relation
(Wells & Coppersmith, 1994):

$$S \approx 10^{(-3.42+0.9M_w)}$$

- Require $S^{\mathbf{BP}} \leq S/10$ to resolve the rupture propagation from BP

Lowest BP frequency:

$$f_{min}^{BP} \approx 10^{(3.31-0.47M_w)}$$

- **Mw 8 ~ 0.35 Hz**
- **Mw 7 ~ 1.02 Hz**
- **Mw 6 ~ 3.02 Hz**

Conclusion

- Linear BP image corresponds to the coseismic slip motion, smoothed by a resolution matrix $\mathbf{F}(\omega)$.

- $\mathbf{F}(\omega)$ helps to quantify the spatial resolution of BP method:
  1. Resolvability $\epsilon_I(\omega)$
  2. Resolvable area

- Lowest BP frequency $f_{min}^{BP} \approx 10^{(3.31-0.47M_w)}$

This work has been submitted to GJI and is under review

Conclusion

Thanks for you attention!

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- $\mathbf{F}(\omega)$ helps to quantify the spatial resolution of BP method:
  1. Resolvability $\epsilon_I(\omega)$
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