Entailment and Monotonicity

Plan for today

• We will focus on entailments among sentences and monotonicity patterns of sentential operators. Due to the lack of time, we discuss non-sentential expressions and operators after the spring break.

• To prepare for the midterm, there will be a short p-set assigned tonight and due on Monday. This p-set together with the one on presuppositions and implicatures count as one single p-set.

1 Entailment

1.1 What is entailment?

• Given any propositions \( p \) and \( q \):
  
  – \( p \) and \( q \) are (semantically/logically) equivalent iff they always have the same value.
  
  – \( p \) and \( q \) are contradictory iff they cannot be simultaneously true.
  
  – \( p \) entails \( q \) iff \( q \) is true whenever \( p \) is true, written as \( p \Rightarrow q \).

    (In other words, \( p \) entails \( q \) iff the implication \( p \Rightarrow q \) is a tautology.\(^1\))

• Special entailments:
  
  – Every proposition is entailed by a contradiction.
  
  – Every proposition entails a tautology.

Exercise: Based on the semantics of propositional logic and predicate logic, identify whether each of the following claim is right or wrong.

(1) a. For any two propositions \( p \) and \( q \):
  
  i. \( p \land q \) entails \( p \);
  
  ii. \( p \land q \) entails \( p \lor q \);
  
  iii. \( p \lor q \) entails \( \neg(p \land q) \);
  
  iv. \( \neg p \) entails \( p \Rightarrow q \).

b. For any two sets \( A \) and \( B \):
  
  i. \( a \in A \) entails \( a \in A \cup B \);
  
  ii. \( a \in A \) does not entail \( a \in A \cap B \).
  
  iii. \( \forall x [x \in A \Rightarrow x \in B] \) entails \( \exists x [x \in A \land x \in B] \).

\(^1\)We are now working on extensional semantics, which interprets a sentence as truth value. Once you work on intensional semantics, which interprets a sentence as a set of possible worlds, be aware that some of the concepts shall be defined differently.
1.2 Entailment test in natural languages

- In natural language semantics, a commonly proposed test for entailment relations is as follows:

  \[(2) \phi \text{ entails } \psi \text{ if and only if } \phi \text{ but not } \psi \text{ is intuitively contradictory.}\]

  For example:

  \[(3) \]
  a. John and Mary left, but Suzi didn’t leave.
  (Not contradictory. Hence “John and Mary left” \(\Rightarrow\) “Suzi left”.)
  b. # John and Mary left, but John didn’t leave.
  (Contradictory. Hence “John and Mary left” \(\Rightarrow\) “John left”.)

  Nevertheless, this test doesn’t always work:

  \[(4) \]
  # John or Mary left, but both John and Mary left.
  (Sounds odd. But “John or Mary left” \(\Rightarrow\) “John and Mary didn’t both leave”.)

  Why this test fails? How do you think?

- The following contradiction test in the form of a question-answer pair is more reliable:

  \[(5) \]
  a. Q: Did John and Mary leave?  
    A: Yes. # Actually, John didn’t leave.
  b. Q: Did John or Mary leave?  
    A: Yes. Actually, they both left.

**Exercise:** Use contradiction tests to identify whether each (a) sentence entails the paired (b) sentence.

\[(6) \]
 a. All of the students left.
 b. Some of the students left.

\[(7) \]
 a. Some of the students left.
 b. Not all of the students left.

\[(8) \]
 a. John’s daughter will come.
 b. John has a daughter.
2 Monotonicity

• Compare the directions of entailments in the following pairs of sentences:

(9) a. Mary is a semanticist. ⇒ Mary is a linguist.

b. Possibly, Mary is a semanticist. ⇒ Possibly, Mary is a linguist.

c. Mary isn’t a semanticist. ⇐ Mary isn’t a linguist.

• Monotonicity pattern of propositional operators:

Let \( \pi \) be a one-place propositional operator, we can define its monotonicity pattern as follows:

\(-\pi\) is an **upward-entailing (UE)** operator iff

for any two sentences \( p \) and \( q \) such that \( p \Rightarrow q \): \( \pi(p) \Rightarrow \pi(q) \);

\(^*\) Possibly ...

\(-\pi\) is a **downward-entailing (DE)** operator iff

for any two sentences \( p \) and \( q \) such that \( p \Rightarrow q \): \( \pi(p) \Leftarrow \pi(q) \);

\(^*\) It is not the case that ...

\(-\pi\) is a **non-monotonic (NM)** operator iff \( \pi \) is neither upward-entailing nor downward-entailing.

\(^*\) S if and only if ...

• More about conditionals: A conditional is DE in its antecedent, but UE in its consequent.

(10) a. If Mary is a semanticist, she can teach LING 83.

\( \Leftarrow \) If Mary is a linguist, she can teach LING 83.

b. If John likes Mary, he will invite her and her mother to the party.

\( \Rightarrow \) If John likes Mary, he will invite her to the party.

**Exercise:** Identify the monotonicity pattern for each of the following propositional operators.

(11) Example: ‘If not that ..., then Mary cannot teach LING 106’ is UE.

a. Method 1: Applying the definition

If Mary isn’t a semanticist, she cannot teach LING 106.

\( \Rightarrow \) If Mary isn’t a linguist, she cannot teach LING 106.

b. Method 2: Operator stacking

\( \mathrm{not\ that} \) is DE, and \( \text{If } ... \text{ then } S \) is DE. Hence \( \text{If } \mathrm{not} \ ... \text{ then } S \) is UE.

(12) a. Maybe it is not the case that ...

b. It won’t be the case that ..., if John likes Mary.

c. If ..., then Mary will not come.

d. If John believes that ..., then he will invite Mary.
3 Extending to non-sentential expressions (Not discussed yet)

3.1 Entailments between predicates

• The following entailment relation between sentences comes from the subset relation between the semantics of the 1-place predicates.

(13) a. Mary is a Chinese student. ⇒ Mary is a student.
b. Mary is a semanticist. ⇒ Mary is a linguist.
c. Mary arrived early. ⇒ Mary arrived.

• Given two one-place predicates \( A \) and \( B \), \( A \Rightarrow B \) iff for any \( x \): \( A(x) \Rightarrow B(x) \).

(14) a. For any individual \( x \), Chinese-student(\( x \)) ⇒ student(\( x \)).
    Therefore, Chinese student ⇒ student.
b. For any individual \( x \), semanticist(\( x \)) ⇒ linguist(\( x \)).
    Therefore, semanticist ⇒ linguist.
c. ...

3.2 Monotonicity of determiners and quantifiers

• We are not ready to determine the monotonicity pattern of an operator that selects a one-place predicate:

(15) Scope of a quantifier:
    a. Some student arrived early. ⇒ Some student arrived.  \( \text{some student is UE} \)
b. Every student arrived early. ⇒ Every student arrived.  \( \text{every student is UE} \)
c. No student arrived early. ⇐ No student arrived.  \( \text{not every student is DE} \)

(16) Restriction of a quantifier:
    a. Some semanticist arrived ⇒ Some linguist arrived.  \( \text{some ( ) arrived is UE} \)
b. Every semanticist arrived ⇐ Every linguist arrived.  \( \text{some ( ) arrived is DE} \)
c. No semanticist arrived ⇐ No linguist arrived.  \( \text{no ( ) arrived is DE} \)

Exercise: Identify the monotonicity pattern of the following quantifiers:

(17) a. exactly three students
    b. not every student
    c. Every participant who _____ got an award

Exercise: For each of the following claims, identify whether it is right or wrong.

(18) a. Negation is a DE operator.
    b. Every is a DE operator.
    c. Conditionals create DE environments.
    d. Any environment containing negation is DE.