Deep Learning for Revenue-Optimal Auctions with Budgets

Zhe Feng
Harvard SEAS

Based on joint work with
Harikrishna Narasimhan (Harvard) and David C. Parkes (Harvard)
History of Auction Design

W. Vickrey
Second Price Auction

1961

E. H. Clarke, 1971

1971

T. Groves, 1973

1973

R. B. Myerson
Optimal Single-item Auction

1981

Manelli & Vincent, 2006
Pavlov, 2011
Hart & Nisan, 2012
Cai et al., 2012a; b
Daskalakis et al., 2016
Yao, 2017

2006 - present

VCG Auction
Auction with budgets

• For bidders
  • Willingness to pay: Valuation
  • Ability to pay: financial (budget) constraints
  • Utility is $-\infty$, when payment is larger than budget

• E.g. Rent an apartment near Harvard
  • Valuation: Amenities, Distance from Harvard, Nearby environment
  • Budget (affordable)
  • Multiple items
Auction with budgets

• For bidders
  • Willingness to pay: Valuation
  • Ability to pay: financial (budget) constraints
  • Utility is $-\infty$, when payment is larger than budget

• For auctioneer
  • Maximize Social Welfare
  • Maximize Revenue
Main question

Design revenue-optimal auctions for budget constrained bidders
Optimal Auction Design with (private) Budgets

• (one item one bidder) Che and Gale, 2000
• (one item two bidders) Malakhov and Vohra, 2008
  • Only one budget constrained bidder
• (one item multiple bidders) Pai and Vohra, 2014
  • BIC mechanism
  • Discrete valuation and budget

Optimal DSIC single-item auction with private budgets is still not fully understood! There are no results for multi-item auctions!
Results Summary

• Apply Deep Learning to design revenue-optimal auctions with private budgets.

• Extend RegretNet framework in [Dütting, Feng, Narasimhan, Parkes, 2017] to handle multi-item, DSIC & BIC setting.

• For settings with analytical solutions, RegretNet recovers almost-optimal auctions

• For settings where optimal solution is unknown, RegretNet outperforms well-known baselines.

• First automated mechanism design for budget constrained auction design.
Auctions with Private Budgets

• $n$ bidders, $m$ items

• Auction contains allocation rule $g$ and payment rule $p$

• Each bidder has a type $t_i = (v_i, b_i) \sim F_i$
  • $F = (F_1, \ldots, F_n)$ is common knowledge
  • Multi-dimensional setting

• Utility function:
  • $u_i(t_i, (t'_i, t_{-i})) = \begin{cases} v_i \cdot g_i(t'_i, t_{-i}) - p_i(t'_i, t_{-i}), & p_i(t'_i, t_{-i}) \leq b_i \\ -\infty, & \text{otherwise} \end{cases}$

  • Non-quasilinear: VCG fails
  • Denote $u_i(t'_i, t_{-i}) := u_i(t_i, (t'_i, t_{-i}))$
Auctions with Private Budgets

• Individual Rationality: $u_i(t_i, t_{-i}) \geq 0$
• Budget Constraints: $p_i(t_i, t_{-i}) \leq b_i$
• Incentive Compatibility
  • Dominate Strategy IC (Strategy Proof): Under budget constraints, no matter what the other bidders report, truth-telling is always the weakly dominate strategy for each bidder. \( \forall i, t, t'_i, u_i(t_i, t_{-i}) \geq u_i(t'_i, t_{-i}). \)
  • Bayesian IC, a weaker version of IC.
• Expected Revenue

\[ \mathbb{E}_{t \sim F} \sum_i p_i(t) \]
Challenges

• Multi-dimensional even for single-item auctions
  • Bidders can misreport both value and budgets
• Non-quasilinear utility function
  • VCG-based auctions fail!
VCG-based auction fails

Run VCG Auction with truncated valuation (tv): 
\[ \min(\text{value, budget}) \]

Not Strategy Proof!
VCG-based auction fails

If win 1 chair, tv = $1
If win 2 chairs, tv = $2

If win 1 chair, tv = $10
If win 2 chairs, tv = $10

Value = $1, Budget = $10
Value = $10, Budget = $10
VCG-based auction fails

If win 1 chair, tv = $1
If win 2 chairs, tv = $2

Win 1 chair
Pay $0 = $10 - $10

Value = $1,
Budget = $10

If win 1 chair, tv = $10
If win 2 chairs, tv = $10

Win 1 chair
Pay $1 = $2 - $1

Utility: $9

Value = $10,
Budget = $10

Utility: $9
VCG-based auction fails

If win 1 chair, tv = $1
If win 2 chairs, tv = $2

If win 1 chair, tv = $5
If win 2 chairs, tv = $10

Win 2 chairs
Pay $2 = $2 - $0
Utility: $18
Data-driven Auction Design

Assume probability distribution for buyer’s type

Generate sample of type from distribution

Use machine learning to discover the optimal auction

- ($5, $9, $10)
- ($15, $2, $7)
- ...
- ($9, $12, $6)
Deep Learning

- Rich Representation (Nonlinearity, Deep vs. Shallow)
- Fast Training Method (SGD, Adam)
- Robust tool chains (e.g. TensorFlow, Pytorch, GPUs)
Data-driven Auction Design

\[
(t^1) \\
(t^2) \\
\vdots \\
(t^N)
\]

**Training**

Tune weights to maximize revenue s.t. incentive constraints and budget constraints

Generalize RegretNet Framework

[Dütting, Feng, Narasimhan, Parkes, 2017]
RegretNet: Architecture

$m$ identical items, $n$ additive bidders, the type of bidder $i$ is $(v_i, b_i)$. Parameters $w$.

Allocation Net

Payment Net

Allocation: $g^w: \mathbb{R}^{2n} \rightarrow \Delta_1 \times \cdots \times \Delta_m$

Payment: $p^w: \mathbb{R}^{2n} \rightarrow \mathbb{R}_{\geq 0}^n$

ReLU units: $relu(x) = \max\{0, x\}$
RegretNet: Metrics

- Deviation from IC (expected ex post regret)
  \[ rg_{t_i} = E_t \left[ \max_{t_i} I(p'_i \leq b_i) [u_i(t'_i, t_{-i}) - u_i(t_i, t_{-i})] \right] \]

- Ignoring measure zero events,
  - IC iff \( rg_{t_i} = 0 \), for each bidder \( i \)

- Expected ex post IR-penalty:
  \[ irp_i = E_t [\max (0, -u_i(t_i, t_{-i}))] \]

- Expected BC-penalty
  \[ bcp_i = E_t [\max (0, p_i(t) - b_i)] \]
RegretNet: Learning Problem

- \((g^w, p^w)\) where \(w\) are parameters in Neural Nets
- Learning problem:

\[
\min_w \mathcal{L}(g^w, p^w) = -E_t \left[ \sum_i p^w_i(t) \right]
\]

s.t. \(\forall i \in [n], \text{rgt}_i^w = 0, \text{irp}_i^w = 0, \text{bcp}_i^w = 0\)
RegretNet: Learning Problem

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• Learning problem:

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\min_w \mathcal{L}(g^w, p^w) = -E_t\left[\sum_i p_i^w(t)\right]
\]

s.t. \(\forall i \in [n], rgt_i^w = 0, \text{irp}_i^w = 0, \text{bcp}_i^w = 0\)
• Train via augmented Lagrangian Method to handle the IR, BC and regret constraints

\[
w^{t+1} := \arg\min_w [\mathcal{L}(g^w, p^w) + \sum_i \lambda_i \cdot rgt_i^w + \frac{\rho}{2} \sum_i (rgt_i^w)^2 + \cdots]
\]

(inner optimization)

\[
\forall i \in [n], \quad \lambda_i^{t+1} := \lambda_i^t + \rho \cdot rgt_i^{w^{t+1}}
\]
• Adaptively tune Lagrange multiplier. In our case, \(\lambda_i\) always increases and we fix \(\rho > 0\).
RegretNet: Empirical Optimization

• Randomly sample $L$ type profiles from distribution, $S = \{t^{(1)}, \ldots t^{(L)}\}$.

• Empirically estimate $\hat{\mathcal{L}}(g^w, p^w) = -\frac{1}{L} \sum_{\ell} p^w(t^{(\ell)})$, similarly for $\widehat{irp}$ and $\widehat{bcp}$.
RegretNet: Empirical Optimization

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• Empirically estimate $\hat{L}(g^w, p^w) = -\frac{1}{L} \sum_{\ell} p^w(t^{(\ell)})$, similarly for $\hat{irp}$ and $\hat{bcp}$.

• To estimate regret,

$$ \hat{rgt}_i^w = \frac{1}{L} \sum_{\ell} \left[ \max_{t_i', t_{-i}'} \mathbb{I}(p'_i \leq b_i)[u_i(t_i', t_{-i}^{(\ell)}) - u_i(t^{(\ell)})] \right] $$

Max over additional fixed samples for each type profile, generated from uniform distribution.

• Run SGD for inner optimization problem
Experiments:

Can RegretNet recover known auction designs?
DSIC mechanism: 1-item, 1-bidder

- $v \sim U[0,1], b \sim U[0,1]$  
  [Che & Gale, 2000]

- [Graphs showing test revenue, test regret, test IR-penalty, and test budget-penalty over the number of iterations, with plots for Optimal Mechanism and RegretNet.]
DSIC mechanism: 1-item, 1-bidder

- \( v \sim U[0,1], b \sim U[0,1] \)  
  [Che & Gale, 2000]
DSIC mechanism: 1-item, 2-bidders

- $v_i \in Unif\{1,2,\ldots,10\}$, one is unconstrained, other: $b_2 = 4$

[Malakhov & Vohra, 2008]

(a) Revenue and regret as a function of solver iterations

(b) Learned allocation rule

(c) Optimal allocation rule
BIC mechanism: 1-item, 2-bidders

- \( v_i \in Unif\{1,2,\ldots,10\} \), one is unconstrained, other: \( b_2 = 4 \)

[Malakhov & Vohra, 2008]
BIC mechanism: 1-item, 2-bidders

• 1-item, 2-bidders, $v_i \sim U[0,1], b_i \sim U\{0.22, 0.42\}$

[Pai & Vohra, 2014]
Experiments:

*Can RegretNet discover new auction designs?*
DSIC mechanism: 4-units, 2-bidders

- $v_i \sim U[0,1], b_i \sim U[0,1]$
DSIC mechanism: 2-item, 2-bidders

- \( v_i \sim Unif\{1,2, ..., 10\}, b_i \sim Unif\{1,2, ..., 4\}, \) unit-demand
Conclusion

• Extend RegretNet framework in [Dütting, Feng, Narasimhan, Parkes, 2017] to revenue-optimal auctions with private budgets (multi-item setting).

• Generalize RegretNet to BIC setting.

• Almost recover the optimal auctions where analytical result exists.

• For the settings where there is no theoretical analysis, our RegretNet outperforms well-known baselines.
Thanks!