Why does democracy need education?

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Abstract Across countries, education and democracy are highly correlated. We motivate empirically and then model a causal mechanism explaining this correlation. In our model, schooling teaches people to interact with others and raises the benefits of civic participation, including voting and organizing. In the battle between democracy and dictatorship, democracy has a wide potential base of support but offers weak incentives to its defenders. Dictatorship provides stronger incentives to a narrower base. As education raises the benefits of civic engagement, it raises participation in support of a broad-based regime (democracy) relative to that in support of a narrow-based regime (dictatorship). This increases the likelihood of successful democratic revolutions against dictatorships, and reduces that of successful anti-democratic coups.

Keywords Democracy · Education · Political participation

1 Introduction

The hypothesis that higher education leads to more democratic politics (Lipset, 1959, 1960) has received a good deal of empirical support (Barro, 1999; Glaeser, LaPorta, Lopez-de-Silanes, and Shleifer, 2004; Papaioannou and Siourounis, 2005). However, the theoretical reasons for this relationship remain unexplored. Indeed, according to (Barro, 1999, p. S182), “given the strength of the Aristotle/Lipset hypothesis as an empirical regularity, it is surprising that convincing theoretical models of this relationship do not exist.” In this paper, we first motivate and then propose one model of a causal impact of education on democracy.

Our starting point is the connection between education and political participation. This connection has been emphasized by Almond and Verba (1989, 1st ed. 1963), who see
education as a crucial determinant of “civic culture” and participation in democratic politics. “The uneducated man or the man with limited education is a different political actor from the man who has achieved a higher level of education (p. 315).” Almond and Verba’s work has influenced both political science (e.g., Brady, Verba, and Schlozman, 1995) and sociology (e.g., Kamens, 1988), and our work can be seen as an elaboration of their ideas using theoretical and empirical tools of economics.

A dramatic place to see the effect of education on political participation is student activism. Students rioted against authority at Oxford, Bologna, and Paris even in the Middle Ages. Martin Luther found the most immediate intense support from the students in Wittenberg and other German universities. Students played key roles in liberal movements and revolutions in Europe in the middle of the 19th century. “If the revolution had a core, it was the young educated elite” (Rander-Pehrson, 1999, p. 145). Student demonstrations played a role in the overthrow of Peron in Argentina in 1955, the Hungarian Revolution in 1956, the downfall of Perez Jimenez in Venezuela in 1958, the resignation of the Kishi government in Japan in 1960, the resistance to Diem in Vietnam in 1963, the anti-Sukarno movement in Indonesia and the toppling of the Rhee government in Korea in 1966, the Prague Spring in 1968, and the downfall of Ayub Khan in Pakistan in 1969. The Tianenmen student uprising of 1989 failed to depose the Communist Party, perhaps because the students got little support in generally uneducated China and were crushed by the troops. Most recently, peaceful demonstrations in which students played a key part helped save democracy in Ukraine against the aggrandizement by the ex-President who stole the election.

It would be incorrect to conclude from these examples that students have a preference for democratic government—perhaps because they value freedom, information, or elections—rather than for political participation. The hep-hep anti-semitic riots in Bavaria in 1819 started when during an academic ceremony an aged professor who had recently come out in favor of civic rights for Jews had to run for his life as angry students assaulted him. Mussolini enjoyed substantial support from students in the young fascist movement. Hitler likewise relied on the Nazi students, who eventually seized control of the universities. In Latin America, students offered strong support to the Che Guevara led communist guerilla movement, no friends of democracy. The evidence that students organize to participate in collective action—democratic or anti-democratic—is much more compelling than the evidence of their preference for democracy. Our goal, then, is to explore more deeply the consequences of what we take to be the primitive connection between education and participation.

In Sect. 2 we briefly review the evidence on education and democracy. Although econometric controversies about this evidence still linger, the weight of recent research suggests that the Lipset hypothesis is valid, and that theories to explain it are indeed called for.

In Sect. 3, we motivate our basic assumption that education leads to higher participation in a whole range of social activities, including politics. This might be so for several reasons. In one view, schooling incorporates indoctrination about the virtues of political participation. A second view sees human capital as actually social capital: schools largely teach students to interact with one another. By improving interpersonal skills, education facilitates civic involvement. Using micro-evidence from both the US and other countries, we document the robust correlation between many forms of civic activity, including political participation, and education. This evidence is most naturally consistent with the second view.

Motivated by this evidence, we present a model of regime stability in Sect. 4. Unlike the literature on regime change that typically focuses on the payoffs to citizens under alternative political regimes (e.g., Bourguignon and Verdier, 2000), our model describes incentives to participate in the political activity itself. These include top-down incentives, such as punishments meted out by political leaders to their presumed supporters who are shirking. But
these also include lateral incentives, provided by participants in politics who encourage or shame their friends and peers to join them. Democracies are more inclusive regimes than dictatorships, so, in the spirit of Olson, they deliver weaker top-down incentives. Large popular movements cannot readily compel their members to vote or demonstrate. But our key assumption is that education raises the benefits of political participation: better educated peers are better at persuading friends to join. As a consequence, although education raises participation in both democratic and dictatorial regimes, the increase is greater for the more inclusive (democratic) regime. When political success is determined by the raw number of supporters, education favors democracy relative to dictatorship.

In this model, the political success of a democracy hinges on having a large number of supporters whose benefits of political participation are sufficiently high that they fight for it even in the absence of direct rewards. Education supplies such supporters and stabilizes democracy. Conversely, in countries with low levels of education, dictatorship or oligarchy is more stable than democracy, because only dictatorships offer the strong top-down incentives needed to induce people to defend them. In the model in Sect. 4, countries with higher levels of education are more likely both to experience a transition from dictatorship to democracy, and to withstand anti-democratic challenges. Moreover, the size of the most successful challenger regime to an existing dictatorship rises with the level of education.

In addition to having some empirical support for its core assumption and delivering the broad empirical predictions documented in Sect. 2, the model has several new implications. It predicts that, in general, education causes the more inclusive groups to dominate politics. In particular, as education increases, groups challenging existing regimes become progressively larger—small coups are replaced by large revolutions. Although they remain to be formally tested, these predictions are consistent with a broad outline of European transition to democracy.

2 The empirical relationship between education and democracy

Across the world, the correlation between education and democracy is extremely high. Figure 1 shows the relationship between the Polity IV index of democracy (Jaggers and Marshall, 2003) and the years of schooling in the country in 1960 (Barro and Lee, 2001). Across 91 countries, the correlation coefficient between these variables is 74%.

Of course, this correlation does not establish causality. Barro (1997, 1999) conducted the initial research on the time-series relationship between education and democracy. In that spirit, consider the evidence on transitions from dictatorship to democracy. Figure 2 shows the raw correlation between the change in the Jaggers and Marshall (2003) democracy score and years of schooling in 1960 (from Barro and Lee) for countries that had low democracy ratings (zero or one) in 1960. This correlation is 66%. If we take the entire sample of countries and regress the change in democracy on initial democracy and initial years of education, we estimate:

\[
\text{Change in Democracy} = 4.13 - .98 \cdot \text{Democracy in 1960} + .84 \cdot \text{Schooling in 1960}
\]  

(1)

1 Przeworski, Alvarez, Cheibub, and Limongi (2000), Barro (1999), Boix and Stokes (2003), Glaeser, LaPorta, Lopez-de-Silanes, and Shleifer (2004), and Papaioannou and Siourounis (2005) also consider the relationship between income and democracy. The conclusion emerging from the controversies is that income does cause transition to democracy, as well as its stability. Our focus, however, is on education not income. Nor do we consider the consequences of democracy, see, e.g., Przeworski and Limongi (1993) and Mulligan, Gil, and Sala-i-Martin (2004).
There are 65 observations in this regression and the $R^2$ is 67%. Initial schooling, even in highly dictatorial regimes, strongly predicts becoming more democratic over time. In contrast, democracy does not predict growth in schooling. We estimate:

\[
\text{Change in Schooling} = 2.80 - 0.28 \cdot \text{Democracy in 1960} + 0.06 \cdot \text{Schooling in 1960}
\]

(2)

There are 68 observations in this regression and the $R^2$ is only 3%. The relationship between initial democracy and changes in years of schooling is shown in Fig. 3. The
evidence suggests that schooling leads to democracy, but there is no evidence that democracy leads to schooling.

This evidence is subjected to more formal specifications by Glaeser, LaPorta, Lopez-de-Silanes, and Shleifer (2004) and Papaioannou and Siourounis (2005). Both studies confirm that education is a strong predictor of transition to democracy. The second study in particular focuses on the third wave of democratization (Huntington, 1991) and shows that education is a powerful predictor of permanent transitions from dictatorship to democracy.

The most recent research in this area considered the possibility that some permanent country characteristics, such as geography or culture, are responsible for producing both education and democracy, and to this end estimated regressions with fixed effects. Acemoglu, Johnson, Robinson, and Yared (2005) suggest that, with country fixed effects, there is no remaining relationship between education and democracy. Their results, however, depend on a short time series and extreme persistence in the education data (Glaeser et al., 2004). Castello-Climent (2006) and Bobba and Coviello (2006) argue that, in this situation, the appropriate technique is to use the Blundell–Bond (1998) system GMM estimator rather than the Arellano–Bond (1991) first-difference GMM estimator. When they do so, they find that education indeed causes democracy even taking account of permanent country effects.

The overall findings thus seem quite favorable to Lipset’s (1960) hypothesis. Education is highly correlated with democracy in both cross-section and most recently estimated panel regressions. The best econometric evidence suggests that this effect is causal.

3 Education and civic participation

Education may promote democracy because it raises the benefits (or reduces the costs) of political activity. In Sect. 4, we take this as an assumption and show how it explains the evidence. In this section, we describe some theories of why education raises civic participation and then present some empirical evidence bearing on them.
3.1 Why should education and civic participation be correlated?

Perhaps the simplest hypothesis explaining the link between education and civic participation is that indoctrination about political participation is a major component of education. In democracies, schools teach their students that political participation is good. One “content standard” listed by the State of California’s Department of Education advocates that students “understand the obligations of civic-mindedness, including voting, being informed on civic issues, volunteering and performing public service, and serving in the military or alternative service.” The original public school movement in the US emphasized preparing students for participation in democracy. This emphasis is not unique to America. Holmes (1979) synthesizes the aims of schools systems around the world. Political aims are often cited as an educational goal: “school work is organized so as to develop democracy in school and consequently in society as a whole” (Sweden), “the Constitution states that a general aim of education is to produce good citizens, a democratic way of living and human solidarity” (Costa Rica), and “an education system that creates knowledgeable, democratic and patriotic citizens is the aim of the Indonesian government.” Perhaps the key implication of the indoctrination hypothesis is that the positive impact of schooling should be particularly pronounced in political rather than all social participation.

A second hypothesis holds that schooling lowers the costs of social interactions more generally. According to this view, a primary aim of education is socialization—teaching people how to interact successfully and productively with others. “Education is one of the most important predictors—usually, in fact, the most important predictor—of many forms of social engagement—from voting to chairing a local committee to hosting a dinner party to trusting others” (Helliwell and Putnam, 2007).

What are some of the possible microeconomic mechanisms behind this phenomenon? Social activities hinge on interpersonal exchange of information. The direct benefit of interaction arises from coordination (Gradstein and Justman, 2002). Coordination requires members of a group to explain and to be able to understand what is to be done, how it will be accomplished, and what the motivations for acting are. Social connection also provides indirect benefits in that, by communicating with others, group members acquire new information that is useful for their private purposes. On the other hand, miscommunication is costly to both groups and individuals. Misunderstandings lead to coordination failures that may prove fatal to a group’s mission. At the individual level, misunderstandings lead to hurt feelings and arguments, not to mention poor decisions.

Education raises the benefit from social participation because it facilitates seamless information exchange. Educated people are better able to express what they know, to inform, and to persuade. They are also better able to acquire new information, to understand, and to learn. Schooling also teaches rules of behavior that make a discussion between educated people both more informative and less likely to degenerate into a quarrel (Bowles and Gintis, 1976). These procedural benefits capture the fundamental value of education as socialization. At every level from primary school to university, education is the acquisition of skills helpful for operating with both knowledge and people, to be able to learn and to teach.

Education textbooks list socialization as a pillar of curriculum design. Driscoll and Nagel (2005) describe several curricular approaches to primary education. Many of these list social outcomes among the goals: “the children will develop cooperative relationships, reflecting both social skills and understanding the perspectives of others” (the Kamii and DeVries approach); “socialization of children. Self-regulation of behavior is necessary to participation in forms of society and in relationship with others” (the Bank Street approach); and “conditions that promote or strengthen relationships between children, and between children
and adults” (the Waldorf approach). Gordon and Browne (2004) write that “a major role for the early childhood teacher is to see that children have enjoyable social contacts and to help motivate children toward a desire to be with others,” because “enhancing social intelligence builds a set of skills that may be among the most essential for life success of many kinds.”

Why do schools spend so much effort on socializing children? An altruistic view might suggest that the ability to work well in social settings is among the most important skills needed to function in society. A more cynical view sees socialized children as easier for their teachers to manage. Whatever the reason, schools in all political and religious regimes devote considerable resources to teaching social cooperation.2

The socialization hypothesis predicts that education should impact all forms of social involvement. Its ability to predict political engagement should be no stronger than that for other forms of social participation. This theory also predicts no difference in the impact of education on social activities in democracies or non-democracies.

These two hypotheses both assume that education causes civic participation. It is at least possible that the link between schooling and education represents selection, not treatment, and that exogenous characteristics that make people tolerant of education also enable them to sit through meetings or wait in line to vote. If this were true, then exogenous increases in schooling would have no impact on overall levels of civic participation. Furthermore, if innate characteristics vary more within than across areas, this view predicts a low (or non-existent) relationship between education and civic participation at the aggregate level.

3.2 Empirical evidence on education and civic participation

Using the World Values Survey (WVS), we begin with cross-country evidence on education and membership in social groups. We exclude countries with the Polity IV autocracy score above 4 from the analysis because these countries force party and other participation. (For example, 25% of the Chinese respondents report membership in the Communist party). Figure 4 presents the results for the available 34 countries. It shows a sharply positive and statistically significant ($t = 3.31$) relationship between education and participation in social groups. This evidence is broadly consistent with our theoretical perspective, but unfortunately does not allow us to distinguish the various hypotheses.

We can do that better with individual-level evidence from the US. There are two primary individual-level sources for information on social activities: the General Social Survey (GSS) and the DDB Needham Lifestyles Survey. Using the GSS, DiPasquale and Glaeser (1999) document a strong positive relationship between education and a variety of social outcomes. College graduates are 27% more likely than high-school dropouts to say that they vote in local elections and 29% more likely to say that they help solve local problems.

College graduates are also more likely to join organizations. Glaeser and Sacerdote (2001) show this to be true for 15 out of 16 forms of group membership: the exception is trade-union membership. Using the WVS, they also find a significant positive relationship between years of education and group membership in almost every country. Education also positively predicts church attendance in the GSS. Fifty percent of American college graduates say that they attend church more than several times per year; 36% of high-school graduates say they attend that often.

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2 By social cooperation we do not mean obedience to authority. Obedience to authority is of course something different that schools also teach, although evidently not as successfully, as the evidence of student—and more generally educated people’s—activism against authority illustrates.
Using evidence from the DDB Needham Lifestyles Survey, we revisit some of those results in Table 1. The DDB Needham Survey is administered over the years 1975–1999 and covers (for many questions) a larger sample than the GSS. We control for basic demographics such as age, race and gender. All of our variables are categorical and take discrete values capturing the frequency of the activity. We normalize each of these variables to have a mean of zero and a standard deviation of one. We also control for income. Because (as in the GSS) income is missing for many observations, we include these observations but code them as having the mean value of income in the sample, and add a dummy which takes on a value of one when income is missing. We also include a dummy for each survey year to capture time trends in social activities. We measure education with two separate dummy variables. The first takes on a value of one if the person is a high-school graduate and zero otherwise. The second variable takes on a value of one if the person is a college graduate and zero otherwise. Our results do not change if we use continuous measures of education.

In each regression, both education variables positively affect participation and both are almost always statistically significant. Regression (1) shows the impact of education on attending church. The effect of being a college graduate relative to a high-school dropout is more than 30 percentage points. Regression (2) shows the large and positive impact of education on attending a class or a seminar. Regression (3) shows a strong association between schooling and self-reported working on a community project. In regression (4), we look at writing a letter to a newspaper, a particularly clear form of civic engagement. Again, the correlation with education is positive, and the effect of college education is particularly strong. Since the mean of this variable is much lower than that of many others, we should not be surprised that the coefficients on schooling are smaller. Regression (5) shows results on contacting a public official. Again, the impact of education is strong. Regression (6) looks at registering to vote, an important measure of political participation in a democracy. Registering to vote (and voting) is particularly strongly associated with years of education. Finally, regression (7) shows that giving someone the finger—an anti-social form of behavior—is negatively associated with years of schooling. These regressions show a pervasive pattern, in
which years of schooling are associated not only with political participation in a democracy, but also with many other forms of social engagement.

Because our model addresses political battles that are often violent outside the US, we now turn to the evidence on more violent forms of group activity. Education and training are closely linked to military discipline and group coherence under fire (Hanson, 2001). Following Keegan (1976) and many others, Hanson argues both that historically military success is primarily the result of troops not flee under fire and that military discipline is itself the result of culture and education. Costa and Kahn (2003) show that illiteracy strongly predicts desertion among Union soldiers in the American Civil War.

Ferguson (1999) looks at the ratio of prisoners of war to total casualties across countries during the First World War. This variable is described by some military historians as a measure of soldiers’ willingness to surrender, as opposed to fight, under fire. Across major combatant countries, the ratio of prisoners to total casualties was the lowest for the UK, the US, and Germany (1.4%, 6.7% and 9%, respectively). These arguably were the best educated combatants. The ratio of prisoners to total casualties was the highest among Russians, Austro-Hungarians and Italians (51.8%, 31.8%, and 25.8%)—the least educated of the major combatants. More standard forms of military history corroborate that these prisoner rates capture general failures of morale.

Finally, two recent studies address the view that the correlation between education and participation is selection. Milligan, Moretti, and Oreopoulous (2004) find that exogenous increases in education due to compulsory schooling laws raise voter turnout. Dee (2004) finds that increases in education accounted for by availability of junior and community colleges have a large effect on subsequent participation in voting. These results suggest that the effect of education on political participation is causal, rather than just the consequence of selection.

Can the evidence help us distinguish between the other hypotheses? The fact that education increases all forms of participation, many of which are deeply apolitical, goes against the political indoctrination hypothesis. That hypothesis is also undermined by the peaceful anti-communist revolutions throughout Eastern Europe and the former Soviet Union, dominated by educated people with years of pro-communist indoctrination under their belts. Given the separation between church and state in the US and UK, and given the often anti-religious sentiment of the French educational establishment, the positive correlation between years of education and church attendance is hard to understand as reflecting political indoctrination.

The theory that best explains all of the facts is the second hypothesis: education is socialization. This theory predicts the universal relationship between education and participation across activities and across countries. Of course, this does not mean that other mechanisms do not also operate, but we are inclined to accept the view that acquiring social capital is a crucial part of acquiring human capital.

4 A model of education and democracy

In this section, we model a channel through which education encourages democracy, both by increasing its stability and by increasing the probability of transition to democracy. The critical assumption, following the evidence in Sect. 3, is that education raises the benefits of political participation. The core insight of the model is that democracy requires support from a broad base of citizens who face only weak incentives to fight for it, while dictatorships offer
### Table 1 Social engagement and education

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1) Attended church or other place of worship</th>
<th>(2) Attended a class or seminar</th>
<th>(3) Worked on a community project</th>
<th>(4) Wrote a letter to the editor</th>
<th>(5) Contacted a public official</th>
<th>(6) Are you a registered voter?</th>
<th>(7) Gave someone the finger while driving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable mean</td>
<td>22.7</td>
<td>4.8</td>
<td>2.4</td>
<td>0.5</td>
<td>2.1</td>
<td>88%</td>
<td>2.1</td>
</tr>
<tr>
<td>High School grad</td>
<td>0.1794</td>
<td>0.181</td>
<td>0.1209</td>
<td>0.0353</td>
<td>0.1044</td>
<td>0.4673</td>
<td>−0.1413</td>
</tr>
<tr>
<td>College grad</td>
<td>(12.22)**</td>
<td>(7.01)**</td>
<td>(8.69)**</td>
<td>(1.88)</td>
<td>(2.15)*</td>
<td>(7.89)**</td>
<td>(3.01)**</td>
</tr>
<tr>
<td>Survey year</td>
<td>−0.018</td>
<td>−0.0169</td>
<td>−0.0069</td>
<td>0.0002</td>
<td></td>
<td>0.04</td>
<td>(1.69)</td>
</tr>
<tr>
<td>Female</td>
<td>0.2515</td>
<td>0.0282</td>
<td>0.0733</td>
<td>0.005</td>
<td>−0.1546</td>
<td>0.1167</td>
<td>−0.2101</td>
</tr>
<tr>
<td>Age</td>
<td>0.0103</td>
<td>−0.0111</td>
<td>0.0056</td>
<td>0.0016</td>
<td>0.0027</td>
<td>0.0119</td>
<td>−0.0122</td>
</tr>
<tr>
<td>Black</td>
<td>0.0983</td>
<td>−0.0549</td>
<td>0.0095</td>
<td>0.0138</td>
<td>−0.1619</td>
<td>0.1167</td>
<td></td>
</tr>
<tr>
<td>Asian</td>
<td>−0.303</td>
<td>(1.84)</td>
<td>(0.47)</td>
<td>(0.57)</td>
<td>(1.64)</td>
<td>(2.10)*</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>−0.1549</td>
<td>(2.74)**</td>
<td>(1.25)</td>
<td>(1.06)</td>
<td>(0.15)</td>
<td>(4.61)**</td>
<td></td>
</tr>
<tr>
<td>Log income in 2000 dollars</td>
<td>0.02</td>
<td>−0.0659</td>
<td>0.052</td>
<td>−0.0181</td>
<td>0.0063</td>
<td>0.1299</td>
<td>−0.0038</td>
</tr>
<tr>
<td>Missing income data</td>
<td>−0.0243</td>
<td>(6.17)**</td>
<td>(8.45)**</td>
<td>(2.29)*</td>
<td>(0.20)</td>
<td>(5.84)**</td>
<td>(0.22)</td>
</tr>
<tr>
<td>Constant</td>
<td>34.7132</td>
<td>34.6147</td>
<td>12.7943</td>
<td>−0.4271</td>
<td>−0.2611</td>
<td>−2.4316</td>
<td>−79.0522</td>
</tr>
<tr>
<td>Observations</td>
<td>47459</td>
<td>18888</td>
<td>47808</td>
<td>30710</td>
<td>3229</td>
<td>3617</td>
<td>6747</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.05</td>
<td>0.07</td>
<td>0.02</td>
<td>0</td>
<td>0.02</td>
<td>0.08</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Notes: Absolute value of t statistics in parentheses. * Indicates significance at 5%; ** Indicates significance at 1%. Data from DDB Needham.
strong incentives to a narrow base of supporters. Education raises the benefits of political participation and draws relatively more people to support democracy.

4.1 Model setup

The country is populated by measure one of homogeneous citizens, each with a human capital level of \( h \geq 0 \). A regime is defined as a set \( G_i \) of insiders, with \( g_i \in [0, 1] \) being the measure of the set, or the size of the regime. We interpret a larger \( g_i \) as a more democratic regime. We call a regime with \( g_i = 1 \) a perfect democracy.

In period zero, there is an exogenous status quo regime \( G_0 \) of size \( g_0 \). In period one, an alternative regime \( G_1 \) of size \( g_1 \) is proposed. Membership in each regime is exogenous. In period two, each individual chooses whether to defend the existing regime, to fight for the new regime, or to stay politically uninvolved. Individuals may not support both regimes. In this model, while each individual takes as given his membership in a particular regime (or in neither), he still chooses whether to participate in politics.

We let \( s_i \in [0, g_i] \) denote the endogenously determined mass of insiders who choose to support regime \( G_i \). The challenger unseats the incumbent if and only if \( \varepsilon_0 s_0 \leq \varepsilon_1 s_1 \), where \( \varepsilon_i \) is a random shock to the effectiveness of each faction’s supporters. The ratio \( \rho = \varepsilon_0 / \varepsilon_1 \) has a continuous probability distribution \( Z(\rho) \) on \( \mathbb{R}^+ \).

Each individual is of measure zero and so does not impact the probability that either regime succeeds. Individuals therefore do not base their political participation decisions on their impact on the outcome. Instead, participation in politics is based on three different forces. First, regimes provide incentives to their members to participate. These incentives take the form of punishing a regime’s insiders who do not fight for it (or, equivalently, rewarding regime insiders who do come out and fight). Second, regime insiders who participate themselves motivate their fellow insiders to join them through persuasion, camaraderie, or peer pressure. We model this as a benefit from participation (equivalently, it can be a cost of non-participation, if your friends shame you when you sit out). We also assume that there are individual-specific costs of participation. In our model, what is crucial is the net benefit of participating in politics relative to not participating, so it does not matter whether either regime-level or peer-level incentives take the form of punishments or rewards.

We formally model a regime’s power to motivate insiders by assuming that insiders who fail to support their regime suffer an expected utility loss described by the continuously differentiable function \( p(g_i) \) such that for all \( g_i \in [0, 1] \)

\[
p(g_i) > 0 \text{ and } p'(g_i) < 0.
\]

Smaller groups impose larger punishments on free-riders: “the greater effectiveness of relatively small groups [. . .] is evident from observation and experience as well as from theory” (Olson, 1965, p. 53). Smaller groups benefit from better monitoring and punishment of transgressors. As Olson (p. 61) writes, “In general, social pressure and social incentives operate only in groups of smaller size.” This assumption sets up the basic tradeoff between smaller and larger regimes. Small regimes provide strong incentives to a small base. Larger (i.e., more democratic) regimes provide weaker incentives but to a larger potential base of supporters.

The threat of punishment (or the promise of rewards) captures the global incentives provided by the leaders to all insiders. We also allow regime insiders who participate to motivate

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3 In Bourguignon and Verdier (2000) political participation depends on education, but education is determined by the initial income distribution and participation incentives are not considered.
their peers to do likewise. While the regime-level motivation should be thought of as leaders threatening members, we think of this local motivation as friends convincing friends to come out and fight. Precisely because of their local nature, these benefits depend not on the aggregate size of the regime, but on the rate of participation \( a_i \in [0, 1] \), which captures the share of friends who turn out to support a regime and provide motivation, or identically the probability that each friend turns out.

We also assume that these benefits of participation are a function of the human capital of regime members, and specifically that they are represented by a twice continuously differentiable function \( b(a_i h) \) such that \( b(0) = 0 \) and for all \( a_i \in [0, 1] \) and \( h \geq 0 \)

\[
\begin{align*}
b'(a_i h) &> 0 \quad \text{and} \quad b''(a_i h) < 0.
\end{align*}
\]

Higher levels of human capital make people better at inducing their peers to participate politically.\(^4\) As discussed in Sect. 3, this reflects the twofold role of education in creating social skills. First, more educated people are better at cajoling, encouraging, motivating, or otherwise persuading others they interact with to join them. Second, more educated people are better able to reap the benefits of social interaction themselves, perhaps because they understand better why they are participating. Socialization covers the twin powers to persuade and to understand, both captured by \( b(.) \). It is more appealing to participate in a collective activity the more educated a person is, and the more educated the other participants are.

Offsetting the global and local incentives is an effort cost \( c \) of political participation, which is identically and independently distributed across all individuals with continuous distribution \( F(c) \). This idiosyncratic cost is realized at the start of period two, after membership in the two regimes has been defined.

4.2 Group equilibrium

Peer incentives for participation determine a social multiplier, which could be understood as a bandwagon effect. The more active members a group already has, the more likely to participate the remaining members are. The participation rate \( a_i \) is then endogenously determined as a function of the exogenous parameters \( g_i \) and \( h \). In a group equilibrium,

\[
\begin{align*}
a_i &= F(p(g_i) + b(a_i h)).
\end{align*}
\]

In principle, strategic complementarity could lead multiple equilibria, some of which would typically be Pareto ranked (Cooper and John, 1988). Although coordination failures may play a part in the empirical determination of turnout, they are not central to our analysis. Moreover, considering a scenario without coordination failures allows us to establish a more robust link between education and participation, before taking into account the role of human capital in resolving coordination failures.

Hence, we make two economically intuitive assumptions on the distribution of costs that guarantee uniqueness of the group equilibrium:

**Assumption 1** \( c \) has a connected support \( C \) that includes the range of \( p(g_i) + b(a_i h) \).

**Assumption 2** \( c \) has a continuously differentiable density \( f(c) \) that is monotone non-increasing: \( f'(c) \leq 0 \) for every \( c \in C \).

\(^4\) There is no loss of generality in having \( h \) enter linearly, because we choose how to measure \( h \). We could write \( b(a_i h(H)) \), where \( h(.) \) is any monotone increasing function and \( H \) is another more natural measure of human capital, such as years of schooling.
The first assumption means that among individuals belonging to a group there are always some choosing to participate and some choosing not to participate, regardless of group size and human capital. The second assumption means that the cost of inducing participation is (weakly) convex: the more supporters a regime already has, the more difficult it becomes for it to attract additional supporters.\textsuperscript{5} We can then prove the following:

**Lemma 1** There exists a unique group equilibrium \( (g_i, h) \in (0, 1) \) such that \( \partial a / \partial g_i < 0 \) and \( \partial a / \partial h > 0 \).

The Lemma is intuitive, and illustrates the natural effect of exogenous parameters on endogenous turnout: smaller groups have higher turnout because top-down incentives are more powerful, while higher human capital induces higher participation at any group size.

### 4.3 Human capital and political competition

Consider a contest between two exogenously formed regimes \( G_0 \) and \( G_1 \), with respective sizes \( g_0 \) and \( g_1 \). The former is the incumbent and the latter is a challenger.

Some individuals may belong only to the incumbent regime \( G_0 \): their mass is denoted by \( \hat{g}_0 \). Some may belong only to the challenger regime \( G_1 \), and their mass is denoted by \( \hat{g}_1 \). Some individuals may be excluded from both regimes, their measure is \( 1 - g_0 - \hat{g}_1 = 1 - \hat{g}_0 - g_1 \). Finally, some people could formally belong to both regimes: the mass of these individuals equals \( \gamma \equiv g_0 - \hat{g}_0 = g_1 - \hat{g}_1 \). The membership overlap between regimes complicates the model and requires us to make assumptions on how people choose, but we need to consider this possibility because, with broadly encompassing democratic regimes, overlaps are inevitable.

In practice, each member of both regimes will show allegiance to one regime only, and will not consider supporting the other even if he formally belongs to it as well. We assume for simplicity that each individual formally belonging to both regimes will affiliate \textit{ex ante} with only one of them (with equal probability). As a consequence, he derives no benefits from interaction with the members of the other regime, nor can he be punished by its leaders. Nonetheless, the leaders have to keep wasting monitoring resources on these defectors, perhaps because they need to monitor (a random sample of) all members even though \textit{ex post} they discover that they cannot punish (or reward) some of them. With this assumption, a regime of nominal size \( g_i \) with measure \( \hat{g}_i \) of exclusive members has an actual basin of support

\[
\hat{g}_i = g_i - \frac{\gamma}{2} = \hat{g}_i + \frac{\gamma}{2} = \frac{g_i + \hat{g}_i}{2}.
\]

All individuals belonging to neither regime (a mass of \( 1 - \hat{g}_0 - \hat{g}_1 \)) abstain from political participation, since they would reap no benefits and only incur costs. Those with effective membership in a regime (a mass of \( \hat{g}_i \)) will consider two options: abstaining, which has a cost \( p(g_i) \), or participating in support of \( G_i \), which has a cost \( c - b(a_i h) \). Their participation rate, therefore, is endogenously determined as the unique group equilibrium of the regime.

\textsuperscript{5} For the most direct illustration, consider the simplest incentive scheme where \( w \) is paid to participants. Since only those agents with a cost \( c < w \) will choose to participate, this determines a turnout \( F(w) \). Hence to achieve a participation rate \( a \) incentives would have to be set at \( w(a) = F^{-1}(a) \). The necessary compensation is monotone increasing in the target participation rate: \( w'(a) = \frac{1}{f(F^{-1}(a))} > 0 \). Moreover, it is (weakly) convex if and only if the density of the cost distribution is (weakly) decreasing: \( w''(a) = -\frac{f'(F^{-1}(a))}{[f(F^{-1}(a))]^3} \geq 0 \) if and only if \( f'(c) \leq 0 \).
The probability that the challenger $G_1$ replaces the incumbent $G_0$ equals

$$\pi = Z \left( \frac{g_1 a (g_1, h)}{g_0 a (g_0, h)} \right).$$

We can prove the following:

**Proposition 1** Consider a contest between two given regimes $G_0$ and $G_1$. The probability that the more democratic regime ($G_1$ if and only if $g_1 > g_0$) succeeds is monotone increasing in the level of human capital $h$.

Proposition 1 is illustrated in Fig. 5. The mechanism underpinning this proposition is intuitive. Participation in support of a regime depends on two different kinds of incentives: those provided by the leadership, and those provided by other participants. Human capital makes the latter more effective, and therefore it is particularly important in eliciting support for groups that are primarily driven by peer persuasion rather than by top-down monitoring. Since the effectiveness of the latter is decreasing in group size, while the former is unaffected, human capital is a more important determinant of support for more democratic regimes.

### 4.3.1 The size and composition of most-threatening challenger regimes

We have shown that, among two rival regimes of given sizes, an increase in human capital makes it more likely that the more inclusive one prevails. In particular, democracy is more likely to be instituted and preserved. We next ask a complementary question: for a given level of human capital, what is the size and composition of a regime that poses the greatest threat to a particular incumbent? This analysis offers another perspective on regime stability.

Following Lemma 1, increasing the size of a group has two opposing effects on its support: broadening the base and decreasing the incentives for that base to participate. As human capital increases, incentives are less and less dependent on group size, so that we can prove the following:
Proposition 2 Consider an incumbent oligarchic regime \( G_0 \) of size \( g_0 \in (0, 1] \). The size \( g_1^* \in (0, 1] \) of the challenger regime most likely to overthrow \( G_0 \) is monotone (weakly) increasing in the level of human capital \( h \).

Proposition 2 is illustrated in Figs. 6 and 7. As human capital increases, the greatest threat to an incumbent regime becomes an ever more democratic regime. There can be a finite level of human capital \( \bar{h}(g_0) \) above which the most dangerous challenger becomes a perfect democracy, \( g_1^* = 1 \).

But what is the composition of the most threatening regime? For a fixed incumbent regime \( G_0 \), the support of a challenger \( G_1 \) depends on two factors: the size of its membership \( g_1 \) and the extent of the overlap of the membership of the competing regimes. Recalling that \( \gamma \) is the measure of overlap, the probability that the challenger \( G_1 \) replaces the incumbent \( G_0 \) equals

\[
\pi = Z \left( \frac{(g_1 - \frac{\gamma}{2}) a (g_1, h)}{(g_0 - \frac{\gamma}{2}) a (g_0, h)} \right)
\]

For a challenger, recruiting members from the incumbent regime rather than among those excluded from it has two opposing effects: it steals support from the incumbent, but it also introduces a wedge between the size of the challenging regime and its own actual basin of support. The resolution of this trade-off comes from a comparison of the sizes of the competing regimes. The smaller regime is more affected by the a priori loss of half of the agents with dual membership. Hence, a challenger regime that is more democratic than the incumbent is more likely to succeed when it includes all members of the incumbent itself. Conversely, a less democratic challenger is more likely to succeed when it includes as few members of the incumbent regime as possible (given its size). Formally, we can prove the following:

Corollary 1 Consider an incumbent oligarchic regime \( G_0 \). The composition of the most dangerous challenger can be characterized as follows:

1. if the most dangerous challenger is less democratic than the incumbent \( (g_1^* < g_0) \), it is minimally overlapping: the size of the group of citizens belonging to both regimes is \( \gamma = \max \{0, g_1^* - (1 - g_0)\} \);
According to Proposition 2, at low levels of education, status quo dictatorships are most effectively challenged by small coups. Indeed, some historical and statistical evidence suggests that challengers to dictatorships in such countries are often bands of disgruntled opponents (Campante and Do, 2005; Finer, 1988; Huntington, 1957). At higher levels of education, the sizes of optimal uprisings against both dictatorship and democracy rise. In Europe during the age of Revolutions, increasingly large groups fought to overthrow the existing regime. Similarly, revolts against democracy, such as the Fascist takeover in Italy in the 1920s or the Nazi movement in Germany, became increasingly broad-based in societies with more education.

The Corollary further tells us that, as human capital increases, not only the size but the nature of the most dangerous challenger changes. When \( h \) is low, an incumbent dictatorship is most likely to be replaced by another small dictatorship that comprises a completely different set of agents: the threat comes not from a subset of the current elite trying to exclude other insiders, but from current outsiders trying to oust them. When \( h \) is high, the most effective challenger is instead a (relatively) democratic regime that does not attempt to remove any of the current insiders, but simply to add more members to the regime. In the limit, as human capital rises, the greatest threat to dictatorship comes from a full democracy, which by definition includes the whole population.

An intermediate case is present when the incumbent regime is large, \( g_0 \in (1/2, 1] \). In this case, the maximum probability of success may come from a challenger that includes all the current outsiders but also a subgroup of current insiders. Needless to say, this case is the only possible one when the incumbent regime is a perfect democracy: then any challenger can include only members of the current regime. For sufficiently high levels of human capital, the highest probability of success is associated with democratic turnover. In other words, both the challenger and the incumbent are perfect democracies, and citizens freely choose to affiliate with either group. Our particular specification, then, delivers the outcome of competition among regimes perfectly committed to democracy at high enough levels of human capital.
4.3.2 Implications

The model provides one explanation for the empirical link between education and democracy, but also a further set of testable implications. It also suggests that, at low levels of education, the greatest threat to narrow regimes comes from other narrow regimes, such as small uprisings and coups. In contrast, at higher levels of education, the greatest threat to narrow regimes comes from broader regimes, and democratic governments fall to other democratic governments. Some anecdotal evidence seems consistent with these implications.

Before 1600, European monarchies were relatively secure against broad-based uprising. Sporadic peasant revolts were generally crushed, and the real threat to a monarch usually came from a close relative offering an alternative monarchy. Starting with the Dutch revolution against Spain and the English revolution, monarchs were increasingly threatened by more broadly based opposition groups. Notably, these two nations had unusually high levels of education. In the late 18th and 19th centuries, France, Germany, and Italy all experienced popular uprisings as well, with large roles played by the more educated citizens. By 1920, monarchies had almost vanished in Europe, as the incumbency advantages of monarchs were wiped out in the aftermath of World War I. But it took another 25 years in Western Europe and another 70 years in Eastern Europe for democracy to become entrenched.

We can also look at history from the perspective of political stability. England was unstable until 1689, with often violent royal successions (De Long and Shleifer, 1993). Four French regimes were toppled by popular revolts between 1789 and 1871, as the levels of education were growing. Similar instability was seen in Germany and elsewhere in the educated Europe. Once the transitions to democracy occurred, well educated democracies were themselves initially unstable. At least at first glance, one sees similar patterns of instability of democracy in Latin America, the former Soviet Union, and other parts of the world.

Education also seems linked to the ability of democracies to defend themselves from dictatorial coups. Popular uprisings during the less educated periods (England 1640, France 1789, Russia 1917, and Germany 1920) were almost invariably followed by dictatorial takeovers. The educated supporters of democracy tried but failed to resist the dictators. As nations became more educated, they also became more successful at defending democracy. In France in the 1870s and 1880s, the Third Republic faced risks from both the monarchists and a popular general such as Boulanger. However, unlike the coups of 1797 and 1851, when the two Napoleons crushed their less organized opponents, in this later period a coalition of educated democrats protected the republic. Such a defense of democracy was even more striking in 1990, when educated Russians engaged in a remarkable campaign to stop an attempted putsch.

In less educated times and places, coups are generally small affairs including only small cadres of nobles or army officers (Campante and Do, 2005). As education grows, effective uprisings (like the American Revolution) became larger. Eventually, large swaths of society are included in attempts to overthrow a regime. Even the Nazi takeover in Germany, which eventually led to a dictatorship, succeeded only after the Nazis had built a broad coalition, including students and other educated Germans. Their earlier attempt at a narrow coup proved an embarrassing failure.

5 Conclusion

The correlation between education and democracy is clear. The reason for this correlation is not. In this paper, we offer one explanation for the correlation.
Our explanation hinges on the connection between education and the costs and benefits of political engagement. Schools socialize young people and political involvement is one form of socialization; a variety of evidence shows a positive connection between education and civic engagement. We model education as raising the benefits of political action when individuals choose to support a more or less democratic regime. In this model, democratic regimes offer weak incentives to a wide base of potential supporters, while dictatorships offer strong incentives to a narrower base. Education increases the society-wide support for democracy because democracy relies on people with high participation benefits for its support. We show that better educated nations are more likely both to preserve democracy and to protect it from coups.

The analysis raises three broader questions that are worth mentioning in conclusion. First, while our model has focused on the effects of education on participation, the analysis applies to all social glue that encourages collective action. For example, the analysis might suggest that ethnic homogeneity is good for democracy (although in cross-country or panel regressions with education controls, ethnic heterogeneity does not matter for democracy). This surely seems plausible, and remains to be explored. Perhaps as interestingly, the analysis suggests a solution of Olson’s free-rider problem in all organizations, and not just in political regimes, namely human capital or other social glue as a motivation to participate.

Second, our analysis runs into the old puzzle, namely why do some dictators invest in education if they know that doing so eventually dooms them? The examples of fallen dictators in East Asia, Eastern Europe, and the former Soviet Union are telling reminders of this risk. There are several possible answers. One is that many dictators face an external threat, and therefore must grow their economies and their armies (including investing in human capital) to counter these threats even if this raises the risk of democratization. A second is that even selfish dictators unconcerned with external threats might derive income from economic growth, and therefore promote education to get richer. A third idea is that all dictators face significant ouster risks, and that it is much better for the dictator’s life for him to be replaced by a democracy in an educated country than by another dictator in an uneducated one. A symmetrical point, of course, is that democracies have an extra incentive to invest in education in order to secure and stabilize themselves against oligarchic coups.

Third, our analysis deals with the size of the most productive coalitions in politics, but Lemma 1 applies to organizations and groups more broadly. The results suggest, then, that higher human capital societies can more efficiently maintain larger organizations in which both top-down and peer-provided incentives play a role. Whether this prediction is true about firms and other institutions and groups remains a question for further research.

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Appendix: Proofs

A. 1 Proof of Lemma 1

For ease of notation, we omit the subscript $i$ denoting group-specific parameters in the analysis of group equilibrium. A group equilibrium is a root of the auxiliary function

$$Q(a; g, h) = F(p(g) + b(ah)) - a$$
which is twice continuously differentiable with respect to $a$, because so are $b$ and $F$. Omitting arguments for the sake of brevity, the first derivative is

$$Q_a = hf b' - 1$$

and the second is

$$Q_{aa} = h^2 \left[ f' (b')^2 + fb'' \right] < 0$$

whose sign follows from Assumption 2.

Assumption 1 implies furthermore that

\begin{align*}
Q (0; g, h) &= F (p (g)) > 0 \\
Q (1; g, h) &= F (p (g) + b (h)) - 1 < 0
\end{align*}

and therefore by continuity there exists at least one root $\hat{a} \in (0, 1)$.

Moreover, $Q (0; g, h) > 0$ implies that at the first root $Q_a (\hat{a}; g, h) < 0$. Concavity then implies $Q_a (a; g, h) < 0 \forall a \geq \hat{a}$, which implies that the root $\hat{a}$ is unique. The condition

$$Q (\hat{a}; g, h) = 0 \Rightarrow Q_a (\hat{a}; g, h) < 0$$

can also be interpreted as showing the stability of the group equilibrium.

By the implicit-function theorem, equilibrium participation is a differentiable function $a (g, h)$ such that $Q (a (g, h); g, h) = 0$. Since

\begin{align*}
Q_g &= fp' < 0 \\
Q_h &= af b' > 0
\end{align*}

its gradient is

\begin{align*}
\frac{\partial a}{\partial g} &= - \frac{Q_g}{Q_a} = \frac{fp'}{1 - hf b'} < 0 \\
\frac{\partial a}{\partial h} &= - \frac{Q_h}{Q_a} = \frac{af b'}{1 - hf b'} > 0
\end{align*}

recalling that $Q_a < 0$ in equilibrium.

A. 2 Proof of Proposition 1

The implicit-function theorem also allows us to compute higher-order derivatives, and among these

$$\frac{\partial^2 a}{\partial g \partial h} = - \frac{Q_a^2 Q_{gh} - Q_h Q_a Q_{ga} - Q_g Q_a Q_{ha} + Q_g Q_h Q_{aa}}{Q_a^3}$$

where

\begin{align*}
Q_{gh} &= af' p' b' \\
Q_{ga} &= hf' p' b' \\
Q_{ha} &= fb' + ahf' (b')^2 + ahf b''
\end{align*}
Therefore,
\[
\frac{\partial^2 \log a}{\partial g \partial h} = \frac{1}{a^2} \left( a \frac{\partial^2 a}{\partial g \partial h} - \frac{\partial a}{\partial g} \frac{\partial a}{\partial h} \right)
\]
\[
= -\frac{1}{a^2 Q_a^3} \left[ a (Q_a^2 Q_{gh} - Q_h Q_a Q_{sa}) - Q_s Q_a Q_{ha} + Q_s Q_h Q_a \right]
\]
\[
= -\frac{1}{a^2 Q_a^3} \left\{ a Q_a (Q_a Q_{gh} - Q_h Q_{sa}) + Q_g [a (Q_h Q_{aa} - Q_a Q_{ha}) + Q_h Q_a] \right\}
\]
\[
= \frac{p' \left( f' b' + h f^2 b'' \right)}{(1 - h f b')^3} > 0
\]

Let \( a_i = a (g_i, h) \): the probability of victory for regime \( G_1 \) over regime \( G_0 \) is
\[
\pi = Z \left( \frac{\tilde{g}_i a_1}{\tilde{g}_0 a_0} \right) = Z \left( \exp \left\{ \log \tilde{g}_1 - \log \tilde{g}_0 + \log a_1 - \log a_0 \right\} \right)
\]
so that
\[
\frac{\partial \pi}{\partial h} = z \left( \frac{\tilde{g}_i a_1}{\tilde{g}_0 a_0} \right) e^{\log \tilde{g}_1 - \log \tilde{g}_0 + \log a_1 - \log a_0} \left( \frac{\partial \log a_1}{\partial h} - \frac{\partial \log a_0}{\partial h} \right)
\]
and thus
\[
\frac{d\pi}{dh} > 0 \iff g_1 > g_0
\]

A. 3 Proof of Proposition 2 and Corollary 1

Recall that the probability of success of a challenger is
\[
\pi = Z \left( \frac{g_1 - \gamma}{g_0 - \gamma} a (g_1, h) \right)
\]
so that the challenger that is most likely to succeed is the maximizer of
\[
M (g_1, \gamma; g_0, h) \equiv \log a (g_1, h) + \log \left( g_1 - \frac{\gamma}{2} \right) - \log \left( g_0 - \frac{\gamma}{2} \right)
\]
subject to
\[
g_1 \in [0, 1] \text{ and } \gamma \in \left[ \max \{0, g_1 - (1 - g_0)\}, \min \{g_0, g_1\} \right]
\]
To begin with, since
\[
\frac{\partial M}{\partial \gamma} \geq 0 \iff g_1 \geq g_0
\]
the optimal value of \( \gamma \) is indefinite if \( g_1 = g_0 \) and it lies in a corner if \( g_1 \neq g_0 \).
Hence the optimal regime size is
\[
g_1^* (g_0, h) = \arg \max_{g_1 \in [0, 1]} \{ \log a (g_1, h) + \lambda (g_1, g_0) \}
\]
where \( \lambda (g_1, g_0) \) is defined by the jointly optimal choice of overlap \( \gamma \).
There are two different cases based on the size of the incumbent $G_0$: small $g_0 \in \left(0, \frac{1}{2}\right]$ or large $g_0 \in \left[\frac{1}{2}, 1\right]$. The first will only have two potential corner solutions for optimal overlap: $\gamma = 0$ and $\gamma = g_0$; the latter will also have the possibility of $\gamma = g_1 - (1 - g_0) > 0$.

1. If $g_0 \in \left(0, \frac{1}{2}\right]$ the optimal overlap is

\[
\gamma (g_1) = \begin{cases} 
0 & g_1 \in [0, g_0) \\
g_1 & g_1 = g_0 \\
g_0 & g_1 \in (g_0, 1]
\end{cases}
\]

and therefore

\[
\lambda (g_1, g_0) = \begin{cases} 
\log g_1 - \log g_0 & g_1 \in [0, g_0) \\
\log \left(g_1 - \frac{g_0}{2}\right) - \log \frac{g_0}{2} & g_1 \in [g_0, 1]
\end{cases}
\]

a continuous function that is monotone increasing in $g_1$ and piecewise concave in $g_1$ for $g_1 \in [0, g_0]$ and $g_1 \in [g_0, 1]$, but with a convex kink at $g_1 = g_0$.

2. If $g_0 \in \left(\frac{1}{2}, 1\right]$ the optimal overlap is

\[
\gamma (g_1) = \begin{cases} 
0 & g_1 \in [0, 1 - g_0) \\
1 - g_0 & g_1 \in [1 - g_0, g_0) \\
2g_0 - 1 & g_1 = g_0 \\
g_0 & g_1 \in (g_0, 1]
\end{cases}
\]

and therefore

\[
\lambda (g_1, g_0) = \begin{cases} 
\log g_1 - \log g_0 & g_1 \in [0, 1 - g_0) \\
\log \left(g_1 - \frac{g_0}{2}\right) - \log \frac{g_0}{2} & g_1 \in [1 - g_0, g_0) \\
\log \left(g_1 - \frac{g_0}{2}\right) - \log \frac{g_0}{2} & g_1 \in [g_0, 1]
\end{cases}
\]

a continuous function that is monotone increasing in $g_1$ and piecewise concave in $g_1$ for $g_1 \in [0, g_0]$ and $g_1 \in [g_0, 1]$, but with a convex kink at $g_1 = g_0$ and a concave kink at $g_1 = 1 - g_0$.

Given any $g_0 \in (0, 1]$, $g_0^* (g_0, h) \in (0, 1]$ is well-defined as the maximand of a continuous function on a compact. Consider two levels of human capital $h_L < h_H$. Suppose that

\[
g_L^* \equiv g_1^* (g_0, h_L) > g_1^* (g_0, h_H) \equiv g_H^*
\]

This implies by definition that

\[
\begin{align*}
\log a (g_L^*, h_L) + \lambda (g_L^*, g_0) & \geq \log a (g_H^*, h_L) + \lambda (g_H^*, g_0) \\
\log a (g_H^*, h_H) + \lambda (g_H^*, g_0) & \geq \log a (g_L^*, h_H) + \lambda (g_L^*, g_0)
\end{align*}
\]

and therefore rearranging

\[
\log a (g_L^*, h_L) - \log a (g_H^*, h_L) \geq \lambda (g_H^*, g_0) - \lambda (g_L^*, g_0) \geq \log a (g_H^*, h_H) - \log a (g_L^*, h_H)
\]

and finally

\[
\log a (g_H^*, h_H) - \log a (g_L^*, h_L) \geq \log a (g_H^*, h_H) - \log a (g_L^*, h_L)
\]

But in the proof of Proposition 1 we established that

\[
\frac{\partial^2 \log a}{\partial g \partial h} > 0
\]

which proves by contradiction that

\[
h_L < h_H \Rightarrow g_1^* (g_0, h_L) \leq g_1^* (g_0, h_H)
\]
References


