

Increasing Returns to Scale *Without* Sorting or Agglomeration Economies

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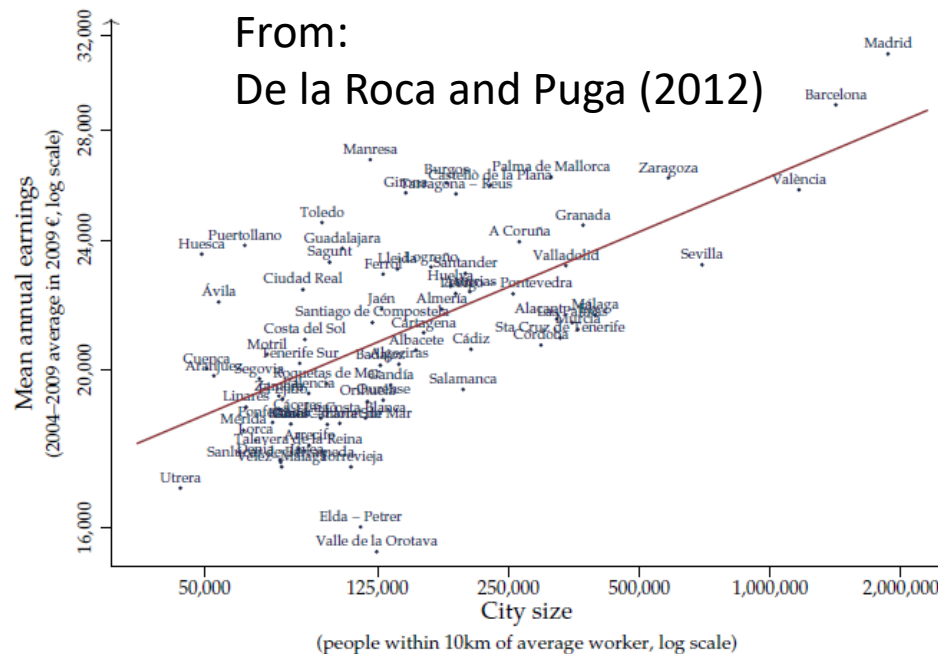
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CONTEXT & RESEARCH STUDY

A well-established fact:

Urban *Increasing Returns to Scale* (IRS)
or
the “city size premium”

- i.e., larger cities offer higher productive advantages than smaller cities
 - Rosenthal-Strange (2004), Duranton-Puga (2004), Henderson (2003), Combes-Duranton-Gobillon-Puga-Roux (2012), Combes-Duranton-Gobillon (2008), Combes-Duranton-Gobillon-Roux (2010), Combes-Duranton-Gobillon-Puga-Roux (2012), Glaeser-Maré (2001), Gould (2007), Baum-Snow-Pavan (2012), Roca-Puga (2012), ...



$$\delta = \frac{\Delta y / y}{\Delta n / n} \approx 0.05$$

Figure 1: Mean earnings and city size

Interpreting Increasing Returns to Scale (IRS)

- Total production:

$$F(\lambda n) > \lambda F(n)$$

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- Several efforts to answer:

- Why are individuals more productive (or earn higher wages) *on average* in larger cities?



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IRS *means* more productive individuals

- Several efforts to answer:
 - Why are individuals more productive (or earn higher wages) *on average* in larger cities?
- Two **general** mechanisms for the *city size premium*:
 - **Sorting** of inherently productive individuals
 - Local (static or dynamic) positive externalities, e.g., **agglomeration economies** which make individuals more productive

(Lots of papers and models)

Empirical Challenges

Handbook of Regional and Urban Economics, Volume 5A
ISSN 1574-0080, <http://dx.doi.org/10.1016/B978-0-444-59517-1.00005-2>

CHAPTER 5

The Empirics of Agglomeration Economies

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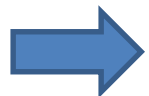
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We claim there is an additional challenge

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- Our contribution:
 - A 3rd mechanism other than sorting or local effects that generates IRS
 - Methodological suggestion to reveal this effect in the estimation of the elasticity.

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When

 - **X**, such as wages, productivity, etc., is “unevenly distributed” (i.e., high inequality), and
 - the sizes of cities are not “large enough”,

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- X , such as wages, productivity, etc., is “unevenly distributed” (i.e., high inequality), and
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Then

- The Law of Large Numbers fails, and
- The aggregate $Y = \text{sum}(X)$ per city displays IRS.

The take-home message of the research

- Wages *are* unequal → change the null model:
 - The expectation from empirical exercises *should not* be the **absence of IRS**.
- The convergence of the law of large numbers must be taken into account when studying IRS.

STATEMENT OF THE PROBLEM

A very simple model

Assumption 1.0: Let a city be defined as the collection of n individuals, $i = 1, \dots, n$. We ignore physical proximity.

Assumption 2.1: Let each citizen i in the city be defined by a large set of innate, not directly observable, characteristics, $\xi_1^{(i)}, \dots, \xi_S^{(i)}$, where $S \gg 1$, and $\xi_s^{(i)}$ are independent and identically distributed (i.i.d.) positive random variables with finite mean and variance, for all $i = 1, \dots, n$ and $s = 1, \dots, S$. The i.i.d. assumption here removes the possibility of any interaction or correlation between individuals.

Assumption 2.2: Let the output of individual i be $X_i = \prod_{s=1}^S \xi_s^{(i)}$. Because of Assumption 2.1, X_i are i.i.d. random variables.

Assumption 3.0: Let the total output of the city be $Y(n) = \sum_{i=1}^n X_i$. Hence, the output of each city is the sum of heterogeneous independent individual contributions.

In few words: In a city with individuals $i=1, \dots, n$, wages are **i.i.d.** lognormal

$$X_i \sim \mathcal{LN}(\ln(x_0), \sigma^2)$$

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Two important characteristics of the lognormal distribution:

(i) Heavy-tailed, but (ii) all moments finite:

$$p_X(x; x_0, \sigma^2) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln x - \ln x_0)^2}{2\sigma^2}}$$

$$E[X] = x_0 e^{\sigma^2/2}$$

$$\text{Var}[X] = (e^{\sigma^2} - 1)x_0^2 e^{\sigma^2}$$

$$E[X^k] = x_0^k e^{k^2 \sigma^2/2}$$

(trivial) analytic results #1

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$\lambda > 1$:

$$\begin{aligned} \mathbb{E}[Y(\lambda n)] &= \mathbb{E}\left[\sum_{i=1}^{\lambda n} X_i\right], \\ &= \sum_{i=1}^{\lambda n} \mathbb{E}[X_i], \\ &= \lambda n \mathbb{E}[X_1], \\ &= \lambda \mathbb{E}[Y(n)]. \end{aligned}$$

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$$\begin{aligned} \mathbb{E}[Y(n)] &= Y_0 n^\beta \\ \text{with } \beta &= 1 \text{ and } Y_0 = \mu. \end{aligned}$$

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$$\mathbb{E}[Y(n)] = Y_0 n^\beta$$

with $\beta = 1$ and $Y_0 = \mu$.

No IRS

In this model, doubling the size of the city ($\lambda = 2$), doubles total expected wages.

In other words, doubling the size of the city does nothing to the expected individual wages.

(not-so-trivial) analytic results #2

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 2. Recognize that in $Y(n) = X_1 + X_2 + \dots + X_i + \dots + X_n$ not all terms contribute “equally” to the sum, but rather the sum is “dominated” by a few, **very large**, terms (i.e., the wealthiest individuals).

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 3. Suppose, as a 1st approximation, that

$$Y(n) \approx M(n),$$

$$\text{where } M(n) = \max\{X_1, \dots, X_n\}$$

(not-so-trivial) analytic results #2

- Let us define the total wages: $Y(n) = \sum_{i=1}^n X_i$
 - Does the $\mathbf{M}(n)$ display IRS?
4. According to the Fisher-Tippett theorem, $\mathbf{M}(n)$ has a distribution that converges to a Gumbel under proper normalization

Analogous to
 $(\sqrt{n}\sigma)^{-1} (Y(n) - \mu n)$
for the CLT

$$c_n^{-1} (M_{\mathcal{LN}}(n) - d_n)$$

5. The term d_n reveals how $\mathbf{M}(n)$ scales with n . Specifically for a LN:

$$d_n = \mathbb{E}[X_1] \exp \left\{ -\frac{\sigma^2}{2} + \sigma \left(\sqrt{2}(\ln(n))^{1/2} - \frac{\ln(4\pi) + \ln(\ln(n))}{\sqrt{8}(\ln(n))^{1/2}} \right) \right\}$$

(not-so-trivial) analytic results #2

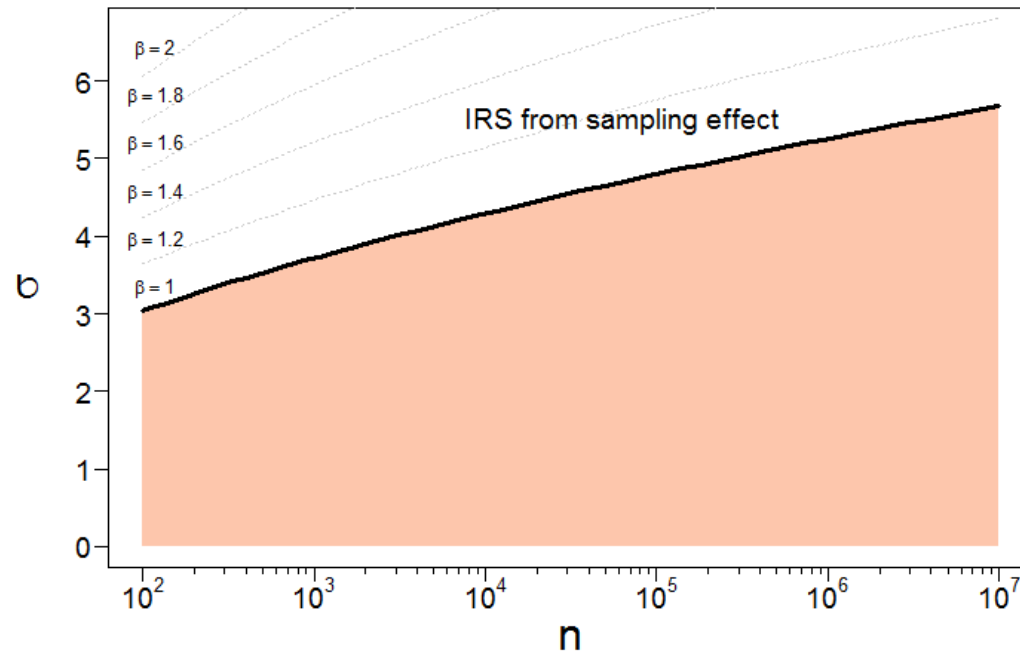
$$\beta = \frac{\partial \ln(Y(n))}{\partial \ln(n)} \approx \frac{\partial \ln(d_n)}{\partial \ln(n)}$$

➡ $\beta(n) \approx \frac{\sigma}{\sqrt{2 \ln(n)}}$ for large σ

SOME INTUITIONS

$$\beta(n) \approx \frac{\sigma}{\sqrt{2 \ln(n)}}$$

**Balance between ‘variance’
and population size**

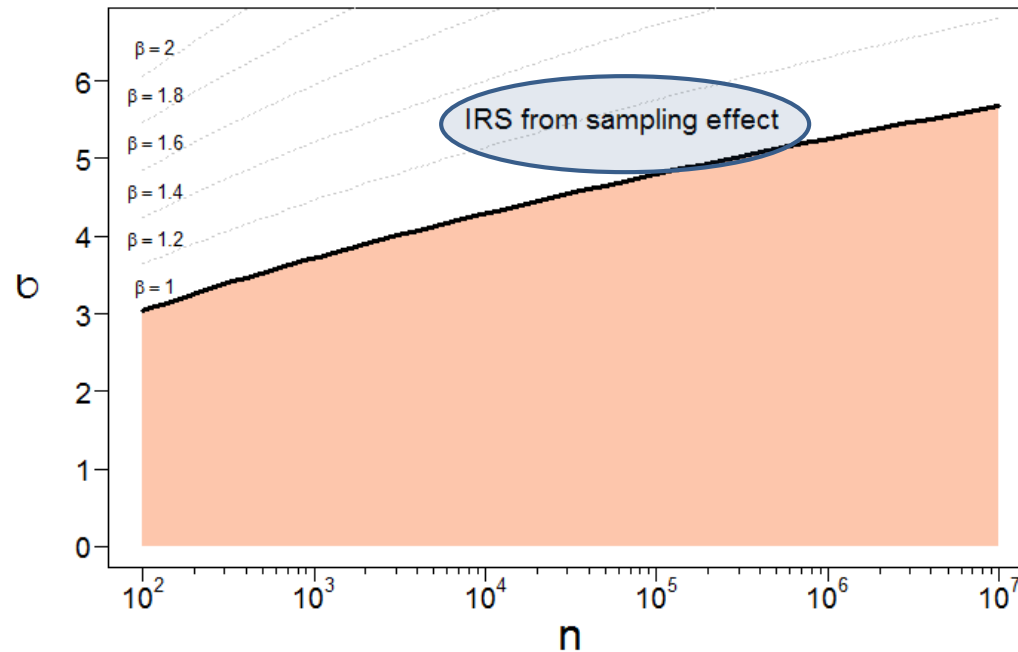


For example:

The U.S. CBSA (micro+metros) have sizes between 10^4 and 10^7 . Hence, we expect a simulation of our model with those city sizes to display IRS for sigmas in the order of 5.

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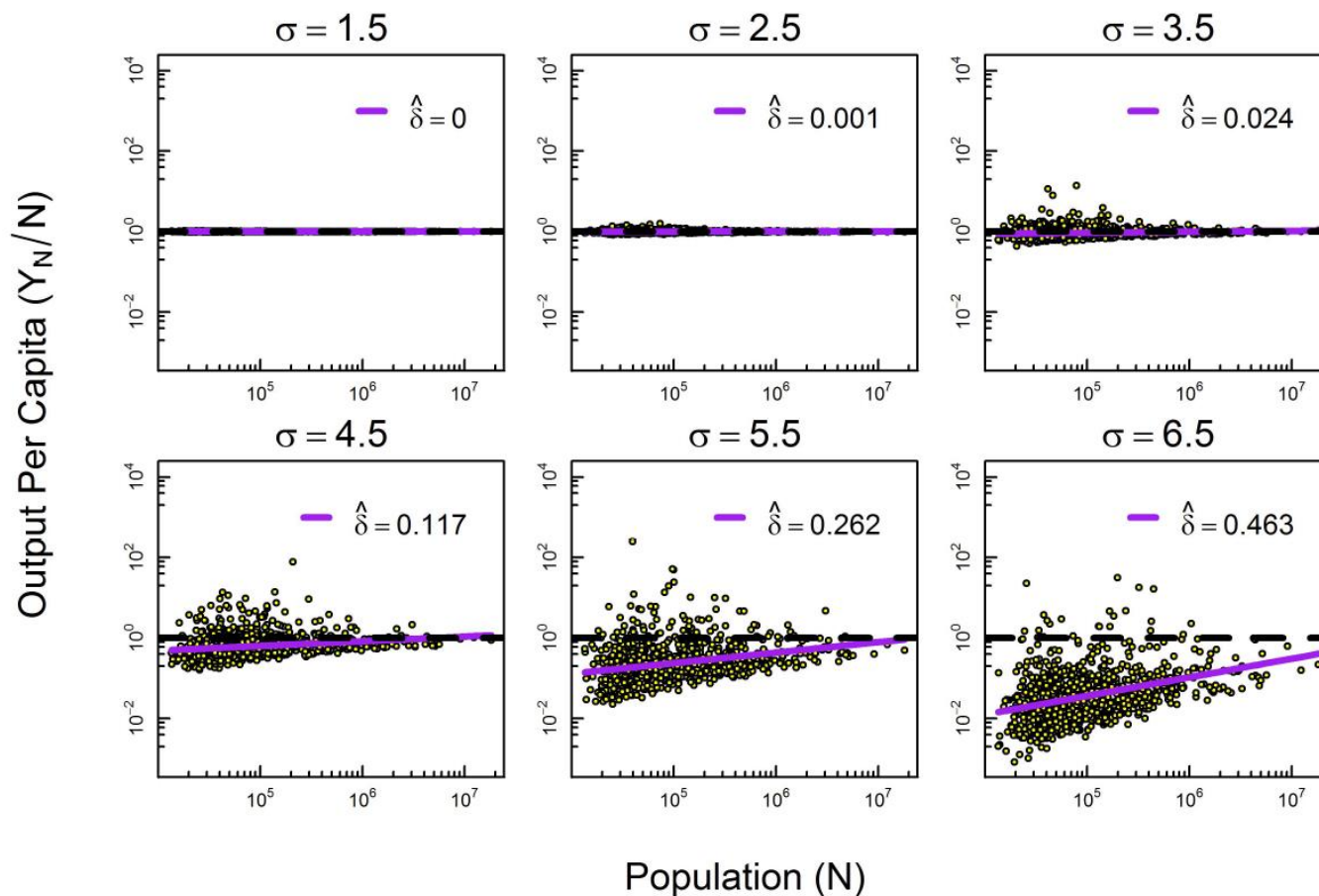
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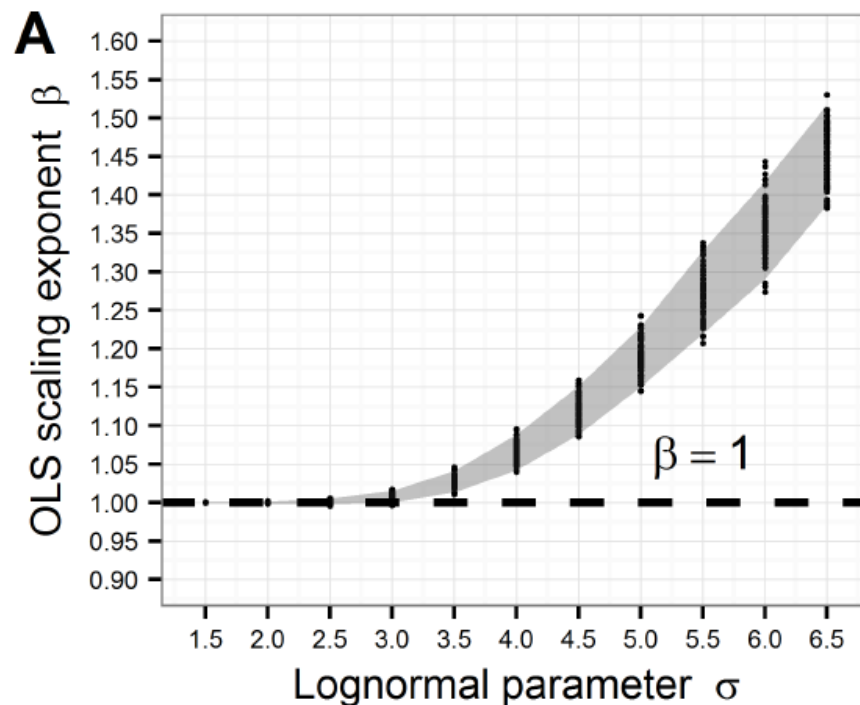
RESULTS

Simulating our model

$$\delta = \beta - 1$$



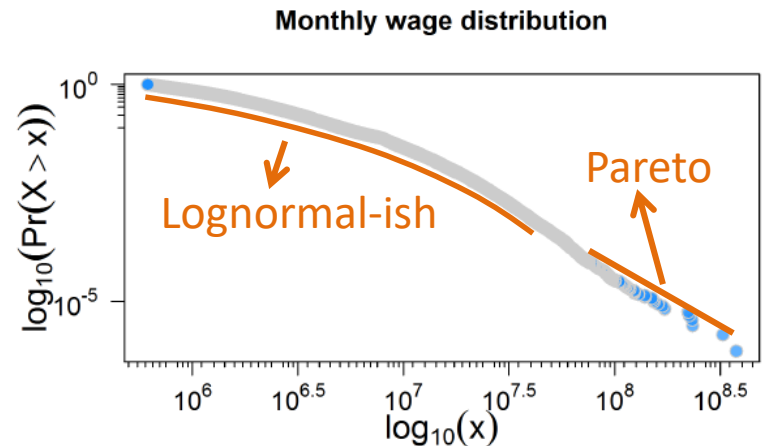
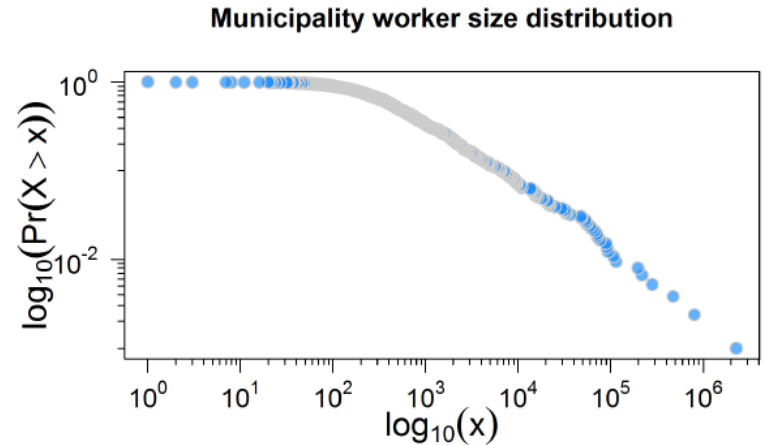
The larger sigma (i.e., variance), the larger the average “city size premium”



$$\beta(n) \approx \frac{\sigma}{\sqrt{2 \ln(n)}}$$

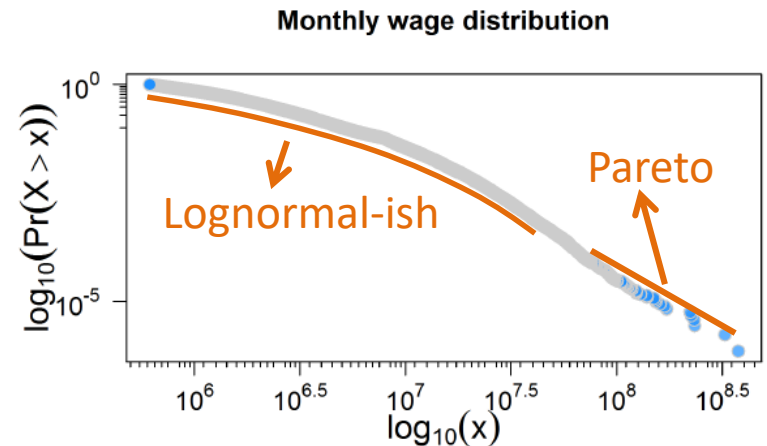
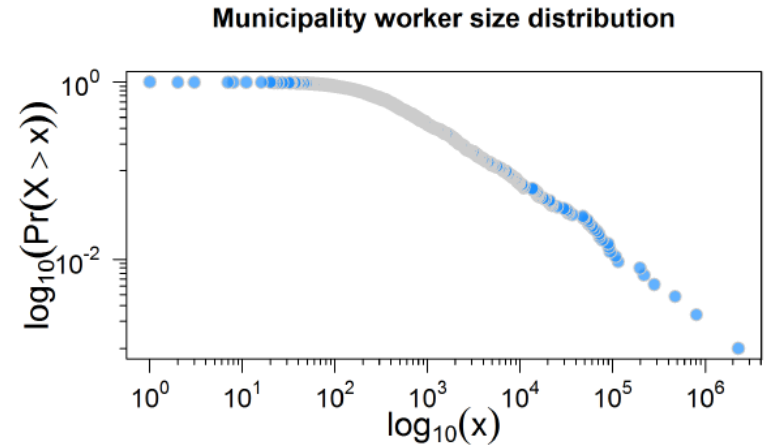
Empirical data

- Colombian Social Security 2014 Dataset
- 122,287,562 total observations (“contributions”)
- 10,535,587 unique contributors
- **6,792,183 workers** (employed or self-employed) that have worked at least one full month and have thus earned at least a full minimum wage during the year.
- **1,127 municipalities**



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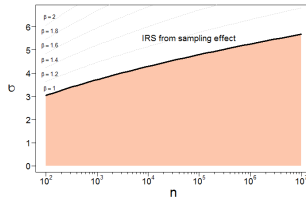
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Testing for IRS as a statistical artifact

$$\beta = \beta^{(\text{sorting})} + \beta^{(\text{aggl. ec.})} + \beta^{(\text{stat. artifact})}$$

- The component in the elasticity to city size coming from the statistical effect should be ***invariant to randomization*** of people across cities.
1. **Randomize** individuals across municipalities.
 2. Re-do the regressions.



Balance between sigma and population size in real data

Table 1: Estimation of parameter σ of average monthly wages per city

Statistic	N	Mean	St. Dev.	Min	Max
$\log_{10}(\text{Worker pop. size})$	555	3.113	0.625	2.464	6.356
real locations σ	555	0.411	0.079	0.222	0.823
randomized locations σ	555	0.612	0.029	0.505	0.746

$$\hat{\sigma} = \sqrt{\frac{1}{n} \sum_{i=1}^n \left(\ln(X_i) - \overline{\ln(X_i)} \right)^2}$$

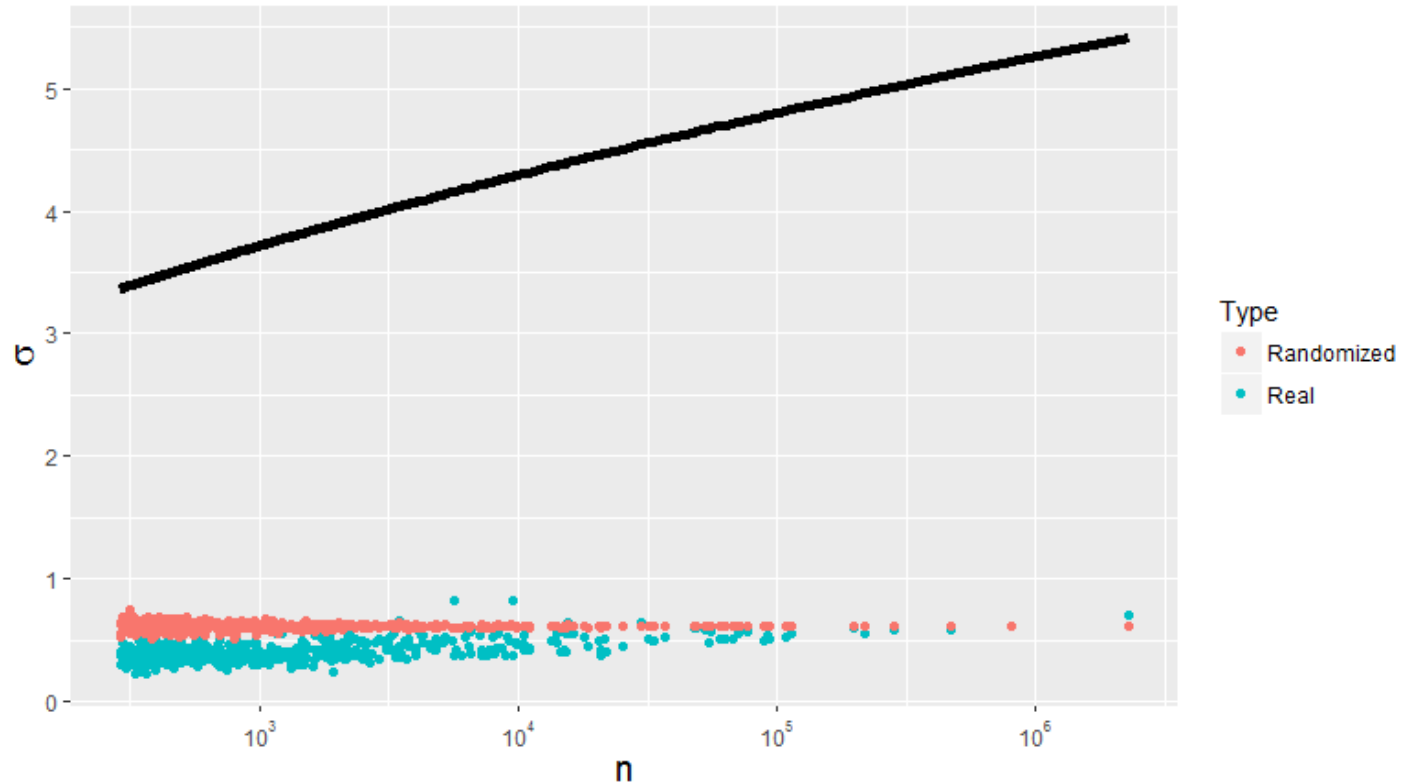


Table 2: Results

	<i>Dependent variable:</i>		
	log(Average monthly wage)		
	Real	Randomized	Randomized ($n \leq 500$)
log(Worker population size)	0.060*** (0.004)	0.001 (0.002)	0.031 (0.035)
Constant	13.315*** (0.029)	14.075*** (0.011)	13.898*** (0.206)
Observations	555	555	172
R ²	0.293	0.001	0.005
Adjusted R ²	0.292	-0.001	-0.001
Residual Std. Error	0.134 (df = 553)	0.053 (df = 553)	0.074 (df = 170)
F Statistic	229.668*** (df = 1; 553)	0.681 (df = 1; 553)	0.792 (df = 1; 170)

Note:

*p<0.1; **p<0.05; ***p<0.01



The effect seems to be negligible in Colombian wages.



CONCLUSIONS

- There is a statistical effect which may (or may not) inflate the city size premium
- We should adjust for that possibility

THANK YOU

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