Invention as cultural accumulation
Evidence from patenting

Joint project with:

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Santa Fe Institute Workshop

"The Complexity of the Patenting System"

Santa Fe, New Mexico

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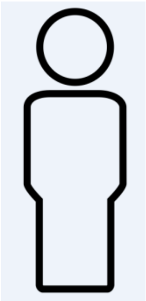
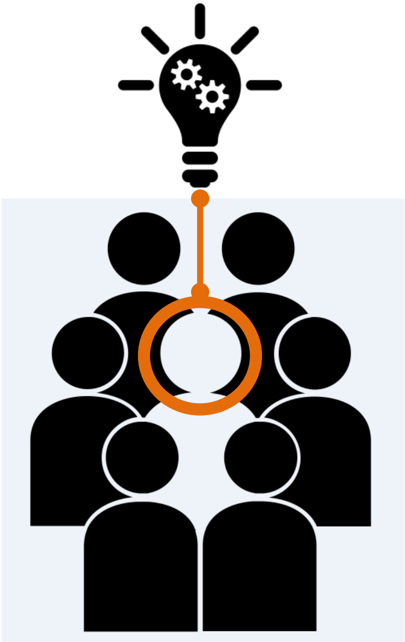
MOTIVATION

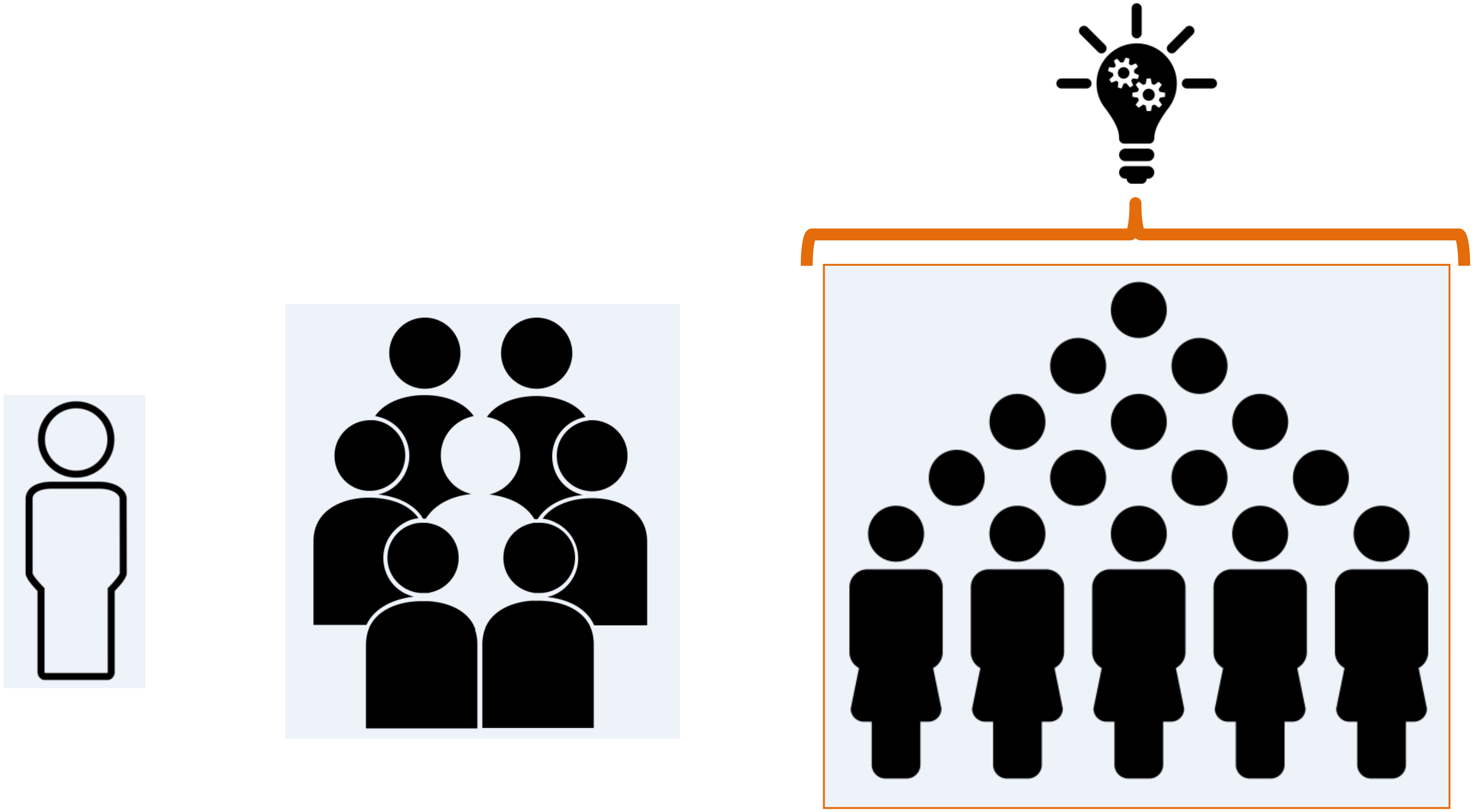
Individual VS. Collective

Individual VS. Collective





Individual VS. Collective



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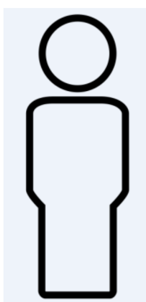
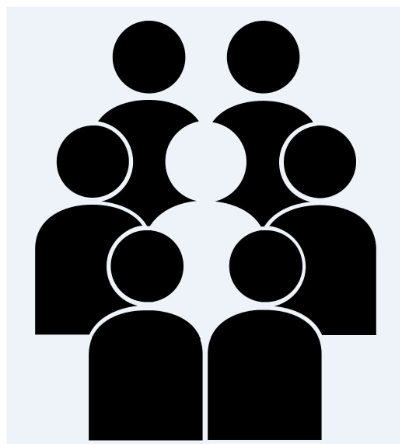
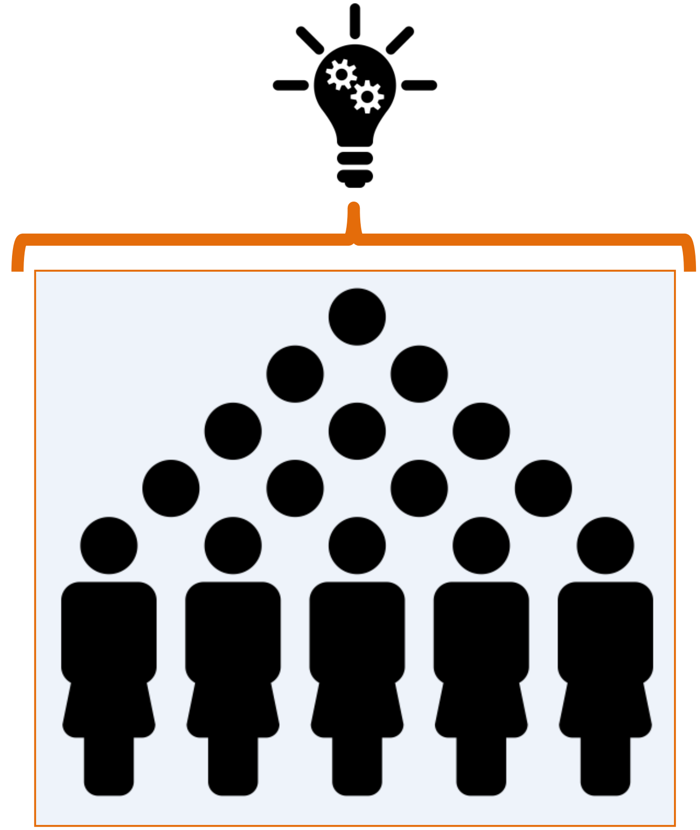


Individual learning

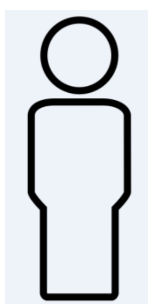


Social learning

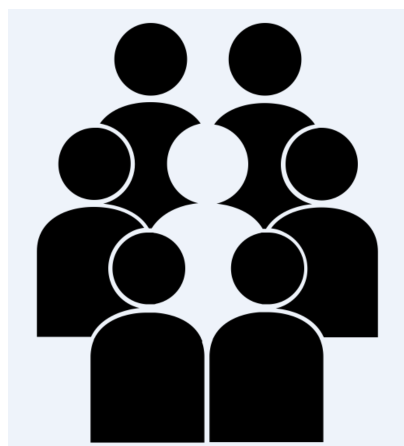


Collective learning

Individual VS. Collective



Individual learning



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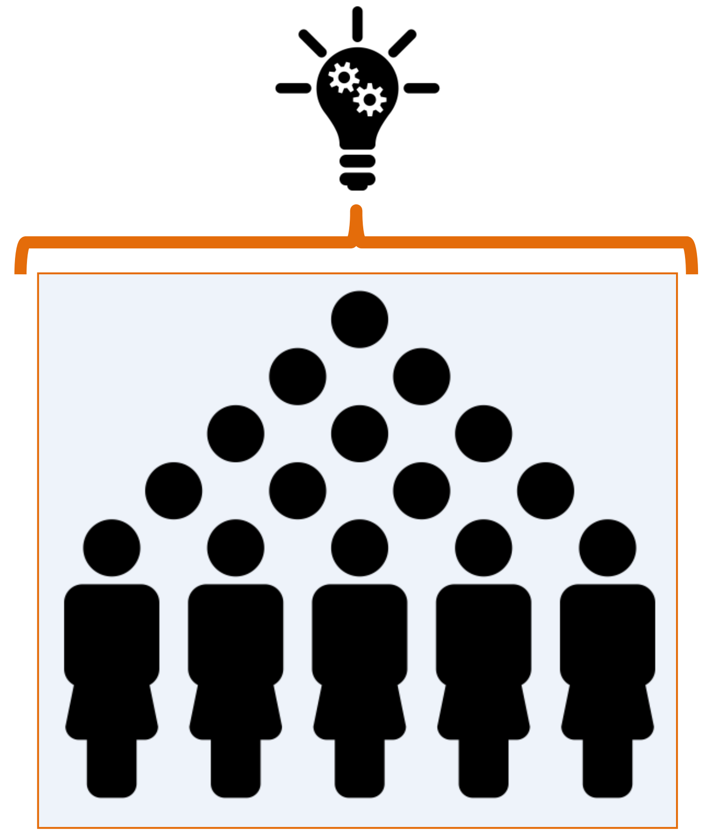
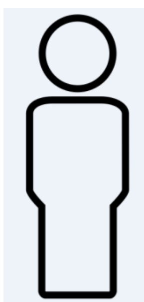
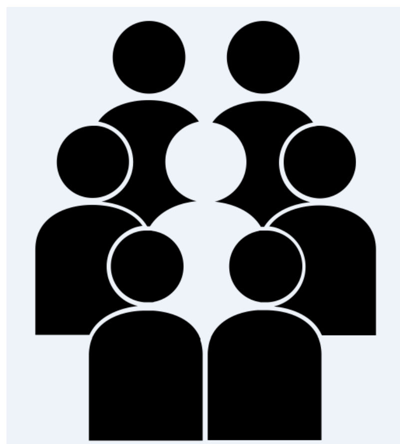


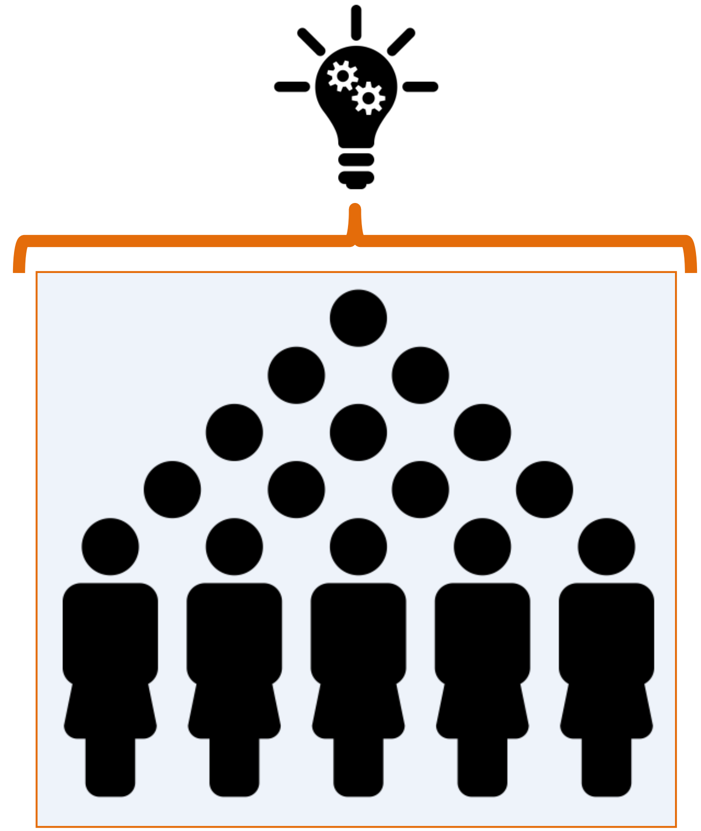
Collective learning

“ Standing on the shoulders of “midgets” ”
(Robert Boyd)

Individual VS. Collective







Individual learning

Social learning

- **Horizonal** transmission of information
- Spillovers

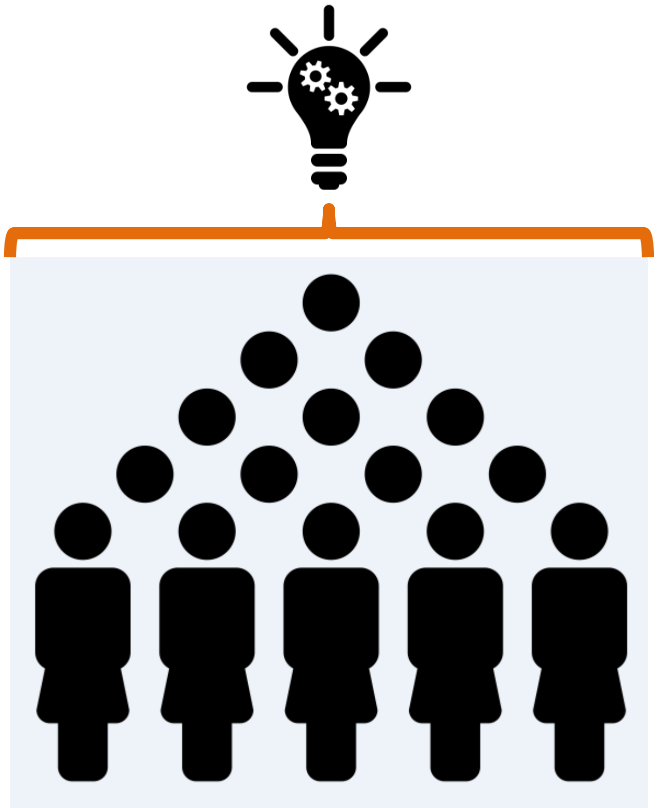
Collective learning

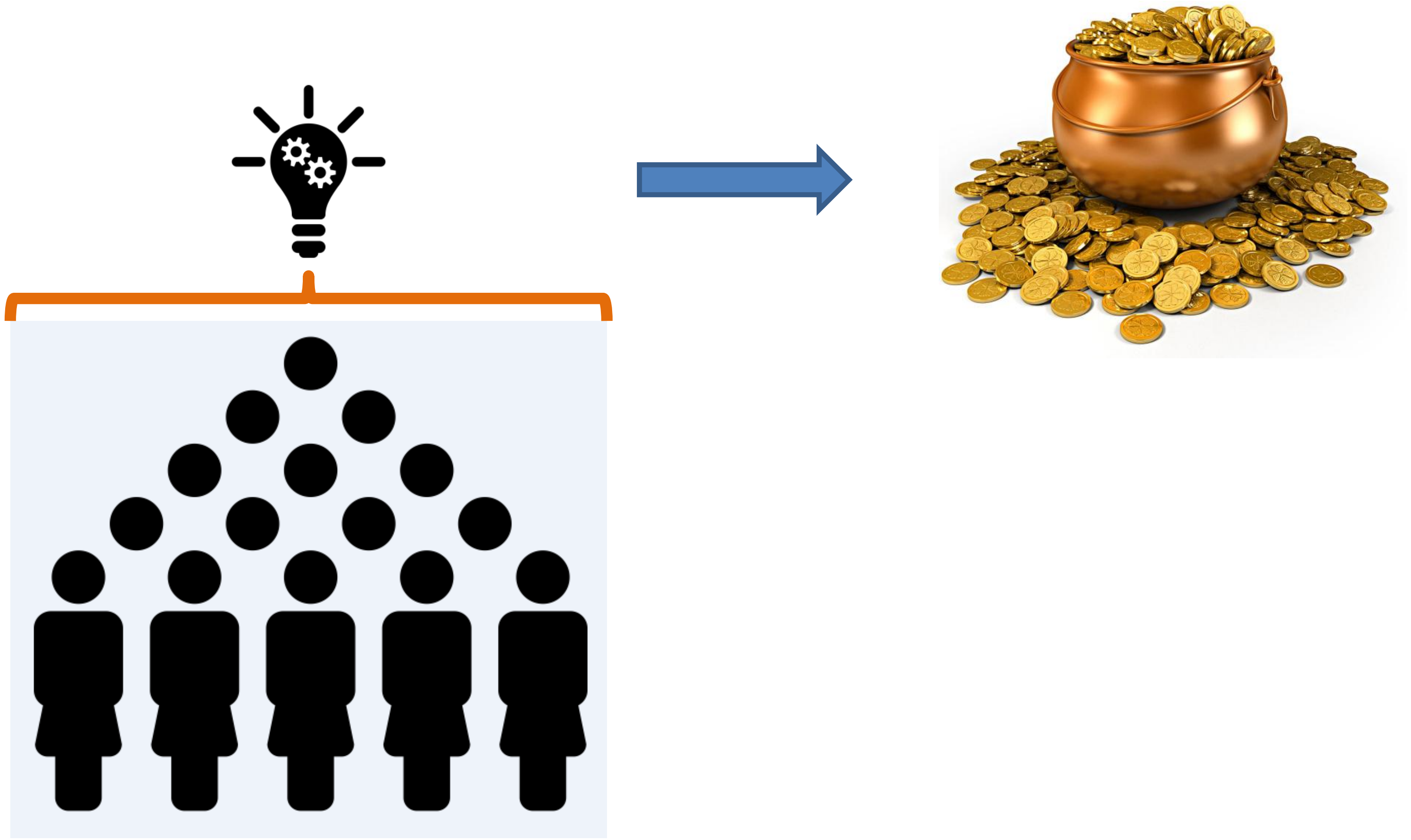
- **Vertical** transmission
- Cooperation
- Complementarities
- **Division** of labor/expertise
- Coordination

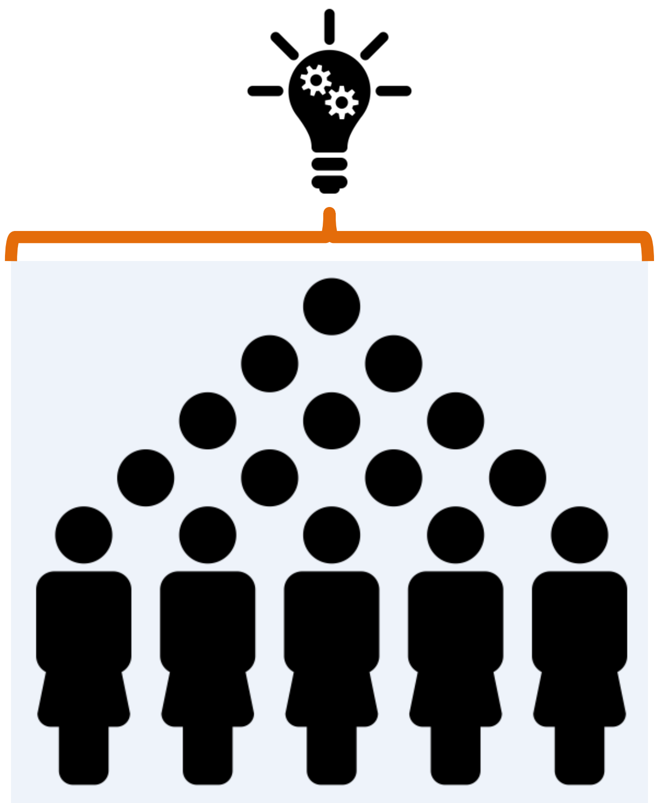
Ideas from Cultural Evolution are central

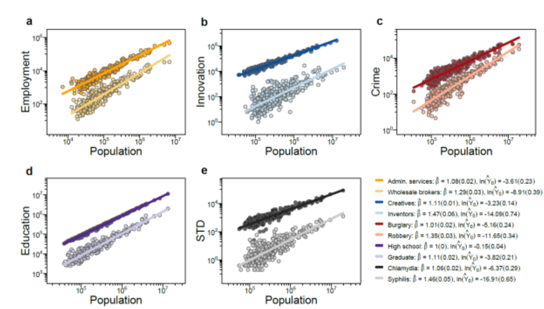
"just as thoughts are an emergent property of neurons firing in our neural networks, innovations arise as an emergent consequence of our species' psychology applied within our societies and social networks... [Societies and social networks] can produce complex designs **without the need for a designer**—just as natural selection does in genetic evolution"

(Muthukrishna and Henrich, 2016, "Innovation in the collective brain")









Gomez-Lievano, Patterson-Lomba, and Hausmann (2016)

The research question

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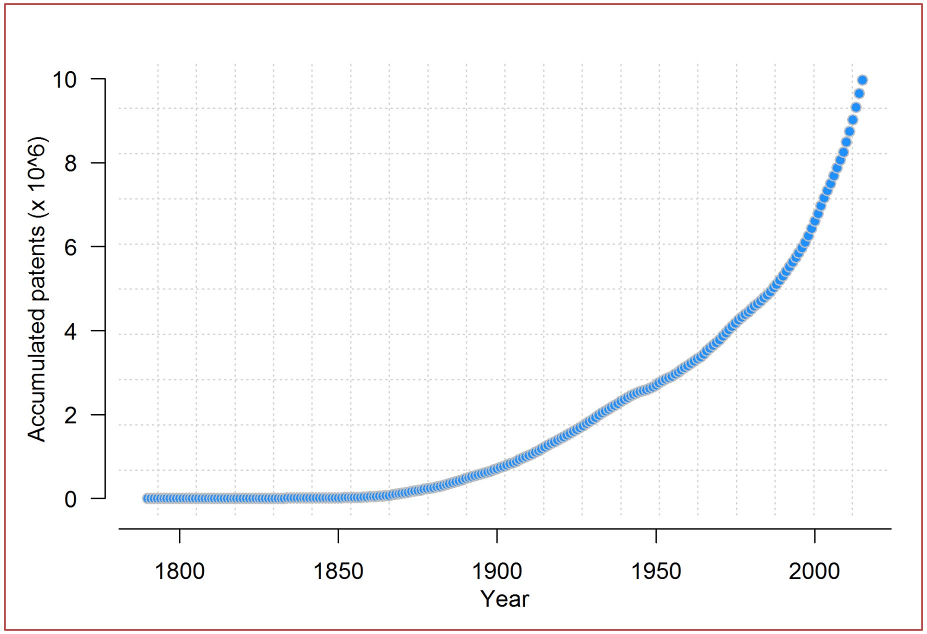
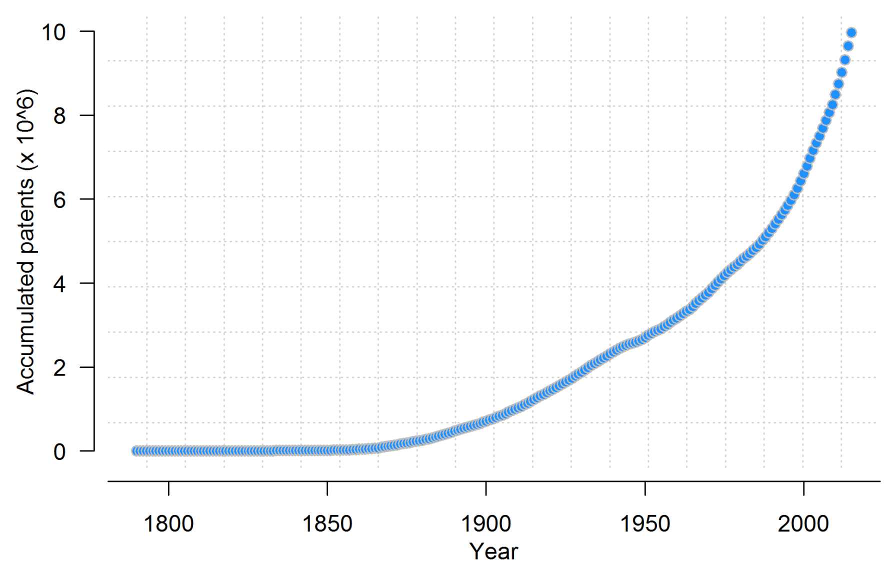


The research question

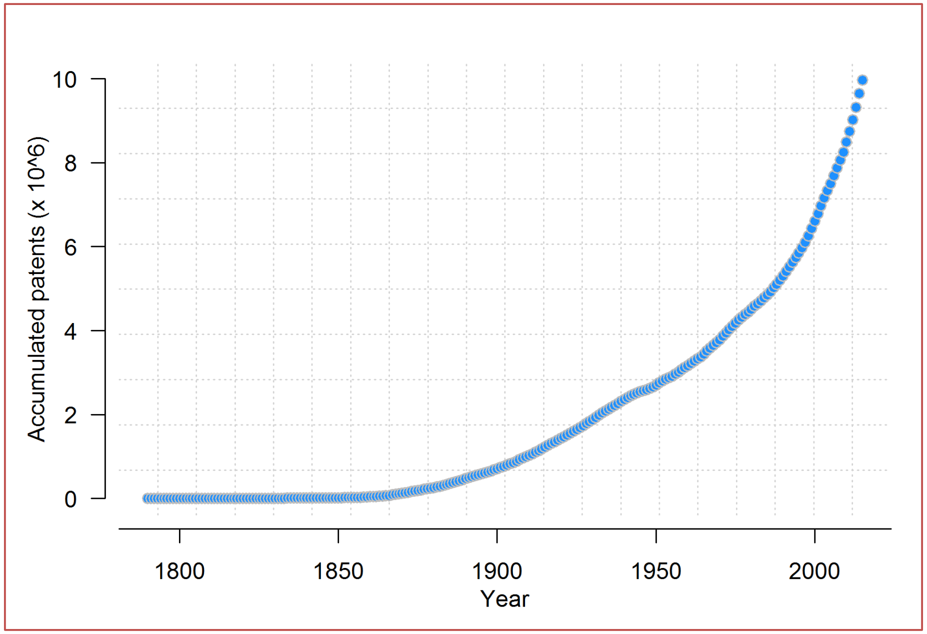
Is this collective learning?



The research question

Is this collective learning?

… or just a **trivial** process of accumulation in one type of historical record…



Related questions

- Should there be a distinct quantitative signal?
- How do we measure collective knowhow?... Are patents a good proxy???
- If we had a measure, Why/How would it grow?
- What would be the **functional form** for such a process?

“Know-how”

- Proposed proxy:
  The **number** of “things” you **know how to do**.

“Know-how”

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  The **number** of “things” you **know how to do**.

> “how many languages do you *know how* to speak?”
>
> “how many different mathematical problems do you *know how* to solve?”
>
> “how many cooking recipes do you *know how* to cook?”

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- “Collective know-how” = the number of things a **collective** knows how to do

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- "Collective learning" = increases of collective knowhow that are **not accounted** by increases in individual know-how

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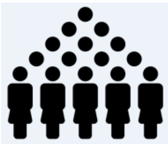
"Know-how"

- Proposed proxy:
  The **number** of "things" you **know how to do**.

- "Collective know-how" = the number of things a **collective** knows how to do, that no individual would know how to do.



" Standing on the shoulders of "midgets" "
(Robert Boyd)

Know-how = size of *Set*

(Olsson 2000, *Knowledge as a Set in Idea Space*)

- Collective learning is a process of accumulation.

Trivial VS. Non-trivial

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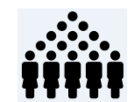
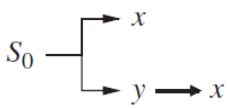
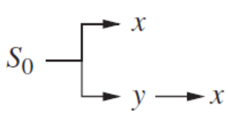
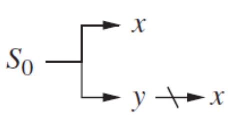
Trivial VS. Non-trivial ?

- Collective learning as a "**self-propelled**" process of **accumulation**.

"culture"	=	collective know-how	
"cultural trait"	=	distinct technology (that can be counted)	
"cultural accumulation"	=	social + collective learning	

(Cavalli-Sforza & Feldman 1981, Boyd & Richerson 1985)

Table 1. Kinds of dependencies of a cultural element, x, upon another cultural element, y. x, y, cultural elements; S_0, culture state without x and y; thicker lines indicate higher probability of transition.

dependence	histories	examples
facilitation		y is a tool, material or knowledge necessary to create x x is a modification of y x is a combination of y and another element (e.g. the harpoon combines spear and rope) y is a social institution that promotes x y is a technology that makes x cheaper
neutral		y is wholly unrelated to x
inhibition		y is a taboo that forbids x y is an alternative to x, e.g. a solution to the same problem

- *If* invention is a process of **cultural accumulation**, there will be **path dependency**, and past and present inventions will **affect future inventions permanently** (positively or negatively).

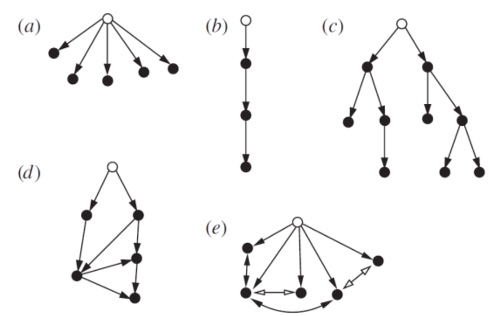


Figure 1. Examples of dependencies between cultural elements: (*a*) independent elements; (*b*) linear succession of elements; (*c*) differentiation of elements; (*d*) pairwise combinations of elements; (*e*) systems of cultural elements (open arrowheads represent inhibitory relationships). The open circle represents a state in which no culture is present.

Figures from: Enquist et al. (2011), "***Modelling the evolution and diversity of cumulative culture***"

The research question… re-stated

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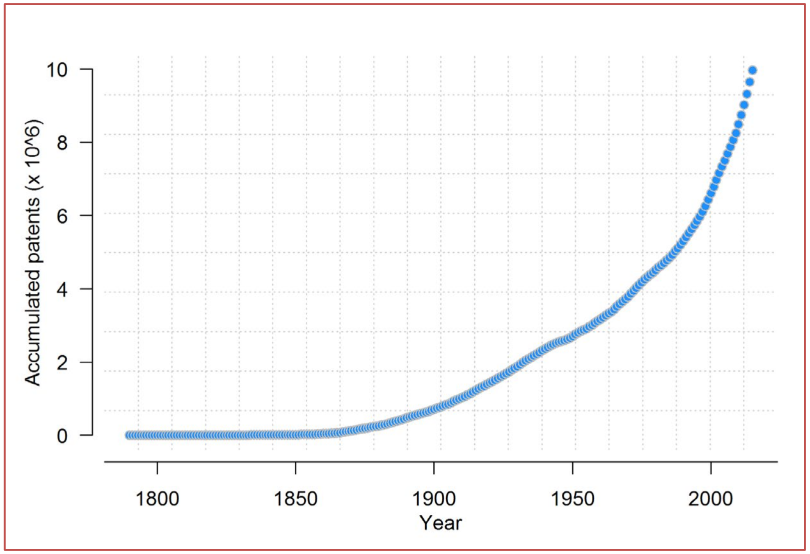
Are patents an instance of **cultural accumulation?**



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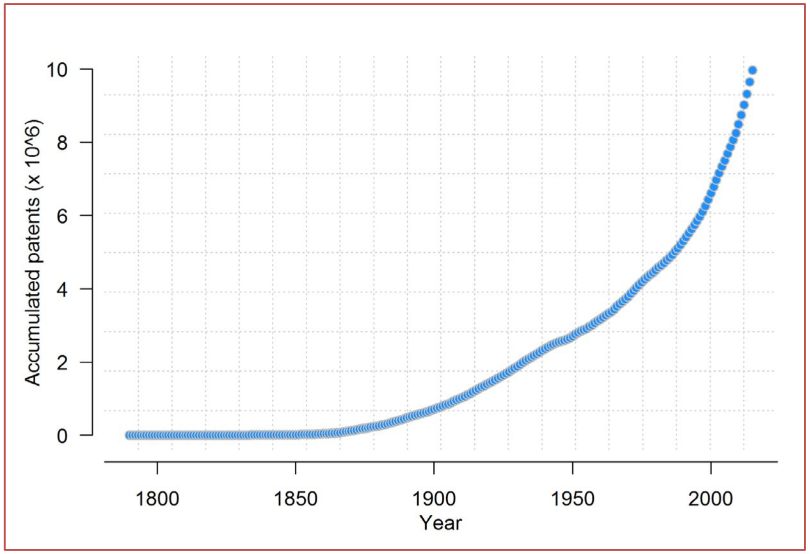

- Affected by previous inventions.

The research question… re-stated

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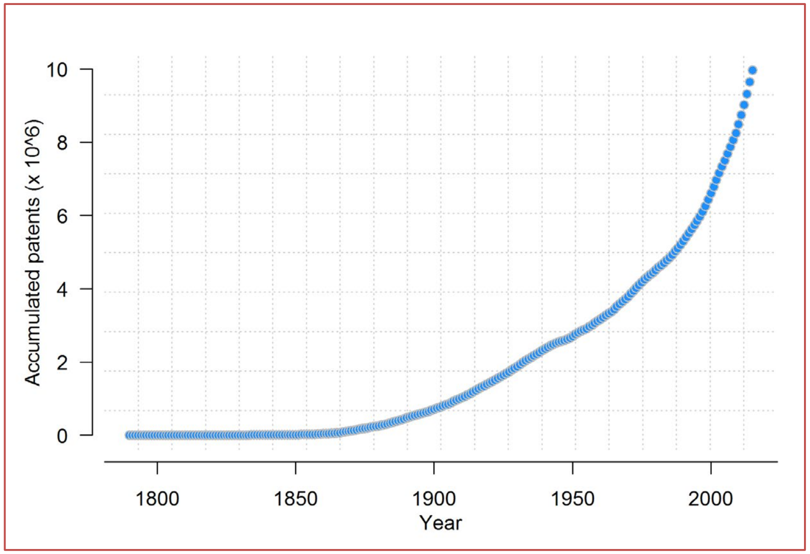

- Affected by previous inventions.
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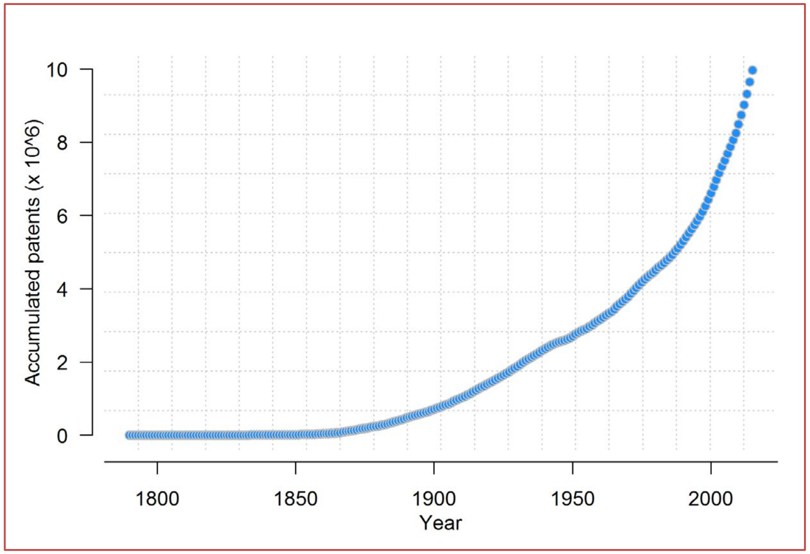

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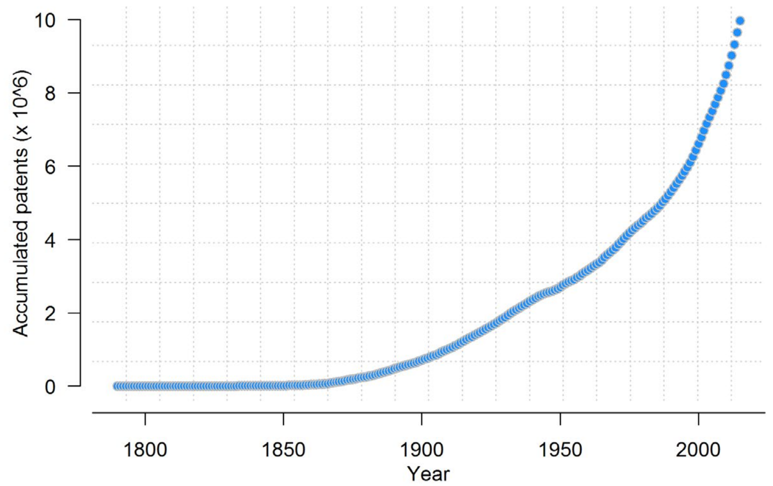


- Affected by previous inventions.
- Path-dependent.
- Non-stationary.
- Self-propelled.

Hypotheses

1. If accumulation of patents is not cultural, but determined by independent contributions, the series of **patents should lack a "unit root"**.

2. If accumulation is cultural, the time-series of patents should display some sort of "memory".

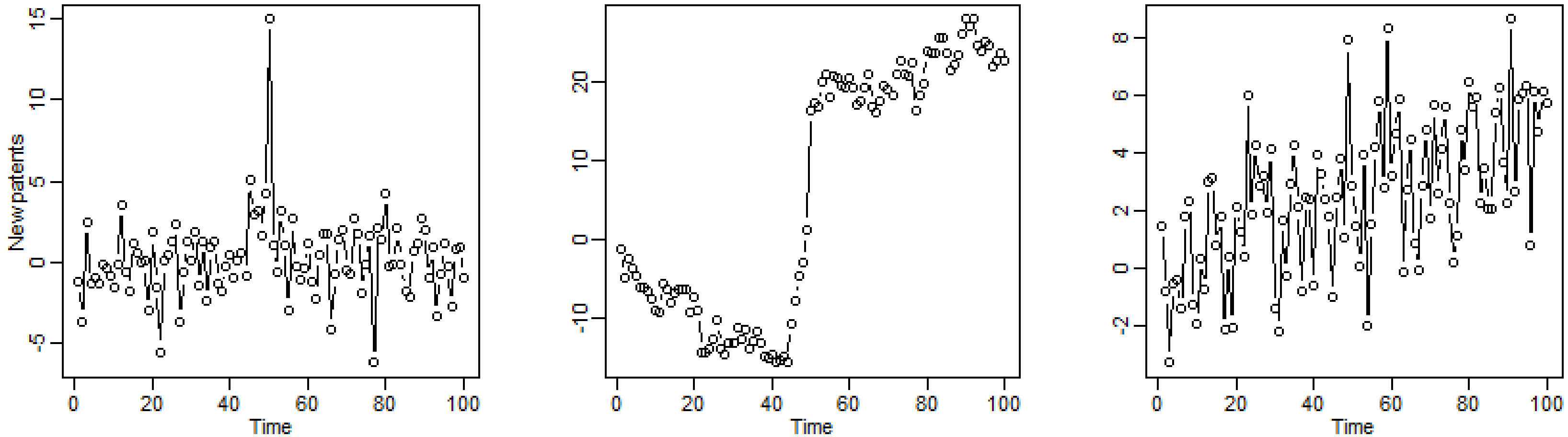
Large literature regarding GNP:
Nelson and Plosser (1982)
Campbell and Mankiw (1987)

USPTO

Total patent applications per year
1840-2015

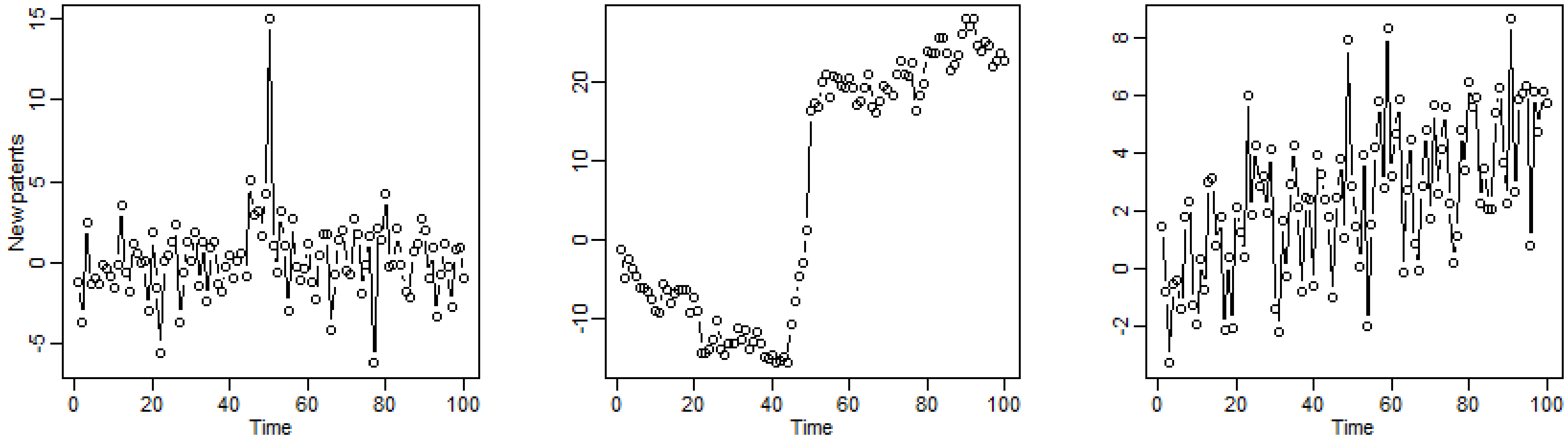
EMPIRICAL RESULTS

Toy Example



$$y(t) = \varepsilon_t, \quad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2)$$

Toy Example



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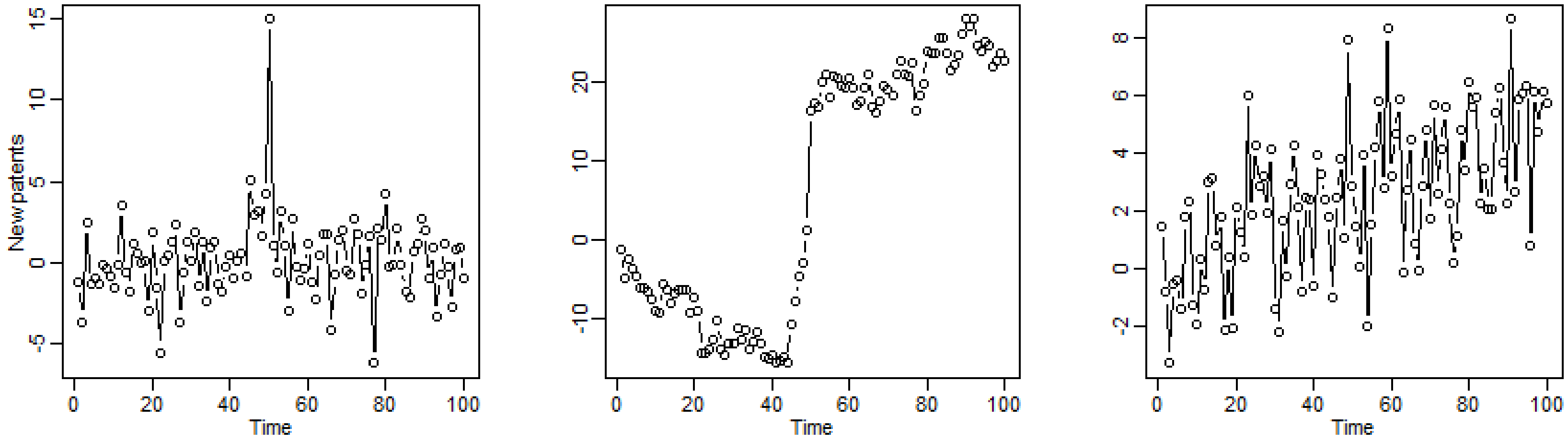

$$y(t) = y(t-1) + \varepsilon_t, \quad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2)$$

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$$y(t) = \varepsilon_t, \quad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2)$$

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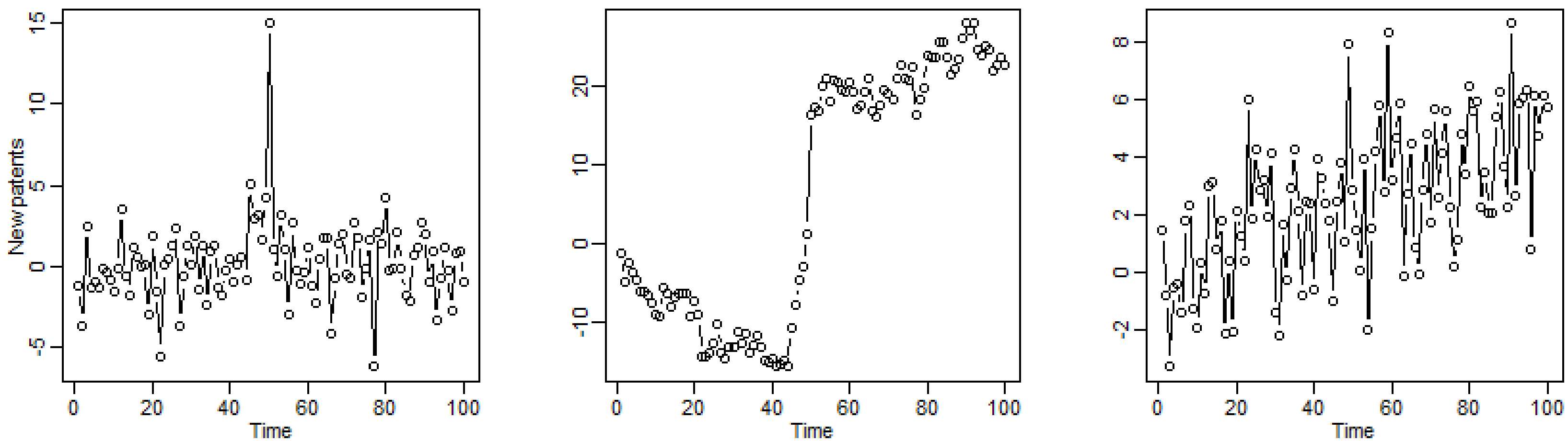

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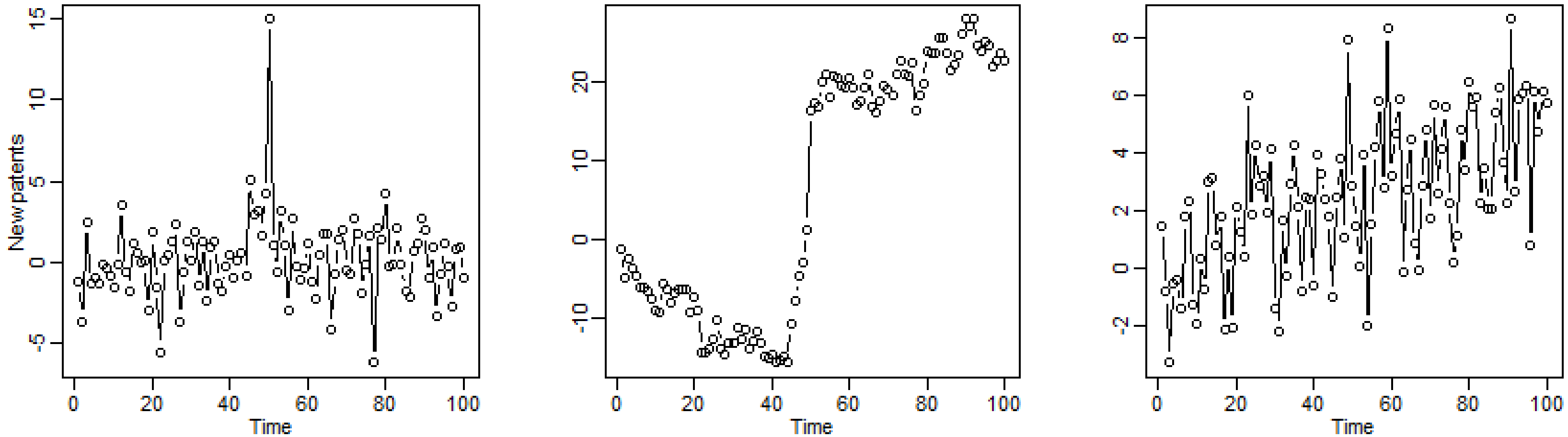
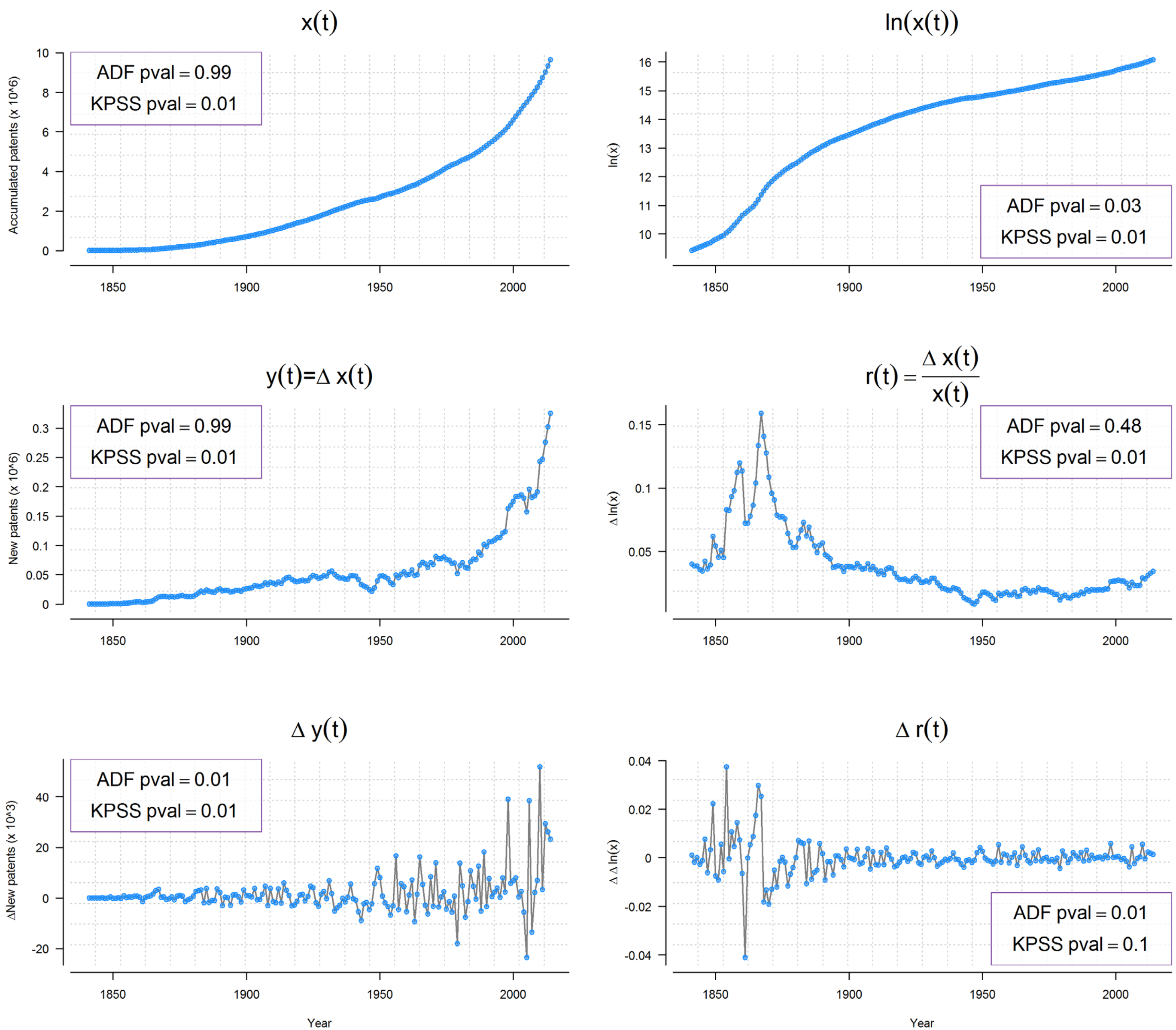


$$\boxed{y(t) = y(t-1) + \varepsilon_t, \quad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2)}$$

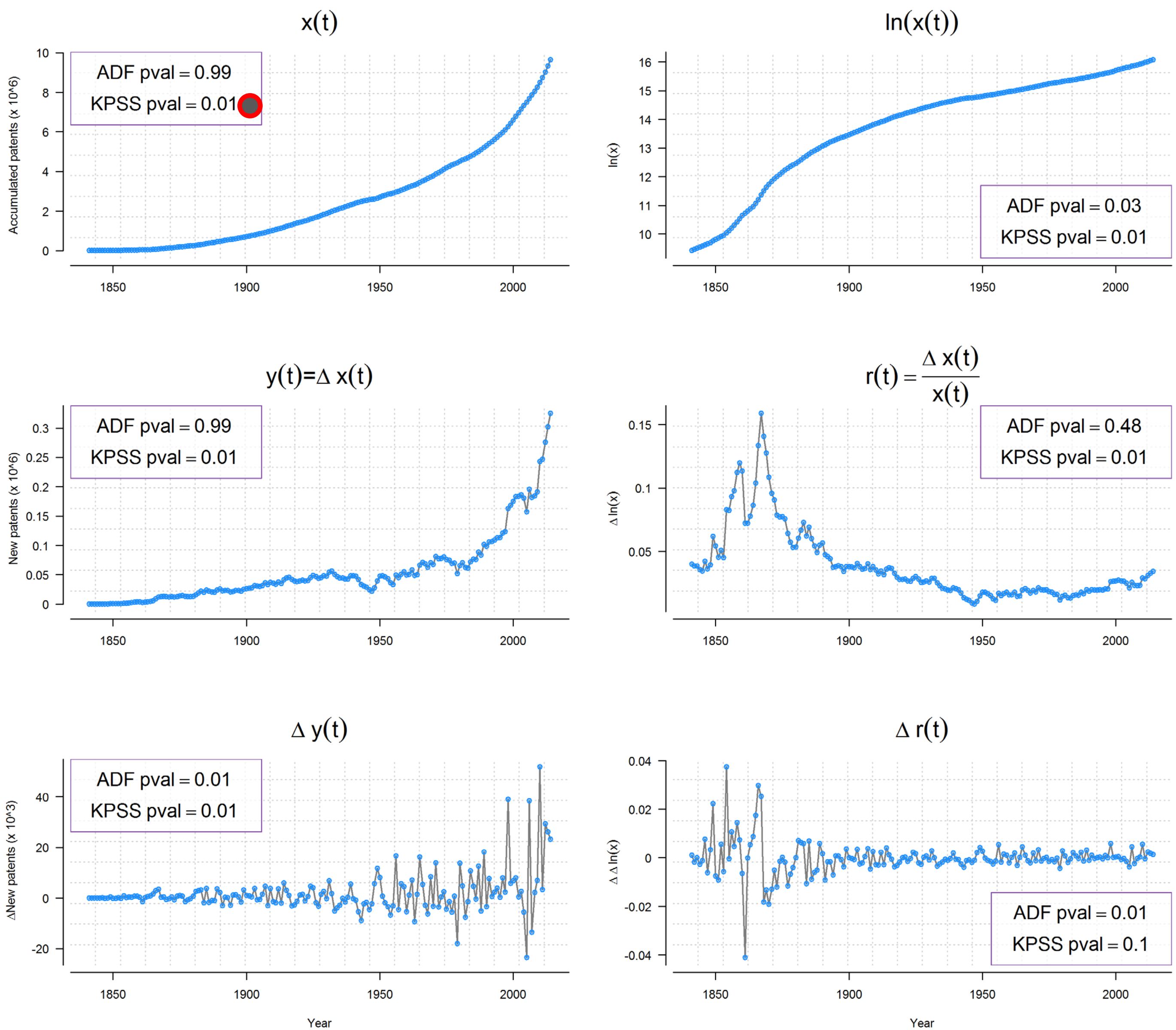
In words…

- When "unit roots" are rejected, we expect some reversion to a trend.
- Conversely, when the series does have a unit root, a shock shifts the series **permanently**.
- When the KPSS trend-stationarity is rejected, the series may behave not-trivially.


x(t)
ADF pval = 0.99
KPSS pval = 0.01
Accumulated patents (x 10^6)
ln(x(t))
ADF pval = 0.03
KPSS pval = 0.01
ln(x)
y(t)=Δ x(t)
ADF pval = 0.99
KPSS pval = 0.01
New patents (x 10^6)
r(t) = Δ x(t) / x(t)
ADF pval = 0.48
KPSS pval = 0.01
Δ ln(x)
Δ y(t)
ADF pval = 0.01
KPSS pval = 0.01
ΔNew patents (x 10^3)
Δ r(t)
ADF pval = 0.01
KPSS pval = 0.1
Δ Δ ln(x)
Year
Year

Augmented Dickey-Fuller Test: H_0 = Unit Root

Kwiatkowski-Phillips-Schmidt-Shin Test: H_0 = Trend Stationarity



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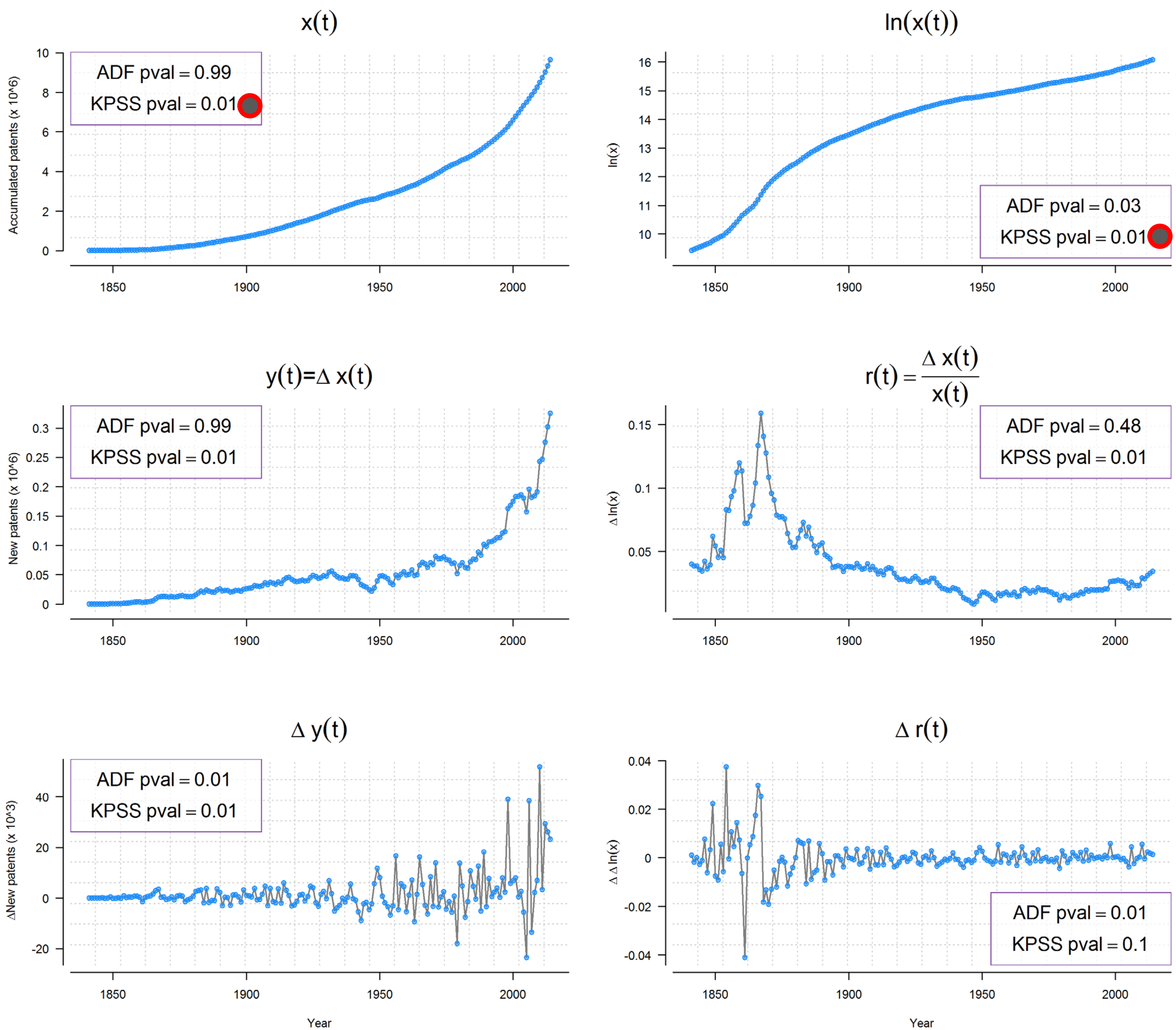

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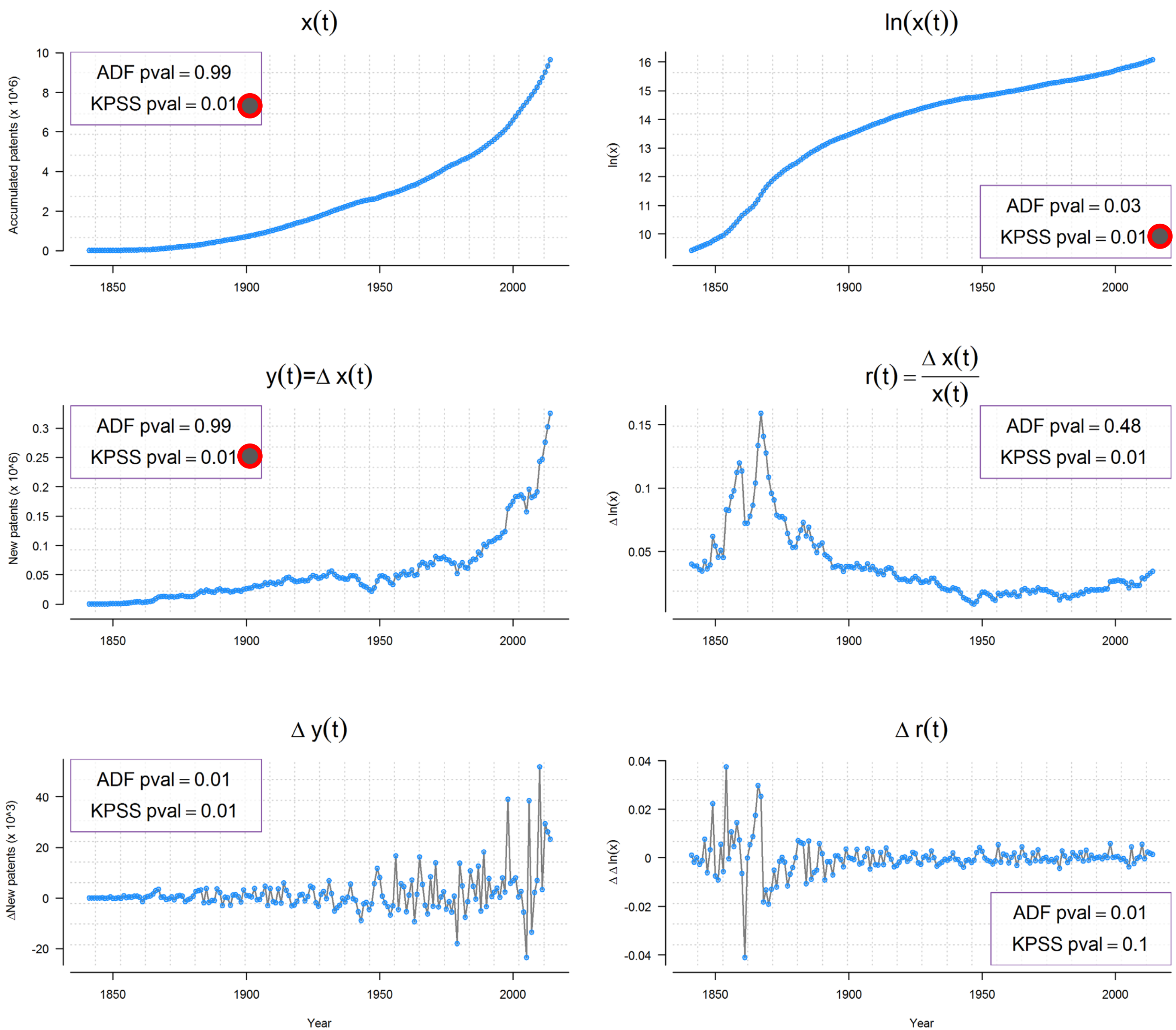

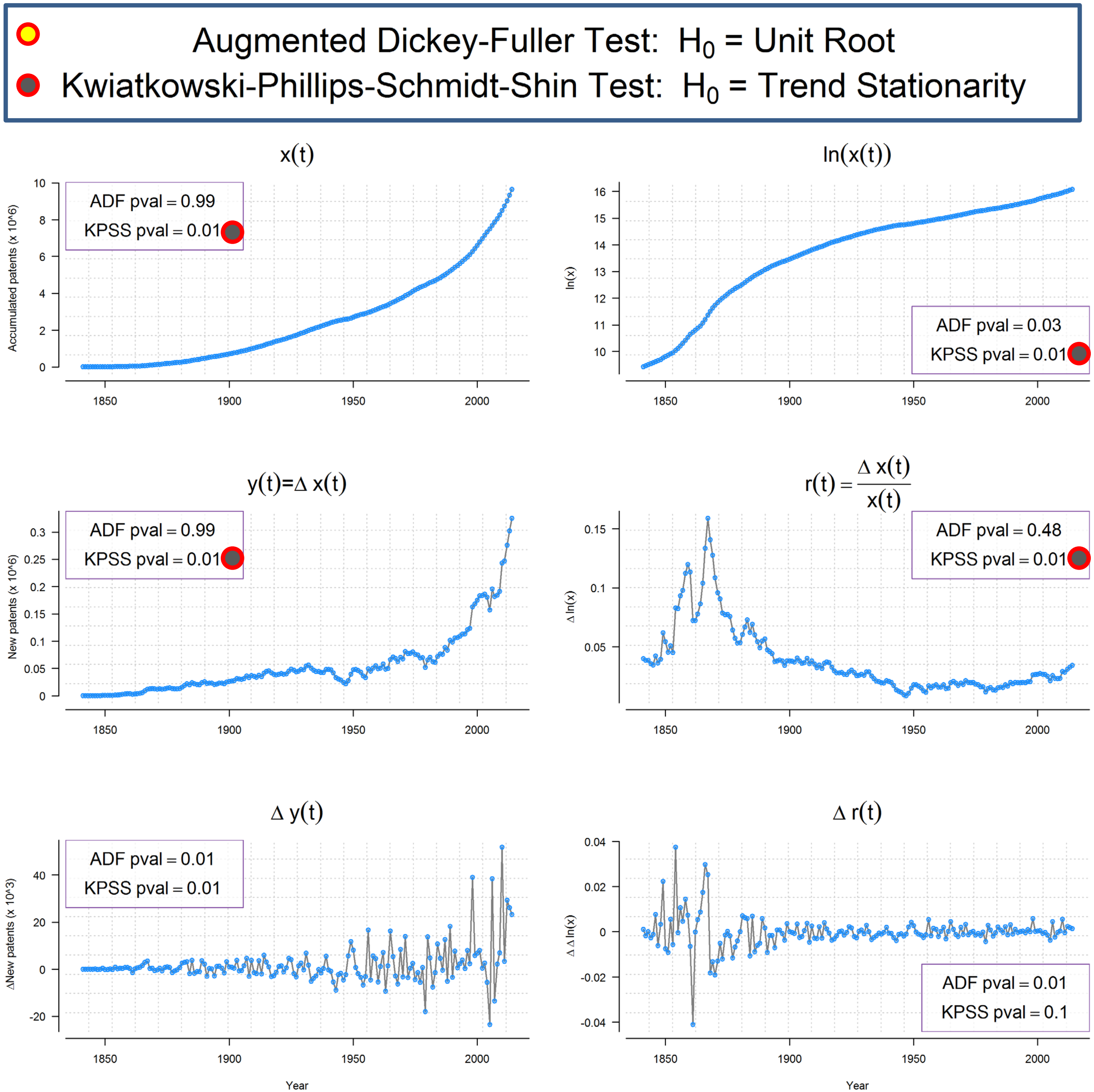

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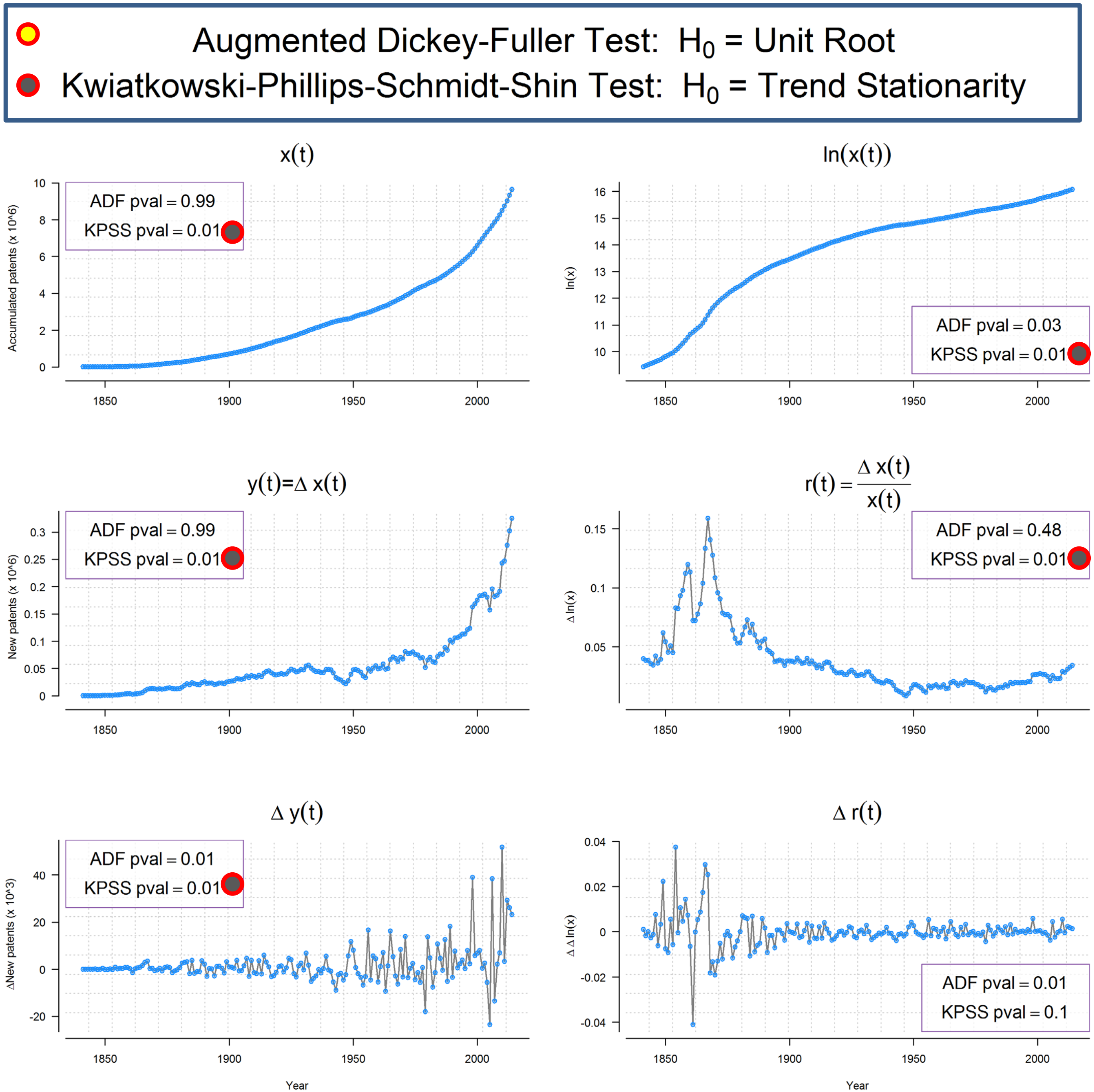

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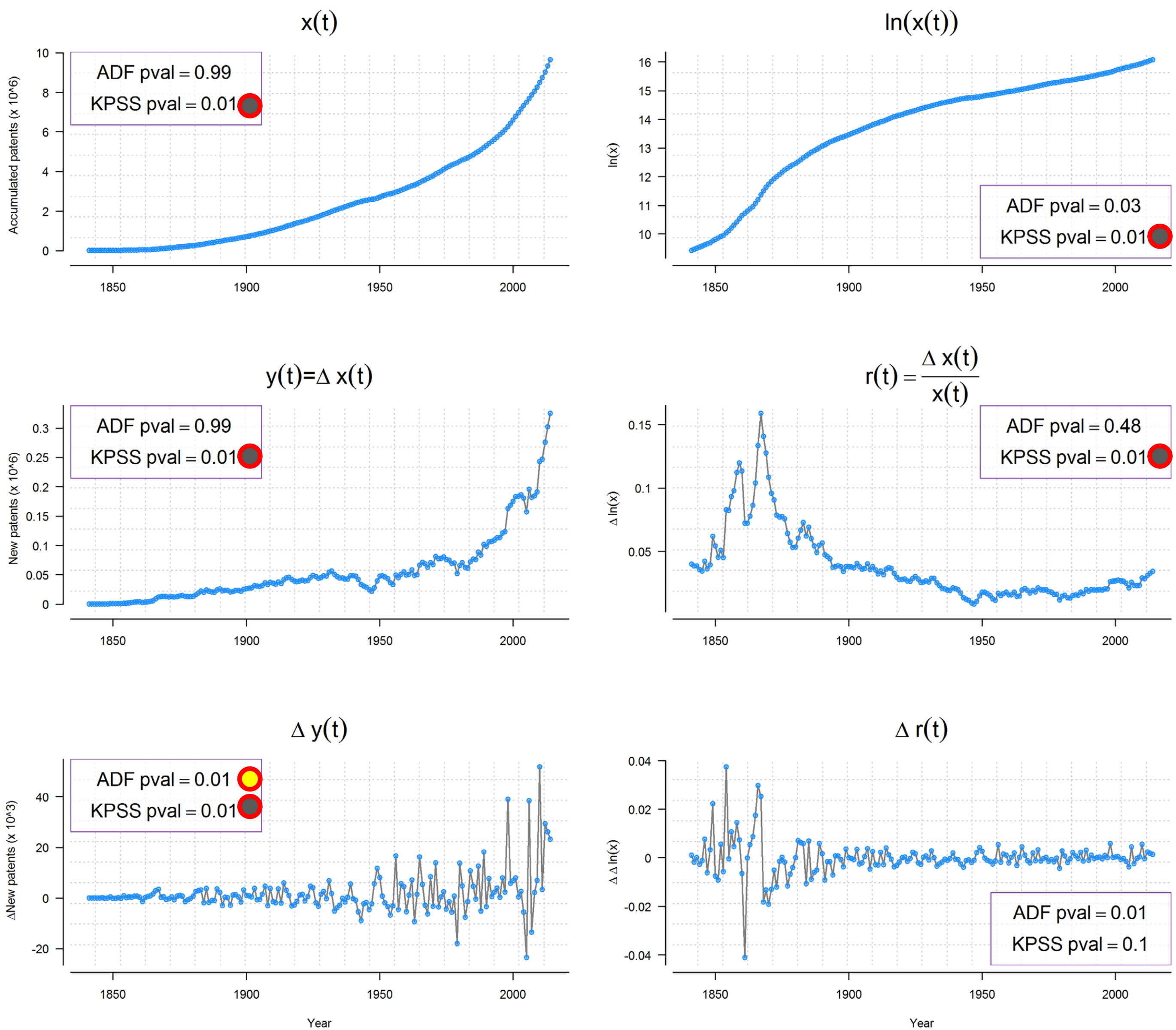

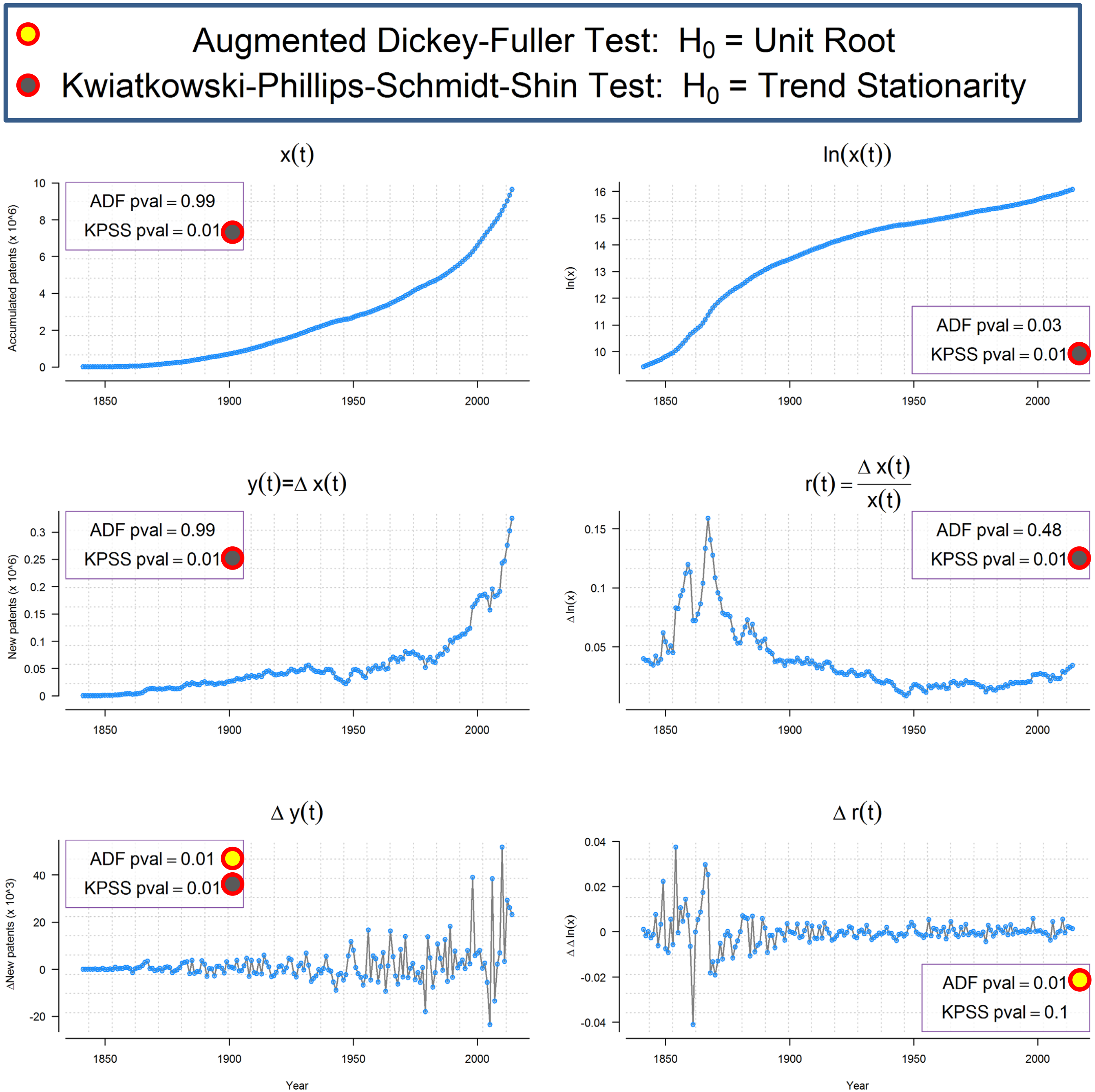

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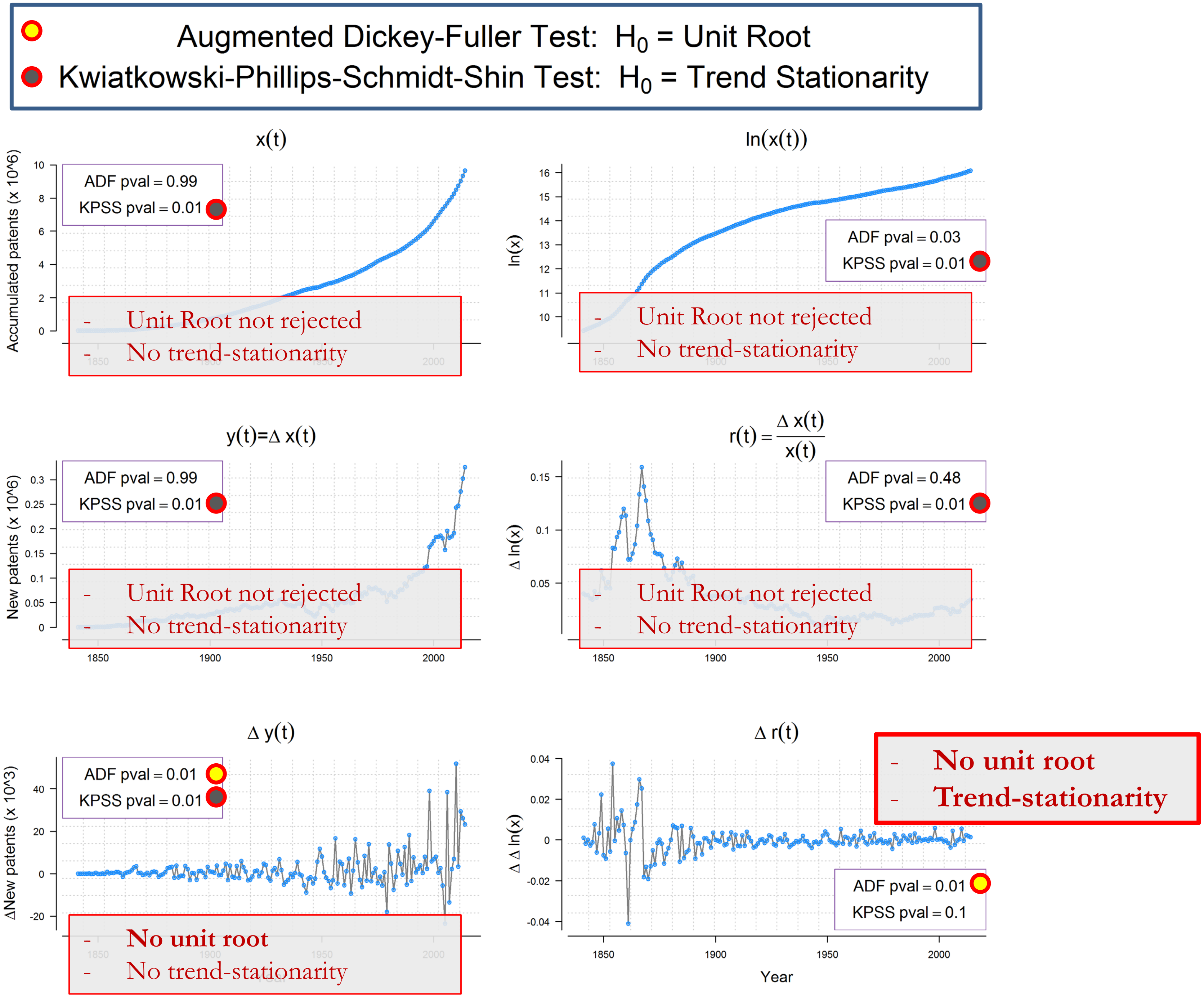

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x(t)
Accumulated patents (x 10^6)
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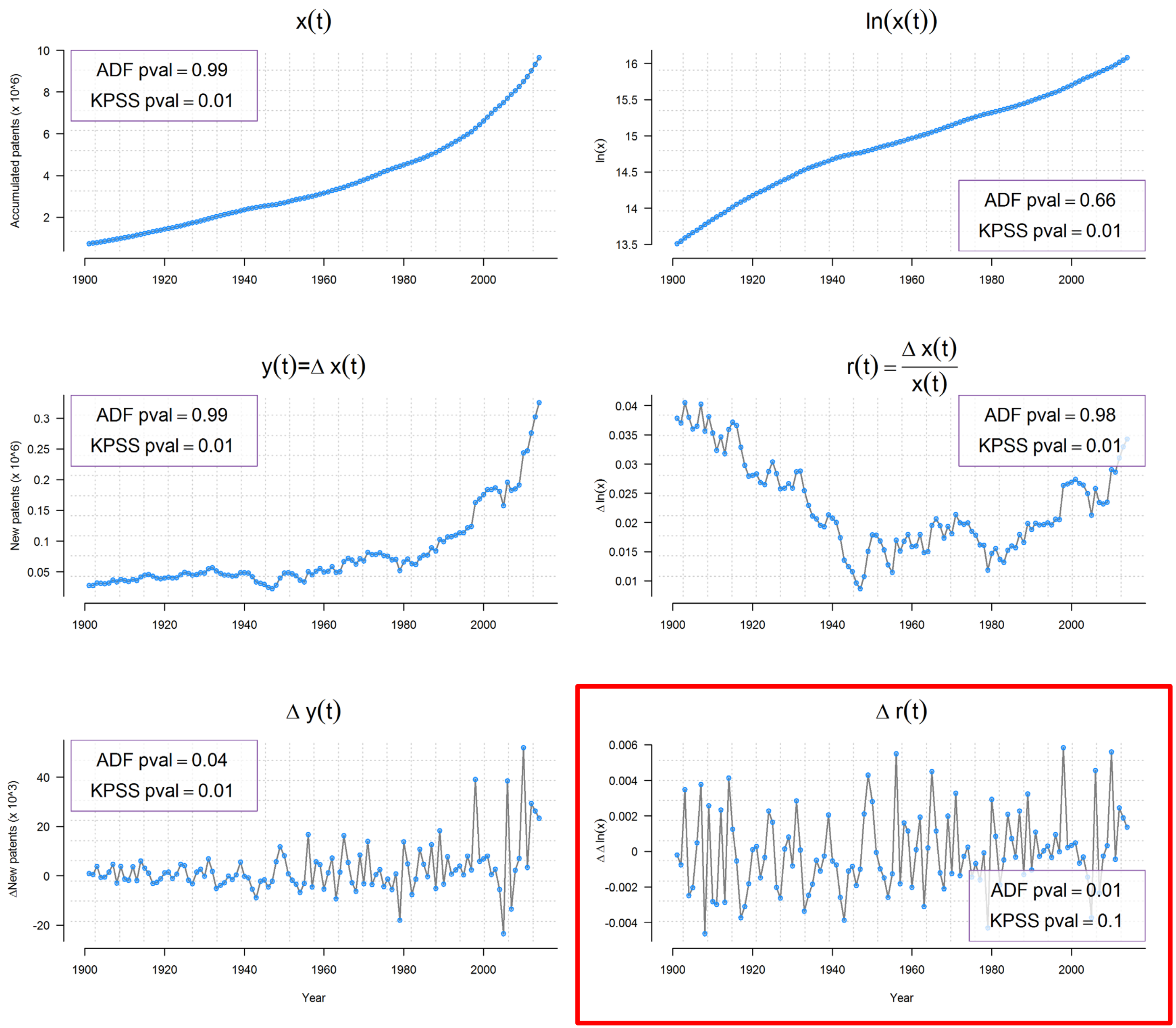
- Unit Root not rejected
- No trend-stationarity
ln(x(t))
ln(x)
ADF pval = 0.03
KPSS pval = 0.01
- Unit Root not rejected
- No trend-stationarity
y(t)=Δ x(t)
New patents (x 10^6)
ADF pval = 0.99
KPSS pval = 0.01
- Unit Root not rejected
- No trend-stationarity
r(t) = Δ x(t) / x(t)
Δ ln(x)
ADF pval = 0.48
KPSS pval = 0.01
- Unit Root not rejected
- No trend-stationarity
Δ y(t)
ΔNew patents (x 10^3)
ADF pval = 0.01
KPSS pval = 0.01
- No unit root
- No trend-stationarity
Δ r(t)
Δ Δ ln(x)
ADF pval = 0.01
KPSS pval = 0.1
- No unit root
- Trend-stationarity
Year

1900-2015

Augmented Dickey-Fuller Test: H_0 = Unit Root
Kwiatkowski-Phillips-Schmidt-Shin Test: H_0 = Trend Stationarity



Implications

- The system has ***memory*** ("Order of Integration = 2")!
- Past shocks seem to have a **multiplicative** effect.

- **Q**: Does this stand as *sufficient* or *necessary* pieces of **evidence** for cultural accumulation?

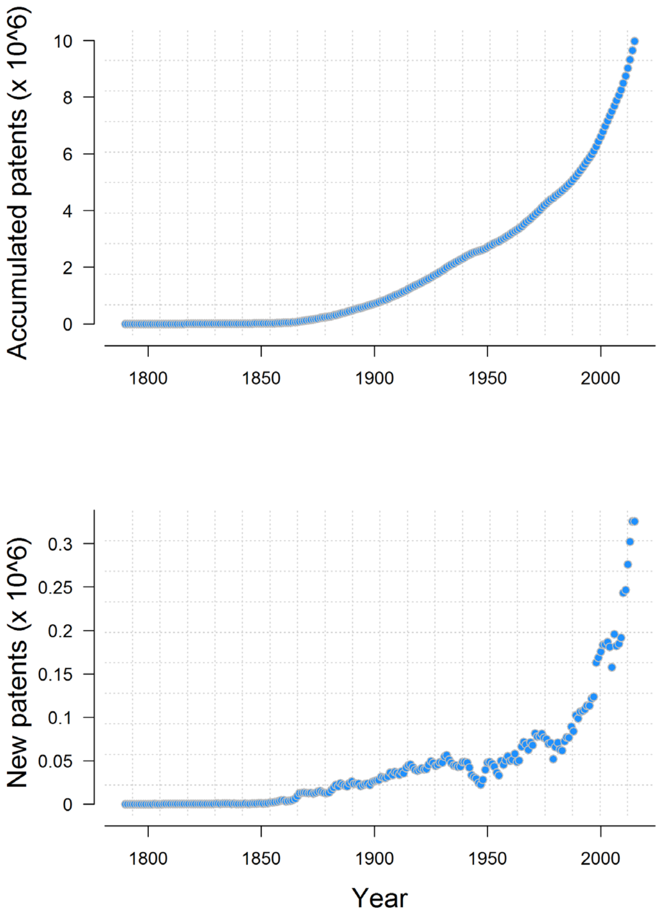
Contingent VS. Deterministic
Micro-structures VS. Macro-structures

- A time-series with "unit roots" can still be a time-series with a deterministic component.

- The contigent (random?) aspect of patenting seems to be consistent with a process of cultural accumulation.

- What about the deterministic component?

What types of *temporal* growth look like these?



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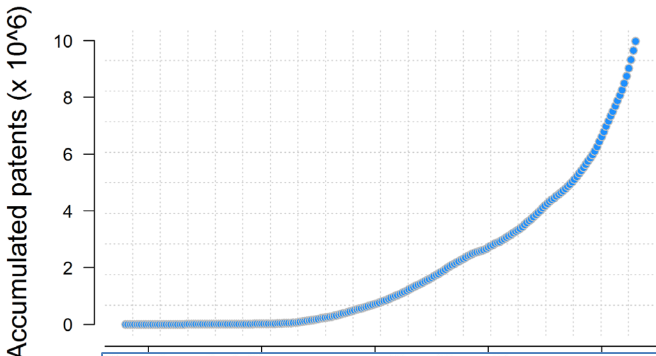
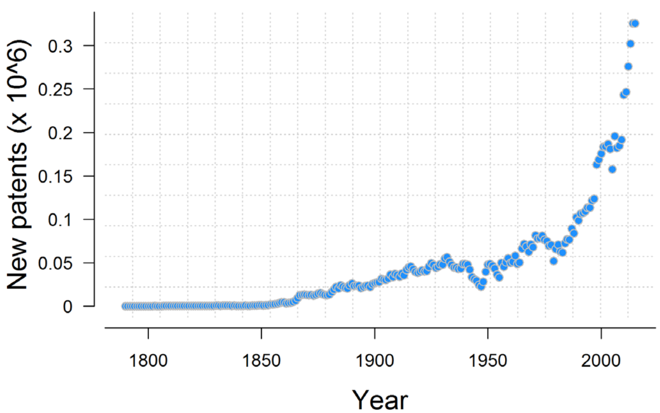


$$\begin{aligned}\ln(x(t)) &= \ln(x_0) + r\ t \\ \ln(\dot{x}(t)) &= \ln(r\ x_0) + r\ t\end{aligned}$$



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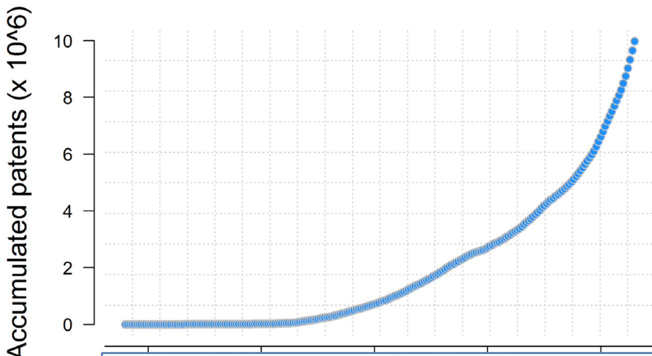

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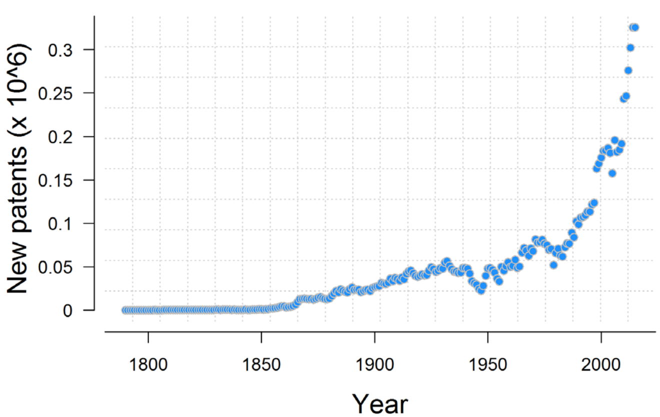

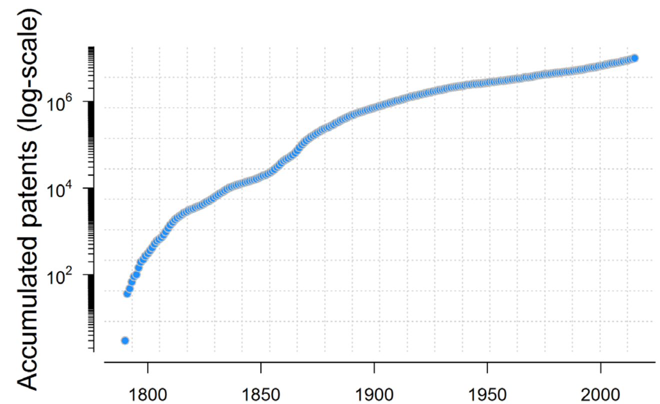

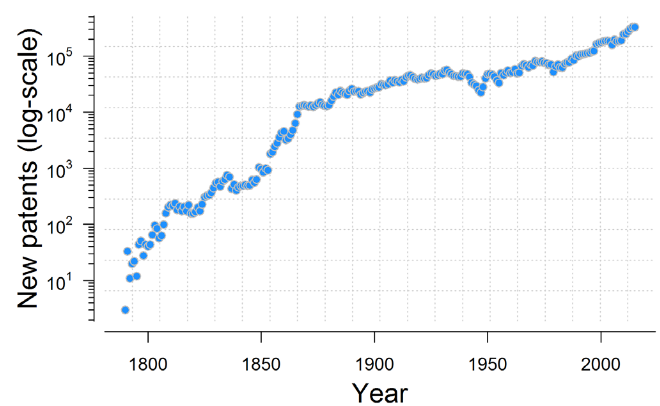

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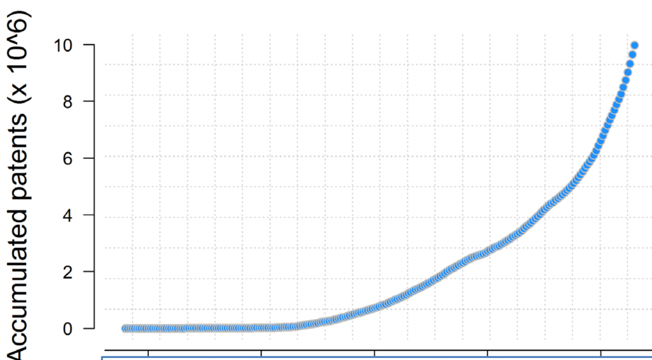

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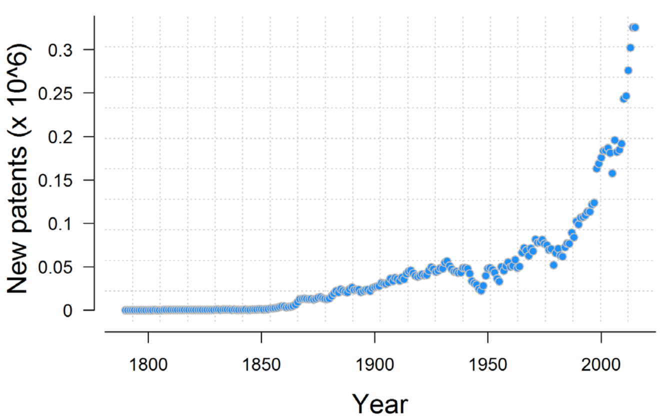
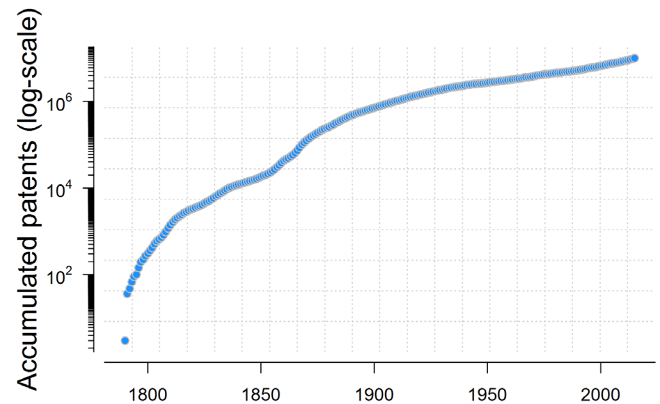
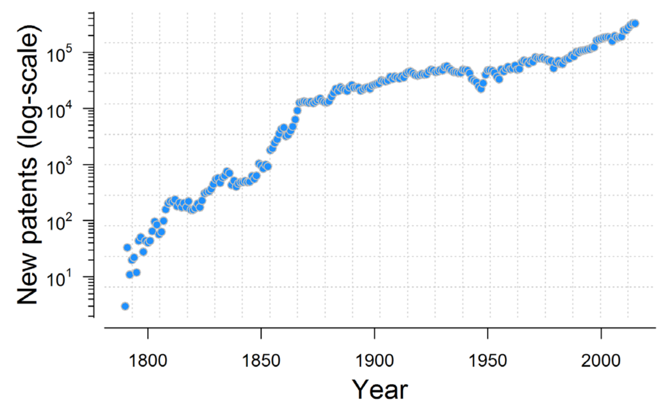


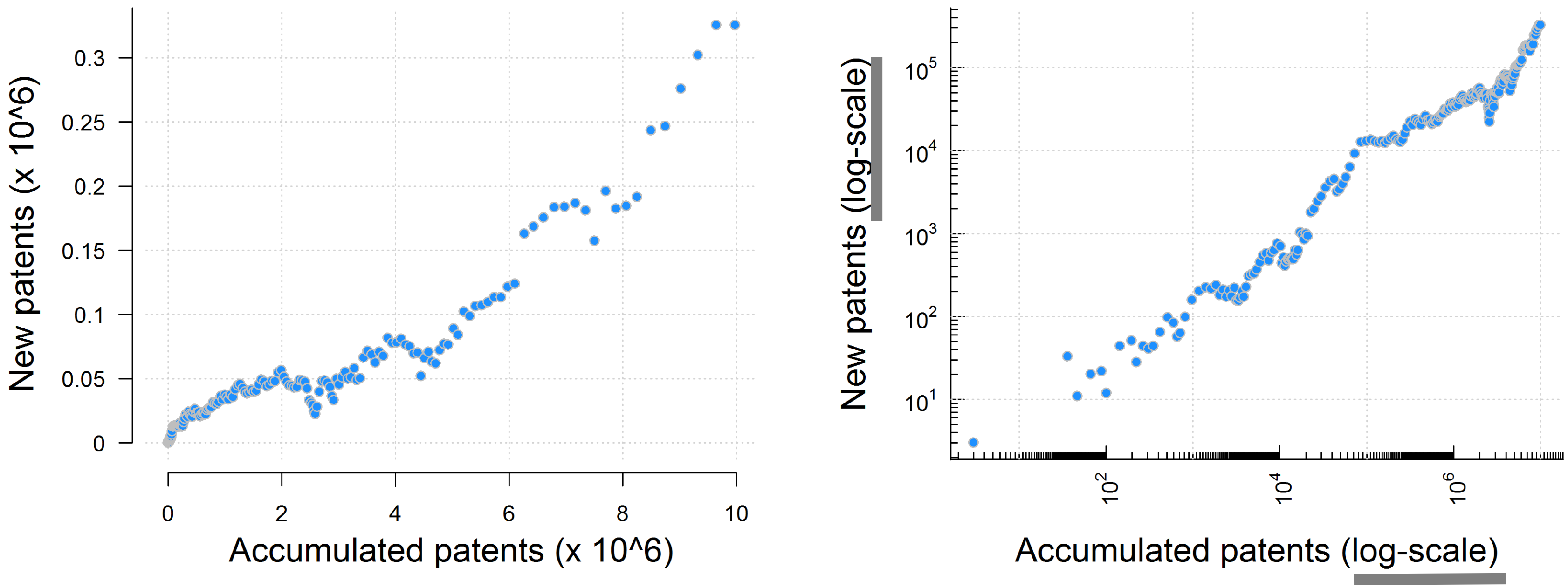




$$\boxed{\dot{x}(t) = r x(t)}$$?

- Maybe, if invention is cultural, time is not measured in “years”, but in the number of elements already in the system.
- In other words, new patents may be a function not of time, but of the number of accumulated patents.

… similarity with "learning curves" in engineering (see McNerney et al., 2011)



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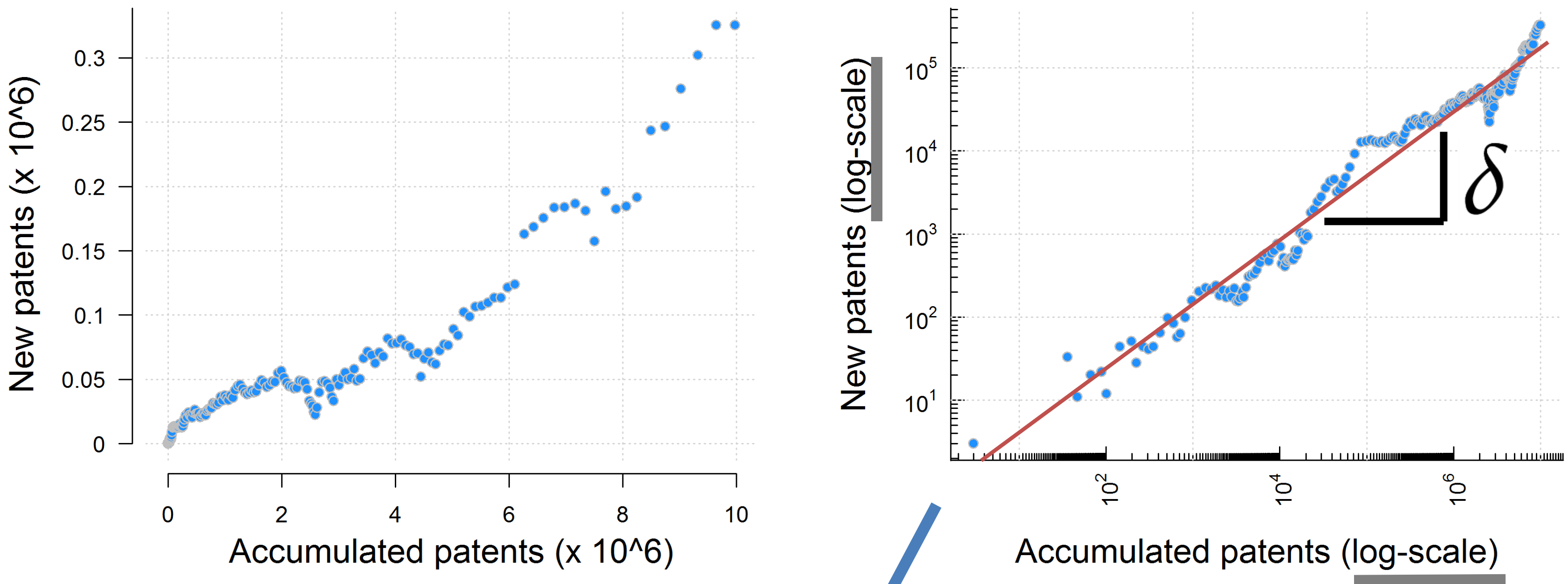

$$\dot{x}(t) = \gamma x(t)^{\delta}$$

Solving: $\dot{x}(t) = \gamma x(t)^{\delta}$

$$\Longrightarrow \quad x(t) = x_0 \left[1 + \left(\frac{\gamma - \gamma\delta}{x_0^{1-\delta}} \right) (t - t_0) \right]^{\frac{1}{1-\delta}}$$

which in turn has three solutions for different values of δ:

$$x(t) = \begin{cases} c_1 \left((x_0/c_1)^{1/\alpha_1} + (t - t_0) \right)^{\alpha_1}, & \text{for } \delta < 1 \quad (1) \\ x_0 \, \mathrm{e}^{\gamma(t-t_0)}, & \text{for } \delta = 1 \quad (2) \\ \dfrac{c_2}{(t_{\text{critical}} - t)^{\alpha_2}}, & \text{for } \delta > 1 \quad (3) \end{cases}$$

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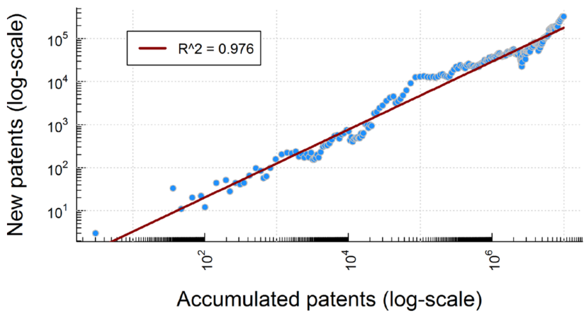
- For the whole period 1790-2015, we get: $\widehat{\delta} \approx 0.789$
  - Overall, sub-exponential.



- However, there seem to be "epochs".

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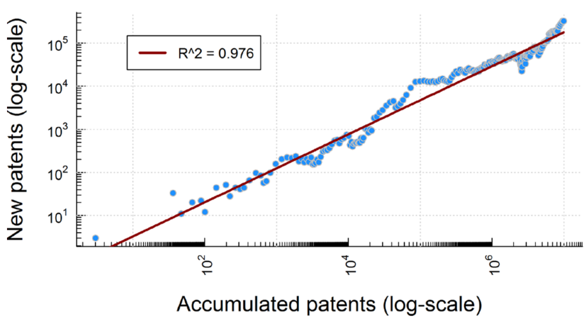

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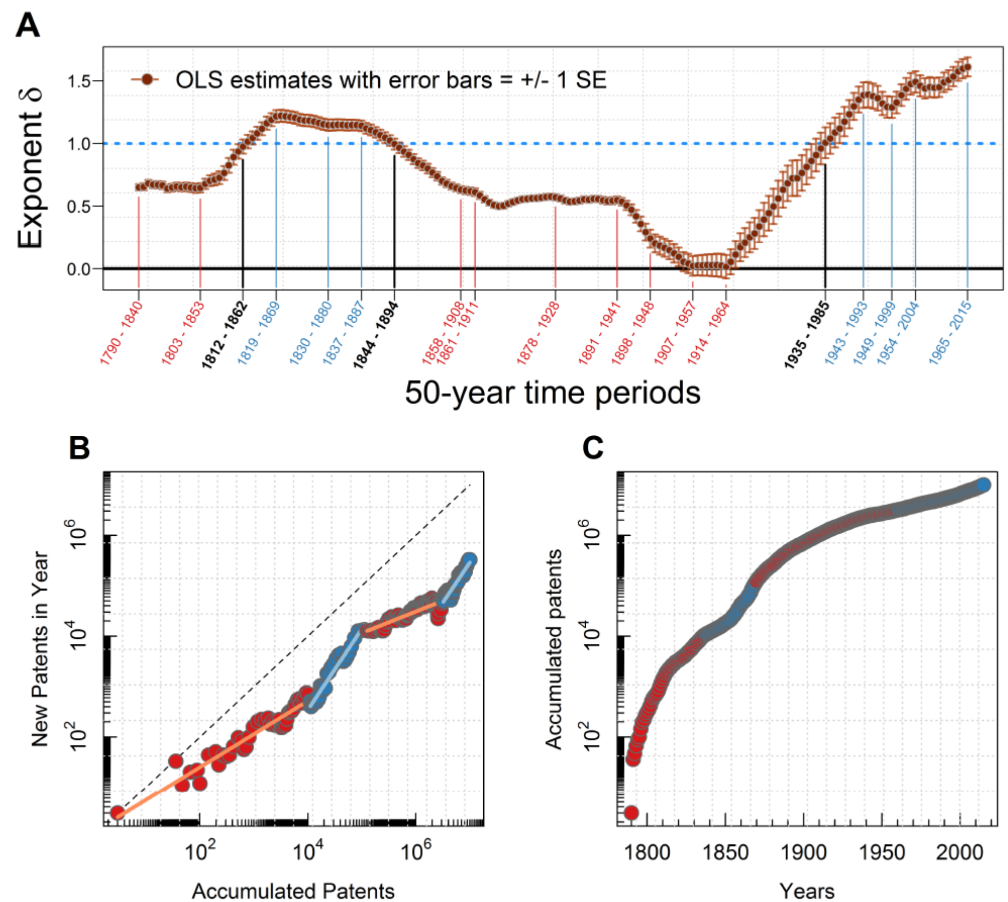
Estimating the exponent over running windows of 50 years, from 1790 to 2015.



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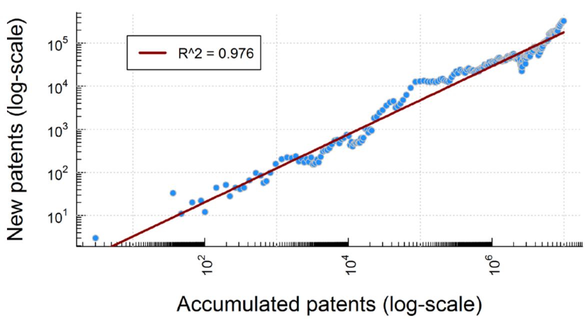

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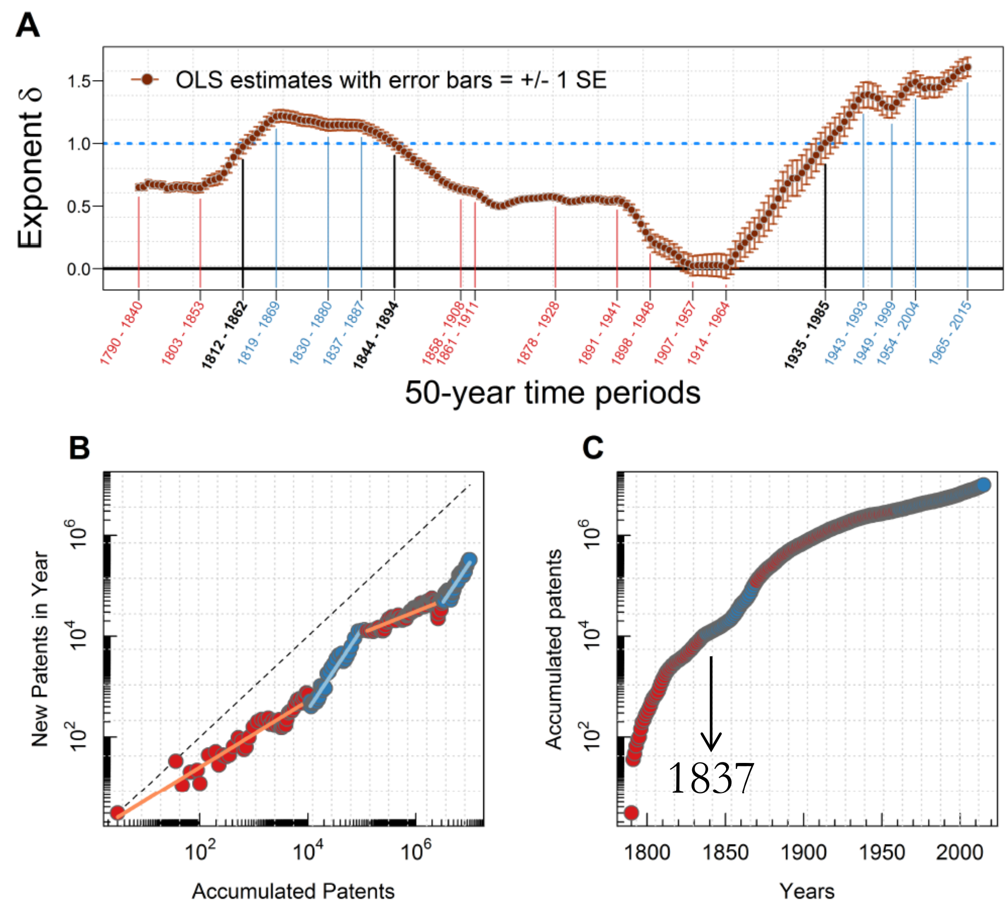

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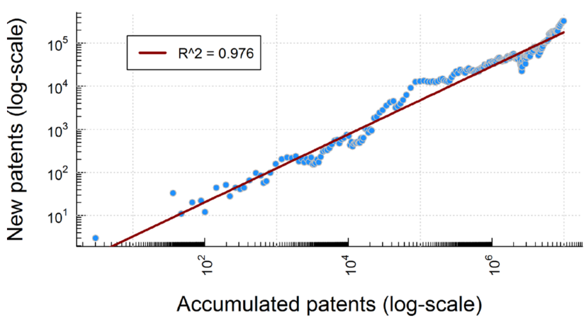

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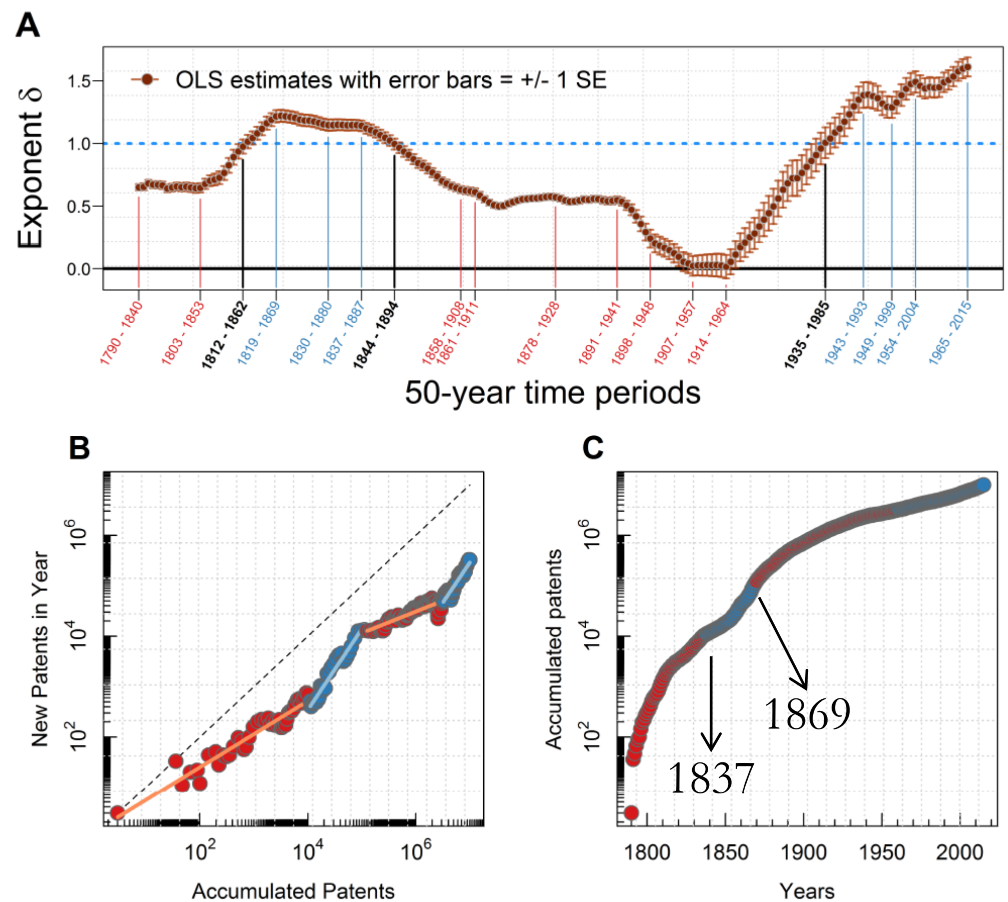

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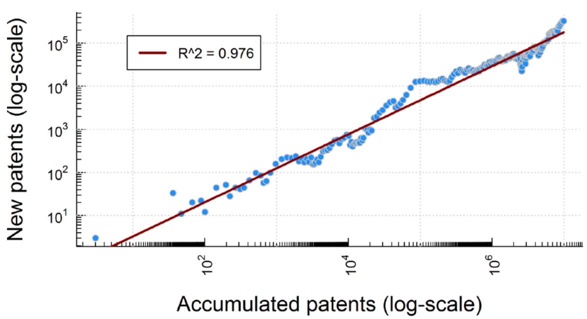

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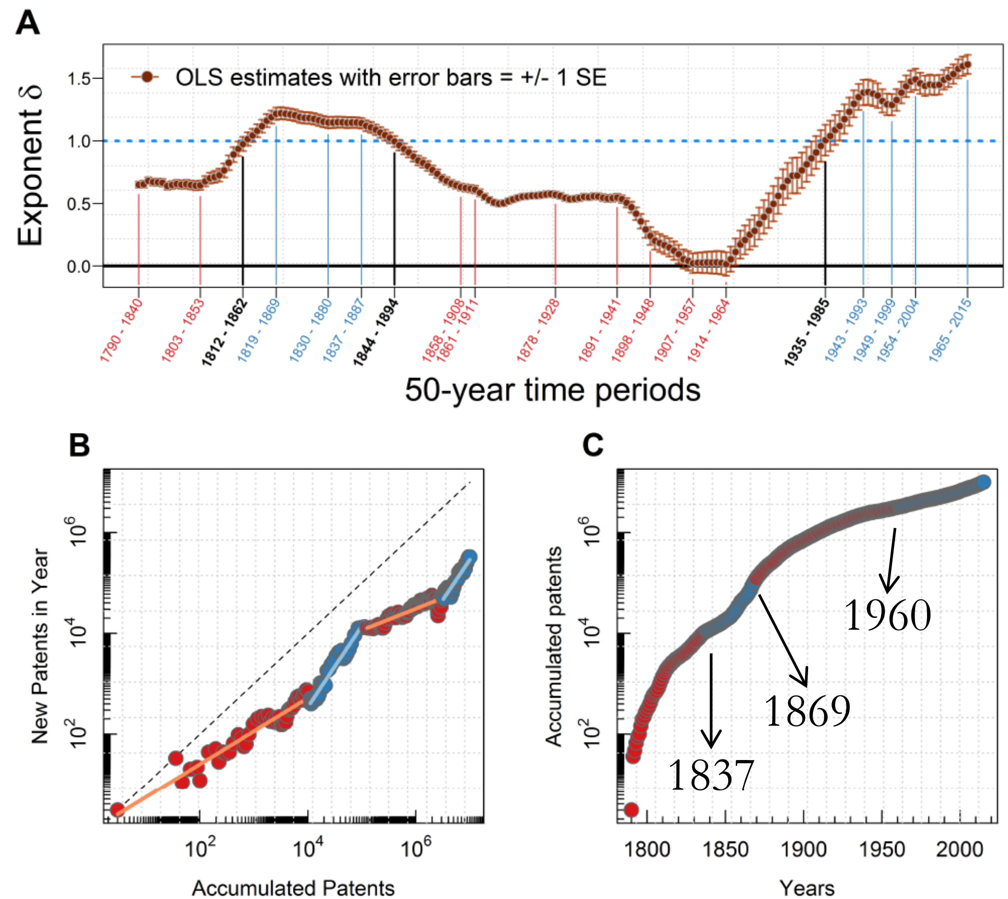

- How can a collective learning process account for the approximately ***sub***-exponential growth, as well as the other regimes?

$$\overbrace{x(t_{n+1})}^{\text{next}} = \overbrace{x(t_n)}^{\text{current}} + \underbrace{\underbrace{x(t_{n-1})}_{\text{old patents}} * \underbrace{(x(t_n) - x(t_{n-1}))}_{\text{recently added patents}}}_{\text{recombination}}$$

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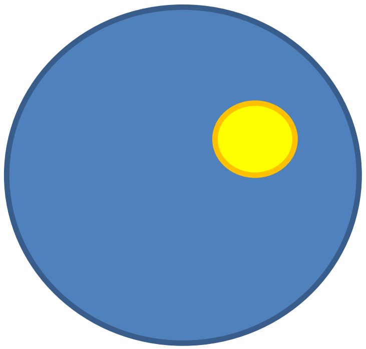

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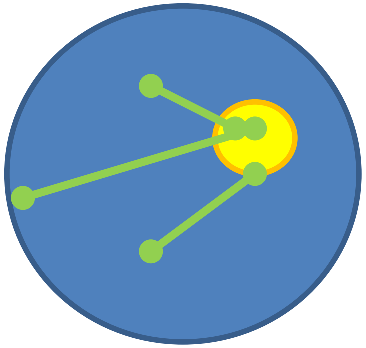

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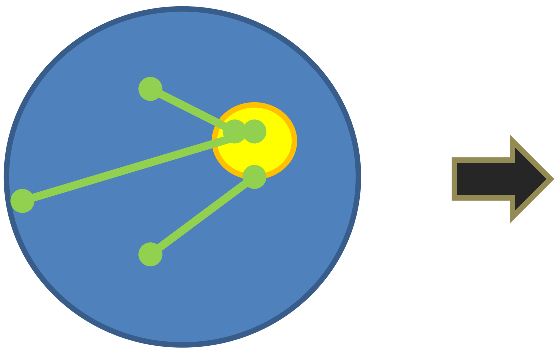

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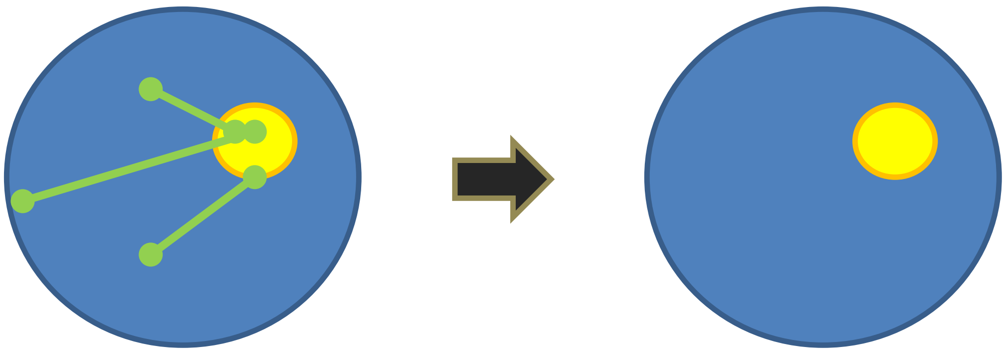

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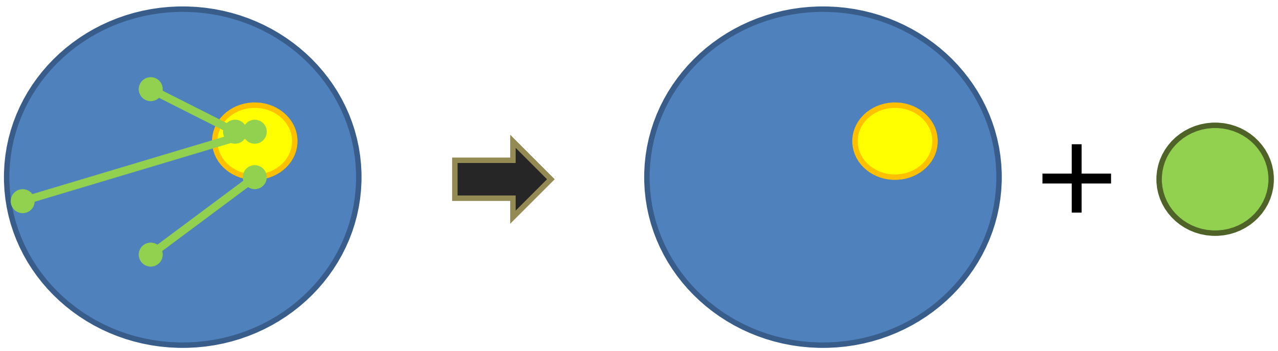

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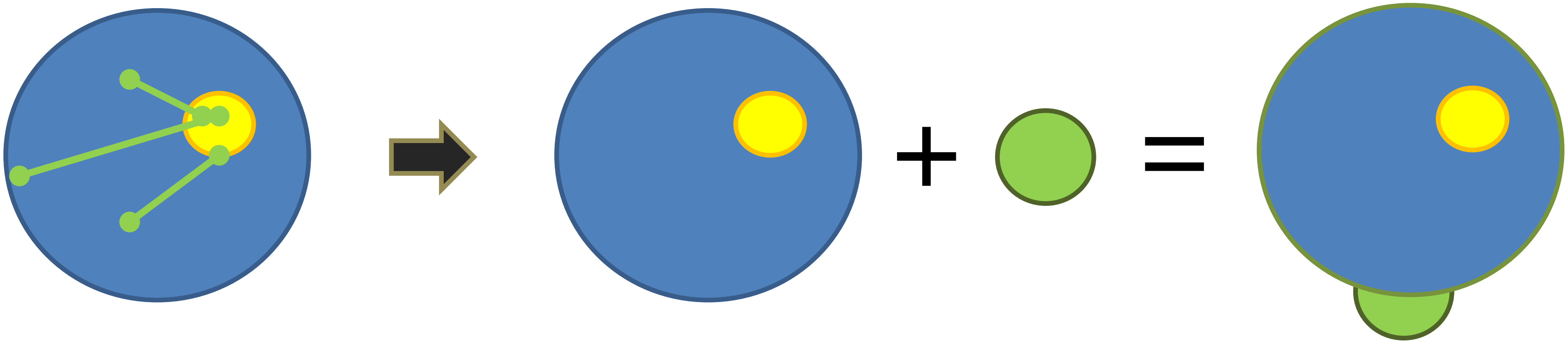

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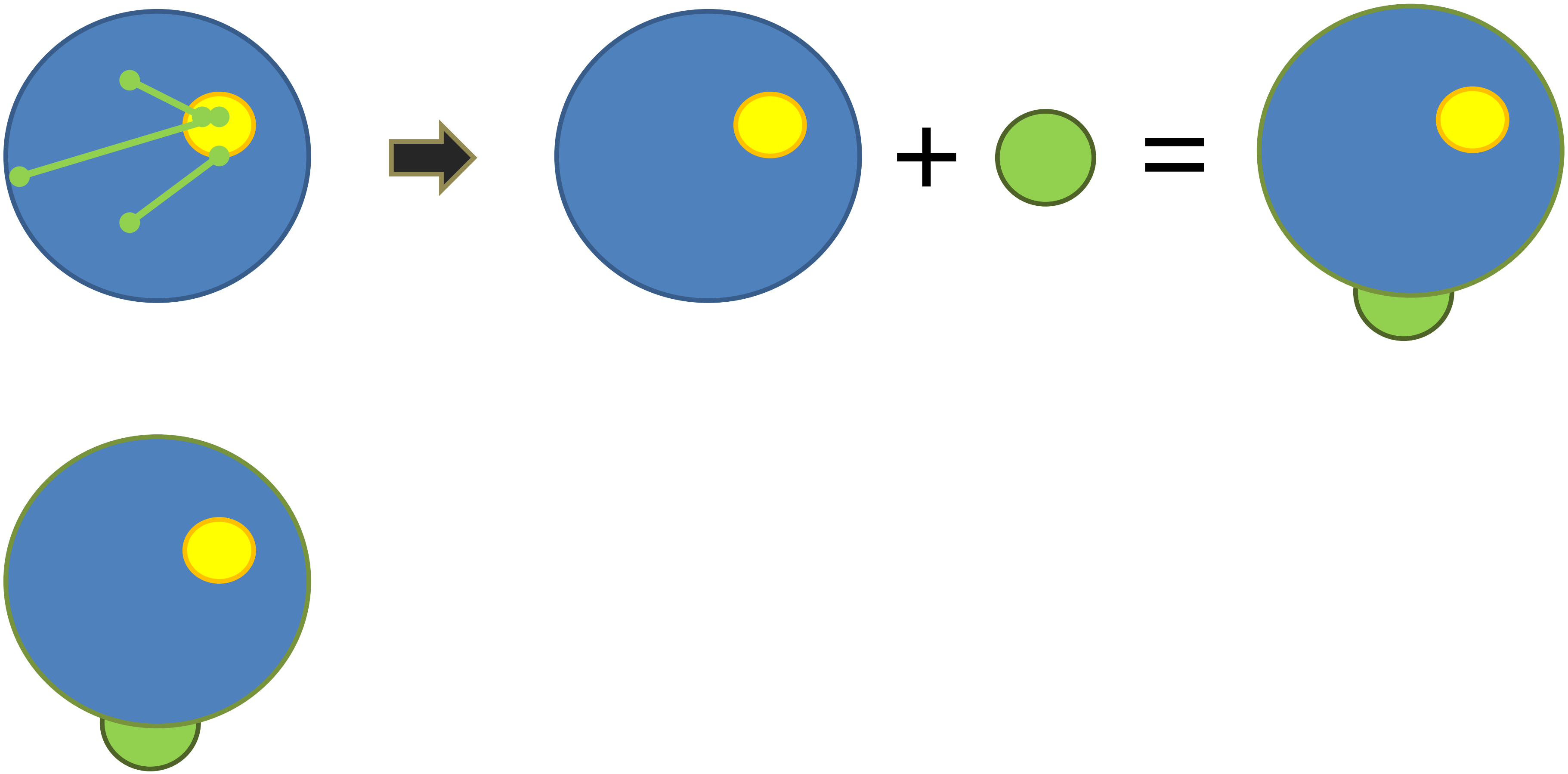

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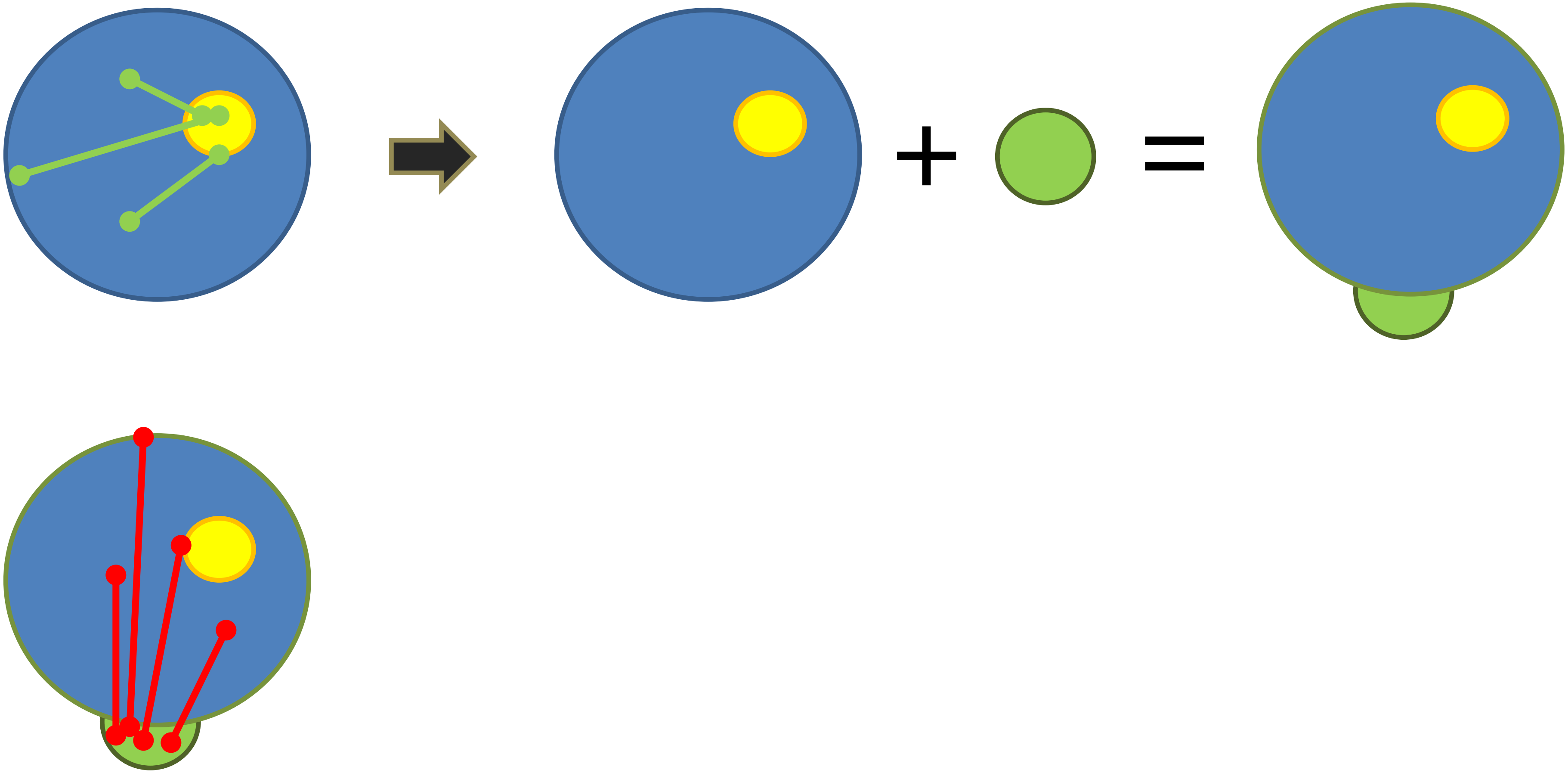

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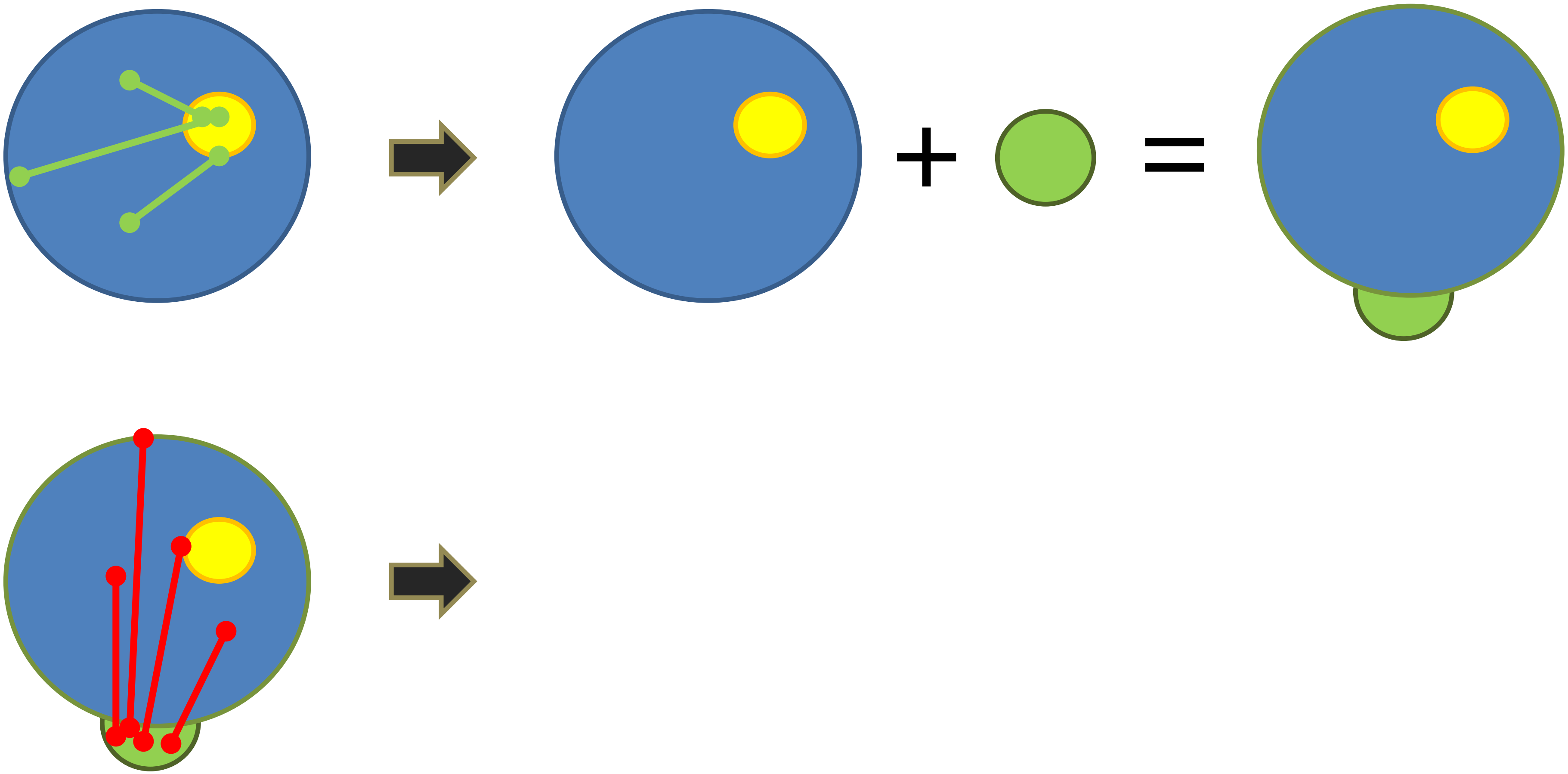

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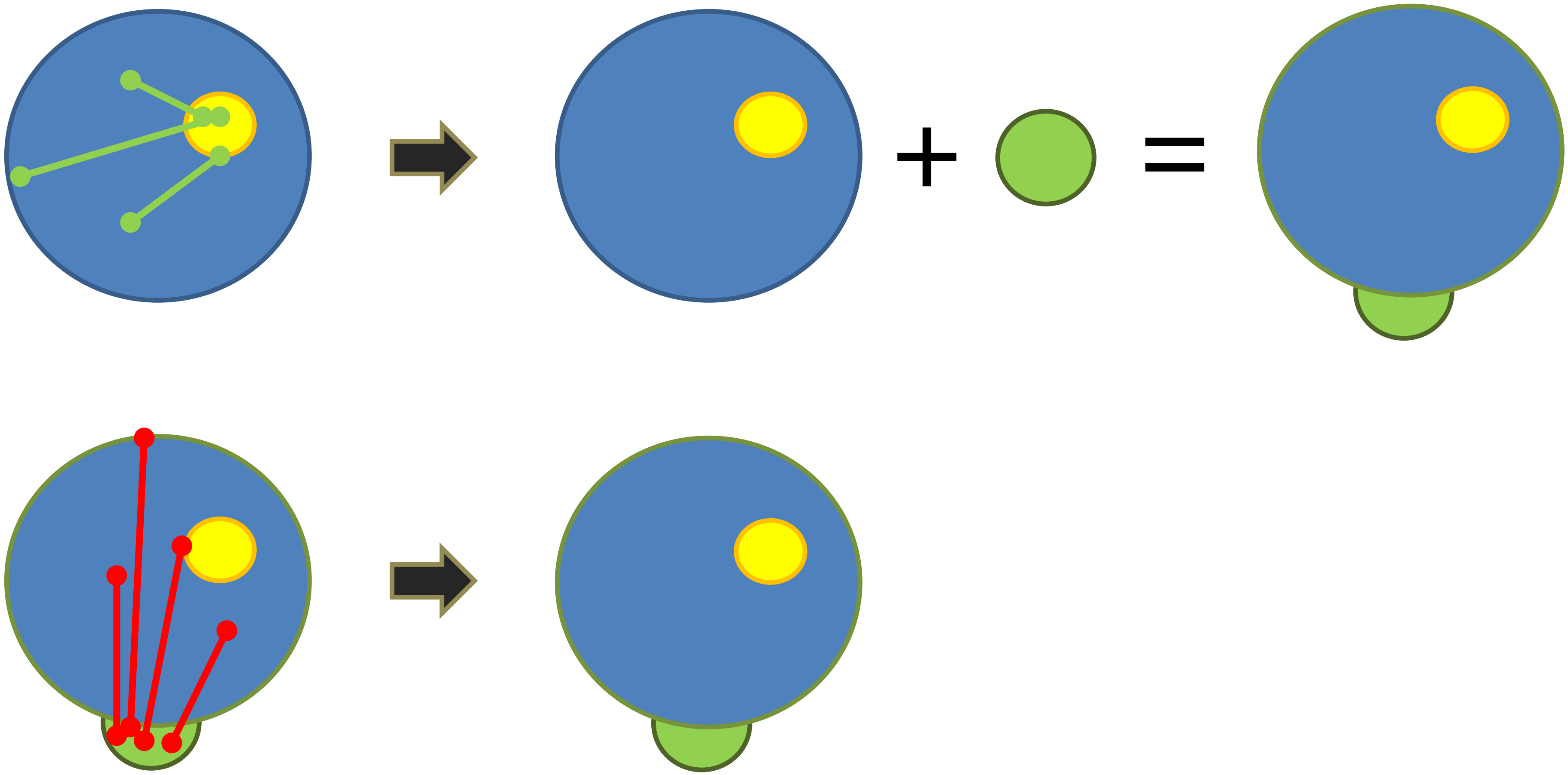

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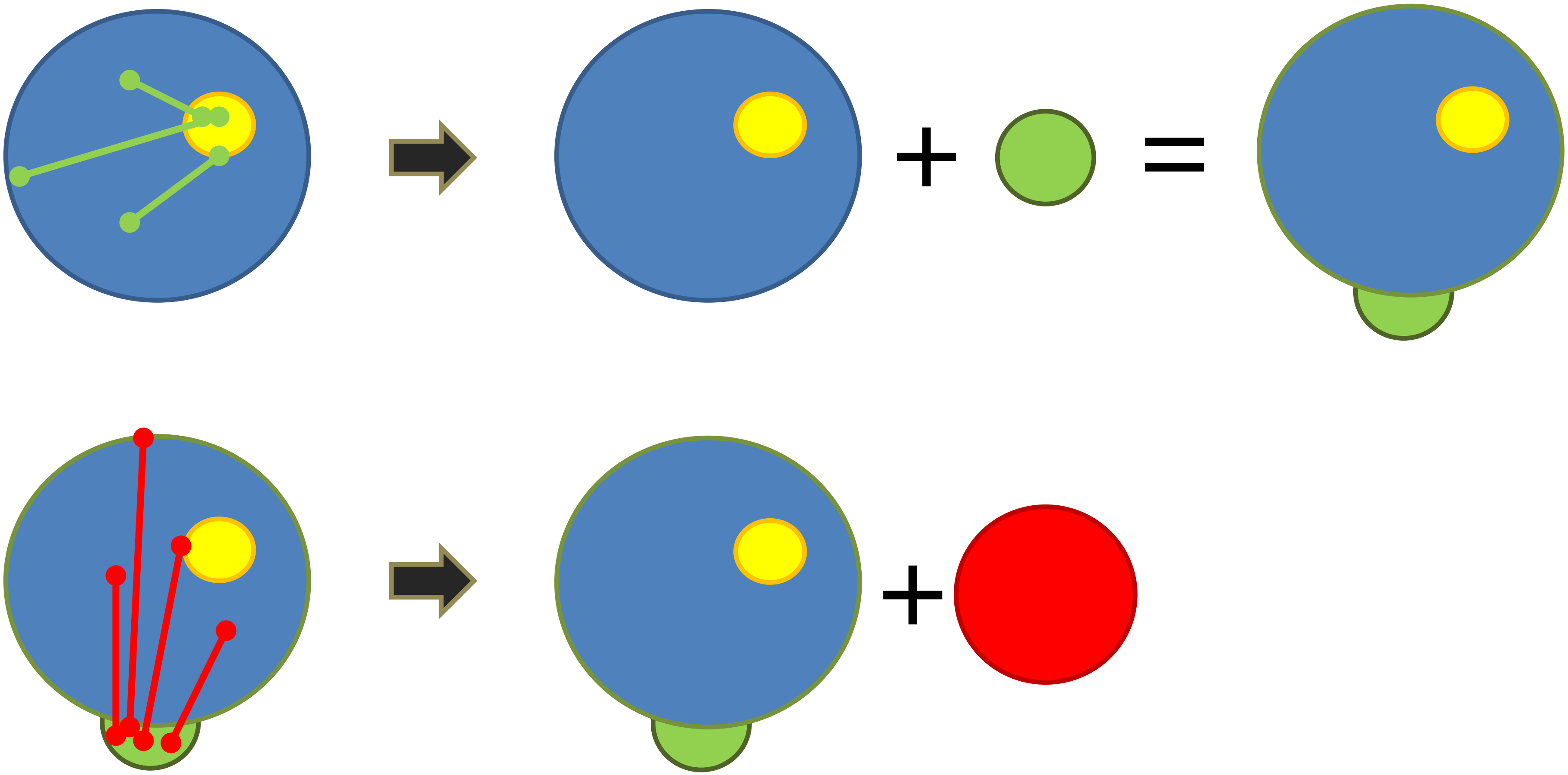

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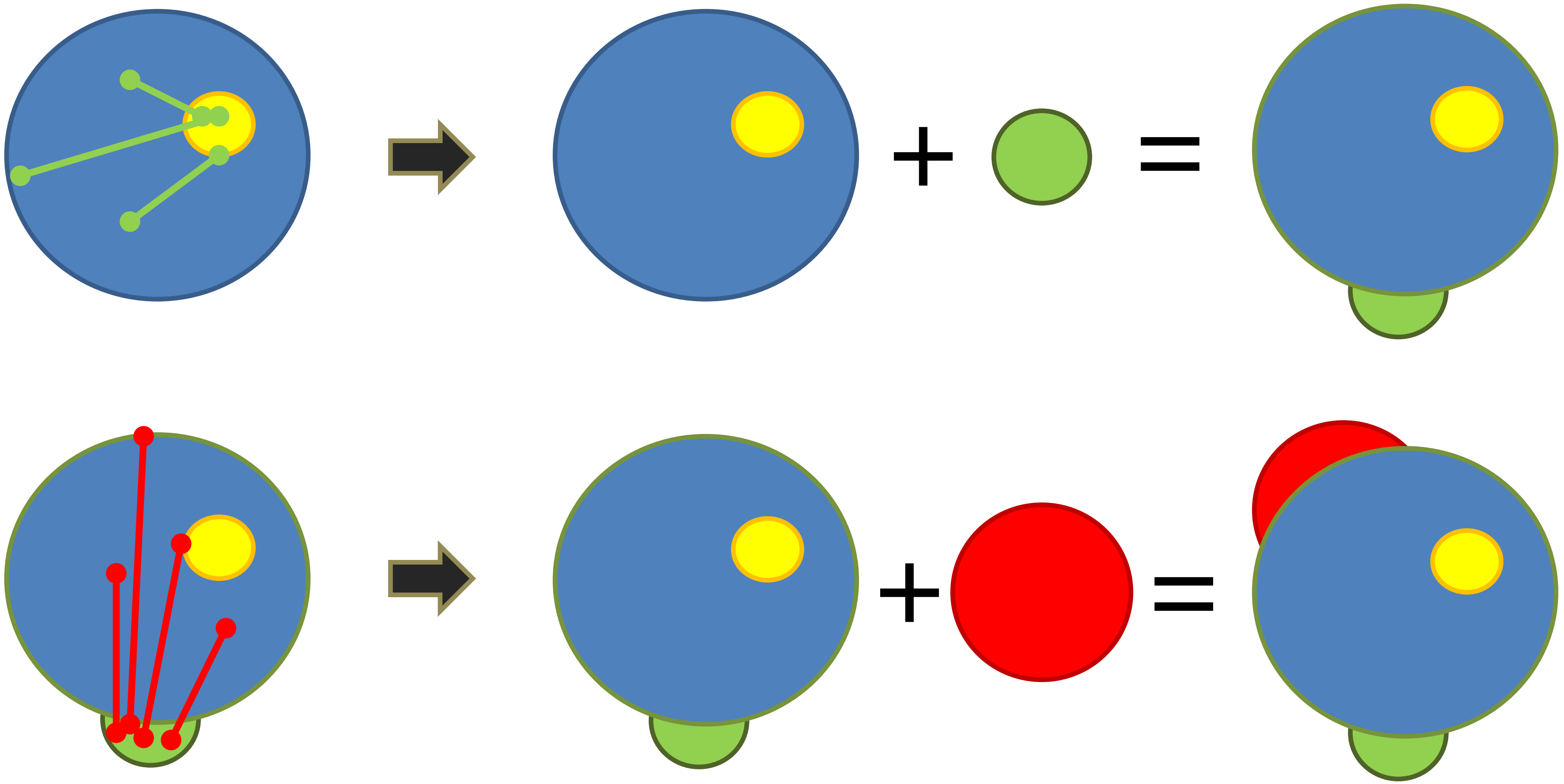

A *very* simple null model of recombination

1. Patents are the only entity being modeled, and the number of patents is denoted by $x(t)$.

2. Patents at any given time can be divided in three: *old* patents, *recently added* patents, and *new* patents.

3. Evolution will happen such that the *new* patents are the pair-wise combinations of the *recently added* patents with the *old* patents. In other words, a combination of an old patent with a recently added patent results in a single novel patent.

In discrete time:

$$\overbrace{x(t_{n+1})}^{\text{next}} = \overbrace{x(t_n)}^{\text{current}} + \underbrace{\underbrace{x(t_{n-1})}_{\text{old patents}} * \underbrace{(x(t_n) - x(t_{n-1}))}_{\text{recently added patents}}}_{\text{recombination}}$$

In continuous time…

Let $y(t) = \dot{x}(t)$.

Generalizing a bit the model so that

- not all re-combinations are possible,
- some patents become obsolete,

The model becomes:

$$\boxed{\frac{\mathrm{d}y(t)}{\mathrm{d}t} = cy(t)(x(t) - 1)}$$

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where $K = \sqrt{2y_0c - c^2}$ and $x_0 = x(0)$.

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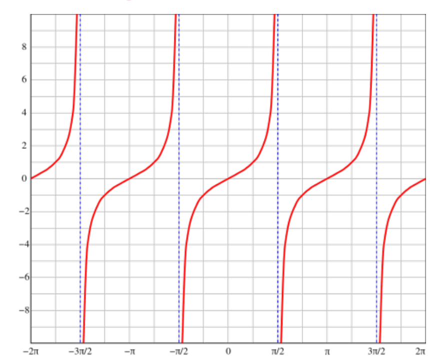
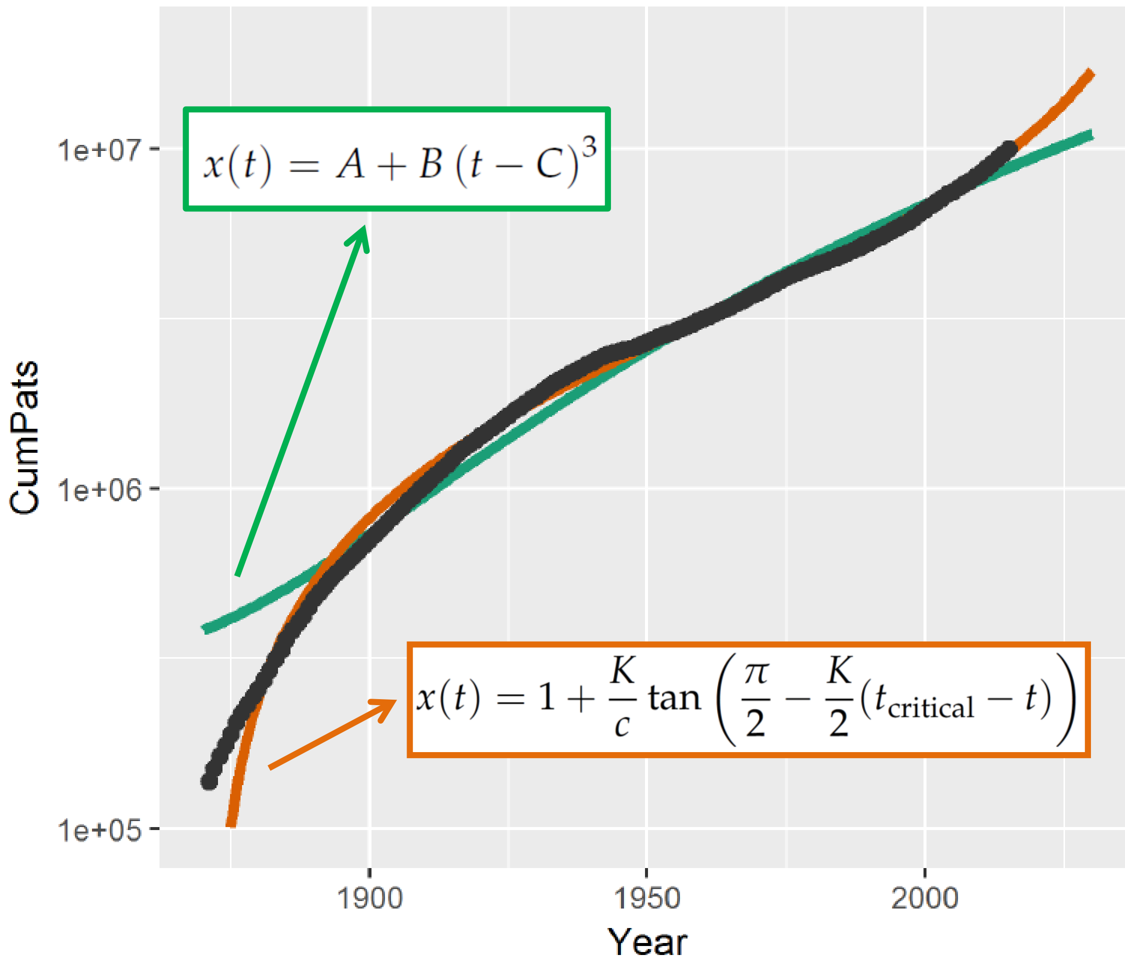
Tangent??



… but does it fit the data?

(analysis done post 1869, after the second regime shift)



Finite-time singularity at $\widehat{t_c} \approx 2051$

Implications

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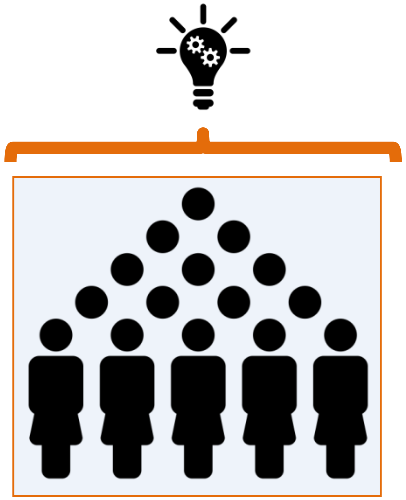
3. Maybe it is the ***sequence*** in which these regimes appear in a time series what is a signal of collective learning.



THANK YOU

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