Distributional Approaches to Understand Patterns of Urban Differentiation

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Deborah Strumsky (SFI)

Kevin Stolarick (MPI)

Vladislav Vysotsky (PDMI, ASU)

The Big Question

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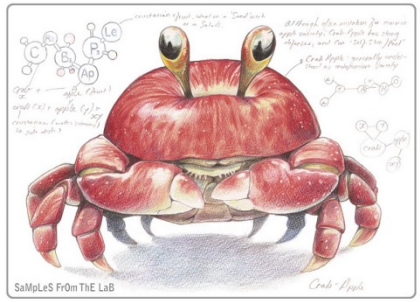

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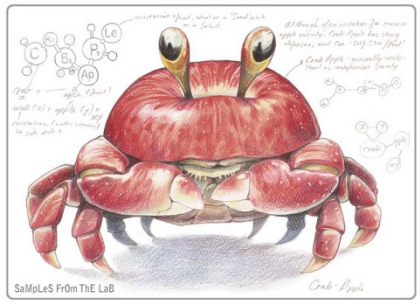

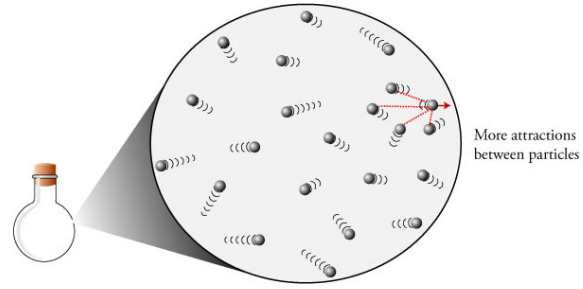

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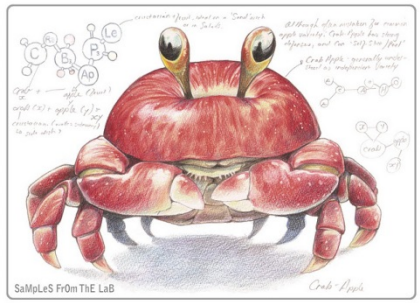
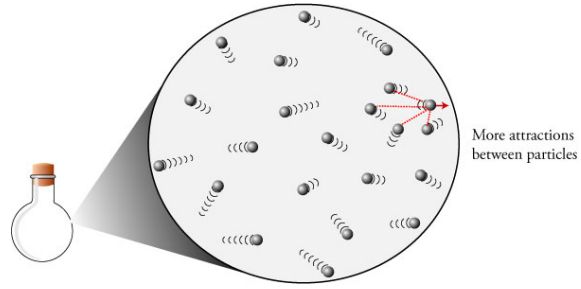
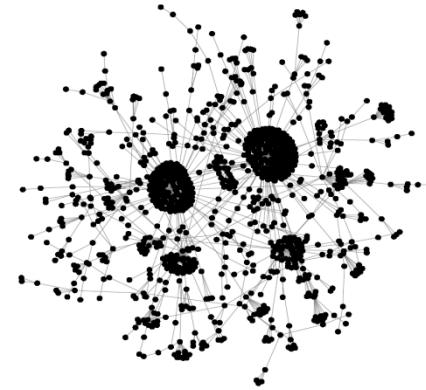


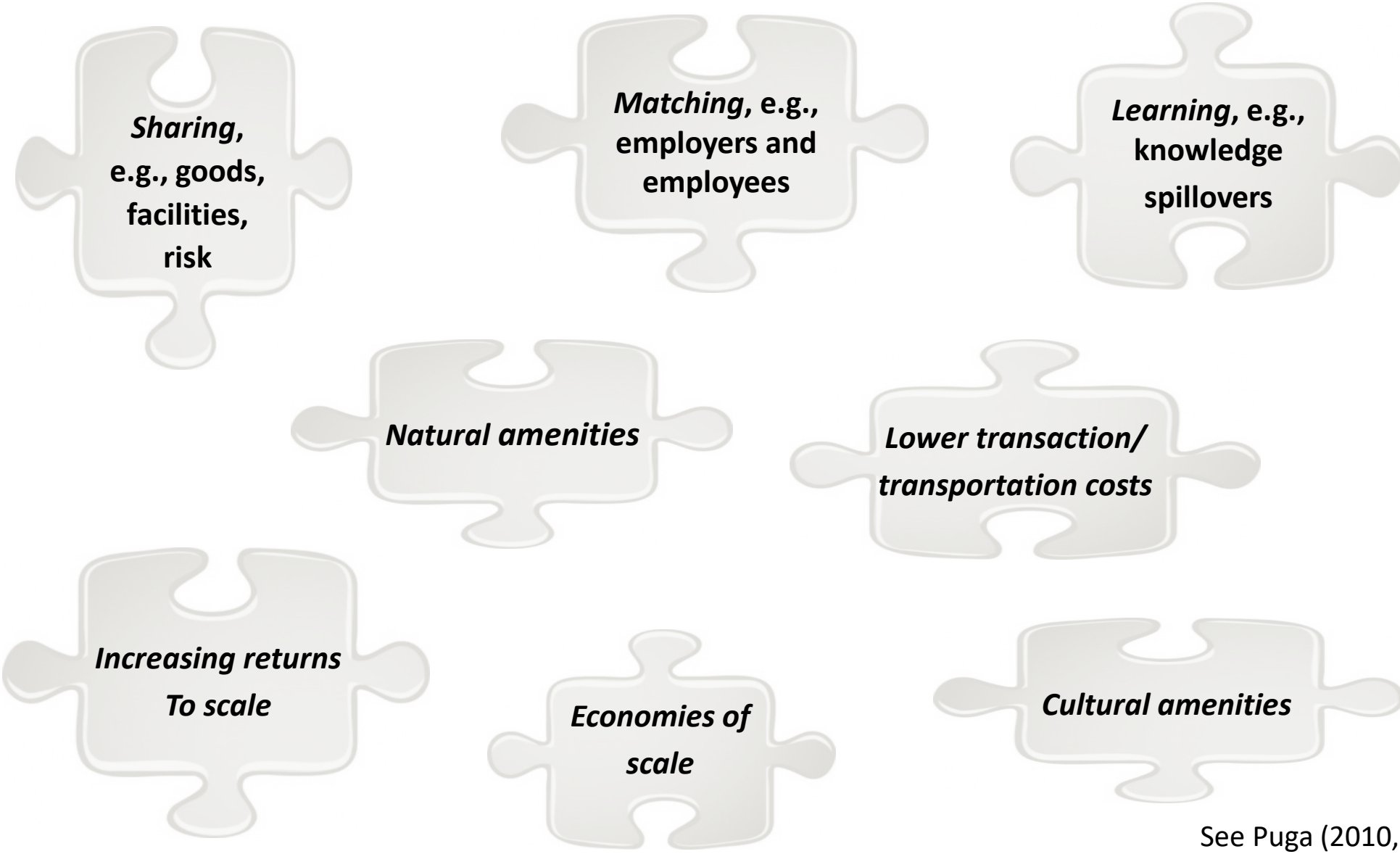




Diversity of: Shapes, Colors, *and* Sizes.

Artist**: Ahmed Farid**. Title: **Urbanization** (Oil on canvas, 2012)
From: http://almasargallery.com/ahmed-farid-urban-diversity-11-may-30-may-2013-solo-exhibition

Diversity and the Puzzle of Cities



See Puga (2010, *JRS*)

Explaining Productivity Differentials

- **Classical models:**

$$\frac{Y}{N} = A\left(\frac{K}{N}\right)^{\kappa}\left(\frac{L}{N}\right)^{\lambda}\left(\frac{H}{N}\right)^{\eta}\left(\frac{M}{N}\right)^{\mu}\cdots\left(\frac{P}{N}\right)^{\pi}, \quad \kappa+\lambda+\ldots+\pi=1$$

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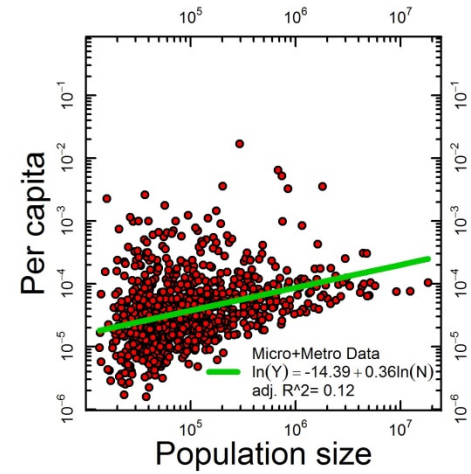

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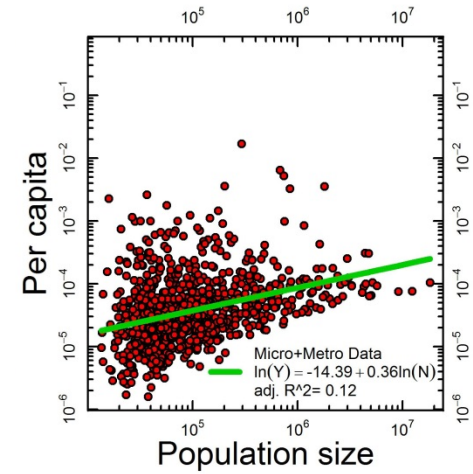

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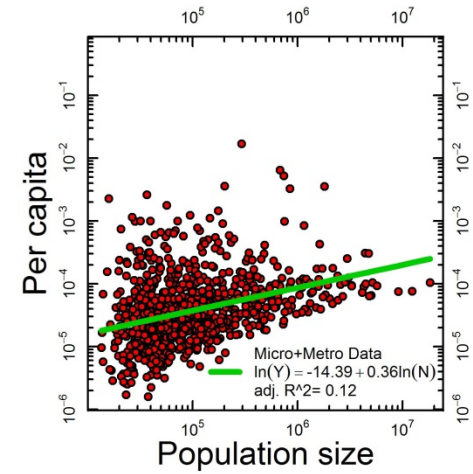

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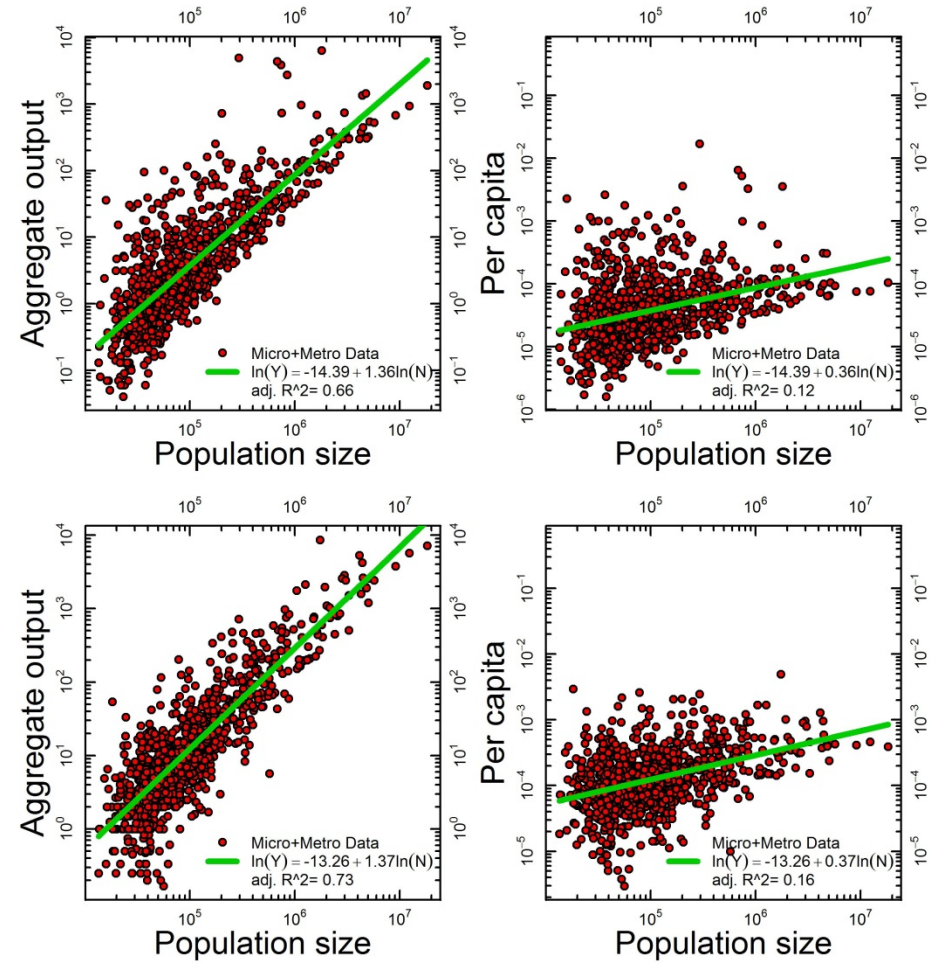

Simple Null Model

Simulated data!

- Real populations,
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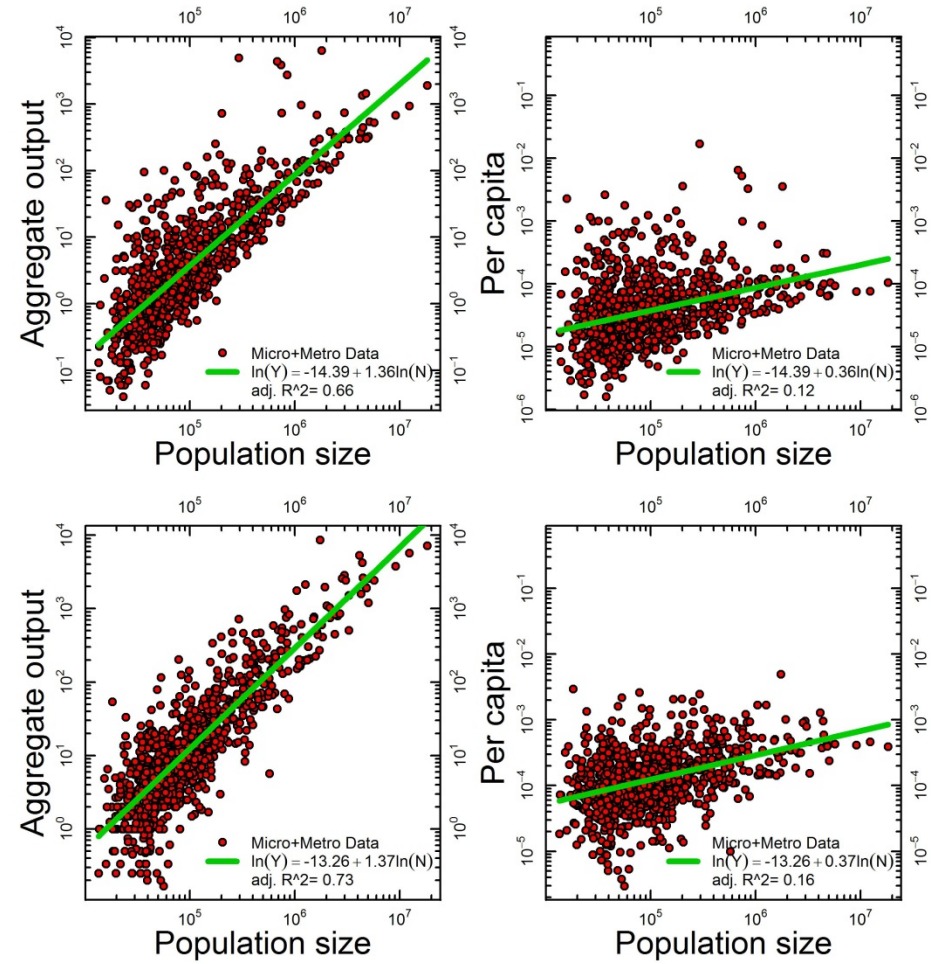

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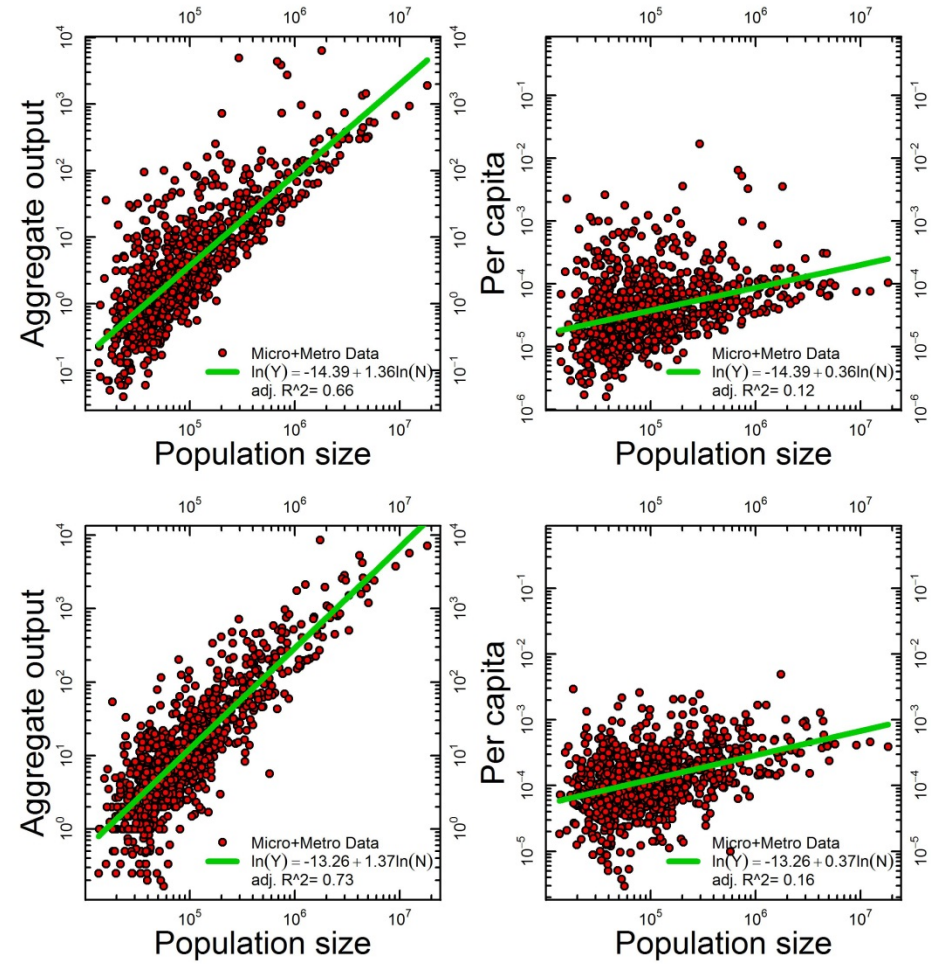

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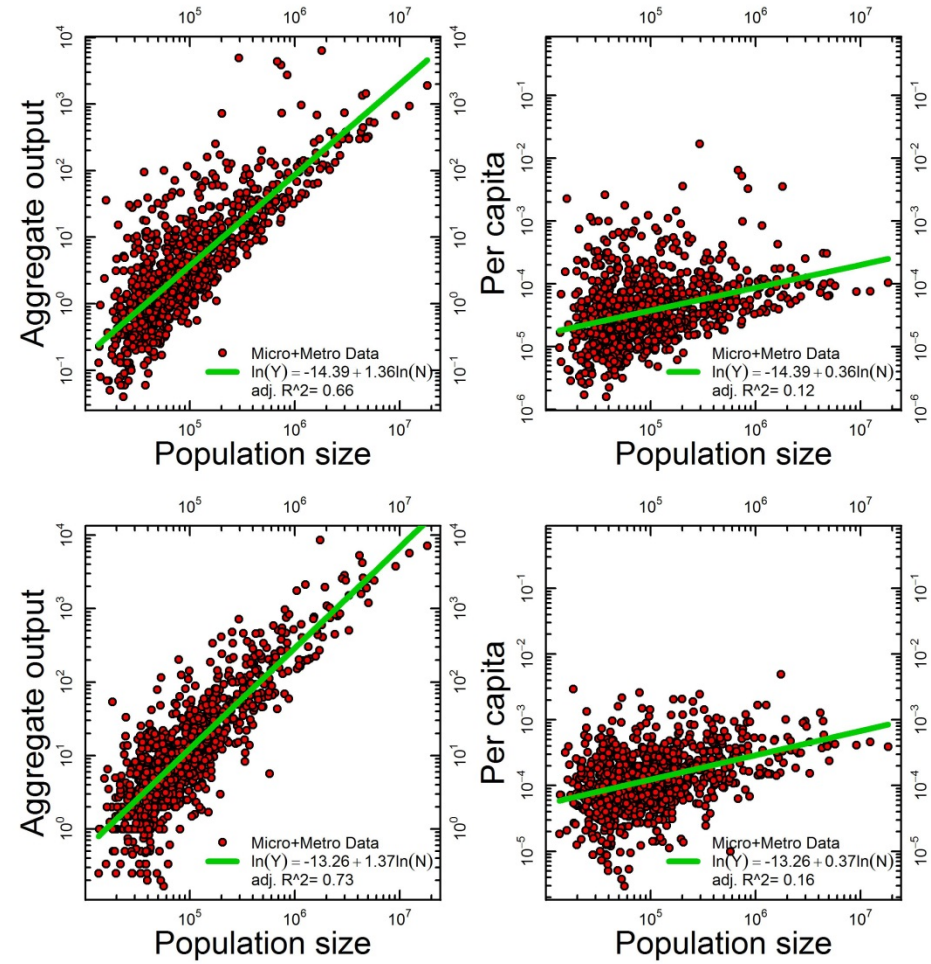

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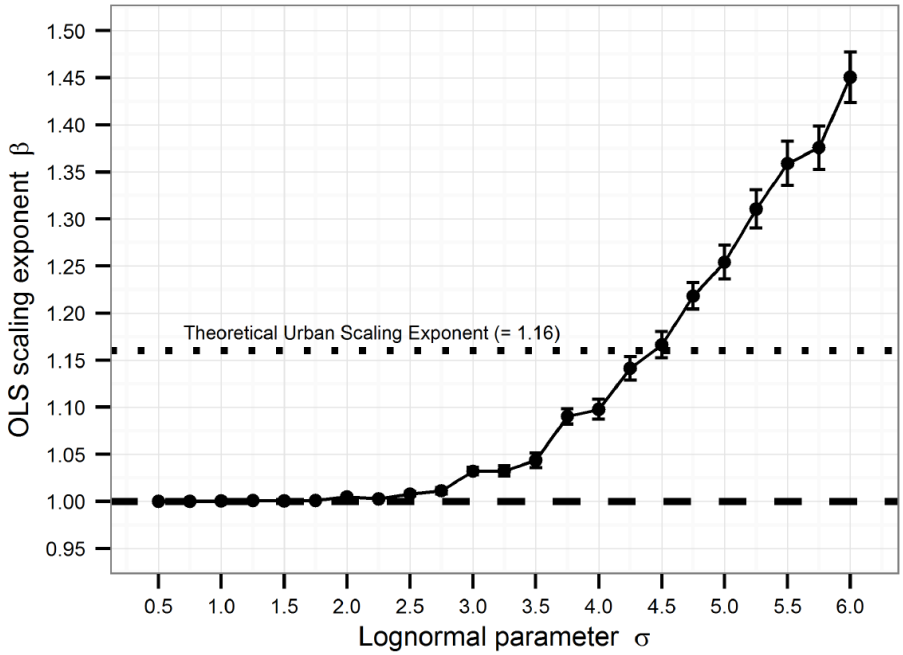

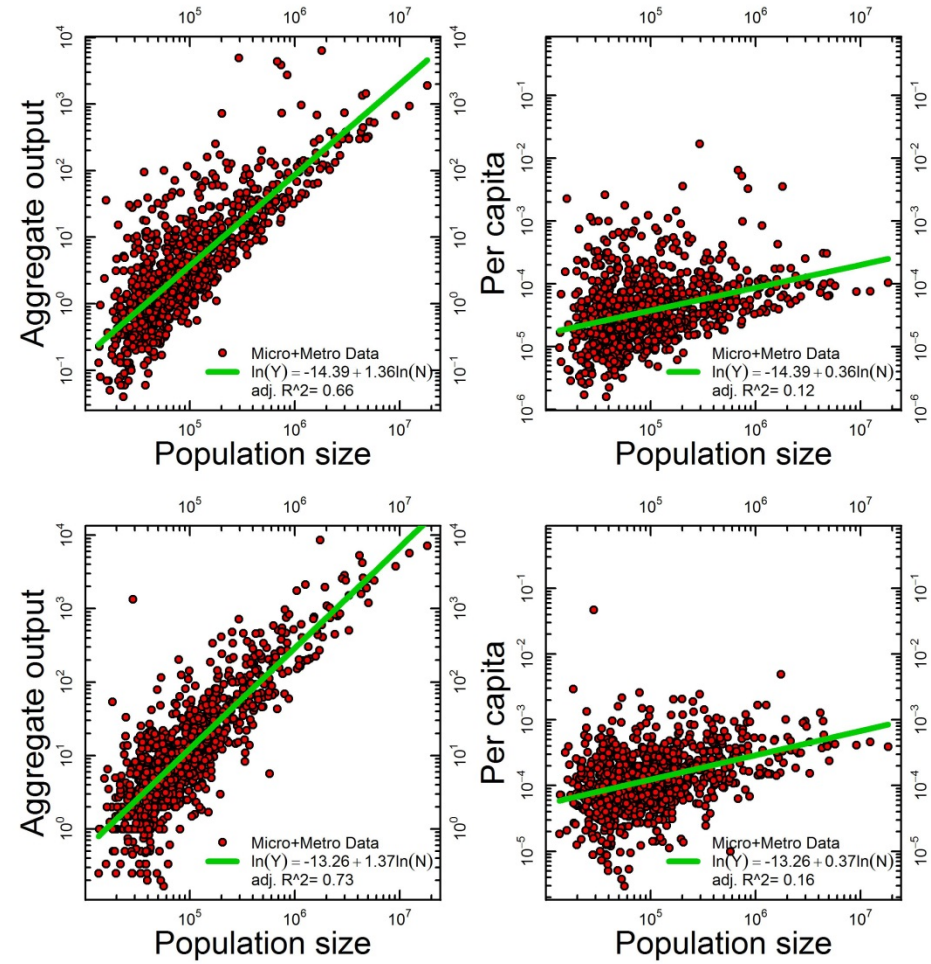

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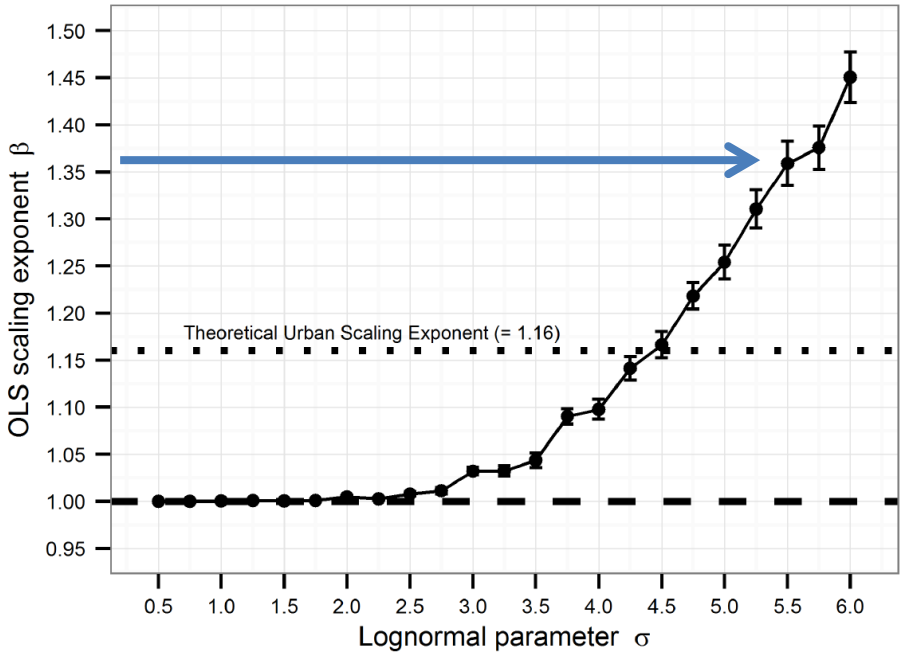

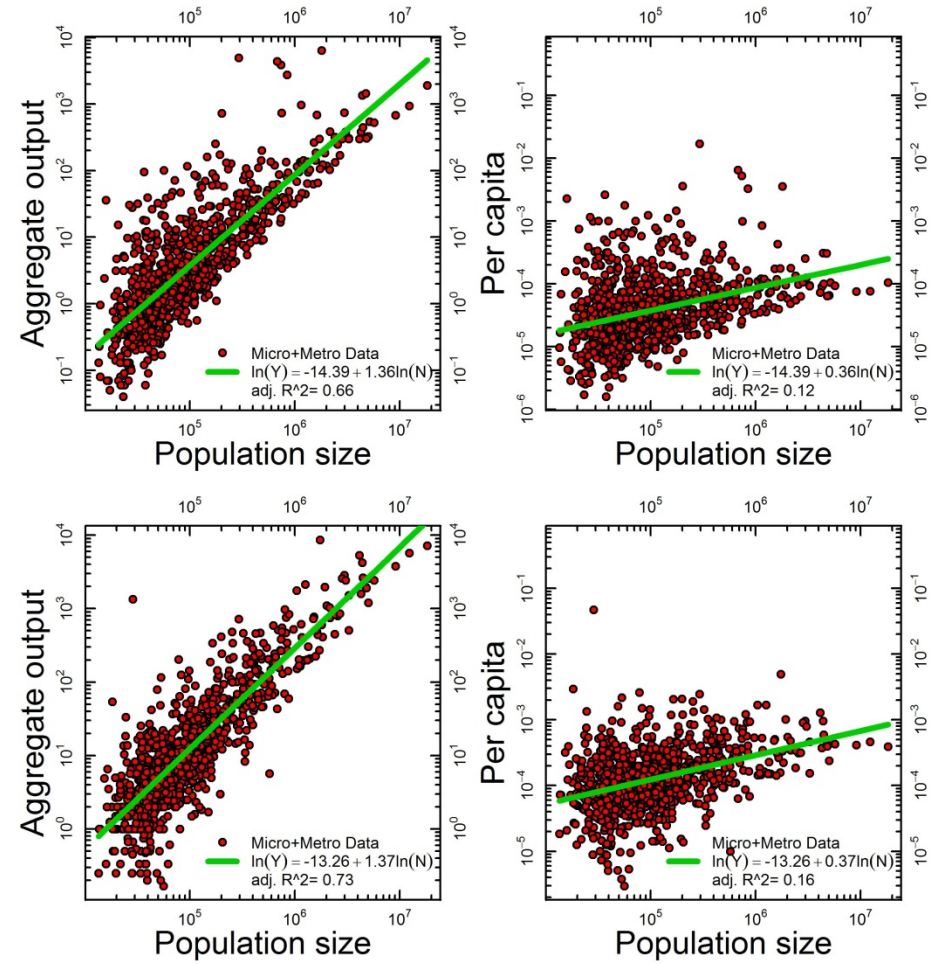

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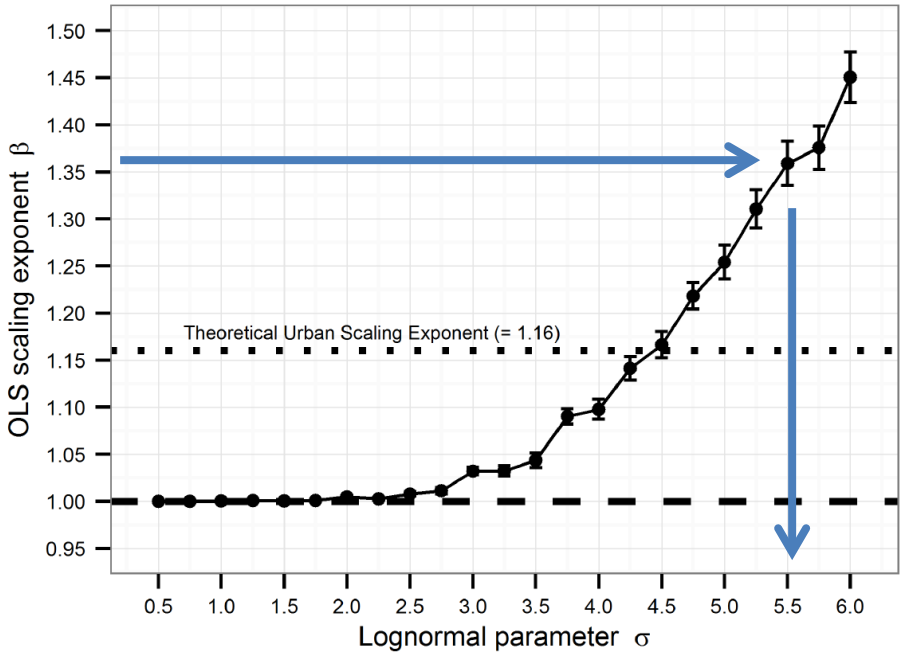
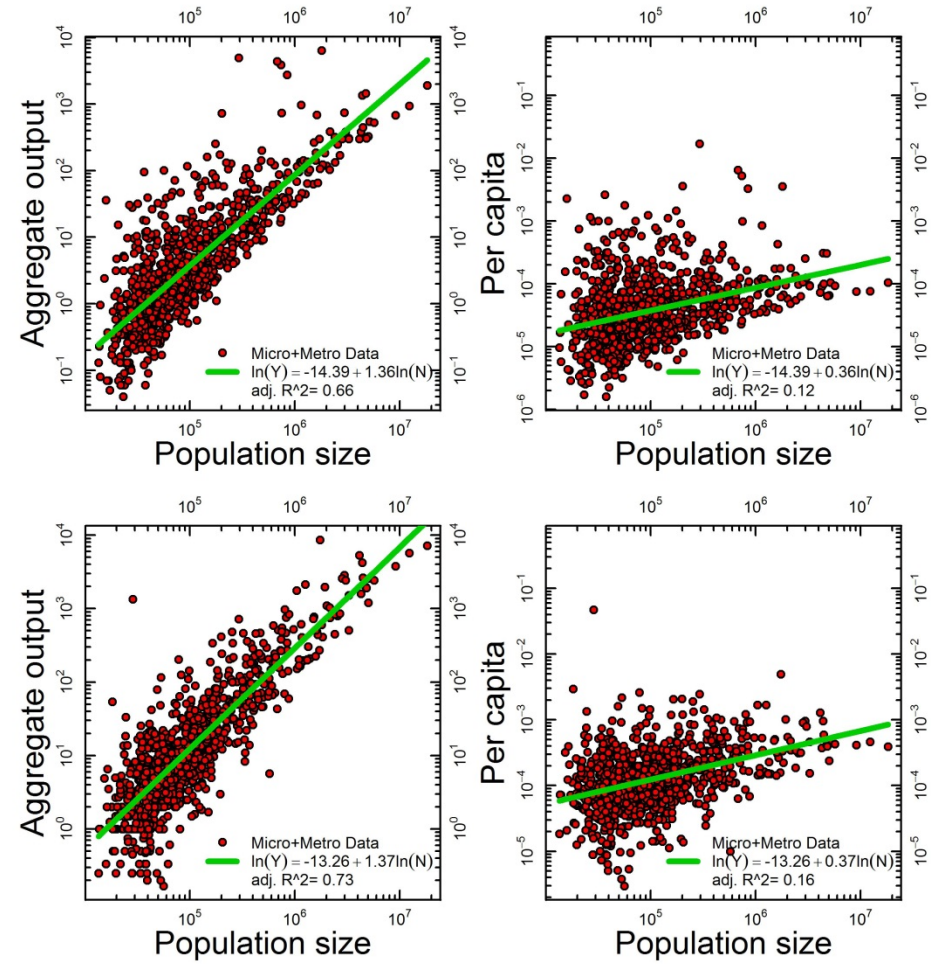


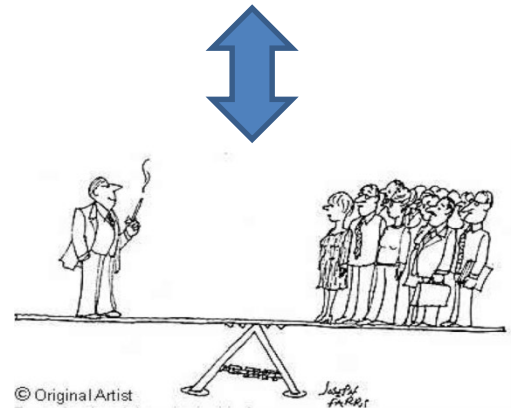


The interplay between city **sizes** and their internal **heterogeneity** determines**:**

- how we should **measure** productivity,
- and the **statistical properties** of the **aggregate output.**

Facing the Heterogeneity

Are cities **finite** systems that **violate** the *Law of Large Numbers*?



Gomez-Lievano, Vysotsky, and Lobo (2014, *in preparation*)

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- "tacit knowledge"
- People move with *less difficulty* than their ideas
  - (See Breschi & Lissoni, 2001a,b, 2004, 2009; Klepper & Sleeper, 2005; Klepper, 2010; Hausmann, Neffke, Otto, 2013)

Conceptually, the economic output of an entire city (a *system-wide* property) should not be understood using the average properties of individuals (e.g., years of schooling)

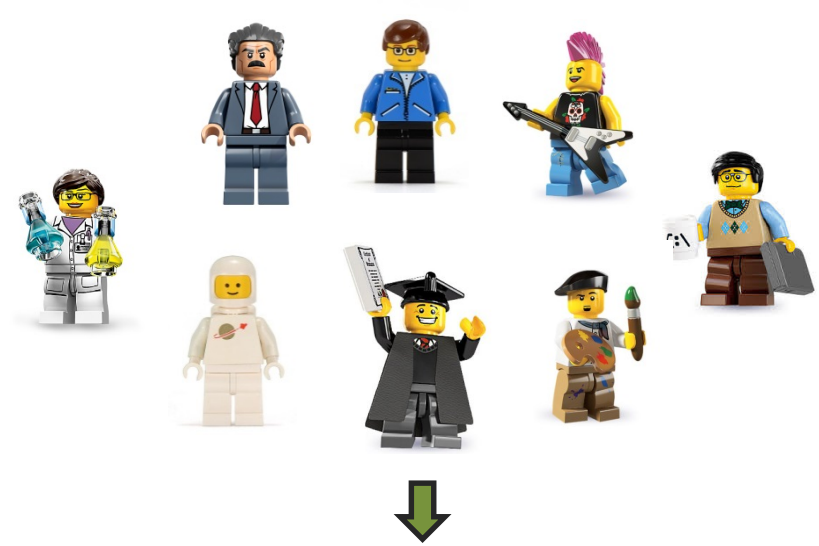
What citizens *do*

- Creative Class (Florida, 2004)

- Inventors

Obstacles to Economic Development

- Are the counts of ‘creatives’ and inventors ***constrained*** in urban areas?

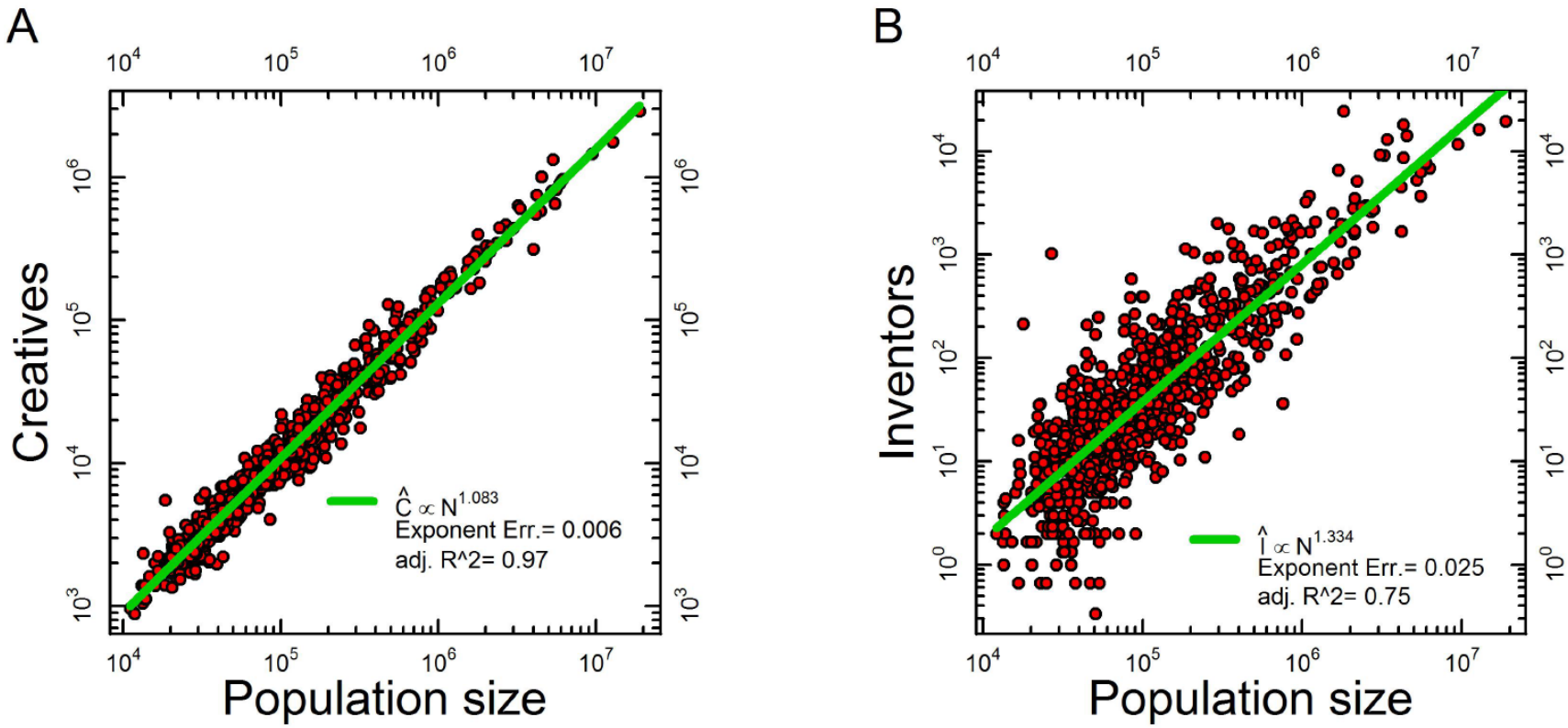


Gomez-Lievano, Bettencourt, Stolarick, Strumsky, Lobo (2014, *in submission*)

Assumption

The ability of a city to “learn” is constrained by its **capacity to attract** creative and inventive individuals (Florida, 1995, *Futures*).

More than 70% explained by urban population



$$E(Y|N) = Y_0 N^{\beta}$$

(Rauch, 1993; Glaeser & Saiz, 2004; Bettencourt et al., 2007)

Methodology & Results: $P(Y,N)=P(Y|N)P(N)$

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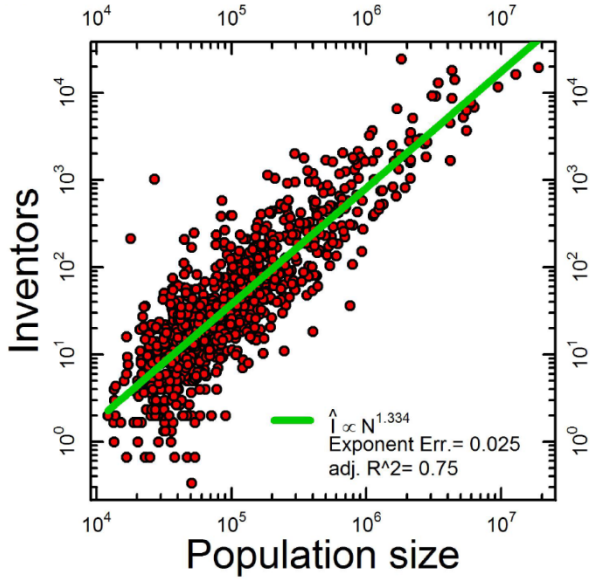

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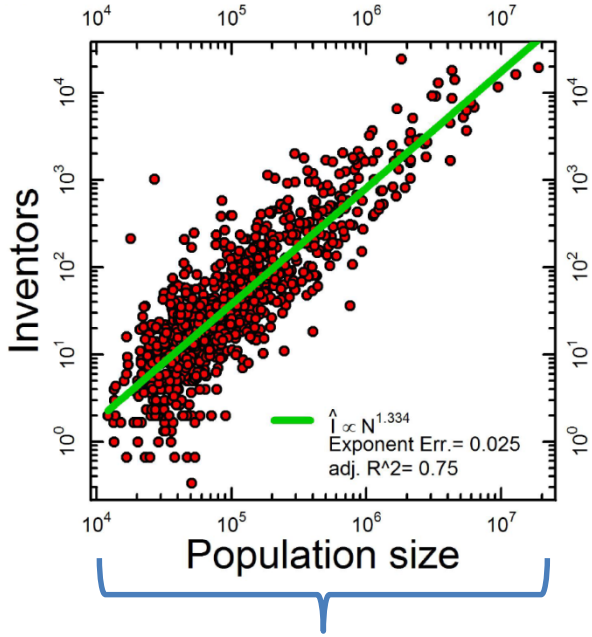

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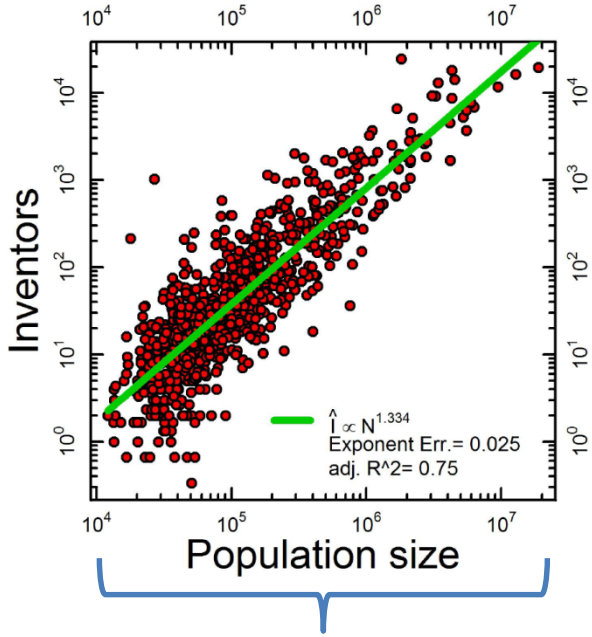

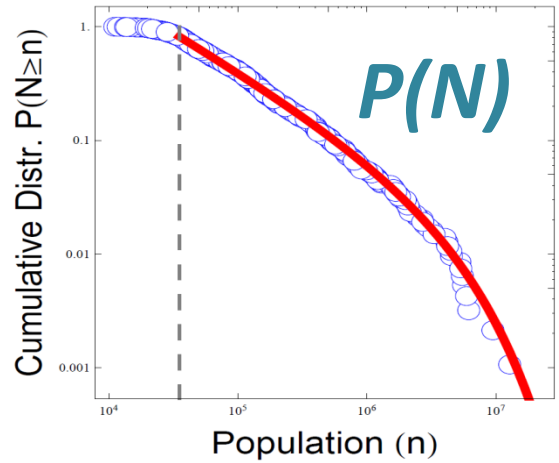

$$p_N(n;\alpha,\beta,\nu,n_{\min}) = C\frac{\mathrm{e}^{-\left(\frac{n}{\nu}\right)^{\beta}}}{\left(\frac{n}{\nu}\right)^{\alpha}}, \quad n \geq n_{\min}$$

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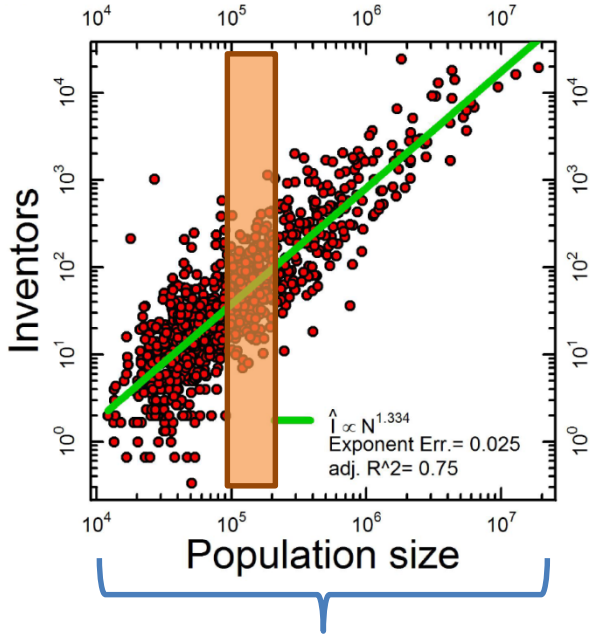

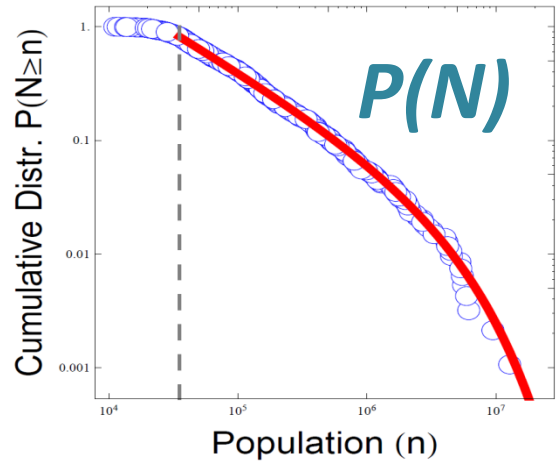

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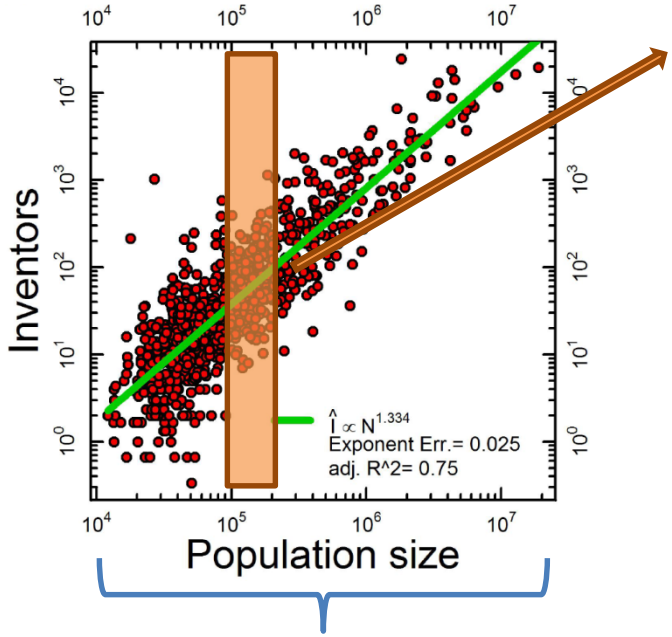

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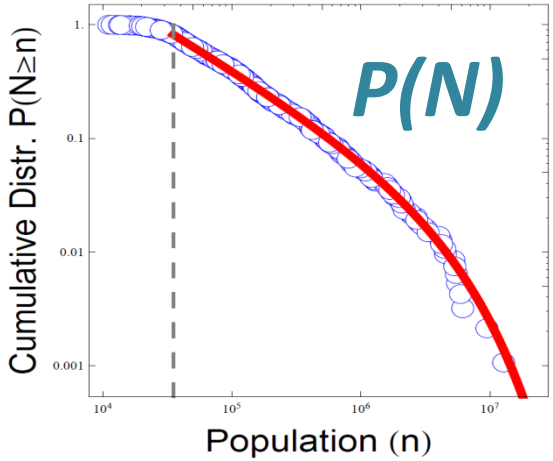

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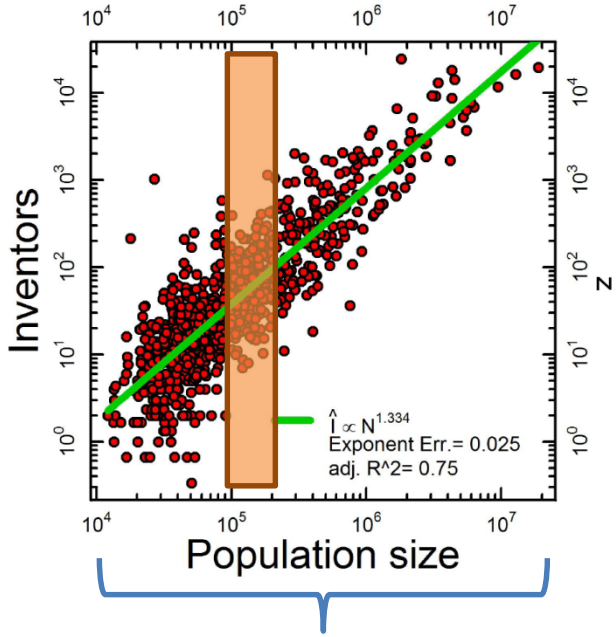

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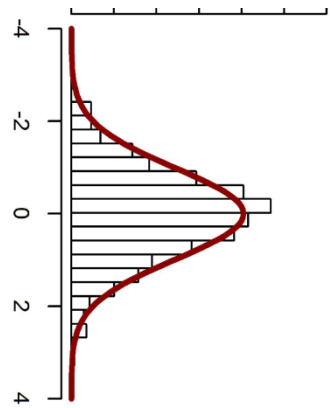

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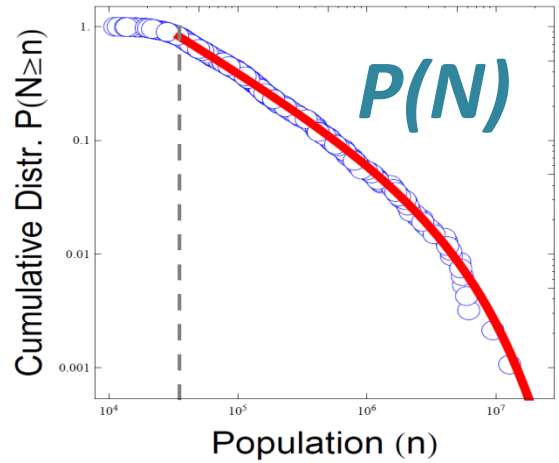

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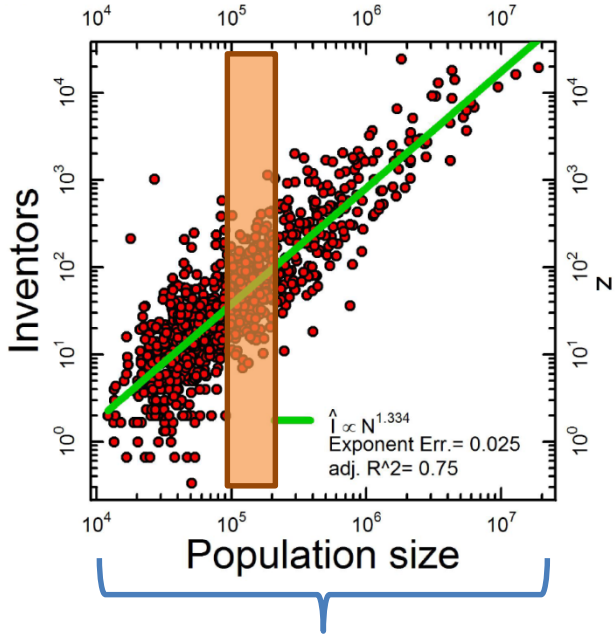

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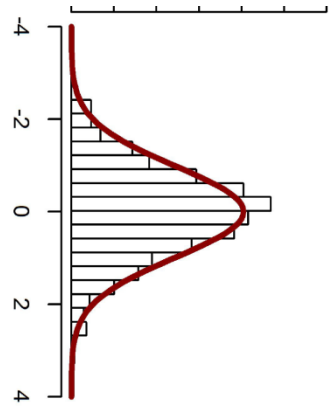

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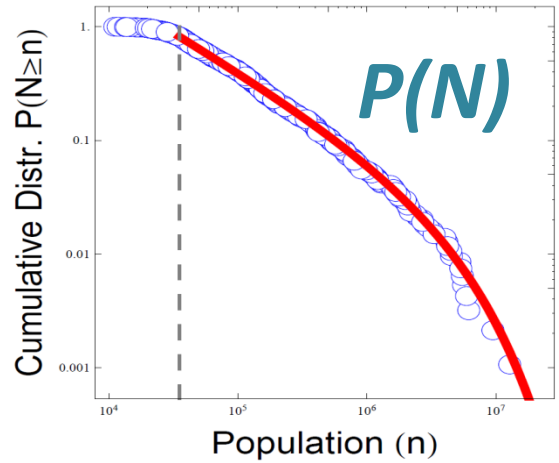

P(N)

$$p_N(n;\alpha,\beta,\nu,n_{\min}) = C\frac{\mathrm{e}^{-\left(\frac{n}{\nu}\right)^{\beta}}}{\left(\frac{n}{\nu}\right)^{\alpha}}, \quad n \geq n_{\min}$$

For a similar study regarding homicides in Brazil, Colombia, and Mexico:
Gomez-Lievano, Youn, Bettencourt (2012, *PLoS ONE*)

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- Jaynes, 2003; Frank, 2009; Frank and Smith, 2011

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- *“Size”, “Heterogeneity” and “Structure”*
- *Policies and Distributions*

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THANKS!

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generate lognormal distributions in output?

Playing with a model of the world

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I expand H&H's model in **two** ways:

1. I will model a **process of acquisition** of skills (pbytes).

2. I will model *what* cities produce, as a **step** to know ***how much*** they produce.


C:\

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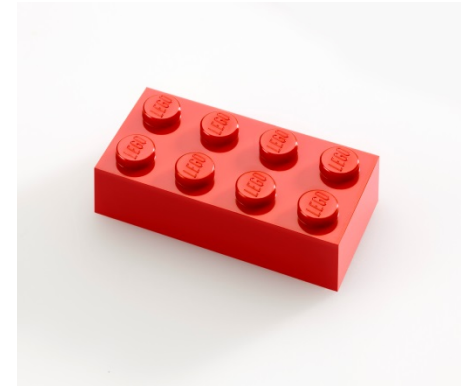

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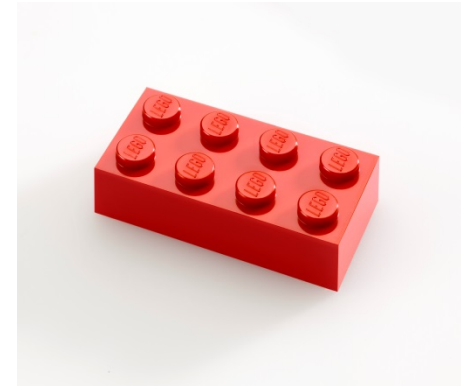

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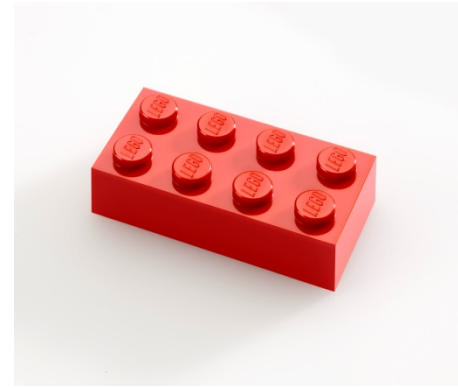
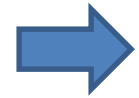
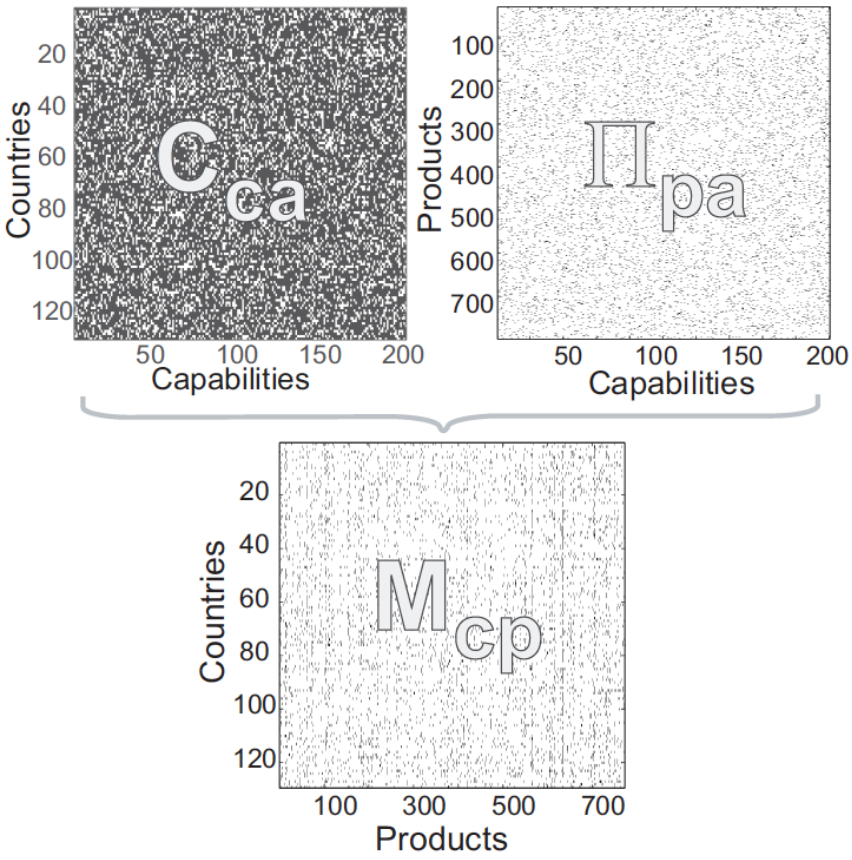




Lego analogy inspired by:
TEDxBoston - César A. Hidalgo - Global Product Space

The Model



From: Hidalgo, C. A. and Hausmann, R. (2009). The building blocks of economic complexity. PNAS, 106(25):10570-10575.

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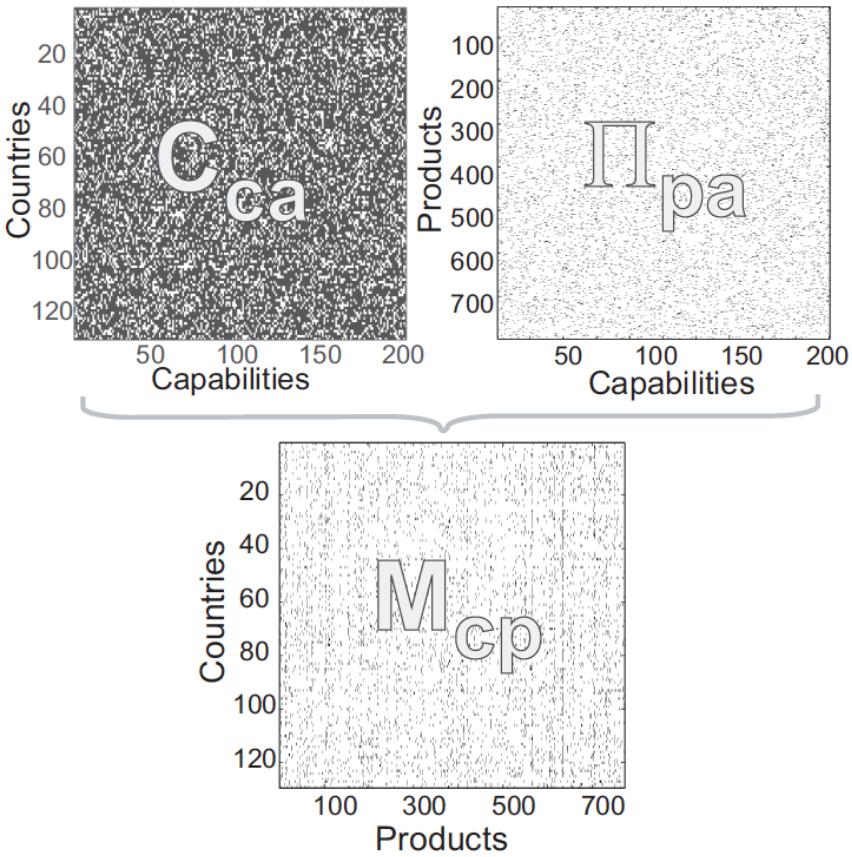

$$X^t = C^t \odot P$$

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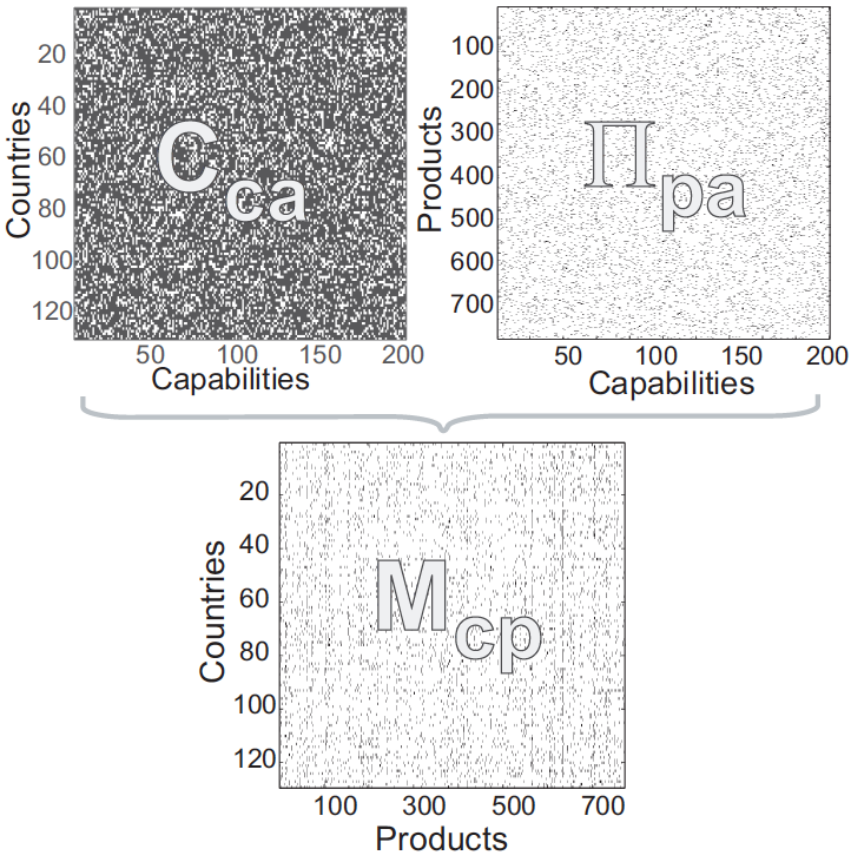

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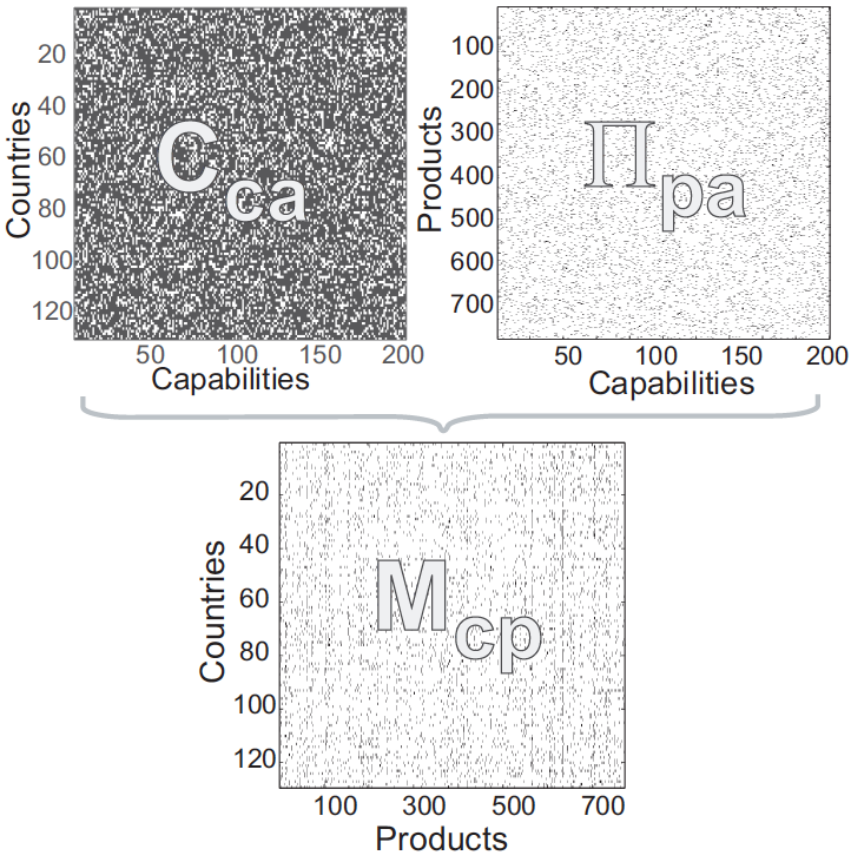

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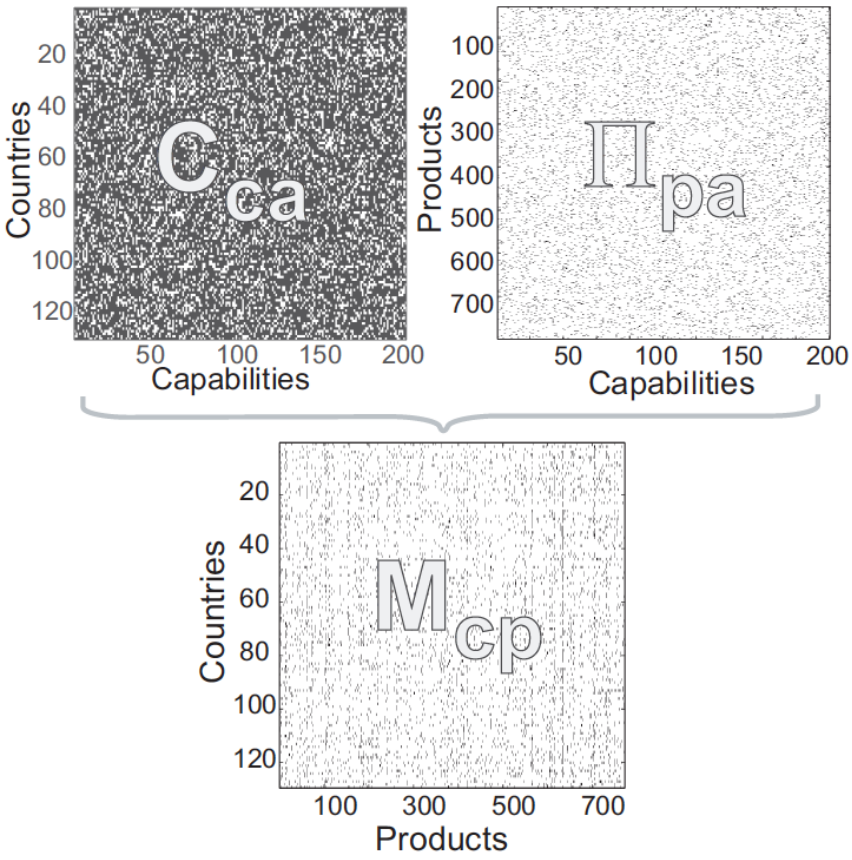

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Model (currently):

1. The matrix ***C*** starts empty.
2. The matrix ***P*** is Bernoulli-filled (with prob. q), and fixed.
3. For each city (i.e., row) in ***C***,
   i. a random skill is added,
   ii. and lost with a certain probability.
4. We calculate the matrix ***X*** of output

Will the model reproduced data?

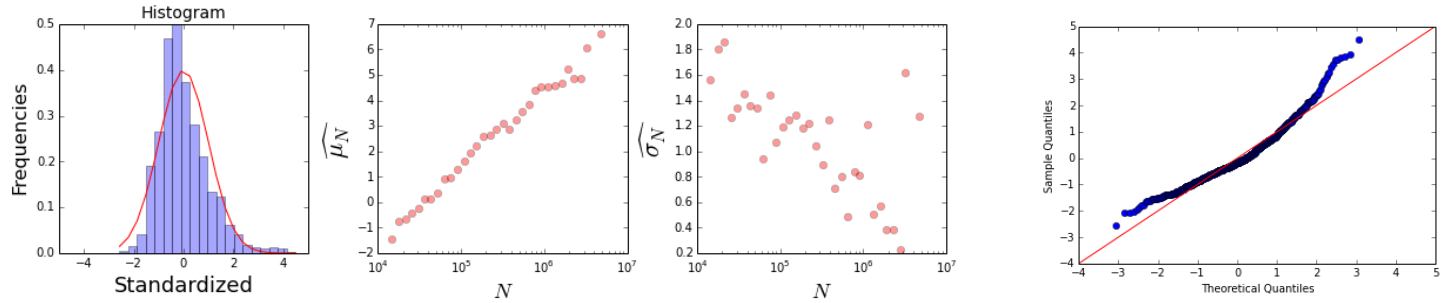
$$y_c^t = \sum_{p=1}^{N_p} X_{cp}^t$$

$$pop_c = \sum_{a=1}^{N_a} C_{ca}$$

$$\Pr\left(y_c^t \mid pop_c\right) \sim \text{Lognormal}$$

Profiles of P(Y|N)

Null Model:



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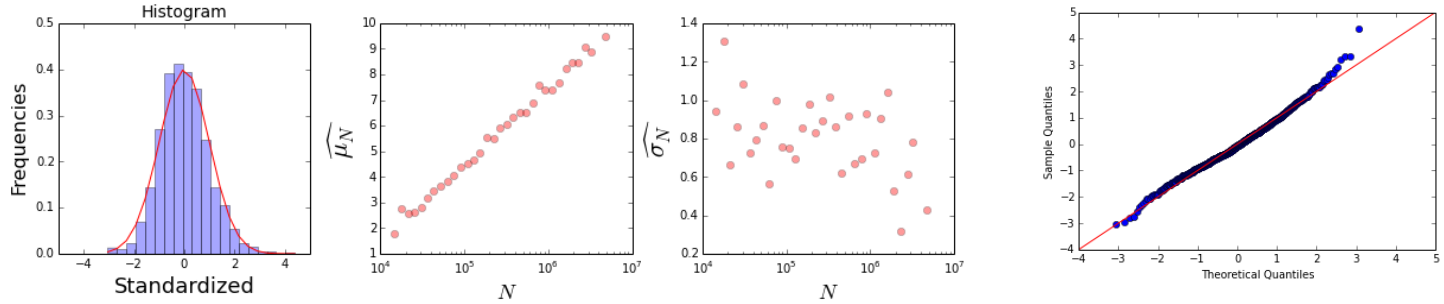


H&H Model

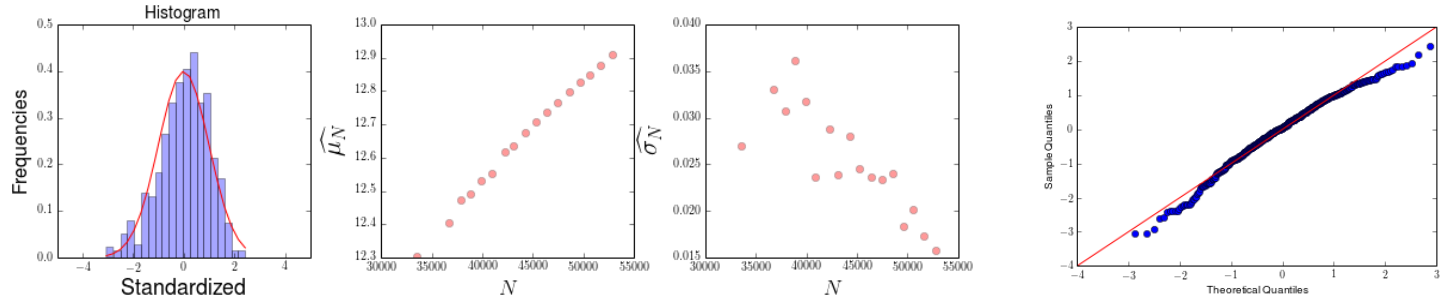


Table 1: US urban areas, sorted by population size, that have counts, in inventors or creatives, that are outside a $z = 3.07$ *sigma* interval around the log-meann, i.e., $y \notin [e^{\mu(n)-z\sigma(n)}, e^{\mu(n)+z\sigma(n)}]$. The numbers shown in the Population and Inventors columns are estimated averages from 2008-2010, although all values representing counts have been rounded to the nearest integer. The value $z = 3.07$ corresponds to a log-deviation such that $1 - \Phi(z) < 1/938$. The random variables and their values are for inventors I and i, for creatives C and c, respectively

Name of Urban Area	Population (n)	Inventors (i)	Most probable I	$P(I \geq i\|n)$	Creatives (c)	Most probable C	$P(C \geq c\|n)$
Los Alamos, NM (Micro)	17,899	213	1	0.	5,502	1,697	0.
Mountain Home, ID (Micro)	26,926	1,027	2	0.	2,801	2,751	0.562
Clewiston, FL (Micro)	39,109	8	4	0.626	1,976	3,848	0.999
Clovis, NM (Micro)	47,009	1	5	0.999	4,504	4,461	0.577
Eagle Pass, TX (Micro)	53,392	1	7	1.	4,328	5,144	0.834
Palm Coast, FL (Metro)	94,755	44	15	0.426	4,040	8,958	1.
Lake Havasu City-Kingman, AZ (Metro)	200,447	43	43	0.82	10,940	21,603	0.999
Merced, CA (Metro)	253,198	37	59	0.924	13,650	27,633	0.999
Ocala, FL (Metro)	330,780	69	87	0.873	17,620	37,218	1.
Durham-Chapel Hill, NC (Metro)	498,511	1,692	155	0.028	128,900	57,900	0.001
McAllen-Edinburg-Mission, TX (Metro)	758,064	36	279	0.999	60,400	87,582	0.963
San Jose-Sunnyvale-Santa Clara, CA (Metro)	1,818,864	24,531	952	0.	396,820	240,275	0.033

Example

C	a1	a2	a3	a4	a5
city 1	1	1	0	0	1
city 2	0	1	0	1	1

P	a1	a2	a3	a4	a5
p1	0	1	0	0	1
p2	0	0	0	1	0

M	p1	p2
city 1	1	0
city 2	1	1

- City 1 has skills {1,2,5}
- City 2 has skills {2,4,5}

- Product 1 requires {2,5}
- Product 2 requires {4}

- City 1 only produces Product 1
- City 2 produces Both

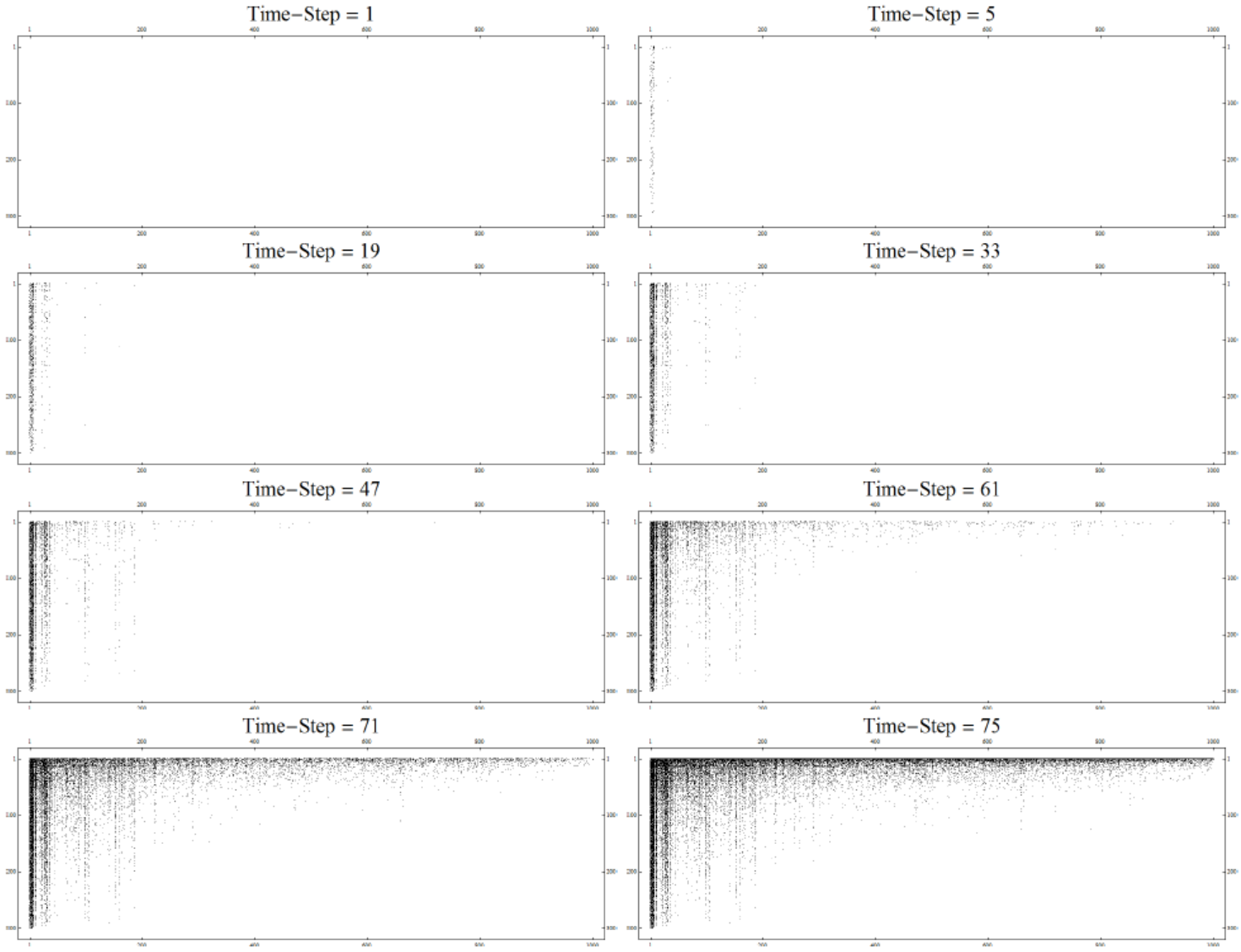
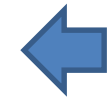


Figure 5: **Model 1: Evolution of the M^{t} matrix of cities versus products.** The model assumes a fixed set of products uniquely defined by their set of capabilities. In each time-step, each city acquires a new random capability previously missing. If the capability allows the city to produce a new product, the capability is fixed, otherwise it is lost (until it acquires it again by chance in the future). Here, $N_c = 300$, $N_p = 1000$, $N_a = 50$, $r = 0$, and $q = 0.13$. The M_{cp} elements are sorted in the same order throughout all steps from most diversified city to the least, and from the most ubiquitous product to the least, as they appear in the last time-step.

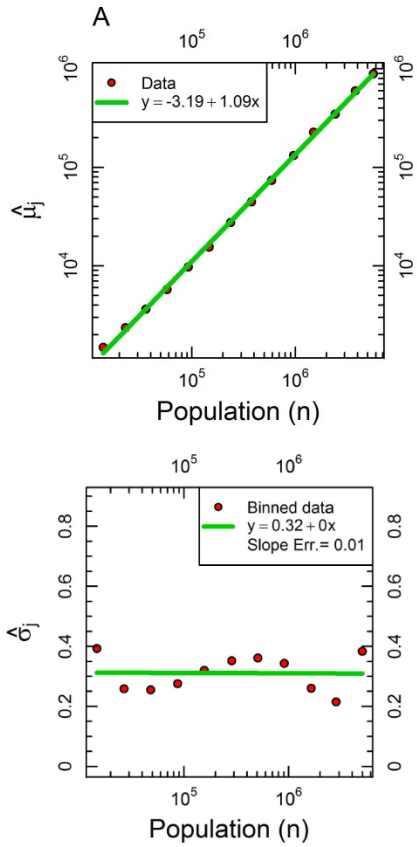
The following relationship has to hold if we want a confidence *(1-α)* that *Y/N* is close to the mean of the process

$$(\alpha\epsilon^2)N^r \geq c(r)\left(N\mathrm{E}\left[y_1^r\right] + N^{r/2}\sqrt{\mathrm{Var}[y_1]}\right)$$

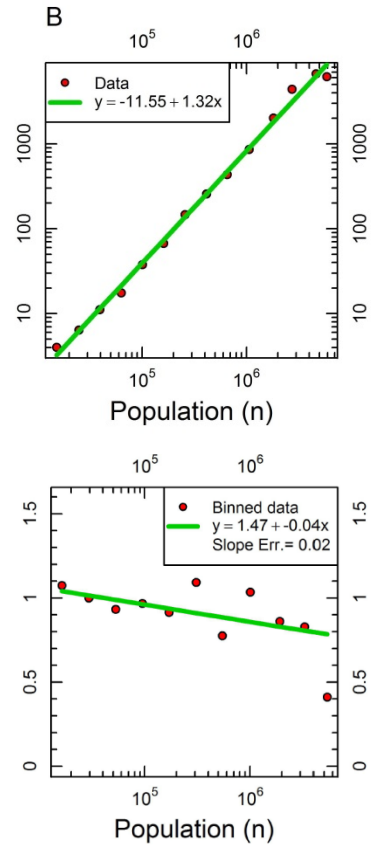


Population size dependence

Creatives



Inventors



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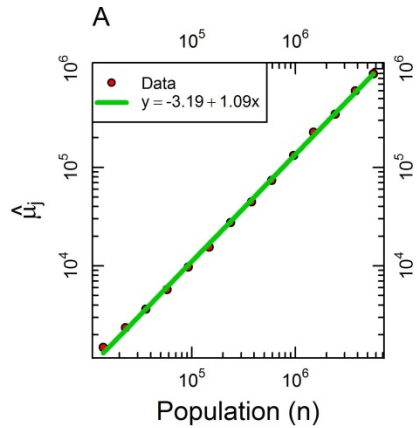

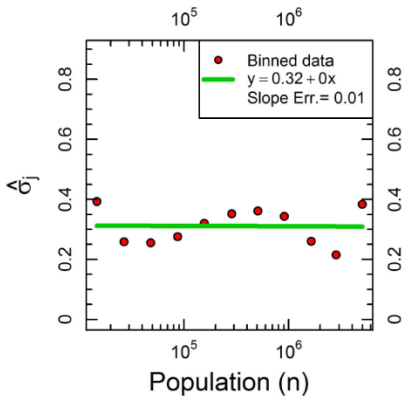

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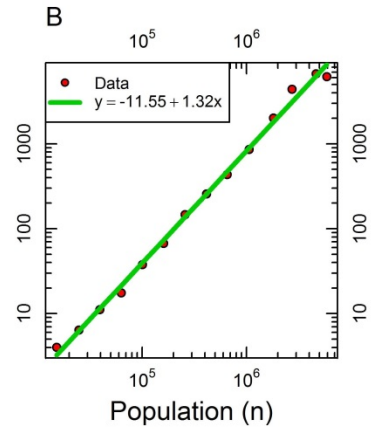

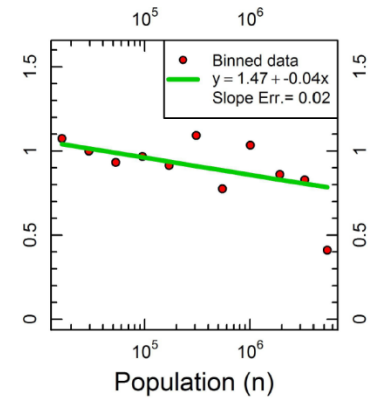

- Note this way we have completely characterized

Population size dependence

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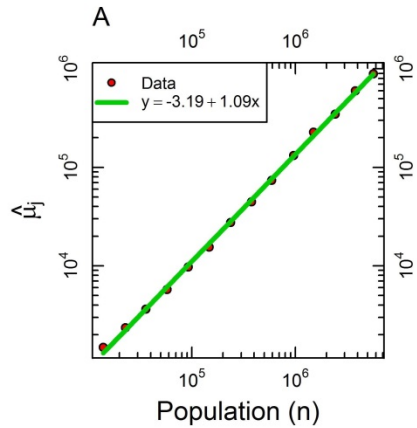

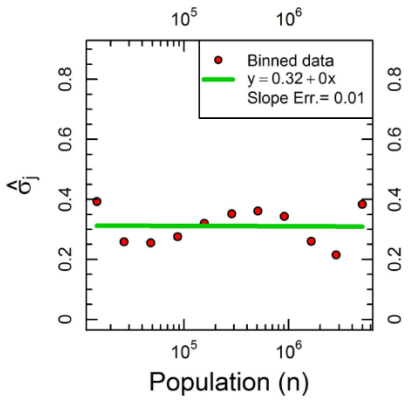

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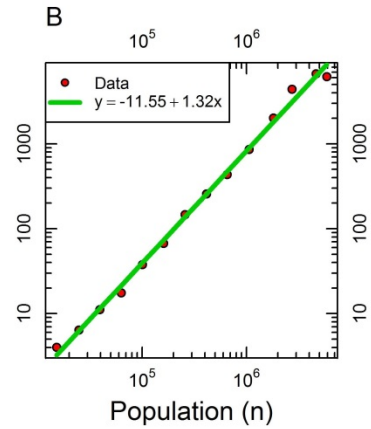
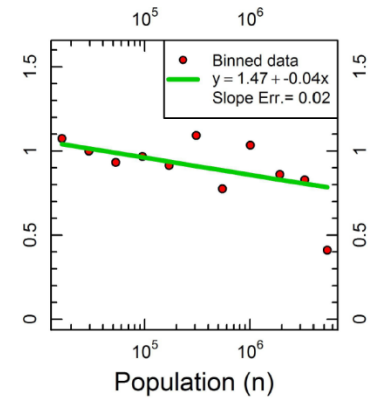




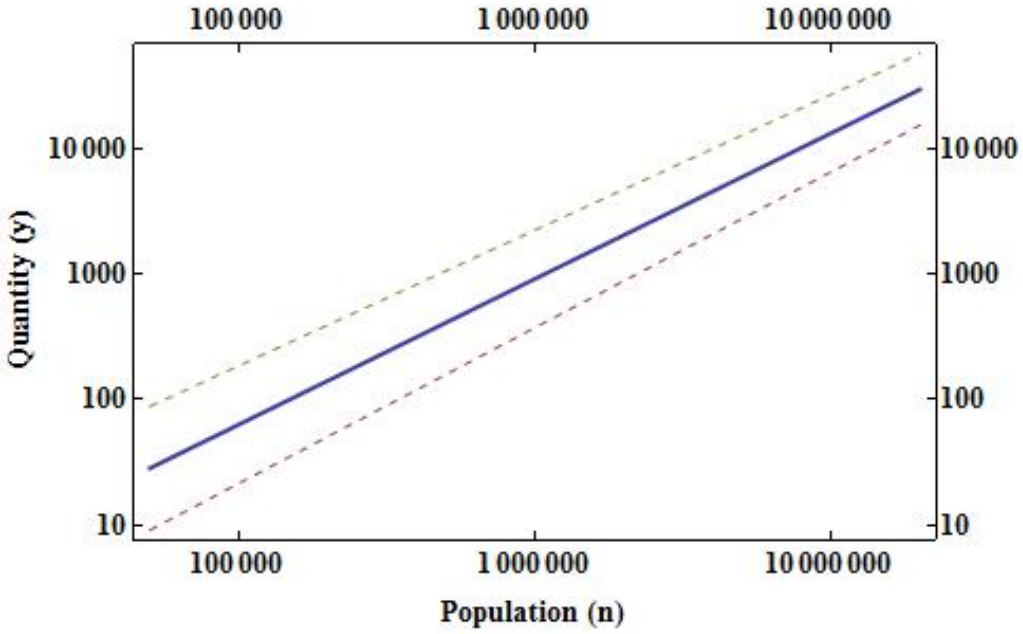
- Note this way we have completely characterized

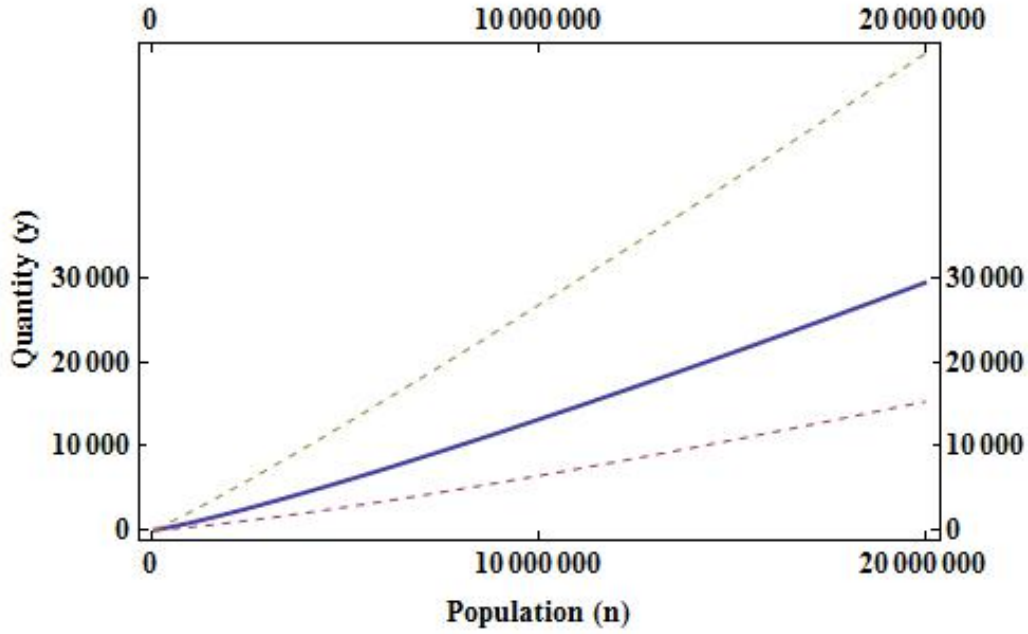
$$P(Y,N)=P(Y|N)P(N)$$

Logarithmic convergence

vs.

Absolute divergence of cities





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PLOS ONE

Urban Economies and Occupation Space: Can They Get "There" from "Here"?

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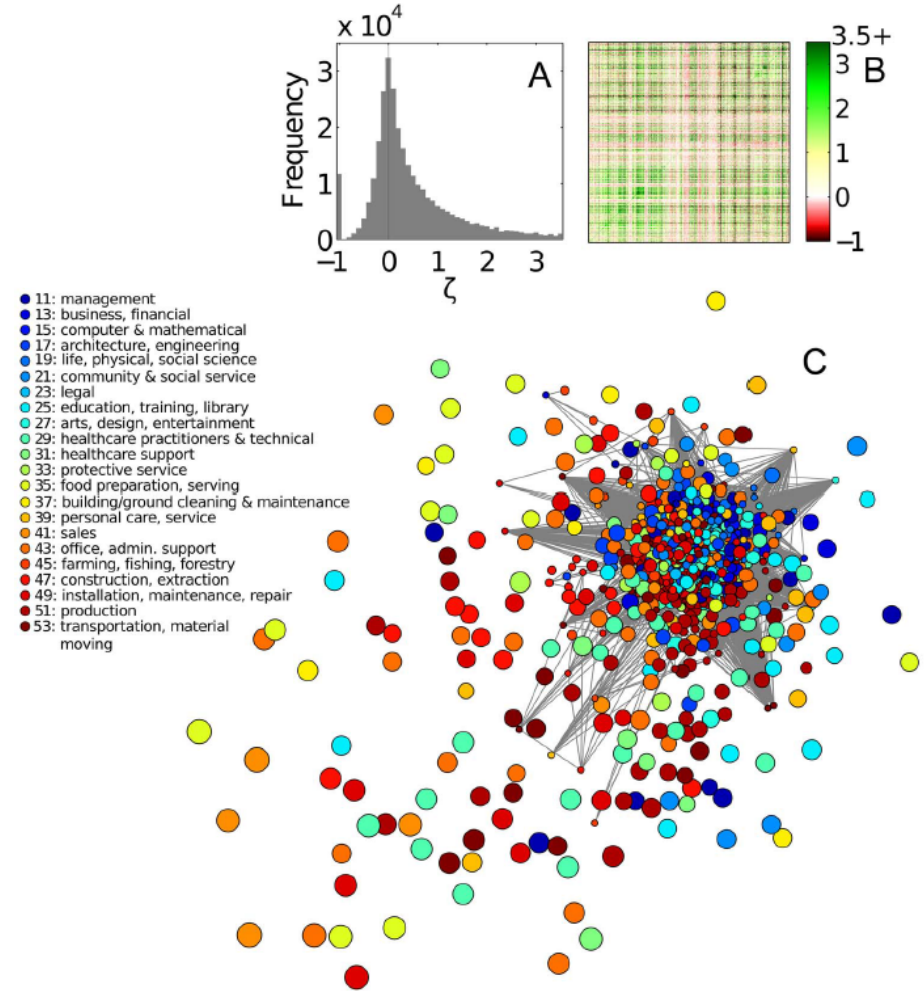
Abstract

Much of the socioeconomic life in the United States occurs in its urban areas. While an urban economy is defined to a large extent by its network of occupational specializations, an examination of this important network is absent from the considerable body of work on the determinants of urban economic performance. Here we develop a structure-based analysis addressing how the network of interdependencies among occupational specializations affects the ease with which urban economies can transform themselves. While most occupational specializations exhibit positive relationships between one another, many exhibit negative ones, and the balance between the two partially explains the productivity of an urban economy. The current set of occupational specializations of an urban economy and its location in the occupation space constrain its future development paths. Important tradeoffs exist between different alternatives for altering an occupational specialization pattern, both at a single occupation and an entire occupational portfolio levels.

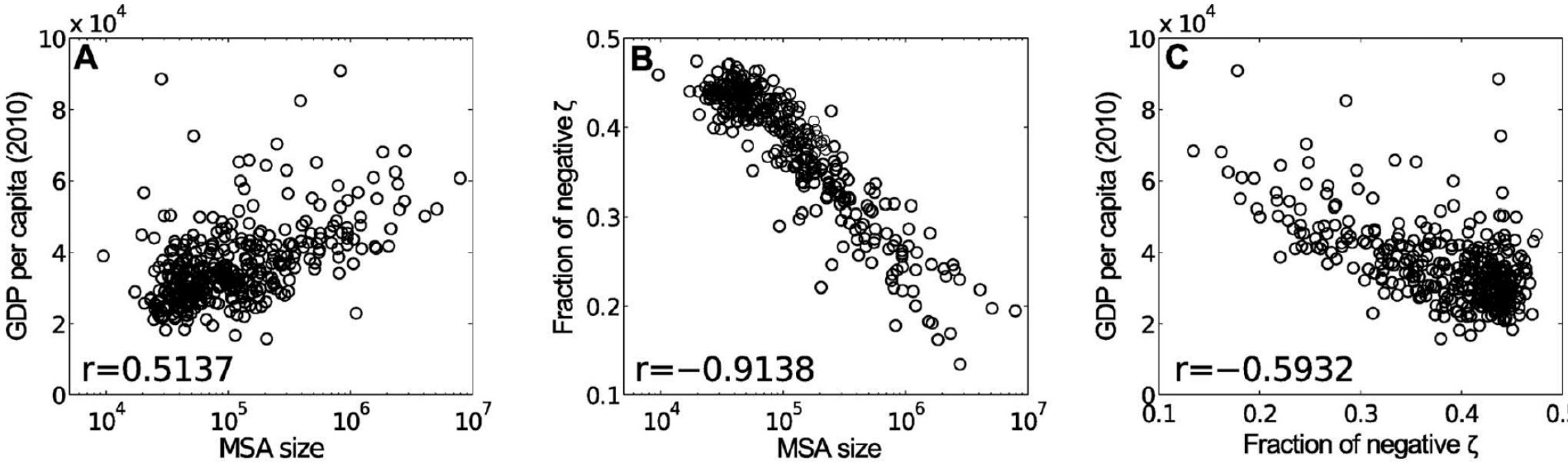
Occupational "interdependencies"

- Definition:

$$\zeta_{ij} = \frac{P[LQ_i^{(M)} > 1, LQ_j^{(M)} > 1]}{P[LQ_i^{(M')} > 1] P[LQ_j^{(M'')} > 1]} - 1$$



“Harmony” between occupations



Scaling relationships

(sensu Stat. Mech.*)*

$$
\begin{aligned}
X_1|N &\sim \mathcal{LN}(\mu_1, \sigma_1^2) \\
X_2|N &\sim \mathcal{LN}(\mu_2, \sigma_2^2) \\
X_2 &= f(X_1) \\
&\Rightarrow X_2 = aX_1^{\beta}, \beta = \frac{\sigma_2}{\sigma_1}
\end{aligned}
$$

$$
\begin{aligned}
X_1 &\sim \mathcal{P}(\alpha) \\
&\Rightarrow X_2 \sim \mathcal{P}(\tau),\ \tau = \frac{\alpha}{\beta}
\end{aligned}
$$