Urban Scaling, Economic Complexity and Cultural Evolution

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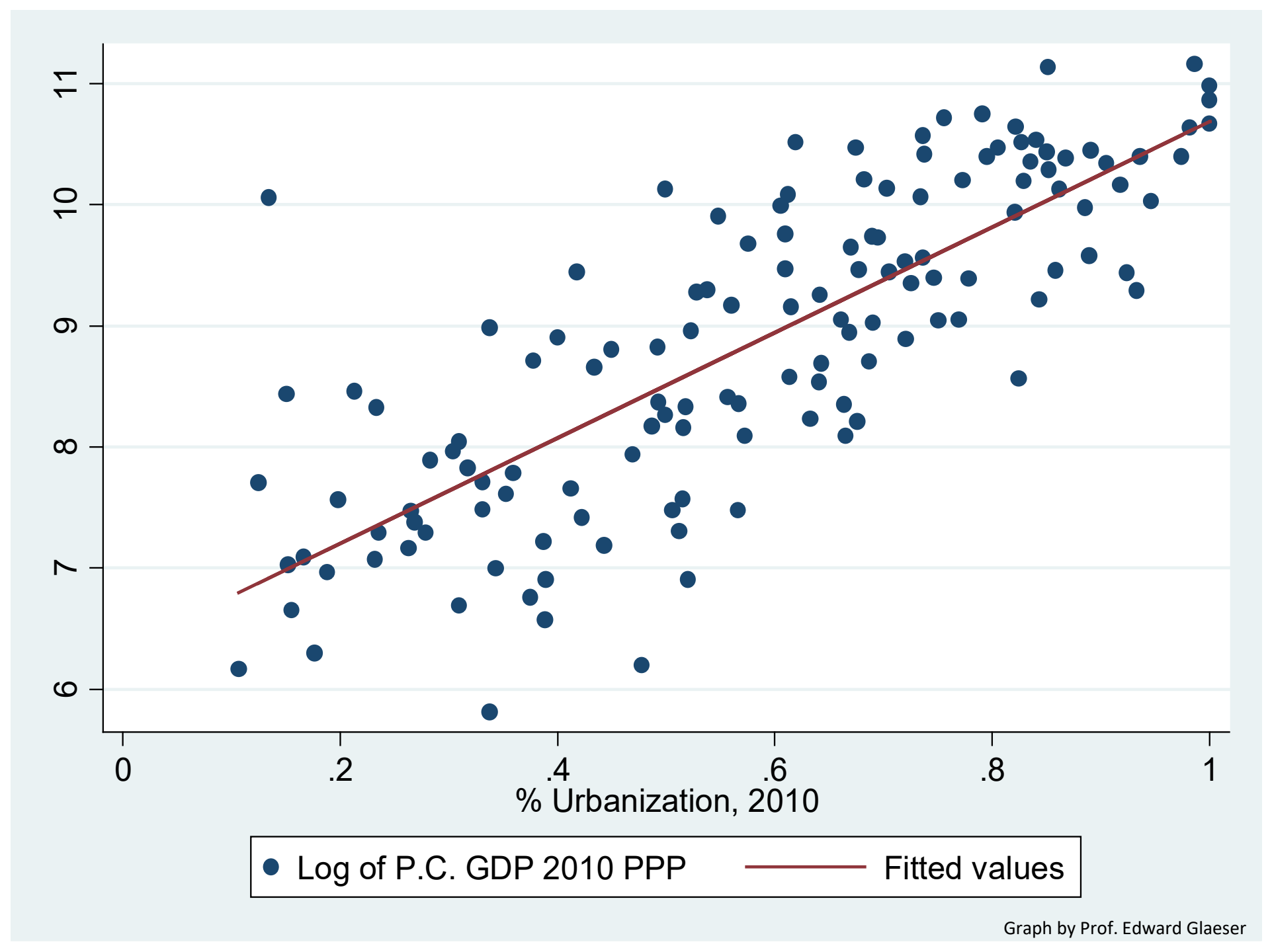
An urban species

https://sites.google.com/a/mmrschool.org/mmr-cities/home


11
10
9
8
7
6
0
.2
.4
.6
.8
1
% Urbanization, 2010
Log of P.C. GDP 2010 PPP
Fitted values
Graph by Prof. Edward Glaeser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Stark differences in levels of urbanization, wealth, innovation, health, crime, etc…

Why?

A presumptuous goal?

Collaborators:
Luis M.A. Bettencourt
Jose Lobo
HyeJin Youn
Rachata Muneepeerakul
Shade Shutters
Deborah Strumsky
Kevin Stolarick
Oscar Patterson-Lomba
Ricardo Hausmann

Three takeaways

1. There is a statistical regularity present in urban systems called “**Urban Scaling**”.

2. Every social phenomenon has a “**complexity**” that summarizes many of its statistical properties.

3. Ideas from **Cultural Evolution** are needed in order to account for the differences in development across cities.

Let’s internalize the types of questions we’re asking

Metropolitan Statistical Area (MSA)	Year	Population	Larceny-theft
Carson City, NV M.S.A.	2010	55,119	3,141

Why *3,141*?

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Why *768*?

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Source: https://www.fbi.gov/about-us/cjis/ucr/crime-in-the-u.s/

Why *20*?

Metropolitan Statistical Area (MSA)	Year	Population	Robbery	Larceny-theft
Carson City, NV M.S.A.	2010	55,119	20	768

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Robbery in Bridgeport

=

Larceny-theft in Carson City ??

Metropolitan Statistical Area (MSA)	Year	Population	Robbery	Larceny-theft
Carson City, NV M.S.A.	2010	55,119	20	768
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Numbers change… but maybe *proportions* do not….

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Numbers change… but maybe *proportions* do not….

Metropolitan Statistical Area (MSA)	Year	Population	Robbery Rate (cases per 100,000 inhabitants)	Larceny-theft (cases per 100,000 inhabitants)
Carson City, NV M.S.A.	2010	55,119	36.3	1,393.3
Bridgeport-Stamford-Norwalk, CT M.S.A.	2010	895,941	110.6	1,226.2

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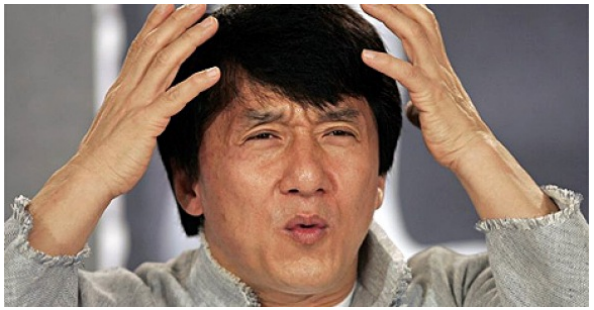

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One approach

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$Y = f_3(X, W, Z)$

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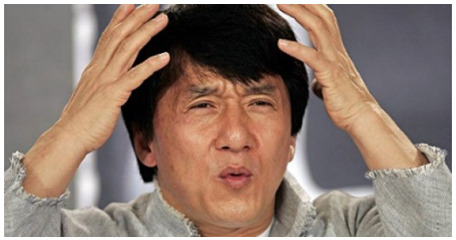


- Collective systems are full of interdependencies, interactions, feedback loops, etc…

A methodological decision

- The conventional approach:

Multivariate regression analysis

(→ Identifying effects/r.h.s.)

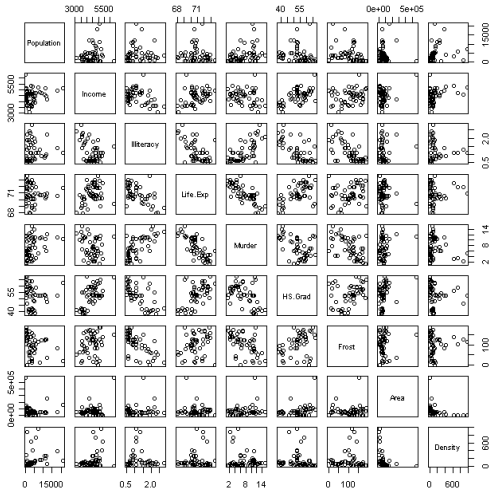


Table 4: Probit Model for Characteristics of Co-Holders

	(1) Co-Holder		(2) Co-Holding > £500		(3) Co-Holding > £1500	
	β / SE	Mfx	β / SE	Mfx	β / SE	Mfx
Age						
18-24	−0.234	−0.061	−0.311	−0.060	−0.849***	−0.105***
	(0.154)		(0.191)		(0.311)	
25-34	0.016	0.004	0.002	0.000	−0.097	−0.012
	(0.101)		(0.111)		(0.128)	
35-44	−0.044	−0.012	−0.048	−0.009	−0.085	−0.010
	(0.096)		(0.104)		(0.117)	
55+	−0.006	−0.001	−0.044	−0.008	0.025	0.003
	(0.105)		(0.116)		(0.134)	
Demographics						
Male (= 1)	−0.016	−0.004	0.035	0.007	0.052	0.006
	(0.068)		(0.076)		(0.088)	
Married / living as married (= 1)	0.019	0.005	0.033	0.006	−0.034	−0.004
	(0.133)		(0.155)		(0.179)	
Dependent children (= 1)	−0.111	−0.029	−0.105	−0.020	0.166	0.020
	(0.084)		(0.092)		(0.102)	
Education						
Education leaving age	0.083	0.022	0.121	0.023	−0.055	−0.007
	(0.249)		(0.284)		(0.335)	
Education leaving age^2	−0.003	−0.001	−0.003	−0.001	0.001	0.000
	(0.006)		(0.007)		(0.009)	
Employmeu						
Employed (= 1)	0.268***	0.070***	0.254**	0.049**	0.303**	0.037**
	(0.093)		(0.109)		(0.136)	
Unemployed (= 1)	−0.016	−0.004	−0.013	−0.002	0.218	0.027
	(0.176)		(0.209)		(0.240)	
Housing						
Homeowner without mortgage (= 1)	−0.369***	−0.096***	−0.070	−0.013	0.006	0.001
	(0.116)		(0.129)		(0.150)	
Homeowner with mortgage (= 1)	0.064	0.017	0.254***	0.049***	0.200*	0.025*
	(0.082)		(0.093)		(0.109)	
Household Finances						
Household income (£10,000s)	−0.081	−0.021	−0.017	−0.003	0.043	0.005
	(0.078)		(0.090)		(0.108)	
Household income^2	0.015	0.004	0.010	0.002	0.010	0.001
	(0.010)		(0.012)		(0.014)	
Household income^3	−0.001	−0.000	−0.000	−0.000	−0.001	−0.000
	(0.000)		(0.000)		(0.000)	
Behavioural Characteristics						
Literacy score (0-3)	0.042	0.011	0.073*	0.014*	0.060	0.007
	(0.035)		(0.040)		(0.046)	
Impulsive spender (= 1)	0.391***	0.102***	0.458***	0.088***	0.518***	0.064***
	(0.083)		(0.091)		(0.102)	
Read financial press (= 1)	0.155**	0.041**	0.147*	0.028*	0.069	0.009
	(0.071)		(0.077)		(0.089)	
Observations	2196		2196		2196	
Pseudo R^2	0.043		0.069		0.095	
LR chi2	91.854		116.344		119.715	
Prob > chi2	0.000		0.000		0.000	
Baseline predicted probability	0.189		0.129		0.083	

* p < 0.1, ** p < 0.05, *** p < 0.01. Standard errors in parentheses.

Note: Omitted groups: *Employmeu:* Student/Housewife/Disabled. *Housing:* Private renter/Social renter. Further controls for spouse employment status.

A methodological decision

- The conventional approach:

  Multivariate regression analysis

  (→ Identifying effects/r.h.s.)

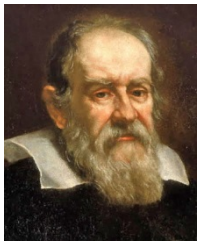
- An alternative approach:

  Scaling analysis

  (→ Identifying mechanisms/l.h.s.)

"Scaling Analysis"

- ***Studying a phenomenon as it changes with scale (e.g., size)***:
  - Regularities
  - Important underlying mechanisms



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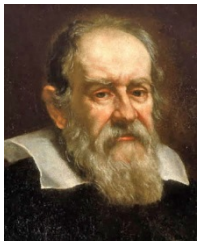

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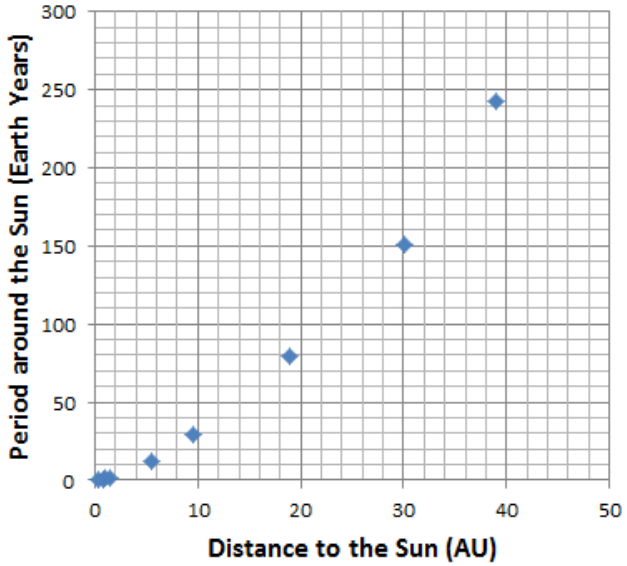

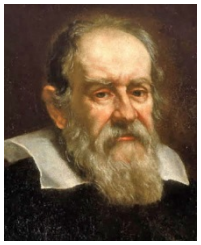

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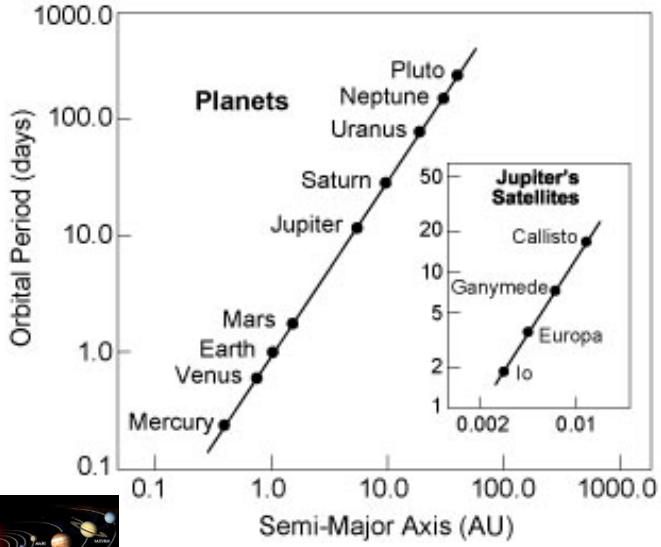

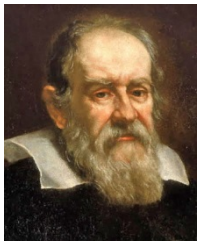

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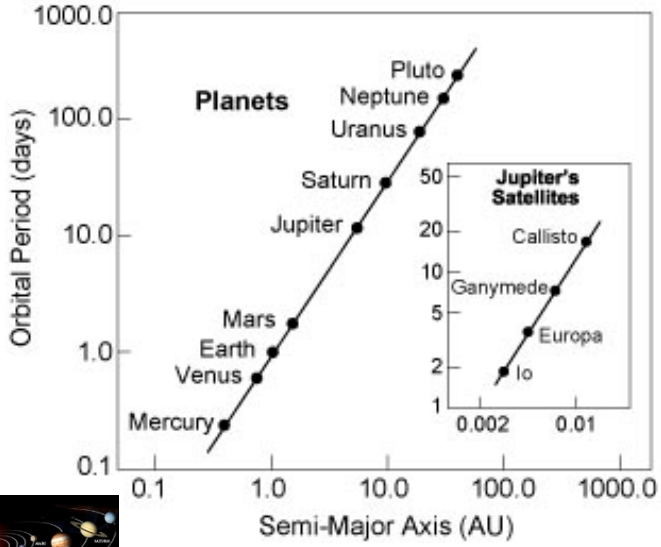
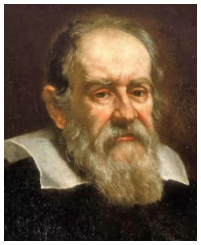


$$T \sim r^{3/2}$$



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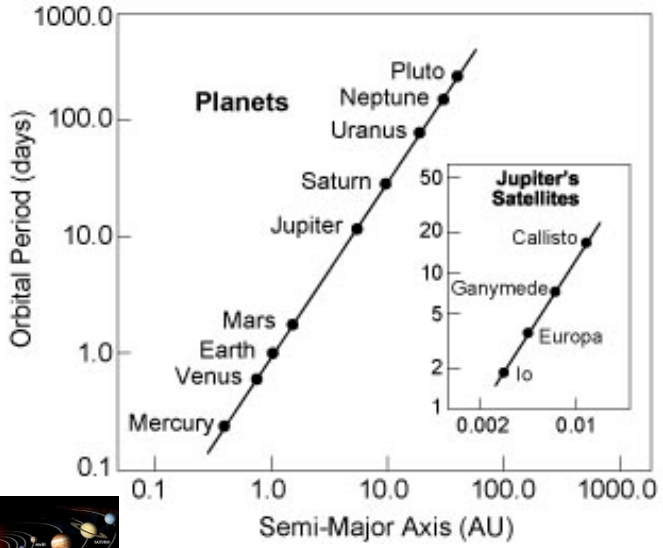

$$T \sim r^{3/2}$$

1. **Slope** of the line: Gravitational interaction decays with the ***square*** of the distance.
2. **Intercept** of the line: Gravitational force is proportional to ***mass***.

If interested…

- "*In the beat of a heart*", John Whitfield (BIOLOGY)
- "*Critical Phenomena in Natural Sciences*", Didier Sornette (PHYSICS & COMPLEX SYSTEMS)
- "*Fractals, Chaos, Power Laws*", Manfred Schroeder (GENERAL, ENGINEERING & PHYSICS)
- "*G.I. Taylor and the Trinity test*", M.A.B. Deakin (*2011*) (HISTORY OF ATOMIC BOMB)

Table 1: Scaling relationships and corresponding theories.

Quantities	**Scaling Law**	**Name**	**Theory**
Orbital period T and distance to the Sun r	$T = T_0\ r^{3/2}$	Kepler's third law	Theory of planetary motion
Average radius of diffusion r and time t	$r = r_0\ t^{1/2}$	Law of diffusion	Theory of Brownian motion
Metabolic rate B and body mass M	$B = B_0\ M^{3/4}$	Kleiber's law	Metabolic Theory of Ecology
Socioeconomic rates Y and population size N	$Y = Y_0\ N^{\beta}$	Urban Scaling Laws	???

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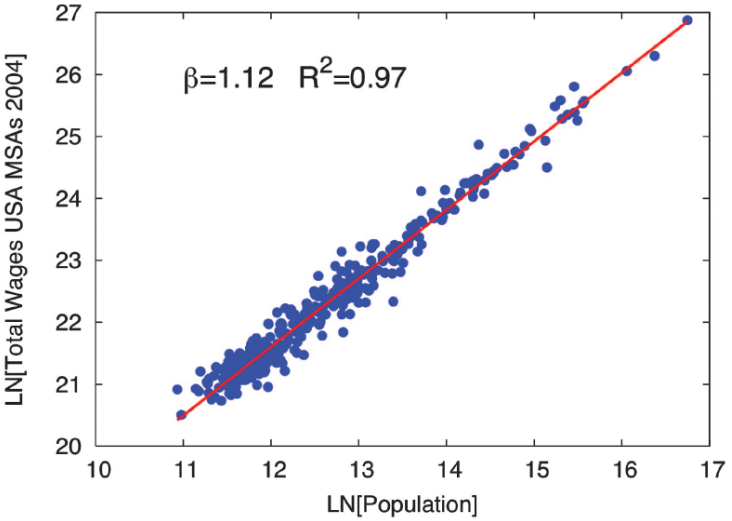
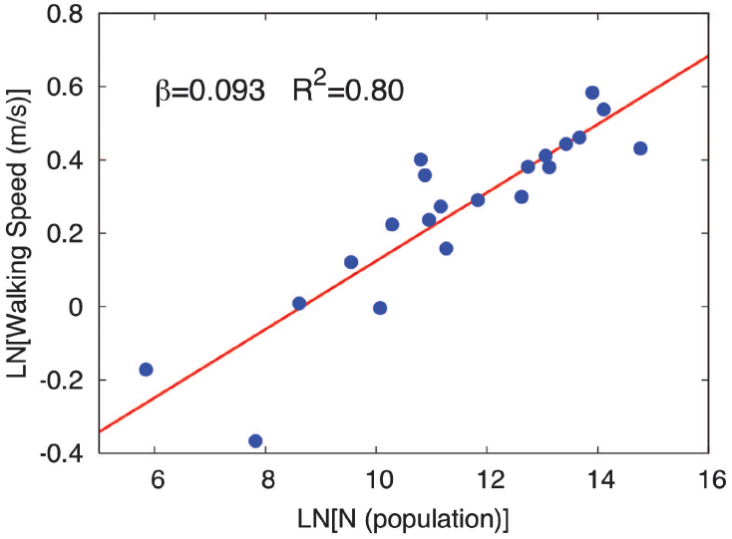
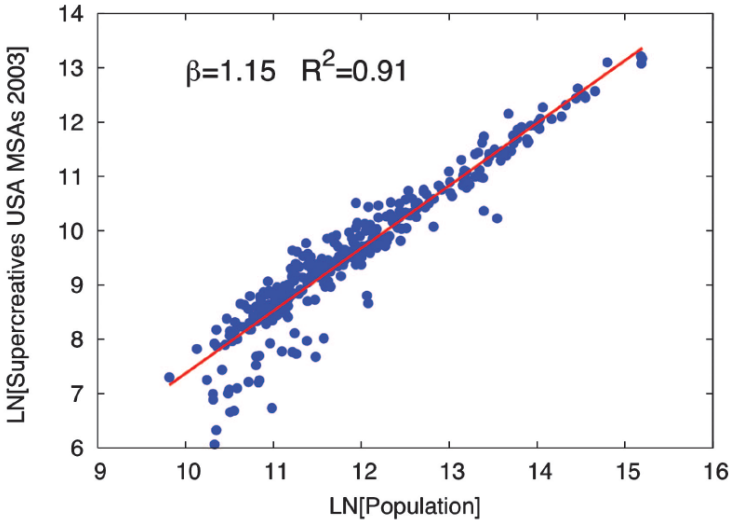
Allometric laws in urban systems

Allometric laws in urban systems







$$Y \sim N^{\beta}$$

Bettencourt, L.M., Lobo, J., Helbing, D., Kühnert, C. and West, G.B., 2007. Growth, innovation, scaling, and the pace of life in cities. *Proceedings of the National Academy of Sciences*, *104*(17), pp.7301-7306.

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PLOS one

The Statistics of Urban Scaling and Their Connection to Zipf's Law

Andres Gomez-Lievano[1]*, HyeJin Youn[2], Luís M. A. Bettencourt[2]

1 School of Human Evolution and Social Change, Arizona State University, Tempe, Arizona, United States of America, **2** Santa Fe Institute, Santa Fe, New Mexico, United States of America

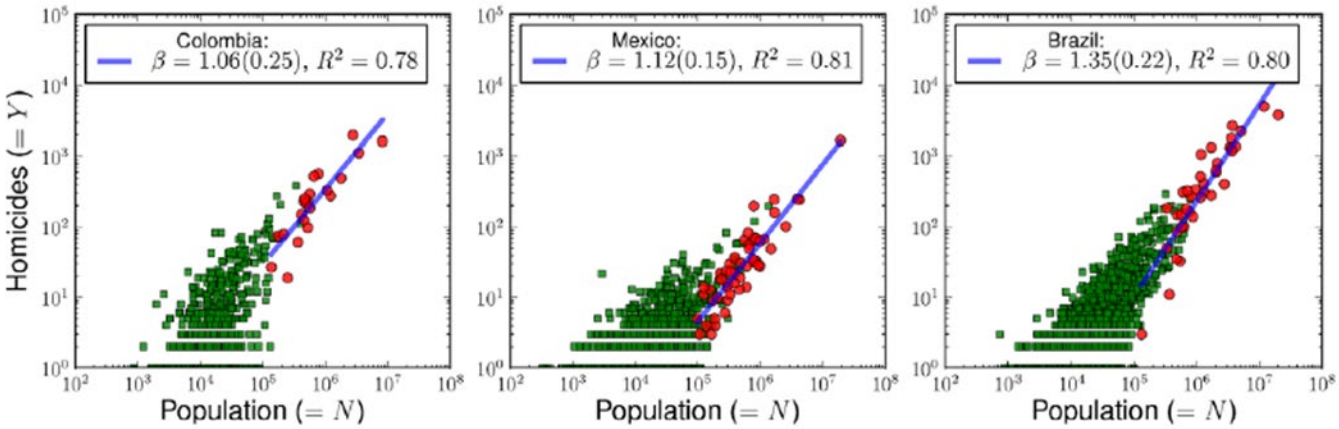

Figure 1. Annual number of homicides in cities of Colombia, Mexico and Brazil versus population size (2007). Large cities are defined in terms of metropolitan areas which are aggregations of municipalities (red circles) while non-metropolitan municipalities are shown separately (green squares). The solid blue line fits only the scaling of homicides for metropolitan areas. Large variations, especially among the smaller population units, and the fact that many municipalities have $Y = 0$ (not shown) prevent a direct scaling analysis. However, it is possible to analyze the data consistently through the estimation of conditional probabilities.

doi:10.1371/journal.pone.0040393.g001

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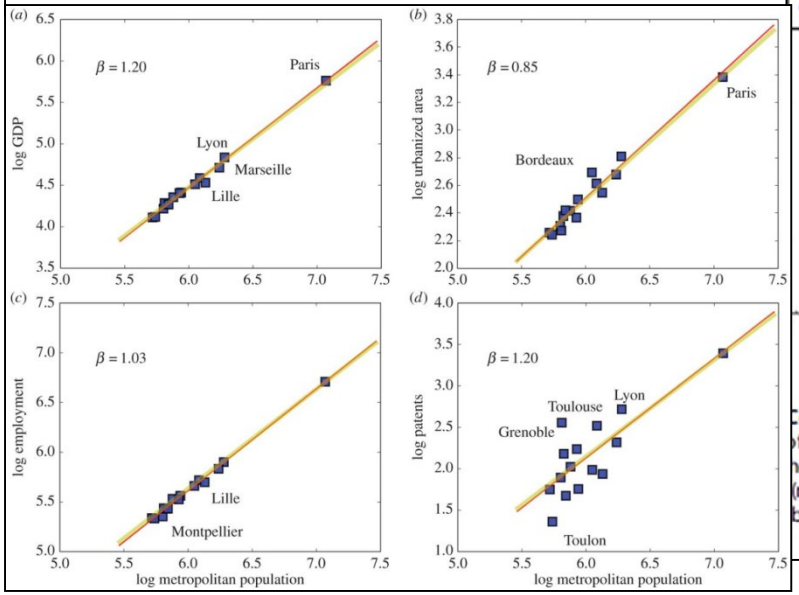
Urban scaling in Europe

Luís M. A. Bettencourt, José Lobo

Published 16 March 2016. DOI: 10.1098/rsif.2016.0005

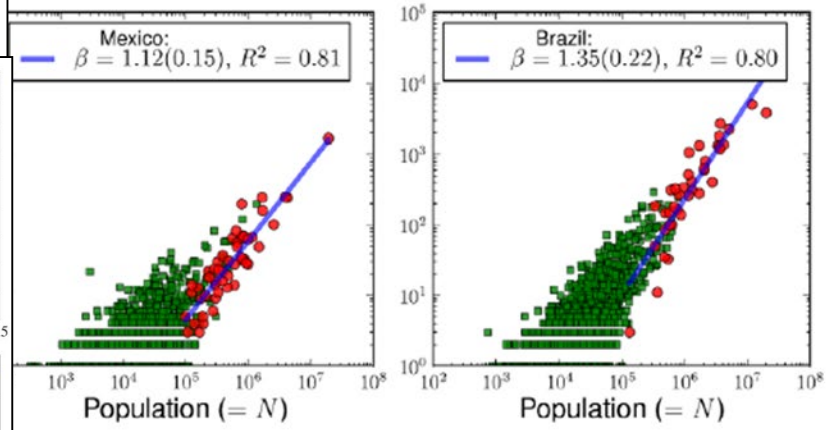


PLOS one

n Scaling and Their Connection to

², Luís M. A. Bettencourt²

University, Tempe, Arizona, United States of America, 2 Santa Fe Institute, Santa Fe, New Mexico, United



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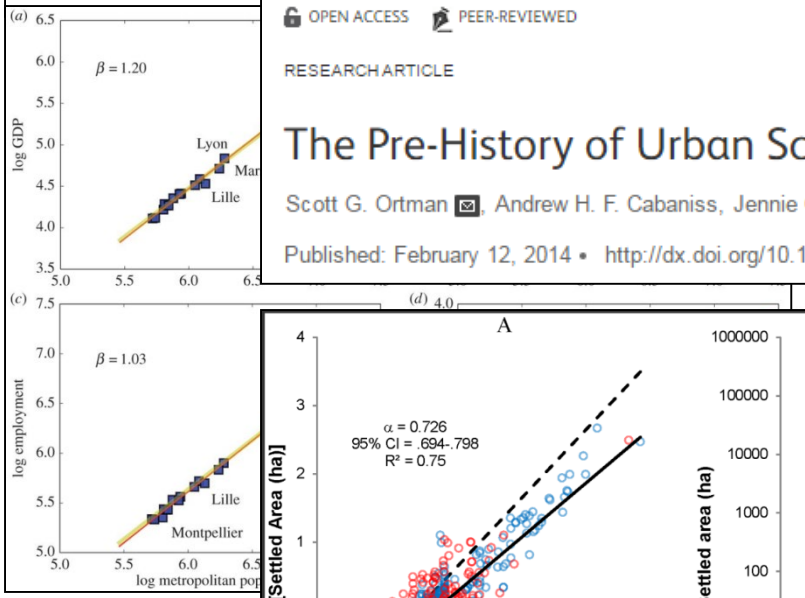

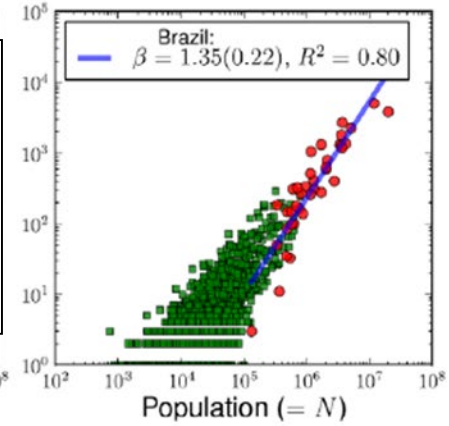

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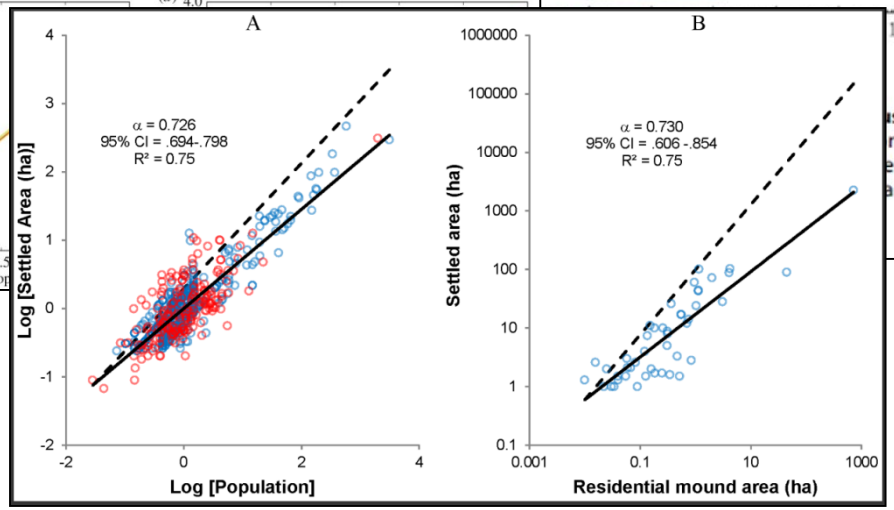
RESEARCH ARTICLE

The Pre-History of Urban Scaling

Scott G. Ortman, Andrew H. F. Cabaniss, Jennie O. Sturm, Luís M. A. Bettencourt

Published: February 12, 2014 • http://dx.doi.org/10.1371/journal.pone.0087902



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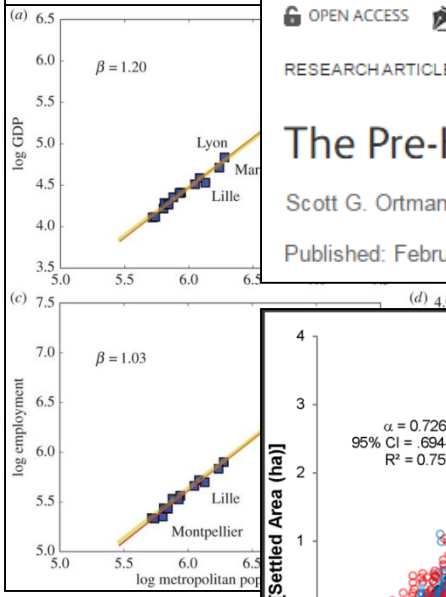
Urban scaling in Europe

Luís M. A. Bettencourt, José Lobo

Published 16 March 2016, DOI: 10.1098/rsif.2016



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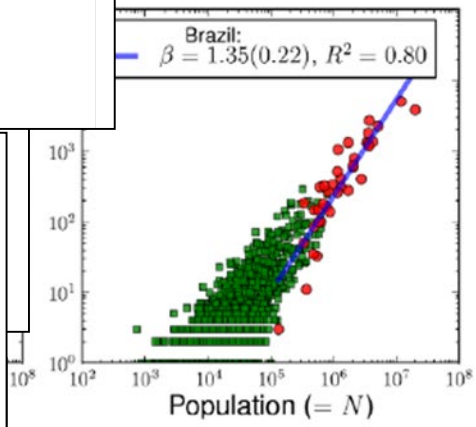

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RESEARCH ARTICLE | ECONOMIC ANTHROPOLOGY

Settlement scaling and increasing returns in an ancient society

Scott G. Ortman[1,2,*], Andrew H. F. Cabaniss[2,3], Jennie O. Sturm[4] and Luís M. A. Bettencourt[2]

+ Author Affiliations

*Corresponding author. E-mail: scott.ortman@colorado.edu

Science Advances 20 Feb 2015:
Vol. 1, no. 1, e1400066
DOI: 10.1126/sciadv.1400066

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The Pre-History

Scott G. Ortman, Andrew

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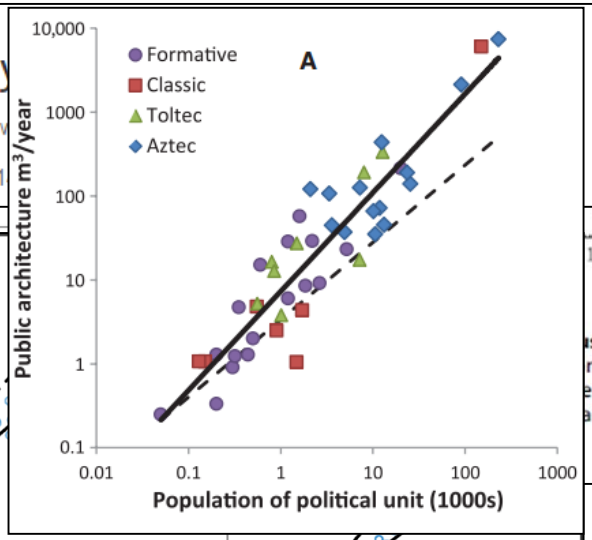

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RESEARCH ARTICLE

Population-Area Relationship for Medieval European Cities

Rudolf Cesaretti[1]*, José Lobo[2], Luís M. A. Bettencourt[3], Scott G. Ortman[3,4], Michael E. Smith[1]

1 School of Human Evolution and Social Change, Arizona State University, Tempe, Arizona, 85281, United States of America, **2** School of Sustainability, Arizona State University, Tempe, Arizona, 85281, United States of America, **3** Santa Fe Institute, 1399 Hyde Park Rd, Santa Fe, New Mexico, 87501, United States of America, **4** Department of Anthropology, University of Colorado Boulder, Boulder, Colorado, 80309, United States of America

* Rudolf.Cesaretti@asu.edu

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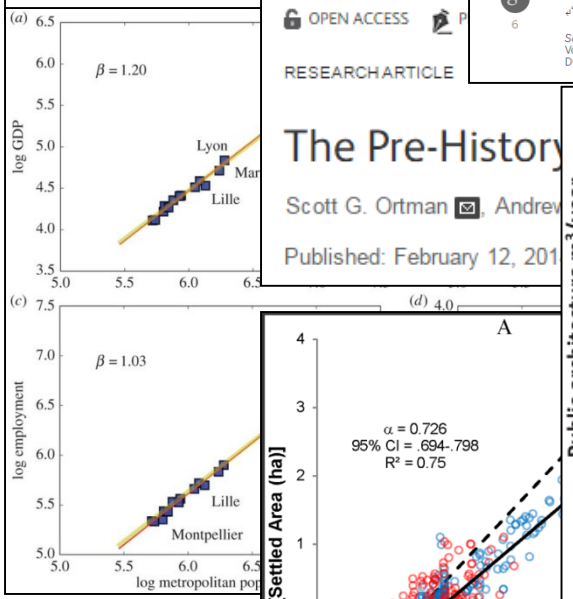

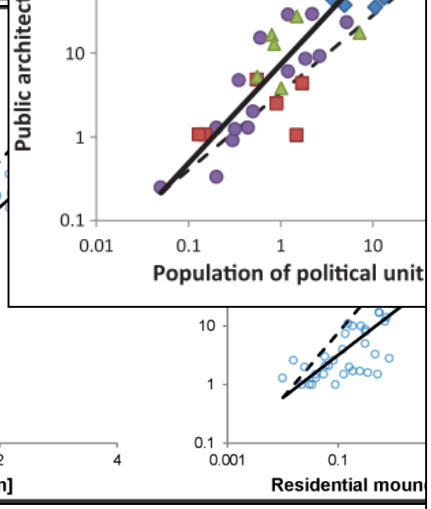

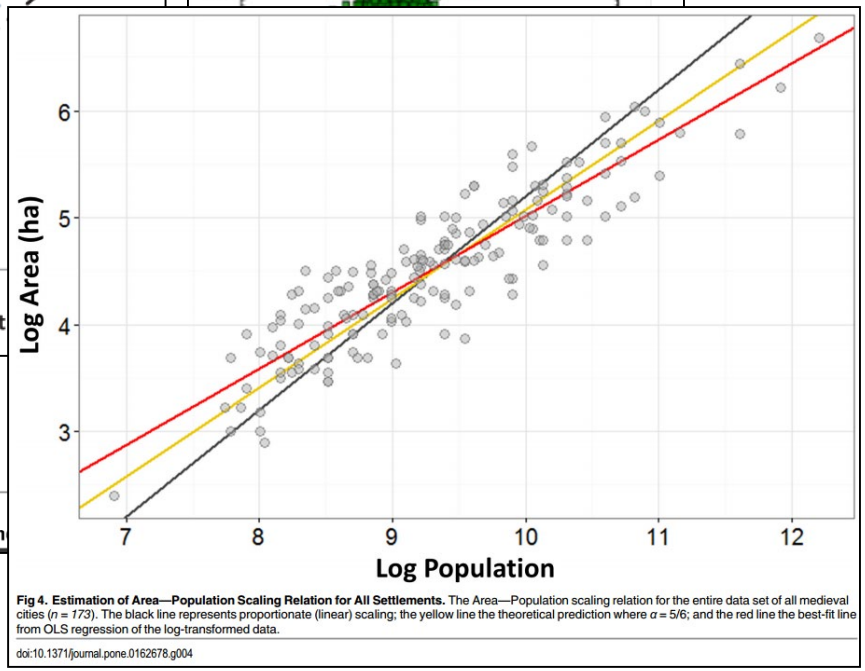

Fig 4. Estimation of Area—Population Scaling Relation for All Settlements. The Area—Population scaling relation for the entire data set of all medieval cities ($n = 173$). The black line represents proportionate (linear) scaling; the yellow line the theoretical prediction where $\alpha = 5/6$; and the red line the best-fit line from OLS regression of the log-transformed data.

doi:10.1371/journal.pone.0162678.g004

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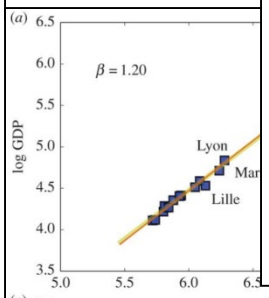

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Scott G. Ortman[1,2,*], An

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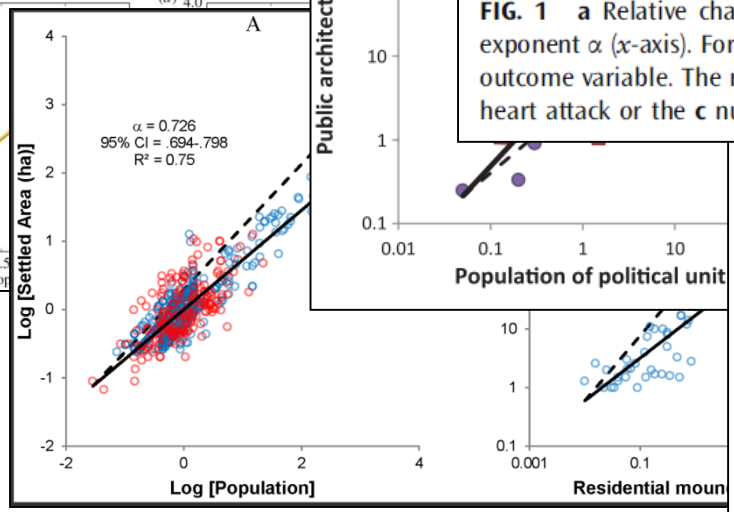


Journal of Urban Health: Bulletin of the New York Academy of Medicine, Vol. 92, No. 5
doi:10.1007/s11524-015-9976-x

The Non-linear Health Consequences of Living in Larger Cities

Luis E C Rocha, Anna E. Thorson, and Renaud Lambiotte

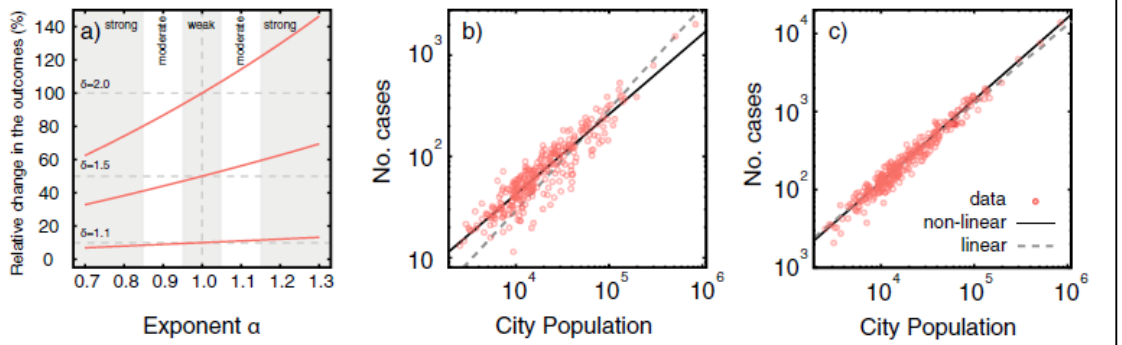
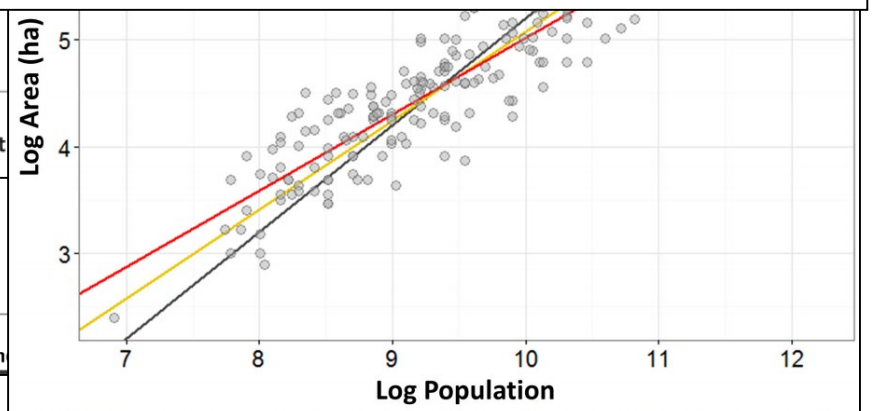


FIG. 1 **a** Relative change in the outcome Q, $(Q_2-Q_1)/Q_1$ (y-axis), for different values of the exponent α (x-axis). For $\alpha=1$, we recover the usual linear relation between population size and the outcome variable. The relation between the population size (x-axis) and **b** the number of deaths by heart attack or the **c** number of abortions (y-axis). Both axes are in log-scale.



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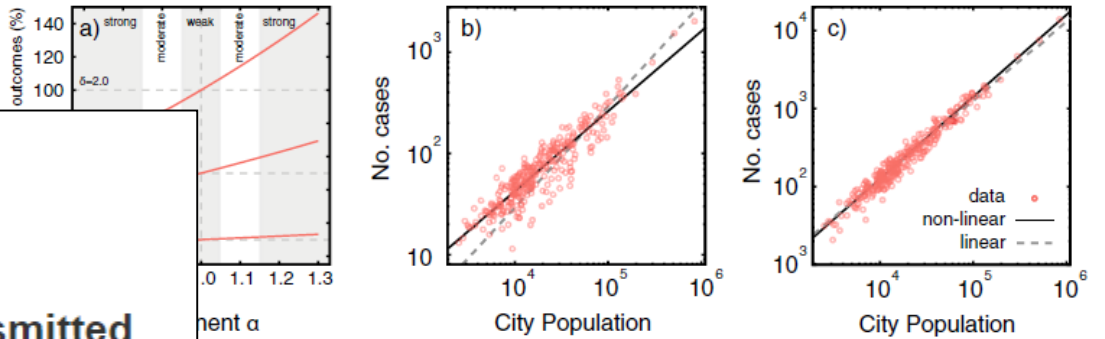

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Sex Transm Infect doi:10.1136/sextrans-2014-051932

Epidemiology

Original article

Per capita incidence of sexually transmitted infections increases systematically with urban population size: a cross-sectional study

Oscar Patterson-Lomba[1], Edward Goldstein[2], Andrés Gómez-Liévano[3], Carlos Castillo-Chavez[4], Sherry Towers[4]

Epidemiology

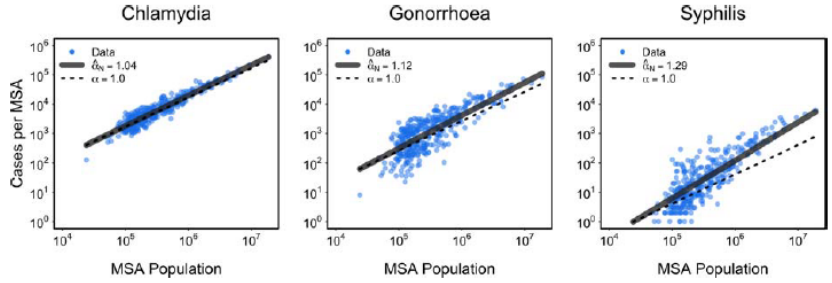
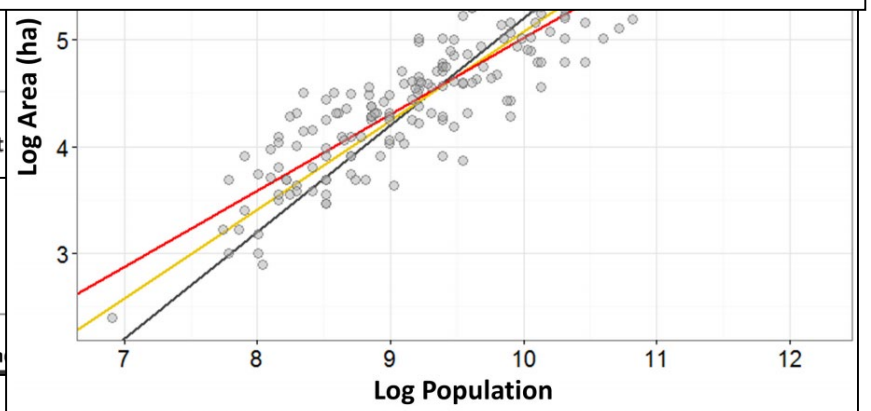


Figure 2 Scaling of STD incidence with MSA population with Negative Binomial regression for chlamydia (left), gonorrhoea (centre) and syphilis (right) using model (4), as reported in Table 1. Comparing the blue lines (with slopes) with the dotted lines (with slope 1) shows the departures from the linear pattern in each case.



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doi:10.1371/journal.pone.0162678.g004

Urban Scaling: $Y = Y_0 N^{(1.1667)}$

Scaling Relation	Exponent	Error	Observations	Region/Nation	Urban Unit	Year
Socioeconomic rates						
GDP	$\beta = 1.13$	[1.11,1.15]	363	USA	MSA	2006
GDP	$\beta = 1.22$	[1.11,1.33]	273	China	Prefectural Cities	2005
GDP	$\beta = 1.10$	[1.01,1.18]	35	Germany	LUZ	2004
income	$\beta = 1.12$	[1.07,1.17]	12	Japan	MA	2005
wages	$\beta = 1.12$	[1.07,1.17]	363	USA	MSA	1969-2009
violent crime	$\beta = 1.16$	[1.11,1.19]	287	USA	MSA	2003
violent crime	$\beta = 1.20$	[1.07,1.33]	12	Japan	MA	2008
violent crime	$\beta = 1.20$	[1.15,1.25]	27; 5,570	Brazil	MA; Municipios	2003-07
new AIDS cases	$\beta = 1.23$	[1.17,1.29]	93	USA	MSA	2002-3
new patents	$\beta = 1.27$	[1.22,1.32]	331	USA	MSA	1980-2001
supercreative jobs	$\beta = 1.15$	[1.13,1.17]	331	USA	MSA	1999-2001
R&D employment	$\beta = 1.19$	[1.12,1.26]	227-278	USA	MSA	1987-2002
Average socioeconomic rates	$\beta = 1.17$	[1.01,1.33]				

Table S3, from:
"The Origins of Scaling in Cities",
Bettencourt, L. M. A. (2013, *Science*)

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The question of Individual vs. Systemic effects

- **Analytic:**
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- Productive individuals self-select into larger urban areas.
- However, additional assumptions would be needed to explain the scaling law.
- It wouldn't explain the disproportionate concentration of crime and disease in larger urban areas.

The question of Individual vs. Systemic effects

- **Holistic:**
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$$Y \propto E \propto N^2$$

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Size and the Density of Interaction in Human Aggregates[1]

Bruce H. Mayhew
University of South Carolina

Roger L. Levinger
Temple University

Size and the Density of Interaction in Human Aggregates
Author(s): Bruce H. Mayhew and Roger L. Levinger
Source: *American Journal of Sociology*, Vol. 82, No. 1 (Jul., 1976), pp. 86-110

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- More individuals engage in productive *as well* as harmful, disadvantageous activities.

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Why do ***different*** urban activities scale ***differently***?!

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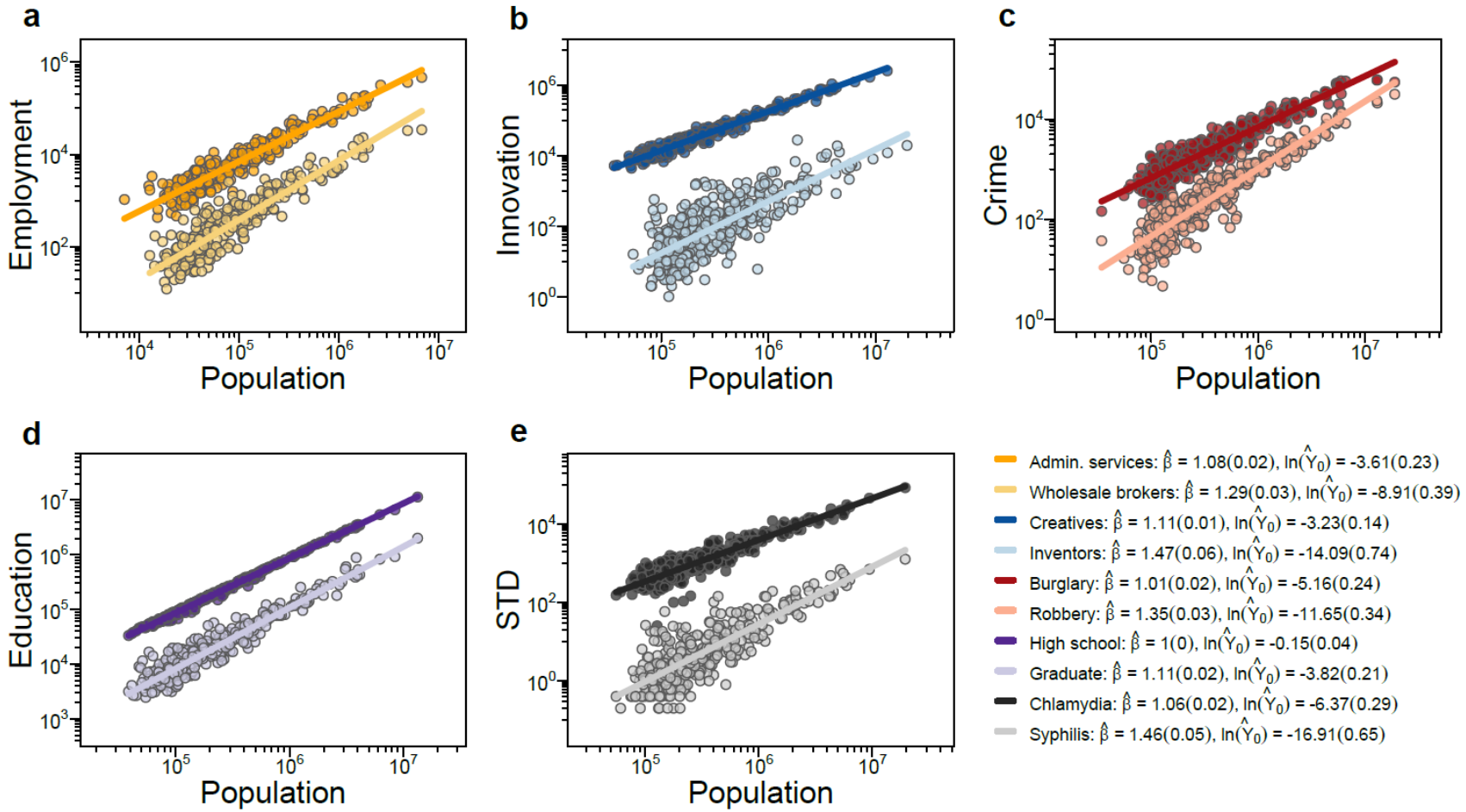
a
Employment
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Admin. services: $\hat{\beta}$ = 1.08(0.02), ln($\hat{Y}_0$) = -3.61(0.23)
Wholesale brokers: $\hat{\beta}$ = 1.29(0.03), ln($\hat{Y}_0$) = -8.91(0.39)
Creatives: $\hat{\beta}$ = 1.11(0.01), ln($\hat{Y}_0$) = -3.23(0.14)
Inventors: $\hat{\beta}$ = 1.47(0.06), ln($\hat{Y}_0$) = -14.09(0.74)
Burglary: $\hat{\beta}$ = 1.01(0.02), ln($\hat{Y}_0$) = -5.16(0.24)
Robbery: $\hat{\beta}$ = 1.35(0.03), ln($\hat{Y}_0$) = -11.65(0.34)
High school: $\hat{\beta}$ = 1(0), ln($\hat{Y}_0$) = -0.15(0.04)
Graduate: $\hat{\beta}$ = 1.11(0.02), ln($\hat{Y}_0$) = -3.82(0.21)
Chlamydia: $\hat{\beta}$ = 1.06(0.02), ln($\hat{Y}_0$) = -6.37(0.29)
Syphilis: $\hat{\beta}$ = 1.46(0.05), ln($\hat{Y}_0$) = -16.91(0.65)

Three takeaways

There is a statistical regularity present in urban systems called "**Urban Scaling**".

$$Y \sim N^{\beta}$$

2. Every social phenomenon has a "**complexity**" that summarizes many of its statistical properties.

3. Ideas from **Cultural Evolution** are needed in order to account for the differences in development across cities.

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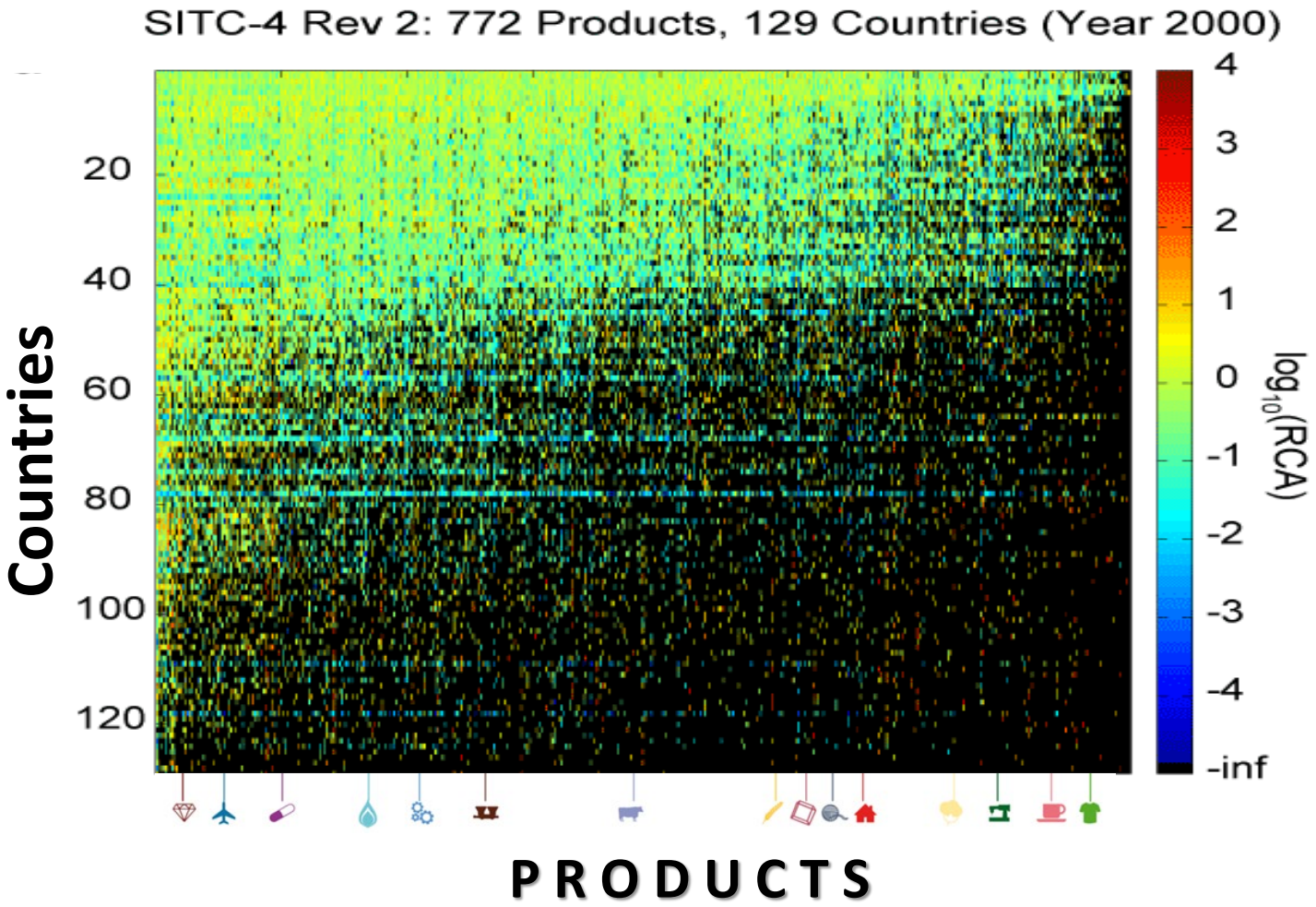
Economics

$y = f($ 

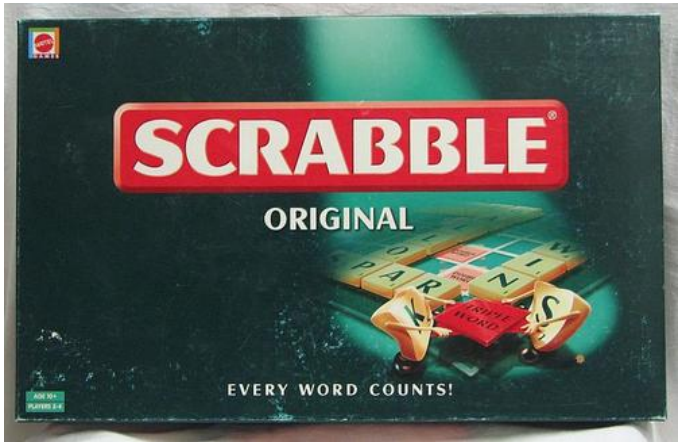
$)$

International trade



Very similar approach to R. Carneiro (1962, 1967, 1970)

… it is the type of pattern you'd expect if countries were playing a game…



Hidalgo, C. A. and Hausmann, R. (2009).
The building blocks of economic complexity. PNAS, 106(25):10570-10575.

Hausmann, R. and Hidalgo, C.A. (2011),
The network structure of economic output. J Econ Growth, 16:309-342.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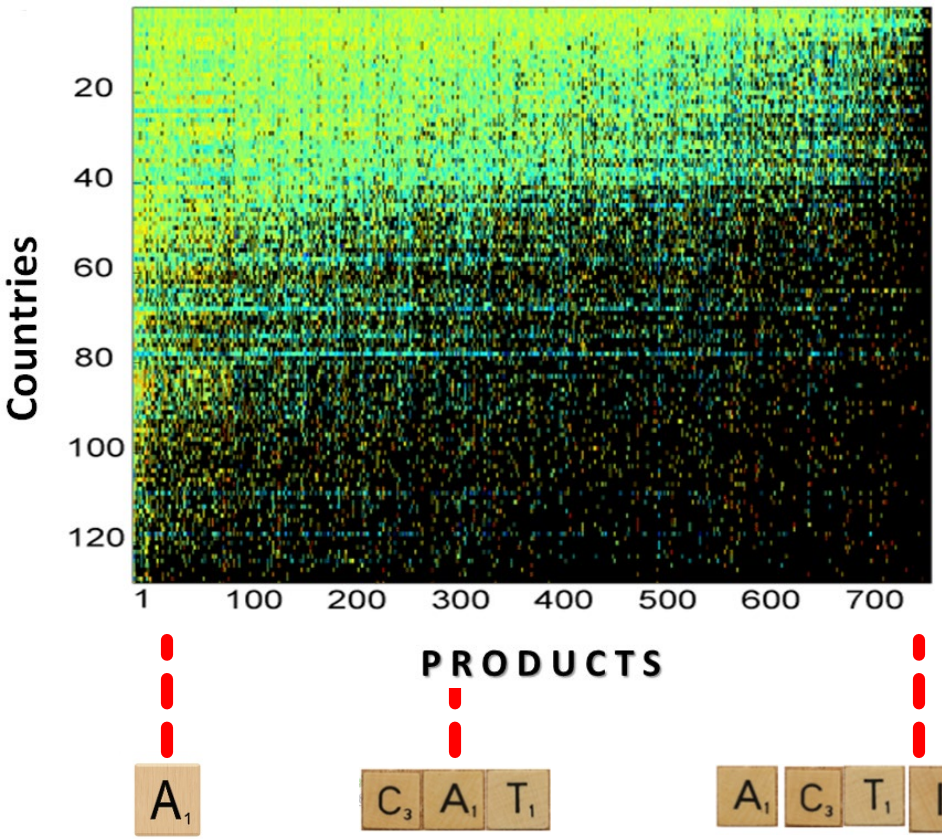
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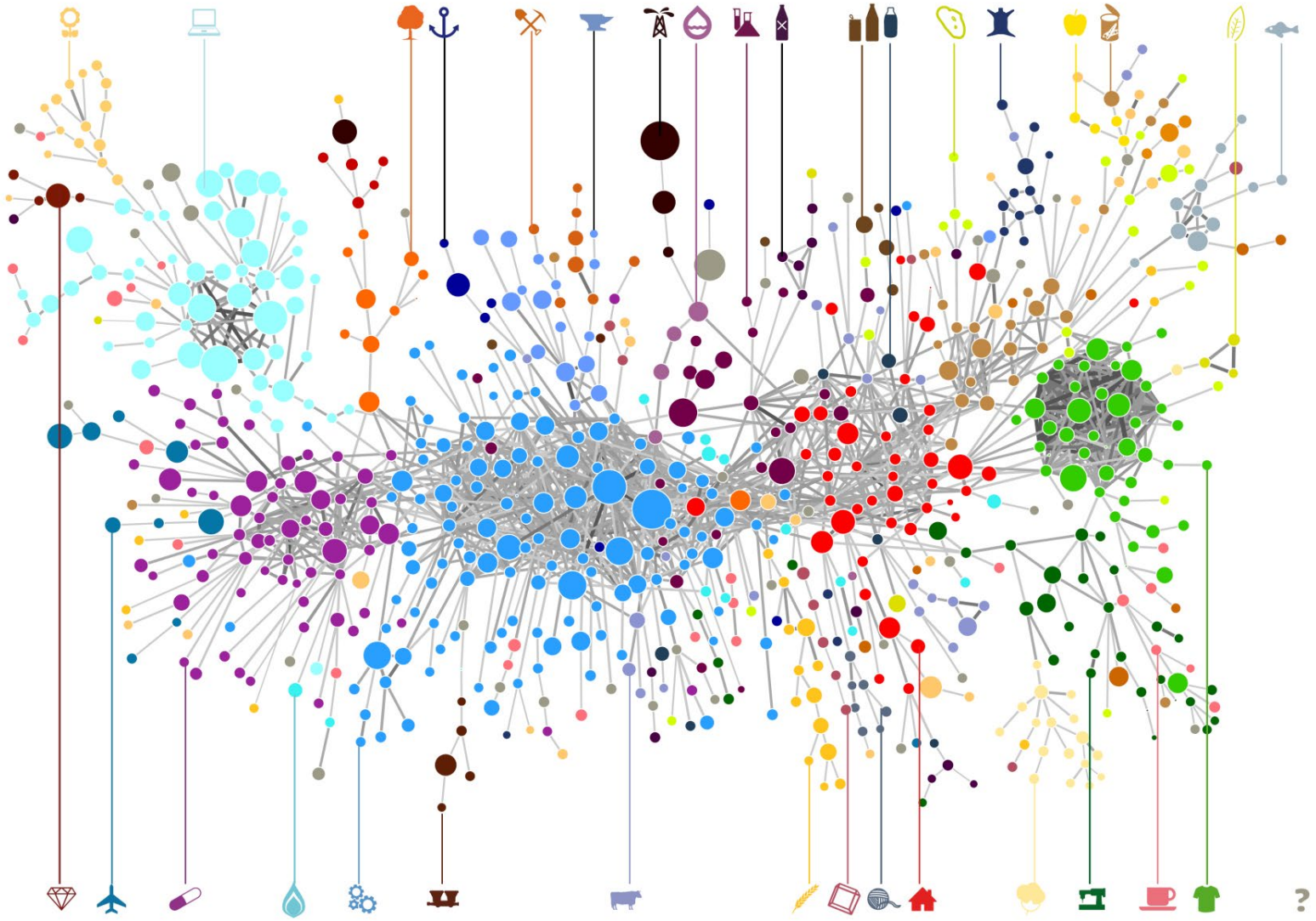
PRODUCTS



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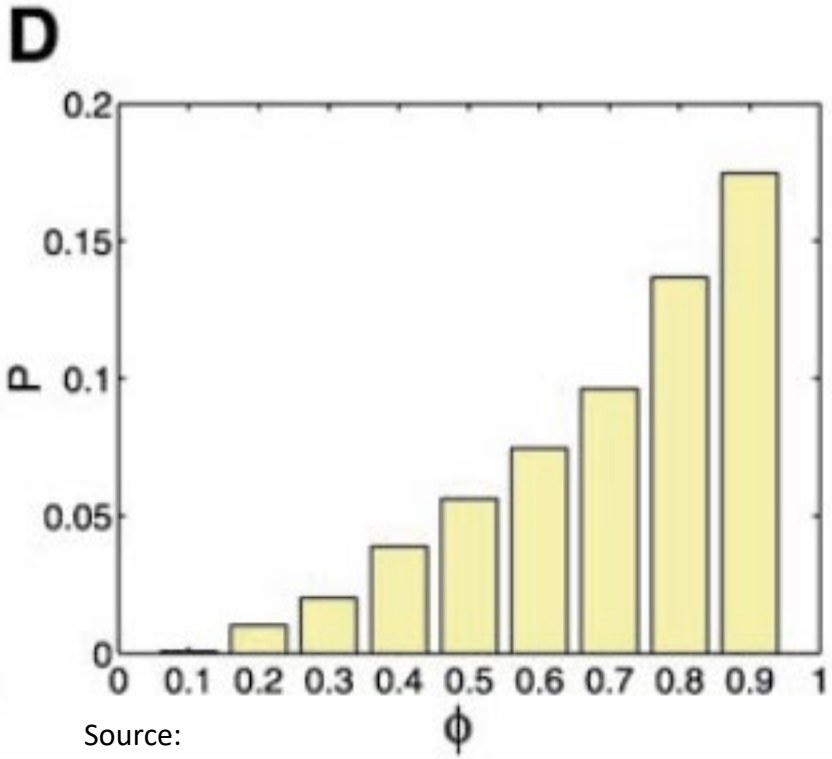
PULP & PAPER
CHEMICALS & HEALTH
AGROCHEMICALS
OTHER CHEMICALS
INOR. SALTS & ACIDS
PETROCHEMICALS
LEATHER
MILK & CHEESE
ANIMAL FIBERS
MEAT & EGGS
FISH & SEAFOOD
TROPICAL AGRIC.
CEREALS & VEG. OILS
COTTON/RICE/SOY & OTHERS
TOBACCO
FRUIT
MISC. AGRICULTURE
NOT CLASSIFIED
TEXTILE & FABRICS
GARMENTS
FOOD PROCESSING
BEER/SPIRITS & CIGS.
PRECIOUS STONES
COAL
OIL
MINING

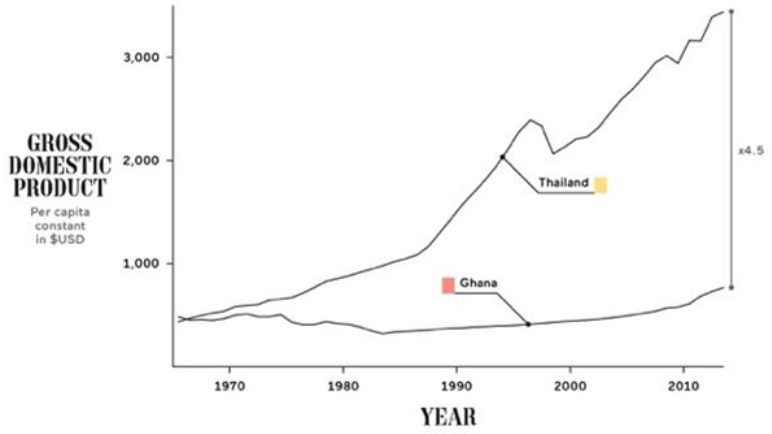
The product space predicts which products will be produced next



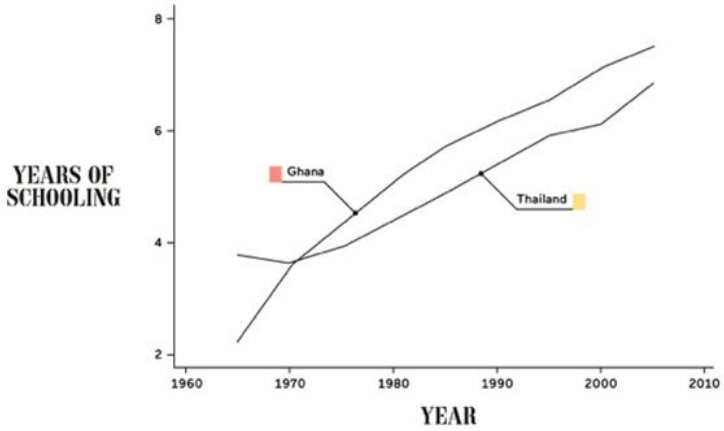
- Entry into new product is easier if you already produce related products
- Products appear in cascades (Klimek/Hausmann/Thurner 2012)

Source:
Hidalgo C.A., Klinger B., Barabási A.-L., and Hausmann R., 2007. The Product Space Conditions the Development of Nations. *Science* (317) pp. 482-487.

Huge divergence in income

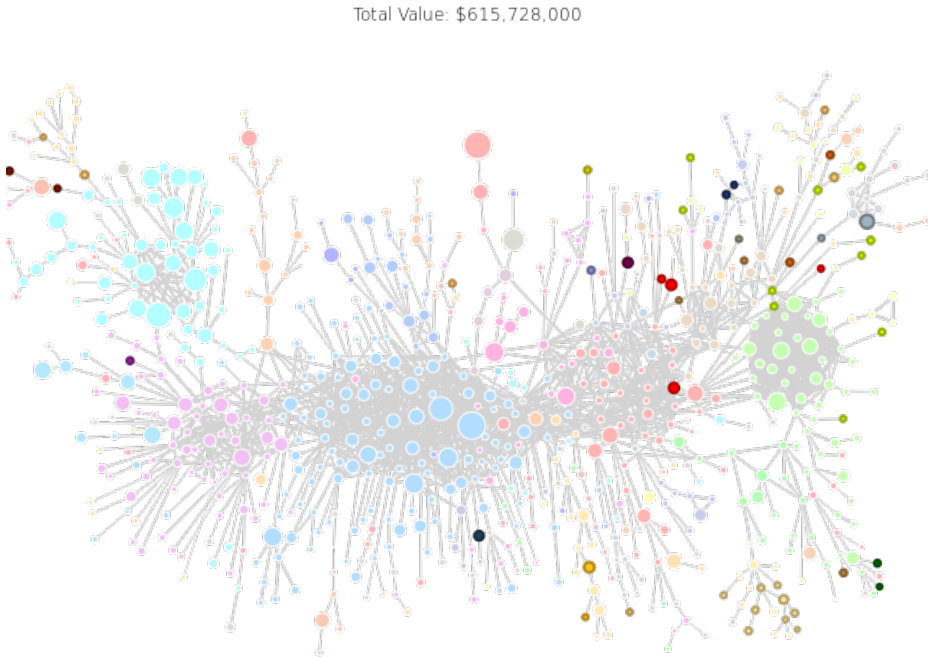


Investment in education

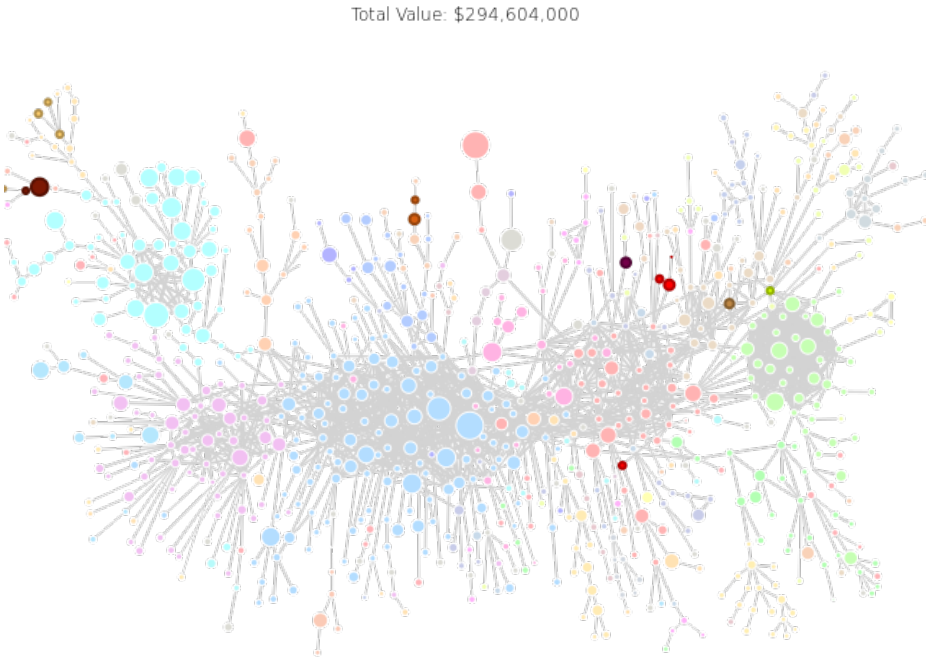


Thailand vs. Ghana in the Product Space

1965

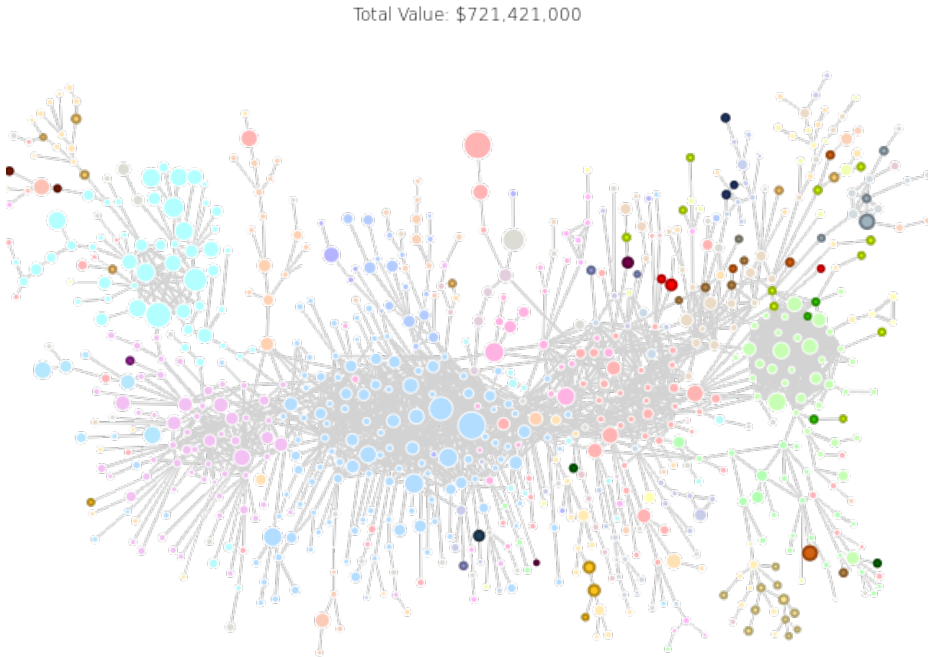


Thailand

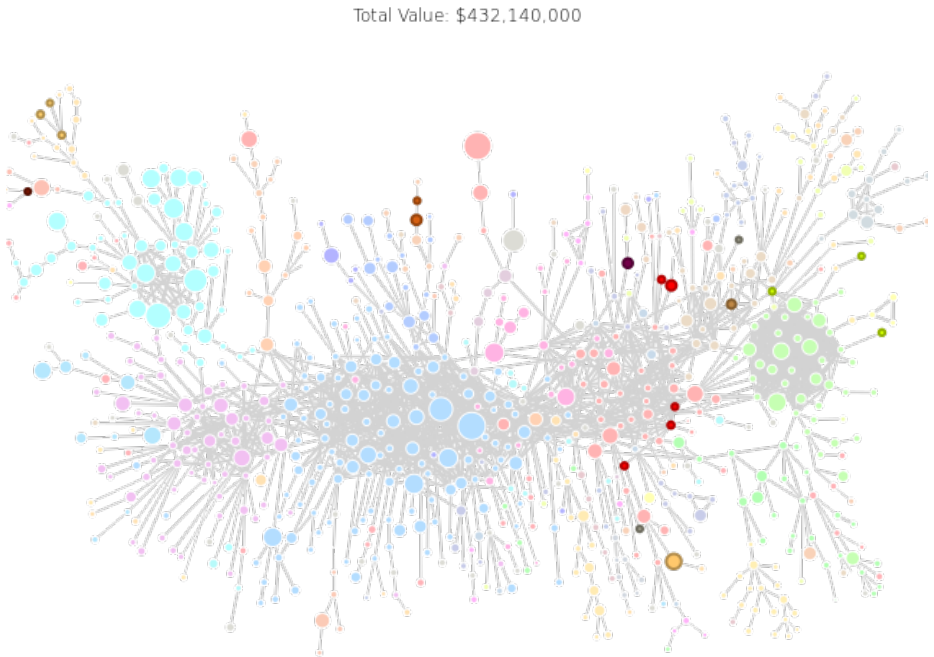


Ghana

1970

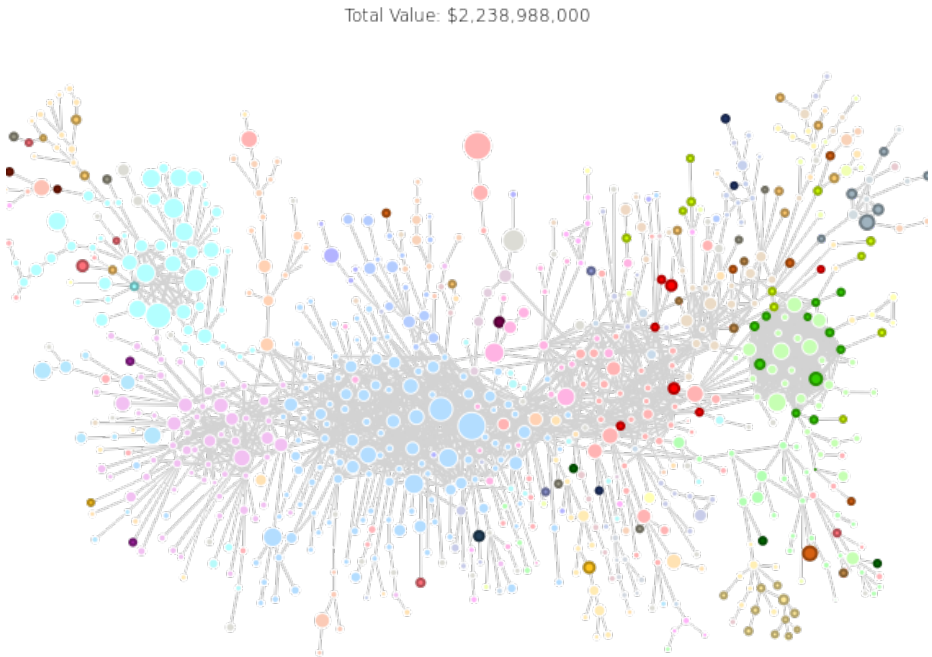


Thailand

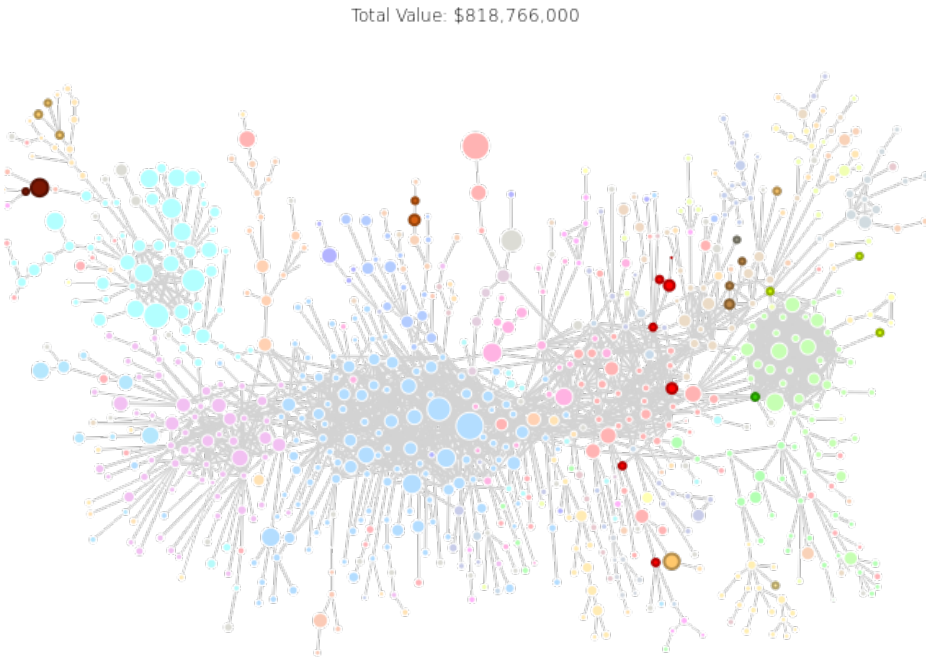


Ghana

1975

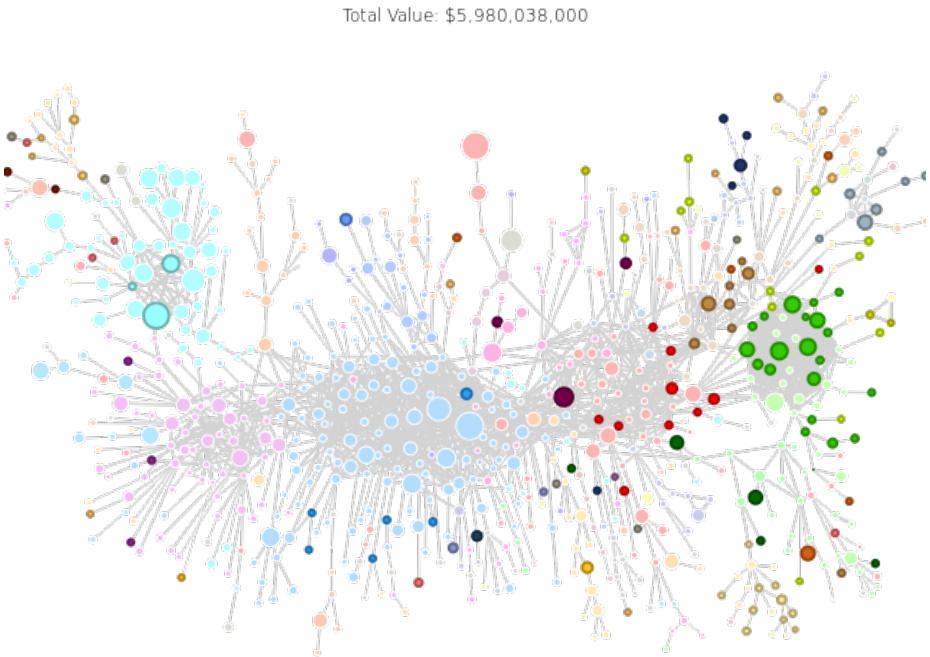


Thailand

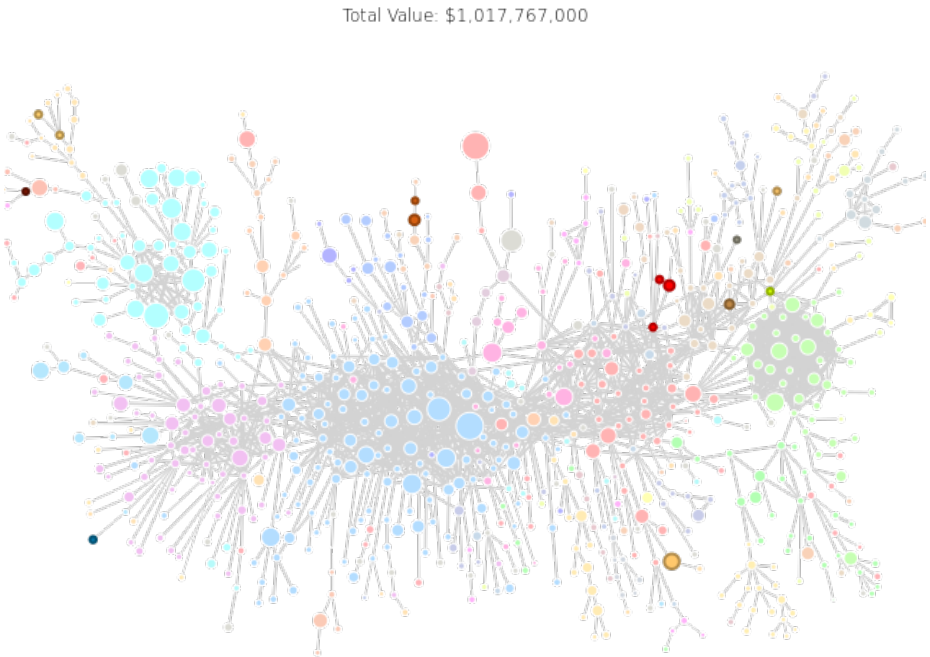


Ghana

1980

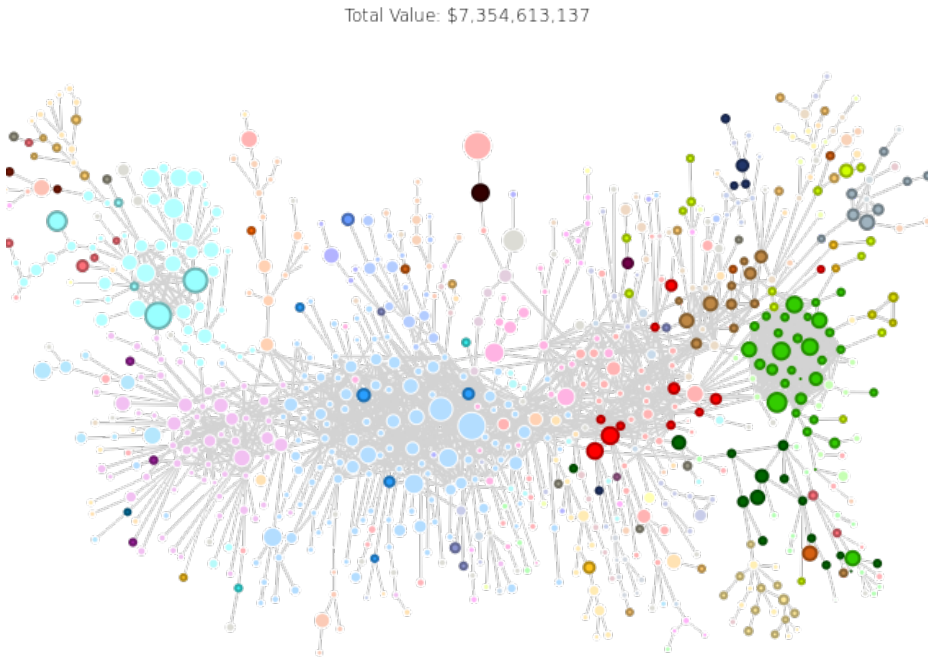


Thailand

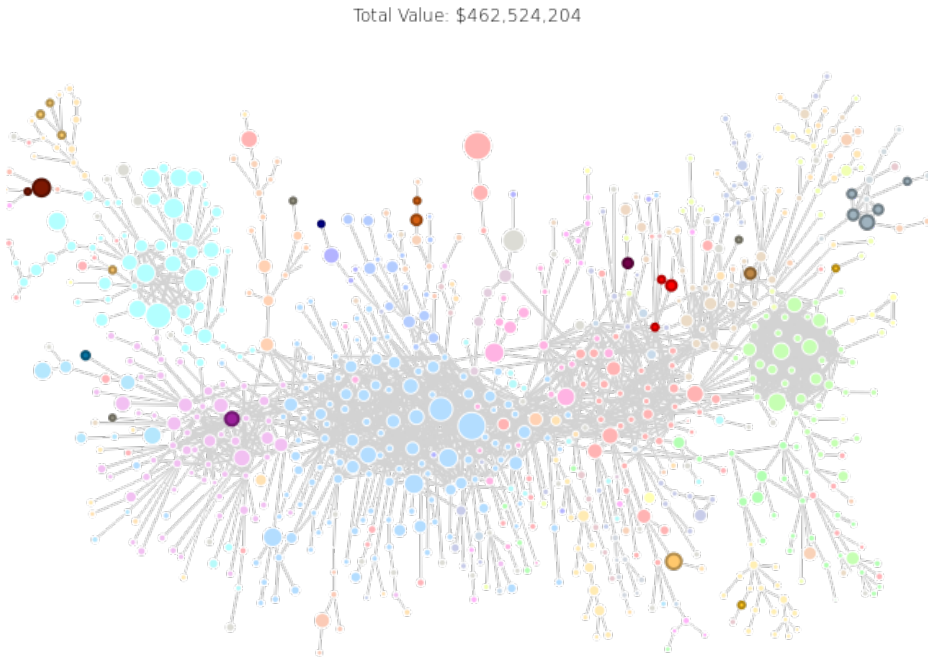


Ghana

1985

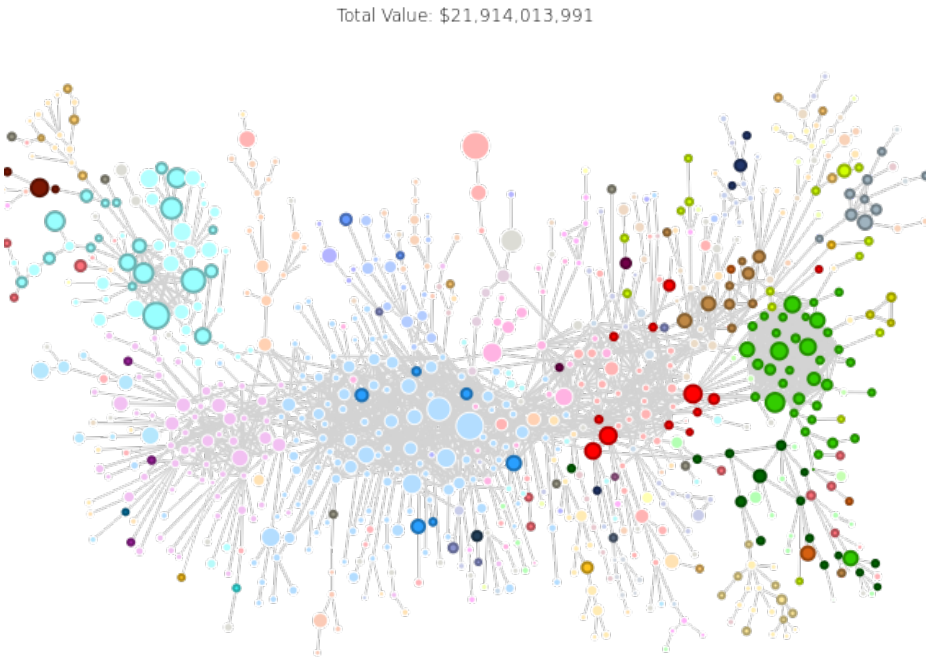


Thailand

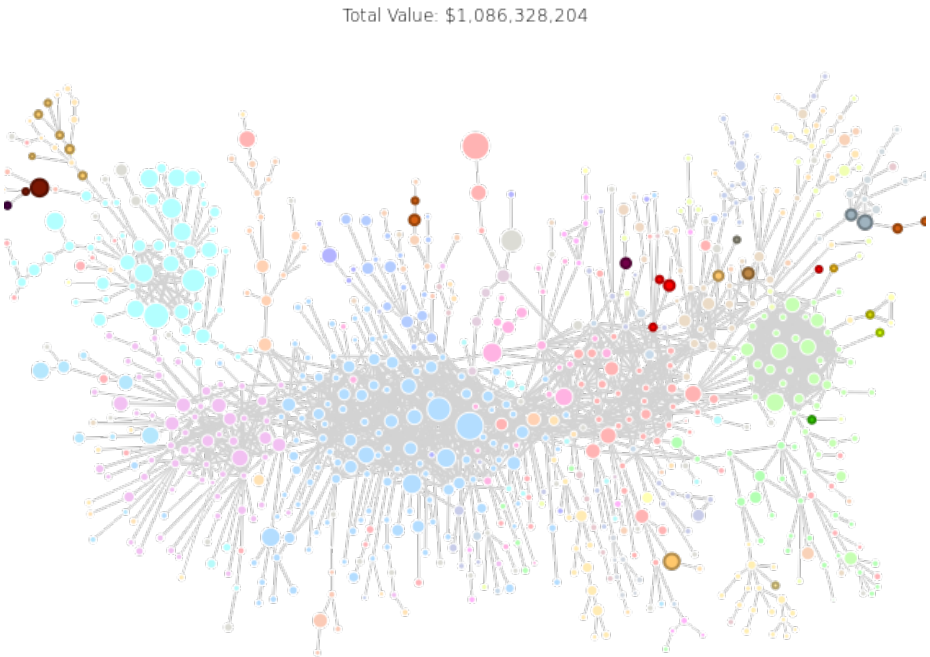


Ghana

1990

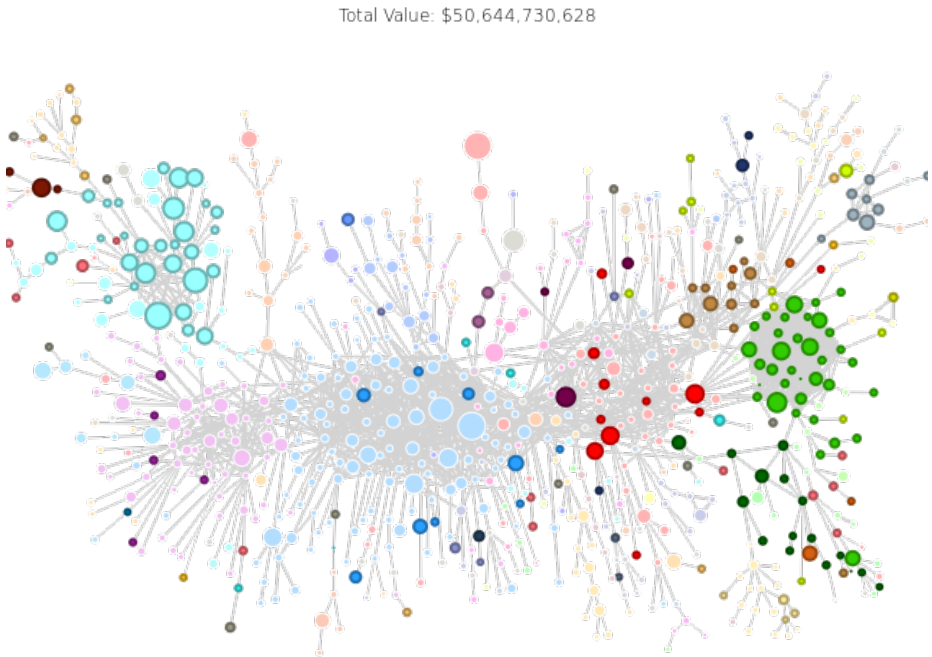


Thailand

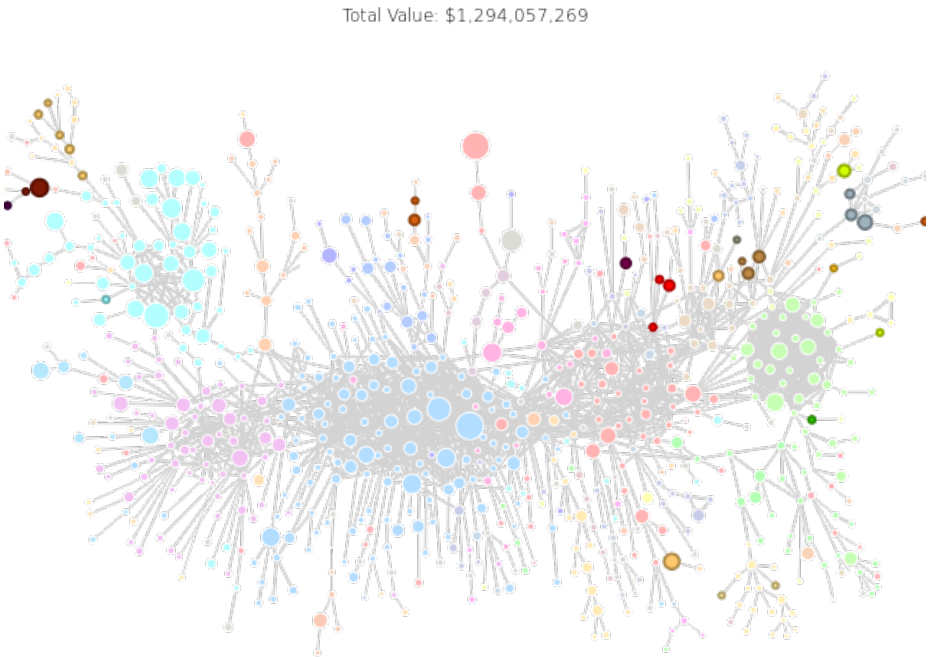


Ghana

1995

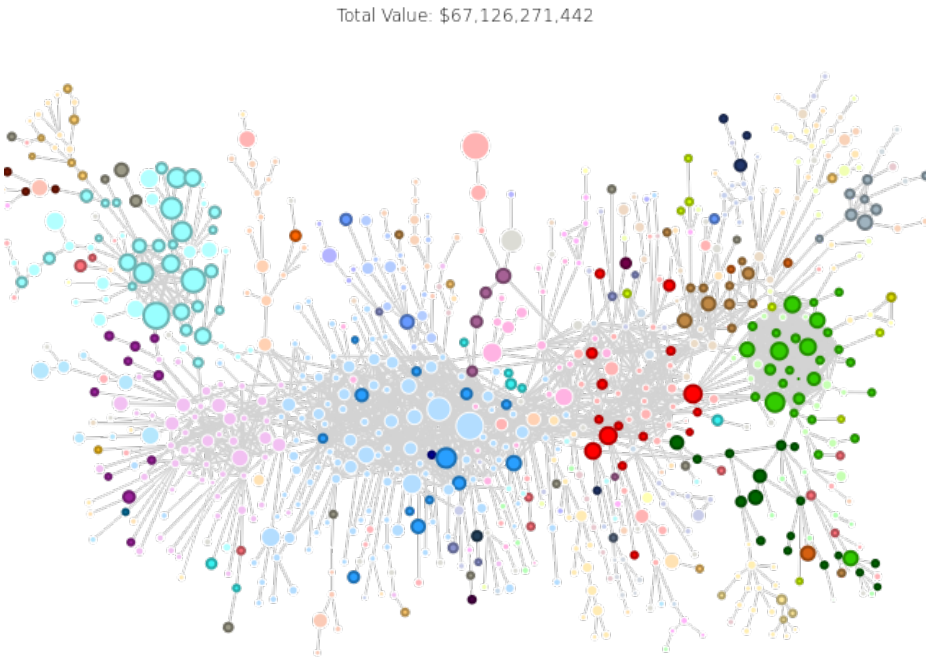


Thailand

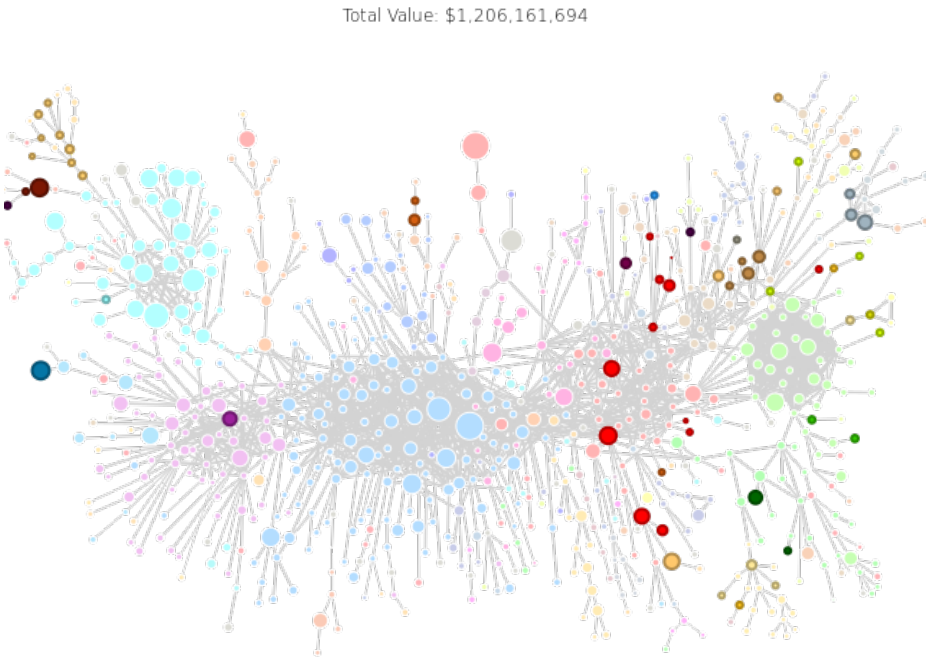


Ghana

2000

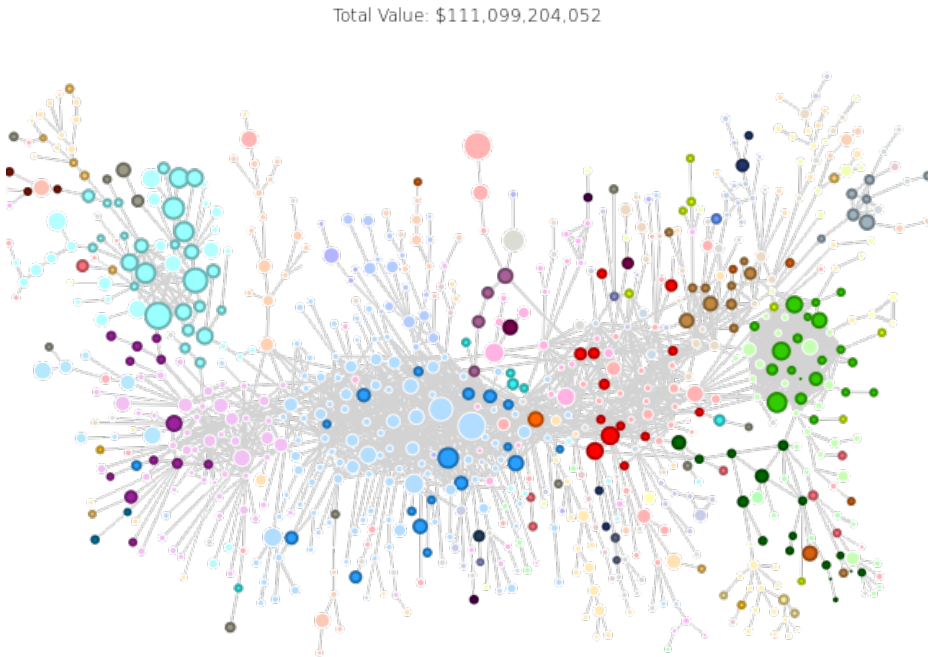


Thailand

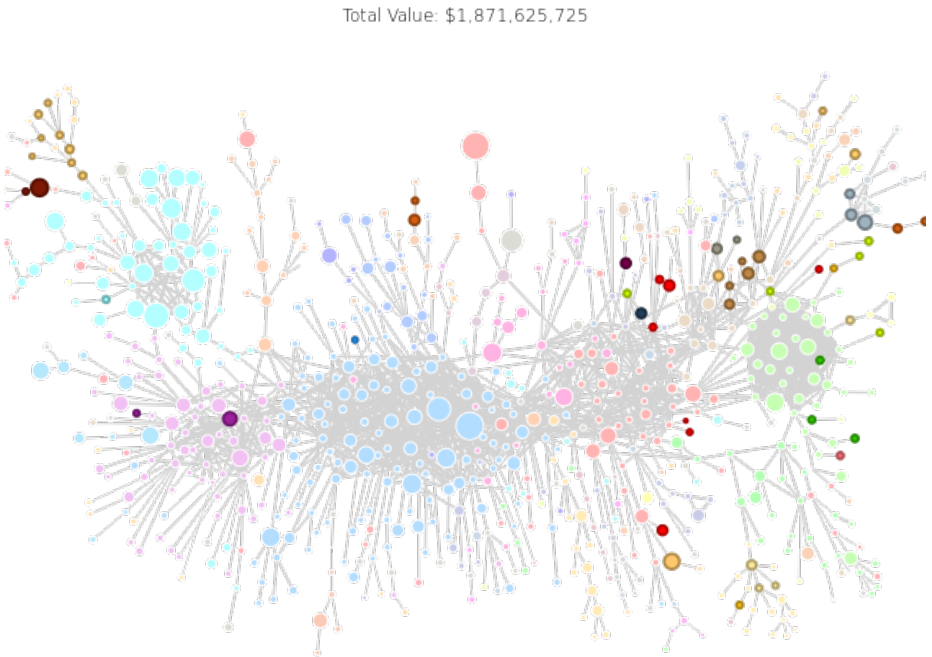


Ghana

2005

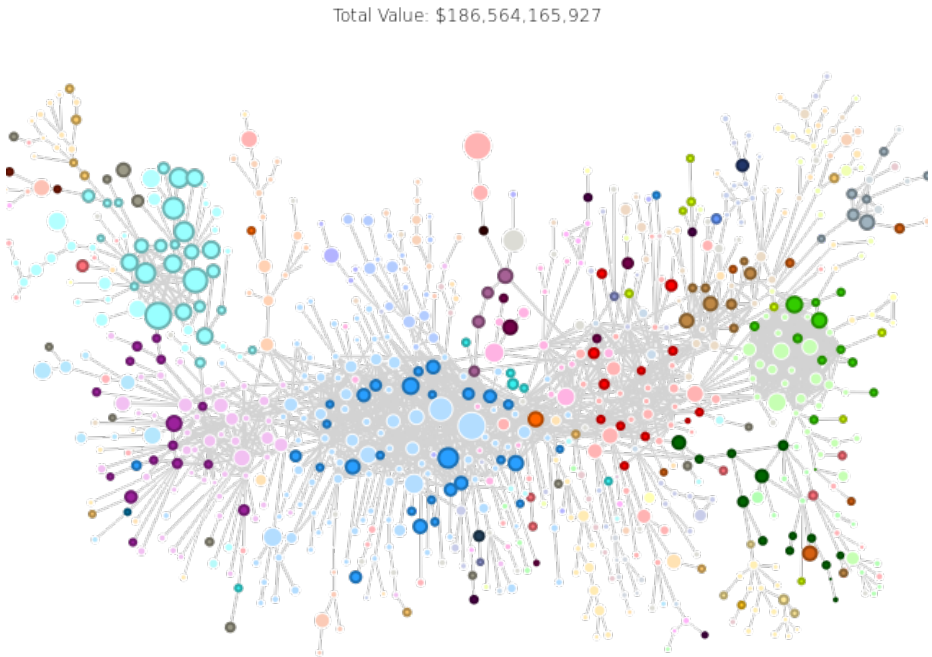


Thailand

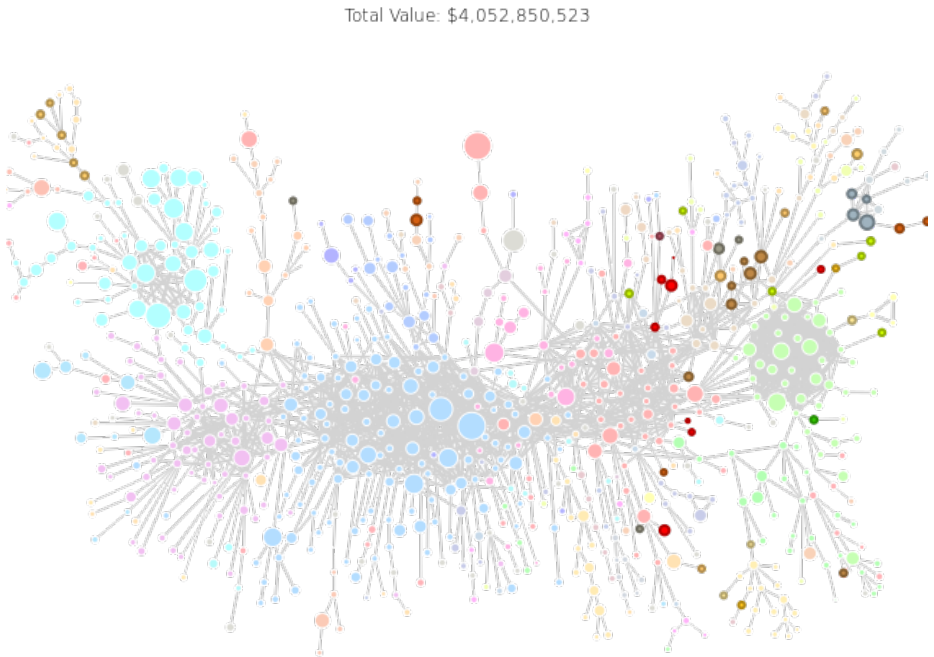


Ghana

2010



Thailand



Ghana

The Theory of Economic Complexity

- Economic processes arise from a multiplicity of factors.

- More complex processes require more factors.
  - # of factors required = **complexity = *q***

- Richer countries have more factors.
  - Endowment of factors = **diversity = *r***

= ***r***

= small ***q***

= large ***q***

= ***r***

C A T

= small ***q***

A C T I N G

= large ***q***

$$\mathbb{E}\left[\frac{Y}{N}\right] = e^{-M(1-r)q}$$

$$= e^{-Mq} e^{Mrq}$$

$$\mathbb{E}\left[\frac{Y}{N}\right] = e^{-Mq}e^{Mrq}$$

Metropolitan Statistical Area (MSA)	Year	Population	Robbery Rate (cases per 100,000 inhabitants)	Larceny-theft (cases per 100,000 inhabitants)
Carson City, NV M.S.A.	2010	55,119	36.3	1,393.3
Michigan City-La Porte, IN M.S.A.	2010	111,553	81.6	2,656.1
Chico, CA M.S.A.	2010	222,130	70.7	1,582.4
Lansing-East Lansing, MI M.S.A.	2010	450,078	79.1	1,763.5
Bridgeport-Stamford-Norwalk, CT M.S.A.	2010	895,941	110.6	1,226.2
Las Vegas-Paradise, NV M.S.A.	2010	1,951,609	240.6	1,580.0
Phoenix-Mesa-Glendale, AZ M.S.A.	2010	4,229,275	124.0	2,400.4
Los Angeles-Long Beach-Santa Ana, CA M.S.A.	2010	12,912,749	189.5	1,428.3
New York-Northern New Jersey-Long Island, NY-NJ-PA M.S.A.	2010	19,042,526	164.2	1,300.3

Source: https://www.fbi.gov/about-us/cjis/ucr/crime-in-the-u.s/

?

- Robbery rates do not seem to increase exponentially with population size.

Three takeaways

- There is a statistical regularity present in urban systems called "**Urban Scaling**". $Y \sim N^{\beta}$
- Every social phenomenon has a "**complexity**" that summarizes many of its statistical properties.

3. Ideas from **Cultural Evolution** are needed in order to account for the differences in development across cities.

Three takeaways

- There is a statistical regularity present in urban systems called "**Urban Scaling**". $Y \sim N^{\beta}$
- Every social phenomenon has a "**complexity**" that summarizes many of its statistical properties.

3. Ideas from **Cultural Evolution** are needed in order to account for the differences in development across cities.

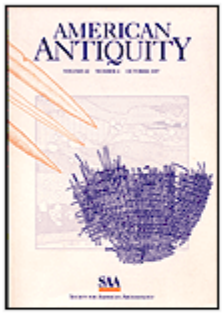
- Robbery rates do not seem to increase exponentially with population size.

Cumulative culture

- Robbery rates do not seem to increase exponentially with population size.

➡ The diversity of factors (i.e., cultural traits) does not change in proportion to population size.

American Antiquity > Vol. 69, No. 2, Apr., 2004 > Demography and Cultu...



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Joseph Henrich

American Antiquity
Vol. 69, No. 2 (Apr., 2004), pp. 197-214

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Subjects: Anthropology | Archaeology

See also:
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Henrich & Boyd (2002),
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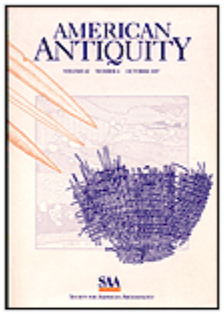

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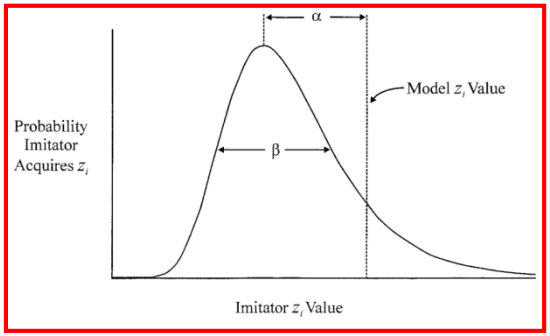

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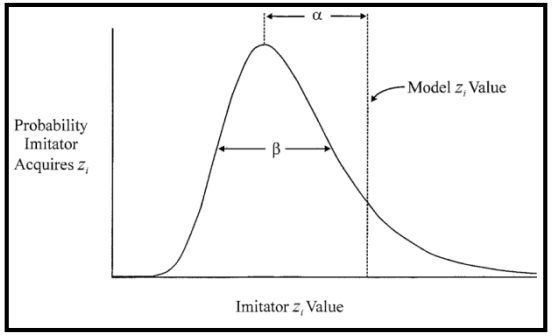
Models of economic complexity

&

Models of cultural evolution



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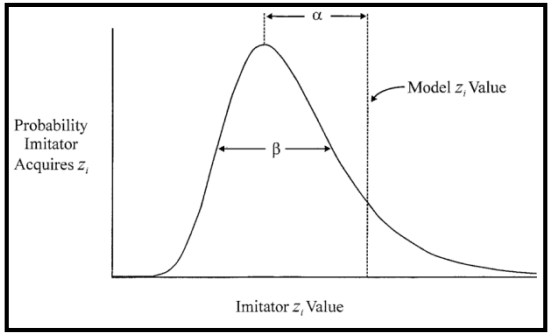


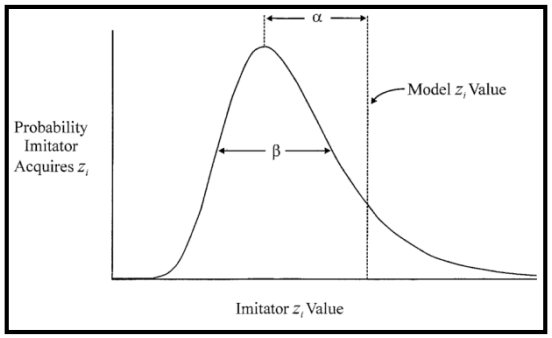
$$\mathbb{E}[Y] = \mathrm{e}^{-Mq} N \mathrm{e}^{Mq\, r(N)}$$

$$r(N) = a + b \ln(N)$$

Models of economic complexity

&

Models of cultural evolution



$$\mathbb{E}[Y] = e^{-Mq} N e^{Mq\, r(N)}$$

$$r(N) = a + b\ln(N)$$

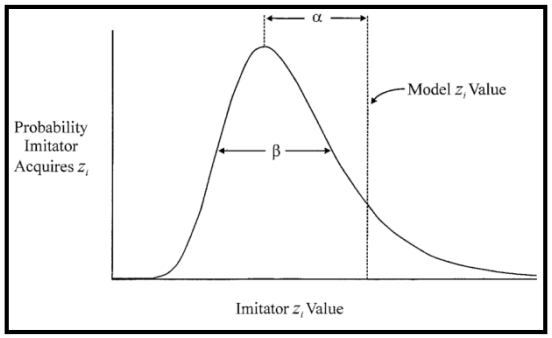
- Urban Scaling: $\mathbb{E}[Y] = e^{-Mq(1-a)} N^{1+Mqb}$

$$= Y_0\, N^{\beta}$$

Models of economic complexity

&

Models of cultural evolution



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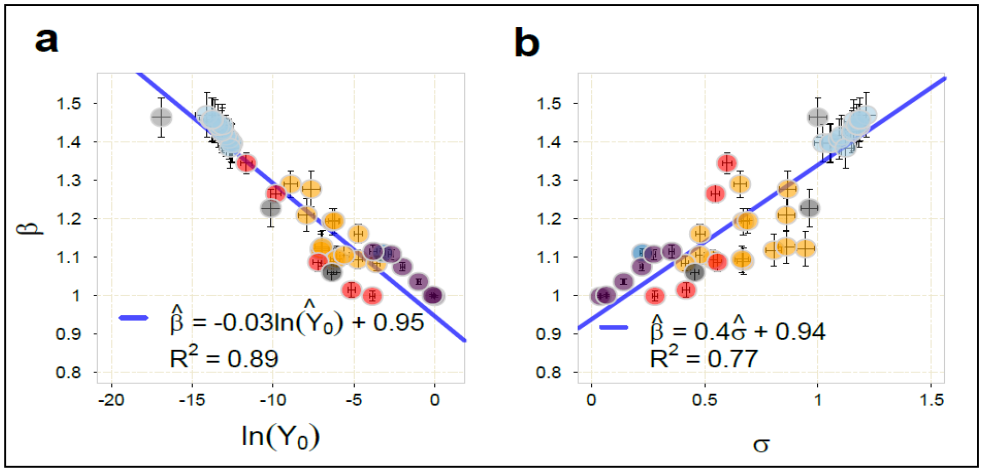


What the Theory Predicts

With the prevalence of a phenomenon in a single city, the theory predicts what the prevalence in the rest of cities is likely to be.

Procedure:

Given coefficients s_1 and s_2 and the populations of all cities n_1, n_2, ...

For a phenomenon of interest:

Pick a random city c with known population size and prevalence:

$$(n_c, y_c)$$

Apply the equations:

$$\beta^{(pred.)} = \frac{1 - s_1 \ln(y_c)}{1 - s_1 \ln(n_c)}$$

$$\ln(Y_0)^{(pred.)} = \frac{1 - \beta^{(pred.)}}{s_1}$$

$$\sigma^{(pred.)} = \frac{\beta^{(pred.)} - 1}{s_2}$$

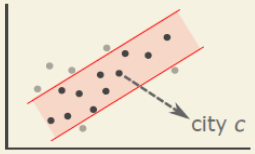
Use the populations n_1, n_2, ..., to predict the prevalence of the phenomenon in the rest of cities within some **prediction bands**:

$$y_i^{\pm} = \exp\left\{\ln(Y_0)^{(pred.)} + \beta^{(pred.)} \ln(n_i) \pm z_\alpha \sigma^{(pred.)}\right\}, \quad \text{for all } i = 1, 2, \ldots$$

To test the predictions, we simulated the **Procedure** 50 times for each phenomenon, for a total of 2150 simulations.

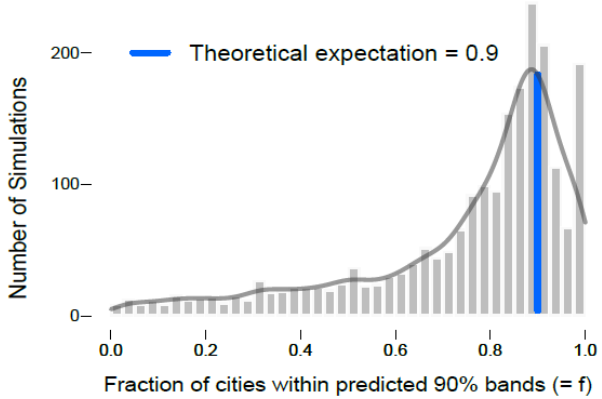
For all phenomena:

1. Apply the **Procedure**
2. Compute the fraction f of cities within the predicted bands



$$f = \frac{\bullet\bullet\ldots}{\bullet\bullet\bullet\ldots + \bullet\bullet\ldots}$$

Results of Simulations



THANK YOU

Contact info:
Email: andres_gomez@hks.harvard.edu
Twitter: @GomezLievano

Learning at Different Levels

- Individuals learn by going to school, by doing, by interacting.

Learning at Different Levels

- Individuals learn by going to school, by doing, by interacting.

- Cities “learn” by attracting new individuals with more diverse skills (Florida, 1995, *Futures*).

Learning at Different Levels

- Individuals ***are limited*** by the amount of knowledge that they can learn;

Learning at Different Levels

- Individuals ***are limited*** by the amount of knowledge that they can learn;

- **Societies, however, <u>are not</u>.**

Example

	Getting a patent requires:
1)	Having a technological problem
2)	Having a solution
3)	Presenting the idea clearly
4)	Applying for a patent
5)	Including corrections from examiners
6)	Satisfying all legal requirements

Example

	Getting a patent requires:	**City *c***	**Person *1***	**Person *2***	**Person *3***
1)	Having a technological problem		X	X	X
2)	Having a solution		X		X
3)	Presenting the idea clearly	X	X	X	
4)	Applying for a patent		X	X	X
5)	Including corrections from examiners	X	X	X	
6)	Satisfying all legal requirements	X	X		
			Person *1*	**Person *2***	**Person *3***
	Gets the patent?		YES	NO	YES

Example

	Getting a patent requires:	City *c*	Person *1*	Person *2*	Person *3*
1)	Having a technological problem		X	X	X
2)	Having a solution		X		X
3)	Presenting the idea clearly	X	X	X	
4)	Applying for a patent		X	X	X
5)	Including corrections from examiners	X	X	X	
6)	Satisfying all legal requirements	X	X		
			Person *1*	**Person *2***	**Person *3***
	Gets the patent?		YES	NO	YES

Example

	Getting a patent requires:	City *c*	Person *1*	Person *2*	Person *3*
1)	Having a technological problem		X	X	X
2)	Having a solution		X		X
3)	Presenting the idea clearly	X	X	X	
4)	Applying for a patent		X	X	X
5)	Including corrections from examiners	X	X	X	
6)	Satisfying all legal requirements	X	X		
			Person *1*	**Person *2***	**Person *3***
	Gets the patent?		YES	NO	YES

City ***c*** has **2 inventors** (from a population of 3).

The Model

- The activity in question requires N_a substeps.
- The substeps provided by the city as a N_ax*1* vector, $\vec{c}$, *1* for the substeps provided, *0* otherwise.
- We represent the substeps that person *j* needs using a N_ax*1* vector, $\vec{p}_j$, with *1* for the missing substeps, *0* otherwise.

City *c*	Person *1*	Person *2*	Person *3*
	X	X	X
	X		X
X	X	X	
	X	X	X
X	X	X	
X	X		

$\vec{c}$	$\vec{p}_1$	$\vec{p}_2$	$\vec{p}_3$
0	0	0	0
0	0	1	0
1	0	0	1
0	0	0	0
1	0	0	1
1	0	1	1

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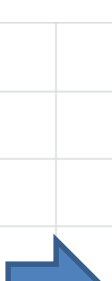
City *c*	Person *1*	Person *2*	Person *3*
	X	X	X
	X		X
X	X	X	
	X	X	X
X	X	X	
X	X		



$\vec{c}$	$\vec{p}_1$	$\vec{p}_2$	$\vec{p}_3$
0	0	0	0
0	0	0	0
1	0	0	1
0	0	0	0
1	0	0	1
1	0	1	1

r

q

The Model

Table 1: Parameters of the model. The parameters M, q and r are in principle phenomenon-dependent.

Parameter	Meaning
$N > 0$	City population size susceptible of participating of a given phenomenon.
$M > 0$	Number of possible factors required for the given phenomenon.
$q \in (0, 1)$	Probability that an individual needs any given factor from the environment.
$r \in (0, 1)$	Probability that the city facilitates any one of the factors to the individual.

- The main parameters, *r* and *q*:
  - *r*: the average "diversity" of the city.
  - *q*: the average "complexity" of the activity.
- Prevalence of the activity in the city: $Y = \sum_{j=1}^{N} X_j$
- E[*Y*] = *f*(*N*, *r*, *q*) = ?