

Transitional Dynamics of the Savings Rate in the Neoclassical Growth Model

Pol Antras*

Department of Economics
Massachusetts Institute of Technology
Cambridge, MA

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Abstract

This paper characterizes the transitional dynamics of the savings rate in the neoclassical growth model. I start with a general formulation with weak assumptions on preferences and technology and go on to fully describe the transitional behavior of the savings rate under particular functional forms. It is shown that under plausible functional forms for preferences and technology, the model is able to explain the hump-shaped behavior of the savings rate observed in most OECD countries in the period 1950-1990. The paper also provides econometric evidence supporting the empirical relevance of the neoclassical growth model in explaining the dynamics of the savings rate both in OECD countries and in a larger cross-section of countries.

1 Introduction

Figure 1 graphs the weighted-average investment rate for a sample of 24 OECD countries over the period 1950-1990.¹ Although the investment rate tended to fluctuate substantially, there is clear evidence of a hump shape in the series. This is confirmed in Figure 2, where the series is detrended, using the Hodrick-Prescott filter, to remove business cycle fluctuations. Different tests on the series corroborated the statistical significance of the hump.² Figure 3 shows that, for most countries, the

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¹The 24 countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, West Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, UK, and USA. The data was taken from the Penn World Tables. In the weighted-average, each country's investment rate was weighted by the ratio of the country's GDP to the aggregate GDP in the 24 OECD countries. Taking a simple average of the investment rates led to a very similar figure.

²I first regressed the investment rate on a constant, a time trend and the square of the time trend. The coefficient of the time trend was positive and significant at the 1% level, whereas that of the squared time trend was negative and also significant at the 1% level. Next, I applied the logistic transformation to the series: $f(I/Y) = \log\left(\frac{I/Y}{1-I/Y}\right)$. This was done to increase the range of

behavior of the investment rate at the country level resembles that of the weighted-average investment rate.

The decline in the savings rate in the 1970s and 1980s has been extensively documented in the literature (e.g., Modigliani, 1993, Schmidt-Hebbel and Serven, 1999a).³ The two most prominent explanations for the fall in national savings have been: (i) the effect of the productivity slowdown of the 1970's, and (ii) the fall in public savings in both the 1970's and 1980's.

The first explanation has been most vigorously pushed by Modigliani (1986, 1993). As he has emphasized in several contributions, the main implication of his life-cycle model is a positive correlation between the growth rate of output and the savings rate. In the stripped-down version of his model, individuals save when they are young and run down their assets after retirement. To the extent that output growth increases the income of the young relative to that of the old, and to the extent that output growth is only explained by productivity growth, the model predicts a positive correlation between the savings rate and productivity growth. Hence, in the framework of his model, the fall in the savings rate that occurred in the 1970's and 1980's can easily be explained by the productivity slowdown that occurred in that same period.

Although highly intuitive, this explanation is flawed in several respects. On the one hand, the assumption that savings are undertaken by the young is counterfactual. This fact is actually one of the main motivations for the new theories of consumption that have enriched Modigliani's original framework with borrowing constraints, precautionary savings motives, and habit formation, among others (see Deaton 1992, or Attanasio 1999 for excellent reviews of this literature).

On the other hand, even if one relied on the stripped-down version of the model, the baby boom of the 1950's and early 1960's would jeopardize the simple explanation for the decline in the savings rate in the 1970's and 1980's. In Modigliani's framework, the effect of population growth on the savings rate is identical to that of productivity growth. Figure 4 shows that following the baby boom in OECD countries, population growth declined dramatically from the late 1960's to 1990. What is most important, however, is that the baby-boomers started playing the role of the young in the life-cycle model around the 1970's and 1980's, precisely when savings rates were starting to fall. This effect could well have counterbalanced the productivity slowdown, in which case the stripped-down life-cycle model would have predicted a rise in the savings rate in the period when it actually fell.

The second explanation for the decline in savings, the fall in public savings, is also problematic. Public-sector savings did fall dramatically in the 1970's and 1980's, but this does not explain why private agents, in anticipation of higher future taxes, did not increase their savings accordingly. In fact, as shown by Barro (1974),

the series from the interval $[0,1]$ to $(-\infty, \infty)$. The transformed series was regressed on a constant, a time trend and the square of the time trend, obtaining the same qualitative results as in the regression with the raw series. Finally, a Chow test was performed on the simple regression of the investment rate on a constant and a time trend. The test identified a significant break in all years around the peak of the hump in Figure 1.

³In a classical contribution, Feldstein and Horioka (1980) documented the close connection between national savings rates and national investment rates, which they interpreted as evidence of limited international capital mobility. Hence, it should not be surprising that the savings rate in OECD countries followed a similar hump-shape pattern for the period 1950-1990.

under certain assumptions, one would have expected private savings to increase by the exact same amount by which public savings fell. The fact the private savings rate remained roughly constant or even fell in this period (c.f. Bosworth, 1990, Schmidt-Hebbel and Serven, 1999b) is thus puzzling and difficult to reconcile even with theories that predict only partial Ricardian equivalence.

Notwithstanding the difficulties explained above, the literature seems to have taken for granted that the life cycle model of Modigliani is better equipped to explain the fall in the savings rate than the alternative permanent income hypothesis model of Friedman (1957), in which agents are infinitely-lived and the life-cycle aspect of consumption is ignored. Certainly, a simplified version of Friedman's model would have predicted an increase in the savings rate following the productivity slowdown. In particular, with quadratic intertemporally additive utility, a constant real interest rate equal to the rate of time preference, and an exogenous and (possibly) random income stream, one can derive from Friedman's model the following so-called "rainy day equation" (c.f. Deaton, 1999):

$$s_t = - \sum_{k=t}^{\infty} E_t \frac{\Delta y_{t+k}}{(1+r)^k} \quad (1)$$

Savings are equal to the discounted present value of expected future falls in income. To the extent that the productivity slowdown anticipated a fall in earnings, it should have fostered an increase, not a fall, in the savings rate.

This paper will show, however, that the fall in the savings rate can be easily rationalized in a simple, well-known model in which agents essentially behave as permanent income consumers: the neoclassical growth model. Unlike in the simple version of Friedman's model that delivers equation (1), in the general equilibrium model developed below, agents' income will be endogenous and will depend on the aggregate evolution of factor prices, which in turn will be affected by capital accumulation originating from the agents' decision to save. The behavior of the savings rate will in general be ambiguous, as it involves the offsetting effects of a substitution effect and an income effect. The income effect relates to the fact that in the transition to the steady state of the economy, income will be below its permanent level. Given the agents' desire to smooth consumption, savings will tend to be low at the beginning of the transition, and will tend to rise as the gap between current and permanent income falls. The substitution effect will work through the agents' holdings of capital. Since the model assumes decreasing returns to capital, the rate of return to capital will be very high in the early stages of the transition providing a high incentive to save. As the economy grows, however, the rate of return falls, and so does the incentive to save. The exact shape of the savings rate along the transition will be determined by the relative magnitude of these two effects, which in turn depend on the particular functional forms imposed on preferences and technology. One of the purposes of this paper is to show that assuming simple and widely used functional forms (e.g. CES technology, Stone-Geary preferences), the neoclassical growth model could naturally explain the hump shapes documented in Figures 1, 2 and 3.

This paper is not the first attempt to look at national savings in the framework of the neoclassical growth model. Carroll and Weil (1994) as well as Barro and Sala-

i-Martin (1995, Chapter 2) analyzed the behavior of the savings rate in the model with constant elasticity of intertemporal substitution (CIES) preferences and Cobb-Douglas technology. In particular, Barro and Sala-i-Martin (1995) demonstrated formally that, in this case, the savings rate is either monotonically increasing or monotonically decreasing along the transition, and that only for high values of the elasticity of intertemporal substitution does the model predict a positive link between income and the savings rate.

Christiano (1989) used the model to explain the hump-shaped pattern of the Japanese national savings rate from the mid 1950s to the late 1980's. In particular, he simulated a neoclassical model with Stone-Geary preferences and Cobb-Douglas technology, obtaining a remarkably good fit of the actual data. In their study of the transitional dynamics of the neoclassical model, King and Rebelo (1993) presented similar simulations with Stone-Geary preferences.

Rebelo (1992) also experimented with Stone-Geary preferences. The focus of his study was to show that, even with perfect capital mobility, the neoclassical model does not necessarily imply that the growth rates of consumption and output are immediately equalized across countries. Although in his study preferences take the same functional form as in Christiano (1989), Rebelo's (1992) model does not deliver a hump-shaped behavior of the savings rate because his choice of technology exhibits a constant marginal product of capital. Instead, he finds a monotonically increasing savings rate.

Recently, Steger (2000) studied the convergence implications of Rebelo's (1992) model. He also simulated the model assuming a technology featuring decreasing but asymptotically constant returns to scale, thus obtaining a hump-shaped pattern similar to the one obtained by Christiano (1989).

A related branch of the literature has introduced habit formation in the neoclassical model. In particular, Carroll, Overland and Weil (2000) present a model in which habit formation causes the intertemporal elasticity of substitution to increase along the transition, in a similar manner as under Stone-Geary preferences. Hence, the qualitative implications for the behavior of the savings rate are analogous to the ones of some of the studies cited above.

In summary, two basic facts emerge from the literature. First, under CIES preferences and Cobb-Douglas technology, either the income effect always dominates the substitution effect or the other way around. Second, allowing for an increasing intertemporal elasticity of substitution seems to weaken the substitution effect in the early stages of the transition, potentially leading to a hump-shape behavior of the savings rate (if the substitution effect dominates later in the transition). One of the contributions of this paper is to identify a second channel through which a hump-shaped savings rate can emerge in the neoclassical model.

The rest of the paper is organized as follows. In section 2, I present a general formulation for the dynamics of the savings rate along the transition of the neoclassical growth model, with very weak assumptions on preferences and technology. In section 3, I fully characterize the transitional behavior of the savings rate under different particular functional forms. Some of the studies discussed above will appear as a particular examples of the general framework developed in section 2. In section 4, I take the model to the data to evaluate its empirical relevance in explaining the

patterns observed for the sample of 24 OECD countries, and also in characterizing saving rates in a larger cross section of countries. Section 5 concludes.

2 General Framework

2.1 Preferences, Technology, and Endowments

The economy admits an infinitely-lived representative agent whose objective is to maximize the utility function:

$$U = \int_0^{\infty} e^{-(\rho-n)t} u(C/L) dt \quad (2)$$

where n is the constant exogenous population growth rate, ρ is the rate at which the agent discounts future utility streams, and C/L is per capita consumption. The function $u(\cdot)$ is assumed to be twice continuously differentiable, increasing and concave, and also satisfies $u'(C/L) \rightarrow 0$ as $C/L \rightarrow \infty$, and $u'(C/L) \rightarrow \infty$ as $C/L \rightarrow \zeta$, for some $\zeta \geq 0$ (to be interpreted as a subsistence level of consumption). The representative agent faces the following budget constraint:

$$\left(\frac{\dot{K}}{L}\right) = r\frac{K}{L} + w - \frac{C}{L} - n\frac{K}{L} \quad (3)$$

where r is the return to capital (net of depreciation) and w is the wage. To avoid Ponzi-game schemes, the agent also faces the following constraint:

$$\lim_{t \rightarrow \infty} \frac{K(t)}{L(t)} e^{-\int_0^t (r(s)-n)ds} \geq 0 \quad (4)$$

On the production side, a continuum of profit-maximizing firms have access to a technology that transforms the capital good (combined with labor) into the consumption or final good. This technology is represented by a production function $Y = F(K, e^{xt}L, A)$ which is assumed to be increasing and concave in the two factors as well as homogeneous of degree one. I allow for labor-augmenting technological change, at rate x , and also introduce a constant multiplicative efficiency parameter, A . Letting $k = \frac{K}{e^{xt}L}$ and $y = \frac{Y}{e^{xt}L}$ denote capital and output per effective worker respectively, the assumptions imply:

$$y = Af(k) \quad (5)$$

To ensure the existence and uniqueness of a steady state equilibrium, it is also assumed that $f'(k) \rightarrow 0$ as $k \rightarrow \infty$, and $f'(k) \rightarrow \xi$ as $k \rightarrow 0$, where ξ is assumed high enough to guarantee a positive level of k in equilibrium.

The representative agent is endowed with k_0 units of capital at time 0 and with 1 units of labor every period, which he supplies inelastically.

2.2 Equilibrium

To solve for the equilibrium of the economy, it is convenient to restate the consumer problem in terms of efficiency units of labor ($c = \frac{C}{e^{xt}L}$):

$$\begin{aligned} \text{Max } U &= \int_0^\infty e^{-(\rho-n)t} u(ce^{xt}) dt \\ \text{s.t. } (ke^{xt}) &= (r-n)ke^{xt} + w - ce^{xt} \\ \lim_{t \rightarrow \infty} k(t)e^{-\int_0^t (r(s)-n-x)ds} &\geq 0 \end{aligned}$$

The present-value Hamiltonian of the consumer program is

$$\mathcal{H} = u(ce^{xt})e^{-(\rho-n)t} + \lambda [(r-n)ke^{xt} + w - ce^{xt}] \quad (6)$$

and the first-order conditions can be combined to obtain the following Euler equation, characterizing the optimal behavior of consumption

$$\frac{\dot{c}}{c} = \frac{1}{\Phi(ce^{xt})} (r - \rho) - x, \quad (7)$$

where

$$\Phi(ce^{xt}) = -\frac{u''(ce^{xt})ce^{xt}}{u'(ce^{xt})} \quad (8)$$

is the inverse of the elasticity of intertemporal substitution. To assure the existence of a steady state we make the following assumption:

Assumption 1 $\lim_{t \rightarrow \infty} \Phi(ce^{xt}) = \theta^*$, for some constant θ^* .

The two other optimality conditions from the consumer problem are the dynamic budget constraint, which we can rewrite as

$$\dot{k} = (r - n - x)k + we^{-xt} - c \quad (9)$$

and the transversality condition,

$$\lim_{t \rightarrow \infty} k(t)e^{-\int_0^t (r(s)-n-x)ds} = 0. \quad (10)$$

On the production side, profit maximization by firms yields the following first-order conditions equating factor prices to their marginal products:

$$r + \delta = Af'(k) \quad (11)$$

$$w = A[f(k) - kf'(k)]e^{xt} \quad (12)$$

Combining equations (7), (11) and (12) above, we obtain the two equations that determine the dynamics of the economy

$$\frac{\dot{c}}{c} = \frac{1}{\Phi(ce^{xt})} (Af'(k) - \delta - \rho) - x \quad (13)$$

$$\dot{k} = Af(k) - c - (n + \delta + x)k \quad (14)$$

Finally, note also that from equation (5), the dynamics of output per effective worker can be characterized by

$$\frac{\dot{y}}{y} = \frac{f'(k)k}{f(k)} \cdot \frac{\dot{k}}{k} \quad (15)$$

2.3 Steady State

Given that technological progress is assumed to be purely labor-augmenting, the steady state corresponds to a situation in which c , k , y , and r are constant while w grows at rate x . Since c and k are constant in the steady state, from equation (13) and Assumption 1, it follows that k^* is implicitly given by,⁴

$$Af'(k^*) = \delta + \rho + \theta^*x \quad (16)$$

while equation (14) implies that steady state consumption is

$$c^* = Af(k^*) - (n + \delta + x)k^* \quad (17)$$

Defining the savings rate s as $1 - c/y$, equations (5), (16) and (17) can be shown to imply:

$$s^* = \frac{(n + \delta + x)}{\delta + \rho + \theta^*x} s_K^* \quad (18)$$

where s_K^* is the steady state capital share, given by

$$s_K^* = \frac{k^* f'(k^*)}{f(k^*)} \quad (19)$$

Finally, the steady state values of y and r are defined by equations (5) and (11), evaluated at the steady state value of capital.

2.4 Savings Rate Dynamics

I will now characterize the dynamics of the savings rate in this economy with general preferences and technology. For most of the discussion below, it is useful to define the variables $z = \frac{c}{y}$ and $\omega = \frac{y}{k}$ and characterize the dynamics in the (z, ω) space. Notice that the savings rate s is equal to $1 - z$, implying $\dot{s} = -\dot{z}$. In words, knowledge of \dot{z} is sufficient to characterize \dot{s} , the dynamic behavior of the savings rate. From equations (7), (14) and (15),

$$\frac{\dot{z}}{z} = \frac{\dot{c}}{c} - \frac{\dot{y}}{y} = \frac{1}{\Phi(ce^{xt})} (Af'(k) - \delta - \rho) - x - \frac{f'(k)k}{f(k)} \left(\frac{Af(k)}{k} - \frac{c}{k} - (n + \delta + x) \right)$$

which, using (18) and the definitions of z and ω , can be rewritten as:

$$\begin{aligned} \frac{\dot{z}}{z}(t) &= s_K(t)\omega(t) \left(z(t) - \frac{\Phi(ce^{xt}) - 1}{\Phi(ce^{xt})} \right) + (\delta + \rho + \theta^*x) \left(s^* \frac{s_K(t)}{s_K^*} - \frac{1}{\Phi(ce^{xt})} \right) + \\ &+ \left(\frac{\theta^*}{\Phi(ce^{xt})} - 1 \right) x \end{aligned} \quad (20)$$

On the other hand, from equations (14) and (15),

$$\frac{\dot{\omega}}{\omega}(t) = \frac{\dot{y}}{y} - \frac{\dot{k}}{k} = (s_K(t) - 1) \left(\frac{Af(k)}{k} - \frac{c}{k} - (n + \delta + x) \right)$$

⁴Hereafter, an asterisk will denote the steady state value of a variable.

or simply

$$\frac{\dot{\omega}}{\omega}(t) = (s_K(t) - 1)(\omega(t)(1 - z(t)) - (n + \delta + x)) \quad (21)$$

Equations (20) and (21) constitute a system of two differential equations in z and ω , and are the basis of my analysis below. In particular, equation (20) states that the transitional behavior of the savings rate depends crucially on the transitional behavior of the capital share and the elasticity of intertemporal substitution. Without imposing specific functional forms on preferences and technology, the behavior of s_K and $\Phi(ce^{xt})$ remain unknown and so does the evolution of the savings rate. In a way, this is a reformulation of the “anything goes” theorem in Boldrin and Montrucchio (1986).⁵

To gain further understanding on the indeterminacy of the savings rate behavior, it is convenient to rewrite equation (20), following Cass (1965), in a slightly different way:

$$\dot{s} = \frac{1}{A [f(k)]^2} [cf'(k)k - \dot{c}f(k)] \quad (22)$$

Equation (22) captures the income and substitution effects discussed in the introduction. When $f'(k)$ is high, the substitution effect will be strong and, *ceteris paribus*, the savings rate will tend to increase. Conversely, a steeper profile of consumption (higher \dot{c}) is associated with a strong income effect, which *ceteris paribus* will make the savings rate fall along the transition. In sum, the shape of the savings rate along the transition will be determined by the relative size of these two effects, which in turn will depend on the particular functional forms imposed on preferences and technology.

In the next three sections, I solve the model for three different combinations of preferences and technology and show how the hump-shaped patterns observed in Figures 1, 2, and 3 can be reconciled with the behavior dictated by the neoclassical growth model, under very simple functional forms widely used in the literature.

3 The Model with particular functional forms

3.1 CIES Preferences and Cobb-Douglas Technology

I first start characterizing the savings rate for the most standard choice of preferences and technology. In particular, the representative consumer’s felicity function is assumed to be of the constant elasticity of substitution (CIES) form:

$$u(ce^{xt}) = \frac{(ce^{xt})^{1-\theta} - 1}{1 - \theta} \quad (23)$$

while the production function is assumed to be Cobb-Douglas:

$$f(k) = k^\alpha \quad (24)$$

⁵The theorem basically states that, under certain assumptions, by an appropriate choice of the utility function and the production function, any twice continuously differentiable function can be implemented as the optimal policy function of a neoclassical economy. Hence, a natural corollary of the theorem is that the savings rate can take any form.

It is straightforward to check that equations (23) and (24) imply:

$$\Phi(ce^{xt}) = \theta \quad \forall t \quad (25)$$

$$s_K(t) = \alpha \quad \forall t \quad (26)$$

so that equation (20) can be rewritten as:

$$\frac{\dot{z}}{z}(t) = \alpha\omega \left(z(t) - \frac{\theta - 1}{\theta} \right) + (\delta + \rho + \theta x) \left(s^* - \frac{1}{\theta} \right) \quad (27)$$

and (21) becomes:

$$\frac{\dot{\omega}}{\omega}(t) = (\alpha - 1)(\omega(1 - z) - (n + \delta + x)) \quad (28)$$

Equations (27) and (28) form a system of two differential equations in z and ω . As shown by Barro and Sala-i-Martin (1995, Chapter 2), the qualitative behavior of the savings rate can be described using analytical methods. I focus instead on a graphical representation of the dynamics, so that the results here can be compared to the ones obtained below with other functional forms.⁶

From equation (27), the $\dot{z} = 0$ locus is defined by:

$$z(\omega) = \frac{\theta - 1}{\theta} - \left(\frac{\delta + \rho + \theta x}{\alpha\omega} \right) \left(s^* - \frac{1}{\theta} \right) \quad (29)$$

and hence is upward sloping if $s^* > \frac{1}{\theta}$ and downward sloping if $s^* < \frac{1}{\theta}$. On the other hand, from equation (28), the $\dot{\omega} = 0$ locus is described by:

$$z(\omega) = 1 - \frac{(n + \delta + x)}{\omega} \quad (30)$$

and hence is always downward sloping. Note also that the slope of the $\dot{\omega} = 0$ locus is always higher than that of the $\dot{z} = 0$ locus.⁷

Figure 5 depicts the dynamics of system for the cases $s^* > \frac{1}{\theta}$ and $s^* < \frac{1}{\theta}$. The steady state corresponds to the intersection of the $\dot{z} = 0$ and $\dot{\omega} = 0$ schedules. Throughout the paper I will focus on the dynamics of an economy that is initially endowed with a low level of capital. Because ω is a decreasing function of k , the economy converges to the steady state from right to left. The direction of the arrows depicted in Figure 5 follows directly from equations (27) and (28). The system exhibits saddle-path stability. The transversality condition and the fact that equation (27) precludes discrete jumps in z , can be used to show that the optimal path of z is unique (c.f. Barro and Sala-i-Martin, 1995).

It is straightforward to see that the direction of the arrows implies that the stable arm is necessarily upward sloping when $s^* > \frac{1}{\theta}$, and necessarily downward sloping when $s^* < \frac{1}{\theta}$. Given that ω falls along the transition, we can conclude

⁶Sala-i-Martin (2000) also follows a graphical approach in studying the CIES/Cobb-Douglas case, but graphs the dynamics in the (z, k) space. My choice of the (z, ω) space stems from its convenience in studying the case with CES technology.

⁷The slope of the $\dot{\omega} = 0$ locus is $\frac{(n + \delta + x)}{\omega^2}$ while that of the $\dot{z} = 0$ locus is $\left(\frac{\delta + \rho + \theta x}{\alpha} \right) \left(s^* - \frac{1}{\theta} \right) \frac{1}{\omega^2} = [(n + \delta + x) - \frac{\delta + \rho + \theta x}{\alpha\theta}] \frac{1}{\omega^2}$ and thus is always lower.

that when $s^* > \frac{1}{\theta}$, z falls monotonically along the transition and thus s increases monotonically along the transition. Conversely, when $s^* < \frac{1}{\theta}$ the savings rate will fall monotonically along the transition. Finally, for the nongeneric case in which $s^* = \frac{1}{\theta}$, the savings rate would remain constant. We can summarize the results in this section in the following proposition:

Proposition 1 *If preferences exhibit a constant elasticity of intertemporal substitution and technology is Cobb-Douglas, then:*

- (i) *if $s^* > \frac{1}{\theta}$, the savings rate will increase monotonically along the transition;*
- (ii) *if $s^* < \frac{1}{\theta}$, the savings rate will fall monotonically along the transition;*
- (iii) *if $s^* = \frac{1}{\theta}$, the savings rate will be constant along the transition.*

Intuitively, when θ is high, the intertemporal elasticity of substitution is low and the income effect dominates throughout the transition, thus leading to an increasing savings rate. When θ is low, the substitution effect dominates and the savings rate falls along the transition. The results imply that unless θ is substantially higher than one, the model has the counterfactual implication of a falling savings rate along the transition. Hall (1988) estimated the intertemporal elasticity of substitution using macroeconomic data for the U.S. and concluded that θ was unlikely to be much below 10.

Nevertheless, the results remain unsatisfactory as they are unable to explain the hump-shaped pattern in Figures 1, 2 and 3. We next turn to a different choice of functional forms that can indeed explain the hump.

3.2 CIES Preferences and CES Technology

In this section, I will introduce only one modification to the framework in the previous section. In particular, preferences are still assumed to be of the CIES type:

$$u(ce^{xt}) = \frac{(ce^{xt})^{1-\theta} - 1}{1-\theta} \quad (31)$$

but technology is described now by the constant elasticity of substitution (CES) production function

$$f(k) = \left[\alpha k^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 0 \quad (32)$$

where σ is the elasticity substitution between capital and labor. When $\sigma = 1$, equation (32) reduces to the Cobb-Douglas case analyzed in the previous section. Here we assume instead that $\sigma < 1$, so that capital and labor are relative complements.

It is straightforward to check that (31) and (32) imply⁸

$$\Phi(ce^{xt}) = \theta \quad \forall t \quad (33)$$

$$s_K(t) = \alpha A^{\frac{\sigma-1}{\sigma}} \omega(t)^{\frac{1-\alpha}{\sigma}} \quad (34)$$

⁸To derive (34) note that $f'(k) = \alpha \left(\frac{f'(k)}{k} \right)^{\frac{1}{\sigma}} = \alpha \left(\frac{\omega}{A} \right)^{\frac{1}{\sigma}}$.

so that equation (20) can be rewritten as

$$\frac{\dot{z}}{z}(t) = \alpha A^{\frac{\sigma-1}{\sigma}} \omega^{\frac{1}{\sigma}} \left(z - \frac{\theta-1}{\theta} \right) + (n + \delta + x) \alpha A^{\frac{\sigma-1}{\sigma}} \omega^{\frac{1-\sigma}{\sigma}} - \frac{\delta + \rho + \theta x}{\theta} \quad (35)$$

and equation (21) becomes

$$\frac{\dot{\omega}}{\omega}(t) = (\alpha A^{\frac{\sigma-1}{\sigma}} \omega^{\frac{1-\sigma}{\sigma}} - 1) [\omega(1-z) - (n + \delta + x)] \quad (36)$$

We again proceed to characterize graphically the dynamics of the system defined by equations (35) and (36). From (35) the $\dot{z} = 0$ locus is now defined by the function

$$z(\omega) = \frac{\theta-1}{\theta} + \left(\frac{\delta + \rho + \theta x}{\alpha \theta A^{\frac{\sigma-1}{\sigma}}} \right) \omega^{\frac{\sigma-1}{\sigma}} - \frac{(n + \delta + x)}{\omega} \quad (37)$$

Taking the first derivative of (37) and remembering that $\sigma < 1$, it is easily verified that the $\dot{z} = 0$ schedule is downward sloping if and only if

$$\omega < \left[\frac{\delta + \rho + \theta x}{(n + \delta + x) \alpha \theta A^{\frac{\sigma-1}{\sigma}} \sigma} \right]^{\frac{\sigma}{1-\sigma}} = \tilde{\omega} \quad (38)$$

and upward sloping if $\omega > \tilde{\omega}$. On the other hand, the $\dot{\omega} = 0$ locus is defined by⁹

$$z(\omega) = 1 - \frac{(n + \delta + x)}{\omega}$$

and hence is always downward sloping. Again, it is possible to show that the slope of the $\dot{\omega} = 0$ locus is always higher than that of the $\dot{z} = 0$ locus.¹⁰

Since we are assuming that the economy converges to the steady state from a low level of capital, for the dynamics of the system it is crucial to determine whether the minimum of the $\dot{\omega} = 0$ locus, $\tilde{\omega}$, is higher or lower than the steady state value of ω , which by equation (34) is given by

$$\omega^* = \left(\frac{s_K^*}{\alpha A^{\frac{\sigma-1}{\sigma}}} \right)^{\frac{\sigma}{1-\sigma}} \quad (39)$$

Using equations (18), (38) and (39), it is straightforward to show that $\tilde{\omega} > \omega^*$ if and only if $s^* < \frac{1}{\sigma\theta}$.

The dynamics for the case $s^* > \frac{1}{\sigma\theta}$ are depicted in the first panel of Figure 6. Since in this case the $\dot{z} = 0$ locus is always increasing in the relevant range, the dynamics are very similar to the ones obtained with Cobb-Douglas technology and $s^* > \frac{1}{\theta}$. In particular, z falls monotonically along the transition and consequently s increases monotonically along the transition.

⁹Equation (36) indicates that $\dot{\omega} = 0$ is also satisfied for $\omega_{\max} = \lim_{k \rightarrow 0} \omega = A \alpha^{\frac{\sigma-1}{\sigma}}$. In words, unlike in the Cobb-Douglas case, the average return of capital is here bounded from above by ω_{\max} , so that the dynamics must be truncated for $\omega > \omega_{\max}$ (see Figure 6 below).

¹⁰The slope of the $\dot{\omega} = 0$ locus is again $\frac{(n+\delta+x)}{\omega^2}$ whereas that of the $\dot{z} = 0$ locus is $\frac{(n+\delta+x)}{\omega^2} - \frac{1}{\sigma} \left(\frac{\delta+\rho+\theta x}{\alpha \theta A^{\frac{\sigma-1}{\sigma}}} \right) \omega^{\frac{\sigma-1}{\sigma}-1}$.

Conversely, when $s^* < \frac{1}{\sigma\theta}$, the $\dot{z} = 0$ locus attains a minimum to the right of ω^* (i.e., in the relevant range), and richer dynamics can emerge.¹¹ Unfortunately, the non-monotonicity of the $\dot{z} = 0$ schedule does not ensure that the optimal path of z exhibits a U-shape (which would correspond to a hump-shaped pattern for the savings rate). For example, panel 2 of Figure 6 presents the dynamics of an economy satisfying $s^* < \frac{1}{\sigma\theta}$ that exhibits a monotonic stable arm, along which the savings rate falls as the economy approaches the steady state.

Panel 3 of Figure 6 shows the dynamics of an economy that does exhibit a hump-shaped behavior of the savings rate. The key difference between this figure and the previous one is that the $\dot{z} = 0$ is assumed to have a higher slope to the right of its minimum, so that the value of z along the $\dot{z} = 0$ schedule surpasses z^* for high enough ω . In particular, provided that k_0 is low enough (i.e., ω_0 is high enough), a sufficient condition for the optimal path of savings to be hump-shaped is that $z(\omega_{\max} = A\alpha^{\frac{\sigma}{\sigma-1}}) > z^*$ where $z(\cdot)$ refers to the $\dot{z} = 0$ locus.¹² Using equations (18), (37) and (39), it can be shown that this will be the case as long as

$$s^* > \left[\frac{1 - (s_K^*)^{\frac{1}{1-\sigma}}}{1 - (s_K^*)^{\frac{\sigma}{1-\sigma}}} \right] \frac{1}{\theta} = \psi(s_K^*) \quad (40)$$

We next summarize the results of this section in the following proposition:

Proposition 2 *If preferences display constant elasticity of intertemporal substitution and technology exhibits a constant and lower than one elasticity of substitution between capital and labor, then:*

- (i) *if $s^* > \frac{1}{\sigma\theta}$, the savings rate will increase monotonically along the transition;*
- (ii) *if $\frac{1}{\sigma\theta} > s^* > \psi(s_K^*)$, for a sufficiently small k_0 , the savings rate will exhibit a hump-shape along the transition;*
- (iii) *if $\frac{1}{\sigma\theta} > s^*$ and $s^* < \psi(s_K^*)$, the savings rate may exhibit a hump-shape along the transition or it may decrease monotonically along the transition.*

The intuition for the results in Proposition 2 is best understood by appealing again to the income and substitution effects discussed in the introduction. By assuming a higher complementarity between capital and labor than the one implied by a Cobb-Douglas specification, in the early stage of the transition the income effect is strengthened and the substitution effect is weakened.¹³ Hence, the CES specification makes the income effect more likely to dominate at low levels of capital. If

¹¹Notice that when $s^* < \frac{1}{\sigma\theta}$, the slope of the $\dot{\omega} = 0$ locus is necessarily negative in the neighborhood of the steady state. Given the dynamics of the system, this ensures that z increases in the last part of the transition. In other words, the savings rate necessarily falls late in the transition.

¹²Given the dynamics of the system, if $z(\omega_{\max} = A\alpha^{\frac{\sigma}{\sigma-1}}) > z^*$, then for ω close enough to ω_{\max} , it must be the case that $\dot{z} < 0$ initially along the optimal path. Hence, the savings rate must initially increase along the transition. Since $s^* < \frac{1}{\sigma\theta}$ implies that the savings rate necessarily falls later in the transition, the dynamics of s are necessarily hump-shaped.

¹³When the elasticity of substitution is lower than one, the marginal product of capital is bounded from above as $k \rightarrow 0$. This weakens the substitution effect when k is low. On the other hand, as Ventura (1997) has shown, the time-series profile of the growth rate of capital is steeper the lower σ (provided that $\sigma < 1$). This reinforces the income effect at low levels of capital.

the parameter values are such that the substitution effect dominates in the neighborhood of the steady state ($s^* < \frac{1}{\sigma\theta}$), then a hump-shape pattern in the savings rate can arise naturally.

Proposition 2 states that the neoclassical growth model can potentially explain the actual behavior of the *average* savings rate for the 24 OECD countries in the period 1950-1990. It remains to be shown, however, that for plausible parameter values, the simulated dynamics of the system resemble those of the actual savings rate. For that purpose we now move to a simple calibration of the model. Table 1 presents the parameter values chose for the two simulations depicted in Figure 7.

Table 1. Parameter Values in the Simulations

	α	δ	ρ	x	n	θ	σ	A	s^*	$s^* - 1/\sigma\theta$
Simulation 1	0.35	0.1	0.02	0.02	0.01	5	0.66	0.5	0.22	-0.08
Simulation 2	0.35	0.1	0.02	0.02	0.01	6	0.7	0.15	0.30	0.06

The parameter α would correspond to the capital share in the case $\sigma = 1$ (i.e., Cobb-Douglas technology). For both simulations we assign a value of 0.35 to α , which is consistent with most studies that interpret k as the stock of physical capital (excluding human capital). The parameter δ is set to 0.1, implying a 10% annual depreciation rate, a figure consistent with NIPA estimates.¹⁴ The discount rate (ρ) and the rate of labor-augmenting technological change (x) are both set at 0.02, which again are values widely used in the literature. The rate of population growth (n) is assumed to be 1% per year, which is very close to the mean population growth rate in Figure 4 (the mean is actually 0.99 %). Following the estimates in Hall (1988), the parameter θ is set at high values: 5 in the first simulation and 6 in the second. The multiplicative efficiency parameter A is a free parameter than can be used to adjust the simulated savings rate to the actual one.

The choice of the elasticity of substitution between capital and labor is worth commenting on. I assume that σ is equal to 2/3 in the first simulation and to 0.7 in the second. These figures are consistent with the time-series estimates obtained by Lucas (1969) and Eisner and Nadiri (1968), which indicated that the elasticity of substitution might be well below one. Cross-sectional estimates of the elasticity have tended to be insignificantly different from one (e.g., Dhrymes and Zarembka, 1970), but Lucas (1969) pointed at several biases inherent in the use of cross-sectional data in the estimation of the elasticity. In a widely cited contribution, Berndt (1976) claimed that the use of higher quality data helped reconcile cross sectional and time series estimates. In particular, a careful construction of the rental rate of capital led him to time series estimates insignificantly different from one. Unfortunately, by not controlling for technological change, Berndt necessarily biased his estimates towards one.¹⁵ In fact, using data similar to the one employed Berndt (1976) but controlling for technological change, Kalt (1978) obtained estimates significantly lower than one (his preferred point estimate is 0.76).

¹⁴Fraumeni (1997) presents estimates of the depreciation rate for different categories of capital goods. Taking a weighted-average of her estimates yields an estimated depreciation rate of 3.1% for structures and 13.3% for equipment. Since the ratio of capital equipment to capital structures seems to exhibit a positive trend (c.f. Greenwood, Hercowitz and Krusell, 1997) an overall estimate of 10% seems warranted.

¹⁵For a discussion on the source of the bias see Antras (2001).

Figure 7 plots simulations 1 and 2 together with the actual behavior of the weighted-average savings rate in the OECD. As shown in Table 1, for the first simulation the savings rate satisfies $s^* < \frac{1}{\sigma\theta}$. Since it is also the case that $s^* > \psi(s_K^*)$, by Proposition 2 we know that for low enough k_0 the dynamics will be hump-shaped. Figure 7 shows that the hump can be quite significant, although it seems unable to explain the entire hump in the actual series. On the other hand, the parameter values for the second simulation satisfy $s^* > \frac{1}{\sigma\theta}$ and thus, by Proposition 2, the simulated savings rate is monotonically increasing.

In conclusion, by assuming a higher complementarity between capital and labor than the one implied by Cobb-Douglas technology, the neoclassical model can generate dynamics for the savings rate that resemble those of the actual OECD economies. The hump in the actual data seems to be more pronounced than the one obtained in the simulations and also seems to occur later in the transition. This suggests that in order to explain the entire hump, one needs to consider other factors ignored in the neoclassical growth model. In particular, maybe part of the hump is explained by the life-cycle pattern in individual consumption, or perhaps it might be the consequence of a failure of full Ricardian equivalence.

This section has shown, however, that a general equilibrium version of the permanent income hypothesis theory is not necessarily at odds with the behavior of the savings rate in the OECD countries over the period 1950-1990.

3.3 Stone-Geary Preferences and Cobb-Douglas Technology

This section presents a second combination of functional forms for preferences and technology that delivers a hump-shaped optimal savings path. Contrary to the previous section, the results of this section are not new in the literature. The contribution of this section is to fully characterize the transitional dynamics of the savings rate for this case, emphasizing that it is just another particular case of the dynamics described in section 2. Previous studies have tended to simply simulate the model without analytically characterizing the transition.

Preferences are now assumed of the Stone-Geary type

$$u(c) = \frac{(c - \bar{c})^{1-\theta} - 1}{1 - \theta} \quad \text{with } \bar{c} > 0 \quad (41)$$

where \bar{c} is interpreted as the minimum subsistence level of per capita consumption. To simplify the algebra it is convenient to assume that there is no technological progress ($x = 0$). To isolate the effect of this particular choice of preferences, I assume, as in section 3.1., that technology is Cobb-Douglas:

$$f(k) = k^\alpha \quad (42)$$

It is easy to show that that equations (41) and (42) imply:

$$\Phi(c) = \theta \cdot \left(\frac{c}{c - \bar{c}} \right) \quad (43)$$

$$s_K(t) = \alpha \quad \forall t \quad (44)$$

Note that, for $c > \bar{c}$, $\Phi(c)$ is a twice differentiable, continuous, monotonically decreasing function of c , satisfying, $\lim_{c \rightarrow \bar{c}} \Phi(c) = \infty$ and $\lim_{c \rightarrow \infty} \Phi(c) = \theta$. In

words, the intertemporal elasticity of substitution is now not constant, but increases with c . Plugging equations (43) and (44) in the general dynamics in equations (20) and (21) yields:

$$\frac{\dot{z}}{z}(t) = \alpha\omega \left(z(t) - \frac{\Phi(c) - 1}{\Phi(c)} \right) + (\delta + \rho) \left(s^* - \frac{1}{\Phi(c)} \right) \quad (45)$$

and

$$\frac{\dot{\omega}}{\omega}(t) = (\alpha - 1)(\omega(1 - z) - (n + \delta)) \quad (46)$$

From equation (45), the $\dot{z} = 0$ locus will now be defined by:

$$z(\omega) = \frac{\Phi(c(\omega)) - 1}{\Phi(c(\omega))} - \left(\frac{\delta + \rho}{\alpha\omega} \right) \left(s^* - \frac{1}{\Phi(c(\omega))} \right) \quad (47)$$

where it is emphasized that, in equilibrium, per capita consumption will be a function of ω .¹⁶ Furthermore, under our assumptions, the policy function $c(\omega)$ is twice continuously differentiable and decreasing in ω , that is, increasing in k .¹⁷ The following lemma characterizes the $\dot{z} = 0$ locus for $\omega \geq \omega^*$ (again I focus on a transition from a low level of capital):

Lemma 1 If $s^* > \frac{1}{\Phi(c^*)}$, the $\dot{z} = 0$ locus is upward sloping for $\omega \geq \omega^*$. If $s^* < \frac{1}{\Phi(c^*)}$ the $\dot{z} = 0$ locus is non-monotonic, attaining a minimum at some $\tilde{\omega} > \omega^*$.

Proof. Differentiation of (47) with respect to ω yields:

$$z'(\omega) = \left[\frac{1}{\Phi(c(\omega))^2} \right] \underbrace{\Phi'(c(\omega))}_{<0} \underbrace{c'(\omega)}_{<0} \underbrace{\left(1 - \frac{\omega^*}{\omega} \right)}_{>0} + \underbrace{\left(\frac{\omega^*}{\omega^2} \right)}_{>0} \underbrace{\left(s^* - \frac{1}{\Phi(c(\omega))} \right)}_{>0 \Leftrightarrow s^* > \frac{1}{\Phi(c(\omega))}} \quad (48)$$

where $\omega^* = \frac{\delta + \rho}{\alpha}$ is the steady state value of ω .

The first term is unambiguously nonnegative for $\omega \geq \omega^*$, whereas the sign of the second term depends on whether the steady state savings rate is greater or smaller than the intertemporal elasticity of substitution.

Assume first that $s^* > \frac{1}{\Phi(c^*)}$ so that $z'(\omega^*) > 0$. Since $\Phi(c(\omega))$ is increasing in ω , it follows that $s^* > \frac{1}{\Phi(c(\omega))}$ for all $\omega > \omega^*$. But this implies $z'(\omega) > 0$ for all $\omega > \omega^*$, and thus the $\dot{z} = 0$ locus is upward sloping for $\omega \geq \omega^*$.

Now let $s^* < \frac{1}{\Phi(c^*)}$. Noticing that the first term in (48) evaluated at the steady state is zero, we obtain $z'(\omega^*) < 0$. On the other hand, note that as $\omega \rightarrow \infty$, $\Phi(c(\omega)) \rightarrow \infty$, and thus for sufficiently high ω , $z'(\omega) > 0$. Let $\bar{\omega}$ be such that for $\omega > \bar{\omega}$, $z'(\omega) > 0$. Since $\Phi(\cdot)$ and $c(\cdot)$ are both twice continuously differentiable, it follows that $z(\omega)$ is continuous and must attain a minimum $\tilde{\omega}$ in the compact set

¹⁶There exists another equilibrium with a level of consumption equal to \bar{c} and no capital. The equilibrium is however unstable and we ignore in the discussion below.

¹⁷Using a dynamic programming approach, the value function of the program could be shown to be twice continuously differentiable and concave in k (c.f. Benveniste and Scheinkman, 1979). The envelope condition, together with the concavity of the utility function would then imply that the policy function $c(k)$ is continuously differentiable and increasing.

$[\omega^*, \bar{\omega}]$. Suppose $\tilde{\omega} = \omega^*$: since $z'(\omega^*) < 0$, there exists a small enough $\varepsilon > 0$ such that $z(\omega^* + \varepsilon) < z(\omega^*)$, contradicting ω^* being a minimum. Hence $\tilde{\omega} > \omega^*$. This completes the proof. ■

Without an explicit knowledge of the functional form of $c(\omega)$ the exact shape of the $\dot{z} = 0$ locus remains unknown. Nevertheless, the assumption below gives a sufficient condition for the $\dot{z} = 0$ locus to be U-shaped.

Assumption 2 There exists at most one $\tilde{\omega}$ such that:¹⁸

$$\frac{\Phi'(c(\tilde{\omega}))c'(\tilde{\omega})}{(\Phi(c(\tilde{\omega})))^2} \left(1 - \frac{\omega^*}{\tilde{\omega}}\right) + \left(\frac{\omega^*}{\tilde{\omega}^2}\right) \left(s^* - \frac{1}{\Phi(\tilde{\omega})}\right) = 0 \quad (49)$$

With this assumption, the $\dot{z} = 0$ attains at most one minimum in $[\omega^*, \infty)$. Numerical simulations indicate that this assumption is very weak. In fact, it could well be that the assumption is not needed, but the fact that no explicit solution exists for $c(\omega)$ makes the proof of this conjecture impossible.

The $\dot{\omega} = 0$ is much simpler to characterize. From equation (46), the $\dot{\omega} = 0$ schedule is such that:

$$z(\omega) = 1 - \frac{(n + \delta)}{\omega} \quad (50)$$

and thus is always downward sloping. It can be shown too, that at the steady state, the slope of the $\dot{\omega} = 0$ locus is higher than that of the $\dot{z} = 0$ locus.¹⁹

The first panel of Figure 8 depicts the dynamics of the system for the case $s^* > \frac{1}{\Phi(c^*)}$. Since the $\dot{z} = 0$ schedule is upwards sloping, the stable arm is necessarily upward sloping. This implies that z must fall in the transition, and that the savings rate must increase monotonically along the transition.

The second panel of Figure 8 presents the dynamics when $s^* < \frac{1}{\Phi(c^*)}$ assuming a low enough k_0 (high enough ω_0). The dynamics are almost identical to the ones in the third panel of Figure 6. If $z(\omega_0) > z^*$, where $z(\cdot)$ refers to the $\dot{z} = 0$ schedule, then the savings rate must necessarily increase at the beginning of the transition. But since $s^* < \frac{1}{\Phi(c^*)}$, the savings will necessarily fall at the end of the transition and thus, under Assumption 2, a hump-shape emerges. Note that in comparison with

¹⁸Noticing that $\frac{1}{\Phi(c)} = \frac{1}{\theta} \cdot \left(\frac{c-\bar{c}}{c}\right) = \frac{1}{\theta} \cdot \left(\frac{c/y - \bar{c}/y}{c/y}\right) = \frac{1}{\theta} \left(1 - \frac{\bar{c}\omega^{1-\alpha}}{z}\right)$, it is possible to restate Assumption 2 in the following terms.

Assumption 2' The system

$$\begin{aligned} \left(s^* - \frac{1}{\theta}\right) \frac{z\omega^*}{\omega} + \frac{\bar{c}}{\theta} \left(\frac{\alpha}{1-\alpha} \omega^{\frac{\alpha}{1-\alpha}-1} + \frac{1-2\alpha}{1-\alpha} \omega^* \omega^{\frac{\alpha}{1-\alpha}-2}\right) &= 0 \\ z = 1 - \frac{1}{\theta} + \frac{\bar{c}\omega^{1-\alpha}}{\theta z} \left(1 - \frac{\omega^*}{\omega}\right) - \frac{\omega^*}{\omega} \left(s^* - \frac{1}{\theta}\right) \end{aligned}$$

has at most one solution in the (z, ω) space.

The system is highly non-linear and it becomes impossible to verify whether assumption 2' is actually needed or not.

¹⁹The slope of the $\dot{\omega} = 0$ locus at the steady state is $\frac{(n+\delta+x)}{\omega^*} = \frac{s^*}{\omega^*}$ whereas that of the $\dot{z} = 0$ schedule is $\frac{s^*}{\omega^*} - \frac{1}{\omega^* \Phi(c(\omega))}$ and thus is smaller.

the CES technology case, now $\omega_{Max} = \infty$. Since the $\dot{z} = 0$ locus is increasing for high enough ω , ω_0 can always be chosen high enough so that $z(\omega_0) > z^*$. This leads us to the following proposition:

Proposition 3 *If preferences are of the Stone-Geary type with a positive subsistence level of consumption and technology is Cobb-Douglas, then:*

- (i) *if $s^* > \frac{1}{\Phi(c^*)}$, the savings rate will increase monotonically along the transition;*
- (ii) *if $s^* < \frac{1}{\Phi(c^*)}$, for a sufficiently small k_0 , the savings rate will exhibit a hump-shape along the transition;*
- (iii) *if $s^* < \frac{1}{\Phi(c^*)}$, but k_0 is not low enough, the savings rate may decrease monotonically along the transition.*

To understand the results in Proposition 3 it is useful to go back to the income and substitution effects discussed above. Introducing a positive subsistence level of consumption (\bar{c}) in the utility function effectively lowers the value of the intertemporal elasticity of substitution at low levels of consumption. In fact, in the neighborhood of \bar{c} the elasticity is very close to 0. Clearly, this weakens the substitution effect in the early stages of the transition, leading to an increasing savings rate. If the parameter values are such that around the steady state the substitution effect dominates (i.e., $s^* < \frac{1}{\Phi(c^*)}$) then the savings rate will start to fall as the economy gets close enough to the steady state. Consequently, the savings rate will first rise and then fall, exhibiting a hump-shape. On the other hand, if the income effect also dominates in the neighborhood of the steady state (i.e. $s^* > \frac{1}{\Phi(c^*)}$), the savings rate will increase monotonically along the transition.

As I pointed out in the introduction, the results in Proposition 3 are not new in the literature. There are several papers that have emphasized that introducing Stone-Geary preferences into the neoclassical growth model can lead to interesting dynamics of the savings rate. In particular, Christiano (1989) used the model to explain the behavior of some macroeconomic variables (including the savings rate) in Japan from the mid 1950s to the late 1980's. The simulations in the paper show that with Stone-Geary preferences and Cobb-Douglas technology, the optimal path of the savings rate can exhibit a hump-shape, but no attempt is made to model the transitional dynamics. Similarly, King and Rebelo (1993) present simulations in which the savings rate displays a hump when preferences are Stone-Geary and discuss the results in terms of the income and substitution effects, but again the dynamics are not fully characterized.

In a related paper, Rebelo (1992) introduced Stone-Geary preferences in the neoclassical model to explain that even with perfect capital mobility, the model does not necessarily imply that the growth rate of consumption and output is immediately equalized across countries. The main difference between Rebelo's (1992) study and the approach in this section is that instead of assuming Cobb-Douglas technology, he assumed an AK technology. Rebelo found that the model delivered a monotonically increasing savings rate. Using the apparatus developed in this paper, it is straightforward to show why in Rebelo's (1992) model the savings rate cannot be hump-shaped. In particular, assume that technology is given by $f(k) = k$ and,

following Rebelo, assume $A > \delta - \rho$.²⁰ Equation (20) now takes the simple form

$$\frac{\dot{z}}{z}(t) = A \left(z(t) - \frac{\Phi(c) - 1}{\Phi(c)} \right) + (\delta + \rho) \left(s^* - \frac{1}{\Phi(c)} \right) \quad (51)$$

Time differentiating (51) we obtain

$$\frac{d \left(\frac{\dot{z}}{z}(t) \right)}{dt} = A \dot{z}(t) + \underbrace{\frac{(A - \delta - \rho)}{[\Phi(c)]^2}}_{>0} \underbrace{\left[-\dot{\Phi}(c) \right]}_{>0} \quad (52)$$

Now assume that $\dot{z}(t) > 0$ for some t . Then equation (52) would imply $\dot{z}(t') > 0$ for all $t' > t$, which is inconsistent with the economy reaching a steady state. Hence it must be the case that $\dot{z}(t) < 0 \forall t$, implying $\dot{s}(t) > 0 \forall t$. Consequently, the savings rate increases monotonically along the transition and cannot exhibit a hump. Intuitively, by assuming a constant marginal product of capital, Rebelo eliminated the substitution effect. With only the income effect remaining, the savings rate necessarily increases along the transition.

Recently, Steger (2000) has studied an extension of Rebelo's (1992) model. In particular, he has simulated the model assuming a technology featuring decreasing but asymptotically constant returns to scale. Contrary to Rebelo's (1992) results, Steger's simulations of the savings rate do exhibit a hump. To see why, assume, as in Steger (2000), that the technology is given by $f(k) = k + k^\alpha$.²¹ Equation (20) becomes now:

$$\frac{\dot{z}}{z}(t) = ((1 - \alpha)A + \alpha\omega) \left(z(t) - \frac{\Phi(c) - 1}{\Phi(c)} \right) + (\delta + \rho) \left(s^* - \frac{1}{\Phi(c)} \right) \quad (53)$$

which is similar to equation (45). It should therefore not be surprising that Steger's simulation displays a hump-shaped savings rate. Intuitively, the crux is that Steger's choice of technology re-introduces the substitution effect into the model.

To close this section I briefly discuss the relevance of this second "theory" of hump-shaped savings in explaining the pattern in Figures 1, 2, and 3. Stone-Geary preferences have been generally introduced in the neoclassical growth model to explain the behavior of some macroeconomic variable in underdeveloped economies. The relevance of the subsistence level of consumption in explaining the behavior of the savings rate in OECD countries seems much more questionable.

It is important to emphasize, however, that the key feature of the Stone-Geary preferences is that they imply an increasing intertemporal elasticity of substitution along the transition. Recently, Carroll, Overland and Weil (2000) have introduced preferences featuring habit formation in the neoclassical growth model. Their felicity function depends on how consumption compares to a habit stock determined by past consumption. They also show that this specification leads to an increasing intertemporal elasticity of substitution along the transition. Their choice of an

²⁰This assumption is necessary to avoid a counterfactual steady state with consumption falling at a constant rate.

²¹Steger's (2000) production function is actually $y = (1 - \tau)[Ak + Bk^\alpha]$. By assuming $f(k) = k + k^\alpha$, we are implicitly imposing $B = A$, but this is clearly not crucial for the discussion below.

AK technology, makes the dynamics of their model resemble substantially those in Rebelo (1992). The results above suggest that if Carroll et al. had assumed a technology exhibiting decreasing returns to capital, the optimal path of the savings rate in their model might well have been hump-shaped (instead of monotonically increasing).

Consequently, it might well be the case that the hump-shape in the savings rate observed in the OECD countries is partly explained by habit formation. This explanation could naturally supplement the one based on the high complementarity between capital and labor, exposed in the previous section. A problem would arise, however, from the fact that as Figure 9 shows, the hump-shape in the savings rate obtained with Stone-Geary preferences seems to occur too early in the transition.²² It remains to be checked whether the model with habit formation would help resolving this counterfactual fact.

4 Empirical Evidence

The purpose of this section is twofold. On the one hand, I try to provide evidence that supports the existence of a hump-shaped pattern in the savings rate of economies experiencing a neoclassical transition. First, I undertake a proper test of the hump-shape hypothesis for a panel containing only the 24 OECD countries. Later, I expand my sample by including a much larger cross-section of countries. The test will be modified to take into account the fundamental differences between the countries in the sample.

The second purpose of this section is to present some evidence supporting the empirical relevance of the theory exposed above. In particular, I investigate the extent to which the variation in the savings rate can be explained by the model, once life-cycle factors are brought into the picture.

Overall, the results in this section should be taken as preliminary.

4.1 Evidence from OECD countries

Figure 1, 2, and 3 presented visual evidence that the savings rate in most OECD countries in the period 1950-1990 featured a hump-shape. In order to map these figures into the model above, we need to make two assumptions: (i) the 24 OECD economies undertook a neoclassical transition starting around 1950 and finishing around 1990; and (ii) the 24 OECD were converging to a common steady state.

The crucial assumption is the first one. It also appears as a reasonable one. A substantial part of the capital stock in some of the countries in the sample was

²²The parameter values in simulations 1 and 2 in Figure 9 are the following:

	α	δ	ρ	x	n	θ	A	s^*	$s^* - 1/\sigma\theta$
Simulation 1	0.3	0.1	0.02	0.02	0.01	2	5	0.24	-0.100
Simulation 2	0.35	0.1	0.02	0.02	0.01	3	5	0.25	0.002

with \bar{c} set equal to half of the output in period 0.

surely destroyed during WWII.²³ In other countries, the war might have had indirect effects through a shift to production of war-related goods.

The second assumption could be dispensed with by increasing the number of covariates in the regressions below to control for the differences in the steady states. I choose not to do so for two reasons. On the one hand, this type of conditional regressions will be discussed in the next section, when the sample is considerably enlarged. More substantively, the assumption of a common steady state does not seem a farfetched one. The convergence literature has emphasized that OECD countries exhibit absolute and not just conditional convergence. In fact, in order to become a member of the OECD, a country has to meet certain institutional and economic criteria. It could even be argued that membership of the OECD is only granted to countries that are expected to converge to the same steady state as the incumbent members.

Under these two assumptions, the model presented in sections 2 and 3 can be taken to data in a straightforward manner. Suppose that the parameter values are such as to expect the savings rate to first increase and then decrease along the transition. Since all countries are assumed to converge to the same steady state, the contemporaneous level of GDP can be used to measure the distance of the economy from its steady state. The model thus predicts that the correlation between the savings rate (and hence the investment rate) and contemporaneous GDP should be positive at low levels of income, and negative at high levels of income.²⁴ A natural econometric specification to test this hypothesis is then

$$s_{i,t} = \alpha + \beta GDP_{i,t} + \gamma GDP_{i,t}^2 + \varepsilon_{i,t} \quad (54)$$

where $s_{i,t}$ and $GDP_{i,t}$ are respectively the investment rate and real GDP of country i at time t . If the theory is valid, one expects to find $\beta > 0$ and $\gamma < 0$. Equation (54) could in principle be run using the annual data from the 24 OECD countries. By doing so, however, the regressions would be capturing business-cycle correlations that are unrelated to the theory developed above. It is therefore convenient to transform the variables into decade averages. We are thus left with a panel of 24 countries and four decades.

Column I of Table 2 presents estimates of equation (54) obtained by the seemingly unrelated regression technique (SUR), which allows for country random effects that are correlated over time. The estimate of β is positive and significant at the 1% level, while that of γ is negative and also significant at the 1% level. Figure 10 presents a graphical representation of the regression. It is evident from the figure that the hump is significant and that it also seems to occur late in the transition (for the fitted values, the maximum savings rate occurs at a real per capita GDP of \$7,411).

²³This was definitely the case of Germany and Japan. To a lesser extent, the war might have deployed part of the capital stock in Italy, France, Belgium, the Netherlands, Luxembourg, Austria and the UK. On the other hand, Spain was dramatically hit by a Civil War, which ended in 1939, leading to an autarkic and oppressive dictatorship.

²⁴Of course this whole exercise is subject to the criticisms raised in Ventura (1997) and Caselli and Ventura (2000). In the previous sections, I developed a theory to explain the time-series pattern of the average OECD savings rate, that I will now test using also cross-sectional evidence. The fact that the savings rate seems to behave very similarly across OECD countries indicates however that cross-sectional comparisons are not misleading here.

There are at least two sources of bias in the coefficients in column I. On the one hand, GDP is clearly not an exogenous variable and is in fact partly determined by investment itself. This calls for the use of instrumental variable techniques. On the other hand, the specification in (54) is very parsimonious and is likely to be subject to an omitted variable bias. This demands the inclusion of more right-hand side variables in the regression. We deal with each of these two problems in turn.

Column II of Table 2 presents the results of running equation (54) using three-stage least-squares (3SLS). GDP and GDP squared average levels for the 1960s, 1970's and 1980's are instrumented with the GDP and GDP squared levels in 1955, 1965 and 1975 respectively. Average GDP and GDP squared in the 1950's is taken as exogenous. Instrumenting does not have a large effect on the coefficients. The estimate of β slightly increases and is still significant at the 1% level, while that of γ is still negative and also significant at the 1% level. There are two ways to read the results: either the choice of instruments is not an adequate one, or maybe the endogeneity problem is simply not sizable. Schmidt-Hebbel and Serven (1999b) ran similar regressions and concluded that the endogeneity problem was not likely to be important.

As mentioned in the introduction, Modigliani has emphasized repeatedly that the single most important implication of his life-cycle model is a positive link running from growth to savings. Furthermore, in the stripped-down version of his model, population growth and productivity growth have identical effects on the savings rate. Finally, one of the propositions in Modigliani (1986) states that the savings rate of a country is entirely independent of its per capita income.

Columns III, IV and V of Table 2 present regressions incorporating these crucial variables in the life-cycle model. Column III confirms the prediction that higher GDP growth and population growth lead to a higher aggregate investment rate. Column IV shows, however, that contrary to Modigliani's prediction, the level of per-capita GDP enters significantly in the regression. More importantly, Column V shows that when both log GDP and log GDP squared are included in the regression, the former enters with a positive and significant coefficient, while the squared term enters negatively and also significantly at the 1% level. In particular, column V partly resolves the second econometric problem mentioned above, namely, the omitted variable bias. The coefficients on β and γ are somewhat smaller, but still highly significant.²⁵

Overall, Table 2 presents evidence that supports the empirical relevance of the theory developed in the previous sections. Correcting for the endogeneity of the regressors and controlling for variables that affect the savings rate under alternative theories does not alter the main conclusions derived from column I. The hump in the savings rate is present in the data and it is potentially explained by a neoclassical transition. The results support the life-cycle model in the sense that higher GDP growth and population growth lead to higher investment rates, but Modigliani's prediction of no effect of the level of GDP is clearly rejected.

To close this section it is worth commenting further on the relation between savings and growth. Carroll and Weil (1994) presented Granger causality tests

²⁵The very low R^2 in all the regressions indicate, however, that the results should be interpreted with caution.

showing that GDP growth rates seem to Granger cause the savings rate, while savings rates do not Granger cause GDP growth rates. They interpreted these results as evidence supporting the life-cycle theory vis a vis the neoclassical growth model. Figure 11 plots together the simulated paths of the savings rate and the growth rate implied by the model in section 3.2., with CIES preferences and CES technology. As is clear from the figure, one must be careful in drawing conclusions from simple Granger causality tests. Although one of the mechanisms implicit in the neoclassical model predicts a causal link between higher savings today and higher growth tomorrow (i.e. the savings rate Granger causes GDP growth), the income and substitution effects explained above may well reverse this “mechanical effect”, to use Carroll and Weil’s (1994) words. If anything, Figure 11 seems to point out at a positive link between higher growth today and higher savings tomorrow (GDP growth Granger causes the savings rate). The Granger causality tests in Carroll and Weil (1994) would not hence be at odds with the theoretical results of this paper.

4.2 Evidence from a larger cross-section

This paper has so far focused on the evolution of the savings rate in 24 OECD countries over the period 1950-1990. In this section, we make a preliminary attempt to assess the empirical validity of the neoclassical model in explaining transitional savings rates across a larger cross-section of countries.²⁶ The empirical strategy is essentially the same as in the previous section. The enormous multidimensional disparities across the countries in the sample makes the assumption of a common steady state much more doubtful, and it does become necessary to include a set of controls in the regressions that follow. The specification will thus now be of the form:

$$s_{i,t} = \alpha + \beta GDP_{i,t} + \gamma GDP_{i,t}^2 + \eta X_{i,t} + \varepsilon_{i,t} \quad (55)$$

where $X_{i,t}$ is a vector of controls. If the variables in X correctly partial out the steady state differences across countries, the model presented above would predict again the estimate of β to be positive and that of γ to be negative.

The choice of the variables to include in X follows very closely the approach in Barro (2000). In particular, in Table 3, I include seven steady-state determinants: (1) the average years of secondary and higher schooling, (2) government consumption over GDP, (3) the growth rate of terms of trade, (4) a rule of law index, (5) the logarithm of the fertility rate, (6) a democracy index, and (7) the inflation rate.²⁷

Columns I and II present estimates of (54) using the SUR and 3SLS techniques respectively. In the IV regression, I also follow Barro (2000) in the choice of instruments.²⁸ The results with this larger cross-section of countries support again the existence of hump-shaped savings along the transition. In particular, the estimate of β is positive and significant at the 1% level, while that of γ is negative and also

²⁶I use a recent update of the Barro-Lee dataset. I thank Robert Barro for kindly providing me with the data.

²⁷The results are robust to including the square of the democracy index, as in Barro (2000).

²⁸The schooling variables and the growth rate of the terms of trade are taken as exogenous. The other variables are instrumented with lags, with the exception of the inflation rate, which again following Barro (2000), I instrument using dummy variables for prior colonial status.

significant at the 1% level. The remaining coefficients are consistent with the results reported in Barro (2000).

In columns III through VI, I enlarge the set of covariates by including variables closely related to a life-cycle view of aggregate savings. This constitutes a preliminary and rough attempt to assess the empirical validity of the model presented in this paper vis a vis the life-cycle model. For this purpose, I sequentially include in the regression the growth rate of GDP, population growth, the percentage of population above age 65 (old-age dependency ratio), the percentage of population below age 15 (young-dependency ratio) and the Gini coefficient.²⁹ As is clear from Table 3, none of these variables enters significantly in the regression, while the coefficients for log GDP and log GDP squared remain highly significant.

The results seem to suggest that once transitional dynamics are taken into account, very little of the variation in savings rate is explained by the life-cycle theory. This contrasts with a vast literature that has found a clear positive link between GDP growth and savings and population growth and savings.³⁰ Column VII of Table 3 sheds some light in the issue. By dropping all the variables in X as well as log GDP and log GDP squared, most life-cycle variables enter the regression significantly. In particular, higher GDP growth and higher population growth are significantly associated with higher savings. The positive and significant sign on the old-age dependency variable is however puzzling and difficult to reconcile with the life-cycle model, in which the larger the proportion of old people, the lower aggregate savings are expected to be.

Overall, the results in this section provide some evidence of the empirical relevance of the general equilibrium model of consumption and savings presented above. Furthermore, it has been shown that “life-cycle” determinants of the savings rate lose most of their explanatory power when “neoclassical” determinants of the savings rate are included in the regression.

5 Conclusion

After a steady increase in the 1950’s, 60’s and early 70’s, the average savings rate in OECD countries experienced a significant decline in the late 1970’s and 1980’s. The recent fall in the savings rate has been associated with the productivity slowdown recorded in most industrialized countries during the same period. In particular, a direct implication of Modigliani’s life-cycle model is a positive correlation between the aggregate savings rate and productivity growth. Conversely, a simplified version of Friedman’s permanent income hypothesis theory would have predicted an increase in the savings rate in reaction to the productivity slowdown.

This paper has demonstrated that the neoclassical growth model, a general equilibrium model in which agents behave essentially as permanent income consumers, can provide an alternative explanation for the observed hump-shaped pattern in the savings rate. In the model, agents’ income is endogenous and is determined by factor prices, which in general equilibrium depend on the extent of capital accumulation.

²⁹In the 3SLS estimation, demographic variables and the Gini coefficient are taken as exogenous, while the growth rate of GDP is instrumented with lags.

³⁰For instance, Modigliani (1986, 1993) and Carroll and Weil (1994).

Inherent in the model is the assumption that following WWII, OECD countries undertook a neoclassical transition featuring steady increases in the capital-labor ratio. Misled by the use of too simple functional forms, namely CES preferences and Cobb-Douglas technology, previous studies have generally regarded the transitional behavior of the savings rate resulting from the neoclassical growth model as largely counterfactual. One of the contributions of this paper has been to show that introducing higher complementarity between capital and labor, an assumption well grounded on empirical evidence, produces dynamics for the savings rate that notably resemble those of the actual economies.

The empirical tests presented in the paper have provided some evidence that the general equilibrium effect identified above might be relevant. In particular, the empirical results seem to (weakly) favor the theory presented in this paper relative to the life-cycle model.

It would be too ambitious, however, to claim that a neoclassical transition explains all of the long-run dynamics of the savings rate in OECD countries. A more thorough study of the observed hump-shape in the savings rate in OECD countries should try to bring other potential explanations into the picture, and try to assess the relative merits of each of them. In particular, it would be interesting to develop a framework that combined some life-cycle features with the general equilibrium effect outlined in this paper. In addition, it should prove fruitful to introduce fiscal policy into the model to allow for part of the hump to be explained by a partial failure of Ricardian Equivalence. This is the next step on my research agenda.

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FIGURE 1. Weighted-Average Investment Rate for 24 OECD countries (1950-1990)

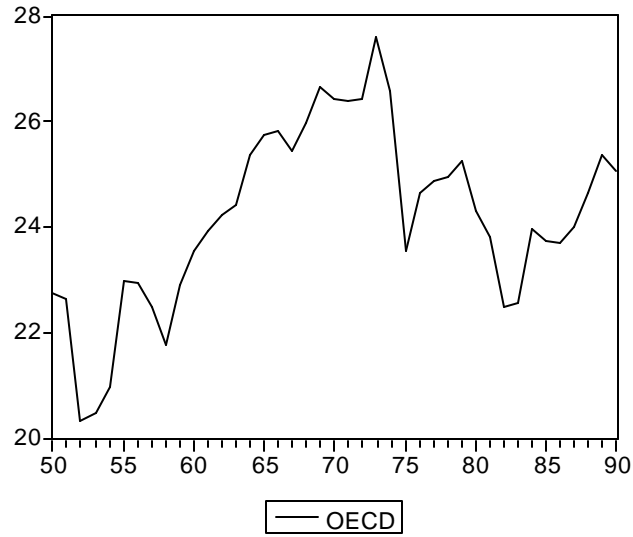


FIGURE 2. Detrended Weighted-Average Investment Rate for 24 OECD countries (1950-1990)

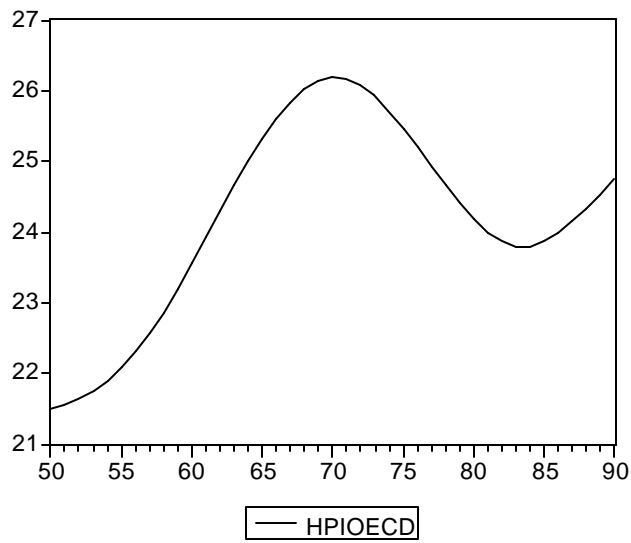


FIGURE 3. Investment Rates for each of the 24 OECD countries (1950-1990)

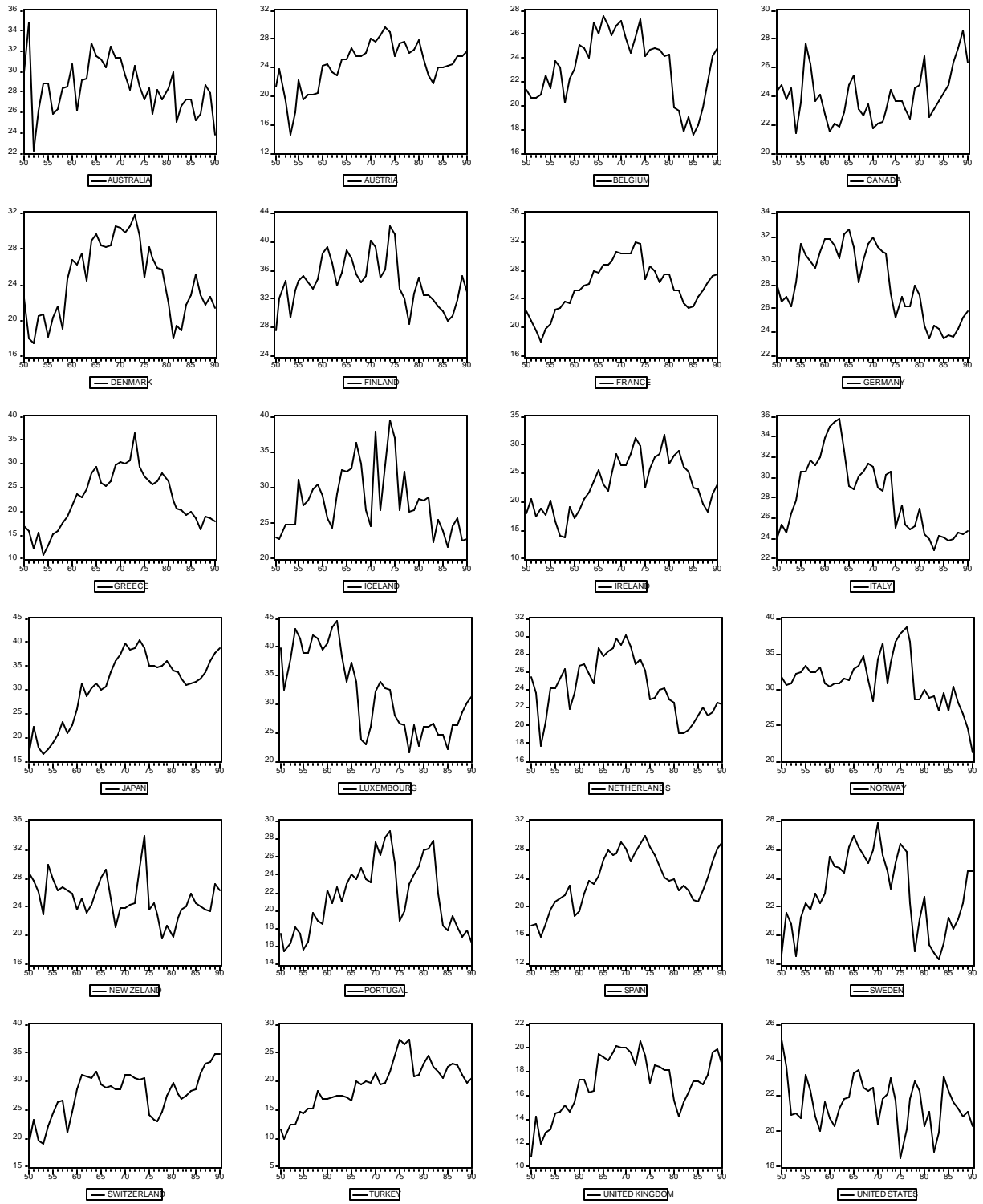


FIGURE 4. Weighted-Average OECD Growth Rate of Population (1950-1990)

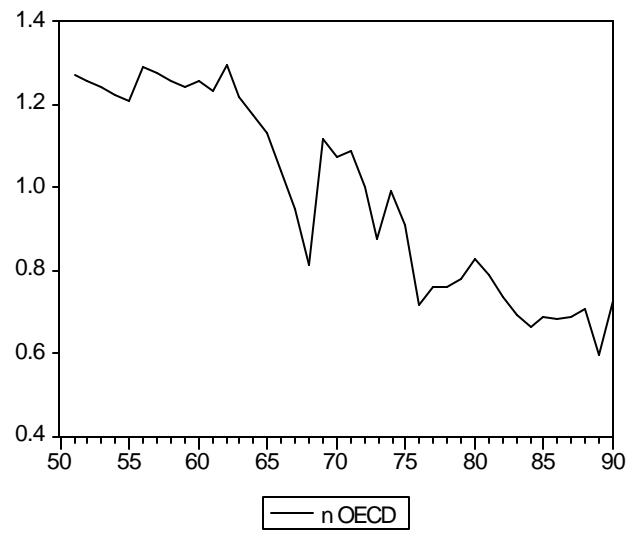


FIGURE 5. Dynamics of z and ω with CIES preferences and Cobb-Douglas technology

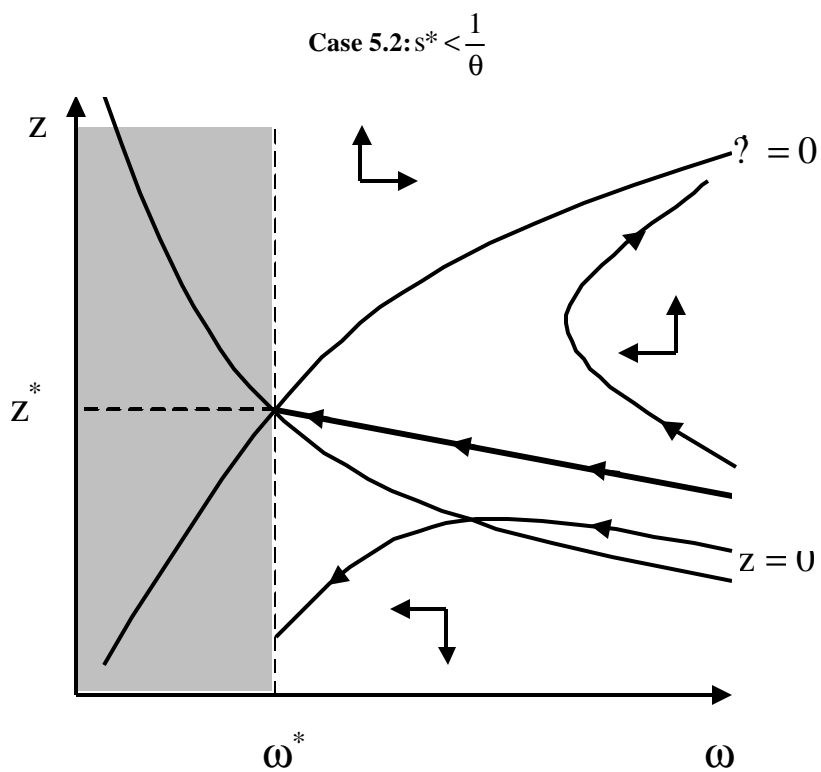
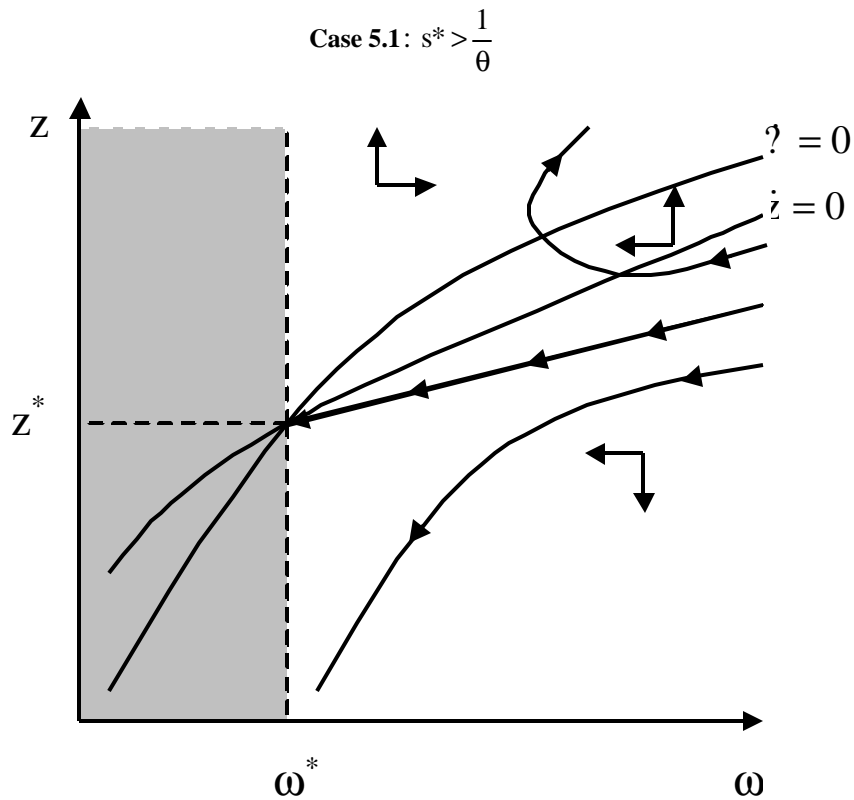
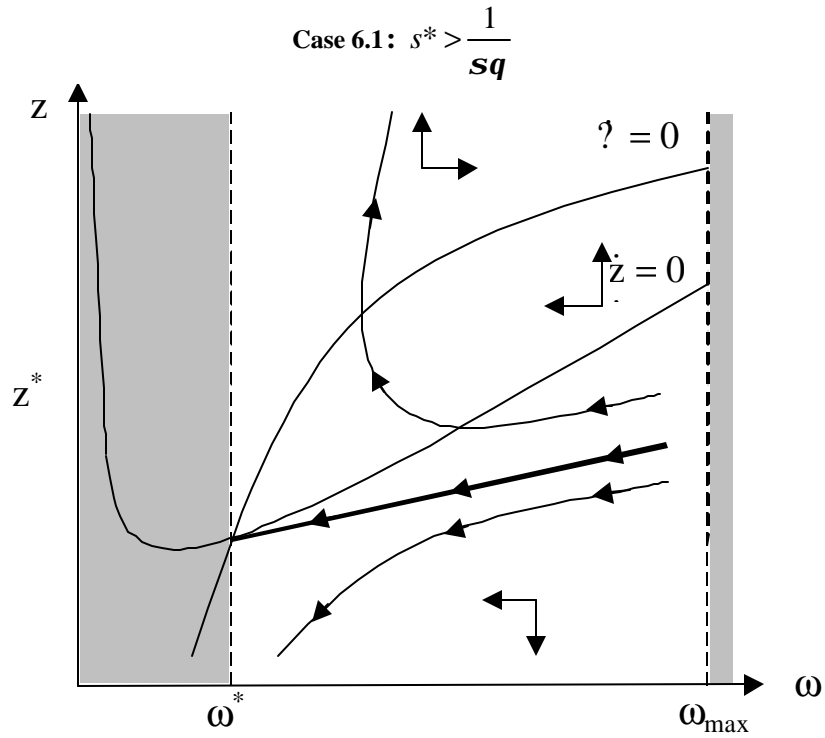
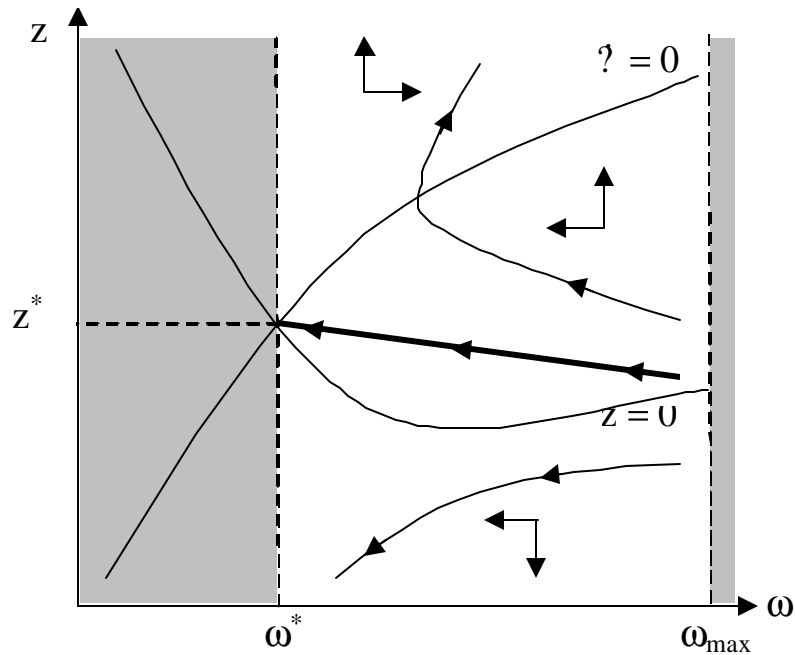


FIGURE 6. Dynamics of z and ω with CIES preferences and CES technology



Case 6.2: $s^* < \frac{1}{sq}$ and $s^* < \psi(s_K^*)$



Case 6.3: $y(s_K^*) < s^* < \frac{1}{sq}$

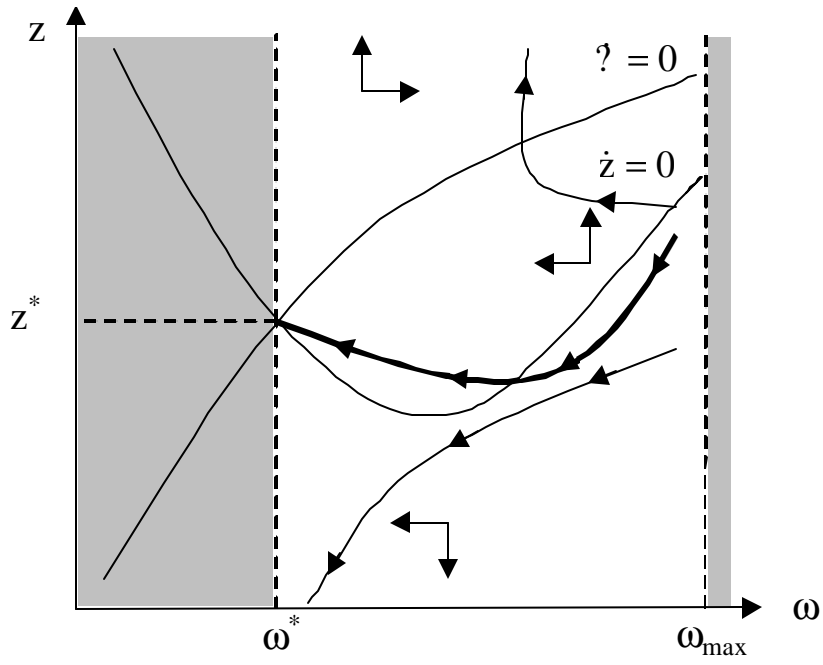


FIGURE 7. Simulations with CIES preferences and CES technology

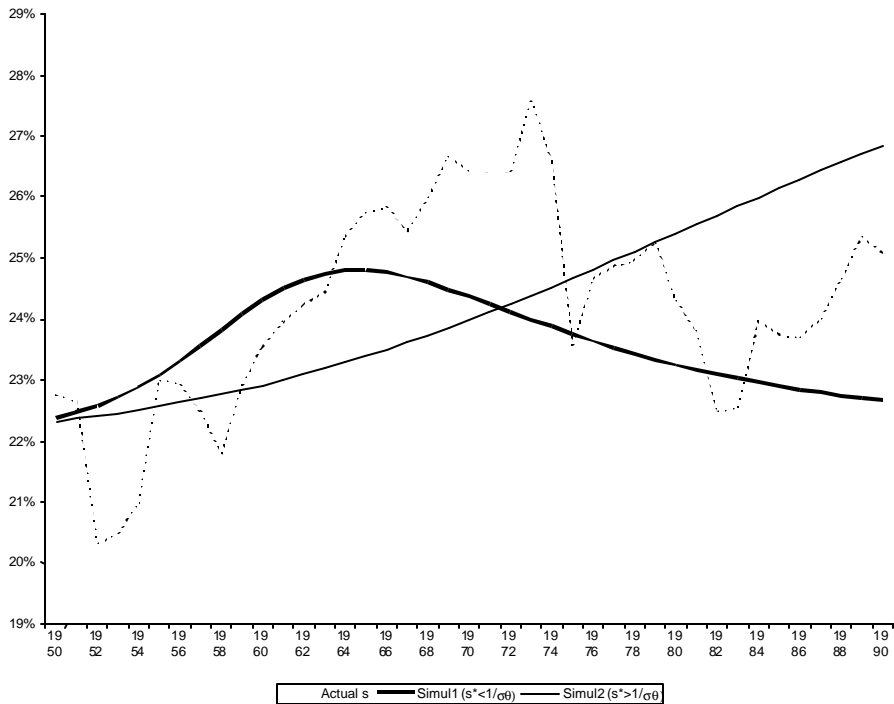


FIGURE 8. Dynamics of z and ω with Stone-Geary preferences and Cobb-Douglas technology

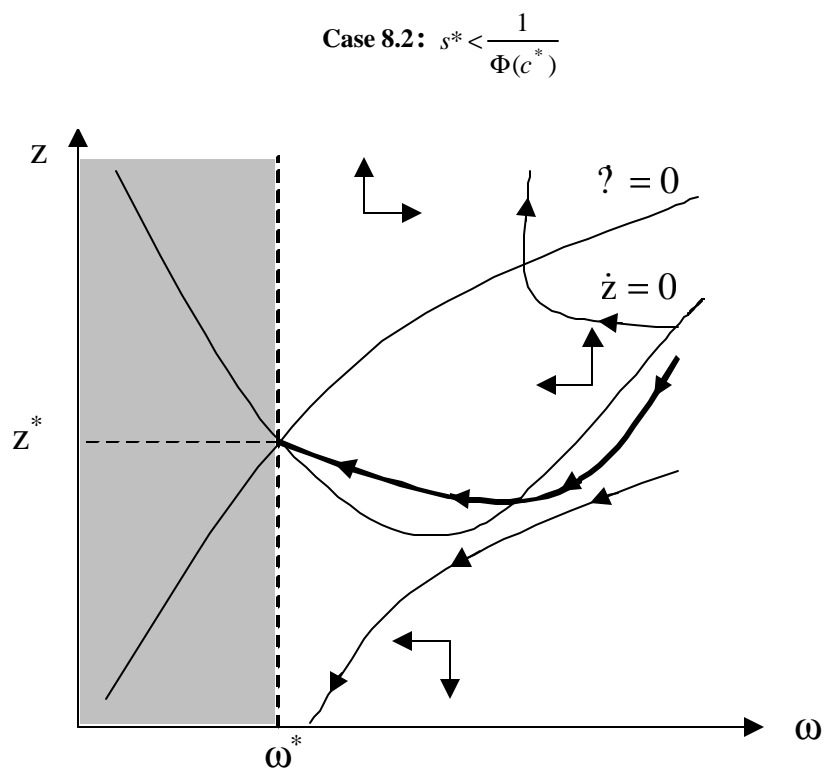
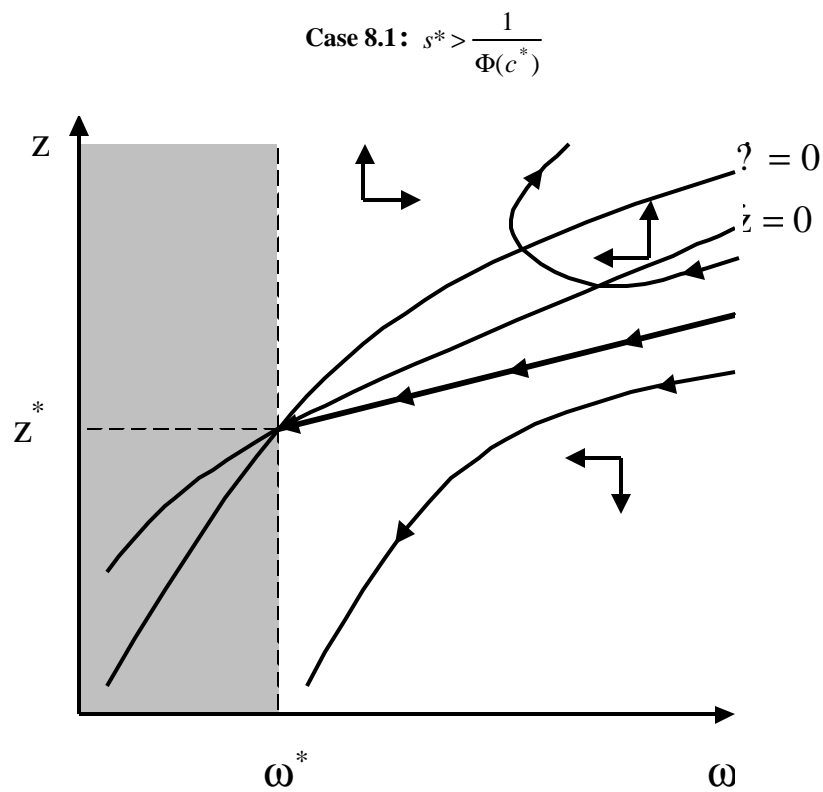


FIGURE 9. Simulations with Stone-Geary preferences and Cobb-Douglas technology

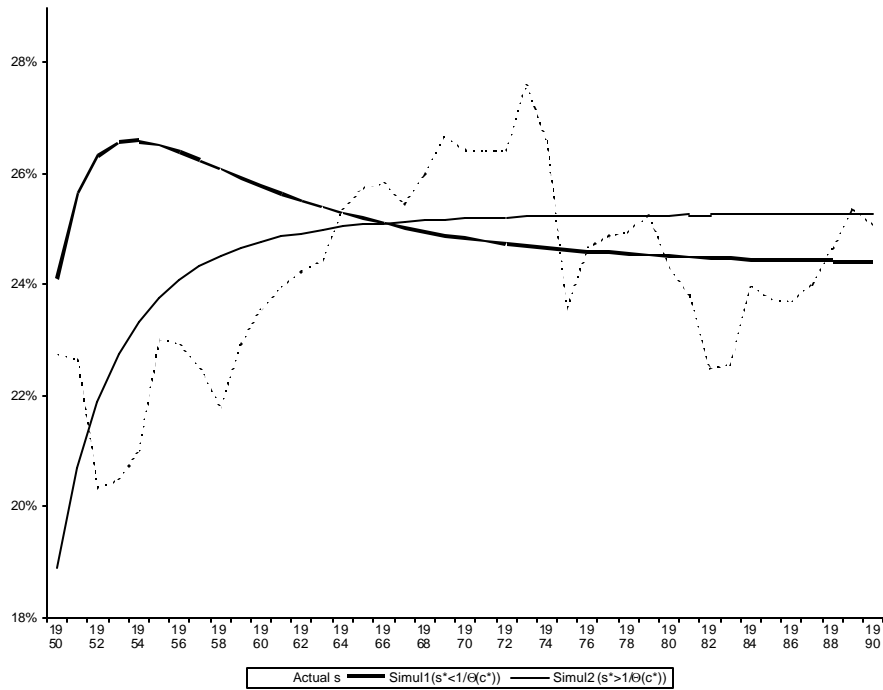


FIGURE 10. Investment Rates vs. Log(GDP) for OECD countries

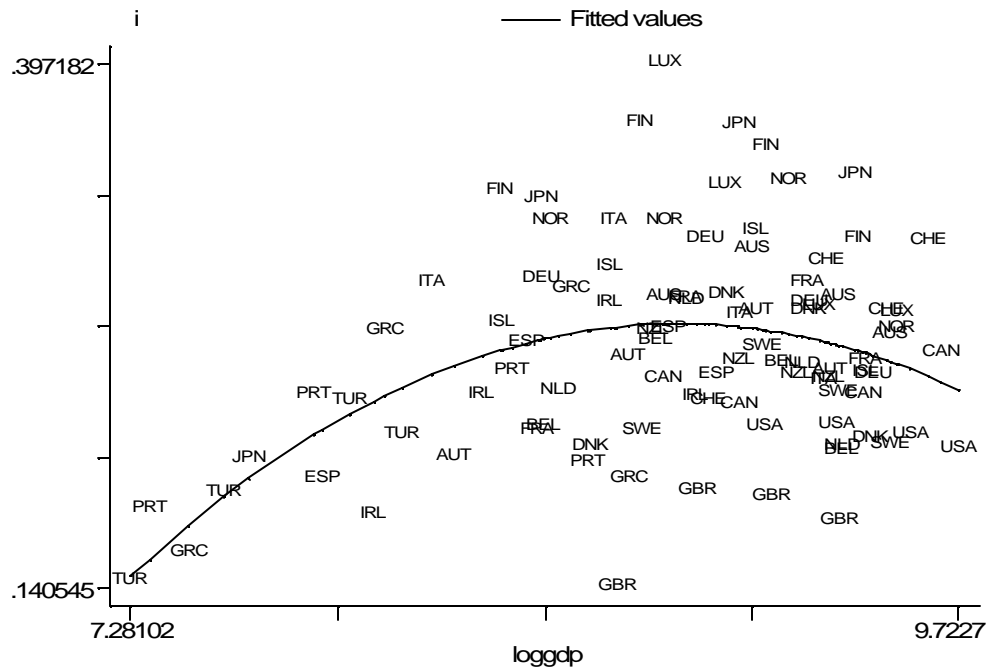


TABLE 2. SUR and 3SLS estimates for the OECD countries

Dependent variable: investment rate	I SUR	II 3SLS	III 3SLS	IV 3SLS	V 3SLS
Log (per capita GDP)	0.998 (0.176)	1.111 (0.189)		0.048 (0.009)	0.713 (0.172)
Log (per capita GDP) squared	-0.056 (0.010)	-0.063 (0.011)			-0.038 (0.010)
Growth rate of per capita GDP			0.901 (0.114)	0.914 (0.115)	0.734 (0.133)
Population growth			2.857 (0.485)	2.854 (0.491)	3.014 (0.507)
Number of Observations	24, 24, 24, 24	24, 24, 24, 24	24, 24, 24, 24	24, 24, 24, 24	24, 24, 24, 24
R ²	0.21, 0.15, 0.12, -0.38	0.16, 0.23, 0.15, -0.48	-0.56, -0.14, -0.20, -0.05	0.13, -0.29, -0.40, 0.25	0.23, 0.01, -0.12, 0.17

Note: The panel contains 4 cross-sections of 24 countries: 1950-60, 1960-70, 1970-80 and 1980-90. GDP is the average of the period. In the 3SLS estimation, 5-year lags are used as instruments for GDP and GDP squares, except for GDP in 1950, which is taken as exogenous. Both the growth rate of GDP and population growth are treated as exogenous. The data was retrieved from the Summers-Heston dataset.

FIGURE 11. Simulated savings rate and growth rate of GDP

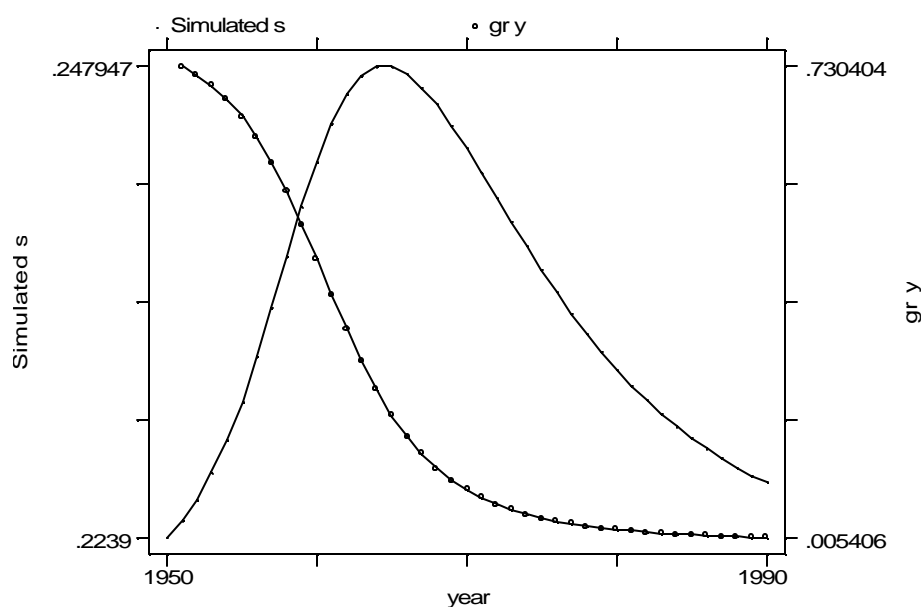


TABLE 3. SUR and 3SLS estimates for the larger cross-section

Dependent variable: investment rate	I SUR	II 3SLS	III 3SLS	IV 3SLS	V 3SLS	VI 3SLS	VII 3SLS
Log (per capita GDP)	0.236 (0.069)	0.224 (0.076)	0.264 (0.094)	0.276 (0.095)	0.272 (0.092)	0.240 (0.114)	
Log (per capita GDP) squared	-0.013 (0.004)	-0.013 (0.005)	-0.015 (0.006)	-0.016 (0.006)	-0.015 (0.006)	-0.014 (0.008)	
Years of schooling	0.000 (0.005)	-0.001 (0.006)	-0.003 (0.006)	-0.003 (0.006)	-0.005 (0.006)	-0.007 (0.007)	
Government consumption/GDP	-0.146 (0.066)	-0.299 (0.072)	-0.254 (0.092)	-0.268 (0.093)	-0.139 (0.082)	-0.388 (0.118)	
Growth rate of terms of trade	0.056 (0.060)	0.055 (0.066)	0.026 (0.082)	0.032 (0.083)	-0.010 (0.071)	0.112 (0.110)	
Rule of law index	0.064 (0.018)	0.063 (0.020)	0.054 (0.020)	0.055 (0.021)	0.037 (0.019)	0.051 (0.027)	
Log (fertility rate)	-0.038 (0.013)	-0.052 (0.014)	-0.039 (0.019)	-0.051 (0.026)	-0.007 (0.025)	-0.039 (0.031)	
Democracy index	-0.025 (0.014)	-0.024 (0.017)	-0.031 (0.017)	-0.027 (0.018)	-0.030 (0.017)	-0.036 (0.023)	
Inflation	-0.011 (0.011)	-0.052 (0.027)	-0.055 (0.031)	-0.062 (0.031)	-0.032 (0.025)	-0.051 (0.035)	
Growth rate of per capita GDP			0.169 (0.389)	0.065 (0.398)	0.554 (0.317)	0.404 (0.431)	2.485 (0.519)
Population growth				0.333 (0.726)	0.718 (0.734)	0.143 (0.994)	1.661 (0.683)
Old-age dependency ratio					0.009 (0.297)	0.128 (0.341)	0.856 (0.334)
Young-age dependency ratio					-0.249 (0.148)	0.017 (0.186)	-0.225 (0.154)
Gini coefficient						0.025 (0.062)	
Number of Observations	82 88 85	79 87 85	79 87 84	79 87 84	79 87 84	53 52 61	116 119 109
R ²	0.52 0.56 0.67	0.51 0.6 0.66	0.51 0.62 0.64	0.51 0.61 0.63	0.54 0.62 0.64	0.45 0.64 0.57	0.42 0.14 0.34

Note: The panel contains 4 cross-sections: 1965-75, 1975-85, 1990-95 and 1980-88. For the instruments in the 3SLS see text or Barro (2000).