Topological Phases with of Bosons with Short Range Quantum Entanglement

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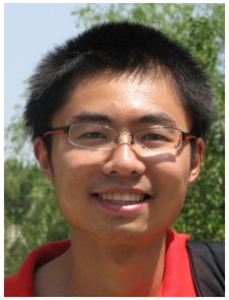
Acknowledgements



YuanMing Lu
(Berkeley)

arXiv:1205.3156 (PRB 2012)
`Integer' topological phases in 2D: A Chern-Simons approach
Yuan-Ming Lu, AV

T. Senthil
(MIT)

arXiv:1209.3058 (PRX 2013)
Physics of 3D bosonic topological insulators:
AV, T. Senthil

Xie Chen
(Berkeley)

arxiv:
SPT phases from decorated domain walls:
Xie Chen, Y.M. Lu, AV.

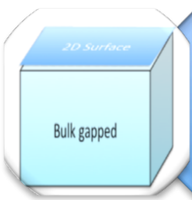
REVIEW: arXiv:1301.0330 Beyond Topological Band Insulators. Ari Turner, AV

Other collaborators: Fiona Burnell, Lukas Fidkowski, Tarun Grover.

OUTLINE

Introduction

2. Field theory of Bosonic Topological Phases in 2+1 Dimensions.



3. Field Theory of Bosonic SPT in 3+1 Dimensions

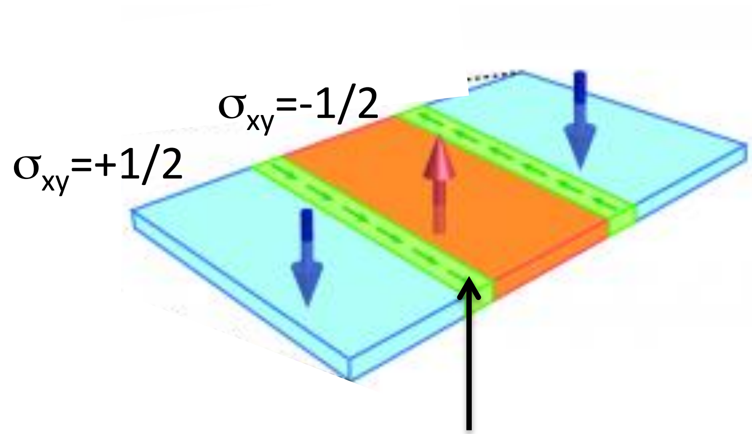
4. Intuitive Description from Decorated domain walls.

Introduction - Free Fermion Topological Insulators

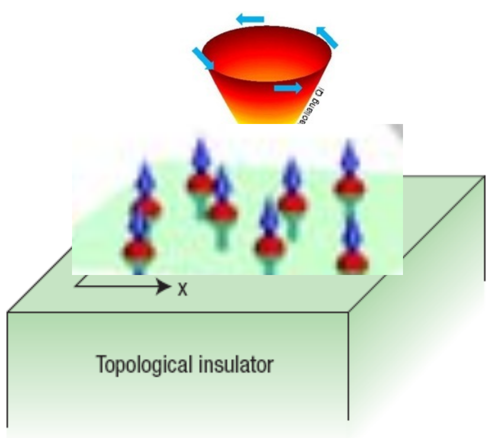
3D Topological Insulators

(Fu, Kane, Mele; Moore, Balents; Fu, Kane; Roy; Hsieh, Hasan, Cava et al.; Xia et al.; Chen et al.; Qi, Hughes, Zhang)

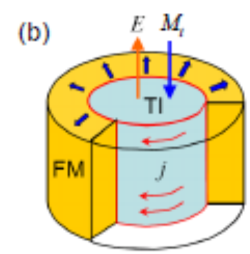
- Bulk gapped. Gapless surface states, not realizable in a 2D system
- Protected by Time reversal and charge conservation symmetry.



One chiral mode on surface domain walls



OR Quantized Magneto-electric polarizability.



$$P = \theta \frac{e^2}{2\pi h} B \qquad \theta=\pi$$

Break time reversal *only* at the surface. Unusual insulating state. "σ_{xy}=1/2"

Similarly 3D Topological Sc. – Thermal analog κ_{xy}.

Interacting Analogs of Topological Insulators

- **New** T. I. like phases that require interactions?
  - Bulk gapped and has *no* exotic excitations. (Short Range Entangled)
  - Laughlin Fractional Quantum Hall
  - Focus on T.I.s of bosons/spins - require interactions.
    - Realization in cold atoms/quantum magnets?
      - 1D Example – AKLT/Haldane spin-1 chain. Eg. Y_2BaNiO_5

`Ising model' of interacting topological phases – new insights.

Eg. `Impossible' Topological order on the surface of 3D Topological phases.

(AV-Senthil, Wang-Senthil, Burnell-Chen-Fidkowski-AV, Metlitski-Kane-Fisher. See talks by Xie Chen and Senthil)

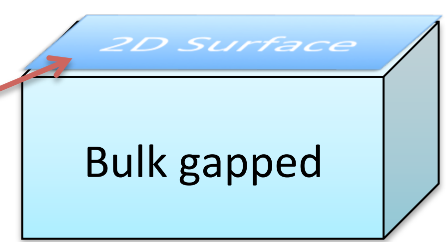
Short Range Entangled Topological Phases of Bosons in D=2,3

- A. Kitaev: E_8 state in d=2. No symmetry.

- Chen, Gu, Liu, Wen (2011): Symmetry protected topological phases.
  - Symmetry group G -> set of topological phases related to group cohomology.

Symmetry → / Dimension ↓	Time Rev. $\mathcal{T}$	Charge Consv. U(1)	Charge Consv. +Time Rev. U(1) |x T
d=1	Z_2	Z_1	Z_2
d=2	$\mathbf{Z_1}$	**Z**	$\mathbf{Z_2}$
d=3	$\mathbf{Z_2}$	$\mathbf{Z_1}$	$\mathbf{Z_2 x Z_2}$

Physical properties? Nature of surface states?



Field Theory Approach to 2D Symmetry Protected Topological Phases

2D Topological Phases: Chern-Simons Approach

- K-matrix description of topological phases.

  (eg. Hierarchy quantum Hall states, bilayer states etc.).

Theoretically `cheaper' method than cohomology.

Physical properties are transparent.

2D Topological Phases: Chern-Simons Approach

- K-matrix directly related to edge states:

$$\mathcal{L} = \sum_{I,J} \frac{K_{IJ}}{4\pi} a^I \cdot \nabla \times a^J \quad \Rightarrow \quad \mathcal{L}_{\text{edge}} = \frac{1}{4\pi} K_{IJ} \dot{\phi}_I \partial_x \phi_J$$

- K: symmetric integer matrix.
- |Det K| = torus degeneracy. Demand:
  1. |Det K|=1 (no topological order)
  2. Diagonal entries are even. (only bosonic excitations)

K=1 not allowed.

‘Bosonic’ Topological Superconductor in d=2.

1. Bosonic ‘topological Sc.’
(like p+ip)

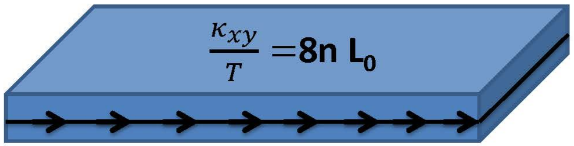
- chiral state, no symmetry.

MIN Dimension: 8x8

- 8 chiral edge modes.

$$K^{E_8} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

- K matrix (E_8 Cartan matrix)
- Kitaev E_8 state.



1. No Symmetry.
$\mathbb{Z}$ classes. Chiral Edge States.
Quantized Thermal Hall Conductivity

Bosonic Integer Quantum Hall Phase

Simplest non-chiral state:

$$\mathcal{L} = \frac{1}{2\pi} a_1 \cdot \nabla \times a_2$$

$$K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

2. Bosonic 'Integer Hall' state: U(1) symmetry. Charge vector: q= (1 n)

- Quantized σ_{xy}=2n (q^2/h).
- EVEN Integers only! (Lu-AV 2012, Senthil-Levin)

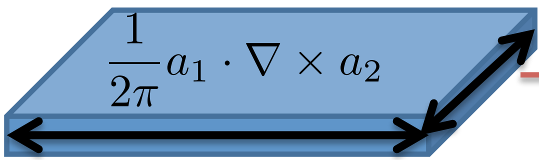
$$q = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

- Edge Luttinger Liquid with 'impossible' symmetry transformations.

Edge States and Symmetry

- Edge - 1D Luttinger Liquid

  But Symmetries act in a way that is *impossible* in 1D. (Lu-AV, Chen-Lu-Wen.)



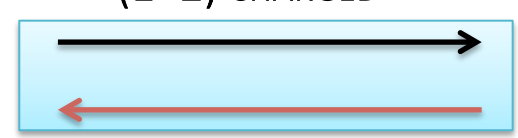
$$\mathcal{L}_{\text{edge}} = \dot{\phi}_1 \frac{\partial_x \phi_2}{2\pi}$$

ϕ_1 Boson phase,

$\rho = \frac{\partial_x \phi_2}{2\pi}$ Boson density.

Usual Tomonaga-Luttinger Liquid	Edge Tomonaga-Luttinger Liquid
$(\varphi) \quad \phi_1 \to \phi_1 + \epsilon$	$\phi_1 \to \phi_1 + \epsilon$
$(2\theta) \quad \phi_2 \to \phi_2$	$\phi_2 \to \phi_2 + \epsilon$
Δ L=Cos ϕ_2 (can be gapped)	*No gapping term*

(1+2) CHARGED

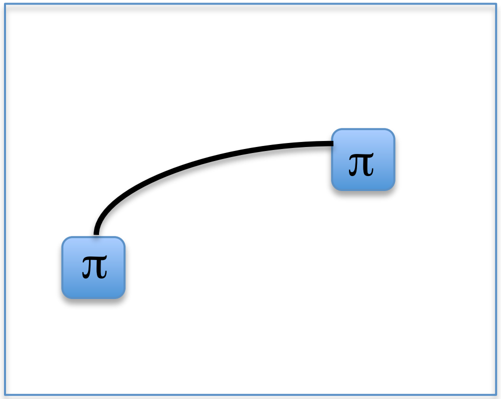


(1-2) NEUTRAL

Topological Phase with Z_2 symmetry.

- Edge theory can be used to extract all universal properties.
  - Eg. Z_2 symmetry (Levin-Gu). $\Psi(\alpha) = (-1)^{N_{\mathrm{domain-wall}}}$

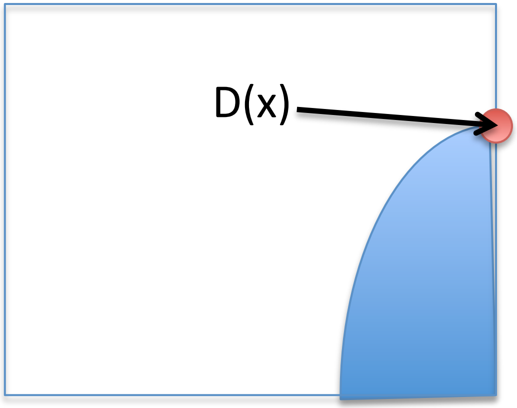
Pi-fluxes. End points of Z2 domain walls.



Pi fluxes are semions.

$$\phi_{1,2} \rightarrow \phi_{1,2} + \pi$$

Or – end domain wall at edge



$$D(x) = e^{i\pi \frac{\int^x dx\, [\partial_x \phi_1 + \partial_x \phi_2]}{2\pi}}$$

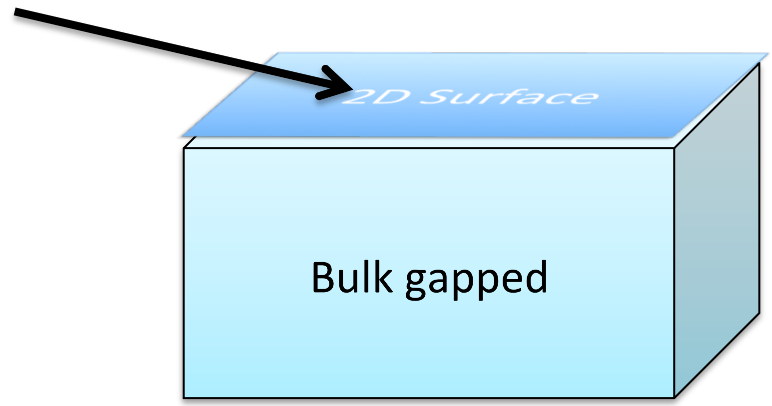
$$= e^{\frac{i}{2}(\phi_1(x) + \phi_2(x))}$$

$$D(x)D(x') = {\color{red} i}\, \mathrm{Sign}(x - x')D(x')D(x)$$

Field Theory of 3D Symmetry Protected Topological Phases

3D BOSONIC `TOPOLOGICAL INSULATORS'

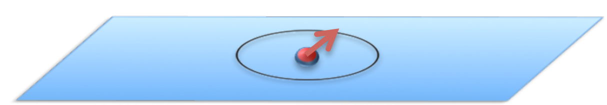
- Surface: 2D Bosons but symmetries act in a special way.



- Example: Symmetry: U(1) (S_z) + *T*
  - Bulk paramagnet.
  - Surface: 2D XY model BUT unusual symmetry action.

$$\langle S^+ \rangle = e^{i\phi_1}$$

$$\text{vortex} = (\psi_\uparrow, \psi_\downarrow)$$

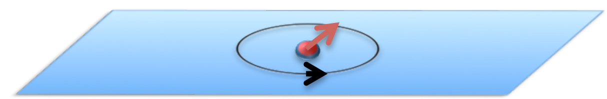


- Describe two SPT phases:
  - PHASE 1: break U(1) on surface.
    Vortices are Kramers pairs. ***T^2 = -1***
  - PHASE 2: break T on surface.
    Insulator with σ_{xy}=1

3D BOSONIC `TOPOLOGICAL INSULATORS' & Magneto-electric Effect

$$\langle S^+ \rangle = e^{i\phi_1}$$

$$\text{vortex} = (\psi_\uparrow, \psi_\downarrow)\; S_z = \pm\frac{1}{2}$$



- Phase 2: Consider [U(1) x U(1)] x *T*
  - *Break* U(1). XY order.
  - Vortices transform under: U(1) x *T*
  - Boson-Vortex duality:
    - *Vortices coupled to gauge field `a'.*

$$\frac{\nabla \times a}{2\pi} = n$$

  - *`External' Fields: A, A*

$$\mathcal{L}_{\text{edge}} = |(\partial - ia + i\frac{\sigma}{2}A)\psi_\sigma|^2 + \frac{\nabla \times a}{2\pi} \cdot A + \mathcal{L}_a$$

Condense: $\psi_\uparrow$ *then* $a = \frac{1}{2}A$ *and* $\mathcal{L}_{em} = \frac{1}{4\pi} A \cdot \nabla \times A$

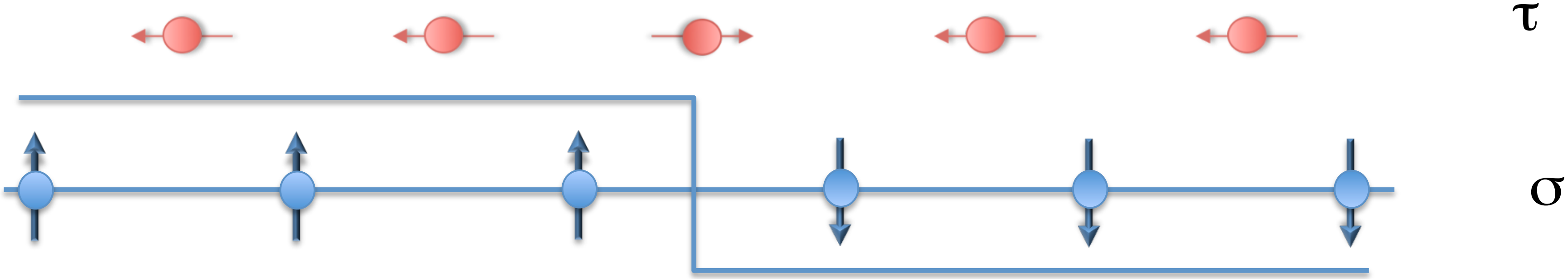
$$\sigma_{xy} = 1 \;\; (\Theta = 2\pi)$$

Two U(1)s needed to obtain fermionic vortices – `framing'.

Xu and Senthil

A PHYSICAL PICTURE of *SOME* SPT phases from DECORATED DOMAIN WALLS

Physical Picture – Condensate of Decorated Defects

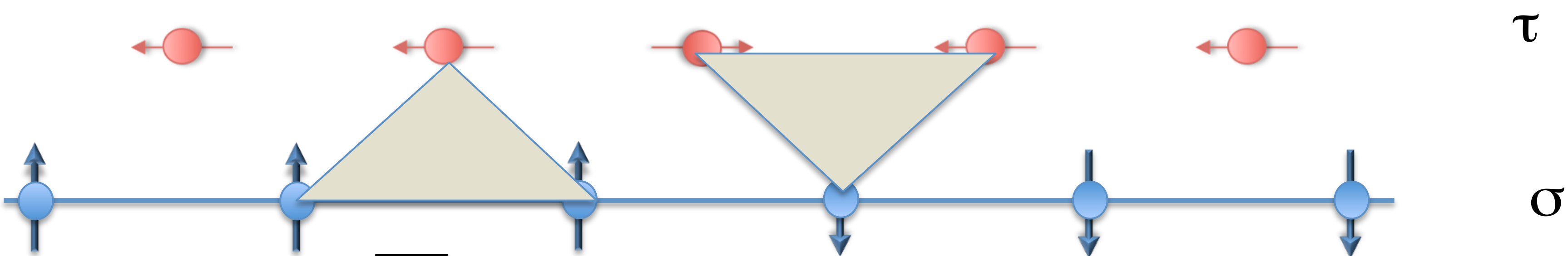


- 1D topological phase with G = $Z_2 \times Z_2$ symmetry.
  - Two Ising models. Rotn. by 180 degrees about X,Y,Z. Analog of Haldane phase.

Condense domain walls of Z_2 with Z_2 charge.

$$H = -\sum_i (\sigma_i^z \tau_{i+1/2}^x \sigma_{i+1}^z + \tau_{i-1/2}^z \sigma_i^x \tau_{i+1/2}^z)$$

Topological Phase from Condensing Decorated Defects



$$H = -\sum_i (\sigma_i^z \tau_{i+1/2}^x \sigma_{i+1}^z + \tau_{i-1/2}^z \sigma_i^x \tau_{i+1/2}^z)$$

Exactly Soluble Hamiltonian (Cluster State) Son, Amico, Vedral

Sum of commuting projectors

$$H = \sum_i (\tilde{\tau}_{i+1/2}^x + \tilde{\sigma}_i^x)$$

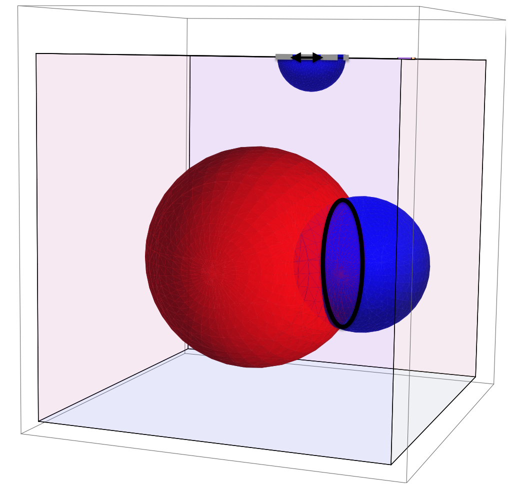
Unique Ground state with periodic boundary conditions

4 fold edge degeneracy with open boundary conditions. $\{\sigma_0^z,\ \sigma_0^x \tau_{1/2}^z\}$

Condensate of domain wall&spin flip =string order parameter:

$$\langle \tau_{1/2}^z \left[\prod_{i=1}^N \mu_i^x \right] \tau_{N+1/2}^z \rangle$$

3D Bosonic Tis from Condensing Decorated Domain Walls



- Symmetries: $Z_2 x Z_2$
- Domain walls of eg. Z_2 in an SPT phase protected by Z_2 symmetry.

1. Wave-function: superposition over domain wall configurations with:

$$\Psi = (-1)^{N_{\mathrm{dw-intersect}}}$$

2. Domain walls on surface carry symmetry protected edge modes.
3. Classification – phases labeled by excluded symmetry: $Z_2 x Z_2$

3D Bosonic Tis from Condensing Decorated Domain Walls

- Symmetries: Z_2xZ_2
- Domain walls of eg. Z_2 in an SPT phase protected by Z_2 symmetry.

4. Topological termination – gapped and fully symmetric state with topological order. Laughlin m=2 state which transforms projectively under Z_2xZ_2 (like S=1/2).

5. Gauging Z_2 : π-flux carries gapless mode. (Z_2 Symmetry enriched topological phase).

Conclusions and Future Directions

Symmetry / Dimension	Time Rev. $\mathcal{T}$	Charge Consv. U(1)	Charge Consv. +Time Rev. U(1) |x T
d=1	Z_2	Z_1	Z_2
d=2	Z_1	Z	Z_2
d=3	Z_2	Z_1	$Z_2 x Z_2$

X Z_2

- Physical description of bosonic topological insulators.
- New topological phase *beyond* group cohomology. Surface related to Kitaev E8 state.
- Simple physical realizations in magnets/Bose-Hubbard models?
- New topological phases of interacting fermions ?