Bargaining over a Burden

How Legislatures Distribute Costs when Benefits are Fixed

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Legislative bargaining is typically modeled as a "Divide-the-Dollar" game, with a divisible benefit and a fixed cost. This framework is a reasonable simplification in many contexts, but it fails to capture the incentives that prevail when a legislature distributes the cost of providing a good with a fixed benefit (i.e. a public good). In this paper, I propose and analyze a model that corresponds to this scenario, and in turn, highlights how bargains over burdens and bargains over benefits differ with respect to the exercise of power, the construction of strategies, and the features of equilibrium distributions. For example, if legislation is considered in multiple stages (e.g. in the House Rule's committee and then on the House floor), supermajority requirements will actually reduce legislative gridlock with respect to the provision of public goods. Also, allowing amendments, under certain circumstances, makes all legislators worse off, and is especially bad for those opposed to the proposal which passes. Uncovering these and other results raise several possible empirical applications, and suggests a new perspective on recent, high-profile Congressional debates. Finally, the results also suggest new reasons to expect that legislators would act quickly to establish durable restrictions on majoritarian impulses: In their absence, bargaining over burdens is likely to lead to outcomes that are extremely unequal, inefficient, and often worse than the legislative status quo for all legislators except those in control of the agenda.

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1 Dividing Dollars vs. Dividing Deficits

"Legislative Bargaining" and Congress' "Grand Bargain"

The 2010 midterms gave Republicans a majority in the U.S. House of Representatives and, once again, ushered in divided government at the federal level in the U.S. One of the key issues in the campaign (at least according to the victorious Republicans) was "fiscal responsibility", with Republicans campaigning to scale back spending and perhaps reform entitlement programs as well. Almost immediately , the issue thorny issue of the national debt limit arose. Raising it required an act of Congress, Republicans pledged not to go along unless they received substantial concessions, and failing to act would potentially destabalize global economic markets.

Almost immediately negotiations began between President Obama's Democratic administration and its allies in the Senate on the one hand, and House Republicans on the other regarding these disagreements. Notably though, there was broad *agreement* on some first-order principles. In particular, both sides agreed about the need for a fiscal consolidation somwhere around the order $4 trillion over ten years.

With this goal shared widely across the political spectrum, hopes of a "Grand Bargain", soon percolated through the Capitol and the press. Though lacking precise definition, this term generally referred to a potential bi-partisan agreement to raise the debt ceiling, reduce fiscal deficits through some combination of spending cuts and tax increases, and reform entitlement programs (primarily Social Security and Medicare) in order to stabalize their long-term fiscal outlook.

Three features appear to have defined Grand Bargain negotiations. First, the negotiating parties agreed more or less about the optimal scale of the fiscal consolidation. That is, agreement on the ideal amount by which to reduce the fiscal deficit was fairly easy to reach.[1] Second, negotiators took the distribution of benefits as relatively fixed. The

[1] See `www.whitehouse.gov/blog/2011/04/13/president-obama-s-framework-4-trillion-deficit-reduction` and `www.nytimes.com/interactive/2011/04/06/us/politics/06budget-doc.html?ref=politics&_r=0` for President Obama and House Budget Committee Chairman Paul Ryan's proposals each claiming to reduce the national debt by about $4 trillion. It should be noted that believing the precision of these estimates requires strong assumptions and that it is often difficult to determine which baseline should be applied to answer the question $4 trillion from *what*

main payoffs to the Grand Compromise were more confident credit ratings, lower debt payments, and general economic confidence. These distribution of thee benefits is more or less inherently fixed because they are derived from inherently public goods, and thus their distribution was outside the scope of bargaining in this context. Finally, the negotiating parties disagreed about the distribution of costs and this is what the legislative bargaining process centered on.[2]

Press accounts at the time described a stream of bargaining sessions and sought to analyze which bargaining parties had the stronger negotiating position. These negotiations carried national and global economic implications, dominated political conversation in some form or another for at least the next year, and emerged as a major issue in the 2012 Presidential campaign. Thus in both name and fact, legislative bargaining was the predominant activity in Congresss during this period.

Under these circumstances, Congressional observers and scholars might naturally look to the extensive political science literature on legislative bargaining to gain insight into the political factors and institutions at play in the "Grand Bargain" negotiations outcome. However, doing so would have revealed a fundamental incompatibility between the theoretical models and the defining features of the "real-world" bargaining process described above. While existing models envison legislators bargaining over the distribution of benefits with costs fixed, the unfolding Congressional process had benefits that were relatively fixed and negotiations centered on the distribution of costs.

Public Goods and Distributive Politics

The fixed costs assumption is ubiquitous in the legislative bargaining literature in which negotiations are framed as "divide-the-dollar" games, and a good is distributed according to the legislators' strategic incentives and procedural prerogatives given the assumed political and economic context.[3] There are many variations on this theme, but in every

[2]See (Peter Wallsten and Wilson 2012), Montgomery and Branigin (2011), and Bai (2012) for contemporaneous accounts of the bargaining process and the general political environment

[3]The canonical application in political science is Baron and Ferejohn (1989). In turn this work was heavily influenced by the economics literature on bargaining which traces back to Rubinstein (1982). Banks and Duggan (2006) has a useful discussion of some of the general features of this literature

application the assumption is that the distribution of costs is fixed, while benefits are determined legislatively.[4]

In this paper I analyze a model that inverts this assumption, in which legislators propose, bargain over, and vote on a distribution of costs, which finances a public good that uniformly benefits all legislators. Thus, the model captures essential features of the "Grand Bargain" example above, and similar settings in which legislators broadly agree about the returns to investing in an available public good, but have conflicting incentives over how to finance it. More colloquially, the model is intended to apply when all legislators want the same thing, but also want other legislators to pay for it.

Such a setting yields several results which stand apart from prevalent patterns in the literature: Supermajority requirements, under certain circumstances, reduce gridlock, more patient legislatures have more difficulty providing and efficiently financing public goods, and allowing amendments may increase distributional inequality and be detrimental to all legislators.

The differences between the results of standard bargaining model and the one here can be boiled down broadly into two categories. First, when costs are distributed, legislators left out of the winning coalition are not simply ignored by the agenda-setter's proposal, as they are when bargaining over a benefit. Instead, the agenda-setter acts on the obvious incentive to impose costs on those legislators whose vote she is not seeking. This incentive has a significant effect on the types of proposals legislators propose and support in equilibrium. In an extension to the baseline model, I show how this incentive is particularly strong when amendments are allowed, which can lead to legislators prefering a closed rule (no amendments allowed) to an open one (amendments possible).

Second, the ability to impose costs implies that, in equilibrium, some legislators can end up worse off than under the status quo of no legislative action. In turn this leads to the counterintuitive result that legislators will frequently be willing to support proposals that leave them worse off than the legislative status quo.[5] In an extension to the baseline

[4]This assumption has roots both in the economic Congressional literatures on the pork barrel and the divergence between economic and political efficiency (Rubinstein (1982), Banks and Duggan (2006), Weingast et al. (1981)).

[5]I will discuss the intuition for this result in more detail in the sections to come. Here, for especially

model, I consider how the inability to avoid such outcomes once a bill reaches the floor (technically once an agenda-setter is given the right to make a proposal) might lead to a two-part agenda-setting process where legislators head off unpopular proposals before they become unstoppable on the floor. Under this extension, some bills that would pass under any socially optimal or Pareto efficient process are blocked, thereby introducing a type of gridlock interval into the analysis. What's more the size of this interval can be functionally derived yielding several interesting comparative static results.

The rest of the paper proceeds as follows. In the next section I discuss the existing bargaining and distributive literature, their underlying assumptions, and the substantive, "real-life" policy debates for which they are ill-suited. In Section 3, I introduce the baseline model and the equilibrium concept used to solve it. After deriving equilibrium conditions I discuss their basic features and how they differ from typical legislative bargaining models. I build on these results in Section 4 by considering two extensions of the model which provides for a pre-floor committee screening process and the possibility of amendments respectively. The final section concludes.

2 Legislative Bargaining and the Fixed Cost Assumption

There is a deep and long-standing literature that models distributive politics and majoritarian bargaining. The foundations of the field were laid in work by McKelvey (1979) and McKelvey (1976) which established that under "pure majority rule", that is in the absence of further assumptions about preferences, agenda-setting institutions, or the dimensionality of the policy space, distributive politics was either degenerate or indeterminate.[6]. The next wave of research both established how institutions could induce stable distributive outcomes, and also illuminated the mechanisms that could result in individually rational legislators to collectively chosing inefficient policies when the costs of particularistic spending were shared (Shepsle (1979), Weingast et al. (1981), Shepsle and Weingast (1981)).

skeptical readers, I would note that Baron (1991a) and Primo (2006) feature models with this result and arrive at it using the same methodology and logic as that employed here.

[6]That is, either any outcome is possible in equilibrium, or no equilibrium is ever possible (Baron 1991a)

The next innovation in the development of this literature came when Baron and Ferejohn (1989) combined a non-cooperative bargaining model with a simplified model of sequential policy-making. Under this approach, stability is ensured primarily because legislators are impatient and agenda-setting power is asymmetric. Since its initial publication, it has served as a template for many hundreds of legislative bargaining applications and remains the default approach today for modeling legislative bargaining, and provides the basic institutional and procedural template for the model I develop in this paper as well.[7]

While the political science literature universally treats cost distributions as fixed in models of distributions and bargaining, the public finance and political economy literatures treat them as endogenous either to be determined on their own or simultaneously with overall expenditures. Thus, there would seem to be a natural connection between the models in this literature and the present task. However, these scholars are not interested primarily in the role that legislaive institutions play in affecting distributions and efficiency. As a result, the dictates of tractability and parsimony lead to pared legislative complexity, with instituions and agenda-setting mostly unmodeled. As a result, these models produce insights that are economic and occasionally political , but not legislative in nature (Alesina (1988), Persson and Tabellini (2002), Alesina and Drazen (1989), Fernandez and Rodrik (1991)).[8]

Returning then to the political science literature, in most applications of the standard bargaining model, not only are costs fixed in distribution, their magnitude is also exogenous. However, several studies consider models in which costs are fixed in their distribution but with their size determined endogenously (e.g. Baron (1991a), Leblanc et al. (2000), Primo (2006)). These approaches are the most similar to the model developed in

[7]A small sample of the applications that build on the canonical Baron-Ferejohn model includes bicameral coalition building, bargaining with a weighted voting procedure, and the interaction between preferencs for public and particularistic goods (Ansolabehere et al. (2003), Snyder et al. (2005),Volden and Wiseman (2007)). Additionally, a number of papers have this template to study questions related to public goods and distributive efficiency. These models are more directly applicable to the approach I take here and so I discuss these in more detail separately.

[8]Typical examples within this subfield, analyze whether social spending or public investment is first cut during a fiscal consolidation (Sanz (2011), Alesina and Ardagna (2010)), or whether large coalitions are needed to support fiscal consolidations (Perotti 1998).

this paper in that they are usually applied to questions related to the provision of public goods and distributive efficiency, evoke legislators' concern about the imposition of costs in the future, and result sometimes in equilibria where legislators find it in their best interest to support a proposal that leaves them worse off than they would be under the legislative status quo. These features and themes, as will be shown in the next section, are central to analyzing the process of bargaining over costs as well.

Previewing results and putting them in context

The above discussion only hints at a small portion of the different models in the literature that can be used to analyze distributive politics and its political-economic consequences. Particularly flexible are the bargaining models which can can be both be used to analyze how legislative institutions interact with features of Congressional politics ranging from the geographic distribution of rivers and dams projects to the timing of the appropriataions cycle and how its "political tempo" interacts with the Senate's system of electoral rotation (Woon and Anderson (2012), Shepsle et al. (2009), Ferejohn (1974). What's more, without too much trouble additional complexity, essentially similar approaches can be used to analyze macroeconomic fiscal phenomena such as the acculmulation and consolidation of national debt as in the works public finance and political-economics work cited previously.

However, there is an important perspective missing from this otherwise robust theoretical landscape. Specifically a model that captures how legislators bargain over the distribution of costs with a fixed benefit, and thus we are at a disadvantage in attempting to understand the role that legislative and procedural institutions play in such processes.

In developing and analyzing such a model, this paper highlights several features of legislative bargaining that are largely obscured when costs are assumed to be fixed. These features, their origins, and their implications are discussed throughout the subsequent sections in greater detail. For now though, I introduce a few of the more important results to highlight the novel, sometimes counterintuitive insights that the cost bargaining model can add to the stable of theoretical results in the existing literature.

Role of legislative "patience" In almost all existing bargaining models, legislative patience - that is the relative weight legislators place on current and future utility - plays a straightforward role. The less legislators value future utility, the more bargaining power is enjoyed by the the agenda-setter, and the less equal is the equilibrium distribution. With a fixed benefit and politically determined cost though, this simple relationship becomes complicated. If legislators project that in future sessions they are likely to be worse off than they are under the status quo, then the agenda-setter's leverage will be higher in legislatures that are relatively patient. An additional complication must also be considered though. Unlike in the standard model, a more powerful agenda-setter does necessarily imply a less equal distribution in equilibrium. With a fixed benefit and negotiated cost, if the agenda-setter is not contributing towards the cost of the good, then the impact of further increases in her bargaining power will depend on modeling assumptions discussed in the next section. Generally though, if the agenda-setter is not contributing towards the cost of the public good, then increasing her bargaining power will lead to more even cost distrbutions across the coalition and opposition blocs.

This result, unlike most others that apply to cost bargaining but not to the canonical model, is also present in some of the previous adaptations found in the extant bargaining literature (See especially Baron (1991a)). Still, the interaction between the legislature patience and the shift in the agenda-setter's bargaining priorities which occurs once she is assured of paying nothing towards the cost of the good, is new, and the role of the discount factor when expected utility is negative is explored in much more detail here than in previous work even where it was present.

Optimal coalition size varies between minimal and unanimous Another result that distinguishes the cost bargaining model from standard ones, relates to optimal coalition size when amendments are allowed (that is, under an open rule). In standard models, coalitions sizes tend towards either unanimity or minimal.[9] Here though, with an open

[9]In Baron (1991a), even though amendments reduce the agenda-setter's leverage, winning coalitions remain, in equilibrium, minimal. In Volden and Wiseman (2007), another paper that considers the tradeoffs between particularistic spending and social efficiency, coalition sizes are either minimal or unanimous. In the original "Divide-the-Dollar" legislature, greater than minimal coalitions were possible, but only for relatively small legislatures, in fairly limited circumstances and only barely

rule, a more dynamic relationship emerges. Optimal coalition size can range between minimal and unanimous depending on the value of the public good, legislators' patience, and the weight they place on net social welfare.

Amendment rules don't always improve outcomes for rank-and-file Partially because of these incentives towards large coalitions, an open-amendment rule does not automatically yield higher expected utility for rank-and-file legislators ex ante. When coalition sizes are particularly large, costs have to be particularly disproportionate in their distribution. This can lower expected utility and will usually do so if the agenda-setter is not contributing towards paying for the public good.

Super-majority requirements can reduce gridlock For a final example, consider that in existing bargaining results, super-majority requirements intuitively lead to more equal distributions as they do in the present model also. However, the implications for cost bargaining are different, since now the uniformity of the cost distributions will, in large part determine the sign of legislators' ex ante expected utility, which in turn determines which projects are enacted and which projects are blocked in legislatures with a two-stage agenda-setting process. Thus, in legislatures with such an insitution, supermajority requirements can actually reduce gridlock.

3 Distributive Model

Model Outline, Assumptions, and Constraints

In this section, I begin the paper's primary task of positing and analyzing a model of legislaive bargaining over the costs of providing a good with a fixed benefit. Many of the basic features, and the notation used to describe it, mimic the existing literature and so hopefully will be familiar to many readers.

exceeded minimal size (Baron and Ferejohn 1989).

Legislative Setting

A legislature with n total members meets to consider a good with a total value of b and from which all legislators derive a benefit equal to $\frac{b}{n}$. The cost of providing this good is standardized to unity. The net benefit of providing the public good is therefore $b - 1$, abbreviated to $\bar{B}$ throughout.

In order to provide the good, the legislature must enact a proposal, X, which finances the good by distributing its cost amongst the legislators. A valid X is therefore an n-length vector, $\{c_1, \ldots, c_i, \ldots, c_n\}$ proposing that legislator i contributes c_i towards the cost of providing good with $0 \leq c_i \leq 1$.

Utility Functions

A legislator's utility under any X is a function of the cost they are required to contribute under an enacted proposal and the value of the shared benefit. Additionally, I assume utility is also a function of the overall efficiency of the cost distribution. This is intended to reflect the fact that, for a public good with a fixed value, legislators' net utility from its provision will depend in part on how efficiently its cost is financed independently of their own contribution to that cost.

To capture this concept I assume that more uniform cost distrbutions are more efficient. This assumption can be justified in more than one way. Most simply perhaps, it is economically valid if the deaweight loss from extracting resources from a single district or state is marginally increasing in the amount taken from that source. Or in other words, if the marginal return to taxing a particular state or district is decreasing from the government's point of view. The key point is that all else constant, each all legislators prefer, at least weakly prefer to share costs evenly.

To encode this assumption, for any proposal X, let $S^2(X) = \sum_{i=1}^{n}(1/n - c_i)^2$ so that S^2 is a proxy for the sample variance of the cost distribution proposed under X,[10] and let α be a free parameter that captures both the weight that legislators attach to overall efficiency and provides linear flexibility to the specification.[11]

[10] Ignoring the statistical bias of dividing the squared summed residucals by n rather than $n - 1$.
[11] Inclusion of the S^2 term and the α parameter may seem a ripe target for elimination via *Occam's razor*.

We can now define the utility for a typical legislator, i, utility under proposal X as a function of $i's$ cost-share under the proposal (c_i), the value of the benefit to be financed b, and the variance of individual costs under the the proposal, $S^2(X)$.[12]

$$u_i(X) = \frac{b}{n} - c_i - \alpha S^2(X) \tag{1}$$

Bargaining Process

In the baselne model, at the start of the session, an agenda-setter, A, is chosen at random and given the opportunity to offer proposal X. In order to pass, X must garner support from a coalition of at least $\mathbb{D}+1$ legislators, with $\frac{n-1}{2} \leq \mathbb{D} \leq n-1$ so that $\mathbb{D}$ is the minimal size of $A's$ coalition not including herself.[13] If enacted, then the cost distribution proposed under X is distributed, and legislators' implied utility under Equation 1 is realized.

If X does not receive the required number of votes then the legislature adjourns and starts the process over in the next session. Legislators discount future utility by factor of δ. I also assume that the legislature can adjourn and reconvene infinitely many times.[14]

Constraints After nature chooses the agenda-setter, A, the game proceeds with A making a proposal that, if it is to pass, must meet two primary constraints. First, the total cost imposed by X must equal the cost of providing the public good standardized to

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Such an argument suggests that the appropriate starting place for a fixed-benefit, legislated-costs model is an assumption that proposals affect legislators' utility solely through direct costs under that proposal which is equivalent to assuming $\alpha = 0$ in Equation 1. Such an assumption generally leads to what I term a "scape-goat" equilibria, in which one legislator pays the entire cost, while all other legislators achieve their bliss point with either certainty or, under an open rule, with probability equal to $\frac{n-1}{n}$. The "scape-goat" strategy is optimal regardless of the other exogenous parameters.(Details regarding this model, in the form of a much earlier draft of this paper, are available from the author). Such equilibria, I would argue, lack on their face the dynamism and conflict of genuine legislative bargaining. Given that leaving it out leads to this unattractive equilibria for essentially all parameter values, and since its economic validity requires only a single weak assumption, I proceed with a model that includes this concept as described.

[12] A brief note on terminology: I use the terms cost and benefit to refer to the corresponding values *prior* to the affect of inefficiency. For example, unless explicitly stated otherwise, when characterizing the "net benefit" of a public good, I intend to reference its value, $\bar{B} = b - 1$. Not the realized benefit net of the inefficiency under the distribution providing the good.

[13] In equilibrium, (and normally in reality) legislators always support their own propoosals

[14] I suspect but have not proven that the results are substantially identical for the case of a finitely-lived legislature.

unity. This requirement is $A's$ budget constraint and restricts valid successful proposals to those in which $c_A + kc_K + (n-k-1)c_L = 1$, where as explained more fully below, k is the size of A's winning coalition, c_K is the cost paid by each member thereof, and c_L is the cost imposed on each member of the losing coalition.[15]

The second critical constraint is that a proposal will only pass if it receives the support of $k \geq \mathbb{D}$ legislators, which establishes an incentive compatibility constraint with respect to the utility realized by $A's$ intended coalition members under her proposal. The form this constraint takes is defined precisely shortly.For now, note that the former restriction always binds, but that the latter is potentially "soft" in two ways. First,A can make a proposal that yields her supporter's utility in excess of what is needed to induce their suppport, or second, she can organize a coalition larger than the minimally decisive one or both.[16].

Strategies and Solution Concept

The equilibrium concept and other assumptions regarding legislators' strategies and behavior are standard: Conditional on the economic and political context, each legislator seeks to maximize her utility given the strategies of her colleagues. Each legislator's strategy consists of the criteria used to determine that legislator's vote in response to any valid proposal and the proposal that that legislator will make if selected as the agenda-setter.

All relevant information is public so that for any proposal X, the typical legislator, i, knows her own cost under that proposal c_i, and the inefficiency of the proposal overall, $S^2(X)$. The benefit ($\bar{B}$) of the public good net of its unit cost is also known to all legislators.

Equilibrium Concept To solve and analyze the model I consider stationary symmetric subgame perfect equilibria which is a standard approach in the literature (Ansolabehere et al. 2003). Informally, this concept has three primary requirements. First, in equilibrium, legislators employ identical strategies in identical subgames (with subgames corre-

[15]I also assume that the public good can't be provided in partial form.

[16]Also each c_i is constrained to be non-negative and less than or equal to total unit cost of providing the good

sponding to legislative sessions in the baseline model). Second, in equilibrium, given the strategies of the other legislators, no legislator has available a profitable deviation from her own strategy, through which she can achieve higher expected utility. Third symmetry requires that legislator i, does not arbitrarily favor or discriminate against any legislator j, when such treatment has no bearing on i's expected utility.[17] Additionally, as is also standard in the literature, I require that legislators choose weakly dominant strategies when multiple strategies yield the same expected utility in equilibrium.[18] An equilibrium then is simply defined as a set of strategies, one for each legislator, where this criteria is fulfilled for each of the strategies in this set.

Conditional on the values of the exogenous parameters, if an equilibrium exists it is defined by a voting criteria, discussed in the next section without the need for additional notation and a cost proposal which I denote as X^*, that meets the equilibrium requirements. Other equilibrium outcomes are therefore functions of X^*, so, for example, the cost paid by legislator i in equilibrium is $c_i(X^*)$.

Preliminary Results

Voting Decision and Continuation Value

The first step towards identifying equilibrium strategies and conditions is to define the optimal voting criteria by which legislators decide which proposals to support or oppose. This also defines the incentive compatibility constraint A must meet in forming her coalition.

Rational legislators vote for a proposal if and only if it offers utility at least as great as their time discounted *continuation value*, defined as the expected utility of rejecting the pending proposal and proceeding to an additional legislative session multiplied by

[17] This is sometimes defined as the requirement that legislators of the same "type" be treated the same.

[18] This assumption precludes unrealistic equilibria in which all or many legislators vote randomly or arbitrarily in turn leading to a situation in which no legislator has an incentive to vote strategically because doing so has no effect on the ultimate passage or rejection of a proposal. State somewhat more rigorously, this requirement says that for legislator i, two strategy profiles σ_1, σ_2, yield the same expected utility given the strategies of other players, but σ_1 yields higher utility for at least one alternative strategy of and σ_2 does not yield higher utility for any strategy profile, then i'schoses σ_i even if it is not a profitable deviation given the prevailing equilibrium.

the associated discount factor of the delay. The equilibrium conditions, as noted in other similar models, therefore imply that each legislators' continuation value is equal to their ex ante expected utility (Baron (1991b), Banks and Duggan (2006)).

Letting V_i be legislator $i's$ continuation value we therefore have that i votes in favor of X if and only if $u_i(X) \geq \delta V_i$. As others have established, if legislators have symmetric utility functions and recongition probabilities are random, then in equilibrium a single continuation value, $\bar{V}$ prevails for all legislators (Baron (1991a), Baron and Ferejohn (1989)). As a result of the single continuation value, and other standard features of the model, any equilibrium proposal can be fully characterized by, at most, three distinct cost functions:[19]

- The cost imposed on members of the winning coalition, c_K, which is paid by the $k \geq \mathbb{D}$ legislators who vote for the bill.
- The cost imposed on members who vote against the proposal, c_L which is paid by an opposition consisting of $n - k - 1$ legislators
- The cost paid by the agenda-setter herself, c_A.

These classes, the legislators' utility functions, and the budget constraint requiring that valid proposals fully pay the unit cost of providing the good make it simple to derive the equilibrium value of $\bar{V}$ as a function of the optimal cost distribution X^*.

$$\bar{V} = \frac{\bar{B}}{n} - \alpha S^2(X^*)$$

Therefore legislator i will vote for proposal X, if and only if $u_i(X) \geq \delta \bar{V}$, or, from the definitions,

[19]Since members have symmetric utility functions functions and the same probability of being chosen as the agenda-setter there is no reason to chose or exclude any particular legislator in a coaltion except for the return the demand for supporting the proposal. Thus, the standard assumption that legislators chose their coalition by randomizing across the set of coalitions that maximize expected utility is employed here.

$$\frac{B}{n} - c_i - \alpha S^2(X) > \frac{\delta\left(\bar{B} - \alpha n S^2(X)\right)}{n}. \tag{2}$$

Equation 2, legislators' utility functions, and the posited constraints are sufficient to derive the following preliminary result:

Result 1

For any X^* (any equilibrium proposal), if $u_k(X^*) > \delta\bar{V}$ or $k(X^*) > \mathbb{D}$ then $k(X^*) = n-1$ and $c_K(X^*) = c_L(X^*)$.

Proof: All Proofs in appendix

This result states that, in equilibrium, if winning coalitions are larger than minimal, or winning coalition members realize utility greater than their time-discounted continuation value (or both), then all legislators other than the agenda-setter must pay the same cost and the distribution will pass unanimously.

To understand the intuition for the result, consider the determination of equilibrium coalition size. In any non-unanimous coalition, winning-coalition members pay less than the opposition. However, for any fixed $c_A^{'}$, $A's$ utility is maximized at $ck = cl = \frac{1-c_A^{'}}{n-1}$ and monotonically increases as the cost shares of the winning and losing coalitions converge. Thus, for any winning coalition with $\mathbb{D} < k < n-1$, and opposition cost share $c_L^{'}$, A could increase her utility by randomly chosing legislator $\hat{k}$ out of the $k(X^*)$ coalition members, setting $c_K < c_{\hat{K}} \leq c_L^{'}$ and reducing the total cost assessed on the opposition bloc by $c_{\hat{k}} - c_K$. Whether or not legislator $\hat{k}$ continues to support it, X still passes with the support of the $k-1 \geq \mathbb{D}$ legislators and the new cost assessments made on $\hat{K}$ and the opposition bloc decrease S^2. Thus for any fixed c_A such a strategy alteration will be a profitable deviation whenever $c_L > 1/n$, or whenever there is a non-unanimous equilibrium coalition.

The intuition for the second leg of the result is very similar. When $k < n-1$, the opposition is paying a disproportionate share of the cost. Therefore, A can decrease $S^2(X)$ without increasing c_A, and without jeapordizing the passage of X by reducing the

difference between the opposition and coalition cost shares.

This result leads to the following observation that can be used to simplify the task of identifying optimal strategies when more than one candidate is feasible.

Observation *For any fixed, self-imposed contribution, A always prefers a unanimous outcome to a minimal one. That is for* $c_A = c_A^{'}$, $u_A \mid (c_A = c_A^{'}, k(X^*) = n-1) > u_A \mid (c_A = c_A^{'}, k(X^*) = \mathbb{D})$

This observation is intuitive and a function of the assumption that legislators care about efficiency and efficiency is increasing in the equality of legislators' cost shares.

Equilibrium Derivation

Possible Equilibrium Classes

Result 1 tells us that that any equilibrium proposal will have a winning coalition that is either minimal or unanimous ($X = X^* \rightarrow k(X) \in \{\mathbb{D}, n-1\}$). For the case of a minimally decisize coalition, it is analytically helpful to first determine whether A contributes positively towards the cost of the good at all, and then, if so, what the optimal contribution amount is.[20] This approach and Result 1 give four possible "classes" of equilibrium proposals, $X^{i,j}$, $i \in \{U, S\}$, $j \in \{0, 1\}$ where $X^{U,j}$ and $X^{S,j}$ denote unanimous and contested proposals, and $X^{i,0}$ and $X^{i,1}$ denote proposals in which $c_A = 0$ and $c_A > 0$ respectively. In words, Result 1 ensures that all equilibrium can be put into one of four categories, depending on whether the agenda-setter contributes or not and whether the optimal coalition is minimal or unanimous.

Intuitive Solution Explanation

In this subsection, I provide an intuitive explanation for the equilibrium derivation. Readers interested primarily in the substantive implications can skip ahead without risk of missing substantively critical insights.

[20] That is, it is helpful to distinguish between interior and corner solutions.

The notation established above permits a fairly intuitive explanation for the derivation of equilibrium results.

First, consider unanimous equilibrium proposals such that $k(X^*) = n-1$, and $X^* = X^{U,j}$. For $j = 0$, the cost distribution, $X^* = \{c_A = 0, c_K = c_L = \frac{1}{n-1}\}$, is immediate given the budget constraint. For $j = 1$, unanimity still implies $c_K = c_L$, and with the the budget constraint we can write $A's$ unconstrained maximization problem as a function of c_A, and solve using standard techniques (precise values given below).

For $X^* = X^{S,1}$, Result 1 requires that $k(X^*) = \mathbb{D}$ and $u_k(X^*) = \delta\bar{V}$. The math then is more more involved but the approach is broadly similar: The budget and incentive constraints allow for A's utility to be expressed as an unconstrained function of c_A, and this function can then be maximized yielding first-order conditions that determine the optimal cost distribution given a minimal winning coalition and $c_A > 0$. Finally, for $X^* = X^{S,0}$, the same approach using the budget constraint and setting $c_A = 0$ gives the value of c_K that must obtain when $u_K = \delta\bar{V}$.

Given these cost distributions, we can then solve for the parameter ranges that would support each distribution in equilibrium. When more than one of the proposal classes is feasible, comparing $A's$ utility under the competing propsals is straightforward and the requirement that A maximize her own utility in equilibrium yields a unique solution.

The result of converting this approach into algebraic terms yields the full specification of equilibrium conditions given below as Proposition 1. The specifications define parameter regions in terms of $\bar{B}$. This is most parsimoniously accomplished by splitting legislatures into "α-types", a high-α type, $\bar{\alpha}$, with $\alpha > \frac{n-1}{2}$, and a low-α type, $\underline{\alpha}$, otherwise.

Proposition 1: Equilibrium Specification

For $\alpha > 0$, $\delta \in [0, 1)$, $\mathbb{D} \in [\frac{n-1}{2}, n-1]$ and $\bar{B} > 0$, a proposal is an equilibrium proposal if and only if it proposes a cost distribution, X^* that adheres to the following conditions:

- When $\alpha > \frac{n-1}{2}$, A's contribution is always positive, ($X^* \in \{X^{S,1}, X^{U,1}\}$, and $c_A > 0$).
  - For $\bar{B} < \widehat{B}(\overline{\alpha})$, coalition size is minimal, and $c_A > 0$, $X^* = X^{S,1}$:
  - For $\bar{B} > \widehat{B}(\overline{\alpha})$, equilibrium coalitions are unanimous and $c_A > 0$. The optimal cost proposal is given by $X^* = X^{U,1}$

- $\widehat{B}(\overline{\alpha}) = \frac{1}{2\alpha}\left(\frac{1}{1-\delta} + \frac{n-1}{2}\right)$

- For $\alpha < \frac{n-1}{2}$, $c_A = 0$ in unanimous coalitions, but $c_A > 0$ if the good's value, $\bar{B}$, is low enough and optimal coalition size is minimal.
  - For $\bar{B} > \widehat{B}(\underline{\alpha})$, winning coalitions are unanimous and $X^* = X^{U,0}$
  - For $\bar{B} < \widehat{B}(\underline{\alpha})$, coalitions sizes are minimal. The optimal cost function then is determined as follows:
    - If $\bar{B} < B^+(\underline{\alpha})$, $c_A > 0$ and $X^* = X^{S,1}$.
    - If $B^+(\underline{\alpha}) < \bar{B} < \widehat{B}(\underline{\alpha})$, $X^* = X^{S,0}$, and $c_A = 0$.
  - $\widehat{B}(\underline{\alpha}) = \frac{\alpha(1-\delta)+1}{(1-\delta)(n-1)}$
  - $B^+(\underline{\alpha}) = \frac{\phi(\alpha(1-\delta)+1)}{(1-\delta)k(\delta(n-1)-n)^2} + \frac{n(n-k-1)(\delta(n-1)-n-1)}{4\alpha(1-\delta)k(\delta(n-1)-n)^2}$
  - $\phi = ((1-\delta)k+1)^2 + \left((1-\delta)^2 k + 1\right)(n-k-1)$
  - $\Omega = k\left(4\alpha(1-\delta)^2\bar{B} - 1\right) + n$

- The optimal cost distributions for each equilibrium proposal class can then be written as

$$X^{S,1} = \begin{pmatrix} c_A(X^{S,1}) \\ c_K(X^{S,1}) \\ c_L(X^{S,1}) \end{pmatrix} = \begin{pmatrix} \frac{2\alpha(1-\delta)+1}{2n\alpha(1-\delta)} - \frac{(n(1-\delta)+\delta)\sqrt{\Omega}}{2n\alpha(1-\delta)\sqrt{\phi}} \\ \frac{\sqrt{\Omega}((1-\delta)k-k+n)}{2kn\alpha(1-\delta)\sqrt{\phi}} + \frac{2k\alpha(1-\delta)-(n-k)}{2kn\alpha(1-\delta)} \\ \frac{1-c_A(X^{S,1})-kc_K(X^{S,1})}{n-k-1}\}) \end{pmatrix}$$

$$X^{S,0} = \begin{pmatrix} c_A(X^{S,0}) \\ c_K(X^{S,0}) \\ c_L(X^{S,0}) \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\sqrt{n-k-1}\sqrt{(n-1)\Omega - k(-2\alpha\delta+2\alpha+1)^2}}{2\alpha(1-\delta)k\sqrt{n}(n-1)} + \frac{k-(n-1)}{2\alpha(1-\delta)k(n-1)} + \frac{1}{n-1} \\ \frac{1-c_A(X^{S,0})-kc_K(X^{S,0})}{n-k-1} \end{pmatrix}$$

$$(X^{U,1} = X^{U,0} = \begin{pmatrix} c_A(X^U) \\ c_K(X^U) \\ c_L(X^U) \end{pmatrix} = \begin{pmatrix} \sup\left(\frac{2\alpha-(n-1)}{2\alpha n}, 0\right) \\ \inf\left(\frac{2\alpha+1}{2\alpha n}, \frac{1}{n-1}\right) \\ c_K(X^U) \end{pmatrix}^{21}$$

Equilibrium Features

These equilibrium conditions lead to several corollaries that are worth stating explicitly.

Corollary 1.1 *When equilibrium coalitions are not unanimous, if c_A is positive, then $c_A(X^*)$ is strictly decreasing in the value of the public good, $\bar{B}$. Similarly, equilibrium inefficiency, $S^2(X^*)$, is strictly increasing in the value of the public good for non-unanimous, equilibrium proposals where $c_A > 0$.*

Corollary 1.2 *When $c_A = 0$, if coalitions are not unanimous then c_L is decreasing and c_K is increasing in $\bar{B}$*

Corollary 1.3 *The viability of a unanimous (minimal) coalition is independent of the exogenously assumed minimal coalition size. That is, $A's$ decision to set $k = (n-1)$ or $k = \mathbb{D}$ does not change as a function of $\mathbb{D}$.*

One of the implications of Result 1 is that when equilibrium proposals receive unanimous support, the agenda-setter is essentially only bargaining between her own competing priorities to minimize personal cost and maximize efficiency. The realized outcome is what A would implement as a a dictator. Thus conditions that lead to these equilibria are on not of much use for understanding genuine legislative conflict. For the remainder of the paper therefore, I mostly focus on on non-unanimous equilibria.[22]

Features of Equilibrium Results

The equilibrium results in Proposition 1 have a number of features that are not immediately obvious and which depart from the standard Divide-the-Dollar model.

- *Imposing Costs, Imposing Negotiators:* As in standard models of benefit bargaining, $A's$ exclusive proposal right lets her leverage her colleagues' impatience for personal advantage. Also though, the model uncovers a second source of bargaining power: As in the standard model, $A's$ optimal strategy her can be seen as the minimal deviation from her most preferred outcome required to obtain the support of a decisive majority. Unlike the standard model though, A can, and often will, meet

[22] Additionally, public goods which are valuable enough to all legislators to support a unanimous equilibrium are by definition easily enacted. Goods that are so valuable and which can be brought to the legislative floor are "low-hanging fruit" and are not material to the hypothesized inefficiency that marks legislatures public good provision.

her coalition's demands for lower costs primarily by increasing the costs borne by the opposition, not the costs she imposes on herself.

- *Shifting priorities:* As Figure 1 shows, A's first motivation is, under most reasonable assumptions, to reduce her own cost to the minimal possible level. Once this is accomplished, any remaining bargaining leverage is applied to increasing overall efficiency. Combined with Proposition 1 and the corollaries this implies that $\bar{B}$ and α always work in opposite directions with respect to A's bargaining power.

- *Net Utility Reductions* In the standard bargaining model, the status quo outcome realized under no enactment plays a major role in determining legislators' strategies. Here though, A's ability to impose costs on all legislators moots the importance of the no-action status quo outcome. That is, because A can impose costs even on legislators who oppose her proposal, net utility reductions are not just possible, they are a key feature of almost all non-unanimous equilibria. What's more, the realization of utility-diminshing outcomes is not limited to the opposition coalition. A's utility maximizing strategy yields no ground towards demands from prospective coalition members that they do at least as well as they would under no legislative action. The key benchmark A needs to beat in her offer to coalition members, therefore, isn't defined by gridlock, but by the utility realized by the opposition bloc, a bloc where but for the luck of the draw, go prospective coalition members if they reject $A's$ offer.

- *Non-monotonic Influence of Discount Factor* Utility reducing equilibrium outcomes and their implications about expected utility in the future implies a non-monotonic relationship between the discount factor (δ), and the bargaining power brought to bear by the agenda-setter. This relationship, introduced in the previous section, is the logical consequence of preferring to receive \$1,000 from your employer today versus next year, but preferring to pay \$1,000 to the IRS next year rather than today.

Figure 1 about here

Equilibrium Existence

Proposition 1 defines equilibrium strategies but does not make explicit whether such equilibria always exist. This question is addressed in Proposition 2.

Proposition 2 *For any $\bar{B} > 0$, a unique stationary equilibrium exists in which the legislature enacts a proposal to provide the public good with no delay.*

This result says that for any $\bar{B} > 0$, there exists a proposal X^* such that $k(X^*) \geq \mathbb{D}$, $u_K(X^*) \geq \bar{V}$, and $u_A(X^*) > \sup\{0, \bar{V}\}$. In words, as long as $\bar{B} \geq 0$, A is able to enact a proposal that is approved by at least a decisive majority and which leaves her better off than under the status quo of no legislative action. Thus, in equilibrium legislation is always enacted to provide for the public good.

A formal proof of the result above is saved for the appendix and, with the exception of the case where $c_A = 0$ and optimal coalition size is minimal, is easily derived from A's indirect utility function as implied by the cost distributions in Proposition 1. The basic intuition is that since $\bar{B} \geq 0$ by assumption, A is always able to achieve *at least* utility equal to zero, since this is what she would receive under an even distribution of costs with $\bar{B}$ at its assumed minimum feasible value and since $A's$ utility is always increasing in $\bar{B}$). Thus, any strategy that diverges from this even distrbution, can only be optimal in equilibrium if the expected utility of doing so is at least as high as under the even distribution.

4 Extensions

In this section, I extend the baseline model model to consider how two different institutional settings might affect cost negotiations. First, I posit a two-step agenda-setting process which would allow legislators to screen out proposals associated with negative expected utility. This yields both surprising theoretical perspectives and tractable empirical predictions regarding the interaction between legislative institutions and the provision of public goods. Second, I analyze an open-rule version of the model where amendments are

possible and consider when and whether such a rule reduces A's bargaining power, and the implied consequences for the rank-and-file legislators. Here I find that while an open rule generally reduces A's bargaining power, it does not eliminate equilibria in which expected net utility are negative, and in some circumstances leads to outcomes that are worse for all legislators than under a closed rule.

Extension 1: Setting the Agenda By Restricting It

The previous section's results indicate that with a fixed benefit and distributed cost, negotiations can frequently lead to outcomes which reduce expected utility relative to no legislative action. The first extension is premised on the assumption that, legislators might seek to restrict the agenda to exclude such proposals.

The Viability and Persistence of Utility Diminishing Equilibria

Equilibria in which legislators support proposals that they prefer to never have seen the light of day, are unusual in the extant literature on legislative bargaining (though, see Banks and Duggan (2006)). Therefore, as an initial matter, it may be helpful to explain in more detail, how and why cost-based bargaining can lead to such outcomes in equilibrium, and why they are not present in comparable studies of distributed benefits.[23]

The main reason that utility-diminishing equilibrium may strike readers as unusual begins with a central difference between cost and benefit negotiations: $A's$ optimal strategy involves exploiting those legislators whose vote she does not need. As a result, given a designated agenda-setter and a non-cooperative legislature, members make their voting decisions not based upon the status quo of no legislative action but against the possibility of being excluded in the losing bloc should the pending proposal be rejected. Thus the outcomes that A needs to "beat" in her offer to potential coalition members isn't the status quo under no legislation but the outcome realized by members of the opposition bloc.[24]

[23] Similar discussions in similar contexts can be found in Baron (1991a) and Primo (2006)

[24] In terms of the algebra and model mechanics this dynamic is captured in the fact that $u_K = \delta\bar{V}$ and $\bar{V}$ in turn is of course, heavily dependent on the utility realized by the losing coalition.

Game theoretic logic not withstanding, equilibrium proposals that are both inescapable and almost universally unwanted, naturally raise the possibility of imposing ex ante restrictions on the agenda-setter. In particular, a process to exclude expected-utility diminishing proposals offers a possible ex ante Pareto improvement - leaving all legislators better off and disadvantaging none since we assume all legislators have the same chance of becoming the agenda-setter. This motivates the consideration of a two-stage agenda-setting process, in which legislators decide whether to proceed with the designation of an agenda-setter after the value of the good is revealed.

Identifying Negative Utility Equilibria

Before considering the details of this process, there is the more straightforward, prior task of defining when negative utility equilibria arise.

Proposition 3 below takes up this task, establishing the precise conditions that must hold for a prospective public good to yield positive, ex ante expected utility. To clarify the exposition of this question, I focus on equilibrium conditions in which $c_A > 0$.[25]

Proposition 3 - Positive and Negative Expected Utility *When all legislators contribute towards the cost of a public good, and coalitions are not unanimous, ex ante expected utility is positive if and only if legislators are willing to pay at least their "fair share" of the public good's cost equal to* $\frac{1}{n}$. *When legislators refuse support for proposals in which they are assessed this amount, expected utility and realized utility for legislators, save the agenda-setter, is negative.*

The simplicity of this result is somewhat surprising. It is not dependent on the value of the public good, the weight legislators place on overall social welfare, the discount factor, or minimally decisive coalition size. Perhaps it can be best understood as the legislative incarnation of "No Free Lunches". If all rank-and-file legislators demand a "deal", then no legislator except for the agenda-setter is going to be able to profit from

[25]This issue of negative expected utility is moot for unanimous coalitions and though there are cases of negative expected utility when $c_A = 0$, the limited additional insight which these cases facilitate is not worth the additional algebraic complexity. Note that $c_A = 0$ implies relatively high values of $\bar{B}$, which, in turn, usually implies that the public good is valuable enough to support equilibria which have positive ex ante expected.

the exchange, even though some legislators get the deal they demand. More cheerily perhaps, the condition in Proposition 3 is not only necessary but also sufficient, so that if legislators *are* willing to pay their share of the public good, then expected utility will be positive and only the minority of legislators left out of winning coalition can potentially realize negative net utility due to legislative action.[26]

Given this result, and legislators' indirect utility and cost functions the following corollary is immediate.

Corollary 3.1 If all legislators contribute towards the cost of the public good in equilibrium, then expected utility, (and therefore the utility realized by the winning coaition) will be positive if and only if the net benefit of the public good ($\bar{B}$) exceeds $\hat{B}$, a critical threshold that is a function of the other exogenous parameters. Therefore, conditional on $0 < c_A$,

$$\bar{B} > \hat{B} \implies \bar{V} = \mathbb{E}[u_i] > 0 \ \ \forall i \tag{3}$$
$$\hat{B} = \frac{n(n-k)(n-k-1)}{4\alpha(\delta n - k)^2}$$

Two-Stage Agenda-Setting

Proposition 3 and the accompanying corollary suggest that, if obstacles to collective action are manageable, and legislators are able to set their own procedural rules, then they will likely establish institutions that allow them to exclude from consideration projects that are worth less than $\hat{B}$.

One way that such a restriction could be realized is to separate the decision to proceed with consideration of a project from the right to make a proposal regarding the distribution of associated costs. Such a partitioning occurs in many real-world legislatures. Most prominently, Committees are often empowered by their chamber's rules to prevent legislation from advancing to consideration by the full chamber (e.g. The Rules Committee

[26]Note also that, if A maximized social welfare, then $\hat{B} = 0$ for any project where $\bar{B} > 0$. However, since $A's$ contribution in a non-unanimous equilibrium is always far less than the cost required under social optimality, this threshold is strictly, and often substantially, greater than zero in equilibrium.

in the House). Extending the baseline model to accommodate a prior agenda stage is, given the results presented above, quite simple.

Two-Stage Model Mechanics Assume a legislature with the following two-stage agenda-setting process. In the first stage a net benefit, $\bar{B}$ is revealed to all legislators but no agenda-setter is designated and no distributive proposal is made. Instead, a randomly chosen subset of legislators is designated as the Committee and tasked with deciding whether to chose an agenda-setter (randomly) and begin the process defined in the baseline model. Alternatively, the Committee can reject the project, leaving all legislators no better or worse than before.[27]

Since the process of chosing an agenda-setter, and the incentives that will govern her actions are anticipated by all legislators, the projects that will be approved for consideration by the full chamber can be inferred directly from Corollary 3.1. Specifically, only those projects which satisfy the condition in 3 are considered.

Therefore, the committee process excludes from consideration a range of public goods that a social welfare maximizing agenda-setter would enact but which a majoritarian legislature can't bring to fruition efficiently. This interval consists of projects with values of $\bar{B}$ such that $0 \leq \bar{B} < \hat{B}$, with $\hat{B}$ defined as in the corollary. Thus, in this two-stage process, $\hat{B}$ quantifies the inefficiency of a majoritarian, partially parochial legislature. Also, in a slightly unorthodox sense, $\hat{B}$ represents the size of a gridlock interval, in that it defines a range of outcomes that a majority of legislators would prefer to the status quo of no action, but which are preempted by the chambers procedural institutions.

We can then analyze the comparative statics presented in Equation 3 to probe the determinants of efficient public good provision under the model.

First, $\hat{B}$ is decreasing in k reflecting that positive expected utility is more readily achieved when larger decisive coalitions are required for passage. That super-majority requirements induce the agenda-setter to act more like a social welfare maximizer is per-

[27]This description is somewhat informal, but a more rigorous outline seems unnecessary. Prior to the designation of an agenda-setter, legislators are essentially identical - in equilibrium they will all have the same preferences over how to dispose of projects so the choice of the Committee and its decision rule are trivial. Of course, its possible to enrich the model by relaxing some of the symmetry assumptions that leads to the total convergence of preferences at this stage.

haps not surprising. However, in the context of possible utility diminishing outcomes, the result suggests that counterintuitively, supermajority requirements may actually mitigate against the under-provision of public goods by lowering the threshold value of such goods that can be considered profitably by the legislature.

Second, $\hat{B}$ is also decreasing in α and therefore in the weight that legislators place on overall efficiency. One might expect that the less legislators care about efficiency, the easier it would be to realize positive utility in equilibrium, since the penalty to inefficiency would be reduced. However, the benefits to rank and file members of caring less about overall efficiency are not enough to make up for the greater leverage A enjoys in such settings. As a result, A achieves higher utility when α is lower but public goods have to be more valuable to yield positive expected utility overall.

Finally, Equation 3 indicates that less patient legislatures more readily achieve positive expected utility and will therefore exclude fewer projects from consideration given the process outlined in this section. This finding runs against those found in most political-economic models of public good provision, where politicians who more heavily weight future utility more willingly invest in public goods. Relatedly, and more directly comparable, in existing bargaining models, higher values of δ reduce the agenda-setter's leverage and are therefore associated with more uniform outcomes. [28]

The intuition for these standard results failing to extend to a setting with fixed benefits and distrbuted costs is that much of A's negotiating leverage comes from legislators' desire to avoid the negative outcome reserved for the opposition bloc. When expected utility is negative, the threat of future negative outcomes is less salient when legislators discount future outcomes at higher rates. Therefore lower discount rates imply that legislators are less motivated to avoid the utility-diminishment that accompanies exclusion from the winning coalition. Thus for higher values of δ, this threat is more salient and therefore A is in a stronger position to compel his coalition to accept higher costs.

A good sense of how these parameters interact to define the gridlock interval captured

[28] In these models, the agenda-setter derives most if not all of his leverage from the fact that with discount factor's less than 1 legislators are willing to accept less than their "fair share" in a given session in order to avoid having to wait to obtain an equivalent good in the future.

envisioned under Proposition 3 can be gleamed from Figure 2, which plots values of $\hat{B}$ as a function of δ for various configurations of α and $\mathbb{D}$. The shaded area indicates the gridlock interval and shows that this interval shrinks with the size of the minimally decisive coalition and as the values of δ and α increase and decrease respectively.

Figure 2 about here

Empirical Predictions These theoretical results can also be converted into empirical predictions. Given how the extension defines the two agenda-setting stages, these predictions can be derived for Congress, for example, if we - oxymoronically perhaps - define the "pre-floor" stage as the consideration of a bill on the floor of the House and Senate. Thus, conference negotiations are the bargaining stage captured in the baseline model, and the vote on the House and Senate floors to send a bill to conference represent the first stage during which the decision is made to endow a conference committee with the agenda-setter's prerogative to make a take it or leave it offer (Shepsle and Weingast 1987). Thus, in its sharpest form, this extension predicts that, given two legislatures with a super and simple majority requirement respectively, *ceteris peribus* there should not be any public good that passes the simple-majority but not the super-majority setting, while the converse should be observed at least occassionally.[29]

Secondly, legislators that more heavily discount future outcomes will support more public goods proposals. This is due to the inverted influence of the discount rate on legislators' incentives when expected utility is negative - a dynamic predicted by the model discussed already.

Unfortunately, these predictions are cross-cutting with respect to Congress and most American bicameral settings. The Senate has both the super-majority coalition requirement and the longer time horizon. Since the former facilitates action, and the latter gridlock the model's empirical predictions are ambiguous.

[29] Additionally, the extension and perspective offered in this section suggest that when legislatures place greater weight on overall social welfare they should also be more likely to approve public goods. There likely are plausibly exogenous changes in this weight - for example, the start of a war is likely to lead legislators to place a higher weight on the overall efficiency of public good provisio - however there are many models that would make similar predictions.

Still, this analysis suggests possible future empirical work. Super-majority requirements evolve over time. To the extent that this evolution occurs independently of the other parameters, the model makes the sharp predictions described above. Less common, but not unheard of, are cases where changes to legislators' time horizons can be identified. It might be easiest to observe such a scenario in a country like Australia, which has a bicameral legislature but one where members' terms of office vary within and between chambers and where one or both chambers can be dissolved. [30]

Extension 2: Open Amendment Process

The extension above highlights that when proposal power is separated from the decision to proceed, net utility diminishing legislation will be blocked. However, this institution does not alter the enacted distribution for any bill. With a two-stage agenda-process, either projects pass as if there was no first stage or don't pass at all. In this section, I consider a second extension of the baseline model, which operates more directly on $A's$ coalition formation and cost distribution incentives.

Specifically I posit a legislative process with an open rule so that amendments to A's initial proposal are allowed. This refinement, when considered in other legislative bargaining applications, is typically found to at least weakly benefit all legislators save the agenda-setter, and precipitate more equal distributions. With a fixed benefit and distributed costs however, these simple relationships no longer generalize. An open-rule, normally and intuitively associated associated with minority protections, will sometimes actually lead to lower expected utility for all members, and disproportionately bad outcomes for legislators in the opposition.

Amendment Process Assumptions To incorporate possible amendments into the model, we need to assume some additional structure on the legislative process. Following the standard in the literature, suppose that after A makes her proposal, another legislator is chosen at random and given the option of either moving $A's$ proposal to a vote or

[30] On the other hand, in Australia at least, there is no clean analogy to the conference committee process in the U.S. suggesting that empirical application would require further refinement.

offering a substitute. If the latter, the new proposer, A', makes a new proposal, which, if approved (as it always will be in equilibrium), restarts the process , with A''s proposal as the pending matter. Under this set up, discounting occurs whenever a new proposal replaces an old one as the pending matter (as when a substitute amendment defeats the original proposal.)

The first result of this procedural change is that equilibrium coalitions can vary between unanimity and minimal. This is intuitive. Under a closed rule, once a coalition is large enough to pass a proposal, adding more members offers no benefit to the agenda-setter. However, with an open rule, each additional supporter reduces the chances of an opposed legislator being recognized to amend and defeat $A's$ optimal proposal. On the other hand, the logic that determines the optimal cost to impose on the winning coalition, conditional on its size, is unchanged. That is, for non-unanimous outcomes, legislators in the winning coalition continue to receive precisely their reservation utility level.

One consequence of the additional considerations with respect to coalition size is that the model becomes considerably more complex. Thus to keep the focus on substantive implications, I illustrate the results of implementing an open rule in a series of observations pertaining to a hypothetical legislature under several representative assumptions regarding political and economic conditions.[31]

Open Rule Observation 1 *Equilibrium coalition size is almost always greater than minimal. In non-unanimous coalitions, winning coalition utility remains equal to legislators' time discounted continuation value.*

As previously alluded to, this first observation indicates that coalition sizes are almost always considerably larger than minimal. Only when the value of the public good just breaks even *and*, the legislature steeply discounts future utility but places a relatively heavy value on overall efficiency will coalition size be minimal.

At first glance, this result is a familiar one - the cannonical legislative bargaining models also show that coalition size will only be greater than minimal if amendments are allowed. However, the magnitude of the affects here are qualititatively larger. For

[31]In the appendix I illustrate that these reslts hold more generally

example, in Baron and Ferejohn (1989), an open-amendment rule is insufficient to induce a greater than minimal winning coalition for a legislature with 51 members at any discount rate. Here however, for a 49 member legislature, and a public good with benefits equal to costs ($\bar{B} = 0$), optimal coalition size is only minimal when α is fairly high, and δ is very low. Otherwise, for other parameter ranges, optimal coalition size quickly grows to much larger than minimal.

The reason for this difference is that in the standard, fixed-cost, model, coalition size only increases if A is willing to give up more of his share of the good. With a fixed benefit and legislated costs on the other hand, A can "buy" a larger coalition, not only by paying a higher cost herself, but, and more likely, by imposing higher costs on the opposition bloc.

Open Rule Observation 2 *With an open-rule, the agenda-setter always pays more and realizes lower utility,* ceteris peribus *than under a closed rule.*

The second consequence of implementing an open rule is that A always pays more and realizes lower utility then she would under a closed rule. The intuition for this is similar to that which applies in extant models of bargaining over benefits. The utility "price" to A of paying less is higher under an open rule because paying more could yield a higher probability of A's proposal being enacted. This change to A's incentives is unconditional. As long as the agenda-setter achieves an outcome superior to her colleagues, the possibility of having this outcome go to another legislator leaves her worse off.

In addition to reducing utility, an open rule also induces a change in A's optimal strategy, which leads to a higher cost for A than under a closed rule.[32]

Open-Rule Observation 3 *Efficiency and aggregate expected utility will often be higher with a closed rule but only if all legislators pay towards the cost of the good.*

Finally, looking at the choice of procedural rules from the perspective of the full legisla-

[32]If we consider an arbitrary positive $c_A^{'}$ that A would make in equilibrium under a closed rule, given an open rule setting instead, $c_A^{'} > 0$ implies an interior solution to A's first order condition, which in turn implies that A's response to a change in the bargaining environment will be combination of both increasing c_A and coalition size. And, since k can only be increased in integer increments, if only one of these components is altered it will be c_A's cost which is continuously defined.

ture, intuition might suggest a preference for an open rule given the preceding discussion regarding its unconditional affect on A's bargaining power. Indeed, in many, perhaps most circumstances, this is what the model predicts. However, the implementation of an open-rule is not a panacea or even an unambiguously attractive option for two reasons. First, while an open-rule often reduces the incidence of utility-diminishing equilibria it does not eliminate them. Second, an open rule often increases expected utility and efficiency but only if all legislators are compelled to contribute in equilibrium.

To appreciate the intuition for these results consider A's three primary incentives in an open rule context. First and foremost, A acts to keep her own costs low. As long as c_A is positive, any improvement in A's bargaining position will be used first to reduce her own costs. When amendments aren't possible, A's only other incentive is to impose costs that are as uniform as possible, with an open-rule, however, A has the additional goal of maximizing coalition size to reduce the probability of a hostile amendment.

As long as c_A is positive, the impulse to reduce self-imposed costs is partially checked by the desire to maintain a large coalition. As a result A is willing to pay a higher cost which ultimately benefits the legislature as a whole. If c_A is equal to zero however, this dynamic is moot, and instead, the motivation to build a larger than minimal coalition actually works against the best interests of the legislature as a whole because it leads A to disproportionately concentrate costs on the opposition, where, under a closed rule, A would instead seek to equalize cost shares in order to increase efficiency. Therefore in such situations distributions will tend to be more efficient with a closed rule.

These dynamics are represented graphically in Figure 3. Each column shows the agenda-setter's cost, the opposition's bloc's (per capita) cost, ex ante expected utility, and optimal coalition size as a function of δ for a public good with the indicated value.[33] The primary pattern to notice is illustrated by the dashed blue line at the value of δ where coalition size reaches minimum as a function of δ. This shows that this value of δ also closely approximates when expected utility under an open process will equal expected utility under a closed process, and which just allows a value of $c_A = 0$ in an open rule

[33]I assume a value of α equal to 7.5.

equilibrium.

Perhaps most noticeably though, it marks where the opposition's costs under closed and open rules dramatically diverge. This makes the point most clearly: Under a closed rule, with $c_A = 0$, increases in A's bargaining power reduce the costs he imposes on the opposition. With an open rule, the affect is reversed, which lowers efficiency and expected utility relative to a closed rule setting.

Thus while efficiency and a larger coalition size may intuitively go together in certain circumstances, here we see that, from A's perspective, the optimal way to increase coalition size is not to propose more efficient cost distributions, but to give a larger coalition a steeper discount relative to the costs imposed on the opposition bloc.

5 Conclusion

Taking a wider view of the equilibrium results presented in Section 4 and the extensions presented in Section 5, a few general points can be made.

First, and most generally, the distributive politics of distributing benefits and distributing costs differ. The theoretical lessons in the extant literature pertaining to goods assumed to have fixed costs and distributed benefits don't map directly into settings where this assumption is reversed. At the same time, a simple review of recent Congressional history indicates that developing a theoretical framework that would apply to such settings could be of substantial value.

The model and extensions considered above highlight two ways in which prevailing bargaining conditions differ when costs are being negotiated. First, in determining the distribution of costs the agenda-setter's leverage is enhanced because of her ability to impose costs that leave legislators worse off than under no legislative action. As a result, legislators can under certain circumstances, and in the absence of further restrictions on agenda, be compelled to vote for legislation that leaves them worse off than under no the status quo of no legislative action.

Second, and relatedly, when equilibria conditions imply that at least some legislators

realize utility reductions, the standard analysis regarding the connection between legislative patience and the provision of public goods is stood on its head. Specifically, more patient legislatures have a harder time efficiently financing public goods and imply that the agenda-setter has greater bargaining power and therefore that the premium which accrues to the agenda-setter is larger.

Two extensions showed these patterns to be generally persistent and that the model could be applied to make predictions regarding the legislative provision of public goods under different institutional settings. First, multiple agenda-setting stages were shown to generate counter-intuitive predictions regarding the role of super-majority requirements and electoral time-horizons in bicameral legislatures. Second open-rule consideration was shown to reduce the disproportionate returns that otherwise accrue to the agenda-setter, but also under certain circumstances lower expected utility for *all* legislators because by increasing the incentives to heavily concentrate costs on a small minority of legislators.

Several paths should be considered in future research. First, on the theoretical side, several assumptions can be relaxed in order to extend the model to richer settings. For example, in the analysis above, legislators are all assumed to have the same discount rate. This is unrealistic in bodies with rotating electoral calendars such as the U.S. Senate. Given the centrality of the discount rate in generating the most interesting outcomes future research can explore how divergent time horizons within a single legislature can be accommodated in a model of fixed benefits and distributed costs.

Similarly, the model assumes that all legislators place the same weight on aggregate social welfare. If some constituencies derive greater benefit from public goods than others then this assumption will obscure how these differences affect legislators coalition formation incentives.

Also, in addition to relaxing restrictions, future research can also explore how additional institutional features affect the legislators' bargaining incentives. For example, how would the overlapping constituencies of bicameral legislatures affect proposals. In particular, one of the primary results of allowing amendments was to increase the incentives to concentrate costs on a small number of unlucky legislators. Do the overlapping con-

stituencies inherent in bicameral settings partially attenuate these incentives? Another variation on this theme would posit a veto player, like a President or a Governor, who could block but not propose legislation.

Empirically, future research can pursue several angles. In addition to those highlighted in the extensions, there is the possibility of applying the model to understanding the evolution of institutions that intended to apply specifically or especially to the public finance issues. For example, in the 1970's most state legislatures developed new rules for the consideration of tax legislation. Of particular relevance given the first extension, several states adopted supermajority requirements to this effect. Contemplating how these rules affect bargaining over costs can potentially help explain why different states adopted different reforms.

Perhaps the biggest take way though is that unchecked majority rule is particularly ruthless when burdens are the subject of legislative bargaining. Thus, legislators most likely had to be confident that checks would indeed be in place before agreeing to organize legislative authority in the first place. While minority protections are of course a staple of democratic government, whether or not specific institutions developed to deal with the dynamics found here is - to my knowledge - an open question that deserves more thought and study.

6 Figures

Figure 1: Equilibria Illustrations

n=49, $\alpha = 5$

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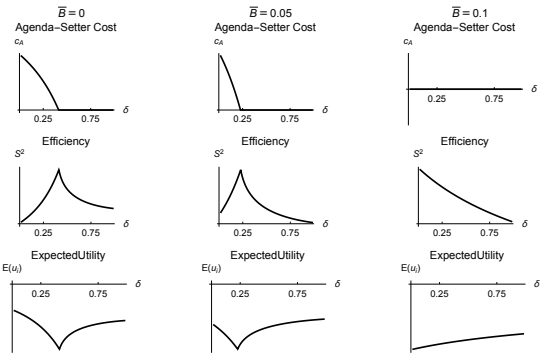
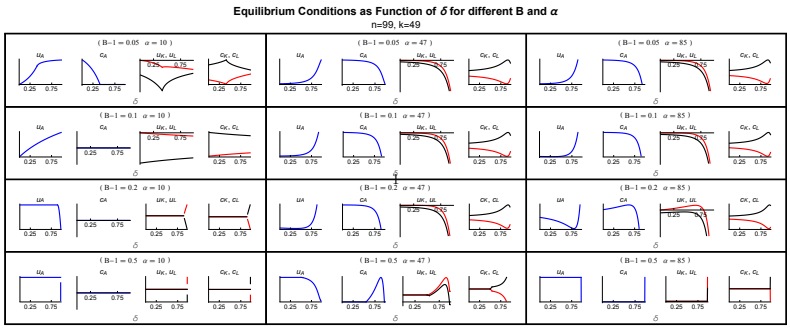
n=49, $\alpha = 5$

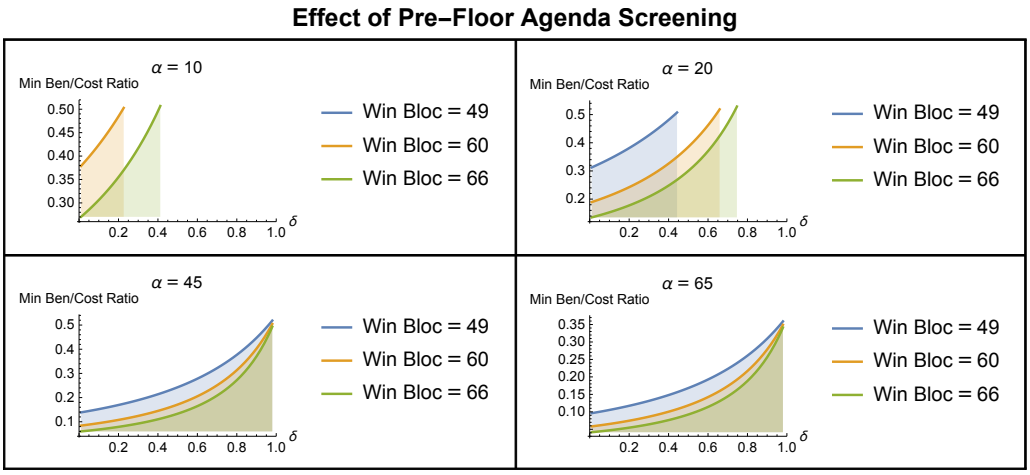




Note: The top panel shows the agenda-setter's cost, equilibrium inefficiency, and ex ante expected utility for three different assumed values of the public good as a function of legislators' discount rate. The bottom panel is shows the same quantities but instead fixes values of the discount rate, and shows quantities as a function of the project's efficient net benefit. Notice the "kink" in both sets of charts when the agenda-setter's cost function reaches zero implying that additional increases in her bargaining power can't be used towards reducingher own contribution towards the cost of the good.

Figure 2: Gridlock Intervals With Two-Stage Agenda

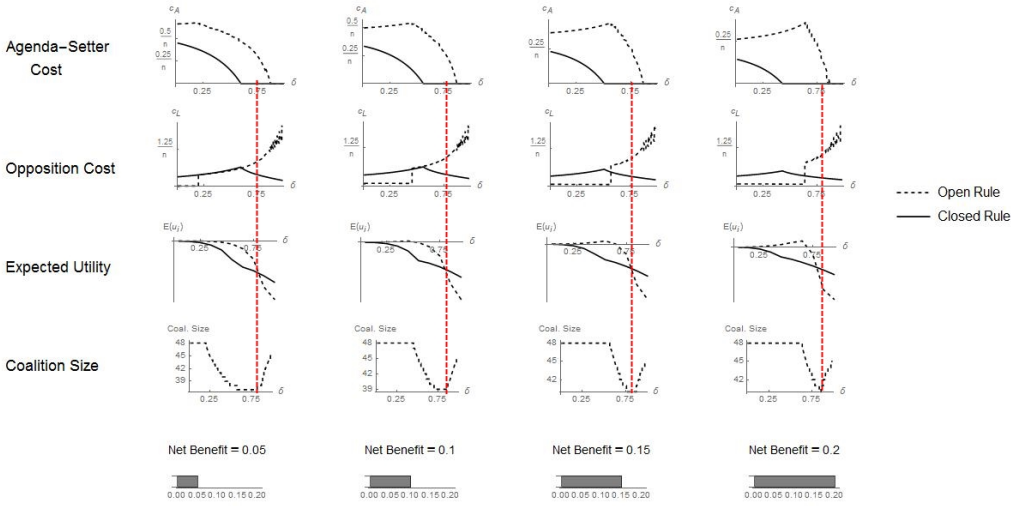
n=99



Note: Shaded area indicates proposals that will not be considered on the floor under the two stage agenda-setting process.

Figure 3: Comparisons of Open and Closed Rule Outcomes

n=49, $\alpha = 7.5$



Note: Dashed red line indicates point at which agenda-setter begins to apply bargaining power primarily to increasing coalition size rather than decreasing her own contribution. Notice how well it approximates the point at which expected utility under open and closed rules is equal.

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