## Matching for Covariate Balance in a Regression Discontinuity Design

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## Overview

## RD requires covariate balance across treatment groups

Matching methods are explicitly designed for this task

Matching on only one covariate, can be used on close U.S. House elections to correct imbalance on many covariates and improve RD estimates
Estimating models across a range of matching and estimation bandwidths gives new perspective on data
Results indicate that imbalance in original data had little effect on incumbency advantage estimates.

## Background and Motivation

- RD design for elections requires that outcomes of close elections are randomly distributed.
- By implication covariates must be balanced across treatment groups.
- Elections in U.S. House post-WWII elections are significantly imbalanced and seem (uniquely perhaps) unsuitable for RD analysis.
The Matching Model
- Assume some incumbents can manipulate results if they are "close enough":
$E\left[Y_{i} \mid\right.$ nc $\left.c_{i}=1, v_{i}<v_{0}\right] \neq 0$.
- Then $V_{i}$ is a function of, latent "true" vote $Z_{i}$ and ability to manipulate that total, $U_{i} \in\{0,1\}$ : $v_{i}\left(z_{i}\right)=z_{i}+u_{i} \Delta, \forall v_{i} \in\left[v_{0}-\Delta, v_{0}+\Delta\right]$,
$D_{i}\left(v_{i}\right)=1\left(v_{i}>v_{0}\right)$
- Matching on $I n c_{i}$ implies

$$
E\left[D_{i} \mid I n c_{i}=1\right]=E\left[D_{i}\right]=E\left[D_{i} \mid I n c_{i}=0\right]
$$

If $Z_{i} \perp$ Inc $c_{i}$ we have

$$
E\left[D \mid I n c_{i}=1, Z_{i}\right]=E\left[D \mid I n c_{i}=0, Z_{i}\right]
$$

- Since $E\left[U_{i} \mid I n c_{i}=0\right]=0$ implies $E\left[D_{i} \mid z_{i}>v_{0}\right]=1$ and $E\left[D_{i} \mid z_{i}<v_{0}\right]=0$ :
$E\left[Y_{i} \mid Z_{i}>v_{0}\right]=E\left[Y_{i} \mid D_{i}=1\right]$,
$\left.E\left[Y_{i} \mid Z_{i}<v_{0}\right]=E\left[Y_{i} \mid D_{i}=0\right] \forall z_{i} \in\left[v_{0}-\Delta, v_{0}+\Delta\right]\right]$
- Thus causal effects from matched samples can be estimated similarly as in an "ideal" RD.

Interpreting the Matching Estimand
Estimation and Inference

## Main assumptions for matching model

- First, manipulation of treatment is a function of observable data features.
- Second, $z_{i}$ is random in some bandwidth.
- Note that both must also hold in "normal" RD too.


## Interpretation and features of model

- An exclusion restriction on $Z_{i}$ is not required for valid causal effects if we assume $E\left[D_{i} \mid U_{i}=1\right]=1$ and $E\left[U_{i} \mid I n c_{i}=0\right]=0$. This is a version of a no defiers assumption.
- Estimand from matching-RD is akin to "LATE for compliers".
- This is the best traditional RD can do also: If $E\left[U_{i}\right]>0$ then matching recovers an unbiased RD estimate for compliers.

Estimates may be sensitive to three model choices

- Bandwidth for local linear regression: $h_{r}$
- Bandwidth within which matching occurs: $h_{m}$ - Sampling variance from matching

Procedure for modeling bandwidth sensitivity: 1 Specify ( $h_{r}, h_{m}$ ) pair
2 Match observations in $\left[v_{0}-h_{m}, v_{0}+h_{m}\right]$ to achieve exact balance on $I n c_{i}$
3 Note proportion of total matches in this bandwidth that are discarded, $p_{m}$
4 Randomly sample $p_{m}$ proportion of data not in $\left[v_{0}-h_{m}, v_{0}+h_{m}\right]$
5 Estimate effects using bandwidth $h_{r}$ (for local linear regression)
6 Repeat 100 times per ( $h_{r}, h_{m}$ ) pair.

Matching creates balance, improves inferences for close U.S. House Elections


Balance Measures, Results and Discussion

## Assessing Balance

- Balance plots: Are covariates within a given margin significantly different on either side of that margin?
Histograms: Are incumbents more likely to be found just above the cutpoint? (Do
incumbents win more close elections then the lose?)
- Balance Trends: As observations approach cutpoint (from right to left) do covariate observations diverge or converge?
- Conditional Independence Tests: In regressions of $Y_{i}$ on $Z_{i}$ is $Z_{i}$ significant a significant predictor after matching?
Balance Frontier: How do bandwidth specifications effect the balance matching achieves across treatment groups?
Estimate sensitivity to bandwidth specifications:
- How do estimated treatment effects vary as a function of $h_{r}$ and $h_{m}$ ?


## Conclusion

Graphs illustrate three main findings
1 Matching within small windows ( $0.5-5 \%$ pictured) significantly effects balance observed near the cutpoint.

2 Balance is not a uniform function of either $h_{r}$ or $h_{m}$. Under some circumstances, wider bandwidths appear to actually reduce bias contrary to expectations.
3Overall, there seems to be little reason to worry that estimates from original data are biased.

