Improving "National Brands":
Reputation for Quality and Export Promotion Strategies

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May 25, 2014

Abstract

This paper studies the effect of firm and country reputation on exports when buyers cannot observe quality prior to purchase. Firm-level demand is determined by expected quality, which is driven by the dynamics of consumer learning through experience and the country of origin's reputation for quality. We show that asymmetric information can result in multiple steady-state equilibria with endogenous reputation. We identify two types of steady states: a high-quality equilibrium (HQE) and a low-quality equilibrium (LQE). In a LQE, only the lowest-quality and the highest-quality firms are active; a range of relatively high-quality firms are permanently kept out of the market by the informational friction. Countries with bad quality reputation can therefore be locked into exporting low-quality, low-cost goods. Our model delivers novel insights about the dynamic impact of trade policies. First, an export subsidy increases the steady-state average quality of exports and welfare in a LQE, but decreases both quality and welfare in a HQE. Second, there is a tax/subsidy scheme based on the duration of export experience that replicates the perfect information outcome. Third, a large reputation shock is self-fulfilling when the economy has multiple steady states. Finally, a minimum quality standard can help an economy initially in a LQE moving to a HQE, but is not necessarily welfare improving.

Keywords: product quality, product differentiation, export promoting, industrial policy, trade.

JEL: F12, F13, L15, L52, O14, O24.

1   Introduction

Why are consumers willing to pay more for indistinguishable "Made in Germany" than "Made in China" products? In which way is building a solid reputation for reliability and quality key to a developing country's economic success – and how can it be achieved? Conversely, can a history of exporting cheap low-quality goods be an obstacle *per se* to national development strategies aimed at upgrading quality over time? These questions find no clear answers in standard models of international trade, which assume that consumers are perfectly informed about the characteristics of every available product and leave no role for country reputations. However, as a large literature on experience goods has shown, starting with Nelson (1970), quality is not fully known to consumers prior to purchase for a wide range of goods. Inferring the quality of a good on offer requires time and is achieved both through search and through experience. For these categories of goods, country-of-origin affects product evaluations and consumers' decisions. Indeed, many survey-based studies in the marketing literature, summarized by Roth and Diamantopoulos (2009), emphasize the role of country-of-origin labels in setting consumer perceptions of quality.

In this paper, we argue that a "national brand image" matters because it provides an anchor for the expected unobservable quality of imports. Consumption decisions, in practice, are based on a limited information set about the characteristics of goods or varieties. To understand the determinants of demand faced by entrants as well as incumbents in an industry, we need to consider the information available to consumers at the time of purchase: information gathered as a result of past consumption experience and word-of-mouth diffusion, but also the country where the good was manufactured. Together, these elements determine perceived quality, which affects consumption more than true quality when the latter is not observable. Specifically, for new and unknown foreign goods, the main piece of information available to consumers is the "made in" label, which indicates the country of manufacturing and creates a key role for national reputations. We call "national reputation" the common component of consumers' perceptions of the quality of goods produced within a given country. Usual examples of such priors are the widespread perceptions that "German goods last a lifetime" or "Chinese goods break down quickly".[1] Country reputations determine the quality that buyers expect from a product before they learn any information specific to this variety. In the long-run, reputations should adjust to the actual average quality of exported goods.

Broadly speaking, we are addressing three main questions. First, how does a poor "national brand" act as a barrier to entry into foreign markets, and how do the dynamics of consumer learning and country of origin reputation interact over time? Second, under which conditions do quality expectations have self-fulfilling effects, keeping some exporting coun-

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[1]Schott (2008) documents that the prices that US consumers are willing to pay for Chinese exports are substantially lower than the prices they are willing to pay for OECD exports in the same product category.

tries into low-quality traps? Finally, which policy instruments can help countries overcome the adverse impact of such information asymmetries?

More specifically, we consider an infinite-horizon two-country model with a continuum of potential foreign exporters heterogeneous in quality, and a constant flow of new potential exporters per period. Each new firm draws a quality parameter from a fixed distribution of firm technologies and has the option to produce a good of this quality. The decision to produce is endogenous: potential foreign exporters decide whether to enter the market and when to exit, taking into account the impact of their decisions on expected future sales. We assume that the cost of producing one physical unit of the good is monotonically increasing in quality, but the cost per quality-adjusted unit is decreasing in quality. Quality is known to firms but not observed by consumers before purchase. Hence, import demand depends on perceived quality, which has two components. Goods imported from a given country are first evaluated according to a country-wide prior, which is determined by the average quality of the country's exports in a long-run industry equilibrium. Importers then learn about the true quality of firms that have exported in the past. The fraction of informed consumers increases with the time a firm has been active on the market. The effect of the country prior will thus prevail for new exporters and fade over time as buyers gain familiarity with individual foreign brands.

We obtain four main results about the equilibrium structure (see Section 2 for a detailed comparison with the literature). First, asymmetric information results in multiple steady-state equilibria with endogenous reputation. Second, there are two types of steady states: a high-quality, high-reputation equilibrium (HQE), and a low-quality, low-reputation equilibrium (LQE). Third, in a LQE, only the lowest-quality and the highest-quality firms are active; a range of relatively high-quality firms are permanently kept out of the market by the informational friction. Finally, there can be multiple equilibria, such that countries with bad quality reputation can be locked into exporting low-quality, low-cost goods.

Our main channel is a distortion in entry and sales due to unobservable quality. Asymmetric information fosters entry by low-quality firms, which earn higher profits than under perfect information by free-riding on high-quality expectations. It depresses profits of the highest-quality firms, forced to incur initial losses in order to reveal information about their type. The combination of fly-by-night export strategies with long-term high-quality strategies results in two types of equilibria with endogenous reputation. In a HQE, imperfect information does not hinder entry of high-quality firms into export markets but generates excess entry by low-quality firms. In a LQE, a range of firms with above-average quality choose never to be active. With costs too high to allow for positive profits in the first periods, and quality too low for initial losses to be recouped with future profits, they do not find it profitable to enter the market.

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We consider various policies that can be implemented by a government to help a country locked in a bad equilibrium. Imperfect information about the quality of imported goods provides a justification for export promotion policies in economies specialized in low-quality products. Some export-led growth strategies for developing countries, pursued in the past by East Asian economies, rely on exporting low-quality, low-cost goods and gradually moving up to higher quality, higher unit value goods. China is attempting to follow the same path. Without policy intervention, though, we show that it may not be feasible if the economy is trapped in a self-fulfilling low equilibrium, in which the country's reputation for low quality prevents high-quality firms from entering export markets. A successful export promotion policy then involves either subsidizing exporters' initial losses, or investing public resources into raising the country's perception abroad.

Our model delivers novel insights about the impact of the following trade policies. First, an export subsidy increases the steady-state average quality of exports and welfare of the exporting country in a LQE, but decreases both quality and welfare in a HQE. A permanent export subsidy raises the incentives to export for all firms, but in a LQE it has a larger effect on high-quality firms which have a longer effective horizon. Conversely, in a HQE, it only leads to additional entry by low-quality firms, which creates a negative reputation externality on all exporters. Second, there is a tax/subsidy scheme based on the duration of export experience that replicates the perfect information outcome. Third, a small reputation shock only has short-lived effects, but a large reputation shock is self-fulfilling when the economy has multiple steady-states. Large positive reputation shocks raise prices received by all firms and allow more firms with above-average quality to enter, thereby driving up the true average quality of exported goods both in the short-run and in the long-run. Negative reputation shocks have the opposite effects: there can be long-term negative consequences of a sudden large drop in reputation, e.g. large product recalls or heavily mediatized consumer safety scandals. Finally, a minimum quality standard can help an economy initially in a LQE moving to a HQE, but is not necessarily welfare improving.

The remainder of this paper proceeds as follows. Section 2 reviews the literature relevant to the present study and shows how our approach leads to new insights. Section 3 lays out our modelling framework. Section 4 analyzes high-quality and low-quality steady-state equilibria with endogenous reputation. Section 5 explores the effects of different policy instruments on quality, reputation and profits. Finally, Section 6 presents additional predictions of our dynamic framework. Section 7 concludes.

2 Relationship to the Literature

Our paper relates to the international trade and industrial organization literature on vertical quality differentiation and asymmetric information. This section briefly explains how our dynamic approach with a continuum of quality types differs from the existing models that consider the effects of asymmetric information on exports.

Informational barriers to entry in international trade have been studied by Mayer (1984), Grossman and Horn (1988), Bagwell and Staiger (1989), Bagwell (1991), Chen (1991), Dasgupta and Mondria (2012) and Chisik (2003). Mayer (1984) was the first to investigate export subsidies in the presence of initially uninformed consumers but did so without modeling explicitly the process of consumer learning and expectations formation, and relied on pessimistic consumer beliefs. Dasgupta and Mondria (2012) develop a two-period model with similar features to ours, where the quality of new exporters is unobservable and that of continuing exporters is known by a fraction of consumers. They, however, focus on the role of intermediaries in providing quality assurance and do not analyze the formation of country reputations. We take the analysis further by endogenizing country reputations in an infinite-horizon setting and characterizing steady-state equilibria.

The articles that are the most closely related to the present paper are Bagwell and Staiger (1989) and Chisik (2003) who both explore the interplay between country reputation, exporting firm quality and optimal trade policy. While our paper builds on these pioneering works, it departs from them both in the assumptions and in the policy implications. First, most of the existing literature stresses the importance of exporters signaling strategy about their quality (either through prices, capacity, or via some other signaling device). While we recognize that signaling issues are important, we choose to abstract from them in order to focus on other effects. In our model, exporting firms have no way to signal their quality, so that information acquisition is entirely driven by the dynamics of consumer learning through experience and country reputation effects. This allows us to introduce a richer quality structure (we have a continuum of quality types, while both Bagwell and Staiger (1989) and Chisik (2003) have only two types) and time structure (we have an infinite horizon model, while Bagwell and Staiger (1989) use a two-period model and Chisik (2003) considers a static game). We are then able to model the process of reputation formation and investigate the transition dynamics of reputation between steady states, between an uninformed prior and the steady state, or following shocks.

Most importantly, these differences in assumptions allow us to develop novel insights about the dynamic interplay between entry and exit of firms, trade patterns, country reputations and trade policies. First, compared to the existing literature, we obtain two regimes (HQE and LQE). Second, in the LQE regime, the non-exporters are in the middle of the quality distribution, while the lowest-quality firms and the highest-quality firms are not precluded

from entering the market. To the extent of our knowledge, our paper is the first to obtain these results. In Bagwell and Staiger (1989) and in Chisik (2003), there are only two types of firms, so by construction there cannot be the hollowing out of the middle that we obtain.

Furthermore, our model delivers novel insights about the dynamic impact of trade policies. In Chisik (2003), export subsidies are of no use because they do not alter the relative payoffs of high-quality and low-quality firms. In our model, high-quality firms face a longer effective time horizon, so that even a subsidy granted indiscriminately to all exporters tilts the incentive to export in favor of high-quality firms. In Bagwell and Staiger (1989), an export subsidy can improve welfare. A subsidy can induce high-quality firms to enter the export market, while absent the subsidy their entry may be blocked by their inability to sell at prices reflecting their true quality. We also obtain a positive effect of an export subsidy in our paper in the LQE case, but with an important difference: in our model, very high-quality firms export even absent the subsidy; the subsidy induces middle-quality firms to export, and the overall effect on average quality is positive. Grossman and Horn (1988) point the fragility of the policy advice in Bagwell and Staiger (1989) where quality is exogenous, like in our paper. They argue that when quality is endogenous, an export subsidy does encourage entry but the marginal entrants are the ones with the greatest incentive to produce low-quality goods, reducing average quality (see Grossman, 1989). However, our model encompasses both effects. While a permanent subsidy does worsen the socially excessive entry of low-quality firms in a HQE, it also alleviates the problem of socially insufficient entry of high-quality firms.[2] In the HQE case, our paper yields a similar result to Grossman and Horn (1988): an export subsidy is detrimental to average quality as it promotes entry of low-quality firms while it does not change the incentives and decisions of high-quality firms.

Next, our new framework allows us to analyze the effect of other trade policies, some of which have been overlooked in the literature. According to Chisik (2003), the correct policy response involves direct investment in education or research and development that can indirectly subsidize high-quality firms, or a quality stamp. In our model, we show that if the government can observe how many periods a firm has been in the market, it can design a tax and subsidy program that similarly changes the relative incentives of high-quality and low-quality firms and improves both quality and reputation. Alternatively, we show that a minimum quality standard can help move to a more favorable equilibrium, but may not be optimal for the exporting country.

Finally, our paper relates to the several early papers (Shapiro, 1983; Riordan, 1986; Farrell, 1986; Liebeskind and Rumelt, 1989) that have studied entry and pricing strategies for experience goods in a closed economy framework. Bergemann and Välimäki (1996, 2006) incorporate

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[2]This would still be the case if we allowed for an endogenous quality choice, although fly-by-night firms would then always choose the minimum quality level.

the experimentation and learning processes by consumers. We develop these insights further by considering the demand for imports, where initial priors depend on country-of-origin, and reputations are built not only for specific firms but also for exporting countries as a whole. It also relates to the industrial organization literature on labels and quality certification, a strand of it assuming that quality is unknown to consumers (see e.g. Perrot and Linnemer, 2000). This literature focuses on the signaling role of labels: labels act as quality signals for individual firms which seek to signal quality and build consumer trust (see e.g. Grossman, 1981; Klein and Leffler, 1981; Milgrom and Roberts, 1986; Lizzeri, 1999), but there are no reputation externalities between firms. Lastly, Tirole (1996) models group reputation as both the outcome and a determinant of individual behavior in a moral hazard setting. His model generates a high- and a low-reputation steady-state equilibrium, but for given initial conditions, there is a unique equilibrium. An interesting result is that a one-time shock can have permanent effects on collective reputation. In this article, we abstract from moral hazard issues but we show that adverse selection can also generate long-run effects of temporary reputation shocks, and we provide a richer equilibrium structure including multiple self-fulfilling equilibria.

3   Model Set-Up

We develop a model with two countries, Home and Foreign. We focus on the industry equilibrium in an export-oriented sector in Home, for which Foreign is the importer. In Home, there is a continuum of potential exporters j of mass 1 being born every period. In Foreign, there is a pool of importers indexed by i, each of which demands one unit of the good. Potential demand for imported goods in Foreign is assumed to be large. We do not model explicitly the domestic market of foreign firms: all firms in the industry produce for export only.[3]

3.1   Firms

Each new potential exporter draws a quality parameter θ from a Pareto distribution $G(\theta)$ with support on $[\theta_m, \infty)$ and shape parameter $\alpha > 1$:

$$G(\theta) \equiv 1 - \left(\frac{\theta_m}{\theta}\right)^{\alpha} \tag{1}$$

We note μ_0 the unconditional expectation of quality draws: $\mu_0 \equiv \frac{\alpha}{\alpha-1}\theta_m$.

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[3]Or, equivalently, markets are segmented. We could easily extend the model to allow firms to serve their domestic market as long as the decisions to enter the domestic and export markets are separable. The key assumption is that there is no information flowing between buyers located in different geographic markets. In particular, a firm having established a reputation in its domestic market would not be able to transfer this reputation to export sales.

Each new firm j has the option to produce a good of quality level $\theta(j)$.[4] At the beginning of every period, firms decide whether to stay active and export, or shut down. Each firm has capacity 1, so that it can choose to sell either one unit or nothing.

If it produces and sells, a firm j of type $\theta(j)$ incurs a cost $w\theta(j) + k$, including both production and transport costs (with $k > 0$ and $0 < w < 1$). k includes all costs that are independent of quality, while $w\theta(j)$ is the portion of costs that increases with the quality of the product (e.g. quality control processes, better intermediate inputs, or more skilled workers). Hence, profits at period $t + s$ of an active firm j born at date t are:

$$\pi_{t+s}(j) = p_{t+s}(j) - w\theta(j) - k \tag{2}$$

where $p_{t+s}(j)$ is the price at which firm j sells its output. The price-setting mechanism is described in the next subsection.

A firm can freely exit at any period and realize zero profits from this period onwards. However if it chooses to exit the export market in a given period, it cannot re-enter later.[5] Moreover, each firm has an exogenous probability $(1 - \delta)$ of suffering from a bad shock that forces it to exit every period, independent of both quality and the firm's age. The probability that a firm still exists from one period to the next, conditional on not choosing to exit, is δ. The exogenous "exit shock" acts as a discount rate, as in Melitz (2003). There is no additional discount rate.

3.2   Buyers

In Foreign, each potential importer demands one unit of the good. Potential demand for imported goods is assumed to be large, in the sense that the market size is sufficient for all Home exporters to find a buyer at a price that does not exceed the expected value of their goods. The true utility from consuming a good of quality level θ is θ but is not observable before purchase. We can think of θ as characteristics that are observed only upon consumption, or for durable goods, as the inverse of the probability of breakdown per period.

At the beginning of every period, each active firm j is randomly matched to a buyer i in Foreign. The firm cannot sell to another importer in that period, nor can the buyer purchase from another exporter before the next period. The firm then sets the price equal to the expected value of the good for its buyer. The indirect utility buyer i receives from the good sold by firm j at time $t + s$ is:

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[4]For simplicity we do not model the choice of quality. We can think of the exogenous quality draw as determined on the domestic market before considering the decision to export, or as a technology blueprint which comes from an R&D process with uncertain outcome: all firms invest the same sunk cost in R&D and randomly, some come up with better quality products than others.

[5]This assumption is inconsequential for the steady-state analysis. It rules out coordination problems among high-quality firms along the transition path.

$$u^i_{t+s}(j) = \theta(j) - p_{t+s}(j) \tag{3}$$

which can be derived from an additively separable utility function where buyer i consumes a numeraire good and one unit of the imported differentiated good. As $\theta(j)$ is not observed, the maximum price that an importer i is willing to pay for the output of firm j at time $t+s$ is given by its expected quality from the perspective of the buyer:

$$p_{t+s}(j) = E^i_{t+s}\left[\theta(j) \,|\, I^i_{t+s}\right] \tag{4}$$

where I^i_{t+s} is the information set of buyer i at time $t+s$. We assume that firms hold all the bargaining power and receive the full expected surplus of the transaction.[6]

There are two types of buyers, informed and uninformed. Uninformed buyers have no information specific to firm j: they do not know its quality or how long it has been an active exporter. The only information at their disposal is the "national reputation", i.e. a prior μ_{t+s} about expected quality among all foreign exporters. μ_{t+s} is common across buyers and is endogenized in Section 4. Informed buyers know the true quality of firm j, either because they have past experience from consumption of good j or because they have received information from another importer who has. The price received by firm j matched with buyer i in period $t+s$ is therefore equal to its quality if i is informed, and to the country's reputation if i is uninformed. Equation (4) can thus be rewritten as:

$$p_{t+s}(j) = \begin{cases} \mu_{t+s} & \text{if } i \text{ is uninformed.} \\ \theta(j) & \text{if } i \text{ is informed.} \end{cases} \tag{5}$$

In the first period when a firm j enters the market, all importers are uninformed about j. Then, if firm j has exported s times in the past, a fraction $\rho(s)$ of buyers are informed, where we make the following natural assumption:

Assumption 1 $\rho' \geq 0$, $\rho(0) = 0$, and $\lim_{s \to \infty} \rho(s) = 1$.

The fraction of informed buyers increases as the firm gains export experience.[7] A possible microfoundation for the $\rho(s)$ function is that importers belong to distinct groups within which information diffusion takes place. The fraction of informed buyers rises according

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[6]Long-term contracts between exporters and importers are ruled out in this setting: all contracts are one-period sales contracts and firms are matched to customers for one period only. In particular, there cannot be price schedules resembling an introductory pricing strategy, whereby buyers would pay a low price in the initial period and offer a sequence of prices contingent on their future consumption experience.

[7]Note that a concave $\rho(s)$ would capture the idea that a firm having already exported is more likely to be matched again with one of its previous buyers or with someone close to a previous buyer in terms of informational diffusion. Hence the rate at which the scope of informed buyers expands would decrease with the number of periods the firm has been on the market. However, the only conditions needed for our analysis is that ρ is monotonically increasing and takes values between 0 and 1.

to the probability of sampling a buyer from an uninformed versus an informed group (See Appendix B for a formal development).

For expositional simplicity we drop the j notation in the next sections and refer to "firm θ" instead of "firm j with quality parameter θ" whenever possible.

3.3 Timing

For a given cohort of firms born at date t, the timing of moves proceeds as follows:

- At t, each new firm draws a parameter θ and decides whether to export or not.

- For each $s \geq 1$, at time $t + s$:

  - Each active firm is matched with a buyer and observes whether the buyer is informed.

  - The price is set at $E_{t+s}[\theta|I_{t+s}]$, and production and sales take place.

  - For each good that was sold, the fraction of informed buyers rises from $\rho(s)$ to $\rho(s+1)$.

  - With probability $(1 - \delta)$, each firm is forced to exit by a bad shock.

  - Firms that survive decide whether to stay active.

  - New firms are born (cohort $t + s$).

3.4 Perfect Information

Under perfect information, all θ are observable by all parties. All firms receive a price p^*_{t+s} equal to true quality regardless of how long they have been exporting: $p^*_{t+s}(\theta) = \theta$ for all s.

Therefore, it follows from (2) that firms are active exporters if and only if $\theta \geq \theta^*$, where the perfect information threshold is defined as:

$$\theta^* \equiv \frac{k}{1 - w} \tag{6}$$

Under perfect information, the model therefore predicts a sorting of firms into non-exporters and exporters similar to that of Melitz (2003). We can define productivity as the inverse of the cost per unit of quality $w + \frac{k}{\theta}$. The firms with the highest quality θ are also those with the highest productivity. Firms above the quality threshold θ^*, or equivalently below the quality-adjusted cost threshold $w + \frac{k}{\theta^*}$, always export and never exit voluntarily, while firms below the threshold never export.

The perfect information threshold is key for our analysis because, under asymmetric information, per-period profits converge over time towards their perfect information value (see

Section 3.5 below). Moreover, from a global welfare point of view, it is not optimal for firms with quality below the perfect information threshold θ^* to enter export markets – in this case, the value of the output to consumers is lower than the opportunity cost of production. However, given that exporters set their price equal to the expected value of the good for the buyers and receive the full expected surplus of the transaction, it can be profitable for firms with quality below the perfect information threshold θ^* to enter the export market temporarily under asymmetric information, increasing welfare in Home but hurting total welfare.

3.5   Imperfect Information: Price and Profits

Under asymmetric information, suppose μ_t is the buyers' prior about the expected quality of a good from the foreign country at time t. This prior is the national reputation or "national brand" and is taken as exogenous by individual firms, although it is endogenous at the country level. We derive its equilibrium value in Section 4, as a function of the quality distribution of exports. The price offered to a firm born at date t is either the country-wide prior if the buyer is uninformed, or its true quality if the buyer is informed. The probability of receiving a price which reflects the firms' true quality increases with the firm's export experience. In the first period in which a firm is active, no buyer has any information specific to the firm, so that the price only depends on the prior:

$$p_{t+1} = \mu_{t+1} \tag{7}$$

Then in the following periods, conditional on the firm still being active, the pricing equation (5) implies that the price received by the firm is set according to the following rule:

$$p_{t+s} = \begin{cases} \theta & \text{with probability } \rho\,(s-1) \\ \mu_{t+s} & \text{with probability } 1 - \rho\,(s-1) \end{cases} \quad \text{for } s \geq 1 \tag{8}$$

where $\rho\,(s-1)$ is the fraction of informed buyers for a firm that has previously exported $(s-1)$ times. In particular, a firm that exports for the first time faces only uninformed importers $(\rho\,(0) = 0)$. As $\lim_{s\to\infty} \rho\,(s) = 1$, the expectation of the price converges to the perfect information price θ over time if the firm stays in the market indefinitely.

The expected profits of the firm in future periods, conditional on remaining active, are the difference between its expected price and its production cost:

$$E_t \pi_{t+s} = (\rho\,(s-1) - w)\,\theta + (1 - \rho\,(s-1))\,E_t \mu_{t+s} - k \tag{9}$$

Expected profits place a larger weight on true quality and a smaller weight on national reputation as the firm gains tenure into exporting. It immediately follows that if reputation is time-invariant, which will be the case in a steady-state equilibrium, a firm with quality above

the country prior $(\theta > \mu)$ expects to realize an increasing sequence of profits over time, while a firm with quality below the country prior $(\theta < \mu)$ expects decreasing profits. For all active firms, if μ is constant, the price is monotonically converging towards θ and per-period profits are monotonically converging towards their perfect information value $(1 - w)\theta - k$.

To ensure that expected profits from repeat purchases – as opposed to initial purchases – are increasing in true quality, we assume that the updating parameter is large enough relative to the cost of producing quality:

Assumption 2 $\rho(1) > w$

In each period t, a firm of quality θ having exported s times in the past stays active if the expected present value of doing so, $PV_t(\theta, s)$, is positive. The firm is free to exit at any future date. Let $T(\theta)$ be the exit date (possibly infinity) that maximizes the firm's intertemporal problem. Then $PV_t(\theta, s)$ is the discounted sum of current and future profits in (9) up to the optimal exit date:

$$PV_t(\theta, s) = \sum_{u=0}^{T(\theta)} \delta^u \left[(\rho(s+u) - w)\theta + (1 - \rho(s+u)) E_t \mu_{t+u} - k \right] \tag{10}$$

where $E_t \mu_{t+u} = \mu_{t+u}$ for all u since there is no aggregate uncertainty. The optimal exit dates and endogenous reputations are derived in the next section.

4 Industry Equilibrium

In this section, we define a steady-state industry equilibrium as one in which national reputation is pinned down by the average quality of a country's exports and the quality distribution is stationary. We characterize the entry and exit patterns in high-quality and low-quality steady-state equilibria and lay out their existence conditions.

4.1 Equilibrium Definition

Country reputations are taken as exogenous by individual firms. In each period t, let $M_t(\theta, s)$ be the number of active firms of quality θ having previously exported s times. Let $N_t(\theta) = \sum_{s=0}^{t} M_t(\theta, s)$ be the total number of active firms of quality θ. We derive $\bar{\theta}_t$ as the average quality of exports across quality levels and cohorts of firms:

$$\bar{\theta}_t = \frac{\int_{\theta_m}^{\infty} \theta N_t(\theta) \, d\theta}{\int_{\theta_m}^{\infty} N_t(\theta) \, d\theta} \tag{11}$$

We assume that country reputations evolve according to the actual quality of exported goods in the previous period:[8]

$$\mu_{t+1} = \mu_t + \eta\left(\bar{\theta}_t - \mu_t\right) \tag{12}$$

where $\eta < 1$ and $\bar{\theta}_t$ is the average quality of foreign firms' exports at period t. Reputation rises (respectively, falls) from one period to the next if the average quality of exported goods in the previous period was higher (respectively, lower) than expected. Setting $\eta < 1$ captures the slow-moving aspect of reputations and only matters for equilibrium stability. Along with $\bar{\theta}_t$, this determines μ_{t+1} according to (11).

A new firm of quality θ is active at $t+1$ if $PV_{t+1}(\theta, 0) > 0$, i.e. if its expected present value of doing so is positive. Hence the number of active new firms per quality level is:

$$M_{t+1}(\theta, 0) = \begin{cases} g(\theta) & \text{if } PV_{t+1}(\theta, 0) > 0 \\ 0 & \text{if } PV_{t+1}(\theta, 0) \leq 0 \end{cases} \tag{13}$$

Among incumbent firms of quality θ having exported s times, $\delta M_t(\theta, s-1)$ survive from period t to period $t+1$. They remain active if $PV_{t+1}(\theta, s) > 0$ in equation (10). Thus the number of active old firms is, for $s \geq 1$:

$$M_{t+1}(\theta, s) = \begin{cases} \delta M_t(\theta, s-1) & \text{if } PV_{t+1}(\theta, s) > 0 \\ 0 & \text{if } PV_{t+1}(\theta, 0) \leq 0 \end{cases} \tag{14}$$

Equations (11), (12), (13) and (14) determine $\bar{\theta}_{t+1}$ and μ_{t+2}. We can then define the industry steady-state as an equilibrium with constant reputation and a constant distribution of quality.

Definition 1 (Steady-State Equilibrium)
$\left\{\mu, \{M(\theta, s)\}_{s,\theta}\right\}$ *is a steady-state equilibrium if and only if:*
(i) For all $\theta \in [\theta_m, \infty)$ and all $s \geq 0$, if $M_t(\theta, s) = M(\theta, s)$ and $E_t\mu_{t+u} = \mu$ for all $u \geq 0$, then $M_{t+1}(\theta, s) = M(\theta, s)$ in (13) and (14);
(ii) If $M_t(\theta, s) = M(\theta, s)$ for all $\theta \in [\theta_m, \infty)$ and all $s \geq 0$, then $\bar{\theta}_t = \mu$ in (11).

Condition (i) ensures that the number of firms in each quality-age segment is constant in the steady state. Condition (ii) states that the average quality resulting from an equilibrium distribution of active firms is equal to the equilibrium country reputation. It guarantees that μ is constant in a steady state. In other words, a steady state with national reputation μ

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[8]This equation is a reduced form for consumers' updating process, where the implied simplifying assumption is that η – which captures the speed at which beliefs are updated – is constant. We adopt this simple rule-of-thumb formulation for beliefs updating rather than modeling explicitly the Bayesian updating process for the sake of simplicity. This hypothesis is not necessary for our main steady-state results, and policy implications and only ensures that equilibrium stability holds under general conditions.

is a rational expectations equilibrium if the average quality of active exporters is equal to buyers' quality expectation. The endogenous entry and exit decisions induced by μ justify the reputation ex post.

4.2 Equilibrium Steady-State Reputation

Let $\overline{\theta}(\mu)$ be the average quality of exports as a function of constant beliefs μ. An equilibrium steady-state reputation is a time-invariant reputation μ such that $\overline{\theta}(\mu) = \mu$.

To compute the fixed point(s) of $\overline{\theta}(\mu)$ (if any), we proceed as follows. First, we characterize firms' entry and exit decision. Second, we compute $M_t(\theta, s)$, i.e. the number of active firms of quality θ having previously exported s times, and thus the average quality of exports as a function of μ. We then derive the existence conditions for each type of equilibrium.

4.2.1 Sorting of Firms into Non Exporters and Exporters

Our analysis distinguishes between two types of equilibria. A "high-quality equilibrium" (HQE) is a steady-state equilibrium where the country reputation μ exceeds the perfect information quality threshold θ^*. A "low-quality equilibrium" (LQE) is a steady-state equilibrium where the country reputation falls short of the perfect information quality threshold θ^*. Formally:

Definition 2 (HQE & LQE)
$\left\{\mu, \{M(\theta, s)\}_{s,\theta}\right\}$ *is a high-quality steady-state equilibrium if* $\mu > \theta^*$ *and* $\left\{\mu, \{M(\theta, s)\}_{s,\theta}\right\}$ *is a steady-state equilibrium according to Definition 1.*

$\left\{\mu, \{M(\theta, s)\}_{s,\theta}\right\}$ *is a low-quality steady-state equilibrium if* $\mu < \theta^*$ *and* $\left\{\mu, \{M(\theta, s)\}_{s,\theta}\right\}$ *is a steady-state equilibrium according to Definition 1.*

The two regimes have different entry and exit patterns, due to the impact of asymmetric information and the dynamics of consumer learning. Compared to the perfect information case, the unobservability of quality initially fosters entry by low-quality firms but depresses profits of the highest-quality firms. In a HQE, a firm with quality equal to the country's reputation would be viable in a perfect information setting. All firms receive high prices as they enter the market, which encourages entry. Therefore, imperfect information does not hinder entry of high-quality firms into export markets but generates excess entry by low-quality firms. On the contrary, in a LQE, a firm with quality equal to the country's reputation would never export in a perfect information setting. Some low-quality firms still profit by free-riding on the country reputation, but a range of firms with above-average quality are permanently kept out of the market by the informational friction. In other words, there is a

hollowing out of the middle of the quality distribution. Proposition 1 establishes formally the sorting of firms into non exporters and exporters in a HQE and in a LQE.

Proposition 1 (Sorting of Firms into Exporting: Two Regimes)
In a HQE:

1. *All entrants are initially active;*

2. *Firms with $\theta < \theta^*$ expect to exit after $T(\theta)$ periods where T is weakly increasing in θ;*

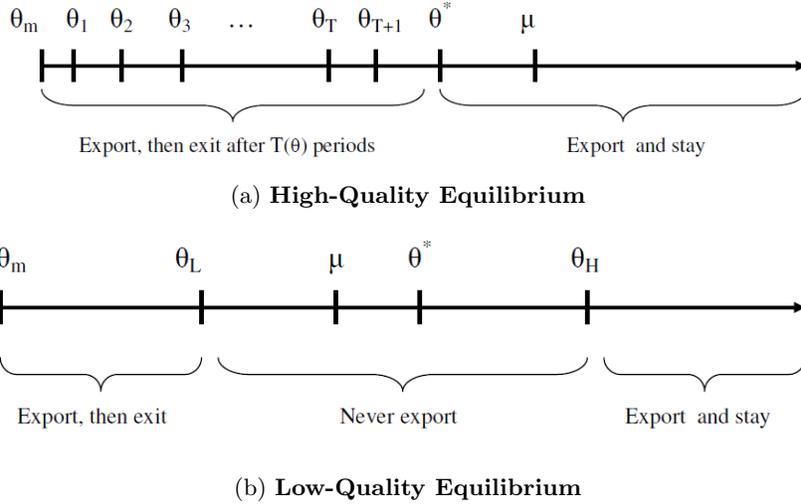3. *Firms with $\theta > \theta^*$ stay in the market until hit by the exogenous shock.*

In a LQE:

1. *Firms with quality $\theta < \theta_L$ enter the export market and exit after selling for one period, where $\theta_L \equiv \frac{\mu - k}{w} < \mu < \theta^*$;*

2. *Firms with quality $\theta > \theta_H$ enter and stay in the market until hit by the exogenous shock, where $\theta_H \equiv \frac{k - \mu(1 - A_\rho)}{A_\rho - w} > \theta^*$ and $A_\rho \equiv (1 - \delta) \sum_{s=0}^{\infty} \delta^s \rho(s)$.*

3. *Firms with quality $\theta_L \leq \theta \leq \theta_H$ never enter the market.*

Proof: see Appendix A.1.

Figure 1 shows the sorting of firms according to their quality parameter in each regime. In a HQE (upper figure 1a), all low-quality firms (below θ^*) find it profitable to enter initially as fly-by-nights as they have low production costs and can therefore reap positive expected profits as long as a small enough number of buyers have information about their type. The higher the country reputation, the higher the price they receive in the first period. However, they become less profitable as consumers gain information about their quality through consumption experience. Low-quality firms thus face a decreasing sequence of expected profits converging to a negative value; they will eventually see their expected present value of profits turn negative and exit. The lowest-quality firms exit first, and θ_T is the highest quality type that exits after selling for T periods – or the lowest quality type that exits after selling for $T + 1$ periods.[9] For high-quality firms (above θ^*), it is always profitable to enter and keep exporting. Firms between θ^* and μ have expected profits declining over time, but positive in every period. Firms above μ have expected profits increasing over time. The highest quality firms incur losses in the initial period but recoup these losses in later periods once enough buyers have received information about their type; their expected intertemporal profits are always positive.

---

[9]Firms below θ^* exit after $T(\theta)$ periods, where $T(\theta)$ is the exit date that maximizes the firm's intertemporal profit (see equation (10)). We can derive $\theta_T = \max \left\{ \frac{k - [1 - \rho(T)]\mu}{\rho(T) - w}, \theta_m \right\}$ for $T \geq 1$ and $\lim_{T \to \infty} \theta_T = \theta^*$ (See Appendix A.1).

(a) **High-Quality Equilibrium**

(b) **Low-Quality Equilibrium**

Notes: The Figure shows the sorting of firms into exporters and non exporters by θ in a HQE (upper figure 1a) and in a LQE (bottom figure 1b).

Figure 1: Sorting of Firms by θ

In a LQE (bottom figure 1b), there is a range of low-quality firms (below θ_L) that gain from the information asymmetry and realize positive first-period profits. However they would make losses if they were to stay active in the second period once buyers have received some information about their quality. These firms therefore exit immediately after selling once. An intermediate range of middle-quality firms $[\theta_L, \theta_H]$ around θ^* never become active exporters. Those with $\theta_L < \theta < \theta^*$ have negative expected profits at all periods, while those with $\theta^* < \theta < \theta_H$ would be profitable in the long run once enough buyers have gathered information about their type. However, for the latter, the present value of their profit stream is negative: losses incurred in the initial periods in order to establish a reputation are not made up for with later expected profits. Hence this range of firms is kept out of export markets by the information asymmetry and the cost of revealing quality. Finally, high-quality firms (above θ_H) are not profitable in the first period when they enter the export market (their quality is higher than the country reputation μ). They nevertheless choose to enter given that their present value of expected profits is positive: expected profits from sales in later periods, when a larger portion of the price reflects true quality, exceed initial losses. The negative profits in their first export periods can be interpreted as investments in building a firm-specific brand name.

This sorting of firms into exporters and non exporters in a LQE, with non-exporters in the middle of the quality distribution, is a new result compared to the existing literature. In particular, the introduction of an infinite horizon and a continuum of quality types, with group reputation endogenously determined by individual firm choices, enables us to uncover this richer pattern compared to two-period or two-type models.

4.2.2 Average Quality

The number $M(\theta, s)$ of active firms of quality θ having already exported s times is derived from Proposition 1 and equations (13) and (14).

High-quality equilibrium In a HQE, the number $M(\theta, s)$ of active firms of quality θ having already exported s times is:

$$
M(\theta, s) = \begin{cases} \delta^s g(\theta) & \text{if } \theta < \theta^* \text{ and } s < T(\theta) \\ 0 & \text{if } \theta < \theta^* \text{ and } s \geq T(\theta) \\ \delta^s g(\theta) & \text{if } \theta \geq \theta^* \end{cases} \tag{15}
$$

so that the total number $N(\theta)$ of active firms of quality θ is $\frac{1-\delta^{T(\theta)}}{1-\delta} g(\theta)$ if $\theta < \theta^*$, and $\frac{1}{1-\delta} g(\theta)$ if $\theta \geq \theta^*$. The steady-state average quality of exports in a HQE as a function of μ and exogenous parameters is given by equation (11) as:

$$
\overline{\theta}(\mu) = \mu_0 \left(\frac{1 - \sum_{T=0}^{\infty} \delta^{T+1} \left[\left(\frac{\theta_m}{\theta_T} \right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_{T+1}} \right)^{\alpha-1} \right]}{1 - \sum_{T=0}^{\infty} \delta^{T+1} \left[\left(\frac{\theta_m}{\theta_{T+1}} \right)^{\alpha} - \left(\frac{\theta_m}{\theta_{T+1}} \right)^{\alpha} \right]} \right) \tag{16}
$$

where $\theta_0 \equiv \theta_m$. The average quality of active firms is higher than the mean of the unconditional distribution of θ, as lower-quality firms exit earlier than higher-quality firms. However, it lies below the perfect information average quality.

From (16), it follows that $\overline{\theta}(\mu)$ is continuously decreasing in μ on $[\theta^*, \infty)$. In words, starting from a reputation μ above θ^*, the average quality of exported goods falls when the reputation μ increases. The intuition is as follows. On the one hand, improving national reputation does not affect the decisions of firms above θ^* to stay or exit, as they are already remaining active as long as possible. On the other hand, it encourages lower-quality firms to stay longer: for firms below θ^*, a higher μ raises all $T(\theta)$, implying that low-quality firms wait longer before exiting the market. In short, the incentives of high-quality firms are not affected but those of low-quality firms result in a higher export duration of firms producing "bad" varieties. Hence, starting from a HQE, raising national reputation μ has a negative effect on actual quality, which ensures that a high-quality steady state is stable.

Low-quality equilibrium In a LQE, the number of active firms of quality θ having already exported s times is given by:

$$M(\theta, s) = \begin{cases} g(\theta) & \text{if } \theta < \theta_L \text{ and } s = 0 \\ 0 & \text{if } \theta < \theta_L \text{ and } s \geq 1 \\ 0 & \text{if } \theta_L \leq \theta \leq \theta_H \\ \delta^s g(\theta) & \text{if } \theta > \theta_H \end{cases} \tag{17}$$

so that the total number $N(\theta)$ of active firms of quality θ is $g(\theta)$ if $\theta < \theta_L$ and $\frac{1}{1-\delta} g(\theta)$ if $\theta > \theta_H$. The steady-state average quality of exports in a LQE as a function of μ and exogenous parameters is given by equation (11) as:

$$\overline{\theta}(\mu) = \mu_0 \left(\frac{1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha-1} + \frac{1}{1-\delta} \left(\frac{\theta_m}{\theta_H}\right)^{\alpha-1}}{1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha} + \frac{1}{1-\delta} \left(\frac{\theta_m}{\theta_H}\right)^{\alpha}} \right) \tag{18}$$

From (17), we can show that $\overline{\theta}(\mu)$ is not necessarily monotonic over $[\theta_m, \theta^*]$. The intuition is as follows. Starting from a reputation μ below θ^*, raising μ has two consequences with opposite effects on average quality. On the one hand, increasing μ enables more firms to realize positive profits from first-period sales: this fosters entry by firms with below-average quality and lowers the expected quality of active firms. On the other hand, increasing μ reduces the loss incurred by high-quality firms before they have been able to signal their quality to buyers, allowing more firms with above-average quality to be active. The net change in the average quality depends on the balance between these two effects. As a result, LQEs can be stable or unstable.

4.2.3 Existence Conditions

A steady-state equilibrium is a fixed point of $\overline{\theta}(\mu)$ defined by equation (18) on $[\theta_m, \theta^*]$ and by equation (16) on $[\theta^*, \infty)$. An equilibrium such that $\mu = \overline{\theta}(\mu) < \theta^*$ is a LQE. An equilibrium such that $\mu = \overline{\theta}(\mu) > \theta^*$ is a HQE. Proposition 2 establishes existence conditions.

Proposition 2 (Existence Conditions)

1. *There is always at least one steady-state equilibrium.*

2. *There is one HQE and an even number of LQE (possibly zero) if and only if $\overline{\theta}(\theta^*) > \theta^*$, i.e. if and only if:*
$$\alpha \left(\frac{\theta_m}{\theta^*}\right) + \frac{\delta}{1-\delta} \left(\frac{\theta_m}{\theta^*}\right)^{\alpha} > \alpha - 1 \tag{19}$$

3. *There is no HQE and an odd number of LQEs (at least one) if and only if $\overline{\theta}(\theta^*) < \theta^*$,*

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i.e. if and only if:

$$\alpha \left(\frac{\theta_m}{\theta^*} \right) + \frac{\delta}{1 - \delta} \left(\frac{\theta_m}{\theta^*} \right)^{\alpha} < \alpha - 1$$

Proof: see Appendix A.2.

Hence, depending on the parameters, the rational expectations steady-state falls into one of two regimes: a LQE or a HQE. The type of equilibrium depends on whether the (not necessarily unique) fixed point of $\bar{\theta}(\mu)$ falls left or right of θ^*. Figure 2 provides a graphical illustration. The figure represents the steady-state average quality of exports $\bar{\theta}(\mu)$ as a function of μ (red line). The black diagonal is a 45 degree line, and we plot a vertical dotted line at the perfect information threshold θ^* (equal to 2.4 in this example). In the upper figure 2a, we illustrate the existence of a HQE: $\bar{\theta}(\theta^*) > \theta^*$ and there is one HQE, μ'_S. This equilibrium is not unique: there are also two LQEs, μ_S and μ_U. On the contrary, in the bottom figure 2b, we illustrate the fact that when $\bar{\theta}(\theta^*) < \theta^*$, there is no HQE. The only equilibria of the economy are LQEs. In this example, there is a unique LQE, μ_S.

(a) **HQE**   (b) **No HQE**
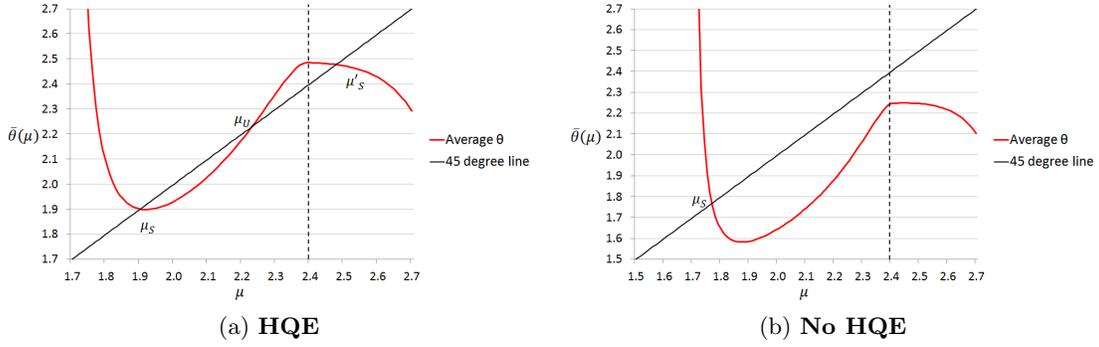
Notes: In subfigure 2a, the fixed parameter values are: $\theta_m = 1$, $\alpha = 2.2$, $\delta = 0.7$, $k = 1.2$, $w = 0.5$. The perfect information threshold is $\theta^* = 2.4$. The steady states of this economy are $\mu_S \approx 1.900$, $\mu_U \approx 2.230$ and $\mu'_S \approx 2.477$. In subfigure 2b, the parameters are the same except $\alpha = 2.4$. The unique steady state of this economy is $\mu_S \approx 1.767$.

Figure 2: Multiple Equilibria

The existence condition (19) for a HQE holds for δ (the probability that a firm still exists from one period to the next) high enough, α (the shape parameter of the distribution) low enough, and/or k (the costs that are independent of quality) and w (the costs that increase with quality) low enough. Figure 3 illustrates this existence condition. In the dark region, condition (19) holds and there exists one HQE. In the light grey region, there is no HQE: the steady-state of the economy is necessarily a LQE. In subfigure 3a, we fix the values of k and w and show that there exists a HQE if α is low enough and δ is high enough. A high δ implies that exogenous exit is relatively less prevalent than endogenous exit, increasing the relative mass of high-quality firms. A low α means that there is high dispersion in the prior distribution of θ and therefore more firms at the right tail of the distribution pushing up
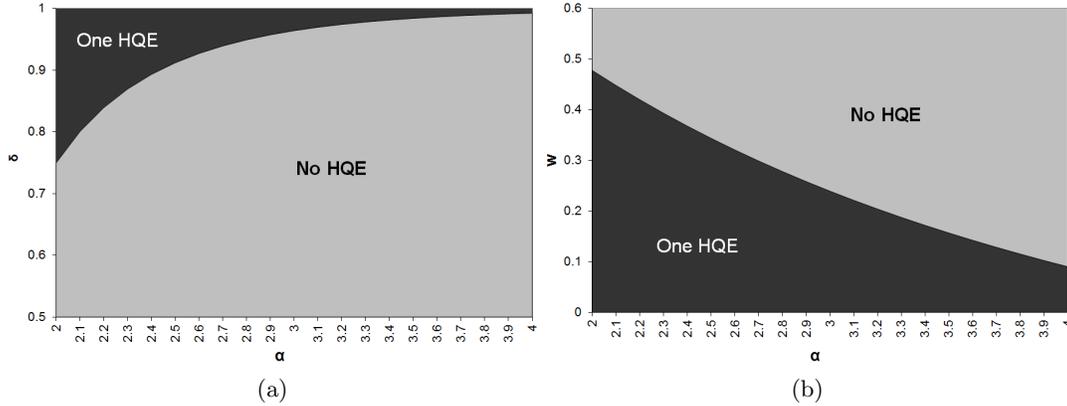
Notes: Fixed parameter values in subfigure 3a: $\theta_m = 1$, $k = 1.5$, $w = 0.5$. Fixed parameter values in subfigure 3b: $\theta_m = 1$, $k = 1.5$, $\delta = 0.95$.

Figure 3: Parameter Values for the Existence of a HQE

the mean. In subfigure 3b, we fix the values of k and δ. There exists one HQE if w is low enough. Intuitively, a low w reduces the relative cost advantage of low-quality firms, as well as the loss incurred in initial periods by high-quality firms. Similarly, a low k also reduces the initial losses of low-quality firms and lowers the perfect information threshold θ^*, expanding the existence region of a HQE.

5 Policy Implications

How can countries improve their "national brand name" – and is it worth it? First-best policies would involve conducting verifiable quality audits or taxing low-quality firms and subsidizing high-quality ones. These policies are not feasible when policy-makers are not better informed than consumers about firms' quality levels. Here, we look at the effects of three main policy instruments on reputation, quality and welfare: (i) export subsidies, (ii) export promotion campaigns creating reputation shocks, and (iii) minimum quality standards.

5.1 Export Subsidies

Export subsidies have been used historically by a number of countries to favor their exporting activities. For example in South Korea, public investment subsidies were tied to exporting activity in the 1970s, as Korean governments were determined to favor the emergence of the country on the international trade scene.[10] We model an export subsidy as a permanent[11]

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[10]Pack and Westphal (1986), Westphal (1990), Levy (1991), Rodrik (1995) and Aw et al. (1998) have documented the importance of government investment subsidies in Korea.

[11]We are comparing the long-run industry equilibria with and without the policy. With a temporary subsidy, if the equilibrium is unique, the economy would return to the initial steady state in the long-run after the subsidy expires.

unanticipated subsidy to fixed export costs, resulting in a lower effective k for active exporters, financed by non-distortionary lump-sum taxes.

Interestingly, export subsidies have opposite effects in the two types of equilibria. This difference stems from the impact of the subsidy on the entry and exit patterns of high-quality versus low-quality firms. In a LQE, an export subsidy leads to higher entry by firms in the middle of the quality distribution. We show that the overall effect on average quality, and thus steady-state national reputation, is positive as long as δ is not too low: starting from a LQE, an export subsidy increases long-run equilibrium quality and the welfare of the exporting country.[12] On the contrary, in a HQE, an export subsidy is actually detrimental to average quality and welfare as it discourages exit by low-quality firms. These results are summarized in Proposition 3.

Proposition 3 (Export Subsidy)

An export subsidy in a LQE increases the steady-state average quality of exports and welfare of the exporting country.

An export subsidy in a HQE decreases the steady-state average quality of exports and welfare of the exporting country.

Proof: see Appendix A.3.

More specifically, in a LQE, a decrease in k induces more relatively high-quality firms to start and continue exporting (lower θ_H) and more relatively low-quality firms to export for one period (higher θ_L). The net effect of this entry response is an increase in average quality, which creates a positive externality on firms that would be exporting regardless of the policy. They receive higher prices on their exports due to improved reputation. New exporters also benefit from the better reputation as well as the subsidy, so that the increase in aggregate profits exceeds the tax cost of the subsidy.[13] The welfare gain is a direct outcome of higher long-run reputation. (Figure A.1 in the online Appendix provides a numerical example of the economy's transition to its new, higher steady state in a case where the LQE is unique.)

On the contrary, in a HQE, a permanent export subsidy lowers average quality by inducing low-quality firms to stay exporters longer, while it does not change the incentives and decisions of high-quality firms which are all already exporters. The number of active low-quality firms increases but the number of active high-quality firms remains unchanged. The lower average quality in turn damages the country's reputation, which decreases the profits of all exporters. Because of this negative externality from the later exit of low-quality firms, the overall increase

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[12]Since there are no domestic consumers in our model and foreign consumers are not taken into account in national welfare calculations, the welfare of the exporting country is composed of exporters' profits and fiscal balance.

[13]In a setting where firms would set prices in a competitive way, we would have to balance this gain against the argument that an export subsidy tends to subsidize foreign consumers.

in aggregate profits of all firms receiving the subsidy is not large enough to cover the cost of the policy, despite a higher volume of sales.[14] (Online Appendix Figure A.2 illustrates the transition to the new steady state in a HQE when an export subsidy is introduced.)

Hence, depending on the kind of equilibrium regime considered (HQE or LQE), an export subsidy could raise or lower welfare in the exporting country. Overall, the desirability of an export subsidy depends on the trade-off between encouraging entry by high-quality firms which are deterred by the cost of establishing a reputation, and inducing entry by low-quality fly-by-night firms. The former impact dominates for countries initially exporting primarily low-quality goods, while the latter prevails for countries that already export a large share of high-quality goods. This result is new compared to the existing literature.[15] It reconciles the argument that export subsidies can help high-quality firms enter a market when they are initially unable to separate from low-quality firms (as in Bagwell and Staiger, 1989) with the criticism by Grossman and Horn (1988) according to which an export subsidy does encourage entry but the marginal entrants are the ones with the greatest incentive to produce low-quality goods. Our model delivers both of these predictions, depending on the initial equilibrium type. It suggests, in particular, that the gain from such policies – or lack thereof – may critically depend on the level of development and export sophistication of the exporting country.

5.2 A Tax/Subsidy Program

Let us now assume that the government is able to observe the "age" of a firm, i.e. the number of periods it has previously exported. Starting from a LQE, is there a tax/subsidy scheme based on the duration of export experience that will replicate the perfect information outcome? It is noteworthy to observe first that a temporary subsidy (subsidizing entrants) can never do better than a permanent subsidy to all exporters. A subsidy that targets new firms only disproportionately benefits low-quality firms, which account for a higher share of entrants than

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[14]Intuitively, we can decompose the welfare effect into two components. For the combination of quality and export experience for which firms are active both with and without the subsidy, the effect is negative: they receive lower prices and the additional profits brought about by the subsidy are taken out of taxes. For the additional periods in which firms below θ^* stay in the market because of the subsidy, their profits fall short of the cost of the subsidy: otherwise, since the price is lower than in the absence of the policy, they would have been exporting without the subsidy. Therefore, the total welfare change is unambiguously negative.

[15]The case for export subsidies is mixed in the existing literature. Brander and Spencer (1985) first introduced the idea of welfare-enhancing subsidies in a Cournot strategic rivalry, and Greenwald and Stiglitz (2006) developed an infant-industry argument for protective trade policy. Mayer (1984) also argues that when actual consumption experiences are required to learn about a commodity's qualities, subsidization of infant-exporters can be justified as a first-best policy measure. Modeling explicitly the process of consumer learning and expectations formation, Bagwell and Staiger (1989) explore the role of export subsidies when foreign goods are initially of unknown quality to domestic consumers; they show that while absent export subsidies entry of high-quality firms may be blocked by their inability to sell at prices reflecting their true quality, export subsidies can break this entry barrier and increase welfare. Flam and Helpman (1987) find that the desirability of export subsidies is ambiguous, depending on the production structure. Demidova and Rodriguez-Clare (2009) show that subsidies improve productivity in a model with heterogeneous firms, but are nonetheless detrimental to welfare due to losses in terms of trade and variety. The main arguments for active trade policy relying on coordination failures and externalities are reviewed and assessed in Harrison and Rodríguez-Clare (2010).

incumbents. As such, infant industry protection in the traditional sense is counterproductive in our model: it would lower average quality, worsen the country's reputation and hurt overall profits. A more promising alternative is to tax entrants and subsidize incumbents, so as to improve the relative payoffs of high-quality firms compared to low quality firms. In order to replicate the perfect information equilibrium, the tax should deter firms below θ^* from entering the market, but the subsequent subsidy should be sufficient to enable all firms above θ^* to earn positive intertemporal profits. However, the design of the tax/subsidy scheme must take into account the fact that if it is successful, the endogenous reputation change also affects the profitability of entry and exit. In fact, we show that when using taxes and subsidies based on age to replicate the perfection information outcome, the policy may actually involve taxing firms in a number of periods and only subsidizing the "oldest" exporters.

Proposition 4 (Taxes and Subsidies based on Export Experience)
The perfect information entry and exit decisions by quality and export experience can be replicated by the following tax/subsidy scheme, where τ_s is the tax (possibly negative) levied on a firm that has previously exported $s - 1$ times:

$$\tau_1 = \left(\frac{\alpha}{\alpha - 1}\right)\frac{k}{1 - w} - k - w\theta_m > 0$$

$$\tau_s = \left[1 - \rho\left(s - 1\right) + \frac{1 - \delta}{\delta}\right]\frac{1}{\alpha - 1}\frac{k}{1 - w} - \left(\frac{1 - \delta}{\delta}\right)\tau_1 \quad for \ \ s > 1$$

The resulting steady-state average quality and reputation are $\left(\frac{\alpha}{\alpha - 1}\right)\theta^*$.

Proof: see Appendix A.4.

The tax/subsidy scheme needs to satisfy three conditions. First, the first-period tax needs to be high enough that no firm below θ^* earns positive profits in the first period. This is ensured by setting τ_1 at least equal to the first-period profits of the lowest quality firm. Second, the taxes on incumbents need to be large enough not to induce any firm below θ^* to enter and stay beyond the first period despite initial losses. Third, the taxes on incumbents also need to be small enough to allow all high-quality firms above θ^* to realize positive expected profits over time. These three conditions are achieved by setting the tax and subsidy rates as in Proposition 4. An important point is that there needs not be a large subsidy for continuing exporters, because the endogenous change in reputation acts as an implicit subsidy for firms which are still active exporters after the policy is put in place. By discouraging the entry of low-quality firms, the tax on entrants improves the country reputation, which allows the firms that remain in the market to charge higher prices. The price effect makes up partially or fully for the effect of the tax on profits, in such a way that the government needs to compensate these firms less through later subsidies.

The large first-period tax can be interpreted as an export license. As long as the government can observe the export tenure of each firm and there is no cost of collecting taxes and distributing subsidies, this tax/subsidy scheme is optimal from a global welfare point of view. However, the impact on domestic welfare (not taking into account consumer surplus in the foreign country) is ambiguous. On the one hand, the low-quality firms that would export without the tax (below the initial θ_L) lose profits. On the other hand, the firms that become exporters because of the policy (initially between θ^* and θ_H) gain as they are now able to realize positive profits; and firms that export in both cases (above θ_H) gain from the improvement in country reputation.

Lastly, a caveat is that this policy may suffer from a time inconsistency problem. Once low-quality firms have decided not to enter, the government has no further incentive to carry through with the announced taxes and subsidies on continuing exporters. This concern can be alleviated by the fact that it is a repeated game. In our model with an infinite horizon and new cohorts of firms every period, if new firms can observe the taxes paid and subsidies received by older generations of firms, the announced policies need to be implemented for the government to maintain its credibility with new entrants and sustain the higher steady-state equilibrium.

5.3   Export Promotion Campaign and Reputation Shocks

An export promotion campaign is an effort to promote the quality of domestic goods abroad, e.g. through advertisements by export promotion agencies or exporters' associations, or by hosting "mega-events" to showcase the country. For instance the Olympic Games in Seoul in 1988, as well as in Beijing in 2008, were explicitly assigned the goal of promoting the hosting country's exports by government officials.[16] Conversely, negative reputation shocks have been analyzed empirically through event studies, such as the negative perception of France in the US at the onset of the Iraq war (Michaels and Zhi, 2010) and recalls of Chinese toys (Freedman et al., 2012). We model an export promotion campaign as a one-shot increase in the national image μ_t from the initial steady state, absent any changes in the underlying quality distribution of firms. More generally, the analysis below applies to reputation shocks – positive or negative – not driven by changes in the quality distribution. We focus on situations in which the economy is initially in a stable low-quality equilibrium.

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[16]Preuss (2004) discusses the Korean case and how the Seoul games were intended to *"raise international awareness of Korean manufactured products' so as to promote Korean exports."* Rose and Spiegel (2011) study the relevance of this argument. They show that hosting the Olympics boosts a country's subsequent exports and find support for the hypothesis that hosting "mega-events" serves as a policy signal.

5.3.1 Unique Steady State

If the economy has only one long-run equilibrium, it must return to this steady state in the long run. The export promotion campaign only has short-run effects on the distribution of quality (Figure A.3 in the online Appendix provides an example of the transition dynamics associated with a positive shock.) An export promotion campaign results in a one-shot increase in national reputation μ_t, starting from the steady state. The initial jump in reputation fosters entry by firms in segments of the quality distribution where they were previously inactive: θ_H decreases and θ_L increases. The net effect of the entry response is a drop in average quality $\bar{\theta}$ immediately after the shock occurs. Thus, the gap between actual and perceived average quality leads national reputation to adjust downwards in the following periods. As the country's reputation moves back down, the range of qualities for which entrants choose to stay inactive widens again, driving average quality back up until it has reverted to its original steady-state value, along with reputation. There are no long-run effects.

5.3.2 Multiple Steady States

If the economy has multiple steady states, there are several low-quality equilibria. Figure 2a above provides an illustration of this case. Assume the country starts in a stable LQE μ_S. If there are no steady states with higher reputation than μ_S, an export promotion campaign has the same effects as when the steady state is unique.

If there exists a steady state $\mu > \mu_S$, there must be an even number of steady states with $\mu > \mu_S$. Let us define $\mu_U > \mu_S$ such that μ_U is a steady state and for all $\mu_S < \mu < \mu_U$, μ is not a steady state. Similarly, define $\mu'_S > \mu_U$ such that μ'_S is a steady state and for all $\mu_U < \mu < \mu'_S$, μ is not a steady state. μ_U is unstable and μ'_S is stable.

Starting in μ_S, a "small" promotion campaign moves national reputation to a level μ_t such that $\mu_S < \mu_t < \mu_U$. The impact of a small campaign is similar to the case with a unique equilibrium: in the long run, the economy returns to μ_S. A "large" promotion campaign moves national reputation to $\mu_t > \mu_U$. Then, the resulting entry by the intermediate range of middle-quality firms that were not exporting before the shock (firms below the initial θ_H and above the initial θ_L) leads to an increase in average quality, magnifying the shock. Actual quality follows reputation in a self-fulfilling manner. Quality and reputation keep rising until the economy settles in the more favorable steady-state μ'_S. In the example of Figure 2a, μ'_S is a high-quality equilibrium. (Figure A.4 in the online Appendix illustrates the transition to the new steady state.) These results are summarized in Proposition 5.

Proposition 5 (Positive Reputation Shocks)

(i) Starting from a unique LQE or from a LQE that has the highest μ among steady states, a one-time positive shock to national reputation μ_t increases aggregate profits and decreases

average quality in the short run, and has no effect in the long run.

(ii) Starting from a stable LQE μ_S such that there exist other steady states above μ_S, a small one-time positive shock ($\mu_t < \mu_U$ as defined above) to national reputation increases aggregate profits and may increase or decrease average quality in the short-run, and has no effect in the long-run.

(iii) Starting from a stable LQE μ_S such that there exist other steady states above μ_S, a large one-time positive shock ($\mu_t > \mu_U$ as defined above) to national reputation increases aggregate profits and average quality both in the short-run and in the long-run.

Proof: see Appendix A.5.

To sum up, a policy which brings about a positive shock to national reputation has only short-lived effects on the quality distribution of exporters and on aggregate profits in a unique steady state or if the shock is small. However, a large shock starting from a low-reputation, low-quality equilibrium is self-fulfilling when the economy has multiple steady states. It encourages entry by high-quality firms. In the segments of the average quality-reputation function where entry by high-quality firms drives up quality more than entry by lower-quality firms drives it down, a one-shot increase in reputation brings about a permanent increase in quality, profits and welfare. To be successful in the long run, an export promotion campaign based solely on improving the country's brand image must therefore induce a large jump in beliefs. A negative reputation shock has the opposite effects, as stated in Corollary 1.

Corollary 1 (Negative Reputation Shocks)

(i) Starting from a unique LQE or from a LQE that has the lowest μ among steady states, a one-time negative shock to national reputation μ_t reduces aggregate profits and increases average quality in the short-run, and has no effect in the long-run.

(ii) Starting from a stable LQE μ'_S such that there exist other steady states below μ'_S, a small one-time negative shock ($\mu_t > \mu_U$ as defined above) to national reputation reduces aggregate profits and may increase or decrease average quality in the short-run, and has no effect in the long-run.

(iii) Starting from a stable LQE μ'_S such that there exists other steady states below μ'_S, a large one-time negative shock ($\mu_t < \mu_U$ as defined above) to national reputation reduces aggregate profits and average quality both in the short-run and in the long-run.

This last result implies that there can be long-term consequences of a sudden large drop in reputation, which moves a country to a less desirable steady-state equilibrium. In particular, large product recalls or heavily mediatized consumer safety scandals concerning exports of one country can permanently affect the structure of its industry, lowering both quality and reputation in the long-run.

5.4 Minimum Quality Standards

Finally, we examine the effect of imposing minimum quality standards. Quality standards have been used by Japan at the end of World War II. At that time, "Made in Japan" goods had a reputation for being cheap low-quality goods. Japanese companies were suffering from an inferior "national brand". To improve this "national brand", both Japanese private companies and the government decided to impose strict quality norms (Matsushita, 1979). They formed export cartels which provided product quality guarantees, by setting product design and quality standards, establishing industry brand names, guaranteeing delivery schedules and mediating disputes between individual exporters and foreign buyers (Dyck, 1992). Providing product quality assurances to importers stimulated growth in exports, improved terms of trade, and was key to establish a reputation for product quality (Lynn and McKeown, 1988).

We model a quality standard by a "quality standard threshold" θ_{MQ}.[17] We assume that if a firm of quality $\theta < \theta_{MQ}$ is controlled, then it has to exit the export market and cannot re-enter later.[18] Inspection is costly for the government and we denote the inspection cost c, where $c = c(\theta_{MQ})$. To the extent that it is more difficult to certify higher quality, it is natural to expect $c'(\theta_{MQ}) \geq 0$.

If we first assume that the controlling cost is negligible and independent of the quality threshold, then it is obvious that from a global welfare point of view, it is optimum to set $\theta_{MQ} = \theta^*$. In other words, it is socially inefficient from a global welfare point of view to let firms of quality θ below θ^* to enter the export market. Is it optimal for the domestic country? For the exporting country, the welfare effect of the minimum quality standard has three components (see Appendix A.6 for more details). Let us call μ_{init} the initial steady-state reputation μ, and define $\theta_{L,\text{init}} \equiv \frac{\mu_{\text{init}} - k}{w}$ and $\theta_{H,\text{init}} \equiv \frac{k - (1 - A_\rho)\mu_{\text{init}}}{A_\rho - w}$. First, for firms with quality $\theta < \theta_{L,\text{init}}$, the effect is unambiguously negative: these firms were exporting as fly-by-night before the introduction of the minimum standard, making positive profits. With the minimum quality standard, they cannot enter the export market. Second, for firms with quality $\theta^* < \theta < \theta_{H,\text{init}}$, the effect is unambiguously positive: these firms were not exporting before the introduction of the minimum standard. With the minimum quality standard, they now enter the export market and remain exporters until they are hit by the exogenous exit shock. The value of their profit stream is positive. Third, for firms with quality $\theta > \theta_{H,\text{init}}$, the effect is also unambiguously positive: these firms receive extra profits brought by a higher reputation. The lower α (the shape parameter of the distribution), the lower the loss from the minimum quality standard. Intuitively, a low α means that there are more firms at the right tail of the distribution, and so relatively less firms that lose from the standard. The

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[17]For the sake of simplicity, we assume that the inspection probability is always equal to 100%.

[18]The minimum quality standard acts as the exogenous exit shock δ, but with an important difference: the controlled firm has to exit the export market *after* it incurs the production cost $w\theta + k$ and *before* it can sell its good, i.e. make any profits. The controlled firms of quality $\theta < \theta_{MQ}$ thus makes losses.

lost profits due to the minimum quality standard also decrease with production costs, while they increase with the initial reputation: the higher the reputation, the higher the profits the fly-by-night firms were able to extract when they exported for one period.

There are two important caveats to this insight. First, even if the controlling process is costless and the minimum quality standard $\theta_{MQ} = \theta^*$ is welfare improving for the exporting country, under certain conditions, it might be in the interest of the exporting country to set the quality standard below θ^*. On the one hand, a lower quality standard reduces the importance of the profits forgone by firms that do not meet the standard. On the other hand, the higher the quality standard, the higher the profits of high-quality firms due to the higher reputation. Moreover, the quality standard has to be high enough for the economy to move from a LQE to a HQE. The second caveat is the following: if the controlling cost to certify quality is too high, one may need to supplement the minimum quality standard with export subsidies. Indeed, it may be too costly for the government to set a minimum quality standard high enough to support a HQE. In this case, it may be optimal to choose a lower quality standard and to subsidize firms (to induce more relatively high-quality firms to start and continue exporting).

6   Additional Predictions

This last section lays out additional empirical predictions of our model on the dynamic patterns of price and turnover rates among exporters.

6.1   Unit Prices

We first characterize the path of prices for a given cohort of firms. At the firm level, there is a "brand premium" for high-quality firms both in a HQE and in a LQE: the price charged increases over time for a given good provided that its quality is higher than the country average. Result 1 establishes that on average, incumbents receive higher prices than entrants, and the average price among a cohort of firms is higher, the longer the cohort has been active on export markets. This result follows from the fact that over time, an increasing fraction of prices reflect firms' true quality parameters, and the average quality of a cohort of firms weakly increases over time as the lowest quality firms exit.

Result 1 (Unit Prices)

In a steady-state low-quality equilibrium, the average unit price charged at $t + s$ by firms born at date t is strictly increasing in s.

In a steady-state high-quality equilibrium, the average unit price charged at $t + s$ by firms born at date t is strictly increasing in s for all s if $\mu > \frac{\alpha}{\alpha-1}\theta_1$ and for $s \geq T\left(\frac{\alpha-1}{\alpha}\mu\right)$ otherwise.

Proof: see Appendix A.7.

These predictions are supported by the findings of Foster et al. (2008) on the behavior of US firms in their domestic market. They show that entering businesses have significantly lower prices than incumbents, and that prices rise with plant age. The empirical literature on pricing strategies is also consistent with this result[19], although there can be other explanations for entrants charging lower prices.[20]

6.2   Hazard rate

A second prediction implied by our model is that firms' exit rates vary systematically with their quality and across cohorts.

Result 2 (Hazard Rate)   *In a steady-state equilibrium, the aggregate hazard rate is weakly decreasing in quality and in firms' export experience.*

Proof: see Appendix A.8.

The first part of Result 2 establishes that across cohorts, the fraction of active firms that exit per period is higher for lower-quality firms, both in a LQE and in a HQE. Low-quality firms exit voluntarily in finite time while high-quality firms only exit when hit by the exit shock. The second part states that the probability of exit, across quality levels, decreases with the age of a cohort. It derives from the fact that the distribution of quality among older cohorts has a higher lower bound than among younger cohorts.

This last prediction is consistent with the findings of Besedes and Prusa (2006) on survival rates in US import relationships at the disaggregated product level. They estimate that the probability that the import relationship will end falls with its duration for differentiated products.

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[19]Firm's usage of introductory pricing strategies is indeed often observed: software is usually offered at a very low price in initial periods whereas updates are expensive; phone companies or network providers have low price deals for new services (see e.g. van Ackere and Reyniers, 1995; Gabszewicz and Garcia, 2007). Benkard (2004) considers the market for wide-bodied commercial aircraft and gives the example of the Lockheed L-1011, whose price was below static marginal cost for essentially its entire 14-year production run, for a total variable loss of approximately 2.5 billion current dollars. Bhattacharya and Vogt (2003) show that in the market for pharmaceutical drugs, the price of a branded drug commonly rises after its patent expires and lower-price generic drugs enter the market.

[20]Introductory pricing strategies, whereby buyers would pay a low price in the initial period and offer a sequence of prices contingent on their future consumption experience, can be rationalized in several classes of models. See e.g. Gabszewicz et al. (1992) who examine first mover advantages in a new product market with sequential entry and show that the first entrant's optimal strategy is to set a low introductory price. Bagwell (1987) shows that in a two-period game between firms and consumers, where firms are privately informed about their individual costs, and consumers must pay a search cost in order to learn a firm's current price, consumers have incentive to use introductory price as a signal of cost.

7 Conclusion

We have shown that when consumers are not fully informed about the quality of what they buy, national reputation matters for exporters. The inability to reveal quality to consumers before purchase distorts the incentives to enter export markets for new firms. Low-quality firms rely on the national brand, while high-quality firms suffer from it. More broadly, unobservable quality tilts the long-run quality composition of an export-oriented industry towards its low end, all the more so as the exporting economy has a poor reputation for quality. In that respect, reputation has self-perpetuating features since future national reputation adjusts to past exports quality. These issues are particularly relevant for developing countries trying to grow into exporting increasingly sophisticated goods. National reputations create history dependence in the range of goods a country can successfully export. A damaged national reputation is a barrier to entry for companies that develop more expensive high-quality products, threatening the success of such a growth strategy. In those cases, we have examined several possible policy responses designed to enhance the quality reputation and welfare of an exporting country.

This analysis suggests several avenues for future empirical research. In our model, the rational expectations steady-state can fall into one of two categories. In a high-quality equilibrium, all firms are able to export: low-quality firms enter initially as fly-by-nights and eventually exit after a given number of periods; high-quality firms enter and keep exporting. In a low-quality equilibrium, an intermediate range of middle-quality firms never become active exporters. This difference in the sorting of firms into exporters and non exporters may be useful for future empirical research to identify low-quality traps. In addition, case studies have already provided evidence of the benefits of a reputation for quality in terms of brand premia (Imbs et al., 2010) and image spillovers across products of the same brand (Sullivan, 1990). We develop these insights further by considering the demand for imports, where initial priors depend on country-of-origin and reputations are built not only for specific firms but also for exporting countries as a whole. Our findings imply that future research could analyze empirically the extent to which country's reputation matters for trade flows.

Going further, our analysis provides a framework for a richer understanding of firms' sourcing decisions through the lens of a strategic use of "made in" rules. Exporters can find it optimal to resort to original equipment manufacturers or depart from the cost-minimizing way of splitting the production process across locations, in order to obtain a favorable country-of-origin denomination. The location of manufacturing and assembly will be decided not only according to cost considerations, but also depending on the regulations surrounding rules of origin, consumer sensitivity to quality, and the degree of asymmetric information in the industry. An extension of our model along these lines would generate testable predictions at the firm level. These topics will be investigated in future research.

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A   Proofs

A.1   Proof of Proposition 1: Sorting of Firms into Exports

A.1.1   High-Quality Equilibrium

Assume there is a steady state equilibrium where $\mu > \frac{k}{1-w}$. First, consider firms with $\theta < \theta^*$ born at date t. Since their expected profits are decreasing with time, they are active in the first period if and only if $E_t \pi_{t+1}(\theta) = \mu - w\theta - k > 0$, which requires $\theta \leq \frac{\mu-k}{w}$. Since $\mu > \frac{k}{1-w}$, it is straightforward that $\frac{\mu-k}{w} > \mu > \theta^*$ which ensures $E_t \pi_{t+1}(\theta) > 0$ for all firms born at t which have quality $\theta < \theta^*$. Hence all such firms enter initially. Also, $\theta < \frac{k}{1-w}$ and $\rho' > 0$ imply that

$$E_t \pi_{t+s}(\theta) = (\rho(s-1) - w)\theta + (1 - \rho(s-1))\mu - k$$

is decreasing in s and $\lim_{s \to \infty} E_t \pi_{t+s} = (1-w)\theta - k < 0$ so all firms below θ^* expect to exit in finite time when their profits turn negative. The expected number of periods a firm θ born at t is active is $T(\theta)$ given by

$$[1 - \rho(T(\theta-1))]\mu > [w - \rho(T(\theta) - 1)]\theta + k$$
$$\text{and} \quad [1 - \rho(T(\theta))]\mu < [w - \rho(T(\theta))]\theta + k$$

The highest quality type θ_T that exits after selling for T periods (or the lowest quality type that exits after selling for $T+1$ periods) is defined by $E_t \pi_{t+T+1}(\theta_T) = 0$, hence

$$\theta_T = \max\left\{ \frac{k - [1 - \rho(T)]\mu}{\rho(T) - w}, \theta_m \right\} \tag{20}$$

and θ_T is increasing with T:

$$\frac{\partial \theta_T}{\partial T} \propto \rho'(T)(\mu(1-w) - k) > 0 \quad \text{as } \rho' > 0 \ \text{ and } \mu > \theta^*$$

Second, consider firms with $\theta^* < \theta < \mu$. These firms expect positive profits at all periods: they have $E_t \pi_{t+s}(\theta)$ monotonically decreasing from

$$\pi_{t+1}(\theta) = \mu - w\theta - k > \mu(1-w) - k > 0 \quad \text{since } \theta < \mu$$

to

$$\lim_{s \to \infty} E_t \pi_{t+s}(\theta) = \theta(1-w) - k > 0 \quad \text{since } \theta > \theta^*$$

Hence firms with $\theta^* < \theta < \mu$ always enter the market and stay until they are exogenously forced to exit.

Finally, consider the highest quality firms with $\theta > \mu$. Such firms have increasing expected

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profits over time. They enter the market if and only if their expected intertemporal profits are positive, which requires:

$$E_t \left(\sum_{s=1}^{\infty} \delta^{s-1} \pi_{t+s} (\theta) \right) = \sum_{s=0}^{\infty} \delta^s \left[(\rho(s) - w) \theta + (1 - \rho(s)) \mu \right] - \frac{k}{1-\delta} > 0$$

or equivalently:

$$\theta > \frac{k - \mu (1-\delta) \sum_{s=0}^{\infty} \delta^s (1 - \rho(s))}{(1-\delta) \sum_{s=0}^{\infty} \delta^s \rho(s) - w} \equiv \theta_H \tag{21}$$

Let us show that $\theta_H < \mu$. Rearranging:

$$\theta_H < \mu \ \ \text{iff} \ \ \mu \left[(1-\delta) \sum_{s=0}^{\infty} \delta^s (1 - \rho(s)) + (1-\delta) \sum_{s=0}^{\infty} \delta^s \rho(s) \right] > w\mu + k$$

or equivalently iff $\mu > \frac{k}{1-w}$ which holds by assumption in the high reputation case. Hence all firms with $\theta > \mu$ always export until they are hit by the exogenous shock.

A.1.2 Low-Quality Equilibrium

Assume there is a steady state equilibrium with $\mu < \frac{k}{1-w}$. First, consider firms with $\theta < \mu$ born at date t. Since their expected profits are decreasing with time, they are active in the first period if and only if $E_t \pi_{t+1} (\theta) = \mu - w\theta - k > 0$, which requires $\theta \leq \frac{\mu - k}{w} \equiv \theta_L$. We can immediately check that $\mu < \frac{k}{1-w} \Leftrightarrow \theta_L < \mu$. Expected second-period profits are

$$E_t \pi_{t+2} (\theta) = (\rho(1) - w) \theta + (1 - \rho(1)) \mu - k < (1 - w) \mu - k < 0$$

since $\theta < \mu$ and $\rho(1) > w$. Hence among firms with $\theta < \mu$, those with $\theta < \theta_L$ are active in the first period and exit afterwards, and those with $\theta_L \leq \theta < \mu$ are never active.

Second, consider firms with $\mu \leq \theta < \theta^*$. These firms have $E_t \pi_{t+1} (\theta) < 0$ since $\theta > \theta_L$, $E_t \pi_{t+s} (\theta)$ monotonically increasing in s since $\theta \geq \mu$, and $\lim_{s \to \infty} E_t \pi_{t+s} (\theta) < 0$ since $\theta < \theta^*$. Thus their expected profits are negative in all periods and they optimally exit after drawing their quality parameter.

Third, consider firms with $\theta > \theta^*$. These firms have $E_t \pi_{t+s} (\theta)$ monotonically increasing in s since $\theta > \mu$, and $\lim_{s \to \infty} E_t \pi_{t+s} (\theta) > 0$ since $\theta > \theta^*$. If they decide to be active in the first period, they expect to remain in the market as long as they survive the exogenous shock. However given $\theta > \theta_L$ they incur a loss in the initial periods. The condition for a firm of type $\theta > \theta^*$ to be active is for intertemporal expected profits to be positive, which requires $\theta > \frac{k - (1 - A_\rho) \mu}{A_\rho - w} \equiv \theta_H$ as derived in (21), where we define $A_\rho \equiv (1 - \delta) \sum_{s=0}^{\infty} \delta^s \rho(s)$. Finally,

$$\theta_H > \theta^* \Leftrightarrow \frac{k - (1 - A_\rho) \mu}{A_\rho - w} > \frac{k}{1-w} \Leftrightarrow k (1 - A_\rho) > (1 - A_\rho) \mu (1 - w)$$

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which is equivalent to $\frac{k}{1-w} > \mu$ and holds by definition in the low reputation case. Hence firms with $\theta^* \leq \theta \leq \theta_H$ are never active and firms with $\theta > \theta_H$ enter the export market and stay active.

Lastly, the equilibrium reputation must satisfy $\mu > k + w\theta_m$. Suppose μ is a steady state reputation and $\mu < k + w\theta_m$. The first period price μ does not cover the production cost of the lowest quality firm, hence no firm below θ^* finds it profitable to enter. It follows that no rational expectations equilibrium can have $\mu < \theta^*$, so there is no LQE with $\mu < k + w\theta_m$. Similarly, if $\theta^* < \mu < k + w\theta_m$, then firms with $\theta < \mu$ would have negative profits in all periods. Hence such firms are never active, and μ cannot be a HQE reputation under rational expectations.

A.2 Proof of Proposition 2: Existence Conditions

Proposition 2 establishes the existence of at least one steady state equilibrium and the possibility of multiple equilibria. We investigate how $\overline{\theta}(\mu)$ varies with μ on $[\theta_m, \infty)$. Then, we compute the fixed points of the function $\overline{\theta}(\mu)$, and show that there can be multiple fixed points.

A.2.1 Average Quality Function

Given Proposition 1 and equation (11), the average quality $\overline{\theta}$ of active firms in a steady state as a function of the country reputation μ is, on $[\theta_m, \theta^*]$:

$$\overline{\theta}(\mu) = \mu_0 \left(\frac{(1-\delta)\left(1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha-1}\right) + \left(\frac{\theta_m}{\theta_H}\right)^{\alpha-1}}{(1-\delta)\left(1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha}\right) + \left(\frac{\theta_m}{\theta_H}\right)^{\alpha}} \right) \tag{22}$$

and on $[\theta^*, \infty)$:

$$\begin{aligned}\overline{\theta}(\mu) &= \mu_0 \left(\frac{1 - \sum_{T=0}^{\infty} \delta^{T+1}\left(\left(\frac{\theta_m}{\theta_T}\right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_{T+1}}\right)^{\alpha-1}\right)}{1 - \sum_{T=0}^{\infty} \delta^{T+1}\left(\left(\frac{\theta_m}{\theta_T}\right)^{\alpha} - \left(\frac{\theta_m}{\theta_{T+1}}\right)^{\alpha}\right)} \right) \\ &= \mu_0 \left(\frac{1 + \sum_{T=\tilde{T}}^{\infty} \delta^{T}\left(\frac{\theta_m}{\theta_T}\right)^{\alpha-1}}{1 + \sum_{T=\tilde{T}}^{\infty} \delta^{T}\left(\frac{\theta_m}{\theta_T}\right)^{\alpha}} \right)\end{aligned} \tag{23}$$

where $\tilde{T}$ is the lowest value of T such that $\theta_T > \theta_m$ in equation (20). At $\mu = \theta^*$, $\theta_L = \theta_H = \theta^*$ and $\theta_T = \theta^*$ for all T. It follows that the function $\overline{\theta}(\mu)$ is continuous at θ^* since both equations

above yield:

$$\overline{\theta}\left(\theta^*\right) = \mu_0 \left(\frac{1 - \delta + \delta \left(\frac{\theta_m}{\theta^*}\right)^{\alpha-1}}{1 - \delta + \delta \left(\frac{\theta_m}{\theta^*}\right)^{\alpha}} \right) \tag{24}$$

A.2.2 Equilibrium Existence

A steady state equilibrium is a fixed point of the function $\overline{\theta}\left(\mu\right)$ defined in A.2.1. We have already established that the function is continuous on $[\theta_m, \infty)$. Let us show in addition that $\overline{\theta}\left(\theta_m\right) > \theta_m$ and $\lim_{\mu \to \infty} \frac{\overline{\theta}(\mu)}{\mu} < 1$.

If $\mu = \theta_m$, no firm below θ^* finds it profitable to export, as national reputation imposes a first-period loss on all firms. Some firms with high enough θ have a positive NPV of future profits and enter. So since θ_m is the lower bound of the prior quality distribution, $\overline{\theta}\left(\theta_m\right) > \theta^* > \theta_m$.

As $\mu \to \infty$, it remains profitable for all firms to stay active, so firms of all qualities continue exporting until hit by the exogenous shock: $T\left(\theta\right) \to \infty$ for all θ. Therefore, $\lim_{\mu \to \infty} \overline{\theta}\left(\mu\right) = \mu_0$ which is finite, so $\lim_{\mu \to \infty} \frac{\overline{\theta}(\mu)}{\mu} = 0 < 1$.

By the fixed point theorem, we have established that $\overline{\theta}\left(.\right)$ has at least one fixed point on $[\theta_m, \infty)$.

A.2.3 Existence Condition for a High Quality Equilibrium

A HQE is a steady state equilibrium with equilibrium reputation above θ^*. Therefore a sufficient condition for the existence of a HQE is $\overline{\theta}\left(\theta^*\right) > \theta^*$ in (24). We then prove that this is also a necessary condition and that if there exists at least one HQE, there is only one HQE.

Let us show that $\overline{\theta}\left(\mu\right)$ is strictly decreasing in μ on $[\theta^*, \infty)$. We can rewrite (23) as:

$$\overline{\theta}\left(\mu\right) = \mu_0 \left(\frac{1 + K\left(\alpha - 1\right)}{1 + K\left(\alpha\right)} \right) \quad \text{where } K\left(\alpha\right) \equiv \sum_{T=\tilde{T}}^{\infty} \delta^T \left(\frac{\theta_m}{\theta_T}\right)^{\alpha}$$

$$\frac{\partial K\left(\alpha\right)}{\partial \alpha} = \sum_{T=\tilde{T}}^{\infty} \delta^T \ln\left(\theta_m / \theta_T\right) \left(\frac{\theta_m}{\theta_T}\right)^{\alpha} < 0$$

Consider a positive change in one of the thresholds, θ_S, leaving unchanged all other thresholds. Then all else equal, average quality rises:

$$\begin{aligned}
\frac{\partial \overline{\theta}}{\partial \theta_S} &= \frac{\delta^S}{\theta_S} \left(\frac{\theta_m}{\theta_S}\right)^{\alpha-1} \left[\alpha \left(\frac{\theta_m}{\theta_S}\right) \left(1 + K\left(\alpha - 1\right)\right) - \left(\alpha - 1\right)\left(1 + K\left(\alpha\right)\right) \right] \\
&= \frac{\delta^S}{\theta_S} \left(\frac{\theta_m}{\theta_S}\right)^{\alpha-1} \left(1 + K\left(\alpha\right)\right) \left[\alpha \left(\frac{\theta_m}{\theta_S}\right) \frac{\overline{\theta}}{\mu_0} - \left(\alpha - 1\right) \right] \\
&= \frac{\delta^S}{\theta_S} \left(\frac{\theta_m}{\theta_S}\right)^{\alpha-1} \left(1 + K\left(\alpha\right)\right) \left(\alpha - 1\right) \left[\left(\frac{\overline{\theta}}{\theta_S}\right) - 1 \right] > 0
\end{aligned}$$

which derives from $\bar{\theta} > \theta^* > \theta_S$ for all S in a HQE. An increase in μ lowers all θ_T given Assumptions 1 and 2 and differentiating:

$$\frac{\partial \theta_T}{\partial \mu} = -\frac{1 - \rho(T)}{\rho(T) - w} < 0$$

Thus, in a HQE, $\bar{\theta}(\mu)$ is a decreasing function:

$$\frac{\partial \bar{\theta}}{\partial \mu} = \sum_{T=\tilde{T}}^{\infty} \frac{\partial \bar{\theta}}{\partial \theta_T} \frac{\partial \theta_T}{\partial \mu} < 0$$

We have proved that $\bar{\theta}(.)$ is strictly and continuously decreasing in μ on $[\theta^*, \infty)$. It follows that $\bar{\theta}(.)$ has at most one fixed point on $[\theta^*, \infty)$. Hence $\bar{\theta}(\theta^*) > \theta^*$ is a necessary and sufficient condition for the existence of a HQE, and if this condition is satisfied, there is only one HQE.

Lastly, we express the condition for $\bar{\theta}(\theta^*) > \theta^*$ as a function of fundamental parameters. At $\mu = \theta^*$, $\pi_t(\theta) < 0$ for all $t > 1$ and $\theta < \theta^*$. Then:

$$\bar{\theta}(\theta^*) = \frac{\int_{\theta_m}^{\theta^*} \theta dG(\theta) + \frac{1}{1-\delta} \int_{\theta^*}^{\infty} \theta dG(\theta)}{\int_{\theta_m}^{\theta^*} dG(\theta) + \frac{1}{1-\delta} \int_{\theta^*}^{\infty} dG(\theta)} = \mu_0 \left(\frac{1 - \delta + \delta \left(\frac{\theta_m}{\theta^*} \right)^{\alpha-1}}{1 - \delta + \delta \left(\frac{\theta_m}{\theta^*} \right)^{\alpha}} \right)$$

So $\bar{\theta}(\theta^*) > \theta^*$ is equivalent to $\mu_0 \left(\frac{1 - \delta + \delta \left(\frac{\theta_m(1-w)}{k} \right)^{\alpha-1}}{1 - \delta + \delta \left(\frac{\theta_m(1-w)}{k} \right)^{\alpha}} \right) > \frac{k}{1-w}$, which after substituting for the value of μ_0 and rearranging yields the following condition for the existence of a HQE:

$$\alpha \left(\frac{\theta_m(1-w)}{k} \right) + \frac{\delta}{1-\delta} \left(\frac{\theta_m(1-w)}{k} \right)^{\alpha} > \alpha - 1$$

Conversely, there is no HQE and there is at least one LQE iff:

$$\alpha \left(\frac{\theta_m(1-w)}{k} \right) + \frac{\delta}{1-\delta} \left(\frac{\theta_m(1-w)}{k} \right)^{\alpha} < \alpha - 1$$

A.2.4 Multiple Equilibria

While there cannot be more than one HQE, we show that the non-monotonicity of $\bar{\theta}(.)$ on $[\theta_m, \theta^*]$ creates the possibility of multiple equilibria. A LQE is a fixed point of $\bar{\theta}(\mu)$ given by (22) on $[\theta_m, \theta^*]$. On this interval, we can show that the sign of $\frac{\partial \bar{\theta}(\mu)}{\partial \mu}$ is indeterminate. Differentiating with respect to each threshold:

$$\frac{\partial \overline{\theta}}{\partial \theta_L} = \frac{\mu_o(\alpha-1)}{1-\left(\frac{\theta_m}{\theta_L}\right)^\alpha + \frac{1}{1-\delta}\left(\frac{\theta_m}{\theta_H}\right)^\alpha}\left(\frac{1}{\theta_L}\right)\left(\frac{\theta_m}{\theta_L}\right)^{\alpha-1}\left[1-\frac{\overline{\theta}}{\theta_L}\right] < 0$$

$$\frac{\partial \overline{\theta}}{\partial \theta_H} = \frac{\frac{1}{1-\delta}\mu_o(\alpha-1)}{1-\left(\frac{\theta_m}{\theta_L}\right)^\alpha + \frac{1}{1-\delta}\left(\frac{\theta_m}{\theta_H}\right)^\alpha}\left(\frac{1}{\theta_H}\right)\left(\frac{\theta_m}{\theta_H}\right)^{\alpha-1}\left[\frac{\overline{\theta}}{\theta_H}-1\right] < 0$$

$$\frac{\partial \overline{\theta}}{\partial \mu} = \frac{\partial \overline{\theta}}{\partial \theta_L}\frac{\partial \theta_L}{\partial \mu} + \frac{\partial \overline{\theta}}{\partial \theta_H}\frac{\partial \theta_H}{\partial \mu} = \frac{\mu_o(\alpha-1)}{1-\left(\frac{\theta_m}{\theta_L}\right)^\alpha + \frac{1}{1-\delta}\left(\frac{\theta_m}{\theta_H}\right)^\alpha} \times \ldots$$

$$\ldots \times \left[\left(\frac{\theta_m^{\alpha-1}}{\theta_L^\alpha}\right)\left[\frac{\overline{\theta}}{\theta_L}-1\right]\left(\frac{1}{w}\right) - \frac{1}{1-\delta}\left(\frac{\theta_m^{\alpha-1}}{\theta_H^\alpha}\right)\left[1-\frac{\overline{\theta}}{\theta_H}\right]\left(\frac{1-A_\rho}{A_\rho-w}\right)\right]$$

$$\frac{\partial \overline{\theta}}{\partial \mu} < 0 \text{ iff } \frac{1}{1-\delta}\left(\frac{1}{\theta_H}\right)^\alpha\left[1-\frac{\overline{\theta}}{\theta_H}\right]\left(\frac{1-A_\rho}{A_\rho-w}\right) > \left(\frac{1}{\theta_L}\right)^\alpha\left[\frac{\overline{\theta}}{\theta_L}-1\right]\left(\frac{1}{w}\right)$$

This condition can be rewritten as:

$$\delta > 1 - \left(\frac{\theta_L}{\theta_H}\right)^{\alpha+1}\left(\frac{\theta_H-\overline{\theta}}{\overline{\theta}-\theta_L}\right)\left(\frac{(1-A_\rho)w}{A_\rho-w}\right)$$

Then note that the bracketed terms are $\frac{\theta_L}{\theta_H} = \frac{(\mu-k)(A_\rho-w)}{w(k-(1-A_\rho)\mu)}$ and $\frac{\theta_H-\overline{\theta}}{\overline{\theta}-\theta_L} = \frac{\frac{k-(1-A_\rho)\mu}{A_\rho-w}-\overline{\theta}}{\overline{\theta}-\frac{\mu-k}{w}} = \frac{w}{A_\rho-w}\left(1-\frac{A_\rho(\overline{\theta}-\mu)}{k-\mu+w\overline{\theta}}\right)$. Therefore $\overline{\theta}(\mu)$ decreases in μ when:

$$\delta > 1 - \left(\frac{\mu-k}{k-(1-A_\rho)\mu}\right)^{\alpha+1}\left(\frac{A_\rho-w}{w}\right)^{\alpha-1}\left(1-\frac{A_\rho(\overline{\theta}-\mu)}{k-\mu+w\overline{\theta}}\right)$$

and increases in μ otherwise. The reason why $\overline{\theta}(\mu)$ needs not be monotonic over $[\theta_m, \theta^*]$ is that μ has opposite effects on $\overline{\theta}$ coming from θ_L and θ_H. Which effect dominates depends on the position of μ as well as the shape parameter α and the survival parameter δ. This non-monotonicity is what gives rises to the possibility of multiple equilibria.

To sum up, we have shown that if condition (19) holds, there is one HQE and an even number of LQEs (possibly zero). If condition (19) does not hold, there is no HQE and there is an odd number of LQEs.

A.3 Proof of Proposition 3: Export Subsidies

A.3.1 Low-Quality Equilibrium

In a LQE, average quality is given by:

$$\bar{\theta} = \mu_0 \left(\frac{(1-\delta)\left(1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha-1}\right) + \left(\frac{\theta_m}{\theta_H}\right)^{\alpha-1}}{(1-\delta)\left(1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha}\right) + \left(\frac{\theta_m}{\theta_H}\right)^{\alpha}} \right)$$

Differentiating with respect to each threshold:

$$\frac{\partial \bar{\theta}}{\partial \theta_L} = \frac{\mu_o (\alpha - 1)}{1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha} + \frac{1}{1-\delta}\left(\frac{\theta_m}{\theta_H}\right)^{\alpha}} \left(\frac{1}{\theta_L}\right)\left(\frac{\theta_m}{\theta_L}\right)^{\alpha-1}\left(1 - \frac{\bar{\theta}}{\theta_L}\right) < 0$$

$$\frac{\partial \bar{\theta}}{\partial \theta_H} = \frac{\frac{1}{1-\delta}\mu_o (\alpha - 1)}{1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha} + \frac{1}{1-\delta}\left(\frac{\theta_m}{\theta_H}\right)^{\alpha}} \left(\frac{1}{\theta_H}\right)\left(\frac{\theta_m}{\theta_H}\right)^{\alpha-1}\left(\frac{\bar{\theta}}{\theta_H} - 1\right) < 0$$

$$\frac{\partial \bar{\theta}}{\partial k} = \frac{\partial \bar{\theta}}{\partial \theta_L}\frac{\partial \theta_L}{\partial k} + \frac{\partial \bar{\theta}}{\partial \theta_H}\frac{\partial \theta_H}{\partial k} = \frac{\mu_o (\alpha - 1)}{1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha} + \frac{1}{1-\delta}\left(\frac{\theta_m}{\theta_H}\right)^{\alpha}} \times \ldots$$

$$\times \left[\frac{1}{1-\delta}\left(\frac{1}{\theta_H}\right)\left(\frac{\theta_m}{\theta_H}\right)^{\alpha-1}\left(1 - \frac{\bar{\theta}}{\theta_H}\right)\left(\frac{1}{A_\rho - w}\right) - \left(\frac{1}{\theta_L}\right)\left(\frac{\theta_m}{\theta_L}\right)^{\alpha-1}\left(\frac{\bar{\theta}}{\theta_L} - 1\right)\left(\frac{1}{w}\right)\right]$$

$$\frac{\partial \bar{\theta}}{\partial k} > 0 \text{ iff } \frac{1}{1-\delta}\left(\frac{1}{\theta_H}\right)^{\alpha}\left(1 - \frac{\bar{\theta}}{\theta_H}\right)\left(\frac{1}{A_\rho - w}\right) > \left(\frac{1}{\theta_L}\right)^{\alpha}\left(\frac{\bar{\theta}}{\theta_L} - 1\right)\left(\frac{1}{w}\right)$$

This condition can be rewritten as

$$\delta > 1 - \left(\frac{\theta_L}{\theta_H}\right)^{\alpha+1}\left(\frac{\theta_H - \bar{\theta}}{\bar{\theta} - \theta_L}\right)\left(\frac{w}{A_\rho - w}\right)$$

Then note that, starting from a steady-state ($\bar{\theta} = \mu$), the bracketed terms are $\frac{\theta_L}{\theta_H} = \frac{(\mu-k)(A_\rho-w)}{w(k-(1-A_\rho)\mu)}$ and $\frac{\theta_H - \bar{\theta}}{\bar{\theta} - \theta_L} = \frac{\frac{1}{A_\rho-w}(k-(1-w)\mu)}{\frac{1}{w}(k-(1-w)\mu)} = \frac{w}{A_\rho - w}$.

Therefore $\bar{\theta}$ decreases in k if and only if

$$\delta > 1 - \left(\frac{\mu - k}{k - (1 - A_\rho)\mu}\right)^{\alpha+1}\left(\frac{A_\rho - w}{w}\right)^{\alpha-1}$$

The RHS is decreasing in μ and α, so this holds for δ not too low, α not too high and an initial μ not too low. Then starting from a LQE, a decrease in k moves up the $\bar{\theta}(\mu)$ function left of the initial μ. The new steady-state equilibrium quality and reputation are necessarily higher. If the steady-state is unique, the new steady-state has higher μ. If there are multiple steady-states, ranked by increasing μ, either the new steady-state has the same rank and

higher μ, or the new steady-state has higher rank and higher μ.

The welfare effect of a subsidy σ ($\sigma = -dk$) has three components. First, for firms with θ parameters such that they sell both without and with the subsidy, the policy adds to their profits the amount it costs to the government, plus the extra profits brought by a higher reputation $\mu' > \mu$. The total effect is unambiguously positive.

Second, for new exporters that enter around θ_L because of the policy ($\theta_L < \theta < \theta_{L'}$), the net benefit NB_L of the subsidy is positive:

$$NB_L = \int_{\theta_L}^{\theta_{L'}} \left(\mu' - w\theta - k + \sigma\right) dG\left(\theta\right) - \int_{\theta_L}^{\theta_{L'}} \sigma g\left(\theta\right) d\theta$$

$$
\begin{aligned}
NB_L &= \left(\mu' - k\right) \int_{\theta_L}^{\theta_{L'}} g\left(\theta\right) d\theta - w \int_{\theta_L}^{\theta_{L'}} \theta dG\left(\theta\right) \\
&= \left[\left(\mu' - k\right) \left(\left(\tfrac{\theta_m}{\theta_L}\right)^{\alpha} - \left(\tfrac{\theta_m}{\theta_{L'}}\right)^{\alpha}\right) - w\tfrac{\alpha}{\alpha-1}\theta_m \left(\left(\tfrac{\theta_m}{\theta_L}\right)^{\alpha-1} - \left(\tfrac{\theta_m}{\theta_{L'}}\right)^{\alpha-1}\right)\right] \\
&= w\theta_m \left[\left(\tfrac{\theta_{L'}}{\theta_L} \left(\tfrac{\theta_m}{\theta_L}\right)^{\alpha-1} - \left(\tfrac{\theta_m}{\theta_{L'}}\right)^{\alpha-1}\right) - \tfrac{\alpha}{\alpha-1}\left(\left(\tfrac{\theta_m}{\theta_L}\right)^{\alpha-1} - \left(\tfrac{\theta_m}{\theta_{L'}}\right)^{\alpha-1}\right)\right] \\
&> \tfrac{w\theta_m}{\alpha-1} \left(\left(\tfrac{\theta_m}{\theta_L}\right)^{\alpha-1} - \left(\tfrac{\theta_m}{\theta_{L'}}\right)^{\alpha-1}\right) > 0
\end{aligned}
$$

where we go from the second to the third line using $w\theta_{L'} = \mu' - k$.

Third, for new exporters that enter around θ_H because of the policy ($\theta_{H'} < \theta < \theta_H$), the net benefit NB_H of the subsidy is also positive:

$$
NB_H = \int_{\theta_{H'}}^{\theta_H} \left(\sum_{t=0}^{\infty} \delta^t \left(\rho\left(t\right)\theta + \left(1 - \rho\left(t\right)\right)\mu' - w\theta - k + \sigma\right)\right) g\left(\theta\right) d\theta - ...
$$

$$
... \quad \tfrac{1}{1-\delta} \int_{\theta_{H'}}^{\theta_H} \sigma g\left(\theta\right) d\theta
$$

$$
\begin{aligned}
&= \tfrac{1}{1-\delta} \int_{\theta_{H'}}^{\theta_H} \left(\left(A_\rho - w\right)\theta + \left(1 - A_\rho\right)\mu' - k\right) g\left(\theta\right) d\theta \\
&= \tfrac{1}{1-\delta} \left[-\left(k - \left(1 - A_\rho\right)\mu'\right) \left(\left(\tfrac{\theta_m}{\theta_{H'}}\right)^{\alpha} - \left(\tfrac{\theta_m}{\theta_H}\right)^{\alpha}\right) + \tfrac{\alpha\left(A_\rho - w\right)}{\alpha-1}\theta_m \left(\left(\tfrac{\theta_m}{\theta_{H'}}\right)^{\alpha-1} - \left(\tfrac{\theta_m}{\theta_H}\right)^{\alpha-1}\right)\right] \\
&= \tfrac{1}{1-\delta}\left(A_\rho - w\right)\theta_m \left[\left(\left(\tfrac{\theta_m}{\theta_{H'}}\right)^{\alpha-1} - \tfrac{\theta_{H'}}{\theta_H}\left(\tfrac{\theta_m}{\theta_H}\right)^{\alpha-1}\right) - \tfrac{\alpha}{\alpha-1}\left(\left(\tfrac{\theta_m}{\theta_{H'}}\right)^{\alpha-1} - \left(\tfrac{\theta_m}{\theta_H}\right)^{\alpha-1}\right)\right] \\
&> \tfrac{1}{1-\delta}\left(A_\rho - w\right)\tfrac{\theta_m}{\alpha-1} \left(\left(\tfrac{\theta_m}{\theta_{H'}}\right)^{\alpha-1} - \left(\tfrac{\theta_m}{\theta_H}\right)^{\alpha-1}\right) > 0
\end{aligned}
$$

So the overall welfare gain is positive.

A.3.2 High-Quality Equilibrium

In a HQE, average quality is given by $\bar{\theta} = \mu_0 \left(\frac{1+K(\alpha-1)}{1+K(\alpha)} \right)$ where $K(\alpha) \equiv \sum_{T=\tilde{T}}^{\infty} \delta^T \left(\frac{\theta_m}{\theta_T} \right)^{\alpha}$ and $\theta_T = \frac{k-(1-\rho(T))\mu}{\rho(T)-w}$. Using the derivations in the proof of Proposition 2, we have:

$$\frac{\partial \theta_T}{\partial k} = \frac{1}{\rho(T)-w} > 0 \quad \text{for all } T > \tilde{T}$$

$$\frac{\partial \bar{\theta}}{\partial k} = \sum_{T=\tilde{T}}^{\infty} \frac{\partial \bar{\theta}}{\partial \theta_T} \frac{\partial \theta_T}{\partial k} > 0$$

Hence a subsidy that lowers k shifts down the $\bar{\theta}(\mu)$ function. As $\bar{\theta}$ is decreasing in μ in the HQE region, the new steady-state equilibrium defined by $\bar{\theta}(\mu) = \mu$ necessarily has lower μ. So average quality and national reputation are higher in the HQE steady-state without subsidies than with subsidies.

A.4 Proof of Proposition 4: Tax/Subsidy Scheme

In this section, we show that the age-dependent tax/subsidy in Proposition 4 sustains a steady-state equilibrium which replicates the perfect information entry and exit patterns by quality level. First, all per-period profits, and therefore all entry or exit decisions, depend on μ, which is itself affected by the policy. If the tax/subsidy scheme induces decisions identical to the perfect information setting, then firms below θ^* never enter and firms above θ^* always enter and stay. The resulting average quality and steady-state reputation would therefore be:

$$\bar{\theta}(\mu) = \mu = \int_{\theta^*}^{\infty} \theta dG(\theta) = \frac{\alpha}{\alpha-1}\theta^*$$

Second, let us show that with $\mu = \frac{\alpha}{\alpha-1}\theta^*$ and taxes and subsidies set as in Proposition 4, firms below θ^* choose never to enter. In the first export period, profits are decreasing in θ. To show that no firm below θ^* can profitably enter as fly-by-night and exit after one period, it suffices that a firm of quality θ_m cannot do so.

$$\pi_{t+1}(\theta_m) = \mu - k - w\theta_m - \tau_1$$

$$= \frac{\alpha}{\alpha-1}\theta^* - k - w\theta_m - \left[\frac{\alpha}{\alpha-1}\left(\frac{k}{1-w} \right) - k - w\theta_m \right] = 0$$

Hence no firm can realize strictly positive first period profits. However, if subsidies are offered in later periods, it could still be the case that some firms below θ^* have an incentive to enter and pay the initial tax in order to benefit from subsidies in later periods. The tax/subsidy combination for $s \geq 2$ ensures that this is not the case. The expected per-period profit of a

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continuing exporter is, for $s \geq 2$:

$$E_t \pi_{t+s}(\theta) = (\rho(s-1) - w)\theta + (1 - \rho(s-1))\frac{\alpha}{\alpha - 1}\theta^* - k - \tau_s$$

Among low-quality firms, this expression is the highest for firms of quality just below θ^*. Then the definition of θ^* yields:

$$E_t \pi_{t+s}(\theta^*) = \theta^* - k - w\theta^* + \left(\frac{1-\delta}{\delta}\right)\left(\tau_1 - \frac{1}{\alpha - 1}\frac{k}{1 - w}\right)$$

$$= \left(\frac{1-\delta}{\delta}\right)\left(\tau_1 - \frac{1}{\alpha - 1}\theta^*\right) = \left(\frac{1-\delta}{\delta}\right)(\theta^* - w\theta_m) > 0$$

Hence if firm θ^* enters, it incurs a loss in the first period and positive profits thereafter. It follows that it never exits voluntarily and its intertemporal expected profits are:

$$E_t \sum_{s=1}^{\infty} \delta^{s-1} \pi_{t+s}(\theta^*) = \pi_{t+1}(\theta^*) + \sum_{s=2}^{\infty} \delta^{s-1}\left(\frac{1-\delta}{\delta}\right)\left(\tau_1 - \frac{1}{\alpha - 1}\theta^*\right)$$

$$= \frac{\alpha}{\alpha - 1}\theta^* - k - w\theta^* - \tau_1 + \tau_1 - \frac{1}{\alpha - 1}\theta^*$$

$$= \theta^* - k - w\theta^* = 0$$

Since the after-tax, after-subsidy profits of continuing exporters are increasing in θ, it follows that firms below θ^* realize negative intertemporal profits regardless of their exit date and never enter.

Third, by the same token, all firms above θ^* have initially negative profits but a positive net present value of their profit stream, and have no incentive to exit voluntarily. Finally, note that τ_s can be rewritten as:

$$\tau_s = \left(\frac{1 - \rho(s-1)}{\alpha - 1}\right)\theta^* - \frac{1-\delta}{\delta}(\theta^* - k - w\theta_m)$$

where the second term is negative. τ_1 is positive and given Assumption 1, τ_s is decreasing in s, τ_2 may be positive or negative, and τ_s negative for s large enough.

A.5 Proof of Proposition 5: Positive Reputation Shocks

This appendix section provides a sketch of the proof of Proposition 5. It essentially relies on the stability of μ_S and μ'_S and the instability of μ_U. For (i) and (ii), let us show that if μ_S is a steady-state LQE and $\frac{\partial \bar{\theta}(\mu)}{\partial \mu} < 0$ at μ_S, then μ_S is a stable equilibrium for $\eta < 1$. Define $\theta_{L,S} \equiv \frac{\mu_S - k}{w}$ and $\theta_{H,S} \equiv \frac{k - (1 - A_\rho)\mu_S}{A_\rho - w}$. At time $t - 1$ the economy is in an initial steady-state

where

$$\mu_S = \overline{\theta}\left(\mu_S\right) = \mu_0 \left(\frac{1 - \left(\frac{\theta_m}{\theta_{L,S}}\right)^{\alpha-1} + \left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha-1}}{1 - \left(\frac{\theta_m}{\theta_{L,S}}\right)^{\alpha} + \left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha}} \right)$$

Suppose $\overline{\theta}$ is locally decreasing in μ. Then for all $\mu_S < \mu < \mu_U$, $\overline{\theta}\left(\mu\right) < \mu$. Now suppose there is a perturbation at time t such that $\mu_t = \mu_S + \varepsilon$, $\varepsilon > 0$ and $\varepsilon < \mu_U - \mu_S$. The entry thresholds at t are:

$$\begin{aligned}
\theta_{L,t} &= \frac{\mu_t - k}{w} = \frac{\mu_S + \varepsilon - k}{w} \\
\theta_{H,t} &= \frac{k - (1-\delta)\sum_{u=0}^{\infty}(1-\rho(u))\mu_{t+u}}{A_\rho - w}
\end{aligned}$$

where $\theta_{H,t}$ is determined by the zero intertemporal profits condition

$$\sum_{u=0}^{\infty} \delta^u \left[(\rho(u) - w)\theta_{H,t} + (1 - \rho(u))E_t\mu_{t+u} - k \right]$$

and the absence of aggregate uncertainty allows us to remove the expectations operator.

Let us conjecture, to be verified, that $\mu_S \leq \mu_{t+u+1} \leq \mu_{t+u} \leq \mu_S + \varepsilon$ for all $u \geq 1$. Then:

$$\begin{aligned}
\theta_{L,S} &< \theta_{L,t+u+1} < \theta_{L,t+u} < \theta_{L,t} \\
\theta_{H,S} &> \theta_{H,t+u+1} > \theta_{H,t+u} > \theta_{H,t}
\end{aligned} \qquad \text{for all } u \geq 1$$

The average quality of exports is determined by the θ_L and θ_H thresholds in the periods after the shock in the following manner: for $u \geq 0$,

$$\begin{aligned}
\overline{\theta}_{t+u} &= \mu_0 \left(\frac{1 - \left(\frac{\theta_m}{\theta_{L,t+u}}\right)^{\alpha-1} + \sum_{l=0}^{u}\delta^{u-l}\left(\frac{\theta_m}{\theta_{H,t+l}}\right)^{\alpha-1} + \sum_{l=u+1}^{\infty}\delta^l\left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha-1}}{1 - \left(\frac{\theta_m}{\theta_{L,t+u}}\right)^{\alpha} + \sum_{l=0}^{u}\delta^{u-l}\left(\frac{\theta_m}{\theta_{H,t+l}}\right)^{\alpha} + \sum_{l=u+1}^{\infty}\delta^l\left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha}} \right) \\
\theta_{L,t+u} &= \frac{\mu_{t+u} - k}{w} \\
\theta_{H,t+u} &= \frac{k - (1-\delta)\sum_{l=0}^{\infty}(1-\rho(l))\mu_{t+u+l}}{A_\rho - w}
\end{aligned}$$

At time t, let us define $\overline{\theta}_t^{perm}$ as the average quality that would prevail if firms expected the shock to be permanent, i.e. if $E_t\mu_{t+u} = \mu_t$ for all $u \geq 0$. We calculate:

$$\overline{\theta}_t = \mu_0 \left(\frac{1 - \left(\frac{\theta_m}{\theta_{L,t}}\right)^{\alpha-1} + \left(\frac{\theta_m}{\theta_{H,t}}\right)^{\alpha-1} + \frac{\delta}{1-\delta}\left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha-1}}{1 - \left(\frac{\theta_m}{\theta_{L,t}}\right)^{\alpha} + \frac{\delta}{1-\delta}\left(\frac{\theta_m}{\theta_{H,t}}\right)^{\alpha} + \frac{\delta}{1-\delta}\left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha}} \right)$$

$$\overline{\theta}_t < \overline{\theta}_t^{perm} = \mu_0 \left(\frac{1 - \left(\frac{\theta_m}{\theta_{L,t}}\right)^{\alpha-1} + \left(\frac{\theta_m}{\theta_{H,t}^{perm}}\right)^{\alpha-1} + \frac{\delta}{1-\delta}\left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha-1}}{1 - \left(\frac{\theta_m}{\theta_{L,t}}\right)^{\alpha} + \frac{\delta}{1-\delta}\left(\frac{\theta_m}{\theta_{H,t}^{perm}}\right)^{\alpha} + \frac{\delta}{1-\delta}\left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha}} \right)$$

as $\theta_{H,t}^{perm} = \frac{k-(1-A_\rho)(\mu_S+\varepsilon)}{A_\rho-w} < \theta_{H,t}$ from the conjecture $\mu_S \leq \mu_{t+u+1} \leq \mu_{t+u} \leq \mu_S + \varepsilon$ for all $u \geq 1$. Also

$$\overline{\theta}_t^{perm} < \overline{\theta}(\mu_S + \varepsilon) < \mu_t$$

The first inequality results from $\theta_{H,S} > \theta_{H,t}^{perm}$. The second inequality comes from $\overline{\theta}(\mu) < \mu$ for $\mu \in (\mu_S, \mu_U)$. Hence $\overline{\theta}_t < \mu_t$ and therefore:

$$\overline{\theta}_t < \mu_{t+1} = \mu_t + \eta\left(\overline{\theta}_t - \mu_t\right) < \mu_t$$

Additionally as long as η is not too close to 1, $\mu_{t+1} > \mu_S$.

We can show, similarly, that in all subsequent periods, $\overline{\theta}_{t+u} < \mu_{t+u}$ as long as $\mu_{t+u} > \mu_S$. Thus $\mu_{t+u+1} < \mu_{t+u}$ for all u and the conjecture that μ_{t+u} follows a decreasing path from $\mu_S + \varepsilon$ to μ_S is verified. In case of a negative shock to μ at time t starting from a steady-state where $\overline{\theta}$ is locally decreasing in μ, the proof is identical with opposite signs. It follows that if μ_S is a steady-state reputation and $\overline{\theta}(\mu)$ is locally decreasing in μ at μ_S, then μ_S is stable. Any positive shock, starting from μ_S, that brings μ_t to a value in (μ_S, μ_U) has no long-run effects as the economy moves back to μ_S.

By the same reasoning, μ_U is unstable. Suppose there is a negative shock to μ starting from

$$\mu_U = \overline{\theta}(\mu_U) = \mu_0 \left(\frac{1 - \left(\frac{\theta_m}{\theta_{L,U}}\right)^{\alpha-1} + \left(\frac{\theta_m}{\theta_{H,U}}\right)^{\alpha-1}}{1 - \left(\frac{\theta_m}{\theta_{L,U}}\right)^{\alpha} + \left(\frac{\theta_m}{\theta_{H,U}}\right)^{\alpha}} \right)$$

where $\theta_{L,U} \equiv \frac{\mu_U-k}{w}$, $\theta_{H,U} \equiv \frac{k-(1-A_\rho)\mu_U}{A_\rho-w}$ and $\overline{\theta}(\mu)$ is increasing in μ at μ_U. At time t, $\mu_t = \mu_U - \varepsilon$, where $\varepsilon > 0$ and $\varepsilon < \mu_U - \mu_S$. We conjecture $\mu_S \leq \mu_{t+u+1} \leq \mu_{t+u} \leq \mu_U - \varepsilon$, which implies $\theta_{L,S} < \theta_{L,t+u+1} < \theta_{L,t+u} < \theta_{L,U}$ and $\theta_{H,S} > \theta_{H,t+u+1} > \theta_{H,t+u} > \theta_{H,U}$ for all $u \geq 0$. Then $\overline{\theta}_{t+u} < \mu_{t+u}$ and thus $\mu_{t+u+1} < \mu_{t+u}$ for all $u \geq 0$.

For part (iii), consider a "large shock", starting from μ_S, as a shock $\varepsilon > \mu_U - \mu_S$ such that if $\mu_t = \mu_S + \varepsilon$, then $\underline{\mu} > \mu_U$, where $\overline{\theta}_t$ is defined as in (ii) and $\underline{\mu}$ is defined below. $\overline{\theta}_{t+u}$, $\theta_{L,t+u}$ and $\theta_{H,t+u}$ are defined as in part (ii). Also, for $\mu_U < \mu < \mu'_S$, we know that $\overline{\theta}(\mu) > \mu$. We can then show that $\overline{\theta}_{t+u}$ is increasing in u as long as $\overline{\theta}_{t+u} < \mu'_S$, and μ_{t+u} is increasing in u for $u \geq \underline{u}$ if $\mu_{t+u} > \mu_U$ for all $u \geq 0$. $\underline{u}$ is the inflexion point of the path of μ_{t+u}, which can initially decrease but is eventually increasing as long as $\mu < \mu'_S$. Define $\underline{\mu} = \mu_{t+\underline{u}}$, a large reputation shock is a shock such that $\underline{\mu} > \mu_U$. It ensures that reputation and average quality

both grow along the transition path until the economy reaches the steady-state μ'_S.

Finally, note that more entry and higher reputation in the long-run imply higher aggregate profits and higher average quality. The latter follows from $\bar{\theta} = \mu$ in the long run. The former results from a higher number of active firms and the fact that the range of firms which are active both with the initial μ and with the higher final μ receive a higher sequence of prices.

A.6 Minimum Quality Standard

Let us assume a minimum quality standard $\theta_{QS} = \theta^*$, such that all the firms with $\theta < \theta^*$ choose not to enter the export market. Thanks to this minimum quality standard, an economy initially in a stable LQE will move to a HQE. In this HQE, all the firms of quality $\theta > \theta^*$ will export. Then the average quality of exports across quality levels and cohorts of firms is $\bar{\theta}_t = \frac{\int_{\theta^*}^{\infty} \theta N_t(\theta) d\theta}{\int_{\theta^*}^{\infty} N_t(\theta) d\theta}$, where the total number of active firms of quality θ is given by $N_t(\theta) = \frac{1}{1-\delta} g(\theta)$. Simple calculations give us $\bar{\theta}(\mu) = \frac{\alpha}{\alpha-1} \theta^*$.

Let us call μ_{init} the initial steady-state reputation μ (before the minimum quality standard), and define $\theta_{L,\text{init}} \equiv \frac{\mu_{\text{init}} - k}{w}$ and $\theta_{H,S} \equiv \frac{k - (1 - A_\rho)\mu_{\text{init}}}{A_\rho - w}$. The welfare effect of the minimum quality standard for the exporting country has three components. First, for firms with quality $\theta < \theta_{L,\text{init}}$, the effect is unambiguously negative: these firms were exporting as fly-by-night before the introduction of the minimum standard making positive profits. With the minimum quality standard, they cannot enter the export market. In the initial LQE, the total number of active firms of quality $\theta < \theta_{L,\text{init}}$ was $g(\theta)$ and given that these firms exported just for one period, their profit was $\mu_{\text{init}} - w\theta - k$. Hence the lost profits due to the minimum quality standard is given by:

$$
\begin{aligned}
\text{Loss}_{\theta < \theta_{L,\text{init}}} &= \int_{\theta_m}^{\theta_{L,\text{init}}} (\mu_{\text{init}} - w\theta - k) \, dG(\theta) \\
&= (\mu_{\text{init}} - k) \int_{\theta_m}^{\theta_L} g(\theta) \, d\theta - w \int_{\theta_m}^{\theta_{L,\text{init}}} \theta \, dG(\theta) \\
&= (\mu_{\text{init}} - k) \left(1 - \left(\frac{\theta_m}{\theta_{L,\text{init}}} \right)^{\alpha} \right) - w \frac{\alpha}{\alpha - 1} \theta_m \left(1 - \left(\frac{\theta_m}{\theta_{L,\text{init}}} \right)^{\alpha-1} \right) \\
&= w\theta_m \left[\frac{\theta_{L,\text{init}}}{\theta_m} - \frac{\alpha}{\alpha - 1} + \frac{1}{\alpha - 1} \left(\frac{\theta_m}{\theta_{L,\text{init}}} \right)^{\alpha-1} \right]
\end{aligned}
$$

Second, for firms with quality $\theta^* < \theta < \theta_{H,\text{init}}$, the effect is unambiguously positive: these firms were not exporting before the introduction of the minimum standard. With the minimum quality standard, they now enter the export market and export and stay until they

are hit by the exit shock. The value of their profit stream is positive and the gain is given by:

$$\text{Gain }_{\theta^* < \theta < \theta_{H,\text{init}}} = \int_{\theta^*}^{\theta_{H,\text{init}}} \left(\sum_{t=0}^{\infty} \delta^t \left(\rho(t)\theta + (1 - \rho(t)) \left(\tfrac{\alpha}{\alpha-1}\theta^* \right) - w\theta - k \right) \right) g(\theta)\, d\theta$$

$$= \tfrac{1}{1-\delta} \int_{\theta^*}^{\theta_{H,\text{init}}} \left((A_\rho - w)\theta + (1 - A_\rho) \left(\tfrac{\alpha}{\alpha-1}\theta^* \right) - k \right) g(\theta)\, d\theta$$

$$= \tfrac{1}{1-\delta} \left[-\left(k - (1 - A_\rho)\tfrac{\alpha}{\alpha-1}\theta^* \right) \left(\left(\tfrac{\theta_m}{\theta^*} \right)^\alpha - \left(\tfrac{\theta_m}{\theta_{H,\text{init}}} \right)^\alpha \right) + \tfrac{\alpha(A_\rho - w)}{\alpha-1}\theta_m \left(\left(\tfrac{\theta_m}{\theta^*} \right)^{\alpha-1} - \left(\tfrac{\theta_m}{\theta_{H,\text{init}}} \right)^{\alpha-1} \right) \right]$$

$$= \tfrac{\theta_m}{(1-\delta)(\alpha-1)} \left[(1 - w) \left(\tfrac{\theta_m}{\theta^*} \right)^{\alpha-1} - \tfrac{\alpha}{\theta_{H,\text{init}}} \left(\tfrac{k}{\alpha} + (1 - A_\rho)(\theta^* - \mu_{\text{init}}) \right) \left(\tfrac{\theta_m}{\theta_{H,\text{init}}} \right)^{\alpha-1} \right]$$

$$= \tfrac{1}{(1-\delta)(\alpha-1)} \left[k \left(\tfrac{\theta_m}{\theta^*} \right)^\alpha - (k + \alpha(1 - A_\rho)(\theta^* - \mu_{\text{init}})) \left(\tfrac{\theta_m}{\theta_{H,\text{init}}} \right)^\alpha \right]$$

Third, for firms with quality $\theta > \theta_{H,\text{init}}$, the effect is also unambiguously positive: these firms receive extra profits brought by a higher reputation (from μ_{init} to $\tfrac{\alpha}{\alpha-1}\theta^*$). For a given reputation μ, the aggregate profit of firms with quality $\theta > \theta_H$ is given by:

$$\int_{\theta_H}^{\infty} \left(\sum_{t=0}^{\infty} \delta^t \left(\rho(t)\theta + (1 - \rho(t))\mu - w\theta - k \right) \right) g(\theta)\, d\theta$$

$$= \tfrac{1}{1-\delta} \int_{\theta_H}^{\infty} \left((A_\rho - w)\theta + (1 - A_\rho)\mu - k \right) g(\theta)\, d\theta$$

$$= \tfrac{1}{1-\delta} \left[-(k - (1 - A_\rho)\mu) \left(\tfrac{\theta_m}{\theta_H} \right)^\alpha + \tfrac{\alpha(A_\rho - w)}{\alpha-1}\theta_m \left(\tfrac{\theta_m}{\theta_H} \right)^{\alpha-1} \right]$$

$$= \tfrac{1}{1-\delta} \left[-k + (1 - A_\rho)\mu + \tfrac{\alpha(A_\rho - w)}{\alpha-1}\theta_H \right] \left(\tfrac{\theta_m}{\theta_H} \right)^\alpha$$

Hence the gain from the higher reputation is given by:

$$\text{Gain }_{\theta > \theta_{H,\text{init}}} = \left(\tfrac{\alpha}{\alpha-1}\theta^* - \mu_{\text{init}} \right)(1 - A_\rho) \left(\tfrac{\theta_m}{\theta_{H,\text{init}}} \right)^\alpha$$

Overall, the total welfare effect of a minimum quality standard is:

$$\left(\tfrac{\alpha}{\alpha-1}\theta^* - \mu_{\text{init}} \right)(1 - A_\rho) \left(\tfrac{\theta_m}{\theta_{H,\text{init}}} \right)^\alpha$$

$$+ \tfrac{1}{(1-\delta)(\alpha-1)} \left[k \left(\tfrac{\theta_m}{\theta^*} \right)^\alpha - (k + \alpha(1 - A_\rho)(\theta^* - \mu_{\text{init}})) \left(\tfrac{\theta_m}{\theta_{H,\text{init}}} \right)^\alpha \right]$$

$$- w\theta_m \left[\tfrac{\theta_{L,\text{init}}}{\theta_m} - \tfrac{\alpha}{\alpha-1} + \tfrac{1}{\alpha-1} \left(\tfrac{\theta_m}{\theta_{L,\text{init}}} \right)^{\alpha-1} \right]$$

From simple computations (derivatives of the lost profits terms Loss $_{\theta < \theta_{L,\text{init}}}$ with respect to various parameters), we obtain that the lower α (the shape parameter of the distribution), the lower the loss from the minimum quality standard. Intuitively, a low α means that there are more firms at the right tail of the distribution, and so relatively less firms that lose from

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the standard. The lost profits due to the minimum quality standard also decrease with the costs (w and k) (remember that $\theta_{L\text{init}} = \frac{\mu_{\text{init}} - k}{w}$). On the contrary, they increase with the initial reputation (before the standard); the higher the reputation, the higher the profits the fly-by-night firms were able to extract when they exported for one period.

A.7 Proof of Result 1

In a LQE, the set of continuing firms is $[\theta_H, \infty)$ from the second period onwards, so the average price $\bar{p}_{t,t+s}^{lqe}$ of cohort t at time $t+s$ is given by:

$$\bar{p}_{t,t+s}^{lqe}\left(\bar{\theta}\right) = \begin{cases} \bar{\theta} & \text{if } s = 1 \\ \bar{\theta} + \rho\left(s\right)\left(\frac{\alpha}{\alpha-1}\theta_H - \bar{\theta}\right) & \text{if } s > 1 \end{cases}$$

As $\bar{\theta} < \theta_H$ in a LQE and $\rho\left(s\right)$ increases in s, it immediately follows that $\bar{p}_{t,t+s}^{lqe}$ increases with s.

In a HQE, the set of active firms of cohort t at time $t+s$ is $[\theta_{s-1}, \infty)$, and their average price is:

$$\bar{p}_{t,t+s}^{hqe}\left(\bar{\theta}\right) = \begin{cases} \bar{\theta} & \text{if } s = 1 \\ \bar{\theta} + \rho\left(s\right)\left(\frac{\alpha}{\alpha-1}\theta_{s-1} - \bar{\theta}\right) & \text{if } s > 1 \end{cases}$$

$\rho\left(s\right)$ and θ_{s-1} increase with s. Immediately following the entry of cohort t, $\bar{p}_{t,t+s}^{hqe}$ may fall with s if the distribution of θ has low variance (α high), such that $\frac{\alpha}{\alpha-1}\theta_1 > \mu$. In this case, there is initially a large mass of firms at the bottom of the distribution of continuing firms and their prices are falling. However, since $\mu < \frac{\alpha}{\alpha-1}\theta^*$, there is some finite s' such that for all $s \geq s'$, $\bar{p}_{t,t+s+1}^{hqe}\left(\bar{\theta}\right) > \bar{p}_{t,t+s}^{hqe}\left(\bar{\theta}\right)$ and thus at each given point in time, the average unit price is higher for older cohorts of firms.

A.8 Proof of Result 2

The first part establishes that across cohorts, the fraction of active firms that exit per period is higher for lower quality firms. In a LQE, the hazard rate is 1 for firms below θ_L and $1 - \delta$ for firms above θ_H. In a HQE, the hazard rate is $\frac{1-\delta}{1-\delta^T}$ for firms between θ_{T-1} and θ_T for all T, which is decreasing in T, and $1 - \delta$ for firms above θ^*.

The second part states that the probability of exit, across quality levels, decreases with the age of a cohort. In a LQE, the hazard rate of cohort t at time $t+s$ is

$$h_{t,t+s}^{lqe} = \begin{cases} 1 - \delta + \delta\frac{G(\theta_L)}{G(\theta_L)+1-G(\theta_H)} & \text{if } s = 1 \\ 1 - \delta & \text{if } s > 1 \end{cases}$$

It falls from $t+1$ to $t+2$ and remains constant thereafter. In a HQE, the hazard rate of

cohort t at $t + s$ is

$$h_{t,t+s}^{hqe} = 1 - \delta + \delta \frac{G(\theta_s) - G(\theta_{s-1})}{1 - G(\theta_{s-1})} = 1 - \delta + \delta \left(1 - \left(\frac{\theta_{s-1}}{\theta_s}\right)^\alpha\right)$$

Since $\frac{\theta_{s-1}}{\theta_s}$ is decreasing in s, $h_{t,t+s}^{hqe}$ falls over time.

B   Informed and Uninformed Buyers

Suppose the population of importers is divided into N equal-sized groups. There is perfect information diffusion within groups but no information diffusion across groups. Thus, if any individual in group n has previously consumed the output of firm j, then all buyers in group n are informed about good j. When firm j is matched with buyer i, $i \in$ Informed if there exists $i' \in n$ such that i' has been matched with j in the past, and $i \in$ Uninformed if there is no $i' \in n$ such that i' has been matched with j in the past. Further assume that the firm observes in any period whether its buyer is informed or not, but not which group the buyer belongs to; hence it does not know the exact proportion of informed buyers in any period but only its expectation.

It follows immediately from this setup that $\rho(0) = 0$. After the firm has exported for one period, one group is informed, so $\rho(1) = \frac{1}{N}$. For each subsequent period, if the fraction of informed buyers after s export periods is $\rho(s)$, then with probability $\rho(s)$, the firm is matched with a buyer in an informed group, and the proportion of informed importers stays at $\rho(s)$ for the next period. With probability $1 - \rho(s)$, the firm is matched with a buyer in an uniformed group; then the fraction of informed importers next period is $\rho(s) + \frac{1}{N}$.

Therefore, the expected fraction of informed buyers is given by the following path: for $s \geq 0$,

$$
\begin{aligned}
\rho(0) &= 0 \\
\rho(s+1) &= \rho(s)^2 + (1 - \rho(s))\left(\rho(s) + \frac{1}{N}\right) = \rho(s)\left(\frac{N-1}{N}\right) + \frac{1}{N}
\end{aligned}
$$

We can check that this function satisfies Assumption 1.

$$
\begin{aligned}
\rho(s+1) - \rho(s) &= \frac{1}{N}(1 - \rho(s)) > 0 \\
\frac{\rho(s+1) - \rho(s)}{\rho(s)} &= \frac{1}{N}\left(\frac{1}{\rho(s)} - 1\right) \quad \text{is decreasing in } s \\
\lim_{s \to \infty} \rho(s) &= \frac{1}{N}\left(1 - \frac{N-1}{N}\right)^{-1} = 1
\end{aligned}
$$

So $\rho(s)$ is increasing in s, rises with s at a falling rate, and converges to 1.