Appendix to “Bad Beta, Good Beta”: Data Construction, Additional Empirical Results, and Robustness Checks

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This appendix to "Bad Beta, Good Beta" (henceforth BBGB, Campbell and Vuolteenaho, 2004) contains background material on our empirical methods and reports the results of robustness checks.

The first section describes in detail how we construct our data. The second section discusses our method for estimating betas. The third section briefly reviews the econometrics of predictive regressions, and then asks whether our findings might be driven by finite-sample bias in the predictive equations of our vector autoregressive model. The fourth section discusses the evolution of betas over time, and asks whether it is reasonable to work with a model in which betas are fixed in each of two subsamples as we do in BBGB. The fifth section asks whether our results would be affected if we estimated a conditional rather than an unconditional asset pricing model. The sixth section explores the sensitivity of the BBGB results to changes in the parameter ρ, which is a constant of loglinearization in our loglinear approximate asset pricing framework. The seventh section asks whether the BBGB results are robust to changes in the data frequency from monthly to quarterly or annual. The eighth section considers alternative VAR specifications with additional explanatory variables.

1 Data construction

We construct the variables that enter our VAR as follows. First, the excess log return on the market (r^e_M) is the difference between the log return on the Center for Research in Securities Prices (CRSP) value-weighted stock index (r_M) and the log risk-free rate. The risk-free-rate data are constructed by CRSP from Treasury bills with approximately three month maturity. Second, the term yield spread (TY) is provided by Global Financial Data and is computed as the yield difference between ten-year constant-maturity taxable bonds and short-term taxable notes, in percentage points. Third, the price-earnings ratio (PE) is from Shiller (2000), constructed as the price of the S&P 500 index divided by a ten-year trailing moving average of aggregate earnings of companies in the S&P 500 index. Following Graham and Dodd (1934), Campbell and Shiller (1988b, 1998) advocate averaging earnings over several years to avoid temporary spikes in the price-earnings ratio caused by cyclical declines in earnings. We avoid any interpolation of earnings in order to ensure that all components of the time-t price-earnings ratio are contemporaneously observable by time t. The ratio is log transformed.

Fourth, the small-stock value spread (VS) is constructed from the data made available by Professor Kenneth French on his web site.[2] The portfolios, which are constructed at the end of each June, are the intersections of two portfolios formed on size (market equity, ME) and three portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoint for year t is the median NYSE market equity at the end of June of year t. BE/ME for June of year t is the book equity for the last fiscal year end in $t-1$ divided by ME for December of $t-1$. The BE/ME breakpoints are the 30th and 70th NYSE percentiles. At the end of June of year t, we construct the small-stock value spread as the difference between the $\log(BE/ME)$ of the small high-book-to-market portfolio and the $\log(BE/ME)$ of the small low-book-to-market portfolio, where BE and ME are measured at the end of December of year $t-1$. For months from July to May, the small-stock value spread is constructed by adding the cumulative log return (from the previous June) on the small low-book-to-market portfolio to, and subtracting the cumulative log return on the small high-book-to-market portfolio from, the end-of-June small-stock value spread.

Our small-stock value spread is similar to variables constructed by Asness, Friedman, Krail, and Liew (2000), Cohen, Polk, and Vuolteenaho (2003), and Brennan, Wang, and Xia (2001). Asness et al. use a number of different scaled-price variables to construct their measures, and also incorporate analysts' earnings forecasts into their model. Cohen et al. use the entire CRSP universe instead of small-stock portfolios to construct their value-spread variable. Brennan et al.'s small-stock value-spread variable is equal to ours at the end of June of each year, but the intra-year values differ because Brennan et al. interpolate the intra-year values of BE using year t and year $t+1$ BE values. We do not follow their procedure because we wish to avoid using any future variables that might cause spurious forecastability of stock returns.

It is important to specify the value-spread variable in terms of log-transformed valuation ratios. In levels, the spread in market-to-book ratios predicts the stock market with a negative relation and the spread in book-to-market ratios with a positive relation. This is simply because these spread variables in levels track the market's overall valuation.

We also construct two sets of portfolios to use as test assets. The first is a set of 25 ME and BE/ME portfolios, available from Professor Kenneth French's web site.

[2] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

The portfolios, which are constructed at the end of each June, are the intersections of five portfolios formed on size (ME) and five portfolios formed on book-to-market equity (BE/ME). BE/ME for June of year t is the book equity for the last fiscal year end in the calendar year $t-1$ divided by ME for December of $t-1$. The size and BE/ME breakpoints are NYSE quintiles. On a few occasions, no firms are allocated to some of the portfolios. In those cases, we use the return on the portfolio with the same size and the closest BE/ME.

The 25 ME and BE/ME portfolios were originally constructed by Davis, Fama, and French (2000) using three databases. The first of these, the CRSP monthly stock file, contains monthly prices, shares outstanding, dividends, and returns for NYSE, AMEX, and NASDAQ stocks. The second database, the COMPUSTAT annual research file, contains the relevant accounting information for most publicly traded U.S. stocks. The COMPUSTAT accounting information is supplemented by the third database, Moody's book equity information hand collected by Davis et al.

Daniel and Titman (1997) point out that it can be dangerous to test asset pricing models using only portfolios sorted by characteristics known to be related to average returns, such as size and value. Characteristics-sorted portfolios are likely to show some spread in betas identified as risk by almost any asset pricing model, at least in sample. When the model is estimated, a high premium per unit of beta will fit the large variation in average returns. Thus, at least when premia are not constrained by theory, an asset pricing model may spuriously explain the average returns to characteristics-sorted portfolios.

To alleviate this concern, we follow the advice of Daniel and Titman and construct a second set of 20 portfolios sorted on past risk loadings with VAR state variables (excluding the price-smoothed earnings ratio PE, since high-frequency changes in PE are so highly collinear with market returns). These portfolios are constructed as follows. First, we run a loading-estimation regression for each stock in the CRSP database:

$$\sum_{j=1}^{3} r_{i,t+j} = b_0 + b_{r_M} \sum_{j=1}^{3} r_{M,t+j} + b_{VS}(VS_{t+3} - VS_t) + b_{TY}(TY_{t+3} - TY_t) + \varepsilon_{i,t+3}, \quad (1)$$

where $r_{i,t}$ is the log stock return on stock i for month t. The regression (1) is reestimated from a rolling 36-month window of overlapping observations for each stock at the end of each month. Since these regressions are estimated from stock-level instead of portfolio-level data, we use a quarterly data frequency to minimize the impact of infrequent trading.

Our objective is to create a set of portfolios that have as large a spread as possible in their betas with the market and with innovations in the VAR state variables. To accomplish this, each month we perform a two-dimensional sequential sort on market beta and another state-variable beta, producing a set of ten portfolios for each state variable. First, we form two groups by sorting stocks on $\widehat{b}_{VS}$. Then, we further sort stocks in both groups to five portfolios on $\widehat{b}_{r_M}$ and record returns on these ten value-weight portfolios. To ensure that the average returns on these portfolio strategies are not influenced by various market-microstructure issues plaguing the smallest stocks, we exclude the smallest (lowest ME) five percent of stocks of each cross-section and lag the estimated risk loadings by a month in our sorts. We construct another set of ten portfolios in a similar fashion by sorting on $\widehat{b}_{TY}$ and $\widehat{b}_{r_M}$. We refer to these 20 return series as risk-sorted portfolios. Both the 25 size- and book-to-market-sorted returns and the 20 risk-sorted returns are measured over the period 1929:1–2001:12.

2 Beta estimation

We estimate cash-flow and discount-rate betas using the fitted values of the market's cash-flow and discount-rate news. Specifically, we use the following beta estimators:

$$\widehat{\beta}_{i,CF} = \frac{\widehat{\text{Cov}}\left(r_{i,t}, \widehat{N}_{CF,t}\right)}{\widehat{\text{Var}}\left(\widehat{N}_{CF,t} - \widehat{N}_{DR,t}\right)} + \frac{\widehat{\text{Cov}}\left(r_{i,t}, \widehat{N}_{CF,t-1}\right)}{\widehat{\text{Var}}\left(\widehat{N}_{CF,t} - \widehat{N}_{DR,t}\right)} \tag{2}$$

$$\widehat{\beta}_{i,DR} = \frac{\widehat{\text{Cov}}\left(r_{i,t}, -\widehat{N}_{DR,t}\right)}{\widehat{\text{Var}}\left(\widehat{N}_{CF,t} - \widehat{N}_{DR,t}\right)} + \frac{\widehat{\text{Cov}}\left(r_{i,t}, -\widehat{N}_{DR,t-1}\right)}{\widehat{\text{Var}}\left(\widehat{N}_{CF,t} - \widehat{N}_{DR,t}\right)} \tag{3}$$

Above, $\widehat{\text{Cov}}$ and $\widehat{\text{Var}}$ denote sample covariance and variance. $\widehat{N}_{CF,t}$ and $\widehat{N}_{DR,t}$ are the estimated cash-flow and expected-return news from the VAR model of Tables 2 and 3 in BBGB..

These beta estimators deviate from the usual regression-coefficient estimator in two respects. First, we include one lag of the market's news terms in the numerator. Adding a lag is motivated by the possibility that, especially during the early years of our sample period, not all stocks in our test-asset portfolios were traded frequently and synchronously. If some portfolio returns are contaminated by stale prices, market return and news terms may spuriously appear to lead the portfolio returns, as noted by Scholes and Williams (1977) and Dimson (1979). In addition, Lo and MacKinlay (1990) show that the transaction prices of individual stocks tend to react in part to movements in the overall market with a lag, and the smaller the company, the greater is the lagged price reaction. McQueen, Pinegar, and Thorley (1996) and Peterson and Sanger (1995) show that these effects exist even in relatively low-frequency data (i.e., those sampled monthly). These problems are alleviated by the inclusion of the lag term. Second, we normalize the covariances in (2) and (3) by $\widehat{\mathrm{Var}}(\widehat{N}_{CF,t} - \widehat{N}_{DR,t})$ or, equivalently by the sample variance of the (unexpected) market return, $\widehat{\mathrm{Var}}\left(r^e_{M,t} - E_{t-1}r^e_{M,t}\right)$.

Under our maintained assumptions, $\widehat{\beta}_{i,M} = \widehat{\beta}_{i,CF} + \widehat{\beta}_{i,DR}$ is equal to the portfolio i's Scholes-Williams (1977) beta on unexpected market return. It is also equal to the so-called "sum beta" employed by Ibbotson Associates, which is the sum of multiple regression coefficients of a portfolio's return on contemporaneous and lagged unexpected market returns. These equalities follow from three points. First, Scholes and Williams (1977) include an additional lead term, which captures the possibility that the market return itself is contaminated by stale prices. Under the maintained assumption that our news terms are unforecastable, the population value of this term is zero. Second, the Scholes-Williams beta formula also includes a normalization. The sum of the three regression coefficients is divided by one plus twice the market's autocorrelation. Since the first-order autocorrelation of our news series is zero under the maintained assumptions, this normalization factor is identically one. Third, the Ibbotson sum beta uses multiple regression coefficients instead of simple regression coefficients. Under the maintained assumption that the news terms are unforecastable, the explanatory variables in the multiple regression are uncorrelated, and thus the multiple regression coefficients are equal to simple regression coefficients.

3 Finite-sample bias

The asset pricing model in BBGB relies on a vector autoregression (VAR) that generates estimates of innovations in two components of market returns: discount-rate news, the discounted value of revisions in future return forecasts; and cash-flow news, the residual component of the current return innovation. There are two well known biases that might affect these VAR estimates.

First, since the work of Kendall (1954) it has been understood that the estimates of persistent autoregressive coefficients are biased downwards in finite samples when the mean of the persistent process must also be estimated. This is relevant for forecasting variables such as the term spread, the price-earnings ratio, and the value spread whose autoregressive coefficients are estimated at 0.879, 0.994, and 0.991 respectively in our monthly VAR system over the period 1929:1–2001:12.

Second, Stambaugh (1999) has pointed out that the estimated coefficients of returns on persistent forecasting variables are biased downward (upward) when return innovations are positively (negatively) correlated with the innovations to the forecasting variables. Related to this, the usual t test for statistical significance of the forecasting variable has incorrect size as pointed out by Cavanagh, Elliott, and Stock (1995). Several authors have documented the important effect of this bias in regressions of stock returns on price-dividend or price-earnings ratios, and have suggested alternative ways to correct it (Campbell and Yogo 2003, Lewellen 2003, Polk, Thompson, and Vuolteenaho 2003, Torous, Valkanov, and Yan 2003).

It seems unlikely a priori that these biases could explain the results reported in BBGB. First, the variability of dicsount-rate news is generated by nonzero predictive coefficients together with large persistence estimates for the explanatory variables. The Stambaugh bias in predictive coefficients increases the variability of dicsount-rate news while the Kendall bias in persistence estimates reduces it, and the sign of the total bias is not clear. Second, the results in BBGB depend on the finding that growth stock returns are correlated with discount-rate news. This in turn depends on the finding that the value spread predicts returns on the market. While the value spread is persistent, its innovations are only weakly correlated with market returns and thus the Stambaugh bias should be modest for this variable.

As a way to explore the potential effects of this bias, we now report the results of a simple Monte Carlo study. We take the estimated VAR coefficients as the true data generating process and generate repeated samples. We use these samples to estimate new VAR systems and calculate various statistics. The difference between the mean of these statistics and the statistic in the data generating process is a measure of bias. Of course, this measure depends on the maintained data generating process so it should be taken merely as indicative in small samples.

Table 1 reports the estimated VAR coefficients (also shown in Table 2 of the BBGB paper) along with the bias in these coefficients estimated from 2500 samples. The bias in each coefficient is shown in curly brackets. The table illustrates both the Kendall bias in the persistent autoregressive coefficients for the forecasting variables, and the Stambaugh bias in the coefficient of stock returns on the price-earnings ratio. The bias in this coefficient is -0.005 as compared with a point estimate of -0.014. There is, however, only a negligible bias in the coefficient of stock returns on the value spread (-0.001 as compared with a point estimate of -0.013). This bias is small because innovations in the value spread are almost uncorrelated with stock returns as shown in Tables 1 and 2 of BBGB.

Table 2 reports statistics that describe the cash-flow and discount-rate news terms (also shown in Table 3 of BBGB), along with the bias in these statistics. There is very little bias in the estimated volatilities of cash-flow and discount-rate news, shown in the top panel. The most important bias shown in this table is upward bias in the negative coefficients of cash-flow and discount-rate news on the value spread. This upward bias makes the estimated coefficients closer to zero than the true coefficients, and thus understates the relevance of the value spread for the news terms. In other words, this bias works against the results reported in BBGB.

Tables 3 and 4 report the estimated cash-flow and discount-rate betas of growth and value stocks in the early and modern sample periods, 1929:1–1963:6 and 1963:7–2001:12. These betas are also shown in Tables 4 and 5 of BBGB. In addition, Tables 3 and 4 here report the biases in the betas. In the early period, value stocks have higher betas than growth stocks; the difference in cash-flow betas is biased downwards, while the difference in discount-rate betas has little consistent pattern. In the modern period, value stocks still have higher cash-flow betas, and the positive difference is biased downwards; but growth stocks now have higher discount-rate betas, and the negative difference is biased upwards. In other words, all the biases in the modern period work to shrink the beta differences across growth and value stocks towards zero. The results in the paper therefore tend to understate these beta differences.

Table 5 studies the effects of these biases on the prices of risk estimated in Tables 6 and 7 of the paper. There is little bias in the early period, and a strong downward bias in the premium for cash-flow beta in the modern period. The main conclusion of BBGB is that cash-flow beta has a much higher premium than discount-rate beta. This conclusion is if anything strengthened by the consideration of finite-sample bias.

4 The evolution of betas over time

BBGB estimates fixed betas in each of two subperiods. In other words betas are assumed to be constant except in 1963, when they change discretely in the middle of the year. An alternative view is that betas might have changed continuously during our sample period. Inferences about the time variation in betas are of course challenging due to the relatively large standard errors of individual portfolios' betas. Table 6 reports the subperiod beta standard errors that take into account the full estimation uncertainty in the news terms (these standard errors are omitted from BBGB to save space).

To explore the possibility that cash-flow and discount-rate betas vary continuously, in Figure 1 we show an alternative view of their time-series evolution. We first estimate a time-series of cash-flow and discount-rate betas for the 25 ME and BE/ME portfolios using a 120-month window. The series in Figure 1 are constructed from the estimated betas as follows: The value-minus-growth series, denoted by a solid line and triangles in the figure, is the equal-weight average of the five extreme value (high BE/ME) portfolios' betas less the equal-weight average of the five extreme growth (low BE/ME) portfolios' betas. The small-minus-big series, denoted by a solid line, is constructed as the equal-weight average of the five extreme small (low ME) portfolios' betas less the equal-weight average of the five extreme large (high ME) portfolios' betas. The top panel shows the cash-flow betas and the bottom panel discount-rate betas. The dates on the horizontal axes are centered with respect to the estimation window.

Two patterns stand out in the top panel of Figure 1. First, for the majority of our sample period, the higher-frequency movements in cash-flow betas of value-minus-growth and small-minus-big appear correlated, the small stocks' cash-flow betas possibly leading the value stocks' cash-flow betas. This pattern is strongly reversed in the 1990's, during which the cash-flow betas of small stocks clearly diverge from those of the value stocks. Second, over the entire period, the cash-flow betas of small stocks drifted down relative to those of large stocks, while the cash-flow betas of value stocks remain considerably higher than those of growth stocks (.15 higher at the beginning of the sample and .17 higher at the end). Much of the variation in these betas occurred during the first decade after World War II, with comparative stability of betas thereafter until the late 1990's.

The bottom panel of Figure 1 shows the time-series evolution of discount-rate betas. The first obvious trend in the figure is the steady and large decline in the discount-rate betas of value stocks relative to those of growth stocks. Over the full sample, the value-minus-growth beta declines from .31 to -.86. There is no similar trend for the discount-rate beta of small-minus-big, for which the time series begins at .37 and ends at .62. As for cash-flow betas, the discount-rate betas of value-minus-growth and small-minus-big strongly diverge during the nineties.

We believe that our practice of simply splitting the sample into two subperiods at 1963:6, and then assuming that the betas are constant for subperiods, is a reasonable and parsimonious approximation of reality. However, to alleviate any concerns that any of our results are due to thise approximation, we perform a number of experiments with time-varying betas in the next section.

5 Conditional pricing results

One concern about the results in BBGB might be that the estimated preference parameters appear rather different in the first and second subsamples. The point estimate of risk aversion, in the model with a restricted zero-beta rate and risk price for discount-rate news, is 3.6 in the first subsample and almost 24 in the second subsample. Even if betas and the variance of the market return have changed over time, one would hope that the underlying preferences of investors have remained stable.

To address this concern, we have estimated a version of our model that allows for changing betas and variances across the two subsamples, but imposes a constant coefficient of relative risk aversion. This model is not rejected at the 5% level, and the implied risk aversion coefficient is approximately six. Also, if we allow for different risk-aversion coefficients for the subsamples, we cannot reject the hypothesis that the two parameters are the same.

Another way to come at this issue, while simultaneously addressing the issue of continuous time variation in betas, is to estimate the preference parameters from a conditional model. We do this using two alternative approaches. Our first approach is illustrated in Figure 2. The figure shows the smoothed conditional premium on $\text{cov}_t(r_{i,t+1}, N_{M,CF,t+1})$ and $\text{cov}_t(r_{i,t+1}, -N_{M,DR,t+1})$, with the ICAPM predicting a premium of γ on the former and a unit premium on the latter. The graph is produced in three steps as follows. First, we run three sets of 45 time-series regressions on a constant, time trend, and the lagged VAR state variables, i.e., three regressions per test asset. The dependent variables in these regressions are simple excess return on the test assets $(R^e_{i,t})$, $(N_{CF,t} + N_{CF,t-1})R^e_{i,t}$, and $(N_{DR,t} + N_{DR,t-1})R^e_{i,t}$. Second, each month we regress the forecast values of excess return on the forecast values of the two covariances, excluding the constant and thus restricting the zero-beta rate to equal the risk-free rate. Third, we plot the five-year moving averages of these cross-sectional regression coefficients in Figure 2.

The lower line in Figure 2 is the estimated risk price for the discount-rate beta, divided by the variance of market returns. If our ICAPM holds exactly, this should be a horizontal line with a height of one. The upper line is the estimated risk price for the cash-flow beta, again divided by the variance of market returns. According to our ICAPM, this should be a horizontal line with a height of γ. The traditional CAPM implies that both lines should have the same height. Figure 2 shows that the scaled price of discount-rate risk has a long-term average very close to one, with substantial variations around this average, while the scaled price of cash-flow risk has a long-term average around six, again with substantial shorter-term variations. During the period 1935–1955 the two lines are close to one another, illustrating the good performance of the CAPM in this period. For most of the period since 1960 the two lines have diverged substantially, but there is no sign of a trend or other low-frequency instability in the risk prices.

Our second approach allows us to perform a formal asset-pricing test while allowing for continuously time-varying betas. We proceed as follows. First, we estimate covariances $\text{cov}_t(r_{i,t},\, N_{CF,t} + N_{CF,t-1})$ and $\text{cov}_t(r_{i,t},\, -N_{DR,t} - N_{DR,t-1})$ for each test asset using a rolling three-year (36 months) window. This three-year window will restrict the subsequent asset-pricing tests to the post-1931:1 period, but we (as always) estimate the market VAR using the full 1928:12-2001:12 sample. We use these rolling covariance estimates as instruments that predict future covariances.

Second, we regress the realized cross products $(N_{CF,t}+N_{CF,t-1})r_{i,t}$, and $(-N_{DR,t} - N_{DR,t-1})r_{i,t}$ on the corresponding lagged $(t-2)$ rolling covariance estimates and portfolio dummies in two pooled regressions. We lag the data by two months to avoid any overlap between the instruments and the dependent variables. On the one hand, the approach is flexible: Portfolio dummies are included to allow for portfolio-specific average covariances. On the other hand, we increase the power of the test by specifying a common predictive coefficient on past covariance across time and portfolios.

We define conditional covariances ($\widehat{\text{cov}}_{DR}$ and $\widehat{\text{cov}}_{CF}$) as the fitted values of those regressions.

Third, in period-by-period cross-sectional regressions, we regress the realized simple excess returns on the fitted conditional covariances, applying the restrictions implied by particular model. The data are aligned such that the realized simple excess returns for month t are regressed on forecasts of cross products for time t, where the forecast is formed using time $t-2$ rolling covariance estimate.

Table 7 shows the results of this exercise using the 25 ME- and BE/ME- sorted portfolios and 20 risk-sorted portfolios as the test assets. As in all the other pricing-test tables, we report the results for an unrestricted factor model, the two-beta ICAPM, and the CAPM. The second column for each model constrains the zero-beta rate (R_{zb}) to equal the risk-free rate (R_{rf}), i.e. the cross-sectional regressions omit the intercept. "Two-beta ICAPM" constrains the coefficient on $\widehat{\text{cov}}_{DR}$ to equal one. "CAPM" constrains the coefficient on $\widehat{\text{cov}}_{DR}$ to equal that on $\widehat{\text{cov}}_{CF}$.

In summary, allowing for continuous time variation in the covariances produces results that are very similar to the sub-period results reported in BBGB. The risk premium on cash-flow covariance is much higher than that on discount-rate covariance. The implied risk aversion parameter is high but reasonable, with point-estimates between 8 and 11. The two-beta model fits very well even with the ICAPM restrictions, while CAPM fits poorly and is rejected by the pricing error tests.

In unreported experiments, we have also reproduced Table 7 for subperiods (while still estimating the VAR using the full-period data.) The two-beta model passes the asset-pricing tests with flying colors, while the CAPM performs poorly in the latter subsample. Furthermore, when using continuously time-varying betas and the covariance (instead of beta) formulation, the preference parameter γ appears quite stable across subsamples. Estimated γ's range from 4 to 16 in the early sample and from 7 to 12 in the modern sample, depending on whether the zero-beta rate is assumed to equal the risk-free rate.

6 Sensitivity of results to changes in ρ

An important parameter in our model is ρ, a coefficient of loglinearization defined by Campbell and Shiller's (1988) approximation of the log return on an asset as a linear function of log prices and log dividends on the asset. The standard formula for ρ is $\rho \equiv 1/\left(1+\exp(\overline{d_t - p_t})\right)$. When the dividend-price ratio is constant, then $\rho = P/(P+D)$, the ratio of the ex-dividend to the cum-dividend stock price.

BBGB follows Campbell (1993, 1996) and applies the Campbell-Shiller method to the wealth of an investor. In this application ρ is linked to the investor's average consumption-wealth ratio. To understand this, consider a mutual fund that reinvests dividends and a mutual-fund investor who finances her consumption by redeeming a fraction of her mutual-fund shares every year. Effectively, the investor's consumption is now a dividend paid by the fund and the investor's wealth (the value of her remaining mutual fund shares) is now the ex-dividend price of the fund. Thus the Campbell-Shiller approximation describes a portfolio strategy as well as an underlying asset and the average consumption-wealth ratio generated by the strategy determines the discount coefficient $\rho = 1/(1 + C/W)$.

BBGB assumes $\rho = 0.95$ per year, corresponding to an average consumption-wealth ratio of 5.3%. This number is similar to the typical payout rate of endowments and foundations. Here, we explore the sensitivity of the BBGB results to variation in ρ between 0.93 (corresponding to an average consumption-wealth ratio of 7.5%) and 0.97 (corresponding to an average ratio of 3.1%). Tables 8 and 9 report estimated beta premiums and cross-sectional R^2 statistics for alternative asset pricing models, for ρ values of 0.93, 0.94, 0.96, and 0.97, over the early and modern subsamples.

As ρ varies the decomposition of market returns into cash-flow and discount-rate news varies, but of course the sum of these two news components must remain unchanged. Thus the value of ρ makes no difference in the CAPM, where both components of the market return are restricted to have the same price of risk. In the two-beta ICAPM the risk price of discount-rate beta is restricted to equal the variance of the market return so this risk price is unaffected by the value of ρ, which only affects the risk price of cash-flow beta and the overall fit of the model. In an unrestricted two-factor model both risk prices may vary with the value of ρ, which implies that the overall fit of the model is relatively insensitive to ρ.

The value of ρ makes very little difference to any of the results in the early subsample. In the modern subsample the fit of the two-beta ICAPM is sensitive to ρ if the zero-beta rate is restricted to equal the Treasury bill rate, because then the zero-beta rate and risk price of discount-rate beta are both restricted so changes in the estimate of discount-rate news affect the fit of the model. The fit of the two-beta ICAPM is much less sensitive to ρ if the model allows a free zero-beta rate, for then it offsets changes in the estimate of discount-rate news with changes in the zero-beta rate. The model with free factor risk prices is very insensitive to ρ and always estimates a price of cash-flow beta much higher than the price of discount-rate beta, supporting the main claim of BBGB.

Overall, the results in Tables 8 and 9 show that the main results of BBGB are robust to reasonable variation in the parameter ρ.

7 Data frequency

BBGB estimates a monthly VAR. Some readers have been curious whether the results would be similar if we had used lower-frequency data. Table 10 reports results for a quarterly VAR and an annual VAR. Asset pricing tests are conducted only over the full sample for annual data, since subsample results are tenuous when we have lower-frequency estimates of cash-flow and discount-rate news. The results are consistent with BBGB in that the estimated premiums for cash-flow betas are always higher than those for discount-rate betas, although the differences are smaller and less statistically significant because they are estimated over the full sample period and low-frequency data rather than the modern subsample and high-frequency data.

We have also performed the subperiod experiments for quarterly data. The subperiod point estimates obtained from quarterly data are very similar to those obtained from monthly data.

8 Sensitivity to changes in VAR state variables

Our basic VAR includes the return on a market stock index, the term spread, the smoothed price-earnings ratio, and the value spread. It omits two other variables that are often used to predict stock returns: the Treasury bill rate and the log dividend-price ratio. In Tables 11 and 12 we report asset pricing tests, for the early and modern subsamples, when we include these other variables in the VAR system. The results are very similar to those reported in BBGB. The BBGB results are, based on our experience, robust to adding many other known return predictors to the VAR system.

The results are also robust to estimating the VAR using real (instead of excess) market returns. Luis Viceira has pointed out that using excess stock returns in the VAR rather than real returns (given the particular version of the Campbell-Shiller loglinearization) is only correct if real interest rates are constant. We do not use real returns for the monthly tests because we believe that the monthly inflation data are too poorly measured and that the real interest rate is relatively constant. However, since the measurement error in inflation is likely to be less severe for quarterly and annual data, we have repeated our quarterly and annual tests using real market returns as the object of the Campbell-Shiller decomposition. The results are very similar to those obtained using quarterly and annual excess returns.

Finally, it should be remembered that our results depend critically on the inclusion of the small-stock value spread in our aggregate VAR system. If we exclude this variable we no longer find a large difference between the cash-flow betas of value stocks and growth stocks. BBGB contains a detailed discussion of various reasons why the small-stock value spread should predict market returns. Further motivation is provided by the ICAPM itself. We know that value stocks outperform growth stocks, particularly among smaller stocks, and that this cannot be explained by the traditional static CAPM. If the ICAPM is to explain this anomaly, then small growth stocks must have intertemporal hedging value that offsets their low returns; that is, their returns must be negatively correlated with innovations to investment opportunities. In order to evaluate this hypothesis it is natural to ask whether a long moving average of small growth stock returns predicts investment opportunities. This is exactly what we do when we include the small-stock value spread in our forecasting model for market returns. In short, the small-stock value spread is not an arbitrary forecasting variable but one that is suggested by the asset pricing theory we are trying to test.

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Table 1: VAR parameter and bias estimates

The table shows the OLS parameter estimates for a first-order VAR model including a constant, the log excess market return (r_M^e), term yield spread (TY), price-earnings ratio (PE), and small-stock value spread (VS). Each set of three rows corresponds to a different dependent variable. The first five columns report coefficients on the five explanatory variables, and the remaining columns show R^2 and F statistics. Bootstrap bias estimates in {curly brackets} are produced from 2500 realizations generated under the assumptions that the estimated VAR is the true data generating process. Sample period for the dependent variables is 1929:1-2001:12, 876 monthly data points.

	constant	$r_{M,t}^e$	TY_t	PE_t	VS_t	R^2 %	F
$r_{M,t+1}^e$	.062	.094	.006	-.014	-.013	2.57	5.34
	{.016}	{-.002}	{.000}	{-.005}	{-.001}		
TY_{t+1}	.046	.046	.879	-.036	.082	82.41	1.02×10^3
	{.021}	{.003}	{-.008}	{-.007}	{.001}		
PE_{t+1}	.019	.519	.002	.994	-.003	99.06	2.29×10^4
	{0.011}	{-.001}	{.000}	{-.003}	{-.001}		
VS_{t+1}	.014	-.005	.002	.000	.991	98.40	1.34×10^4
	{.009}	{-.000}	{.000}	{.000}	{-.006}		

Table 2: Bias in cash-flow and discount-rate news

The table shows the properties of cash-flow news (N_{CF}) and discount-rate news (N_{DR}) implied by the VAR model of Table 1. The upper-left section of the table shows the covariance matrix of the news terms. The upper-right section shows the correlation matrix of the news terms with standard deviations on the diagonal. The lower-left section shows the correlation of shocks to individual state variables with the news terms. The lower right section shows the functions ($e1' + e1'\lambda, e1'\lambda$) that map the state-variable shocks to cash-flow and discount-rate news. We define $\lambda \equiv \rho\Gamma(I - \rho\Gamma)^{-1}$, where Γ is the estimated VAR transition matrix from Table 1 and ρ is set to .95 per annum. r^e_M is the excess log return on the CRSP value-weight index. TY is the term yield spread. PE is the log ratio of S&P 500's price to S&P 500's ten-year moving average of earnings. VS is the small-stock value-spread, the difference in log book-to-markets of value and growth stocks. Bootstrap bias estimates in {curly brackets} are produced from 2500 realizations generated under the assumptions that the estimated VAR is the true data generating process.

News covariance	N_{CF}	N_{DR}	News corr/std	N_{CF}	N_{DR}
N_{CF}	.00064	.00015	N_{CF}	.0252	.114
	{.00004}	{.00007}		{.0005}	{.0085}
N_{DR}	.00015	.00267	N_{DR}	.114	.0517
	{.00007}	{.00008}		{.0085}	{.0002}
Shock correlations	N_{CF}	N_{DR}	**Functions**	N_{CF}	N_{DR}
r^e_M shock	.352	-.890	r^e_M shock	.602	-.398
	{-.027}	{.000}		{-.003}	{-.003}
TY shock	.128	.042	TY shock	.011	.011
	{-.003}	{.002}		{.000}	{.000}
PE shock	-.204	-.925	PE shock	-.883	-.883
	{-.028}	{-.002}		{-.005}	{-.005}
VS shock	-.493	-.186	VS shock	-.283	-.283
	{.153}	{.060}		{.064}	{.064}

Table 3: Bias in cash-flow and discount-rate betas, early sample

The table shows the estimated cash-flow ($\widehat{\beta}_{CF}$) and discount-rate betas ($\widehat{\beta}_{DR}$) for the 25 ME- and BE/ME-sorted portfolios. "Growth" denotes the lowest BE/ME, "value" the highest BE/ME, "small" the lowest ME, and "large" the highest ME stocks. "Diff." is the difference between the extreme cells. Bootstrap bias estimates in {curly brackets} are produced from 2500 realizations generated under the assumptions that the estimated VAR of Table 1 is the true data generating process. Estimates are for the 1929:1-1963:6 period.

$\widehat{\beta}_{CF}$	Growth	2	3	4	Value	Diff.
Small	.53	.46	.40	.42	.49	-.04
	{-.26}	{-.27}	{-.22}	{-.25}	{-.31}	{-.05}
2	.30	.34	.36	.38	.45	.16
	{-.17}	{-.18}	{-.18}	{-.20}	{-.25}	{-.08}
3	.30	.28	.31	.35	.47	.18
	{-.15}	{-.12}	{-.11}	{-.17}	{-.22}	{-.07}
4	.20	.26	.31	.35	.50	.30
	{-.07}	{-.11}	{-.11}	{-.17}	{-.22}	{-.07}
Large	.20	.19	.28	.33	.40	.19
	{-.07}	{-.07}	{-.10}	{-.12}	{-.15}	{-.08}
Diff.	-.33	-.26	-.12	-.09	-.10	
	{.15}	{.11}	{.09}	{.05}	{.05}	

$\widehat{\beta}_{DR}$	Growth	2	3	4	Value	Diff.
Small	1.32	1.46	1.32	1.27	1.27	-.06
	{.09}	{-.13}	{-.03}	{-.06}	{.02}	{-.07}
2	1.04	1.15	1.09	1.25	1.25	.21
	{-.02}	{-.06}	{-.06}	{-.03}	{-.00}	{.02}
3	1.13	1.01	1.08	1.05	1.27	.14
	{-.10}	{-.03}	{-.07}	{-.02}	{-.01}	{.09}
4	.87	.97	.97	1.06	1.36	.49
	{-.01}	{-.05}	{-.01}	{-.00}	{-.01}	{-.00}
Large	.88	.82	.87	1.06	1.18	.31
	{-.04}	{-.01}	{.12}	{.08}	{.10}	{.07}
Diff.	-.45	-.64	-.43	-.21	-.08	
	{.19}	{.17}	{.12}	{.08}	{.10}	

Table 4: Bias in cash-flow and discount-rate betas, modern sample

The table shows the estimated cash-flow ($\widehat{\beta}_{CF}$) and discount-rate betas ($\widehat{\beta}_{DR}$) for the 25 ME- and BE/ME-sorted portfolios. "Growth" denotes the lowest BE/ME, "value" the highest BE/ME, "small" the lowest ME, and "large" the highest ME stocks. "Diff." is the difference between the extreme cells. Bootstrap bias estimates in {curly brackets} are produced from 2500 realizations generated under the assumptions that the estimated VAR of Table 1 is the true data generating process. Estimates are for the 1963:7-2001:12 period.

$\widehat{\beta}_{CF}$	Growth	2	3	4	Value	Diff.
Small	.06	.07	.09	.09	.13	.07
	{-.01}	{-.02}	{-.04}	{-.04}	{-.06}	{-.05}
2	.04	.08	.10	.11	.12	.09
	{.01}	{-.02}	{-.04}	{-.04}	{-.05}	{-.05}
3	.03	.09	.11	.12	.13	.09
	{.01}	{-.01}	{-.03}	{-.04}	{-.03}	{-.04}
4	.03	.10	.11	.11	.13	.10
	{.03}	{-.01}	{-.02}	{-.02}	{-.01}	{-.04}
Large	.03	.08	.09	.11	.11	.09
	{.04}	{.01}	{.01}	{-.01}	{-.01}	{-.04}
Diff.	-.03	.02	-.01	.02	-.01	
	{.11}	{.10}	{.08}	{.09}	{.08}	

$\widehat{\beta}_{DR}$	Growth	2	3	4	Value	Diff.
Small	1.66	1.37	1.18	1.12	1.12	-.54
	{-.32}	{-.24}	{-.19}	{-.21}	{-.22}	{.09}
2	1.54	1.22	1.07	.96	1.03	-.52
	{-.20}	{-.16}	{-.14}	{-.10}	{-.11}	{.10}
3	1.41	1.11	.95	.82	.94	-.47
	{-.14}	{-.11}	{-.10}	{-.05}	{-.10}	{.05}
4	1.27	1.05	.89	.79	.87	-.41
	{-.11}	{-.09}	{-.04}	{-.01}	{-.04}	{.08}
Large	1.00	.87	.74	.63	.68	-.33
	{-.09}	{-.05}	{-.00}	{.01}	{-.04}	{.05}
Diff.	-.66	-.50	-.44	-.49	-.44	
	{.14}	{.13}	{.10}	{.11}	{.08}	

Table 5: Bias in factor premia

The table shows premia point estimates and bias estimates for an unrestricted factor model, the two-beta ICAPM, and the CAPM. The test assets are the 25 ME- and BE/ME- sorted portfolios and 20 risk-sorted portfolios. The second column per model constrains the zero-beta rate (R_{zb}) to equal the risk-free rate (R_{rf}). Estimates are from a cross-sectional regression of average simple excess test-asset returns (monthly in fractions) on an intercept and estimated cash-flow ($\widehat{\beta}_{CF}$) and discount-rate betas ($\widehat{\beta}_{DR}$). Bias estimates in {curly brackets} are produced from 2500 simulated realizations assuming that the estimated VAR is the true data generating process.

1929:1-1963:6	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	.0042	0	.0023	0	.0023	0
	{-.0013}	{0}	{.0012}	{0}	{.0002}	{0}
$\widehat{\beta}_{CF}$ premium (g_1)	.0173	.0069	.0083	.0148	.0051	.0067
	{-.0193}	{-.0100}	{-.0046}	{.0035}	{.0005}	{.0009}
$\widehat{\beta}_{DR}$ premium (g_2)	-.0003	.0066	.0041	.0041	.0051	.0067
	{.0061}	{.0026}	{-.0000}	{-.0000}	{.0005}	{.0009}
1963:7-2001:12	**Factor model**		**Two-beta ICAPM**		**CAPM**	
R_{zb} less R_{rf} (g_0)	.0009	0	-.0009	0	.0069	0
	{.0058}	{0}	{.0061}	{0}	{.0012}	{0}
$\widehat{\beta}_{CF}$ premium (g_1)	.0529	.0572	.0575	.0483	-.0007	.0051
	{-.0568}	{-.0461}	{-.0589}	{-.0347}	{-.0013}	{.0005}
$\widehat{\beta}_{DR}$ premium (g_2)	.0007	.0012	.0020	.0020	-.0007	.0051
	{-.0005}	{.0051}	{-.0000}	{-.0000}	{-.0013}	{.0005}

Table 6: Subperiod betas for the 25 ME and BE/ME portfolios

The table shows the estimates of cash-flow betas ($\widehat{\beta}_{CF}$) and discount-rate betas ($\widehat{\beta}_{DR}$) for Davis, Fama, and French's (2000) 25 size- and book-to-market-sorted portfolios for the two subperiods (1929:1-1963:6 and 1963:7-2001:12). The standard errors (in parentheses) take into account the full estimation uncertainty in the news terms.

1929:1-1963:6

$\widehat{\beta}_{CF}$	Growth		2		3		4		Value		Diff.	
Small	.53	(.28)	.46	(.24)	.40	(.23)	.42	(.22)	.49	(.25)	-.04	(.07)
2	.30	(.18)	.34	(.29)	.36	(.18)	.38	(.20)	.45	(.24)	.16	(.08)
3	.30	(.18)	.28	(.27)	.31	(.18)	.35	(.19)	.47	(.24)	.18	(.08)
4	.20	(.14)	.26	(.26)	.31	(.17)	.35	(.19)	.50	(.26)	.30	(.12)
Large	.20	(.14)	.19	(.14)	.28	(.16)	.33	(.20)	.40	(.24)	.19	(.11)
Diff.	-.33	(.15)	-.26	(.11)	-.12	(.09)	-.09	(.05)	-.10	(.05)		

$\widehat{\beta}_{DR}$	Growth		2		3		4		Value		Diff.	
Small	1.32	(.31)	1.46	(.28)	1.32	(.26)	1.27	(.25)	1.27	(.28)	-.06	(.15)
2	1.04	(.20)	1.15	(.20)	1.09	(.20)	1.25	(.22)	1.25	(.26)	.21	(.11)
3	1.13	(.19)	1.01	(.18)	1.08	(.18)	1.05	(.20)	1.27	(.25)	.14	(.09)
4	.87	(.15)	.97	(.17)	.97	(.18)	1.06	(.20)	1.36	(.27)	.49	(.14)
Large	.88	(.14)	.82	(.15)	.87	(.16)	1.06	(.20)	1.18	(.25)	.31	(.13)
Diff.	-.45	(.20)	-.64	(.17)	-.43	(.13)	-.21	(.09)	-.08	(.10)		

1963:7-2001:12

$\widehat{\beta}_{CF}$	Growth		2		3		4		Value		Diff.	
Small	.06	(.24)	.07	(.19)	.09	(.16)	.09	(.14)	.13	(.14)	.07	(.13)
2	.04	(.24)	.08	(.18)	.10	(.14)	.11	(.13)	.12	(.14)	.09	(.13)
3	.03	(.22)	.09	(.15)	.11	(.13)	.12	(.12)	.13	(.13)	.09	(.14)
4	.03	(.20)	.10	(.15)	.11	(.12)	.11	(.11)	.13	(.12)	.10	(.12)
Large	.03	(.14)	.08	(.12)	.09	(.11)	.11	(.10)	.11	(.10)	.09	(.09)
Diff.	-.03	(.11)	.02	(.10)	-.01	(.08)	.02	(.08)	-.01	(.07)		

$\widehat{\beta}_{DR}$	Growth		2		3		4		Value		Diff.	
Small	1.66	(.26)	1.37	(.21)	1.18	(.17)	1.12	(.16)	1.12	(.15)	-.54	(.14)
2	1.54	(.25)	1.22	(.19)	1.07	(.16)	.96	(.14)	1.03	(.15)	-.52	(.14)
3	1.41	(.23)	1.11	(.16)	.95	(.14)	.82	(.13)	.94	(.14)	-.47	(.15)
4	1.27	(.21)	1.05	(.15)	.89	(.13)	.79	(.13)	.87	(.14)	-.41	(.14)
Large	1.00	(.15)	.87	(.13)	.74	(.12)	.63	(.11)	.68	(.11)	-.33	(.11)
Diff.	-.66	(.14)	-.50	(.13)	-.44	(.10)	-.49	(.11)	-.44	(.10)		

Table 7: Asset-pricing tests with time-varying betas

The table shows premia estimated from the full 1932:1-2001:12 sample for an unrestricted factor model, the two-beta ICAPM, and the CAPM. The test assets are the 25 ME- and BE/ME- sorted portfolios and 20 risk-sorted portfolios. The test is performed as follows. First, we estimate covariances $\text{cov}_t(r_{i,t}, N_{CF,t} + N_{CF,t-1})$ and $\text{cov}_t(r_{i,t+1}, -N_{DR,t} - N_{DR,t-1})$ for each test asset using a rolling three-year (36 months) window. Second, we regress the realized cross products $(N_{CF,t}+N_{CF,t-1})r_{i,t}$, and $(-N_{DR,t} - N_{DR,t-1})r_{i,t}$ on the corresponding lagged $(t-2)$ rolling covariance estimate and protfolio dummies in two pooled regressions. We define conditional covariances ($\widehat{\text{cov}}_{DR}$ and $\widehat{\text{cov}}_{CF}$) as the fitted values of those regressions. Third, in period-by-period cross-sectional regressions, we regress the realized simple excess returns on the fitted conditional covariances, applying the restrictions implied by particular model. Fourth, we report the time-series average coefficients in the table. The second column per model constrains the zero-beta rate (R_{zb}) to equal the risk-free rate (R_{rf}), i.e. the cross-sectional regressions omit the intercept. "Two-beta ICAPM" constrains the coefficient on $\widehat{\text{cov}}_{DR}$ equal to one. "CAPM" constrains the coefficient on $\widehat{\text{cov}}_{DR}$ equal that on $\widehat{\text{cov}}_{CF}$. $\widehat{R}^2$ is from a cross-sectional regression of average portfolio return on the average covariances. Standard errors and critical values [A] are conditional on the estimated news series and (B) incorporating full estimation uncertainty of the news terms. The test rejects if the pricing error is higher than the listed 5% critical value.

Parameter	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf}	.0039	0	.0012	0	.0055	0
Std. err. A	[.0021]	N/A	[.0023]	N/A	[.0021]	N/A
Std. err. B	(.0021)	N/A	(.0025)	N/A	(.0021)	N/A
$\widehat{\text{cov}}_{CF}$ premium	11.55	12.91	10.46	14.71	.77	2.96
Std. err. A	[4.08]	[4.18]	[4.21]	[8.49]	[1.06]	[.77]
Std. err. B	(7.75)	(7.69)	(6.47)	(12.65)	(1.09)	(.79)
$\widehat{\text{cov}}_{DR}$ premium	-.36	1.28	1.00	1.00	.77	2.96
Std. err. A	[1.23]	[.98]	N/A	N/A	[1.06]	[.77]
Std. err. B	(1.54)	(1.41)	N/A	N/A	(1.09)	(.79)
$\widehat{R}^2$	75.71%	67.44%	69.43%	60.60%	36.75%	24.18%
Pricing error	.0090	.0110	.0083	.0143	.0260	.0252
5% critic. val. A	[.0137]	[.0192]	[.0292]	[.0742]	[.0157]	[.0194]
5% critic. val. B	(.0170)	(.0244)	(.0294)	(.1576)	(.0152)	(.0191)

Table 8: Sensitivity to changes in rho, early subsample

The table shows premia estimated from the 1929:1-1963:6 sample for an unrestricted factor model, the two-beta ICAPM, and the CAPM. The test assets are the 25 ME- and BE/ME- sorted portfolios and 20 risk-sorted portfolios. The second column per model constrains the zero-beta rate (R_{zb}) to equal the risk-free rate (R_{rf}). Estimates are from a cross-sectional regression of average simple excess test-asset returns (monthly in fractions) on an intercept and estimated cash-flow ($\widehat{\beta}_{CF}$) and discount-rate betas ($\widehat{\beta}_{DR}$). The panels vary $\rho = [0.93, 0.94, 0.96, 0.97]$.

$\rho = 0.93$	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	.0042	0	.0023	0	.0023	0
$\widehat{\beta}_{CF}$ premium (g_1)	.0168	.0065	.0074	.0123	.0051	.0067
$\widehat{\beta}_{DR}$ premium (g_2)	-.0021	.0070	.0041	.0041	.0051	.0067
$\widehat{R}^2$	48.29%	40.26%	45.59%	39.13%	44.52%	40.26%
$\rho = 0.94$	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	.0042	0	.0023	0	.0023	0
$\widehat{\beta}_{CF}$ premium (g_1)	.0170	.0069	.0077	.0133	.0051	.0067
$\widehat{\beta}_{DR}$ premium (g_2)	-.0013	.0066	.0041	.0041	.0051	.0067
$\widehat{R}^2$	48.21%	40.26%	45.70%	38.70%	44.52%	40.26%
$\rho = 0.96$	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	.0041	0	.0024	0	.0023	0
$\widehat{\beta}_{CF}$ premium (g_1)	.0175	.0067	.0090	.0172	.0051	.0067
$\widehat{\beta}_{DR}$ premium (g_2)	.0006	.0067	.0041	.0041	.0051	.0067
$\widehat{R}^2$	47.92%	40.26%	46.03%	36.63%	44.52%	40.26%
$\rho = 0.97$	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	.0040	0	.0025	0	.0023	0
$\widehat{\beta}_{CF}$ premium (g_1)	.0177	.0066	.0101	.0212	.0051	.0067
$\widehat{\beta}_{DR}$ premium (g_2)	.0016	.0067	.0041	.0041	.0051	.0067
$\widehat{R}^2$	47.71%	40.26%	46.26%	33.60%	44.52%	40.26%

Table 9: Sensitivity to changes in rho, modern subsample

The table shows premia estimated from the 1963:7-2001:12 sample for an unrestricted factor model, the two-beta ICAPM, and the CAPM. The test assets are the 25 ME- and BE/ME- sorted portfolios and 20 risk-sorted portfolios. The second column per model constrains the zero-beta rate (R_{zb}) to equal the risk-free rate (R_{rf}). Estimates are from a cross-sectional regression of average simple excess test-asset returns (monthly in fractions) on an intercept and estimated cash-flow ($\widehat{\beta}_{CF}$) and discount-rate betas ($\widehat{\beta}_{DR}$). The panels vary $\rho = [0.93, 0.94, 0.96, 0.97]$.

$\rho = 0.93$	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	.0007	0	-.0016	0	.0069	0
$\widehat{\beta}_{CF}$ premium (g_1)	.0487	.0516	.0325	.0239	-.0007	.0051
$\widehat{\beta}_{DR}$ premium (g_2)	-.0033	-.0032	.0020	.0020	-.0007	.0051
$\widehat{R}^2$	53.15%	52.86%	11.17%	9.38%	3.10%	-61.57%
$\rho = 0.94$	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	.0008	0	-.0025	0	.0069	0
$\widehat{\beta}_{CF}$ premium (g_1)	.0506	.0542	.0498	.0317	-.0007	.0051
$\widehat{\beta}_{DR}$ premium (g_2)	-.0013	-.0011	.0020	.0020	-.0007	.0051
$\widehat{R}^2$	52.67%	52.29%	32.13%	25.76%	3.10%	-61.57%
$\rho = 0.96$	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	.0010	0	.0022	0	.0069	0
$\widehat{\beta}_{CF}$ premium (g_1)	.0555	.0608	.0503	.0842	-.0007	.0051
$\widehat{\beta}_{DR}$ premium (g_2)	.0029	.0037	.0020	.0020	-.0007	.0051
$\widehat{R}^2$	51.41%	50.69%	50.41%	-7.20%	3.10%	-61.57%
$\rho = 0.97$	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	.0012	0	.0045	0	.0069	0
$\widehat{\beta}_{CF}$ premium (g_1)	.0587	.0654	.0325	-.0078	-.0007	.0051
$\widehat{\beta}_{DR}$ premium (g_2)	.0053	.0064	.0020	.0020	-.0007	.0051
$\widehat{R}^2$	50.54%	49.47%	42.36%	-371.76%	3.10%	-61.57%

Table 10: Sensitivity of the asset-pricing tests to data frequency

The table shows estimated premia for an unrestricted factor model, the two-beta ICAPM, and the CAPM. The test assets are the 25 ME- and BE/ME- sorted portfolios and 20 risk-sorted portfolios. The second column per model constrains the zero-beta rate (R_{zb}) to equal the risk-free rate (R_{rf}). Estimates are from a cross-sectional regression of average simple excess test-asset returns (monthly in fractions) on an intercept and estimated cash-flow ($\widehat{\beta}_{CF}$) and discount-rate betas ($\widehat{\beta}_{DR}$). The first panel use quarterly data (1929:3-2001:12) and the second panel uses annual data (1930:5-2001:5). The thrid and fourth panels use subsamples of quarterly data, with the break point at 1963:6. The VAR that genererates the news terms is always estimated from the full sample.

Full quarterly	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	.0119	0	.0029	0	.0090	0
$\widehat{\beta}_{CF}$ premium (g_1)	.1321	.1251	.0492	.0658	.0114	.0185
$\widehat{\beta}_{DR}$ premium (g_2)	-.0097	.0022	.0115	.0115	.0114	.0185
$\widehat{R}^2$	59.91%	42.48%	36.34%	34.56%	27.99%	17.60%
Full annual	**Factor model**		**Two-beta ICAPM**		**CAPM**	
R_{zb} less R_{rf} (g_0)	.0269	0	.0058	0	.0004	0
$\widehat{\beta}_{CF}$ premium (g_1)	.4908	.4439	.3555	.3851	.0988	.0991
$\widehat{\beta}_{DR}$ premium (g_2)	.0059	.0390	.0496	.0496	.0988	.0991
$\widehat{R}^2$	75.61%	73.21%	73.12%	72.85%	61.28%	61.27%
Early quarterly	**Factor model**		**Two-beta ICAPM**		**CAPM**	
R_{zb} less R_{rf} (g_0)	.0104	0	.0086	0	.0093	0
$\widehat{\beta}_{CF}$ premium (g_1)	.0370	.0268	.0112	.0386	.0143	.0209
$\widehat{\beta}_{DR}$ premium (g_2)	.0069	.0001	.0159	.0159	.0143	.0209
$\widehat{R}^2$	47.10%	37.05%	46.33%	35.17%	46.55%	36.50%
Modern quarterly	**Factor model**		**Two-beta ICAPM**		**CAPM**	
R_{zb} less R_{rf} (g_0)	.0008	0	-.0096	0	.0227	0
$\widehat{\beta}_{CF}$ premium (g_1)	.1900	.4439	.2102	.1203	-.0045	.0175
$\widehat{\beta}_{DR}$ premium (g_2)	-.0019	.0390	.0077	.0077	-.0045	.0175
$\widehat{R}^2$	69.95%	69.90%	56.65%	41.31%	5.21%	-69.67%

Table 11: Alternative VAR specification, early sample

The table shows premia estimated from the 1929:1-1963:6 sample for an unrestricted factor model, the two-beta ICAPM, and the CAPM. The news terms are estimated using a VAR model that includes T-bill yield and log dividend yield in the VAR state vector, in addition to the variables in the base-case specification (market's excess return, term yield spread, log price-earnings ratio, and the small-stock value spread). The VAR estimation period is 1928:12-2001:12. The test assets are the 25 ME- and BE/ME- sorted portfolios and 20 risk-sorted portfolios. The second column per model constrains the zero-beta rate (R_{zb}) to equal the risk-free rate (R_{rf}). Estimates are from a cross-sectional regression of average simple excess test-asset returns (monthly in fractions) on an intercept and estimated cash-flow ($\widehat{\beta}_{CF}$) and discount-rate betas ($\widehat{\beta}_{DR}$). Standard errors and critical values [A] are conditional on the estimated news series and (B) incorporating full estimation uncertainty of the news terms. The test rejects if the pricing error is higher than the listed 5% critical value.

Parameter	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	.0038	0	.0024	0	.0024	0
% per annum	4.57%	0%	2.90%	0%	2.82%	0%
Std. err. A	[.0028]	N/A	[.0025]	N/A	[.0028]	N/A
Std. err. B	(.0028)	N/A	(.0030)	N/A	(.0028)	N/A
$\widehat{\beta}_{CF}$ premium (g_1)	.0175	.0056	.0086	.0160	.0051	.0068
% per annum	21.03%	6.72%	10.36%	19.23%	6.14%	8.12%
Std. err. A	[.0292]	[.0285]	[.0183]	[.0170]	[.0046]	[.0034]
Std. err. B	(.0403)	(.0371)	(.0305)	(.0615)	(.0046)	(.0034)
$\widehat{\beta}_{DR}$ premium (g_2)	.0002	.0071	.0041	.0041	.0051	.0068
% per annum	.26%	8.52%	4.87%	4.87%	6.14%	8.12%
Std. err. A	[.0093]	[.0078]	[.0005]	[.0005]	[.0046]	[.0034]
Std. err. B	(.0098)	(.0078)	(.0005)	(.0005)	(.0046)	(.0034)
$\widehat{R}^2$	47.31%	40.24%	45.87%	37.64%	44.82%	40.21%
Pricing error	.0120	.0127	.0120	.0134	.0126	.0126
5% critic. val. A	[.020]	[.021]	[.022]	[.30]	[.021]	[.027]
5% critic. val. B	(.020)	(.024)	(.031)	(.085)	(.021)	(.027)

Table 12: Alternative VAR specification, modern sample

The table shows premia estimated from the 1963:7-2001:12 sample for an unrestricted factor model, the two-beta ICAPM, and the CAPM. The news terms are estimated using a VAR model that includes T-bill yield and log dividend yield in the VAR state vector, in addition to the variables in the base-case specification (market's excess return, term yield spread, log price-earnings ratio, and the small-stock value spread). The VAR estimation period is 1928:12-2001:12. The test assets are the 25 ME- and BE/ME- sorted portfolios and 20 risk-sorted portfolios. The second column per model constrains the zero-beta rate (R_{zb}) to equal the risk-free rate (R_{rf}). Estimates are from a cross-sectional regression of average simple excess test-asset returns (monthly in fractions) on an intercept and estimated cash-flow ($\widehat{\beta}_{CF}$) and discount-rate betas ($\widehat{\beta}_{DR}$). Standard errors and critical values [A] are conditional on the estimated news series and (B) incorporating full estimation uncertainty of the news terms. The test rejects if the pricing error is higher than the listed 5% critical value.

Parameter	Factor model		Two-beta ICAPM		CAPM	
R_{zb} less R_{rf} (g_0)	-.0006	0	-.0039	0	.0069	0
% per annum	-.76%	0%	-4.66%	0%	8.26%	0%
Std. err. A	[.0030]	N/A	[.0032]	N/A	[.0026]	N/A
Std. err. B	(.0034)	N/A	(.0038)	N/A	(.0026)	N/A
$\widehat{\beta}_{CF}$ premium (g_1)	.0262	.0247	.0281	.0154	-.0007	.0050
% per annum	31.47%	29.65%	33.67%	18.49%	-.84%	6.05%
Std. err. A	[.0116]	[.0095]	[.0118]	[.0129]	[.0032]	[.0022]
Std. err. B	(.0201)	(.0224)	(.0206)	(.0222)	(.0032)	(.0022)
$\widehat{\beta}_{DR}$ premium (g_2)	-.0012	-.0014	.0020	.0020	-.0007	.0050
% per annum	.88%	1.44%	2.43%	2.43%	-.84%	6.05%
Std. err. A	[.0035]	[.0033]	[.0002]	[.0002]	[.0032]	[.0022]
Std. err. B	(.0075)	(.0084)	(.0002)	(.0002)	(.0032)	(.0022)
$\widehat{R}^2$	66.88%	66.62%	47.10%	32.95%	3.16%	-63.32%
Pricing error	.0179	.0183	.0281	.0327	.0593	.0887
5% critic. val. A	[.035]	[.049]	[.062]	[.097]	[.031]	[.044]
5% critic. val. B	(.044)	(.087)	(.072)	(.313)	(.031)	(.044)

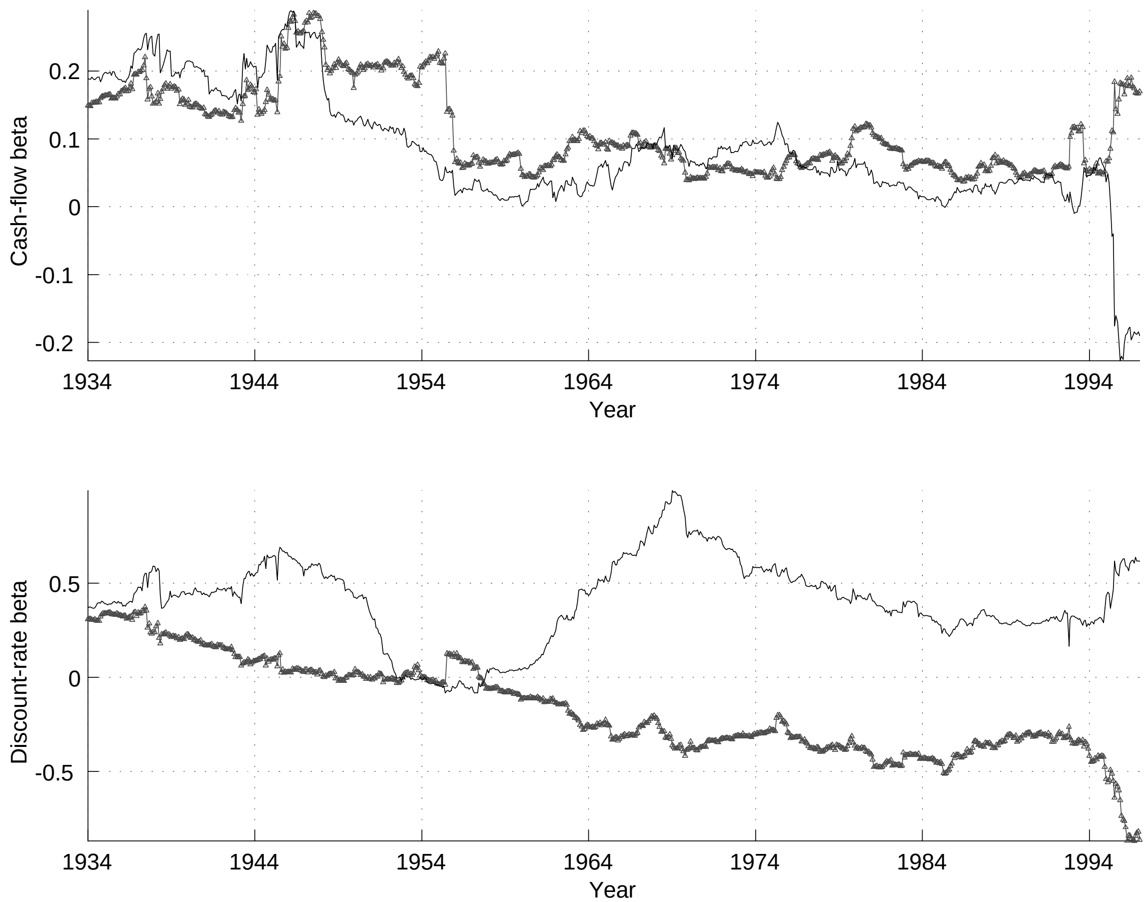

Figure 1: Time-series evolution of cash-flow and discount-rate betas of value-minus-growth and small-minus-big.

First, we estimate the cash-flow betas ($\widehat{\beta}_{CF}$) and discount-rate betas ($\widehat{\beta}_{CF}$) for the 25 ME and BE/ME portfolios using a 120-month moving window. The value-minus-growth series, denoted by a solid line and triangles, is then constructed as the equal-weight average of the five extreme value (high BE/ME) portfolios' betas less that of the five extreme growth (low BE/ME) portfolios' betas. The small-minus-big series, denoted by a solid line, is constructed as the equal-weight average of the five extreme small (low ME) portfolios' betas less that of the five extreme large (high ME) portfolios' betas. The top panel shows the estimated cash-flow and the bottom panel estimated discount-rate betas. Dates on the horizontal axis denote the midpoint of the estimation window.

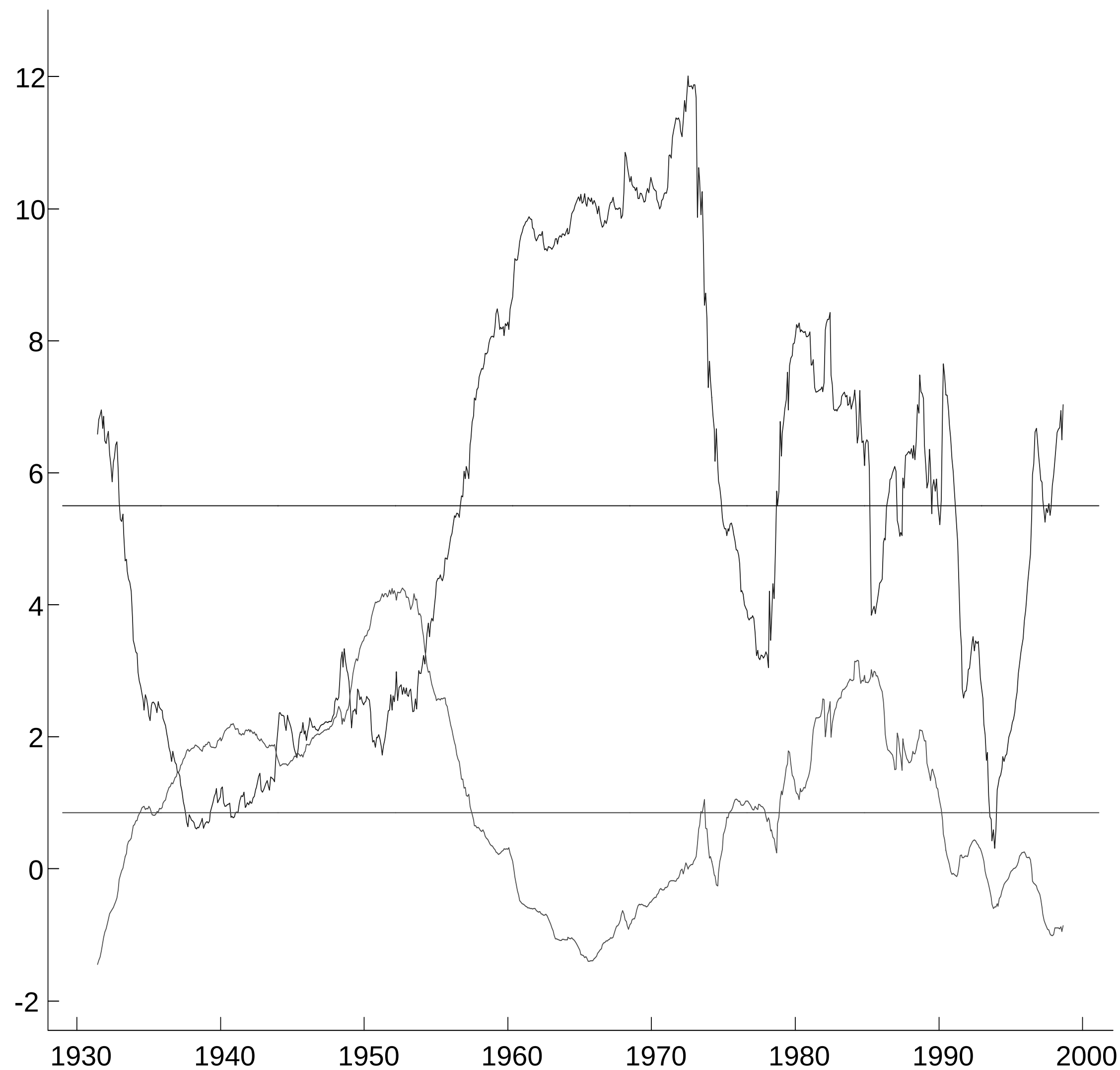

Figure 2: Conditional risk premia for cash-flow and discount-rate betas.

We show the smoothed conditional premium on β_{CF} (top line) and β_{DR} (bottom line), both scaled by the market's conditional volatility. The horizontal lines are time-series averages. First, we run three sets of 45 time-series regressions on a constant, time trend, and the lagged VAR state variables, where the dependent variables are (1) excess return on the test assets ($R^e_{i,t}$), (2) $(N_{CF,t} + N_{CF,t-1})R^e_{i,t}$, and (3) $(N_{DR,t} + N_{DR,t-1})R^e_{i,t}$. Then, each month, we regress the fitted values of (1) on the fitted values of (2) and (3), and plot the five-year moving averages of these cross-sectional coefficients.