

Banks as Safety Multipliers: A Theory of Safe Assets Creation*

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Abstract

Why do banks hold so many safe assets such as government bonds? I argue that the economic role of banks is to multiply safety, and their holdings of public debt help them achieve that goal. In a environment with endogenous collateral constraints and multiple assets, risk-neutral banks issue debt securities to cater to the safety demand of risk-averse investors. Private safety creation requires banks to hold on their balance sheets government bonds, whose returns are negatively correlated with macroeconomic shocks. When heterogeneity in risk aversion is large and public debt supply low, there is a safety multiplier in the sense that lowering even more the supply of public debt induces banks to delever, hurting private debt supply. In this regime, public debt crowds-in private investment, because a shortage of public debt constrains risk-sharing. In a dynamic version with maturity choices, I endogeneize the negative correlation of public debt. The expectation of a flight-to-safety transforms long-term securities into hedging instruments. The private equilibrium is constrained inefficient due to an issuance externality. The economy lacks long-term securities, as private agents do not internalize the benefits of the negative beta of their own liabilities. Public debt is non-neutral but there is an interior optimal level of public securities, as their hedging properties deteriorate with their supply.

The model interprets the ongoing European debt crisis as a shortage of public safe assets. Public debt and private debt positively comove in Europe, contrary to the US. Simultaneously, banks increased their holdings of safe public debt to back private debt. As for asset pricing, the spread between public debt yield and private debt yield reveals bank leverage and can be used as a macroprudential tool. In an open economy environment, sovereign risk hurts aggregate private leverage but domestic banks become the natural holders of domestic public debt.

JEL Classification: G12, G23, G32, D8, L14.

Keywords: financial intermediation, risk sharing, safe assets, public debt, sovereign risk.

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1 Introduction

Why do banks hold so much public debt that yields so little? Banks hold 15% of their assets in safe securities on their balance sheet (Figure 2). This paper is motivated by banks' holdings of large quantities of mundane securities, even at an accounting loss.

Traditional theories of banking usually revolve around the idea that banks are able to mitigate some type of agency frictions. In the [Gorton and Pennacchi \(1990\)](#) tradition, there is asymmetric information about the quality of the projects undertaken by entrepreneurs, and banks exhibit a comparative advantage in screening and monitoring loans to these projects. For instance, [Holmstrom and Ordonez \(2013\)](#) argues that banks business is all about being able to 'keep secret' about these loans, i.e. issue information-insensitive securities against these loans. It is hard to apply this line of argument to universal banks, which hold substantial amounts of highly liquid, marketed and researched securities such as government bonds. For these holdings, banks do not have an intrinsic comparative informational advantage on the market.

In this paper I develop a view of banks as insurers against aggregate shocks. Banks are in the business of producing safe liabilities, and they use public debt as an *input* to their safety production function. Public debt has two desirable features: a safe terminal payoff, and an interim value that is negatively correlated with aggregate shocks. The output of banking arises on their liability side, whereas their asset side is merely the juxtaposition of inputs that maximize the safety output. This interaction between public debt and private debt, the former being an input to the production of the latter, has crucial positive and normative implications related to the macroeconomic shortage of safe assets.

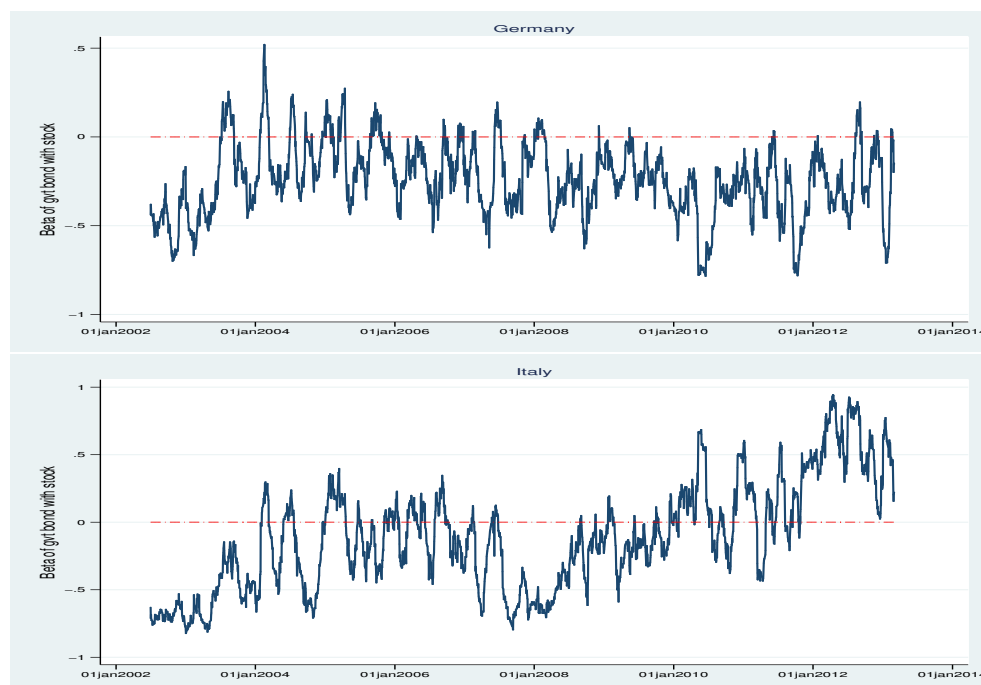


Figure 1: What is a safe asset? Daily betas of government bonds with DJ EUROSTOXX 50, Germany (top) and Italy (bottom).

The model of endogenous leverage laid down in this paper aims at capturing three novel stylized facts. The first one concerns banks' balance sheets. Figure 2 splits the aggregate balance sheet of the Eurozone financial sector in two categories of assets: loans and fixed assets that are mainly positive beta with the stock market, and securities and holdings such as gold that are mainly negative beta with the stock market ('safe assets'). The figure illustrates that negative beta holdings by European banks are substantial, but also that these holdings did increase with the ongoing Eurozone crisis. This is puzzling, as one could think that in stress times, safe assets ownership gets more concentrated in risk-averse hands, i.e. moves away from banks' balance sheets to household portfolios. Figure 1 computes the beta of German and Italian 10 year government bonds with the European stock market index DJ EUROSTOXX 50. As both countries belong to the same monetary union, it controls for expectations about the monetary policy stance, i.e. the Neo-Keynesian channel emphasized in Campbell et al. (2013b). And still, the two assets exhibit radically different behavior during the Eurozone crisis: German public debt exhibits now an even more negative beta, whereas Italian public debt has turned sharply positive. I interpret this as Italian public debt losing its safe asset status, in line with my definition of safe assets.¹

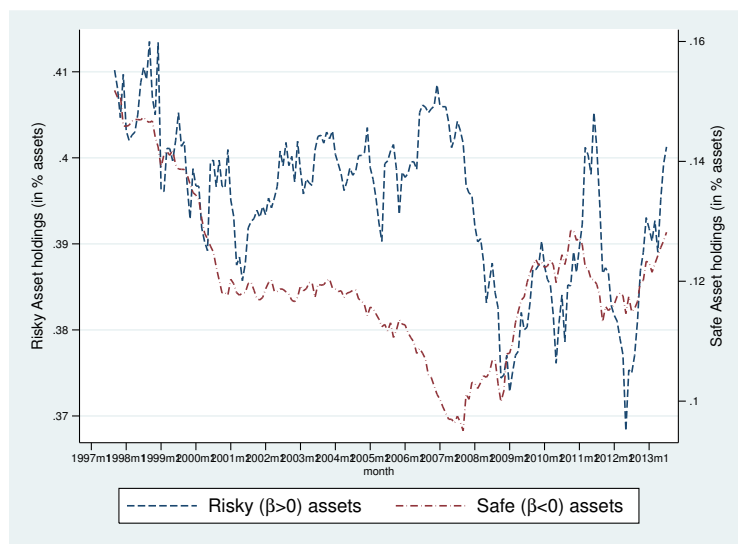


Figure 2: First stylized fact: European banks' portfolio composition, holdings divided according to their β .

The second stylized fact is concerned with the pricing of safe assets. I take the view in this paper that no asset is entirely riskless, but that there is an inelastic supply of quasi-safe assets. Critically I draw the distinction between *public* safety (public debt) and *private* safety (private bank debt). This leads me to consider two distinct safe prices and safety premia: one on public debt and one on bank debt. Figure 3 computes the spread between these two yields. This figure illustrates that this spread progressively decreased during the

¹Section 4 of this paper microfound the beta of public debt in an open economy model, and rationalizes these patterns along with the redomestication of public debt currently at play in Europe.

run up to the crisis, even turning negative, before sharply bouncing back in the midst of the crisis. This spread is exactly the carry trade banks are doing when they hold public debt and finance these holdings by bank debt. A micro investigation of the same carry trade is shown by Figure 24 in the Appendix. In this figure, I additionally subtract from the carry trade the operating expenses of running the bank pro rata the safe asset holdings. This figure shows that the median European bank loses almost ten million euros on its safe asset holdings.

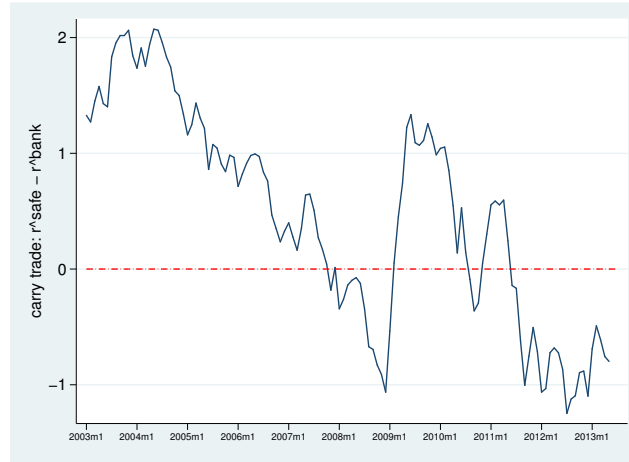


Figure 3: Second stylized fact: negative carry trade by banks on safe public debt: $SMI = r^{safe} - r^{bank}$.

The third and last stylized fact the model captures is a macroeconomic one. I compute two aggregate measures: one is the stock of safe Eurozone public debt, the other is the stock of Eurozone bank short-term debt. Figure 4 shows these two time-series, scaled by GDP. This figure eyeballs a positive comovement of private debt with public debt in Europe. This stands in contrast with the result obtained by [Krishnamurthy and Vissing-Jorgensen \(2013\)](#) for the United States, in which they show there is a negative comovement between these two aggregates. This stylized fact motivates the intuition that in Europe, for limited participation reasons, the creation of private safe assets is even more needed. This leads to a larger banking system in size and a positive comovement of private debt with public debt.

The three stylized facts, respectively related to the financial sector balance sheet, the financial sector income statement and to monetary aggregates, are rationalized in the model of private safety creation developed in this paper. Banks produce private safe assets in a general equilibrium environment. The macro inspiration comes from [Caballero and Farhi \(2013\)](#), which emphasizes the shortage of safe assets as a key macroeconomic imbalance. The authors make the point that public debt, being a bearish asset, plays a central role in mitigating this shortage by a mechanism that they call *safety multiplier*. The present paper microfounds this mechanism by putting banks at the heart of the creation of private safe assets: increasing the supply of public debt enables banks to lever up more, and this increases the endogenous supply of private debt. The dynamic version of the model also microfounds why public debt has a negative beta. In my environment,

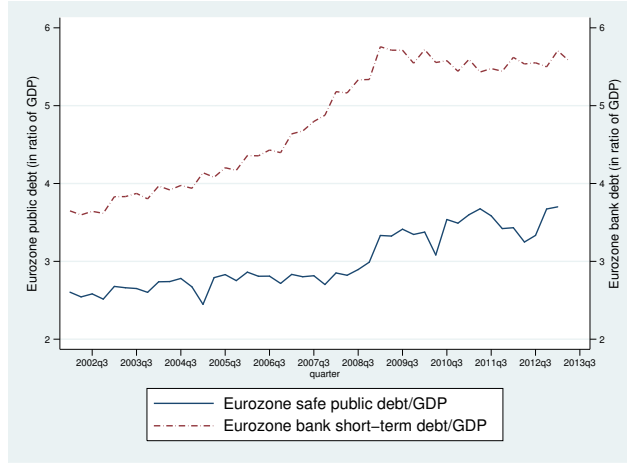


Figure 4: Third stylized fact: comovement of public safe assets and private safe assets in Europe.

a shortage of public safe assets triggers a recession, not through a New-Keynesian demand channel as in Caballero and Farhi (2013), but through a supply channel caused by bank deleveraging. The diversification motive has some commonality with Gennaioli et al. (2013b) model of shadow banking, but applied to banks in general. In Gennaioli et al. (2013b), by diversifying away idiosyncratic risk, securitized debt is made entirely riskless. In my macro environment, there are only two assets, so the law of large numbers is ineffective to create safety. It is the endogenous correlation properties of public debt that enable banks to produce safe assets.

The safety multiplier mechanism critically relies on two ingredients: risk-aversion heterogeneity and incomplete markets. Risk-aversion heterogeneity is a parsimonious way to capture the distinction between active wealth (risk-neutral banks) and passive wealth (risk-averse investors). Only with incomplete markets banks' leverage is determinate and depends on the supply of public debt. If markets were complete, risk-neutral banks would be able to fully insure risk-averse investors, so equilibrium leverage would always be equal to the net worth of risk-averse investors. There would be no safety multiplier.

Putting banks at the heart of the creation of safe assets has two key normative implications. The private competitive equilibrium without public debt is constrained inefficient. Banks under-provide insurance when the economy lacks long-dated securities. The issuance of long-term public debt improves welfare by facilitating intragenerational risk-sharing. Thanks to their negative beta properties, long-term securities exert a *positive* externality. They are an attractive input for safety production, but potential issuers of long-term debt do not internalize it. Nevertheless, issuing too much public debt destroys its own hedging properties, and this can eventually hurt welfare. As a result, there exists a finite optimal level of public safe assets in the economy.

The second normative implication of the model relates to the economic role of universal banks. Narrow banking regulations such as Glass-Steagall, which call for a split between banks' securities arm and retail arm, are harmful from a safety-creation standpoint. Indeed, such a split prevents banks from leveraging the hedging properties of public debt, implying a lower level of bank debt, and a lower level of private safe assets in the economy. Under Glass-Steagall, banks under-provide insurance to the risk-averse investors.

Finally, an open-economy version of the model introduces heterogeneity in sovereign risk. Public debt

is then priced according to its relative sovereign risk compared to other public debts. This open economy environment rationalizes Figure 1, in which the *relatively* safer public debt exhibits a negative beta whereas the relatively riskier public debt has positive beta. This environment proposes a pure asset pricing, moral-suasion and moral-hazard free, perspective for why domestic debt gets redomesticated on domestic banks' balance sheets in sovereign crises. The leveraging ability of domestic banks is determined at the margin by the flight to safety of domestic investors. This implies that domestic banks become the natural holders of domestic debt under sovereign risk heterogeneity.

Model and theoretical results I develop a model of endogenous leverage based on risk aversion heterogeneity and endogenously incomplete markets. I do not resort to heterogeneity in beliefs disagreement as Geanakoplos (2009), as I explicitly introduce multiple assets than can be used as collateral for recourse debt. It is hard to discipline the beliefs of different agents on different assets.² Compared to asset pricing models with heterogeneity, such as Dumas (1989), I introduce equilibrium default through a limited liability constraint.³ Risk-neutral banks partially insure risk-averse lenders against macroeconomic shocks. I embed this model of endogenous leverage in a general equilibrium environment in which the supply of public debt is perfectly inelastic and there are no deep pockets investors. In this economy, too many savings are chasing too few *safe* assets. On the other hand, the supply of risky assets is perfectly elastic. Through their leverage decision, banks control the endogenous supply of private safe assets. The partial equilibrium effects of the agents' portfolio choices interact with the general equilibrium channel of a Walrasian market for public debt.

Under low supply of public safe assets and high heterogeneity in risk-aversion, the economy features a *safety multiplier* mechanism. It is optimal for risk-neutral banks to hold public safe assets and bundle them with risky assets. Doing so creates more safety in the economy than the endowment of public debt would do alone. When public safe assets become scarce, they can become so expensive that banks prefer to delever. This impairs the supply of private safe assets, as well as less investment in the real economy. This credit crunch is caused by a shortage of public debt. In such situation, the proportion of safe assets on banks' balance sheets rise, as they are best used in risk-neutral hands to trigger the safety multiplier. Hence there is a *pecking order* in public debt ownership: having them on the balance sheets of banks not only hedges risky investment, but at same time enables private safety creation. When the risk averse agents have limited access to the risky technology (Europe), equilibrium leverage is higher and the parameter region under which the economy features a safety multiplier is wider.

The welfare analysis is carried out in a stochastic environment of overlapping long-lived generations. The negative beta of long-term securities is microfounded by the expectations of a flight-to-safety in low aggregate states. The government and private agents are facing the exact same maturity choice between short-term debt and long-term debt. The decentralized equilibrium is constrained inefficient because private agents do not issue enough long-term securities. The reason is that they do not fully internalize the positive effects of having a high supply of negative beta assets in the economy in order to crowd-in aggregate investment.

²It is unclear whether optimists on the stock market should also be optimists on negative beta assets.

³Without limited liability, risk-neutral banks would fully insure risk-averse investors and the safety multiplier disappears. This could be seen as a particular case of the generic result of Krishnamurthy (2003) of irrelevance of balance sheet recession under complete contracting (see Di Tella (2013) for a continuous-time formulation). In my environment, limited liability is key to endow banks with a role in portfolio construction.

Asset pricing implication: the Safety Mismatch Index The model delivers endogenous closed-form solutions for two safety premia: one on public debt and one on private debt. The spread between the two is the carry trade made by the banks on public debt. I show this carry trade increases with public debt beta and decreases with leverage. I argue that this spread is a relevant welfare and financial stability indicator. It reveals the fragility of banks' balance sheets, as well as the extent to which the economy is exposed to a sovereign debt crisis that feeds into a banking crisis. I call this spread the Safety Mismatch Index and show its predictive power on the Eurozone sovereign crisis. The empirical analysis confirms the key predictions of the model on monetary aggregates. I interpret the European crisis as a shortage of *public* safe assets, which deprives banks of their leveraging ability.

Key related literature My paper connects two strands of literature: models of endogenous leverage à la Geanakoplos (1997) and general equilibrium macro models à la Caballero and Farhi (2013). The latter also studies the effect of a fixed supply of safe assets, but does not feature optimizing banks. Taking into account the inelastic supply of riskfree assets is a key departure from the standard asset pricing approach of Campbell and Viceira (2002), which focuses on exogenous changes of risk preferences to pin down the riskfree rate. My paper also contributes to the banking literature about what banks do. It suggest a view of banks as private safety creators, alternative to the agency view of the firm: e.g. Diamond (1984), Diamond and Rajan (2001), and Tirole (2003) for a unified theory of banking relying on agency frictions. Furthermore, I argue that treating bank debtholders as risk-neutral is counterfactual with the vision that these holders are 'passive money.' As a result, I focus on the main heterogeneity between active money and passive money emphasized in Caballero and Farhi (2013) and Gennaioli et al. (2013b), and I introduce it in an asset-pricing model with endogenous leverage. Kashyap et al. (2002) sees banks as liquidity economizers, whereas I see them as safety multipliers. Diamond (1984) banks engage in idiosyncratic diversification, whereas my financial intermediaries engage in diversification of aggregate shocks. My model can also be seen as providing microfoundation to the view in Philippon (2012) of banks as service providers to households.⁴ On the creation of safe assets, all the previous theoretical models are assuming *substitutability* between public safe assets and private safe assets: Gorton and Metrick (2012), Gourinchas and Jeanne (2012), Sunderam (2012) and Greenwood et al. (2010). On the contrary, my model exhibits crowding-in. Empirically, I document the stylized fact of a positive comovement between US Treasuries and bank debt supplies in Europe, whereas Krishnamurthy and Vissing-Jorgensen (2013) shows there is substitutability in the US. Compared to Campbell et al. (2013b), I do not emphasize the *nominal* properties of bonds, but their relative *safety* properties, in order to analyze their negative beta. On the normative side, contrary to Stein 2012 who argues there is too much private safe assets, my model hints at a lack of public safe assets. The stochastic OLG model used for the normative analysis is reminiscent of Ball and Mankiw (2007), but allows for within-generations heterogeneity and maturity choices. Compared to Woodford (1990) and Gale (1990), my environment features both intergenerational risk sharing (between generations) and intra-generational risk-sharing (within a generation). The interplay between the two is at the core of the constrained inefficiency result.

The paper is organized as follows. Section 2 presents the static environment, in which banks create safe assets and public debt crowds-in private investment. Section 3 solves for the dynamic version of the model,

⁴In his empirical investigation, Philippon (2012) uses exogenous weights to value the services provided by banks to households. Similarly, DeAngelo and Stulz (2013) have in mind a liquidity premium on bank debt. My model provides rigorous microfoundations to their insights, by pinning down banks production function of safety to their portfolio choice.

when agents face a maturity choice. Section 4 carries out a normative analysis of public debt issuance and financial regulation. It shows the private equilibrium is constrained inefficient, and how public debt issuance can achieve a Pareto improvement. As far as financial regulation is concerned, the crowding-in of private investment by banks holdings of public debt gives a rationale for a relatively favorable prudential treatment of these holdings. Section 5 analyzes an open economy extension to shed light on the redomestication of public debt. Section 6 turns to the empirical analysis. Section 7 discusses the results in light of the literature and concludes.

2 A Model of crowding-in by public debt

I develop a model of endogenous leverage in an environment of risk aversion heterogeneity, multiple assets and limited liability. Even if it shares some flavors with models of beliefs disagreement (Geanakoplos 1997, Simsek (2013)), it is instead casted with risk aversion heterogeneity. This is motivated by the focus of this paper on asset multiplicity, and the view of banks as macro insurers.⁵

This section investigates the two following issues:

- *How to improve welfare by enabling agents to share risks.*
- *How the supply of an additional asset improve this risk-sharing and enables more investment in the initial asset.*

2.1 The safety multiplier argument in a nutshell

Before developing the dynamic CARA-normal environment, I start by a static four-states example to illustrate the safety multiplier argument. Why would banks ever hold the public safe assets on their own balance sheets instead of letting these safe assets being held in risk-averse hands?

Consider only two dates: $t = 0, 1$, four equally plausible states. Assume that a $t = 1$ risky payoff for the technology and consider a public security that is imperfectly negatively correlated with the technology:

$$technology = s_K = \begin{bmatrix} 8 \\ 6 \\ 4 \\ 2 \end{bmatrix} \text{ and } public\ debt = s_B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}$$

By bundling the two assets ('self-diversification'), a risk-averse investor would get:

$$aggregate = \begin{bmatrix} 9 \\ 8 \\ 7 \\ 5 \end{bmatrix}$$

The mean of this payoff is 7.25 and its volatility is 2.19. But actually, a risk-neutral bank can help improving the Sharpe ratio effectively faced by the risk-averse investor. Assume the risk-neutral bank and

⁵Fostel and Geanakoplos (2008) argue that a reasonable assumption consists in treating optimists for one asset as also optimists for the other assets, and this is how they obtain contagion. On the contrary, I do not want to take a stand on who is more optimistic on a given class of assets, so I claim that risk-aversion heterogeneity is a more parsimonious modeling device.

the risk-averse investor enter 7 units of a financial contract that promises $\bar{s} = 1$ at $t = 1$, and that the risk-neutral agent holds both assets on its balance sheet. The state-contingent value of the risk-neutral agent portfolio is *aggregate*. Thus he defaults on its contract if and only if one of the two lowest states realize. As a result, the state-contingent payoff faced by the investor, by the means of the financial contract ('delegated diversification'), is now:

$$contract = \begin{bmatrix} 7 \\ 7 \\ 7 \\ 5 \end{bmatrix}$$

This state-contingent payoff has a mean of 6.5 and its volatility is 0.75. By bundling the two assets and issuing risky debt against it, the bank has been able to significantly improve the Sharpe ratio faced by the risk-averse agent. This asset will be traded in equilibrium. Formally, the equilibrium is defined by the risk-neutral maximization of the bank and the mean-variance maximization of the investor, under their respective budget constraints, as well as two market clearing conditions, where B is the exogenous supply of public debt (price q_B) and D is the endogenous supply of private debt (price q_D):

$$x_B^A + x_B^P = B \text{ and } x_{1A}^P = y^A$$

The portfolios $\{i^A, x_B^A, y^A\}$, $\{i^P, x_B^P, x_{1A}^P\}$, and the two prices q_B and q_D are the endogenous variables. Denoting $D = q_D y^A$ the value of private debt, Appendix B.1 shows that this economy features a safety multiplier:

$$\frac{\partial D}{\partial B} > 0$$

This example shows that the only ingredients needed are limited liability and risk-aversion heterogeneity. From a situation of market incompleteness (2 assets and 4 states), agents endogenously decide to *partially complete* the markets. Defaultable debt is a 3rd asset that enables to attain the constrained efficient allocation. The net worth n^A of the risk-neutral bank gives an additional rationale for leverage (cross-subsidization), but does not play a role in the safety multiplier mechanism.

What critically misses on this example is the endogeneity of the default threshold, which is actually a choice variable for the bank (with endogenous choice of the default threshold and discrete states, it suffices to check that it is indeed optimal for agents to choose a default threshold between the third and fourth state). The full-fledged model below solves for the optimal default threshold jointly with the portfolio choices. Contrary to the simplified model, the price p_B of public debt then depend on its supply B . In that case, the safety multiplier implies a crowding-in of private investment by the supply of public debt, whereas, when the price p_B does not depend on the supply B as in the simplified model, the resource constraint implies that private investment is always crowded-out by public debt.

2.2 Static environment

There are two periods $t = 0, 1$.

Agents' preferences Each generation is populated by a continuum of agents of two types. There is a mass 1 of banks (type A: active) and a mass 1 of investors (type P: passive). Banks are risk-neutral whereas investors are risk-averse, with a coefficient of absolute risk aversion of γ^P .⁶ For banks, the risk-neutrality assumption captures enhanced sophistication, diversification opportunities or bailout expectations.⁷ Risk-neutrality should be seen as a normalization, as what matters is the *differential* of risk aversion between the two populations of agents. The CARA parameter of investors captures this differential. CARA preferences are more constraining than HARA or Epstein-Zin, often used to shed light on the risk-free rate puzzle. Agents' types are common knowledge. At birth, the mass of risk-neutral agents A is endowed with n^A of numeraire, whereas the mass of risk-averse investors P is endowed with n^P .

Demographics There are overlapping generations of such agents. Each generation lives three periods.⁸ The heterogeneity within generations is kept constant over time. Every new born of type A is endowed with n^A of numeraire, agents of type P with n^P .⁹ Not having the wealth distribution as a state variable makes the model bloc-recursive. The notation G_t^i with $i \in \{A, P\}$ refers to an agent of type i that belongs to the generation that was born in period $t - 2$ and dies in period t .

Technology There is only one exogenous asset to invest in: a risky linear technology. This technology is short-term: investing one unit at t yields an uncertain dividend at $t + 1$: s_{t+1} . I assume that this payoff follows a random walk:¹⁰ $s_{t+1} \sim N(s_t, \sigma_1)$. Thus s_t is the aggregate state of the economy. All agents have the same beliefs of the shock distribution. The technology is in perfectly elastic supply, so it has an exogenous linear cost p_K in numeraire.¹¹ The ratio p_K/s_t should be thought as the time-varying Tobin- q of the model. There is no riskless storage technology.

Second asset There is a second asset, called long-term public debt, which is in fixed supply. We assume in the static model In a second-moments approximation, the prices at $t + 1$ of the technology and long-term public debt are multivariate normal:

$$\begin{bmatrix} s_{t+1} \\ q(s_{t+1}) \end{bmatrix} \sim N \left(\begin{bmatrix} s_t \\ \mathbb{E}_t[q(s_{t+1})] \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_K^2 & \rho\sigma_K\sigma_B \\ \rho\sigma_K\sigma_B & \sigma_B^2 \end{bmatrix} \right)$$

where σ_K is the exogenous volatility of technology, σ_B is the endogenous volatility of interim price of long-term debt, and ρ is their correlation. One should think of the long-term debt volatility as low ($\sigma_B < \sigma_K$)

⁶I have also solved a version of the model with risk-averse banks, with CARA coefficient γ^A . The results of this paper are robust to this extension, as long as $\gamma^A < \gamma^P$. The model generalizes to a continuum of types γ^i , and then features assortative matching in the competitive equilibrium: the surplus of each bilateral match is endogenous to agents outside options. Therefore the least risk averse agent is matched with the least risk-averse agent above the endogenous cutoff $\bar{\gamma}$ that decides who the lenders are: $[\bar{\gamma}; \gamma^{max}]$.

⁷All these reasons point towards value maximization by banks. For the latter microfoundation, it will amplify the destructive effects of sovereign risk shown in the model extension, as arguably sovereign risk would in this case hurt bailout ability of the sovereign, hence increasing banks' risk aversion.

⁸Beyond breaking the First Welfare Theorem, the OLG structure then mutes undesirable discounting and long-run wealth effects.

⁹It is within-generation heterogeneity in preferences, not in endowments as Sargent-Wallace (1982) and Smith (1988).

¹⁰Qualitative results apply to general Markov chains.

¹¹The price ϕ can be normalized to 1. Results are robust to decreasing return to scale technology, i.e. a partially elastic offer curve. On the other hand, having an inelastic supply of capital (supply of one Luca's tree) renders the quantity of aggregate risk in the economy constant, and this shuts down the crowding-in effects of public debt. A weaker version of the safety multiplier and constrained inefficiency results can be recovered in the latter case when allowing for young consumption.

and its correlation with the technology as negative ($\rho < 0$), as is shown in the recursive equilibrium. The expectation on technology is the aggregate state: $\mu_K = s_t$, whereas the expectation on long-term debt is the expectation of its market price tomorrow: $\mu_B = \mathbb{E}_t[q(s_{t+1})]$.

Private financial contracts At the same time, agents can borrow or lend to each other by trading on securities markets. One unit of security is a promise to pay \bar{s} of numeraire at maturity $t + h$. Contrary to Geanakoplos and Zame (2013), this contract does not specify a given level of collateral. This intends to capture risky recourse debt. *Risky* implies that agents can default on their contract, and they will do so as long as the payoff on their portfolio (their asset side) is realized below the sum of all the promises contracted by the agent. *Recourse* implies that a borrower cannot pledge only a part of its balance sheet. When the borrower defaults, the lender has recourse to seize the entire balance sheet assets of the borrower. This matches the empirical fact that, in practice, most short-term debt is recourse.¹²

Each security \bar{s} is traded on a Walrasian market, which clears at a price $q_{\bar{s}}$. An agent selling y units of such contract is therefore able to raise $D = q_{\bar{s}} * y$ at $t = 0$, against the promise of paying back $\bar{S} = \bar{s} * y$ at $t = 1$. Define the rate of return on bank debt as the ratio of the promise to its price: $r_{\bar{s}} = \frac{\bar{s}}{q_{\bar{s}}} = \frac{\bar{S}}{D}$.¹³ Primary and secondary debt are fungible, i.e. any promise issued by a given agent G_{t+j}^θ at a given maturity $t + h$ trades on the same Walrasian market, which clears at price $\left\{ q_{\frac{G_{t+j}^\theta}{\bar{s}_{t+h}}} \right\}_{h \leq j}$. y denotes a short-term promise (issued one period before maturity), whereas $y^{primary}$ denotes a long-term promise (issued two periods before maturity).

Timing At $t=0$, banks and investors make their portfolio choice: investment in the two assets, and they write financial contracts between themselves. At $t=1$, the returns on assets realize. Agents potentially default if their wealth is lower than their promises. In that case, the lenders seize the entire balance sheet of the borrower in default. Finally, all agents consume their wealth.

2.3 Benchmark: equilibrium without financial contracts

Without financial contracts, there is no default, and the equilibrium simply derives from the portfolio choices of the bank and the investor, and the market clearing of public debt¹⁴. The bank optimization:

$$\underset{\{X, \{x_{\bar{s}}\}\}}{\text{Max}} L^B = \mathbb{E}_0 \left[(X'S) 1_{\{X'S \geq \int x_{\bar{s}} \bar{s}\}} \right] + \lambda \left[n^B - X'P \right]$$

¹²Geanakoplos (2009) features secured non-recourse debt. In practice, most of the so-called secured debt such as repurchase agreement contracts (repo) includes an additional claim to the balance sheet of the issuer in case of collateral shortfall, which makes them in effect recourse. In case of default, the lenders seize the whole balance sheet of the agent in default. See Weymuller (2013) for an analysis of the idiosyncratic drivers of the market for secured debt.

¹³This General Equilibrium approach is not equivalent to the Principal-Agent approach where the borrower and the lender bargain over the loan contract. Theorem 1 of (Simsek, 2013) (equivalence with full bargaining to the borrower) does not apply due to the absence of a riskless technology. Nevertheless, appealing to the first theorem of welfare, both environment are constrained efficient, so they trace the same Pareto frontier. The Walrasian equilibrium is therefore equivalent to a Principal-Agent economy with a specific bargaining power. The Walrasian treatment is more transparent, as it restricts the space of ex-ante transfers that could be achieved through the terms of the contract.

¹⁴If there was no public debt, the portfolio of the bank would merely be pinned down by its budget constraint: $x_1 = \frac{n^B}{p_1}$; and the portfolio of the risk-averse investor would also not interact with the bank's one: $y_1 = \frac{n^L}{p_1}$

pins down the price of public debt by equating the expected returns on the two assets: $\frac{p_2}{p_1} = \frac{\mu_2}{\mu_1}$.

The investor maximization:

$$\text{Max}_{\{Y, \{y_s\}\}} L^L = -\mathbb{E}_0 \left[e^{-\gamma_L(Y'S)} \right] + \mu \left[n^L - Y'P \right]$$

Using $\mathbb{E}_0 \left[e^{-\gamma_L(Y'S)} \right] = e^{-\gamma_L(\mu_1 y_1 + \mu_2 y_2) + \frac{1}{2} \gamma_L^2 (y_1^2 \sigma_1^2 + 2\rho y_1 y_2 \sigma_1 \sigma_2 + y_2^2 \sigma_2^2)}$, and using the price of public debt derived above, we obtain:

$$y_1 \left(\frac{\sigma_1^2}{\mu_1} - \rho \sigma_1 \sigma_2 \right) = y_2 \left(\frac{\sigma_2^2}{\mu_2} - \rho \sigma_1 \sigma_2 \right)$$

Which, combined with the investor budget constraint, pins down its portfolio choice. The bank is the residual investor of public debt (through market clearing), and finally the budget constraint of the bank pins down its holdings of the technology.

In the case public debt is a perfect storage asset ($\sigma_2 = 0$ and $\mu_2 = 1$) we get: $y_1 = 0$. In that case, the investor invests all its wealth in the storage asset and we have $n^L = p_2 y_2$ and $y_2 + x_2 = B$.

- If $B < n^L \frac{\mu_1}{p_1}$: in that case, $x_2 = 0$: Only the investor holds the storage asset. There is cash-in-the-market pricing: the expected return on the storage asset is lower than the expected return on the technology. The differential can be interpreted as a safety premium;
- If $B \geq n^L \frac{\mu_1}{p_1}$: in that case $x_2 > 0$ and $p_2 = \frac{p_1}{\mu_1}$. Expected returns are equalized, and the share of the storage asset held by the investor is pinned down by its budget constraint: $y_2 = n^L \frac{\mu_1}{p_1}$. Moreover: $x_2 = B - n^L \frac{\mu_1}{p_1}$ and $x_1 = \frac{1}{p_1} (n^B + n^L) - \frac{1}{\mu_1} B$.

In the general case, there are the same two regimes, with the cutoff: $B \geq \frac{n^L}{p_1} \frac{1}{\frac{\mu_2}{\mu_1} + \frac{\sigma_2^2 / \mu_2 - \rho \sigma_1 \sigma_2}{\sigma_1^2 / \mu_1 - \rho \sigma_1 \sigma_2}}$. Under the

assumption that $\frac{\sigma_2^2}{\mu_2} < \frac{\sigma_1^2}{\mu_1}$ (i.e. neighborhood of $\sigma_2 = 0$), this cutoff increases with ρ . As a result, when the safe asset has better hedging properties ($\rho < 0$) then the bank does hold more of this safe asset.

Without financial contracts, there is no default, and the equilibrium simply derives from the portfolio choices of the bank and the investor, and the market clearing of public debt.

We see on the resource constraint and on expected return equality that there is crowding-out.

2.4 Equilibrium with financial contracts

In the static model, I consider here the generation G_{t+1} : the middle-aged at period t . Its agents do not face any maturity choice: they only can issue short-term debt. However they can invest in all 3 active markets beyond the technology: within-generation short-term promise¹⁵, next-generation short-term promise and next-generation long-term promises. Without loss of generality, I analyze a decentralized equilibrium with zero supply of public debt.

I conjecture a *contract equilibrium* in which, within each generation, risk-neutral banks borrow from risk-averse investors, and all agents have non-degenerate portfolio holdings in all assets. The CARA-normal environment enables to derive the equilibrium closed-form, even with equilibrium default.

¹⁵It is the same as buying back long-term promises if this investment is negative.

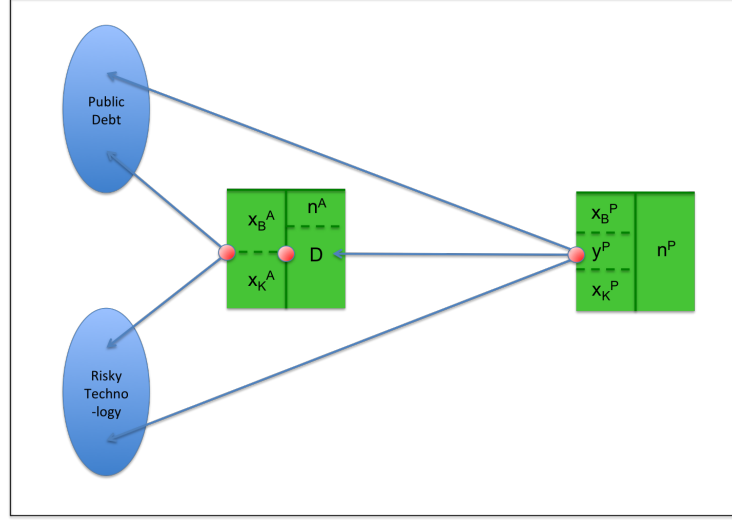


Figure 5: Contract equilibrium representation. Within-generation agents are in green and assets are in blue. Red bullets indicate, from left to right: bank portfolio choice, bank leverage choice and lender portfolio choice.

The net worth of middle-aged agents is the result of their young portfolio decisions and of the realization of the aggregate state t . Middle-aged agents can be thought as liquidating their entire portfolios, including of long-term debt, at period t market prices, before entirely reinvesting the proceeds. Therefore the post-liquidation net worth $n_t^{G_{t+1}^\theta}(s_t)$ of the middle-aged is the state variable that encodes all the previous decisions of the generation.

Program of middle-aged risk-neutral agents Denote \bar{S} the sum of all promises and its portfolio choice X^{A_m} :

$$\bar{S} = \int \left(y_{\bar{s}_{t+1}}^{G_{t+1}^A} + y_{\bar{s}_{t+1}}^{primary} \right) \bar{s} d\bar{s}$$

$$X^{A_m'} = \begin{bmatrix} i_t^{G_{t+1}^A} & x_{\bar{s}_{t+2}}^{G_{t+1}^A} & x_{\bar{s}_{t+1}}^{G_{t+1}^A} \end{bmatrix}$$

The bank pre-default portfolio realization u_{t+1} is:

$$u_{t+1} = X^{A_m'} S_{t+1} - \bar{S} - \tau_{t+1}^A \quad (1)$$

The program of the bank then writes:

$$\begin{aligned} \underset{\{X^A, \bar{S}\}}{\text{Max}} \quad & W_{t+1}^{G_{t+1}^A} \left(n_t^{G_{t+1}^A}(s_t), s_t; y_{\bar{s}_{t+2}}^{primary}(s_{t-1}) \right) = \mathbb{E}_t \left[u \mathbf{1}_{\{u \geq 0\}} \right] \\ \text{s.t.} \quad & X^{A_m'} P \leq n_t^{G_{t+1}^A}(s_t) + \left\{ \bar{S} - \int y_{\bar{s}_{t+1}}^{primary} \bar{s} d\bar{s} \right\} \frac{q_{\bar{s}_{t+1}}^{G_{t+1}^A}}{\bar{S}} \end{aligned} \quad (2)$$

Out-of-generation trades are with a representative agent of the other generation as counterpart: G_{t+2} . Indeed, the equilibrium can be broken down into two sequential (but interacting) problems: risk-sharing

between generations, then risk-sharing within the generation. The legacy stock of long-term promise becomes fungible with short-term promises. P denotes the price vector at t of assets and S their $t + 1$ realization. For the technology, price is ϕ and realization s_{t+1} . For securities, 'prices' are Walrasian prices at t and 'realizations' are prices at $t + 1$ in history s_{t+1} . The realization of the short-term promise on the outside generation G_{t+2} is risky debt payoff: $\text{Min} \left(X^{A_m} S, x_{\bar{s}_{t+1}}^{G_{t+1}^A} \bar{s}_{t+1}^{G_{t+2}} \right)$. The key feature of the full equilibrium is that long-term promises issued by generation G_{t+2} are negative beta, thus appealing in the portfolio choice of generation G_{t+1} agents.

In the multivariate normal environment, u verifies:

$$u \sim N \left(\mu_u, \sigma_u^2 \right)$$

with $\mu_u = X' \mu - \bar{S} - \tau_{t+1}^A$ and $\sigma_u^2 = X' \Sigma X$. The objective function of the bank writes:

$$W_{t+1}^{G_{t+1}^A}(s_t) = \mu_u \Phi \left(\frac{\mu_u}{\sigma_u} \right) + \sigma_u \phi \left(\frac{\mu_u}{\sigma_u} \right)$$

Banks expected utility increases in a convex fashion with the mean of the pre-default payoff mean, and it increases with its variance: this risk-shifting motive arises from the limited liability friction.

Program of middle-aged risk-averse investors Due to the recourse feature of the debt contract, the investor seizes the entire balance sheet of the bank when the bank is in default. Such banks can belong to generation G_{t+1} (within generation) or generation G_{t+2} (cross-generation). Denote by $\bar{S}^{G_{t+2}}$ the sum of promises by banks of generation G_{t+2} , it is seen as an out-of-the-generation risky payoff. The investment in within-generation short-term promise $x_{\bar{s}_{t+1}}^{G_{t+1}^P}$ is captured by \bar{S} .

Denote the investor's portfolio choice X^{P_m} :

$$X^{P_m'} = \begin{bmatrix} i_t^{G_{t+1}^P} & x_{\bar{s}_{t+2}}^{G_{t+1}^P} & x_{\bar{s}_{t+1}}^{G_{t+1}^P} \end{bmatrix}$$

The investor pre any-default portfolio realization v_{t+1} is:

$$v_{t+1} = X^{P_m'} S_{t+1} + \bar{S} - \tau_{t+1}^P$$

The program of the investor then writes:

$$\begin{aligned} \text{Max}_{\{X^P, \bar{S}\}} W_{t+1}^{P_m} \left(n_t^{G_{t+1}^P}(s_t), s_t \right) &= -\mathbb{E}_t \left[e^{-\gamma^P(u+v)} 1_{\{u < 0\}} + e^{-\gamma^P v} 1_{\{u \geq 0\}} \right] \\ \text{s.t.} \quad X^{P_m'} P + \bar{S} \frac{q_{\bar{s}_{t+1}}^{G_{t+1}^A}}{\bar{S}} &\leq n_t^{G_{t+1}^P}(s_t) \end{aligned} \quad (3)$$

In the multivariate normal environment, this objective function can be written:

$$W_{t+1}^{P_m}(s_t) = -e^{-\gamma \mu_v + \frac{1}{2} \gamma^2 \sigma_v^2} \left\{ e^{-\gamma \mu_u + \frac{1}{2} \gamma^2 (\sigma_u^2 + 2\rho_{uv} \sigma_v \sigma_u)} \left\{ 1 - \Phi \left(\frac{\mu_u}{\sigma_u} - \gamma (\rho_{uv} \sigma_v + \sigma_u) \right) \right\} + \Phi \left(\frac{\mu_u}{\sigma_u} - \gamma \rho_{uv} \sigma_v \right) \right\}$$

I start by solving for the equilibrium when the technology is the only potential investment: the supply B of next generation promises (as well as public debt) is set to zero. This is the case if there is no maturity choice. The following lemma shows that the optimal contract is risky debt.

Lemma 1. *When there is no outside asset in the economy beyond the technology, the risk-averse investors lend all their wealth to the risk-neutral banks through one financial contract: risky debt.*

The proof in Appendix B.1 makes use of the first theorem of welfare. The problem can be broken on one hand on optimal level of aggregate investement and on the other hand on the optimal risk-sharing between risk-neutral and risk-averse agents. For aggregate investment, the resource constraint pins it down, as, in this benchmark, there are no other assets to invest in:

$$p_K (i_t^A + i_t^P) \leq n^A + n^P$$

As for risk-sharing, the constrained efficient allocations are such that risk-averse agents enjoy a constant consumption as long as the technology shock realizes above this consumption level. When the shock realizes below this threshold, the risk-averse agents consume all the $t = 1$ wealth of the economy. Any of such allocations is implemented by a risky debt contract, with face value the desired constant consumption level.

I am interested in the equilibrium leverage (i.e. within-generation risk-sharing), and how it varies with the supply of negative beta assets (i.e. the supply of long-term promises by generation G_{t+2}). There is a safety multiplier when the two quantities covary positively.

The application of the first welfare theorem carries through in the environment with two assets: the optimal allocation is implemented by a decentralized equilibrium in which risk-averse investors lend all thier wealth to banks through risky debt. But introducing public debt, taken for now as an exogenous fixed supply B ,¹⁶ make the ex ante investment depend on the endogenous price $q_{B,t}$ of long-term debt. Indeed, the resource constraint of the economy is now:

$$p_K (i_t^A + i_t^P) + q_{B,t} B \leq n^A + n^P$$

Thus the endogenous total value of the public safe asset ($q_{B,t} B$) crowds out private investment in the technology. If there is a **Laffer** effect, i.e. if the value ($q_{B,t} B$) decreases with respect to the supply B , then issuing more long-term debt B actually crowds in investment.

The within-generation equilibrium is defined by 8 endogenous variables: for each of the two agents, their investment in the technology i_t^θ , their investment in the long-term debt $x_{B,t}^\theta$ and their position in the intrageneration risk-sharing contract \bar{S}_t^θ , as well as the endogenous price for long term-debt $q_{B,t}$ and the endogenous price of the risk-sharing contract $q_{\bar{S},t}$. And the equilibrium is characterized by the 8 independent equations: for each of the two agents, one portfolio choice and one leverage choice, as well as two budget constraints and the two market clearings for long-term debt and for the risk-sharing contract. Appendix B.2 solves for the equilibrium fully closed-form.

As banks are risk-neutral, the equilibrium price $q_{\bar{S},t}$ of the risk-sharing contract (bank debt) does not directly depend on the quantity traded in this contract \bar{S}_t . It is illustrated in Figure 6, where the slope of bank supply curve is constant and equal to this equilibrium price $q_{\bar{S},t}$. The rate on bank debt is given by:

$$r^{bank} = \frac{\bar{s}}{q_{\bar{S}}} = \frac{\mu_B}{q_B} \frac{1 - \frac{\mu_K}{\mu_B} \frac{\sigma_B}{\sigma_K} X(i_t^A, x_{B,t}^A; \rho)}{1 - \frac{p_K}{q_B} \frac{\sigma_B}{\sigma_K} X(i_t^A, x_{B,t}^A; \rho)} \quad (4)$$

¹⁶In the notation of the dynamic model, we will have: $B = y_{\bar{s}_{t+2}^{G_{t+2}^A}}^{primary}$ and $q_t^B = q_t^{\bar{s}_{t+2}^{G_{t+2}^A}}$.

where X is an endogenous measure of bank balance sheet correlation: $X(i_t^A, x_{B,t}^A; \rho) = \frac{\rho + \frac{\sigma_B x_{B,t}^A}{\sigma_K i_t^A}}{1 + \rho \frac{\sigma_B x_{B,t}^A}{\sigma_K i_t^A}}$. This metrics increases with asset correlation ρ if and only if $\frac{\sigma_B x_{B,t}^A}{\sigma_K i_t^A} < 1$. In this case, X increases with ρ from -1 to 1. The ratio $\frac{\sigma_B x_{B,t}^A}{\sigma_K i_t^A}$ controls the concavity: when it tends to zero, the mapping $X(\rho)$ is linear.

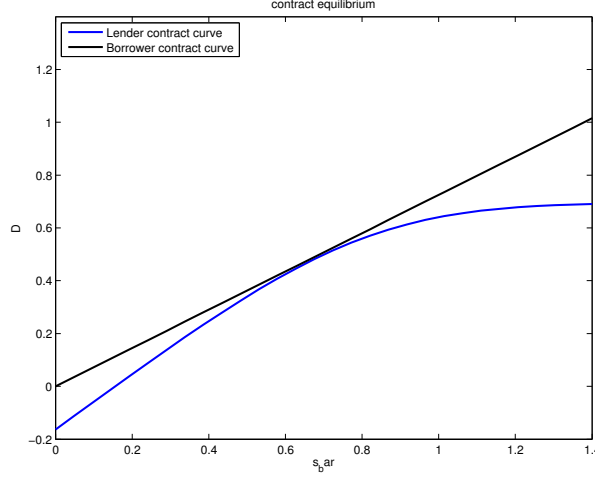


Figure 6: Debt market equilibrium: leverage D w.r.t. promise \bar{s} .

Appendix B.2 characterizes the equilibrium in terms of only two endogenous variables (μ_u, σ_u) : the mean and the volatility of the pre-default bank payoff. The solution strategy is as follows. The bank budget constraint and optimality conditions form a quadratic system in bank asset holdings $(i_t^A, x_{B,t}^A)$. Solving this system delivers (x_K^A, x_B^A) as non-linear functions of (μ_u, σ_u) . The definition of σ_u then delivers a functional $F_{MVF}(\mu_u, \sigma_u) = 0$: a bank mean-variance frontier. In parallel, the equilibrium on the debt market delivers a debt-pricing functional $F_{debt}(\mu_u, \sigma_u) = 0$.

From bank's perspective, its risk-shifting motive deters from holding any negative beta assets. However there is a countervailing force: holding negative beta asset makes its balance sheet less risky, which relaxes its endogenous collateral constraint, hence enabling to lever more. Consider banks portfolio choice condition:

$$p_K \sigma_B \left(\sigma_B x_{B,t}^A + \rho \sigma_K i_t^A \right) - q_B \sigma_K \left(\sigma_K i_t^A + \rho \sigma_B x_{B,t}^A \right) = \left(\frac{\mu_K}{p_K} - \frac{\mu_B}{q_B} \right) p_K q_B \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}$$

As long as the endogenous the long-term debt rate $r_{safe} = \frac{\mu_B}{p_{B,t}}$ is lower than the risky rate $r_K = \frac{\mu_K}{p_K}$, the right-hand side is positive. It implies that $\frac{\sigma_B x_{B,t}^A}{\sigma_K i_t^A} < 1$ can be satisfied only if $\pi > 1$ where $\pi = \frac{p_K \sigma_B}{p_B \sigma_K}$. And in that case, the portfolio condition imposes:

$$1 > \frac{\sigma_B x_{B,t}^A}{\sigma_K i_t^A} > \frac{1 - \rho \pi}{\pi - \rho}$$

The right hand side decreases with ρ : a low correlation restricts more the portfolio choice due to the risk-shifting motive. Having $\pi = \frac{c_K \sigma_2}{p_B \sigma_1} > 1$ does not prevent $\sigma_B < \sigma_K$, as long as assets expectations are chosen such that $r_K > r_{safe}$ but $p_K > q_B$.

Assumption 1. *I make the following PE parameter restriction:*

$$\frac{\sigma_B}{\sigma_K} > \frac{q_B}{p_K} > \frac{\mu_B}{\mu_K}$$

It puts a range on the safe asset price q_B , which translates on bounds on the supply of public safe asset B in general equilibrium.

Lemma 2. *Partial Equilibrium existence*

There exists a contract equilibrium if and only if the safe asset volatility σ_B verifies Assumption 1. In this equilibrium, risky debt is the optimal contract and is the only traded financial contract.

Leverage then is determinate: Modigliani-Miller fails without resorting to any agency frictions. Despite complete markets, the limited liability frictions shapes the optimal contract to be risky debt. Hence equilibrium features limited risk-sharing and equilibrium default.¹⁷

The volatility of long-term debt must be high enough for the contract equilibrium to exist. If $\sigma_B = 0$ (i.e. a riskless storage technology such as money), passive agents all fly to money, and do not find attractive to lend to banks. In the full equilibrium, long-term debt volatility comes from interest rate risk.¹⁸

The mean-variance frontier of the bank $F_{MVF}(\mu_u, \sigma_u) = 0$ is non-degenerate despite banks being risk-neutral. The MVF implicit mapping $\sigma_u \xrightarrow{MVF} \mu_u$ is concave, whereas the *debt* implicit mapping $\sigma_u \xrightarrow{debt} \mu_u$ (from $F_{debt}(\mu_u, \sigma_u) = 0$) is an increasing first-order linear function. Endogenous default makes the banks effectively risk-averse. The equilibrium variables have the following comparative statics with respect to correlation and price of long-term debt:

$$\mu_u = f \left(\begin{matrix} \rho & , & q_B \\ (-) & & (+) \end{matrix} \right) \text{ and } \sigma_u = g \left(\begin{matrix} \rho & , & q_B \\ (-) & & (-) \end{matrix} \right)$$

Lemma 3. *A lower safe asset price and lower beta increases the probability default of the bank.*

That is, when the safe asset is a cheaper and better hedge, banks choose to lever up more and to take more risk. Lower ρ make bank lever up and take more risk, whereas lower q_B make bank lever up more and take less risk.

Effect of a negative correlation ρ Banks have enhanced leverage ability when ρ is low. The safe asset holdings of banks are thus pinned down by the trade-off between the traditional risk-shifting motive (dislikes low ρ) and the debt pricing by investors (likes low ρ). Hedging properties of public debt help the within-generation risk-sharing agreement.

The General Equilibrium endogenizes the safe asset price q_B through the market clearing:

¹⁷Default happens in the low aggregate states, and not in high-income states, a counterfactual feature of Alvarez and Jermann (2000).

¹⁸Equilibrium is then unique: there are *not* two equilibria, one with cheap debt, high leverage and good diversification, and another one with expensive debt, low leverage and poor diversification.

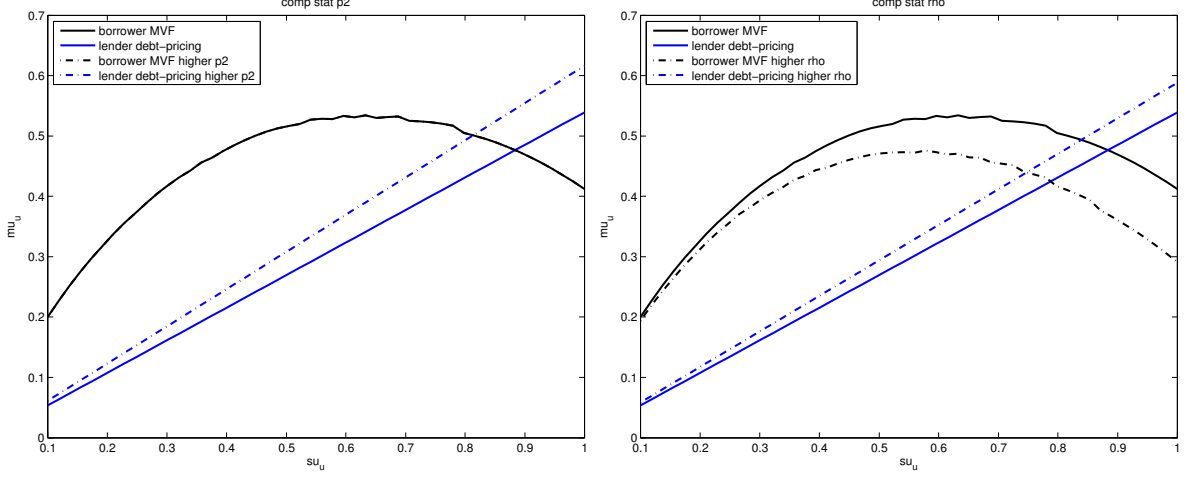


Figure 7: Long-term debt as a hedge: impact of its price and its correlation on bank leverage.

$$x_B^A + x_B^P = B$$

Combining the market clearing condition with the two budget constraint eliminates equilibrium leverage D and recovers the resource constraint:

$$q_B B + p_K \left\{ x_K^A(\mu_K; p_B) + x_K^P(\mu_K; p_B) \right\} = n^A + n^P \quad (5)$$

Thus the safe asset price q_B depends on the equilibrium only through the level of aggregate investment $x_K^A + x_K^P$. This is the heart of the safety multiplier: more expensive long-term debt can deter investment through a GE effect that overcomes the portfolio choice. Assumption 1 and the resource constraint imply a general equilibrium parameter restriction on B for the contract equilibrium to exist:

$$\frac{\mu_K}{\mu_B} \left\{ \frac{n^A + n^P}{p_K} - (x_K^A + x_K^P) \right\} > B > \frac{\sigma_K}{\sigma_B} \left\{ \frac{n^A + n^P}{p_K} - (x_K^A + x_K^P) \right\}$$

A necessary condition is:

$$\frac{B}{n^A + n^P} < \frac{\mu_K}{\mu_B p_K}$$

The closed-form expression for safe asset price q_B enables to prove the following corollary.

Corollary 1. *Existence in General Equilibrium*

There exists a within-generation contract equilibrium if and only if the out-generation safe asset supply B is low enough with respect to aggregate wealth $n^A + n^P$.

The existence does not need any short sale constraints. Limited liability implies an endogenous collateral constraint. Only under a contract equilibrium the aggregate wealth $n^A + n^P$ and the wealth distribution n^A/n^P are priced in the safe asset q_B . A 'safe asset shortage' should qualify a situation in which long-term debt supply is very low with respect to passive wealth n^P : a savings glut of anxious wealth.

2.5 Comparative statics

2.5.1 The safety multiplier

The model explains why risk-neutral banks would ever hold negative beta assets: they have an *endogenous collateral value*, which depends on their correlation with the rest of bank's balance sheet. Long-term debt holdings on bank balance sheet makes bank short-term debt less risky through their hedging property. It decreases the equilibrium default threshold, and this is efficient given the risk aversion of investors. By bundling long-term safe assets with risky assets, banks are able to create more private short-term safe assets. Doing so, it satisfies the risk-averse demand for safety. The first comparative statics captures the macroeconomic puzzle highlighted on Figure 4: a positive comovement of long-term (public) safe assets B and private short-term safe assets D .

Proposition 1. *Complementarity between private safe assets and public safe assets*

When the safe asset supply B is low enough, the supply of private short-term debt D comoves with the supply of long-term debt: $\exists B^* | \forall B < B^*, \frac{\partial D}{\partial B} > 0$.

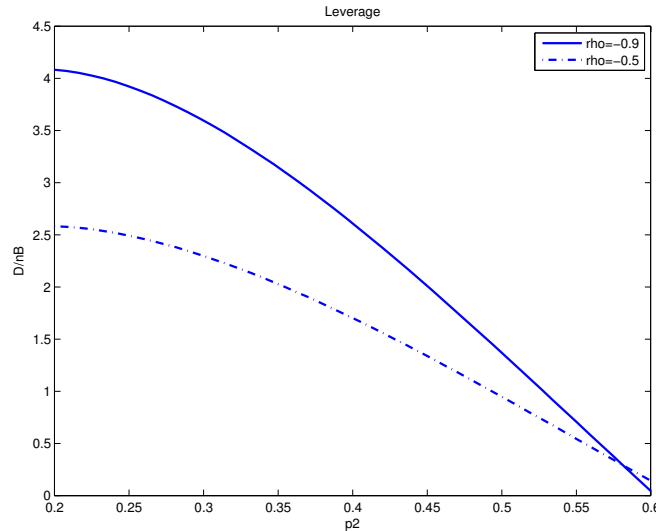


Figure 8: The safety multiplier: positive comovement of public and private debt (PE figure).

There is a safety multiplier when public debt supply B is low enough. In that case, a shortage of public debt leads banks to delever, because the input 'government debt' is too expensive for the safety production function of banks. Proposition 1 shows there is *crowding-in* of private safety by public safety in this regime.

The intuition goes as follows. Banks leverage decision trades off the benefit of leverage with its cost. The latter is determined by the lender's outside option, which itself depends on the price of the safe asset. When the latter is high, the lender prefers to lend to the bank. This is a *crowding-out* effect: lower supply of public debt calls for higher supply of private debt. However, when B is low enough, this effect is overturned by a *crowding-in* effect. From banks perspective, an expensive public debt input makes them scale down safety

production, i.e. less leverage. They decide to lever less as soon as the increase in input price (public debt) swamps the increase in output price (private debt).

The effect can be seen in partial equilibrium by decomposing leverage, where \bar{S} is banks total short-term promises:

$$D(q_B) = \frac{1}{r_{bank}^{(+)}} * \bar{S}^{(-)}$$

A more expensive public debt induces banks to diversify less. Bank debt is then made riskier, which makes it more expensive from banks view. In turn, it leads banks to issue less promises \bar{S} . If the latter endogenous response is strong enough, the combination of the two effects leads to less equilibrium $t = 0$ leverage D . Finally, the safety multiplier can be seen coming from the role of volatility dampener of banks. The ratio of volatilities $\sigma(bank\ debt) / \sigma_B$ is less than one. However, when public debt is expensive, this ratio increases. Banks are hindered in their volatility transformation function.

Effect of risk aversion heterogeneity This safety multiplier mechanism is stronger for a high degree risk aversion heterogeneity.

Corollary 2. *Higher investors risk-aversion leads to higher equilibrium leverage and a safety multiplier for a larger set of the parameter B: $B^* = f\left(\gamma^P\right)_{(+)}$.*

The first part is counter-intuitive, as it seems to say that risk-averse agents invest in a bank debt that is riskier when they are more risk averse. The reason is that the risk-aversion parameter γ^P captures the *differential* of risk attitudes among agents. The optimal risk-sharing agreement features a larger flat part when this differential is higher.

The second part of the corollary comes from the fact that, given that equilibrium leverage is high under high risk aversion, the economy is then more responsive to the safety multiplier mechanism. The scarcity of public safe assets activates more the crowding-in than the crowding-out forces.

Effect of wealth distribution (n^A, n^P) The safety multiplier is also stronger when banks are badly capitalized: n^A/n^P low. This comes from the fact that leverage is slightly *procyclical* in the present model: $\frac{\partial(D/n^A)}{\partial n^A} > 0$. The first order is linear but second order terms of D are convex in n^A . In economic terms, the bank caters even more the safety demand of risk-averse investors when they relatively less capitalized. Similarly, the cutoff B^* broadens when passive wealth n^P is abundant: $B^* = f\left(n^P\right)_{(+)}$. The safety multiplier mechanism is stronger when there is an *anxious-savings glut*.

2.5.2 Crowding-in of private investment

We turn now to the portfolio composition of banks: real investment in the technology vs. holdings of safe assets. The real economy implication of the safety multiplier is a safe-asset driven credit crunch. In this economy with endogenous leverage, banks do not risk shift in stress times. In these stress times of low

supply of public safe assets, the latter are so expensive for banks that they decide to lever up *less*. Total risk-bearing capacity is hindered. The collateral damage on their asset side is an overall crunch of investment in the technology.

Proposition 2. *Crowding-in of private investment*

Lowering the supply of public debt decreases aggregate investment in the risky technology: $\frac{\partial(x_K^A + x_K^P)}{\partial B} > 0$.

That a lower level of public debt in the system triggers a credit crunch is not a priori straightforward.¹⁹ Indeed, it makes the public debt more expensive and induces the banks to rebalance their portfolio toward the other asset, the risky asset, which implies a *crowding-out* of private investment by public debt holdings. Crowding-in arises when the need of the hedging properties of public debt for leverage purposes dominates the portfolio rebalancing force. Appendix B.3 also shows that the mapping $x_1(B)$ is increasing concave: the crunch is exacerbated when B shrinks close to 0.

Proposition 2 can be seen as a beneficial Laffer effect of public debt issuance. In the regime of interest, increasing public debt supply B decreases its ex ante value $p_B B$, and the resource constraint (5) then implies crowding-in of aggregate investment. The credit crunch has a counterpart on bank safe assets holdings.

Proposition 3. *Bank safe asset holdings*

Lowering the supply of public debt increases bank holdings of safe assets: $\frac{\partial x_B}{\partial B} < 0$.

This might be the most surprising result: an increase in the price of the public safe asset leads banks to *increase* their holdings in this asset. This comes from a General Equilibrium effect. An exogenous decrease in B makes it a scarce and sought-after asset with desirable hedging properties. Proposition 3 shows that the marginal buyers for such asset actually are banks, who needs it for a double purpose: hedge their risky investment and relax their collateral constraints. Hence, in times of safe assets shortage, public safe assets are more valuable on banks balance sheets than on passive balance sheets.

Proposition 3 puts forward a *pull* theory of banks holdings of public debt: banks are asking for this public debt, as an input in their safety production function. It is alternative to the *push* theory of financial repression, in which banks are forced to hold public debt by moral suasion from the Treasury. The negative comovement of banks holdings of public safe assets with their aggregate supply is confounded by the two theories. However, the pull theory I develop also predicts a positive comovement of banks holdings of public debt with banks leverage, whereas the push theory of financial repression predicts the contrary. Stylized facts documented in section 6 provide support to the former. In basic supply-demand framework, the fact that both price q_B and quantity x_B^A increased in the safe-asset credit crunch shows that the demand curve did move up.²⁰

¹⁹My channel of a credit supply crunch is complementary to Caballero and Farhi (2013) safety trap, in which the recession is engineered through a New-Keynesian demand channel.

²⁰Zhang (2013) uses households risky assets ratio to predict returns in the US. Theoretically, we can formulate a 'Jacklin critique' to the financial repression argument. With anonymous trading of public debt, the financial repression argument does not hold: optimists will always find it profitable to sell this public debt to risk-averse agents. Contrary to the Jacklin argument, it is not between patient and impatient households shortcutting the bank, but between the bank and the investor shortcutting the government. Here is a profitable deviation which is to circumvent the bank (HH lending directly to the government). So financial repression cannot explain government holdings for sovereigns with deep secondary markets. Now, a long-term contract between sovereign and private agents is sustainable. As the value of this security increases when the value of the risky asset decreases, the optimists will now find it profitable to hold to it in its portfolio, so no more profitable deviation. It is an interesting case of *contagion of commitments*: the government endogenously do not default, preserving a high price for safe debt, diminishing the default threshold of banks.

Finally, the ratio $\frac{x_B^A}{D}$ captures the bank role in the safety multiplier. Contrary to conventional wisdom about the Liquidity Coverage Ratio, banks should not see holdings of safe assets as a constraint, but as an economic force that constitutes an integral input in their macroeconomic role of safety producer.

2.5.3 Asset pricing implications: the two safety premia

As there is not an elastic supply of riskless asset in the environment, a safe rate can only be defined in relative terms. There are two endogenous safe rates: the yield on public debt and the yield on bank debt. I define safety premia taking as reference the exogenous rate of return r_K on the risky technology. The *public safety premium* and the *private safety premium* are:

$$\Sigma_{public} = r_K - \frac{\mu_B}{p_B} \text{ and } \Sigma_{private} = r_K - \frac{\bar{S}}{D}$$

I define the Safety Mismatch Index as the spread between the two premia:

$$SMI = r_{safe} - r_{bank} = \Sigma_{private} - \Sigma_{public}$$

The SMI is the opposite of the endogenous credit spread on banks. It can also be seen as the spread between the *Liquidity Value* and the *Collateral Value* in this collateral constrained economy, using the language of Geanakoplos and Zame (2013).

Proposition 4. *The Safety Mismatch Index.*

Under Assumption 1, the Safety Mismatch carry trade decreases with public debt supply B and increases with public debt beta ρ . Furthermore:

$$r_{safe} - r_{bank} < 0 \Leftrightarrow \rho < -\frac{\sigma_B x_B^A}{\sigma_K x_K^A}$$

The model delivers a negative carry trade on public safe asset: the collateral value dominates the liquidity value. As long as the correlation of public debt with the stock market is low enough, reach for yield is stronger on public safe assets than on private safe assets, despite lower payoff volatility of the latter. This is due to the double role played by the public safe asset when held by banks: hedge the risky investment and back private debt.

The carry trade SMI depends on equilibrium balance sheet quantities only through the correlation metrics X that captures the diversification of banks balance sheets. Under Assumption 1, X is an increasing monotonic transformation of asset correlation ρ and of $\frac{\sigma_B x_B^A}{\sigma_K x_K^A}$. Leverage is high when ρ and $\frac{\sigma_B x_B^A}{\sigma_K x_K^A}$ are low, hence X is a sufficient statistics that captures high equilibrium leverage and higher default probability when low. This translates into low SMI. The latter therefore is a macroprudential market-based tool that reveals aggregate leverage. It is the default risk counterpart of LMI for liquidity risk. It can also be used to track the effect of public debt supply experiments on private bank leverage D .

Relation to bank profitability Banks expected profits increases both in μ_u and σ_u . In times of low SMI, banks' expected profits are higher.

Corollary 3. *Bank profits are higher for a higher supply of public safe assets.*

This is a direct implication of the safety multiplier mechanism. The same force, higher public safe asset price, that leads to a lower equilibrium leverage in safe asset shortage also leads to lower equilibrium bank expected profits.

2.6 Discussion

Limited liability required for a safety multiplier The very parsimonious friction of non-negative consumption at $t = 1$ leads to an economy featuring a safety multiplier. There is no need of any market incompleteness à la Allen and Gale (1994).²¹ Agents can trade in a full set of Arrow-Debreu securities, but in equilibrium, only one contract is traded, risky debt: markets are endogenously incomplete, but are a priori complete. Without the limited liability friction, private would be riskless and public debt issuance would therefore have no traction on private debt issuance²². As banks are essentially doing pooling and tranching in my model (pooling public safe and risky asset, and tranching to issue the private safe asset), a natural question is to ask whether the asset pricing implications are just an application of Modigliani-Miller. This intuition is incorrect because of the limited liability friction (non-negative consumption), which breaks full insurance. Compared to the standard Arrow-Debreu economy, the Inada condition is relaxed by assuming risk neutrality on bankers.

Compared to Holmstrom and Tirole (1998), the CARA-normal environment discards the need of a 3-period timing with a liquidity shock at $t = 1$. In their environment, public debt is purely a store of value. Therefore, with an exogenous liquidity shock, it is intuitive that ‘public debt’ (i.e. cash) should be held by active wealth (entrepreneurs/banks). What my model shows is that when public debt is at tension between two needs: production insurance *and* safety consumption, there exists a *pecking order* of public debt ownership: first bank-entrepreneurs should hold it on their balance sheet, before being held directly in passive hands. By holding public debt on their balance sheets, bank-entrepreneurs fulfill two roles: they are able to insure their technology (the macro shock) and at the same time their private debt then synthetically provides safety and therefore can act as an (imperfect) substitute to public safety. Counter-intuitively, this arrangement strictly dominates having the public debt owned directly by passive hands.

Finally, I differ from Diamond and Dybvig (1983) by focusing on aggregate shocks and not idiosyncratic liquidity shocks. In the latter, banks and depositors enter an optimal contract. Intermediaries both have a liquidity pooling and liquidity insurance role. But the intertemporal liquidity insurance role is not robust to asset spot markets (Jacklin critique, as formulated in Farhi et al. (2009)): as long as there is a spot market for the long-term asset, depositors prefer to invest directly in the risky long-term technology than entering the deposit contract (bank ‘long-term’ debt). It completely unravels the role of intermediaries in liquidity provision: financial intermediaries would not exist, and all the assets, including the risky long-term ones, would be in the hands of households. In my environment of *safety provision* and not liquidity provision, the spot market does not unravel the role of intermediaries: it is robust to the Jacklin critique.

²¹Their chapter in Gale (1990) investigates the efficient design of public debt. Early contributions that risk-sharing could be facilitated by public debt trace back to Weiss (1979). However, all these papers do not entertain the mechanism of public debt as an input to private debt, which is at the core of the present paper.

²²Full insurance can never be attained under the limited commitment friction as long as $\rho > -1$ and $n^B < \infty$. In that case, bank debt is never entirely riskless, even with the promise \bar{s} arbitrarily close to 0. Therefore, private bank debt can never dominate public debt.

Why a safety multiplier in Europe and not in the US: limited participation The model is solved with full participation of all agents in all markets. Risk-averse agents can carry out some diversification themselves, by directly bundling in their own hands public debt and the risky technology. However, as long as the public debt beta $\rho > -1$, they cannot perfectly hedge the macro shock through their own portfolio choice of the technology and public debt. As a consequence the flat part of bank debt still has its appeal. The three assets: public debt, private bank debt and the technology are jointly held by risk-averse investors. Bank leverage is therefore determinate.

In the case of limited participation, i.e. when the risk-averse agents are prevented from investing directly in the risky technology, the safety multiplier mechanism is strengthened. Having the risk-averse investors doing directly some diversification dampens the safety multiplier. The cutoff B^* in Proposition 1 is determined by the tension between two forces: lender portfolio choice which tilts towards crowding-out, and debt safety creation which tilts towards crowding-in. Relaxing the limited participation constraint strengthens the portfolio choice force, as now, the synthetic asset {technology+public debt} can exist and is a better substitute to {private debt} than {public debt} alone. The debt safety creation motive, which is entirely driven by bank portfolio and leverage choice, is not affected by limited participation. As a consequence the cutoff with direct access of passive wealth to the technology is lower than the cutoff in the limited participation environment, i.e. there is a larger parameter region with a safety multiplier under limited participation than under full participation:

$$B^*_{full\ participation} < B^*_{limited\ participation}$$

This comparative statics helps rationalize why Europe behaves differently than the US, i.e. why there is empirically a safety multiplier in Europe and not in the US. As a consequence, it reconciles my empirical stylized fact of positive comovement of private debt with public debt in Europe, whereas [Krishnamurthy and Vissing-Jorgensen \(2013\)](#) shows that private debt and public debt negatively comove in the United States. I argue that this can be explained by applying my limited participation environment to Europe and applying the full participation environment just described to US. Indeed, it is extremely well documented that disintermediated instruments such as Private Equity and Venture Capital are much more developed in the US than in Europe. As a consequence, Europe is much more of the limited participation environment, and I just showed that in this environment there is a safety multiplier for a larger parameter region: crowding-in of private debt by public debt. The higher equilibrium leverage in that case is consistent with the pervasive role played by European banks in the financing of the real economy: 80% of the financing is intermediated by banks and not the corporate bond market (instead of 20% in the US).

Moreover, the Appendix derives:

$$B^* \left(\begin{matrix} \rho, \sigma_K \\ (-) \quad (+) \end{matrix} \right)$$

The first comparative statics rationalizes the time serie: public and private debt comove more when public debt is actually negative beta (it is a recent phenomenon). The second comparative statics also helps to rationalize the cross-country Europe vs. US: public debt and private debt comove more when the risky technology is riskier.

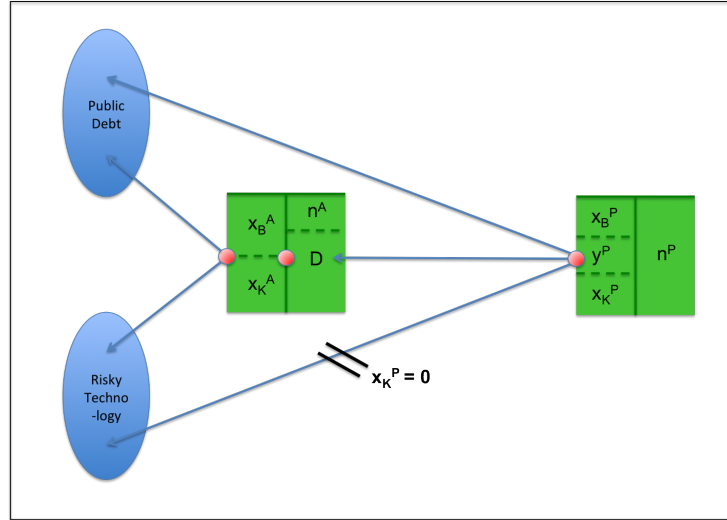


Figure 9: Contract equilibrium representation under Limited Participation. Agents are in green and assets in blue. Red bullets indicate, from left to right: bank portfolio choice, bank leverage choice and lender portfolio choice.

Are banks in the business of creating safety in the long or in the short horizon? In practice, banks create safety on their liability side at different maturities. The model endogenously endows banks with a maturity transformation role. In equilibrium they decide to issue liabilities of shorter maturity than the one of their asset holdings.

One could argue that deposits exhibit long-term liability aspects, given their stickiness. However, the overall cost of funding is weighted average of deposits costs and wholesale funding costs.²³ The marginal cost of funding is pinned down in the latter. On this wholesale funding it is clear that empirically, banks are in the business of creating short-horizon safety. Furthermore, it can be argued that a bulk of securities holdings by banks is not mark-to-market in practice. This is not an issue for the relevance of the model, as, as long as there is some short-term debt to be repaid, banks will in effect mark to market their balance sheet by getting out and rolling over their holdings. As these debt instruments are unsecured as argued in the static model, negative beta holdings have an input role on the asset side of banks.

The dynamic model predicts that risk-neutral agents hold the long-end of public debt and risk-averse agents hold the short-end. Indeed, in line with the view of a scarcity of public safe assets (i.e. their inelastic supply), the model delivers a pecking order in the ownership of public debt. Risk-neutral and risk-averse agents compete for public debt ownership on Walrasian markets. Short-term public debt (T-bills) is the dominating safe asset in the economy. The risk-averse agents value it the most. Therefore in equilibrium, T-bills are owned by risk-averse investors and they yield the lowest. What comes next on the safety ladder is long-term public debt (T-bonds). As these ones are endowed with the negative beta property, they enjoy a 'double coincidence of will' when put on the balance sheet of the risk-neutral agents. At the same time this hedges their investment in the risky technology, and, by relaxing the endogenous collateral constraint, this enables the creation of private short-term debt. Banks do reach for yield in the sense that, contrary to the

²³Figure 16 shows that, for the Eurozone, the deposit base is 10trn€ and the debt from wholesale funding is 2trn€.

risk-averse investors, they prefer T-bonds (with higher yield) on T-bills.

Wealth effects The recent macro-finance literature analyzes the non-linearities due to financial frictions: Krishnamurthy and He (2013) and Brunnermeier and Sannikov (forthcoming) using an agency friction, Mendoza (2010) using an exogenous collateral constraint, and Cao (2013) using beliefs disagreements. Adrian and Boyarchenko (2012) is the only model solved closed-form, using a *VaR* friction. All these papers feature the wealth distribution as states variables. Thus they are all after the interaction of wealth effects with the financial constraint. On the contrary, my environment analyzes how safe asset shortage interacts with limited liability constraints. Their interaction jointly explains the negative beta of public debt and high bank leverage. Amplifying mechanisms arising with wealth effects do not make any normative case as they are constrained efficient environments (no market failure), whereas my environment does make a normative point, as fleshed out in the next section.

Even if wealth effects are not a key ingredient of the interesting dynamics, the endogenous beta formula 18 shows that the flight to safety is more aggressive when n^B increases faster than n^L , which implies lower beta and higher bank leverage. Thus, when keeping track of wealth effects, the dynamic model would be able to generate leverage cycles at business cycle frequencies.

3 Dynamic environment: endogenous beta of public debt

The model is a stochastic overlapping generations model under aggregate technology risk. Contrary to the canonical OLG, risk is not on endowments, but on the technology in perfectly elastic supply. It is cast in a discrete infinite horizon framework, each period is indexed by t .²⁴

Demographics There are overlapping generations of such agents. Each generation lives three periods.²⁵ The heterogeneity within generations is kept constant over time. Every new born of type A is endowed with n^A of numeraire, agents of type P with n^P .²⁶ Not having the wealth distribution as a state variable makes the model bloc-recursive. The notation G_t^i with $i \in \{A, P\}$ refers to an agent of type i that belongs to the generation that was born in period $t - 2$ and dies in period t .

Government A government \hat{B} finances public spending by raising taxes and issuing public debt. The latter plays the role of a second asset from private agents' perspective, albeit endogenous and in fixed supply.

Public debt securities are non state-contingent assets, which promise to one unit of numeraire at maturity. Public debt can be long-term: it can be issued at period t and pay back only at period $t + 2$. The government then issues public debt at different maturities $h = 1, \dots, H$. Taking $H = 2$ exactly matches the horizon of private agents, and therefore do not endow the government with undue advantage on agents.²⁷ Denote

²⁴The purpose of the dynamic model is to endogeneize the beta of safe assets. Microfounding the beta of public debt paves the way to the welfare analysis. The model is the exact repeated sequence of the static game considered in the example above, with a broader set of available contracts to allow for a maturity choice of agents.

²⁵Beyond breaking the First Welfare Theorem, the OLG structure then mutes undesirable discounting and long-run wealth effects.

²⁶It is within-generation heterogeneity in preferences, not in endowments as Sargent-Wallace (1982) and Smith (1988).

²⁷I restrict the government ability to issue long-term debt to only two periods in order to respect the spirit of constrained efficiency. In the theory of the 2nd-Best, the government has the same instruments as private agents. In my environment, this means allowing the government for the exact same maturity choice as the agents.

B_t^h the outstanding stock, at date t , of public debt maturing at date $t + h$. Government consumption g_t is exogenous. To avoid asymmetric tax treatment across generations, only the old are taxed.²⁸ There can be asymmetric tax treatment within generation, but I assume the tax schedule (τ^A, τ^P) is not optimized upon by the government, perhaps because of informational frictions on the types (A, P) , which renders fiscal policy less agile than public debt issuance policy. Fiscal policy then merely tracks the public debt issuance policy by balancing the government budget.

For each residual maturity of public debt $h = 1, \dots, H$, a Walrasian market opens at each period t . All agent of all generations have access to these markets, and primary debt and secondary debt is fungible: a government promise for date $t + h$ has the same price, because it is traded on the same market, which clears at price $\{\hat{q}_t^h\}_h$. As a h period promise at t becomes a $h - 1$ period promise at $t + 1$, the government budget writes:

$$g_t + \sum_h \hat{q}_t^h (B_{t-1}^h - B_t^h) \leq \tau_t^A + \tau_t^P \quad (6)$$

A *riskless financial policy* is a collection $(\{\hat{q}_t^h\}_h, \tau_t^A, \tau_t^P)$ that satisfies the government budget constraint at each history s^t . Nevertheless, I also allow for the possibility of endogenous government default.²⁹ To see this, consider the case of an extremely bad aggregate shock (or an extremely high public spending shock). Given the fiscal policy choice of only taxing the old, the proceeds the government can raise in this case is null. Moreover, as will become clear through the safety multiplier mechanism elicited in this paper, the economy is in a Laffer regime for public debt, hence the government cannot raise additional revenue by issuing more public bonds. The government is then forced into default. On other words, the government has endogenously limited fiscal capacity.

When a long-term public debt is issued at t , it carries *interest rate* risk: at period $t + 1$, its price will be determined mark-to-market, and this price is uncertain from t perspective.³⁰ An investor with a one-period ahead horizon faces a portfolio choice in three stochastic potentially correlated assets: the technology, the short-term public debt and the long-term public debt.³¹

Timeline To rule out Ponzi schemes, private agents can issue promises only at the horizon of their life span: one- or two-period ahead when young, and only one-period ahead when middle-aged. The sequence of actions is as follows.

- At each period t , the government rolls over its debt according to its financial policy. The government issues $B_{t-1}^h - B_t^h$ units of new public bonds with the residual maturity $t + h$.

²⁸Whether to tax the young, the middle or the old generation is innocuous, as wealth effects do not play a role in the safety multiplier.

²⁹The technical motivation is to rule out a trivial strategy for the social planner to issue unlimited level of public short-term debt. The economic motivation is to analyze in Section 5 the interplay between sovereign credit risk and bank credit risk in an open economy environment.

³⁰Contrary to the safe asset literature (Gourinchas and Jeanne (2012), Gorton and Ordonez (2013)), and in line with Caballero and Farhi (2013), my paper insists on a defining characteristics of public debt: its negative beta with macroeconomic shocks.

³¹As a consequence of potential sovereign default, short-term public debt is not fully riskless.

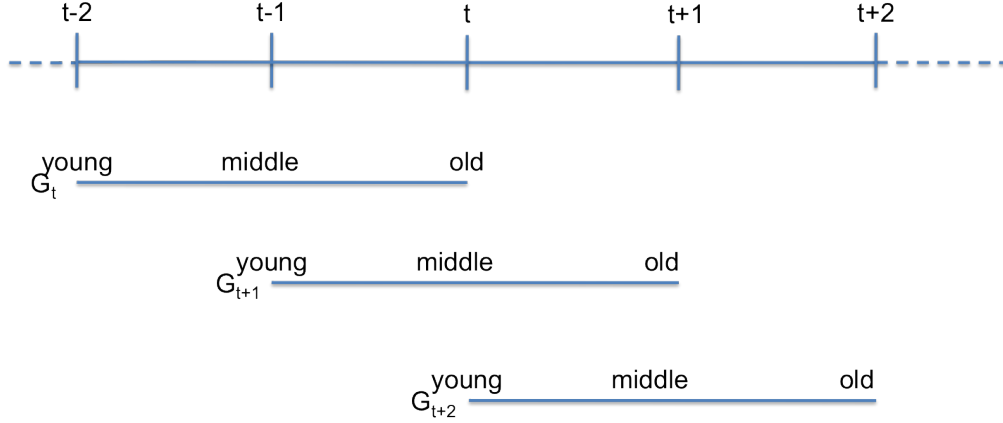


Figure 10: Demographics and timeline.

- At each period t , with macro state s_t , a new generation of agents $G_{t+2}^{\theta \in \{A, P\}}$ is born with endowments (n^A, n^P) . The young generation make portfolio choice decisions by investing in the technology: agents of type $\theta \in \{A, P\}$ invest $i_t^{G_{t+2}^\theta}$ in the technology. At the same time they trade on the Walrasian markets for legacy promises: public debt and claims on former generation balance sheets, as well in the primary markets for new promises issued at t , whether within or between generations. We denote $\bar{s}_{t+h}^{G_{t+j}^\theta}$ the number of units of promise they buy on market for a given promise $\bar{s}_{t+h}^{G_{t+j}^\theta}$. They also create primary markets for promises on their own balance sheets by *issuing* long-term debt and short-term debt. We denote these sort positions in their own long-term and short-term promises $y_{\bar{s}_{t+2}}^{G_{t+2}^\theta}$ and $y_{\bar{s}_{t+1}}^{G_{t+2}^\theta}$ respectively.
- At the following period $t+1$, the same agents becomes middle-aged. The risky technology then pays off s_{t+1} , and their portfolio of promises (long and short) can be mark-to-market with the Walrasian markets that open at period $t+1$. Maturing promises are settled by the actual payment of the promise or by the issuer defaulting. In the latter case, any holder of a promise seizes the total balance sheet $\{\text{technology payoff} + \text{residual promises}\}$ of the agent in default, pro-rata the promise. Subsequently, these middle-aged agents make new decisions of investment in the short-term risky technology $i_{t+1}^{G_{t+2}^\theta}$ and rebalance their portfolio of promises, and can open primary markets for promises by issuing short-term debt.
- Finally, at period $t+2$, the agents of this generation $G_{t+2}^{\theta \in \{A, P\}}$ become old. Before their death, they receive the payoff s_{t+2} of the technology and they liquidate their portfolio of promises, and consumes these proceeds.

Markets At any period t , there are 3 markets open for private promises.³²

³²The identity of the issuer generation has to kept track of, due to the recourse feature of the promises. A priori it would be $3 * \text{card}(\theta)$ potential markets, but given the one dimensional heterogeneity within-generation, promises are traded only in one direction, hence only 3 markets are actively traded.

- **Secondary market for long-term debt**, i.e. promise $\bar{s}_{t+1}^{G_{t+1}^\theta}$ issued by the middle, clearing at price $q_{\bar{s}_{t+1}^{G_{t+1}^\theta}}^{G_{t+1}^\theta}$:³³

$$x_{\bar{s}_{t+1}^{G_{t+1}^\theta}}^{G_{t+1}^{\theta'}} + x_{\bar{s}_{t+1}^{G_{t+1}^\theta}}^{G_{t+2}^\theta} = y_{\bar{s}_{t+1}^{G_{t+1}^\theta}}^{G_{t+1}^\theta} + y_{\bar{s}_{t+1}^{G_{t+1}^\theta}}^{primary} \quad (7)$$

- **Primary market for short-term debt**, i.e. promise $\bar{s}_{t+1}^{G_{t+2}^\theta}$ issued by the young, clearing at price $q_{\bar{s}_{t+1}^{G_{t+2}^\theta}}^{G_{t+2}^\theta}$:

$$x_{\bar{s}_{t+1}^{G_{t+2}^\theta}}^{G_{t+1}^\theta} + x_{\bar{s}_{t+1}^{G_{t+2}^\theta}}^{G_{t+2}^\theta} = y_{\bar{s}_{t+1}^{G_{t+2}^\theta}}^{G_{t+2}^\theta} \quad (8)$$

- **Primary market for long-term debt**, i.e. promise $\bar{s}_{t+2}^{G_{t+2}^\theta}$ issued by the young, clearing at price $q_{\bar{s}_{t+2}^{G_{t+2}^\theta}}^{G_{t+2}^\theta}$:

$$x_{\bar{s}_{t+2}^{G_{t+2}^\theta}}^{G_{t+1}^\theta} + x_{\bar{s}_{t+2}^{G_{t+2}^\theta}}^{G_{t+2}^\theta} = y_{\bar{s}_{t+2}^{G_{t+2}^\theta}}^{primary} \quad (9)$$

On the other hand, for government promises, there are only two markets, as the primary market for short-term public debt and the secondary market for long-term public debt are fungible. Despite being called the market for short-term public debt, it includes the legacy long-term public debt that matures next period.

- **Market for short-term public debt**, i.e. for public promise of $1_{t+1}^{\hat{G}}$ at $t + 1$, which clears at price \hat{q}_{t+1} :

$$x_{1_{t+1}^{\hat{G}}}^{G_{t+1}^\theta} + x_{1_{t+1}^{\hat{G}}}^{G_{t+2}^\theta} = \hat{B}_t^1 \quad (10)$$

- **Market for long-term public debt**, i.e. for public promise of $1_{t+2}^{\hat{G}}$ at $t + 2$, which clears at price \hat{q}_{t+2} :

$$x_{1_{t+2}^{\hat{G}}}^{G_{t+1}^\theta} + x_{1_{t+2}^{\hat{G}}}^{G_{t+2}^\theta} = \hat{B}_t^2 \quad (11)$$

3.1 Young agents portfolio and maturity choices

We now turn to generation G_{t+2} . Agents optimize twice over the course of their life: when young and when middle-aged. When young, they face a meaningful maturity choice. The analysis in section 3.1. enables to derive the indirect utilities of the two middle-aged agents as a function of history s^t : $V_{t+1}^{G_{t+1}^A} \left(n_{t+1}^{G_{t+1}^A}(s_t), s_t; y_{\bar{s}_{t+2}^{G_{t+2}^\theta}}^{primary}(s_{t-1}) \right)$ and $V_{t+1}^{G_{t+1}^P} \left(n_{t+1}^{G_{t+1}^P}(s_t), s_t \right)$.³⁴ These give the marginal values of wealth for each agent of the generation G_{t+1} at t in history s^t . But when young, agents optimize their expected utility over payoff *two* periods ahead, at $t + 2$.³⁵ The middle-aged optimization is not a sideshow as, at period t , the market clearings *jointly* involve the portfolio choices of the two generations.

³³I allow for $y_{\bar{s}_{t+1}^{G_{t+1}^\theta}}^{G_{t+1}^\theta} < 0$, which corresponds to the buy back of the legacy stock $y_{\bar{s}_{t+1}^{G_{t+1}^\theta}}^{G_{t+1}^\theta primary}$ of long-term promises. The stock

$y_{\bar{s}_{t+1}^{G_{t+1}^\theta}}^{G_{t+1}^\theta primary}$ is held at the beginning of the period by the old generation G_t . They are selling in order to consume before dying.

³⁴The problem is not stationary in n^B and n_{mid}^B : the first is exogenous, the second is state-contingent (s_{t-1}, s_t) by bloc-recursivity.

³⁵Allowing for consumption even when young and middle-aged beyond when old is innocuous. It then suffices to collapse the time and state dimensions in one same dimension. With CARA preferences, intertemporal elasticity of substitution and risk-aversion are equal, hence result about state-smoothing generalizes to consumption-smoothing. Epstein-Zin relaxation is left for further research.

Program of young risk-neutral agents Denote \bar{S}^{ST} the sum of all short-term promises, \bar{S}^{LT} the sum of all long-term promises:

$$\bar{S}^{ST} = \int y_{\bar{s}_{t+1}^{G_{t+2}^A}} \bar{s} d\bar{s}$$

$$\bar{S}^{LT} = \int y_{\bar{s}_{t+2}^{G_{t+2}^A}}^{primary} \bar{s} d\bar{s}$$

The only assets they can invest in are the technology and the short-term promises of the middle-aged G_{t+1} agents. Its portfolio choice is then $X^{A_y'}$:

$$X^{A_y'} = \begin{bmatrix} i_t^{G_{t+2}^A} & x_{\bar{s}_{t+1}^{G_{t+2}^A}} \end{bmatrix}$$

The portfolio choice $X^{A_y'}$ and the short-term promises only impact the value function through the interim net worth of the bank at $t + 1$:

$$n_{t+1}^{G_{t+2}^A}(s_{t+1}) = X^{A_y'} S_{t+1} - \bar{S}^{ST} \quad (12)$$

Whereas the long-term promises \bar{S}^{LT} only impact the value function through the long-run payoff u_{t+2} . The program of the bank then writes:

$$\begin{aligned} \{X^{A_y'}, \bar{S}^{ST}, \bar{S}^{LT}\} \quad W_{t+2}^{G_{t+2}^A}(s_t) &= \mathbb{E}_t \left[V_{t+2}^{G_{t+2}^A} \left(n_{t+1}^{G_{t+2}^A}(s_{t+1}), s_{t+1}; y_{\bar{s}_{t+2}^{G_{t+2}^A}}^{primary}(s_t) \right) \right] \\ \text{s.t.} \quad X^{A_y'} P &\leq n^A + \bar{S}^{ST} \frac{q_t^{G_{t+2}^A}}{\bar{s}} + \bar{S}^{LT} \frac{q_t^{G_{t+2}^A}}{\bar{s}} \end{aligned} \quad (13)$$

Program of young risk-averse investors The only assets they can invest in outside the generation also are the technology and the short-term promises of the middle-aged G_{t+1} agents. Its portfolio choice $X^{P_y'}$:

$$X^{P_y'} = \begin{bmatrix} i_t^{G_{t+2}^P} & x_{\bar{s}_{t+1}^{G_{t+2}^P}} \end{bmatrix}$$

The out-generation portfolio choice $X^{P_y'}$ and the long positions in *both* the short-term and long-term promises only impact the value function through the interim net worth of the investor at $t + 1$:³⁶

$$n_{t+1}^{G_{t+2}^P}(s_{t+1}) = X^{P_y'} S_{t+1} + \text{Min} \left(\bar{S}^{ST}, n_{t+1}^{G_{t+2}^A}(s_{t+1}) \right) + \bar{S}^{LT} \frac{q_t^{G_{t+2}^A}}{\bar{s}} \quad (14)$$

The program of the investor then writes:

$$\begin{aligned} \{X^{P_y'}, \bar{S}^{ST}, \bar{S}^{LT}\} \quad W_{t+2}^{G_{t+2}^P}(s_t) &= \mathbb{E}_t \left[V_{t+2}^{G_{t+2}^P} \left(n_{t+1}^{G_{t+2}^P}(s_{t+1}), s_{t+1} \right) \right] \\ \text{s.t.} \quad X^{P_y'} P &+ \bar{S}^{ST} \frac{q_t^{G_{t+2}^A}}{\bar{s}} + \bar{S}^{LT} \frac{q_t^{G_{t+2}^A}}{\bar{s}} \leq n^P \end{aligned} \quad (15)$$

³⁶Due to the presence of Walrasian markets, the investors sell all their long-term promises before reinvesting at $t + 1$ (mark-to-market).

Comparing the programs of the young banks and the young investors, we observe that the choice variables \bar{S}^{ST} and \bar{S}^{LT} are not redundant: even if they are part of the same debt raising at t , given that their payoff happen at different times (t and $t + 1$) and that agents have heterogeneous marginal values of wealth, the maturity choice is well defined.

3.2 Decentralized Equilibrium

I first analyze on the environment abstracting from any government optimization. In this case, the financial policy $\left(\left\{\hat{B}_t^h\right\}_h, \tau_t^A, \tau_t^P\right)$ is taken as exogenous.³⁷ I define all the Walrasian recursive equilibria of this economy. I focus on stationary Markov equilibria. Given the bloc-recursive structure of the environment, these equilibria can be defined with a unique state variable: the aggregate shock $s^t \leftarrow (s_{t-1}, s_t)$.

Definition 1. A stationary Markov equilibrium is a collection in each history s^t of portfolio investments $\left\{i_t^{G_{t+j'}}(s^t), x_{\bar{s}_{t+h}}^{G_{t+j}}(s^t)\right\}_{1 \leq h \leq j, j' \leq H}$, primary issuances $\left\{y_{\bar{s}_{t+h}}^{G_{t+j'}}(s^t)\right\}_{1 \leq h \leq j, j' \leq H}$, a vector of public debt prices $\{\hat{q}_{t+h}(s^t)\}_{1 \leq h \leq H}$ and a vector of private debt prices $\left\{q_{\bar{s}_{t+h}}^{G_{t+j}}(s^t)\right\}_{1 \leq h \leq j \leq H}$ such that:

- i) All agents of all generations optimize.
- iii) Markets for private promises clear at each residual maturity.
- iv) Markets for public promises clear at each residual maturity.

These equilibria have the flavor of the collateral-constrained equilibria of Geanakoplos and Zame (2013),³⁸ but the financial assets traded are different, as borrowers' debt here is *recourse*. Another difference is that there is a priori a continuum of contracts that could be traded: one for each state s of the continuum. Markets therefore are complete.³⁹ Although a priori, an infinite set of securities $(q_{\bar{s}}, \bar{s})$ is available to agents, only one will be traded in the equilibrium of interest given the low level of heterogeneity (only two types of agents).⁴⁰ The economy features endogenous market incompleteness: despite having a complete spanning of financial assets, agents' positions are restricted by their endogenous collateral constraints arising from limited liability. However, due to the recourse feature of unsecured debt, my economy is 'more complete' than the Geanakoplos one. As a result, this environment can be seen as an intermediate case between the Arrow-Debreu and the Geanakoplos economies, less complete than the former but more complete than the latter.

The full equilibrium is solved by backward induction. First I characterize the solution of the portfolio choices of the middle-aged generation, taking the supply of legacy long-term debt as given. Then I move backward to characterize the joint decision of portfolios and maturity choices by the young generation. I prove the existence of a recursive (i.e. time-homogeneous) Markov equilibrium. Such stationary equilibrium

³⁷I allow for irresponsible financial policies: the financial policy does not have to be riskless in the sense of satisfying the government flow of funds at each history s^t . Hence the equilibrium features non-zero probability of government default.

³⁸It is closer to the liquidity-constrained equilibrium than the debt-constrained equilibrium of Kehoe and Levine (2001). In the latter, as in Hellwig and Lorenzoni (2009), default is strategic and the existence of equilibrium hinges on the self-enforcement of debt. These two papers do not feature equilibrium default.

³⁹As markets are complete, there is no need to engage in market design as Athanasoulis and Shiller (2001).

⁴⁰This is fortunate, as it circumvents the possibility of a discontinuity in agents budget sets as in Hart (1975). Restricting agents from consuming at $t = 0$ mutes down consumption smoothing and conveniently avoids difficulties on equilibrium existence.

satisfies the following properties: ergodicity, conditional spotlessness, and compatibility with arbitrary initial conditions.

Lemma 4. *There exists a recursive Markov equilibrium for any given financial policy $(\hat{B}^h, \tau^A, \tau^P)$.*

Proof. The proof is a direct application of Duffie et al. (1994). □

Despite the existence of the equilibrium, the above lemma does not ensure a non-degenerate equilibrium in which assets that pay no dividends have non-zero value. As a matter of fact, Duffie et al. (1994) notes that “we do not know whether coexistence of with- and without-dividend assets is possible in a stochastic economy without population growth, either with or without ergodicity”.⁴¹ The following proves that there exists an equilibrium with coexistence of risky and safe assets in the stochastic economy. In such equilibrium risk-averse agents lend to risk-neutral agents and in which long-term debt has non-zero value.

To show existence of such contract equilibrium, I proceed in two steps. First I take the portfolio and maturity choices of young as given, and solve for the equilibrium leverage (risk-sharing agreement) within the middle-aged generation. Second, I use this portfolio choice to compute middle-aged value functions, before fully solving for the young portfolio and maturity choice.

3.3 Dynamic equilibrium characterization

Starting from the results of the static model (‘within generation equilibrium’), I move backward to the program of the young and the inter-generational full equilibrium at period t . The interaction between the young and the middle aged generations adds two features to the model: endogenous $t + 1$ price functional for long-term debt, and endogenous long-term debt supply through the maturity choice of the young. I analyze these two equilibrium features sequentially. I first characterize the solution of the two above program and the market clearings taking the next generation issuance quantities as given. The latter act as an out-of-generation supply of assets, from generation G_{t+1} perspective.

Long-term debt endogenous price functional In the time-homogeneous Markov equilibrium, the price $q_{s_{t+2}^{G_{t+2}}}^{interim}(s^{t+1})$ of long-term debt is fully endogenous. This key feature of the model enables to derived a formula for the endogenous correlation of long-term debt with aggregate risk (its ‘beta’). I solve for the fixed point in the long-term debt price functional, using a heuristic approach drawing on the ‘static’ pricing by the middle-aged derived in the above section 3.3.2. I still take here the supply of long-term debt as exogenous $B = x_{s_{t+2}^{G_{t+2}}}^{G_{t+2} \text{ primary}} + \hat{B}$ (private and public long-term debt).

By fungibility, the realized marked-to-market price at $t + 1$ of long-term is the same as one of a short-term promise issued by the same risk-neutrals, the ones of generation G_{t+2} . At $t + 1$, such promise can be bought by the middle-aged G_{t+2} risk-averse agents, or by the young G_{t+3} young risk-averse agents. Let first focus on the first type of buyers, the within generation risk-averse agents. In this case, the $t + 1$ price on the market for this promise is given by the debt market equilibrium solved in the above section 3.3.2:

$$q_{s_{t+2}^{G_{t+2}}}^{interim}(s^{t+1}) = q_{s_{t+2}^{G_{t+2}}}^{t+1}(s^{t+1}) \quad (16)$$

⁴¹Scheinkman (1980) shows there does not exist such equilibrium in deterministic economies. Spear and Srivastava (1986) and Spear et al. (1990) entirely characterize the structure of equilibrium in stochastic OLG models.

So in effect we have to mappings that relate the long-term debt functional with the short-term debt functional: the one that gives the price of long-term debt at t as a function of the price of short-term debt at $t + 1$ (equation 16), and the one that gives the price of short-term debt at t as a function of the price of short-term debt at $t + 1$ (equation 4), which can be written formally:

$$q_{\bar{s}_{t+1}}^t(s^t) = g\left(\left\{q_{\bar{s}_{t+2}}^{interim}(s^{t+1})\right\}_{s^{t+1}}\right) \quad (17)$$

The heuristic solution goes as follows. I take the bank debt price functional $q_{\bar{s}}(\mu_K)$ that solves the static model, and develop it in two orders of μ_K . I then analytically compute the multivariate second moments of this functional with respect to the underlying shock $s_t \equiv \mu_K$: its mean $\hat{\mu}_B$, its volatility $\hat{\sigma}_B$ and its correlation $\hat{\rho}$. This leads to three equations in the three unknowns $(\hat{\mu}_B, \hat{\sigma}_B, \hat{\rho})$, whose fixed point gives the second moments of the fixed point functional $q_{\bar{s}_{t+2}}^{G_{t+2}}(s^t)$. This is tantamount to working locally to make the following multivariate normal (2^{nd} -order moments) approximation valid:

$$\left[s^t, q_{\bar{s}_{t+2}}^{G_{t+2}}(s^t)\right] \sim N([\mu_K \hat{\mu}_B], [\sigma_K \hat{\sigma}_B \hat{\rho}])$$

This heuristic approach uses the implicit characterization of the safe asset price functional from the resource constraint:

$$p_K x_K \left(q_{\bar{s}_{t+2}}^{G_{t+2}}(s^t); s^t\right) + q_{\bar{s}_{t+2}}^{G_{t+2}}(s^t) B = n^A + n^P$$

where $x_K(s^t) = x_K^A(s^t) + x_K^P(s^t)$ is aggregate investment. A second-order expansion in s^t of the equilibrium value of investment in the static model leads to an implicit expression of the endogenous beta:

$$\hat{\rho} = -1 + 4(s^t)^2 \left(\left(\frac{\partial_{\mu_K} x_K}{\frac{B}{p_K} + \partial_{p_B} x_K} \right)^2 \frac{\partial_{p_B p_B} x_K}{\partial_{\mu_K} x_K} - 2 \frac{\partial_{\mu_K p_B} x_K}{\frac{B}{p_K} + \partial_{p_B} x_K} \right)^2$$

Appendix B.3 shows the existence of a triplet $(\hat{\mu}_B, \hat{\sigma}_B, \hat{\rho})$ satisfying the fixed point of this equation that defines correlation, as well two additional equations from the definition of long-term debt mean and volatility: $\hat{\mu}_B$ and $\hat{\sigma}_B$. It leads to the solution in the second-order approximation for the endogenous beta of public debt in the recursive equilibrium:

$$\hat{\rho} = -1 + \left(\frac{B}{n^A} \frac{[\hat{\sigma}_B^2 p_K^2 + \sigma_K^2 \hat{q}_B^2]^2}{2\sigma_K^4 p_K^2 \hat{q}_B^2} \right)^2 \quad (18)$$

This expression for the endogenous beta of public debt is interesting in many respects. First, the beta is indeed negative for low levels of B . In the recursive equilibrium, the flight to safety enjoyed by public debt *endogenously endows* this security with an hedging property. It is the expectation of a flight to safety in the low states of tomorrow that endows public debt with negative beta. Ex ante, this enables (within-generation) safety creation. In the canonical Samuelsonian treatment, money is valued today if people expect it to have value tomorrow. This is a deterministic argument. In contrast, public debt has value in my environment because of its endogenous hedging properties.

This flight to safety is amplified by the safety multiplier. A higher level of bank net worth n^A commands a stronger safety multiplier effect. In this regime of high n^A , in the states s^t of low technology productivity, not

only the bank does rebalance aggressively away from technology towards the public debt and at the same time delevers. Private safe assets supply then dwindles and investors also rebalance towards the public debt. Thus when n^A is high, the two portfolio rebalance compounds towards a flight to the public debt. Beta of public debt thus decreases with bank net worth.

Second, the following lemma characterizes the dependence of the negative beta to the supply of public debt. This result is key for the normative analysis.

Lemma 5. *In the stationary Markov equilibrium, the beta of public debt increases with the supply of long-term debt B .*

A scarce supply of public debt makes the flight to safety it enjoys more aggressive. Subsequently, the hedging properties of public debt are enhanced by its scarcity. The candidate heuristic equilibrium derived above is shown to be an equilibrium, using this property of public debt beta. It leads to the dynamic counterpart of the static contract equilibrium.

Lemma 6. *A stationary Markov equilibrium in which risk-averse agents lend to risk-neutral agents (contract equilibrium) exists only if B is low enough with respect to aggregate wealth.*

Proof. The sketch goes as follows. A low enough B creates imply a highly negative beta ρ^{endo} through the flight to safety. It also implies volatility on the safe asset $\hat{\sigma}_B$. We then appeal to Lemma 3. \square

A corollary of this lemma is that, in the dynamic case, the comovement of private debt supply with public debt supply is ambiguous. On the one hand, increasing public debt supply triggers the safety multiplier mechanism described in the static model, and this creates a positive comovement force. On the other hand, the increase of public debt supply also destroys its hedging properties. The latter leads banks to choose a lower equilibrium leverage, thus a lower endogenous supply of private debt. This trade-off is characterized below, in the context of the normative analysis.

Private maturity choice: endogenous supply of long-term debt The last element of the environment to endogeneize is B : the supply of long-term debt. This is carried out by considering the maturity choice at t of the young generation. The following lemma shows that when facing their maturity choice, risk-neutrals agents (banks) choose more short term debt than long term debt. The inefficiency ('too much' short term) is only shown in Section 4.

Proposition 5. *Banks face a meaningful maturity choice: both short-term debt and long-term debt are issued.*

The basic intuition goes as follows. The banks of G_{t+2} will issue the two types of securities at t , as there always is an endogenous price for the two. However, the two securities cater two different types of lenders. Short-term debt is sold within the generation, to cater to the risk-averse of this generation. Long-term debt is sold to the other generation active in trading, the middle-aged one G_{t+1} , as an outside-generation hedging asset. Young investors and middle-aged agents do not have the same one-period ahead risk-sharing needs, therefore the two contracts are not redundant.⁴² As a middle-aged bank, a negative beta asset is of particular interest, as the only out-of-the-generation security is short-term debt, which is entirely bought up by the young risk-averse investors (they outbid the middle-aged bank).

⁴²They do not have the same payoff profile: short-term debt has $\text{Min}(\bar{S}, X'S(s_{t+1}))$, whereas long-term debt has $q_{s_{t+2}}^{interim}(s^{t+1})$.

The maturity choice is driven in the current environment by the design of two different risk-sharing contracts and the catering to two distinct populations of lenders. It is a different mechanism from He and Xiong (2011) and Diamond and He (2013), in which the maturity choice is driven by the non-stationarity of the exogenous shock. In He and Xiong (2011) long-term debt then is always dominated by either short-term debt or cash hoarding. If optimists are very optimistic, they use short-term debt because leverage incentives overwhelm rollover risk. If optimists are not that optimistic, they prefer to hoard cash in order to wait for a degradation of the state. This behavior strongly hinges on a *mean-reversion* assumption, engineered through the beliefs structure. On the contrary, my environment features persistent shocks. In this case, the cash hoarding strategy is always dominated by leverage, and both short-term and long-term debt are issued.

4 Optimal supply of public debt and financial regulation

In this normative analysis, I explore if issuing public debt can lead to a Pareto improvement compared to the decentralized equilibrium without public debt. In order to endow public debt with a welfare role, the competitive equilibrium needs to be shown constrained inefficient.⁴³

4.1 Constrained inefficiency

Constrained efficient allocations As the environment features only two types of agents (risk-neutral A and risk-averse P), constrained efficient allocations trace a Pareto frontier in the space of the indirect utilities $\left(V^{G_{t+2}^A}, V^{G_{t+2}^P}\right)$. Under constrained efficiency, the planner directly chooses consumption allocations. In the spirit of Rawls and Ball and Mankiw (2007), I consider a Social Planner that treats all the future generations, as of period $t = 0$, under the veil of ignorance. I also assume it weights all generations uniformly. However, the within-generation heterogeneity is known to the planner, and let denote β^A the weight on risk-neutral agents and β^P the weight on risk-averse agents. Therefore the Social Planner chooses the state-contingent history of consumptions and history of aggregate investment to maximize the following welfare function under the resource constraints at each history s^t and the non-negativity of consumption and investment:⁴⁴

$$\begin{aligned} \text{Max}_{\{\{c^\theta(s_t)\}_{s^t}, \{i(s_t)\}_{s^t}\}} \quad & \sum_{t \geq 0} \left(\beta^A \mathbb{E}_0 \left[c_{t+2}^A(s_t, s_{t+1}, s_{t+2}) \right] + \beta^P \mathbb{E}_0 \left[-e^{-\gamma^P c_{t+2}^P(s_t, s_{t+1}, s_{t+2})} \right] \right) \\ \text{s.t.} \quad & (\lambda_{t+2}) \quad c_{t+2}^A + c_{t+2}^P + i_{t+2} \leq n^A + n^A + \frac{s_{t+2}}{p_K} i_{t+1} \\ & (\mu_{t+2}^\theta) \quad 0 \leq c_{t+2}^\theta \\ & (v_{t+2}^\theta) \quad 0 \leq i_{t+2} \end{aligned}$$

Appendix proves the following lemma.

Lemma 7. *Constrained efficient allocations are characterized by efficient risk-sharing within generation and a level of aggregate investment $i_t^* = a s_t$ where the optimal rule a is defined.*

⁴³i.e. whether the planner does better than the decentralized equilibrium, using the same instruments as the market.

⁴⁴Recall that agents only consume when old. In the private equilibrium, consumption depends on the three aggregate shocks endured by the agent over its lifespan: $c_{t+2}^\theta(s_t, s_{t+1}, s_{t+2})$.

The Pareto frontier is then traced by deriving the indirect utilities, as of date 0 of the two agents of the first generation G_2 . The ratio β^P / β^A controls the risk aversion of the Social Planner. As long as it is not zero the planner has some willingness to redistribute wealth across states.

Undeprovision of long-term securities in the decentralized equilibrium The stochastic OLG structure of the model brings in the classic violation of the First Welfare Theorem, caused by the infinite value of the aggregate endowment. What is novel in the present environment is a constrained inefficiency when agents face a maturity choice. Even when allowed to share risk with one period-ahead generation, they do not issue the same securities the planner would.

To see this, first observe that the planner is able to engineer Pareto improvement by manipulating the level of investment. This opens the avenue to increase *both* the expected returns and the level of within-generation risk-sharing, hence weakly enhancing the indirect utility of the two agents.

There is some long-term debt issuance in the decentralized equilibrium. It cannot be zero, as there must be the market 9 for this debt. But there is not enough of it. This is due to the fact the issuance of long-term debt exhibit strategic complementarities: an **issuance externality**. In terms of allocations, beyond the intragenerational risk-sharing analyzed in the static model, there is willingness to share risk inter-generationally: generation G_t is exposed to shocks s_{t-1} and s_t , generation G_t is exposed to shocks s_t and s_{t+1} and generation G_t is exposed to shocks s_{t+1} and s_{t+2} . So there is some willingness to smooth risk across these periods, beyond smoothing across states. The first market, the secondary market 7 for long-term debt, is used to share risk between G_{t+1} and G_{t+2} of their risk at the $t + 2$ horizon, whereas the two other markets, the primary market 9 for long-term debt and the market 8 for short-term debt issued by young, are used to share risk between G_{t+1} and G_{t+2} of their risk at $t + 1$ horizon.

The externality arises from the fact that G_{t+1} and G_{t+2} share too much $t + 1$ risk but not enough $t + 2$ risk. In the choice of maturity when young, i.e. does the young bankers issue $x_{s_{t+1}}^{G_{t+1} \text{ primary}}$ or $x_{s_t}^{G_{t+1}}$. When they do so they do not internalize the fact that their own decision on the primary market at $t - 1$ will impact the market clearing 7 on the secondary market of its own public debt (the same as its new issuance). The market clearing on the primary market 9 for long-term debt takes care of its Walrasian role, but does not internalize at the subsequent market clearing on the secondary market 7 for long-term debt. The pecuniary Walrasian role works well only to equate the *concomitant* quantity choices: $y_{s_{t+1}}^{G_{t+1}} + x_{s_{t+1}}^{G_{t+2}} + y_{s_{t+1}}^{G_{t+2}}$ and $x_{s_{t+1}}^{G_{t+1}}$, but not in a retroactive way.

All 3 markets clear, so there will be a bit of all debt, but *not enough* long-term debt, due to the non-internalization of the safety multiplier mechanism $\frac{\partial (x_K^A + x_K^P)}{\partial x_{s_{t+2}}^{G_{t+2} \text{ primary}}} > 0$. This what the planner takes into account:

in the f.o.c. for $x_{s_{t+1}}^{G_{t+1} \text{ primary}}$ it does plug in the market clearing condition 7.

Proposition 6. *The private competitive equilibrium is constrained inefficient: the decentralized equilibrium underprovides long-term debt: $x_{s_{t+1}}^{G_{t+1} \text{ primary}} < \left[x_{s_{t+1}}^{G_{t+1} \text{ primary}} \right]^{planner}$.*

Proof. By inspection of the f.o.c.: the planner would like to have more investment: both $x_{s_{t-1}}^{G_{t+1}}$ and $x_{s_t}^{G_{t+1}}$ (i.e. at the two periods $t - 1$ and t). Especially at t , investment is crowded-in with low price $q_{s_{t+2}}^{G_{t+2}}$ by the resource

constraint $(x_K^A + x_K^P) c_K + x_{\bar{s}_t^{G_t}}^{G_t \text{ primary}} q_{\bar{s}_{t+2}^{G_t+2}} \leq n^P + n^A$. The price $q_{\bar{s}_{t+1}^{G_t+1}}$ on 7 is too high. On the other hand,

the planner takes into account this supply effect: $\frac{\partial \left(q_{\bar{s}_{t+1}^{G_t+1}} x_{\bar{s}_t^{G_t}}^{G_t \text{ primary}} \right)}{\partial x_{\bar{s}_t^{G_t}}^{G_t \text{ primary}}} < 0$. \square

The basic intuition is that the planner can engineer a Pareto improvement by *crowding-in* investment in the current period by borrowing from *two* periods ahead. Compared to the competitive equilibrium, he makes risk-neutrals happier by increasing levered returns *and* risk-agents happier by lowering the state of default. Banks prefer to issue short-term debt than long-term debt, and doing so they starve the economy from negative-beta assets. Banks do not internalize the hedging properties of long-term debt, which could have been used as an input by another agent to create more safety. In other words, the appealing risk characteristics of its own long-term liabilities are not internalized by the bank.

Discussion: the source of the issuance inefficiency The inefficiency rises not from an overinvestment in the risky asset (Lorenzoni (2008)), nor from an overinvestment in the safe asset (Hart and Zingales (2013)), but from a too short maturity structure of private claims. I coin this externality an *issuance externality*, which is a bit different from the terms-of-trade vs. collateral externalities in Davila (2011) topography of pecuniary externalities.⁴⁵ It is akin to the latter, except that the externality does not arise from a direct ‘price in the constraint’ kind of effect: my model therefore does not feature direct pecuniary externalities. Indeed, the only friction, limited liability $c \geq 0$, does not feature any ‘price in the constraint’. The effective endogenous collateral constraint is the result of the combination of limited liability and bank portfolio. The issuance externality I uncover here also comes from a pecuniary effect, but less direct. Consider the ex-ante resource constraint of the economy:

$$q_{\bar{s}_{t+1}^{G_t+1}} (s^t) x_{\bar{s}_{t+1}^{G_t+1}}^{G_{t+1} \text{ primary}} + c_K x_K (s^t) \leq n^A + n^P$$

When choosing to supply the economy with $x_{\bar{s}_{t+1}^{G_{t+1}}}^{G_{t+1} \text{ primary}}$ legacy long-term debt, the primary issuers of generation G_{t+1} do not take internalize the crowding-in role that their own long-term liabilities play through the safety multiplier mechanism. Increasing $x_{\bar{s}_{t+1}^{G_{t+1}}}^{G_{t+1} \text{ primary}}$ decreases the value $q_{\bar{s}_{t+1}^{G_{t+1}}} (s^t) x_{\bar{s}_{t+1}^{G_{t+1}}}^{G_{t+1} \text{ primary}}$ by the **Laffer** effect analyzed in the static model. As can be seen on the resource constraint, issuing more long-term securities $x_{\bar{s}_{t+1}^{G_{t+1}}}^{G_{t+1} \text{ primary}}$ would crowd-in real investment $x_K (s^t)$, leading to a Pareto improvement, but private agents do not take this aggregate channel into account in their private maturity choice.

The government then becomes the natural provider of long-term securities. This is *not* just driven by the superior taxation power of the government. It is driven by the fact that, in a competitive equilibrium, long-term securities are used as input in the production of short-term securities. This leads to the pecking order: government issues long-term, private sector issues short-term. Public debt improves welfare because its supply impacts the creation of private safe assets. The government is able to manipulate bank leverage through its public debt issuance policy. Constrained inefficiency means that welfare improvement can be

⁴⁵Eduardo initially coined his terms-of-trade externality ‘risk-sharing’, but the mechanism he has in mind is non-equality of MRS, which can hold even in a deterministic environment. On the contrary, my issuance externality entirely hinges on the stochastic environment and equilibrium default, so it can be seen as a risk-sharing externality.

achieved by endowing the social planner with the *exact* same issuance capability than the private agents: the available maturities are the same for public debt as for private debt⁴⁶.

Market incompleteness is the feature of the economy that breaks the [Scheinkman \(1980\)](#) generic efficiency result. Indeed, when there is no public debt, the risk-neutral banks underprovide insurance to the risk-averse investors. This is due to a lack of well-diversified collateral on their balance sheet. Having endogenous leverage arising from limited commitment breaks his Modigliani-Miller-like neutrality theorem.⁴⁷

This *intragenerational* inefficiency has nothing to do with the intergenerational inefficiency due to overaccumulation of capital (dynamic inefficiency) of [Diamond \(1965\)](#) or [Gale \(1990\)](#).⁴⁸ The intergenerational inefficiency arises from intergenerational limited participation: all generations cannot trade with each other. Whereas in my setup, generations do trade with each other on the secondary market for public debt. In my model of limited commitment, public debt enhances intragenerational risk-sharing.

4.2 Implementation: optimal issuance of public debt

As the positive part of the dynamic model just showed, by playing around with the supply B of public debt, the government is able, even in the decentralized economy, to manipulate the price $q_{s_{t+2}}^{G_{t+2}}$, hence the leverage i.e. the risk-sharing decision between the two agents. Public debt enables the planner to move aggregate wealth *across states*, and not only across periods as stressed out by the OLG literature. Recall the resource constraint:

$$p_K (i_t^A + i_t^P) + q_{B,t} B \leq n^A + n^P$$

By issuing more long-term public debt B , through the Laffer effect it decreases the value $q_{B,t} B$, hence crowding in aggregate investment. The dynamic model tells us that $\mathbb{E} \left[\left\{ r^{safe}(s_t) - r^K(s_t) \right\} \right] < 0$ and $Cov \left[r^K(s_t), \left\{ r^{safe}(s_t) - r^K(s_t) \right\} \right] < 0$. So issuing public debt can improve the risk profile, but there is a cost which is to crowd out investment. The cost is parametrized by $q_{s_{t+2}}^{G_{t+2}}$ whereas the gains are parametrized by $Cov \left[r^K(s_t), \left\{ r^{safe}(s_t) - r^K(s_t) \right\} \right] < 0$. There will be a Pareto improvement as long as $B q_{s_{t+2}}^{G_{t+2}}$ decreases and still $Cov \left[r^K(s_t), \left\{ r^{safe}(s_t) - r^K(s_t) \right\} \right] < 0$.

I now derive the optimal financial policy (B, τ^A, τ^P) of the government, and show how it can implement a welfare improvement. The welfare criterion I use here is the indirect utility of the risk-averse investors (the 'grandmas').⁴⁹ Putting all the Pareto weight on risk-averse agents enables to focus on the safety creation role of banks. The government maximizes with respect to its financial policy their indirect utility under the competitive equilibrium.⁵⁰

⁴⁶Allowing the government to issue state-contingent debt would obviously lead to even greater welfare improvements.

⁴⁷Modigliani-Miller-like neutrality results all comes from a redundancy in the linear space spanned by the assets.

⁴⁸A key difference with [Gale \(1990\)](#) is that his budget constraints at the time of trade already involve the returns on the securities. In effect, he rules out limited commitment and default. On the contrary, I make clear that, at the trading period, the budget sets are bounded by net worth, whereas the securities promises arrive only at the ex post period. Compared to [Fischer \(1983\)](#) and [Peled \(1985\)](#), I do not have risk on the endowments, but on the assets.

⁴⁹I do not take into account distributional concerns and mutes any active role of the tax scheme. [Bhandari et al. \(2013\)](#) solves for the optimal policy trading-off redistribution and tax distortions. Their Ramsey problem exhibits a Ricardian irrelevance of the level of public debt. My stance for public debt is even stronger, as I show that the optimal policy features high levels of public debt. This said, I share with them the emphasis on *who* holds the public debt. The safety multiplier makes it more valuable in risk-neutral hands.

⁵⁰[Lorenzoni and Werning \(2013\)](#), in the tradition of [Calvo \(1988\)](#), claims that the government policy choice is the ex ante value of debt

$$\text{Max}_{\{B\}} \bar{W}_t^P \left(x_K^P(s^t), \left\{ x_{B,h}^P(s^t) \right\}_h, \left\{ y_{s_{t+h}}^P(s^t) \right\} \right) \text{ s.t. equilibrium}$$

The government takes as a constraint its own budget constraint (flow of funds). In a stationary recursive equilibrium we have the identity: $\hat{B}_{t-1}^h = \hat{B}_t^{h-1}$. Appendix B.7 derives the indirect utility and investigates its comparative statics in the supply of long-term public debt S^K . The following expression signs the welfare criterion with respect to an increase in public debt supply, taking the beta as given:

$$\bar{W}_t^P \propto \underset{(+)}{r_t^{safe} n^P} + \left\{ \underset{(+)}{r_t^{bank}} - \underset{(+)}{r_t^{safe}} \right\} \underset{(+)}{D_t} + \underset{(-)}{\frac{1}{2} \mu_{u,t}} - \underset{(+)}{\frac{1}{2} \sqrt{\frac{2}{\pi}} \sigma_{u,t}} \quad (19)$$

The effect of increasing public debt supply is broken down in different channels. The beneficial effect on the first term $r_t^{safe} n^L$ is a purely net worth effect. It is a **wealth channel**. This benefit is entirely muted by the necessary tax adjustments it implies. Satisfying the government flow of funds with a lower price \hat{q}^h of public debt forces to increase taxes. I shut down any redistributive role of taxes. This is tantamount to *not allowing* for any ex-ante transfers between agents. I choose $\tau^A = 0$ by consistency with the welfare criterion. I shut down any role of fiscal policy: monetary dominance, the fiscal policy is here only to balance government flow of funds. In this case, the tax burden exactly undoes the wealth effect of the increase in public debt supply.⁵¹ This is tantamount to the Woodford neutrality critique of the portfolio rebalance theory.

Nevertheless, public debt still is able to improve welfare through its *indirect* effect on the creation of private safe assets. This effect is captured by the second term of 19: a **safety multiplier channel**. In the language of Weitzman (1974), public debt supply has a price effect on the SMI spread and a quantity effect on bank leverage. The latter is beneficial due to the safety multiplier result (Proposition 1). The former price effect comes from the result that SMI decreases with public debt supply (Proposition 4), and that investor welfare 19 features the opposite of SMI (i.e. bank credit spread). The last two terms are adjustment terms that capture the cost of bank default from the investor perspective. They penalize the private debt, which traces back to public debt through the safety multiplier. This **default channel** is not innocuous, and is reminiscent of the traditional fire-sale externality which leads to cost of overborrowing. But here, they are only the third channel happening in conjunction with the wealth channel and the safety multiplier channel. The latter completely overturns the case of overborrowing by the private economy. That public debt leads to a Pareto improvement shows that the private equilibrium features a marginal inefficiency, in the sense that it does not rely on Pareto ranked multiple equilibria. The Pareto improvement can be thought as: public debt does crowd in real investment (Proposition 2). This makes *both* banks and investors happier, the first for an expected return reason, the second for a safety supply reason. This is how *ex-post* Pareto improvements can be achieved, a much more demanding task than not ex-ante ones (from the sum of utilities, but this then leaves room to arguing about the Pareto weights).

($\hat{q}^h B$) and not the ex post value (B). However, in practice, auctions ran by treasuries always announce a *face value* of government debt to raise, which is tantamount to choosing the ex post value B .

⁵¹This Ramsey problem of public debt issuance implicitly takes into account the *cost* for the sovereign of having a lower price for the public debt: from this perspective, having low public debt price \hat{q}^h is harmful. Shutting down the redistributive effects of taxation mutes Krueger and Perri (2010) mechanism of using taxation (public risk sharing) to overcome imperfect private risk sharing. The latter finds it hard to obtain crowding-in of private insurance by public insurance, whereas my model does feature such crowding-in for a large parameter region.

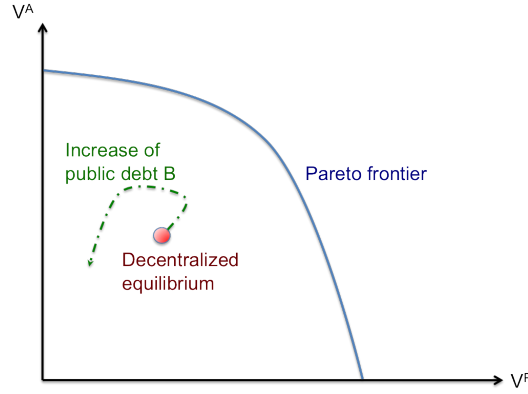


Figure 11: Pareto improvement by optimal public debt issuance.

Finally, there is a countervailing negative effect of public debt arising in the endogeneity of public debt beta. This force goes on the exact *opposite* direction than the static effect just described: increasing public debt supply increases its beta, hence it decreases bank leverage D , and also increases SMI, hence decreases bank credit spread $\{r_t^{bank} - r_t^{safe}\}$. In economic terms, flooding the economy with public debt hurts its hedging property. The final proposition of the paper qualifies the exact dependence of welfare with respect to the supply of long-term public debt.

Proposition 7. *Optimal level of long-term public debt*

In the dynamic model, there exists an interior optimal level of public debt $B^{optimal}$.

Proposition 7 endows public debt with a powerful role in regulating leverage. As pointed out by Davila (2011), when no transfers are allowed, capital regulation does *not* lead to a Pareto improvement (the traditional discussion misses accounting for agents heterogeneity). In contrast, public debt issuance is shown here to lead to a Pareto improvement, by manipulating leverage.

However, the dynamic harmful effect of issuance ends up by overwhelming this static beneficial effect. Intuitively, the static effect is bounded upper. The safety multiplier mechanism is concave: the resource constraint 5 imposes that private leverage cannot grow as fast as public debt supply. Hence the static beneficial effects of public debt will start to fade away. On the other hand, the dynamic negative effects of issuance do not fade away, as the beta of public debt increases monotonically with its supply.

The beneficial role of public debt works through a *market mechanism*, in which banks freely decide how much to issue of private safety (bank debt). This is different from Holmstrom and Tirole (1998) in which the government has the taxation power of circumventing the exogenous collateral constraints of private agents (not a constrained inefficiency). The decomposition 19 also helps to distinguish the beneficial role of public debt from what Gale (1990) has in mind for intergenerational risk-sharing. In my model public debt leads to welfare improvement not only when real rates are negative (safe price above 1). Finally, the Pareto improvement relies on the investment of private agents. The government is able to manipulate the level of investment merely by issuing long-term debt. Therefore, it can be argued that the Pareto improvement engineered this way requires less public intervention than the social security system designed

in Ball and Mankiw (2007), in which the government engages himself in investment, or even than their lighter implementation, which involves safe debt holdings by the government and time-varying social benefits.⁵² The present argument in favor of public long-term debt is novel. Indeed, the strategy of Angeletos (2002) is inoperational here given the continuum of states. It is also an interesting counterpart to the Aguiar and Amador (2013) argument for short-term debt for incentive reasons. Taken together, the two results makes clear that long-term debt is beneficial for *hedging purposes*, but at the expense of government repayment incentives. Arellano and Ramanarayanan (2012b) also features a hedging-incentive trade-off in the issuance of long-term debt. However, their hedging motive is partial equilibrium in nature as they do not consider a closed economy. On the contrary, my hedging motive is desirable even in GE as it crowds-in investment.

Compared to Greenwood et al. (2010) gap-filling theory (which stresses the crowding out), my theory advocates for the issuance of *long-dated* public securities. This normative recommendation on public debt maturity choice is in line with the empirical supply of Eurozone public debt. The short-end of public debt is quantitatively much smaller than the long end (1trn€ vs. 7.5trn€). If agents were long-lived, they would reach for short-term public debt to avoid interim volatility. They would still underprovide long-term negative beta securities.

As Corollary 3 informs us that bank expected utility also increases with the supply of public debt (due to an increase in equilibrium expected returns), the public debt financial policy of Proposition 5 does lead to a Pareto improvement. Everyone takes advantage of a higher supply of public debt in the economy. The model does point towards a *positive externality* of long-term securities, which private agents do not internalize. Issuing long-term public debt is a constrained efficient way to achieve Pareto improvements.

This dynamic model with endogenous leverage leads to policy recommendations in line with Caballero and Farhi (2013), but more nuanced. The present model helps to qualify their prescriptions. First, Proposition 5 can be interpreted as Quantitative Easing being good only up to a certain extent.⁵³ That SMI also reveals the welfare impact of public debt enables the Treasury to conduct experiments of debt issuance and by tracking the response on SMI, can calibrate the effective level of optimal public debt. Second, Operation Twist (i.e. removing long-duration assets from the economy to prop down long-term real rates) has negative welfare effects through the safety multiplier by starving the economy from long-term public debt. This is contrary to Stein (2012), where there is an exogenous collateral constraint which gives room to a pecuniary externality due to exogenous credit constraints and this implies *too much* private safe asset creation. In the safety multiplier mechanism, the private equilibrium does *not* produce enough private safe assets.⁵⁴

Finally, public debt is *not* as any other asset. It is *not* collateralized, so it can be seen as the *ultimate collateral*, the very starting point of any collateral chain. This is why the government should not take over the banking sector, and to the private safety creation itself: for the model to be efficient, there needs to be two distinct sectors (government and banks), for the liabilities of the former to be used as a hedge by the latter. As such,

⁵²They conclude that “negative indexation and government ownership of capital seem to be the only mechanisms that allow current capital risk to be shared optimally with future generations.” Long-term public debt issuance should be thought as an effective third avenue to enhance risk allocation.

⁵³Long-term government bonds can also help on the monetary side (see Sheedy (2013)). So the two objectives, debt to GDP stabilization and safety creation are not orthogonal.

⁵⁴This result is also different from Barro (1974) and Angeletos (2002), in which the benefits of public debt are traded off against the costs of distortionary taxation. My perspective is pure asset pricing, in which the value of public debt relies in its hedging property. It also complements Cochrane’s view that Treasury should go long to hedge its interest rate risk: insurance property of long maturities. My model takes the perspective of the private agents, and argues they need long-term Treasuries to produce safety.

even if the economy is exposed as a whole to only one univariate shock, having the government sector and banking sector enables the synthetic second asset, public debt, to be used as a hedge by the banks in their safety production.

4.3 Financial regulation

4.3.1 The cost of narrow banking

The safety multiplier beneficial effect uncovered in the previous section critically needs risk-neutral banks to be able to issue unsecured debt. The interesting comparative statics here is with respect to the *size* of the banking sector.

In the model it is captured by net worth of banks n^A . In the limit of no banks: $n^A = 0$ the safety multiplier is entirely muted, and risk-averse investors invest all their wealth in the aggregate portfolio. It is straightforward to see that the welfare of the risk-averse agents increases with the size of the risk-neutral sector. Even without accounting for the welfare of banks, the economy benefits from the presence of banks for their private safety creation role.⁵⁵ The welfare improvement coming from the presence of risk-neutral banks is equal to:

$$\Delta W^P = \left(r^{bank} - r^{safe}\right) D + \gamma^P \frac{1}{2} \sigma_B^2 \frac{r^{safe}}{\mu_B} D - (0.4\sigma_u - 0.5\mu_u)$$

Corollary 4. *Universal banks are beneficial to risk-averse investors: $\bar{W}_i^P \left(\frac{n^A}{(+)}\right)$.*

This corollary isolates the beneficial role of private safety creation of banks. It echoes [Gennaioli et al. \(2013a\)](#) finance as preservation of wealth. With asset pricing and endogenous leverage, my model describes *how* the financial system is able to create private money by transforming risk. A natural implication of Corollary 4 is that Glass-Steagall act type regulations have a cost. By breaking up the banks and splitting their positive beta from their negative beta part of the balance sheet, these regulations prevent the bank to create private safe asset by taking advantage of the hedging property of public safe assets⁵⁶. Thus, according to this theoretical treatment, the universal banking model is welfare improving⁵⁷. As a result, for safety creation, narrow banking and bank structural reforms⁵⁸ can harm social welfare in the exact same way as the Glass-Steagall act. It prevents banks from issuing safe bank debt. Empirically, [Berger et al. \(2013\)](#) show that capital injections and regulatory interventions have a costly persistent negative effect on liquidity creation, in line with the theory developed here.

⁵⁵My model offers a more benign view of leverage than recent macroprudential academic and policy. The ex ante macroprudential regulation literature insists on the negative pecuniary externality of leverage (fire sales), so concludes to the necessity to curb leverage. This is an artefact of having an exogenous collateral constraint. This argument misses that bank leverage does fulfill an economic role of safe asset supply. This is made clear with the present environment, in which the only friction is limited liability.

⁵⁶This bright side of universal banking must be traded off against the moral hazard cost of too big to fail.

⁵⁷A reminiscent intuition is present in the Jacques de Larosiere report on financial stability. An easy complement normative investigation should quantify the cost of market incompleteness. If markets were complete, banks would never default and therefore their debt would be riskless and risk-averse investors would fully invest in it. This is the first-best. It is the other limiting case, opposite to the world without banks $n^B = 0$.

⁵⁸Volcker and Vickers proposals, as well as a number of bank reforms such as the French bank law voted in July 2013, aim at separating market-based activities from bank deposits. However different banking structural reforms have different levels of restrictions on market activities. If the ring-fencing is too constraining, it can prevent universal banks from playing their economic role of diversification and supplier of private safe assets.

A final argument to consider is why the government would not take over the whole banking system and private public and former private safety by doing the tranching itself. The answer is negative and the argument relies in the endogenous hedging properties of government debt. From the government perspective, if it merges its own balance sheet with the balance sheet of the financial sector, it faces only one univariate shock: the macroeconomic shock. The government is therefore unable to effectively hedge this shock, as it does not have access to any other asset. Whereas as long as the financial sector balance sheet is different from the government balance sheet, government liabilities become the **support** of the next period flight to safety. By combining this endogenous claim with the risky technology, an autonomous financial sector is able to create synthetic safety.

4.3.2 The prudential treatment of banks sovereign exposures

There has been recent voices warning against excess holdings of public debt by banks and calling for an evolution of the prudential regulation to better capture sovereign risk. In the context of the model, a risk-weight on public debt would increase the marginal cost of holding public debt, and would therefore gradually decrease equilibrium bank holdings of public debt. A large exposure limit on sovereign debt would constrain these holdings in a more rigid manner, and could lead to a suboptimal allocation if the decentralized implementation of the optimal allocation requires banks to hold a higher amount of public debt than the limit. Here again, the benefits of public debt holdings need to be traded off against the moral hazard issues arising from the nexus between banking risk and sovereign risk.

5 Extension of the model in an open economy

The goal of this extension is two-fold. First, it investigates the impact of sovereign risk on the safety multiplier mechanism elicited in the main model. That is, I investigate the *supply side* of public safe assets, after having analyzed in detail its demand side in the previous section. Second, it opens the avenue to analyze the open economy environment counterpart of the closed economy model, in which two countries only differ by their fiscal capacity (i.e. sovereign risk).

The key insight of this section is that sovereign risk hurts the safety multiplier by introducing a force towards higher endogenous beta ρ . The following qualifies this intuition. The dynamic model with endogenous beta is therefore critically required to address this issue.

5.1 Introducing sovereign risk

I model sovereign default through a notion of fiscal capacity, and not through strategic default: default is *suffered*, not strategic. This is a realistic assumption, having the Eurozone debt crisis in mind⁵⁹. As a result, there is now in the economy private *and* public equilibrium defaults.

⁵⁹The shutdown drama of fall 2013 in the US is another illustration that sovereign defaults are driven by *non-strategic* factors in developed countries. The fact that it made public debt long-term yields go up and short-term public debt go down reinforces the safety pecking order exhibited in the previous section: sovereign risk only hurts long-term public debt, not the short end, due to the overall scarcity of safe assets.

Precisely, I extend the dynamic infinite horizon model in the following way. I model fiscal capacity as default threshold \bar{s} on the macro TFP shock below which the country defaults. At each period $t > 0$, there is sovereign default if and only if:

$$\text{public default at period } t \text{ i.f. } s^t < \bar{s}$$

In the states of the world in which of public default, the consol becomes worthless. To compensate the holders of public debt, the troika manages to secure to them an amount that is linear to the macro TFP shock: κs^t . κ is a measure of troika efficiency. The payoff of the public debt is, and illustrated on Figure 12:

$$\tilde{p}_B^t = \kappa s^t 1_{\{s^t < \bar{s}\}} + p_B^t 1_{\{s^t \geq \bar{s}\}}$$

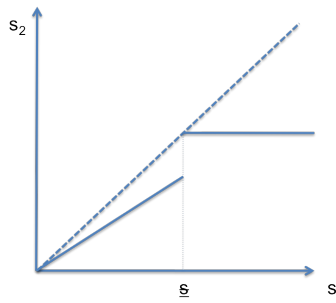


Figure 12: Public debt payoff with respect to macro shock s^t .

Until now, fiscal capacity was taken as fixed and very high. In the general case, Appendix B.8. derives that in this extension, the endogenous beta of public debt is equal in a second-order approximation, not to Equation 18 anymore, but to:

$$\tilde{\rho} = -1 + \frac{\left(\frac{B}{n^A} \frac{[\hat{\sigma}_B^2 p_K^2 + \sigma_K^2 \hat{p}_B^2]^2}{2\sigma_K^4 p_K^2 \hat{p}_B^2} \right)^2 + \left(1 + \kappa \frac{1}{\tilde{p}_B} \left(s^t - \frac{B}{n^A} \frac{[\hat{\sigma}_B^2 p_K^2 + \sigma_K^2 \hat{p}_B^2]^2}{2\sigma_K^4 p_K^2 \hat{p}_B^2} \right) \right) \Phi \left(\frac{\bar{s} - s^t}{\sigma_K} \right)}{1 + \kappa \frac{1}{\tilde{p}_B} \frac{B}{n^A} \frac{[\hat{\sigma}_B^2 p_K^2 + \sigma_K^2 \hat{p}_B^2]^2}{2\sigma_K^4 p_K^2 \hat{p}_B^2} \Phi \left(\frac{\bar{s} - s^t}{\sigma_K} \right)}$$

The increasing dependence of $\tilde{\rho}$ to sovereign risk \bar{s} and the comparative statics of the main model with respect to ρ yield the following lemma.

Lemma 8. *Fragility of the safety multiplier to sovereign risk*

In the closed economy, leverage (private safety D) and real economy lending (x_K) decrease with sovereign risk \bar{s} .

The intuition is that sovereign risk not only increases the volatility of public debt (and lowers its expected return), but more interestingly increases the endogenous correlation of public debt with the macro TFP shock. As a result, it destroys the hedging properties of public debt, and doing so it hinders the safety multiplier mechanism. Sovereign risk also alters the Safety Mismatch Index.

An interesting aspect of introducing sovereign risk in an endogenous leverage model is that it goes *against* the expropriation channel of contagion from sovereign risk to corporate risk. Bai and Wei show that the

expropriation contagion channel holds for the general corporate sector. My model shows that, as far as the *banking sector* is concerned, the contagion can go the other way: higher sovereign risk can lead banks to delever so much that they become *less* risky.

5.2 Environment

There are two countries: North and South. I introduce one unique source of heterogeneity between the two: their fiscal capacity. North has a larger fiscal capacity: $\bar{s}^{South} > \bar{s}^{North}$. The risky technology is the same for both countries. As a result, there are three assets: the risky technology, North public debt and South public debt. Due to sovereign risk we have in equilibrium: $\sigma_B^{South} > \sigma_B^{North}$, $\rho^{South} > \rho^{North}$, $p_B^{South} < p_B^{North}$. The agents are homogeneous in both countries: same endowments and same preferences. The only degree of specificity is the limited participation assumptions: risk-averse investors have access only to their local bank debt and to their local bond market⁶⁰. This local investor base will endogenously drive heterogeneous behavior between North bank and South bank in terms of leverage and portfolio choice.

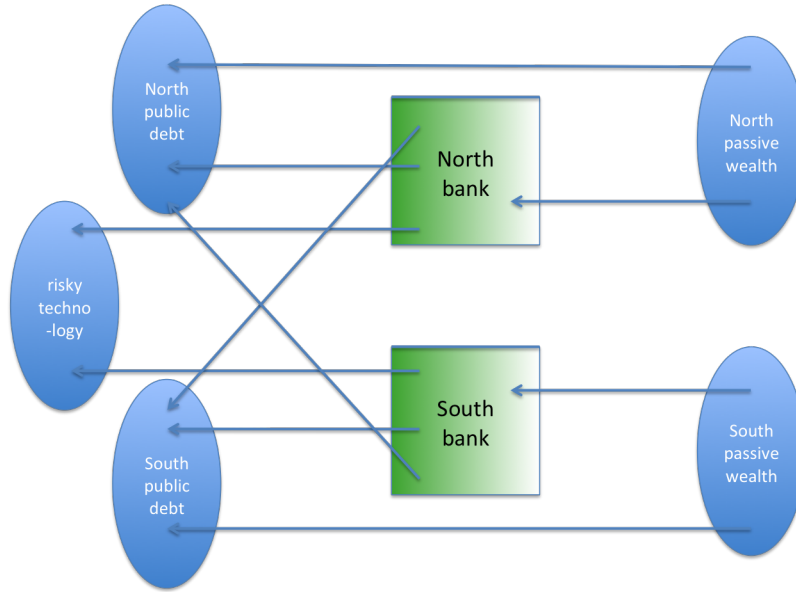


Figure 13: Open economy environment.

5.3 Equilibrium

Consider a South bank. Label Asset 2 the South bond and Asset 3 the North bond: we have $\rho_3 < \rho_2$. The balance sheet correlation metric $X(\rho_2, \rho_3)$ is now:

⁶⁰This is empirically relevant despite a slight quiet run on Greek banks from Greek investors. The assumption of special access to the local base can be relaxed. What matters is that *banks* have better access to the risky technology and the other bond market. It is empirically verified that cross-border holdings of sovereign debt are mainly owned by foreign *banks*, not final investors.

$$X_{OE} = \frac{(x_2\sigma_2 + \rho_2x_1\sigma_1)}{(x_1\sigma_1 + \rho_2x_2\sigma_2 + \rho_3x_3\sigma_3)}$$

This balance sheet correlation measure is generically higher than in the closed-no sovereign economy. As a result the carry trade on South sovereign bond:

$$r^{bond} - r^{bank} = (r_K - r^{bond}) \frac{\mu_K}{c_K} \left(\frac{1}{1 - \frac{c_K}{p_B} \frac{\sigma_B}{\sigma_K} X_{OE}} - 1 \right)$$

can now turn positive for the South bank, as long as ρ_2 high enough such that the carry trade increases with X_{OE} (therefore with ρ). This positive carry trade for *South banks* is at the source of the following proposition.

Proposition 8. *Redomestication of public debt.*

In the open economy, South banks hold more South bonds: $x_B^{South} > x_B^{North}$. The heterogeneity increases with sovereign risk differential.

Proof. See Appendix B.8. □

We can interpret this proposition as sovereign risk strengthening the risk shifting channel and weakening the safety multiplier channel (debt pricing channel). What is really interesting is that sovereign risk strengthens the risk shifting motive and weakens the endogenous leverage mechanism: i.e. higher sovereign risk, due to the outside option of investor local base makes bank debt cheaper so that bank decides to lever up *more*, and they do so using the risky public debt (which is cheaper than the North public debt): a version of risk shifting. The economic intuition can be grasped with a perturbation argument. Start with a symmetric equilibrium, with same level of sovereign risk and increase by ϵ the sovereign risk of the South country. South bond has worse hedging properties, banks should fly to North bond. But: South bond has higher volatility, disliked by the local investors. Thus South investors have higher need of bank debt. Thus South banks have access to cheap credit; they lever up. As South bond is marginally cheap compared to North bond, *they use the South bond* to hedge its additional risky investment.

As a result, my model captures redomestication of South sovereign debt in Eurozone documented by [Acharya and Steffen \(2013\)](#) (they do not explain why *South banks* are better positioned to do this greatest carry trade). Comparing my model to [Uhlig \(2013\)](#): his is a moral hazard model of the South free-riding on bailout expectations from the North/ECB. Mine is a pure Asset Pricing perspective with endogenous leverage, in which due to GE South banks are the natural marginal holders of South public debt. I do not need any financial repression type of argument. The domestication is an efficient asset pricing outcome. The fragmentation of government bond markets is aligned with fragmentation of bank debt markets. Global banks are also portfolio *choosers* between domestic debt and foreign debt. So foreign banks have a key role in the safety multiplier applied to domestic debt. There is an eviction effect due to foreign debt in the safety creation. I thus microfound a home bias in government bond holdings: domestic debt is actually a better hedge of domestic equity than foreign debt.⁶¹ Redomestication is due to the apparition of sovereign risk heterogeneity which made Eurozone switch from the closed economy model to the open economy model. My model of private safety creation rationalizes why banks hold public debt on their balance sheet, and as

⁶¹Maybe also perhaps because of a relational contract between domestic banks and domestic sovereign, the outside option being to invest in foreign debt and accepting foreign investors as debtholders (this would be in a model of strategic default).

a result it microfounds the diabolic loop: the increased sensitivity of bank default risk to sovereign risk.⁶² In normal times, banks help the sovereigns to refinance cheaply (high public debt price). However, due to the contagion of default risks from public to private (a contagion of commitments), this financing broke up.

5.4 Normative implications in open economy

A narrative of the Eurozone crisis as a closed economy can treat sovereign risk as a neglected risk on which investors did load and which, when it did come to mind, destroyed the safety multiplier mechanism and triggered deleverage and credit crunch. Figure 15 illustrates how the beta of South public debt did turn sharply positive during the Eurozone crisis. This sheds light on a dark side of the safety multiplier: private safety creation incentivizes investors to neglect sovereign risk.

Make public debt cheap or expensive? The last step is to keep in mind that, through bailout expectations, there exists a feedback loop from banks to sovereigns: i.e. an increasing causality from bank leverage D to sovereign risk \bar{s} . In a recent FT tribune (October 1, 2013), Jens Weidmann (Bundesbank) argues that the banks holdings of public debt are dangerous should be 'taxed' through increased capital requirements or a large counterparty exposure regulation. His normative thinking is all about this feedback loop and its crowding out effect on real investment. On the contrary, I argue that there are benefits of these relatively safer holdings by banks, through the crowding-in mechanism of this paper. As an implication, Weidmann argues for *expensive* public debt (low yields), whereas in my world of safe asset shortage, I argue for a *cheap* public debt.

Also, as described in the positive model, the safety multiplier mechanism increases the interconnection between banks and sovereigns (diabolic loop). The full-fledged normative analysis should also keep in mind that there is another cost to public debt issuance due to the safety multiplier: exposure to the sovereign neglected risk. The normative analysis in the Open Economy model raises a flag at the Outright Monetary Transactions (OMT)⁶³: they have the undesirable effect of propping up p_B^{South} , which in consequence hurts private leverage.

Eurobonds In the open economy environment, Eurobonds is a beneficial policy, as this increases the supply of low sovereign risk public debt disproportionately more than it increases the sovereign risk of the junior part of the South bond. Contrary to Hellwig-Philippon who advocate for Eurobills and not Eurobonds, the present environment argues for issuance of long-term maturities. See Davila and Weymuller (2013) for a security design approach to the optimal amount of Eurobond issuance. EFSF and ESM are doing exactly the same from an economic standpoint: wash away the idiosyncratic sovereign risk in order to enhance the supply of public safe assets B . Furthermore, splitting the Eurozone has negative consequence on the welfare of risk-averse investors as long that this break up implies a hindered access of banks to foreign public debt.

⁶²It is from sovereign risk to bank risk (Greek crisis), so my model is alternative to Gennaioli-Martin-Rossi. It does not feature the reverse causality, from bank risk to sovereign risk (Ireland crisis) through bailout expectations (Acharya-Schnabl, new Bruner). I stick to a pure asset pricing model without resorting to any moral hazard friction and shows how it can rationalize all the stylized facts of the Eurozone crisis.

⁶³<http://www.ecb.europa.eu/press/key/date/2013/html/sp130902.en.html>

6 Empirical analysis

I investigate in this section how far my simple theory of private safety creation brings us to rationalize the main monetary aggregates in Europe and in the US. The essential insight of the model is that private debt quantities are driven by the risk characteristics (i.e. the beta) of public debt. A calibrated version of the model captures the aggregate patterns of private money in Europe.

The theory has rich empirical predictions regarding the balance sheet of the financial sector. I focus on how safe asset holdings and leverage comove with the supply of public safe assets in the economy, and argue these comovements are consistent with the safety multiplier mechanism and not with alternative theories of financial repression, bailout expectations and term premium trades.

6.1 Calibration of the safety multiplier on the Eurozone

6.1.1 Measure of the supply of public safe assets S^b

The criterion is to consider all the assets available to investors, and select only the ones that are negative beta: $\rho < 0$. The candidates for assets with such property are public debt and gold. Within public debt, I rigorously apply the $\rho < 0$ criterion: as soon as the public debt beta turns positive, it is excluded from the S^b measure. Conceptually, a positive beta asset joins back the risky technology status: it is close to being just one more risk asset as any other. I measure the supply of safe assets by accounting for all the Eurozone public debt that is negative beta with the DJ EUROSTOXX 50 (proxy for the common risky technology).

The daily betas of 10-year government bonds of all 17 Eurozone countries with the European stock index (DJ EUROSTOXX 50) are computed on a 30-day rolling window⁶⁴:

$$\beta_k = \frac{\text{cov}(R_t^{bk}, R_t^e)}{\sigma(R_t^{bk})\sigma(R_t^e)}$$

Betas are plotted on Figures 14 and 15. These stylized facts are newer than just the yield disconnect widely documented (Figure 22). The betas figures that North betas became even more negative over the crisis, whereas South betas turned sharply positive, in accordance with the open economy version of the model.

In the spirit of the model, I then construct a composite measure of public safe asset supply by summing all the public debt that is negative beta:

$$S^b = \sum_{\text{Eurozone country } k} 1_{\{\beta_k < 0\}} S^k$$

where S^k is the nominal stock of public debt of country k . A smoother version of the measure penalizes the positive beta public debts without completely excluding them: $S^b = \sum_{\text{Eurozone country } k} S^k \left(1 - e^{-\text{Max}(0, \beta_k)}\right)$.

6.1.2 Construction of a balance sheet of the European financial sector D, x_1, x_2

Similarly on the demand side of assets, I split the assets side of the financial sector balance sheet in two categories: the positive beta asset (the risky technology) and the negative beta assets (the safe assets holdings).

⁶⁴I rely on daily quoted yields for the 17 MU countries. The daily returns of stock and bonds are computed as $r_t = \ln(\frac{P_t}{P_{t-1}})$ where P_t is the price of the stock index or the price implied by the yield on 10 year government bond. Source: Global Financial Database.

Data come from the Monetary ECB reports, which enable to construct an aggregate balance sheet for the financial sector of the Eurozone. The universe I am considering are all Monetary and Financial Institutions of the Euro area (17 countries), excluding the Central Banks of the Eurosystem. In the case of Europe, as the central bank does not implement its monetary policy through open market operations⁶⁵ but through reverse repo operations, ECB balance sheet works as the negative of a bank balance sheet. I therefore subtract its balance sheet from the European financial sector balance sheet.

The typology of assets is carried out with respect to their beta: loans, shares and other equity are positive beta, whereas securities and remaining assets (such as gold) are negative beta. Fixed assets are zero beta.

Assets side All loans except to gvt are risky assets. For securities, I only have the breakdown between securities to gvt and to other euro area residents. I treat both categories as safe assets, with keeping in mind it is an approximation for securities to non-gvt (but even for the latter category, beta would be overall negative). Also for gvt securities, there is a split between $\beta < 0$ and $\beta > 0$ between I split it between Loans and securities and shares to MFIs are contracts internal to the financial sector, therefore I do not take them into account to avoid double counting. External assets/liabilities are net assets and are therefore included on the asset side, as a risky asset.

In the end, the risky asset holdings measure includes:

$$x_1 = \text{Loans to euro area residents (excluding MFI and gvt)} + \text{shares and other equity} \\ + \text{securities to gvt whose } \beta > 0 + \text{net external assets}$$

Whereas, the safe assets holdings measure includes:

$$x_2 = \text{Loans to gvt} + \text{securities to gvt whose } \beta < 0 \\ - \text{deposits of gvt in MFIs} + \text{securities to euro area residents (excluding MFI and gvt)}$$

As a robustness test, I also use data from stress tests ran by the European Banking Authority, which annually disclosed sovereign debt holdings of the 91 largest European banks. This enables to compute an alternative time-series for x_2 .

Liabilities side On the liabilities side, all deposits count as private money (M1) and is therefore included in the private money measure D . From deposits I only exclude MFIs deposits (which is double counting) and central government deposits (which are negative position of the financial sector in central government liabilities). MMF shares are net liabilities and are therefore included in D . Remaining assets/liabilities are slightly net liabilities, so are also counted as D . Finally, debt securities⁶⁶ up to 2 years are counted in D , whereas debt securities issued over 2 years are counted as bank net worth (sticky liabilities). Debt securities include Commercial Paper (CP). Capital and reserves are naturally counted also as bank net worth n^B (bank equity). Compared to KVJ, I do *not* distinguish between backed (M1) and unbacked (M3-M1) bank debt, as I argue that given the recourse feature of most secured funding (e.g. repo), the two types of securities are

⁶⁵In this case, as for the FED, its balance sheet should be subtracted from the *public* safe asset supply.

⁶⁶There should not be double-counting of debt securities held by other MFIs.

more substitutes than what is thought. I treat fixed assets as negative bank net worth (it is a side show in my model).

$$D = \text{Deposits of euro area residents (excluding MFI and gov)} + \text{net MMF shares} \\ + \text{debt securities and CP up to 2 years} + \text{net remaining liabilities}$$

Whereas bank net worth ('risk-neutral wealth') is:

$$n^B = \text{Capital and reserves} + \text{debt securities and CP over 2 years} - \text{fixed assets}$$

Figures of these quantities are in Appendix A.2. Data are in million euros and collection is end of period. All quantities will be scaled by Eurozone GDP that quarter (which was € 2.2trn in 2011q3).

6.1.3 Calibration results

I argue in this section that my model of private safety creation squares with the European aggregates just calculated. All quantities in trillions of euros⁶⁷. A complication arises from the threatment of the corporate sector balance sheet. The asset side of the corporate balance sheet is a juxtaposition of cash holdings and actual risky $\beta > 0$ investment. I net out the former by carving out the cash part, and merge it with risk-averse net worth.

Table 1 gives the exogenous parameters estimated from available sources, and the equilibrium variables delivered by the calibrated model. The latter are confronted with observable data. The success of the calibrated version of the model to square endogenous portfolios provides support for the risk-aversion heterogeneity view of banking developed in this paper.

6.2 Time-series tests

In this subsection I provide suggestive evidence of Proposition 2,3,4: when the supply of safe assets shrinks, bank leverage shrinks, real lending shrinks and bank holdings of safe assets increases. As a motivating evidence that banks act as insurers to risk-averse investors, the mark-to-market value ($p_1x_1 + p_2x_2$) of the overall EU bank balance sheet is remarkably stable.

6.2.1 Proposition 2: positive comovement of public and private debt

I argue here that the European banking crisis (deleveraging) has been caused by a *shortage of public safe debt*, following the loss of safe asset status of Southern Europe government bonds. This tests for the safety multiplier comparative statics of Proposition 1: $\frac{\partial D}{\partial S^b} > 0$, with public safe assets S^b and private debt D computed as just described. This test aims at answering the following question: when the supply of public safe assets in Europe shrank, did privately safe asset creation stepped in, as an equilibrium outcome, in order to satiate the demand for safety, as hinted by Gorton and Metrick (2012)? Or in the contrary, did it even more crunch private safety, according to my safety multiplier mechanism?

⁶⁷I thank Gabriel Zucman for wealth estimations in the Eurozone. Estimations of MFI vs. households wealth are computed based on <http://sdw.ecb.europa.eu/reports.do?node=100000161>.

I provide suggestive evidence of the quantity comovement between public safety and private safety in Europe, hence putting forward the safe asset shortage as a key cause of the Eurozone current recession. The empirical tests of the model revolve around the impact of public debt beta on private leverage. Not only more public debt, but also ‘better’ hedging qualities of public debt under the form of lower beta, enables the financial sector to sustain high leverage. I use sovereign risk as an instrument to identify this negative beta channel of public debt on private debt.

It replicates what happened in Europe over the last decade: a decrease in the supply of safe public debt. It triggers a non-conventional deleveraging of banks and a credit crunch due to the safety multiplier. There are three regimes in the time period: increase in safe asset supply (2001-2007), even more so with Keynesian stimuli (2008-2010). Then for 2010-2013, there is a split: the total amount of Euro public debt increases, but the amount of it which is safe (no-GIPSI) actually decreased. On a longer time horizon, my model could also shed light the double leverage cycle: public debt and private debt (both domestic and external) documented in Reinhart and Rogoff 2008 and Jorda et al. 2013. One is used as an input to the production of the other, this is why they comove positively.

Beyond the suggestive evidence of positive comovement from 20, the model calls the following specification:

$$D_t = \delta + \gamma_1 S_t^b + \gamma_2 \beta_t + \epsilon_t$$

and the new empirical predictions are: $\gamma_1 > 0$ and $\gamma_2 < 0$. All quantities variables are scaled by GDP. Results of this specification are given in Table 2 and illustrated in Figure 20. I always include a trend regressor to absorb the economy growth effect.

Sovereign risk as an instrument to the causality of beta on private leverage Naturally, the above specification is plagued with reverse causality and omitted variable issues, especially on γ_2 ⁶⁸. To mitigate these concerns, I instrument public debt beta with sovereign CDS. This empirical strategy is inspired by the new result of the model: sovereign risk should impact private leverage only through public debt beta⁶⁹.

Therefore the IV regression I run is, on a first stage, public debt beta on sovereign CDS (as a measure of sovereign risk):

$$\beta_t = \delta + \hat{\gamma}_1 sovCDS_t + \epsilon_t$$

And then use the resulting $\hat{\beta}_t = \hat{\gamma}_1 sovCDS_t$ in the second stage:

$$D_t = \delta + \gamma_2 \hat{\beta}_t + \epsilon_t$$

The model predicts $\hat{\gamma}_1 > 0$ and $\gamma_2 < 0$. Sovereign CDS is obtained from Markit. These results help to distinguish my theory from moral hazard theories of the sovereign-bank diabolic loop, who have no predictions on bank leverage. The IV strategy takes advantage of the exogenous variations in the supply of safe assets in Europe⁷⁰ in order to identify the impact of public debt beta on bank leverage. It could also be

⁶⁸For γ_1 , causality of S^b on D is more reasonable, as long as stay in a Greek style crisis and not an Irish style crisis.

⁶⁹This is a test of the fragility of the safety multiplier to sovereign CDS. I abstract from the reverse feedback loop of implicit guarantees (the Irish style sovereign crisis) to focus on Greek style sovereign crisis: sovereign risk is causal to bank risk.

⁷⁰cf. <http://www.bbc.co.uk/news/business-15748696>. Global pool of triple A status shrinks 60% (FT March 26).

used, in an IO approach, to trace down the demand curve for safe assets. Here a key issue is the sovereign risk heterogeneity within Europe. This calls to refine the IV specification, taking into account the open economy. It is tantamount to a test of the open economy model.

6.2.2 Propositions 3 and 4: bank portfolio composition

Credit crunch Proposition 3 says that lending to the real economy decreases when the supply of safe assets S^b shrinks and its hedging properties get worse (higher β). I therefore compute x_1 from ECB data as the sum of all Monetary and Financial Institutions loans to the real sector (and HH – should be fixed) as described above, and then run the following specification:

$$x_{1,t} = \delta + \gamma_1 S_t^b + \gamma_2 \beta_t + \epsilon$$

where the model predicts: $\gamma_1 > 0$ and $\gamma_2 < 0$. Results are given in Table 2.

Banks safe asset holdings The last quantity test concerns Corollary 3: banks safe asset holdings increase when safe asset supply shrinks. I compute a measure of safe asset holdings x_2 by banks as the sum of bank holdings of public debt. I also do the same for risk-averse investors (money market funds) to compute y_2 . I then run the following specification⁷¹:

$$x_{2,t} = \delta + \gamma_1 S_t^b + \gamma_2 \beta_t + \gamma_3 \beta_t * S_t^b + \epsilon$$

where the model predicts: $\gamma_1 < 0$ and $\gamma_2 > 0$. The coefficient $\gamma_2 > 0$ on beta indicates that having better hedging properties (low β) make banks to need less of public debt in order to ensure the safety of their debt, hence leveraging up. This is a defining test of my theory against other diabolic loop theories (financial repression, bailout expectations). *asset pricing* properties of public debt drive these holdings, not *preferences* from investors. Uhlig (2013) and Acharya-Drechsler-Schnabl (2013)'s Irish style bank to sovereign crisis do not have prediction relating asset pricing properties of public debt to asset holdings. Similarly, I reject the financial repression hypothesis for France and Germany⁷².

I also test the additional predictions of the model on banks asset holdings. When public debt beta is low enough, banks are able to ensure the safety of their private debt even with a small amount of safe assets on their balance sheet. Therefore the banks safe assets holdings negatively comove with the hedging properties of public debt. I test this prediction, along with the safe asset driven credit crunch prediction: real lending positively comoves with the hedging properties of public debt.

The model therefore correctly predicts that in crises times, banks become the natural holders of government debt. The empirical analysis identifies this reconcentration from risk-averse agents (insurance companies, pension funds) to risk-neutral agents (banks) in Europe.

A confounding explanation could be that safe asset holdings are just liquidity hoarding by banks, driven by a precautionary motive. But this precautionary motive driven by regulation does not explain safe asset holdings that are well above the liquidity regulation (Liquidity Coverage Ratio). It could also be merely driven by regulation (capital requirements through the Risk Weight or the recent High Quality Liquid

⁷¹Another specification could focus on safe asset holdings by risk-averse investors (i.e. non banks): y_2 in the model. E.g. insurance companies are large holders of government debt for anti-transformation purposes.

⁷²Deep secondary markets exist for these government bonds. Moral hazard and bailout expectations by banks do not seem to hold.

Asset regulation). This is not the case, as safe asset holdings far exceed these requirements. Also, capital requirements might be more innocuous than what the banking industry claims, as they would anyway be willing to hold safe assets to hedge the positive beta part of their balance sheet.

6.3 Mechanism: cross-sectional tests

This section intends to prove that South banks did redomesticate due to moral suasion, whereas North banks play the diversification role. To do this, I run cross-sectional tests of banks easiness to refinance (bank CDS) on a measure of the quality of the diversification of their balance sheet: $\frac{\sigma_2 x_2}{\sigma_1 x_1}$. The punchline of the paper is how government debt is used as an input in safety creation.

For these cross-sectional tests, I use the EBA stress-tests data. These are extremely granular snapshots of the balance sheet of the 90 largest European banks. In these cross-sectional tests, in order to compute x_1 and x_2 from EBA data, I use *Exposure at Default* (EAD). For on-balance-sheet transactions, EAD is identical to the nominal amount of exposure. For off balance sheet, it is modeled by the bank itself. This is the relevant measure of $p_1 x_1$ and $p_2 x_2$.

I explore here how the holdings of sovereign debt impacts funding costs *in the cross-section* of banks. A bank like BNP holds 15% of its total EAD in its portfolio of sovereign debt. I want to show that the more diversified this one is, the better its refinancing cost will be (i.e. low CDS). For each bank i , I compute the holdings of safe public debt:

$$x_{2,i} = \sum_{\text{Eurozone country } k} 1_{\{\beta_k < 0\}} x_{k,i}$$

Figure 21 explores the explanatory power in the cross-section of bank CDS of either risky public debt holdings and safe public debt holdings. It illustrates that *safe asset holdings* have far more explanatory power in the cross-section than risky asset holdings. The regression table confirms this.

$$CDS_i = \delta + \gamma_1 x_{2,i} + \epsilon_i$$

It shows that it is really the *safe public debt holdings* (i.e. the object of interest of this paper), and not the *risky debt holdings* (i.e. Acharya-Steffen object of interest) that are driving the cross-section of bank CDS. This result is *not* inconsistent with the carry trade behavior of banks. Nevertheless, it does qualify the cross-section.

Finally, the safe asset holdings of banks are much more important than the cash pilings of the corporate sector (Apple is just an outlier regarding its cash reserves, which is not representative of the corporate sector as a whole).

6.4 Asset pricing tests

The model has sharp empirical predictions on the price of the public safe asset (government debt) and private safe asset (bank debt). This focus on the safe asset price is a departure from the literature, which mainly focus on the risky asset price, i.e. the equity premium (see Zhang (2013) for a recent example).

6.4.1 Public safety premium

Lemma 3 tells us that the price of the public safe asset should increase with its hedging properties on the risky asset, and the more so the more the safety mechanism D is at work. As a result I run the following specification on German government yields:

$$r_t^{safe} = \delta + \gamma_1 \beta_t + \gamma_2 \beta_t * D_t + \epsilon$$

where the model predicts: $\gamma_1 < 0$ and $\gamma_2 < 0$. Results of this specification are given in Table 4 using yields, and these yields are graphed in Figure 22.

As a byproduct, this specification enables to estimate the safety premium on Euro public debt. The interaction term $\gamma_2 \beta_t * D_t$ helps to capture the complementarity effect of public safe assets, through the safety multiplier mechanism. It also implies that the safety premium is even more important in countries where the safety multiplier is at work, such as in Europe.

I can also test how this safety premium depends on *who* owns the debt, banks or households, i.e. run the specification:

$$r_t^{safe} = \delta + \gamma_1 n^B + \gamma_2 n^L + \epsilon$$

where n^B is bank net worth and n^L is risk-averse net worth. Results of this specification are also given in Table 4.

6.4.2 Safety Mismatch Index and predictive regression

An insight of this paper is that is not one but two safety premia to consider: one on public debt and one on private debt. I focus here on the spread between the public safety premium and the private safety premium, which is equal to $r^{bank} - r^{safe}$ and which I call the Safety Mismatch Index (SMI).

I use for r^{safe} the weighted yield on all the public debt that is deemed as safe according in the definition employed in section 5.1.1. of construction of the stock of public safety. Here again, I emphasize that the focus of this study is *safe public debt* holdings, and not *risky public debt* holdings (which is Acharya-Steffen focus).

The empirical counterpart of r^{bank} is computed as the weighted average of the deposit rate (source: ECB) and the wholesale funding rate (computed as the deposit rate+CDS spread). Weights are notional amounts of deposits and wholesale fundings of the Eurozone banking balance sheet.

Proposition 4 derives an analytical expression for this SMI, which shows that SMI decreases with public debt beta β , and the more so the more the bank balance sheet is initially diversified: $\frac{\sigma_2 x_2}{\sigma_1 x_1}$ high. I consequently run the following specification:

$$SMI = r^{safe} - r^{bank} = \delta + \gamma_1 \beta_t + \gamma_2 \left(\frac{\sigma_2 x_2}{\sigma_1 x_1} \right)_t + \gamma_3 \beta_t * \left(\frac{\sigma_2 x_2}{\sigma_1 x_1} \right)_t + \epsilon$$

where the model predicts: $\gamma_1 > 0$, $\gamma_2 < 0$ and $\gamma_3 < 0$. Conclusive results of this specification are given in Table 5, and SMI is graphed in Figure 23.

$\gamma_1 > 0$ says that better hedging properties of the public safe asset (low β) induces banks to lever up more, hence a low SMI. $\gamma_2 > 0$ says that a improperly diversified bank balanced sheet ($\frac{\sigma_2 x_2}{\sigma_1 x_1}$ closer to 0 than 1) also implies a low SMI. $\gamma_3 > 0$ says that these two effects reinforce each other. The γ_2 effect can also be interpreted

as saying that the *concentrated ownership* of public debt is priced in SMI: low bank safe asset holdings implies improperly diversified bank balance sheets, hence low SMI.

Finally, I run the predictive regression:

$$D_t = \delta + \gamma_1 SMI_{t-1} + \epsilon$$

Hence it shows that SMI is a strong negative predictor of financial crises. Figure 3 suggested this property, and it is confirmed in Table 6.

6.4.3 Impact on bank profitability

Finally, low SMI hits negatively banks expected profitability. It is in the spirit of the Fed board paper about bank profitability to interest rate innovations. I therefore run as final specification, where the model predicts: $\gamma_1 > 0$:

$$\Pi_t^B = \delta + \gamma_1 SMI_t + \epsilon$$

Banks profitability is computed from Bankscope data, as

$$\Pi^B = \text{interest income} - \text{refinancing cost} - \text{operating expenses}$$

Refinancing costs are using data from Bankscope ('interest expense' entry) and banks' CDS (in bps) as a better measure of refinancing cost. I am matching *dividend gains* and not *capital gains*. I also explicitly take into account the operating costs of running a bank. The model is able to handle constant returns to banking, i.e. I do not need to rely on a convex cost of banking to pin down endogenous leverage. Therefore adding operating costs is innocuous in the model.

Figure 24 gives the SMI part of banks profits, i.e.:

$$\begin{aligned} \Pi^B|_{\text{safe assets}} = & \text{yield} * \text{securities holdings} \\ & - \text{refinancing cost} * \text{securities holdings} / \text{total assets} \\ & - \text{operating expenses} * \text{securities holdings} / \text{total assets} \end{aligned}$$

7 Discussion

7.1 Comparison with existing literature

7.1.1 Role of banks

The traditional banking literature sees the role of banks as mitigating agency frictions: Diamond 1984, Diamond and Rajan 2001, or more recently as mitigating the adverse selection due to information acquisition: Holmstrom and Ordonez 2013. This literature is not suited to explain the large holdings of liquid, publicly traded and 'safe' securities by banks.⁷³ Theories of banking on information asymmetry and banks superior

⁷³The large holdings of government securities also challenge the view that financial integration helped global banks economize on their liquid assets holdings (Castiglionesi et al. forthcoming).

ability to keep secrets do not apply to bank holdings of highly liquid and scrutinized assets such as government debt.

DeAngelo and Stulz 2013 and Philippon 2012 insist on the liquidity/safety creation role of banks. However they do not model for it, even though Philippon 2012 features production function in reduced-form for the supply of safe assets⁷⁴). These articles do not consider public debt as an already safe asset and its role in safety creation, and do not feature risk-aversion. More generally, macroeconomic models take the cost of financial intermediation (the spread between lending and borrowing rates) as exogenous⁷⁵. Landier et al. 2012 and Begenau et al. 2012 insist on the interest rate exposure as the key feature of banks business. However they do not have a model of banks optimizing behavior, the action still comes from the asset side and not the liability side of banks. They do not explain *why* banks have this interest rate exposure, and what the implications are for monetary policy. In my model interest rates are equilibrium outcomes, not causal variables. Bianchi and Bigio (2013) takes the role of banks as granted (loan supply) and do not model a safety mismatch. The literature on liquidity hoarding views cash as a commitment device. Most recently, Calomiris et al. (2013), following Calomiris and Kahn (1991), lays down a theory of cash holdings of banks as a discipline device against a moral hazard friction. The present model does not rely on an agency friction to explain 'safe asset' holdings by banks. Generally, the theoretical literature has found it puzzling to explain why banks would hold assets yielding lower returns than the deposit rate.

In the present model, the role of banks is to create safety by portfolio construction. They choose their asset holdings as inputs to safety creation. Therefore the key engine of banking is diversification. The role of banks is *pooling* and *tranching* in a General Equilibrium environment, with an enforcement friction: banks cannot commit to repay debt. Contrary to Gennaioli et al. 2013b, I analyze banks in general and not shadow banking, and I consider macro shocks and not only micro idiosyncratic shocks that are washed away by a law of large numbers. In my model, bundling a bearish asset with a bullish asset enhances the value of bank collateral. When the safe asset is government securities holdings, sovereign risk acts the neglected risk. It is also reminiscent of Gennaioli et al. (2013a): finance helps to preserve wealth. My model does microfound *how* banks are able to preserve wealth by issuing securities with quasi-flat payoff, which caters to risk-averse investors. This role of banks as *safety creators* is alternative to Kashyap et al. 2002 model of banks as *liquidity providers*. It has the distinctive prediction of positive comovement of public and private safety. It insists not on the *immediacy* of government debt holdings but on its *hedging* value. Similarly, it insists on safety transformation by banks, less on their liquidity transformation. Liquidity (deposits, repo, MMF) is a bit different as safety: liquidity is immediacy (promise of cash redemption), as in Diamond and Dybvig (1983). Similarly, Gale and Ozgur (2005) studies capital structure when banks insure against liquidity shocks, which is different from the role they have in my environment of insuring against macro shocks. I do not need to resort to preference shocks, which are hard to map to a primitive. On the contrary it has similarities with Gorton and Pennacchi 1990 whose purpose of banking is liquidity creation, but the latter is in an asymmetric information partial equilibrium environment. I share with Geanakoplos 1997 and Simsek 2013⁷⁶ the focus on General Equilibrium effects of heterogeneity and the analysis of the determinants of leverage introduce a second asset, which I call 'safe asset', in exogenous fixed supply. I also depart from beliefs heterogeneity, and I use instead risk-aversion heterogeneity to obtain endogenous leverage. Such environment is more tractable

⁷⁴He exogenously posits an intermediation requirement for government bonds of 1/10.

⁷⁵Such as in Curdia and Woodford (2009).

⁷⁶Cao (2013) is a dynamic version of the original Geanakoplos environment. However the equilibrium is solved numerically.

to handle multiple assets.

7.1.2 Sovereign debt

Traditional literature on sovereign debt focuses on issues such as sustainability, reputation and default (Hellwig-Lorenzoni (2012), Cole-Kehoe (2002), Krueger-Perri (2010) and Aguiar et al. (2013)). It mostly ignores the key role of banks portfolio choice in their pricing. I argue that banks are key players that decide of the price of domestic debt by pinning down the price of safety. Moreover, in an international environment, global banks adds an additional layer of endogeneity through their portfolio choice between domestic debt and foreign debt. On the other hand, the recent literature on global banks (Schnabl (2009), Ivashina et al. 2012, Bruno and Shin 2012) does not focus on the role of their sovereign debt holdings.

Recent literature linking sovereign debt and bank debt focuses on bailout expectations to microfound the interconnection between bank risk and sovereign risk. Sovereign debt is solely treated as a *store of value*: Acharya et al. (2011), Mengus (2012) and Gennaioli et al. (forthcoming). These papers feature a 3-period model in which real investment is prohibited in the first period and sovereign debt is used to transfer wealth from the first to the second period. On the contrary, I do not introduce bailout expectations but I simply treat public debt as a given asset class. I circumvent the ad hoc 3-period timeline of public debt models, as my agents' simultaneously hold government debt and risky projects. Furthermore, in Gennaioli et al. (forthcoming), government debt features procyclical returns, which is counterfactual with the negative beta of public debt. Bolton and Jeanne (2011) exogenously posits the collateral value of government debt, whereas I microfound from its hedging properties. Arellano and Ramanarayanan (2012a) emphasizes the role of public debt as an insurance against interest rate fluctuation *from the government perspective*. Broner et al. (2013) is a model of crowding out of real investment whose mechanism relies on the presence of secondary markets for sovereign debt. They assume that foreign investors do not have *investment* access to local equity market, whereas I restrict them from *funding* access in the local market. Reinhart and Sbrancia 2011 puts forward financial repression as the key reason of government debt holdings by banks. Angeletos et al. (2013) is a normative analysis of the liquidity role of public debt for debt issuance, but as in Bolton and Jeanne (2011) they take the collateral value of public debt as exogenous. Martin and Ventura (2012) construct a model that has a similar feature to mine, that a bubble emerges for its collateral properties. However, they do not relate its value to the value of the risky asset and its impact on leverage. Broner et al. (2010) insists on the role of secondary markets for public debt to discipline the sovereign, I insist on their role to provide safe assets.

On the empirical side, Acharya and Steffen (2013) documents that European banks invested heavily in sovereign bonds of countries of the Eurozone periphery and interpret this as moral hazard. They also document a redomestication of domestic debt ownership, and interpret it as an effect ECB funding collateral requirements. However, this last interpretation does not explain why foreign banks could not do the same type of carry trade. On the contrary, in my open economy environment, heterogeneity in holdings is driven by special access to local funding. Bofondi et al. (2013) documents that foreign banks substituted to the credit crunch of domestic banks in Italy. They do not analyze the portfolio holdings of securities of these global banks. Uhlig (2013) is a moral hazard model of the South free-riding on the expectation of a North bailout (through the ECB). Its model presents however the counterfactual feature that in equilibrium, North banks hold zero South banks, at odds with Acharya and Steffen (2013) evidence. In contrast, the pure asset pricing perspective developed here enables to rationalize cross-border holdings of sovereign debt.

7.1.3 Safe assets

Traditionally, the literature treats safe assets as stores of value. Woodford (1990), Holmstrom and Tirole (1998), Caballero and Krishnamurthy (2008), Kocherlakota (2007) and Kocherlakota (2009) are all environments in which the government improves welfare by relaxing through public debt exogenous collateral constraints imposed on the private sector. On the contrary, exogenous financial frictions are absent from my environment. The government improves welfare through the safety multiplier. The rational bubble literature insists on the scarcity of asset supply in general. On the contrary, I insist on the scarcity of *safe* assets. My environment endows public debt with an even more crucial liquidity role, through the safety multiplier. More recently, (Yared, 2013) introduces liquidity shocks in a Woodford (1990) and obtains substitutability between public debt and private debt. On the contrary, I obtain complementarity between the two.

In the macroeconomic literature, Caballero and Farhi 2013 emphasizes the shortage of safe assets as a key macroeconomic force that can lead to a safety trap. They develop a valuation framework to assess the policies remedies to this safety trap. In contrast, my model features optimizing banks with endogenous leverage. Bank leverage is jointly determined at equilibrium with the safe asset price. Compared to Gorton and Metrick (2012), I emphasize the negative beta property of safe assets, whereas in the latter safe assets are defined as keeping a constant value over time. My 'safety' definition, i.e. assts with negative beta, is similar to the one used by Maggiori (2013) for the analysis of the dollar currency in an international context. That the government should issue negative beta securities is an idea mentioned in Pagano (1988). On the crowding out of private debt by public debt, Krishnamurthy and Vissing-Jorgensen (2013), Gorton and Metrick (2012), Gorton and Ordenez 2013⁷⁷ and Gourinchas and Jeanne 2012 emphasize the substitutability between private and public safe assets. My crowding-in comes from endogenous leverage and public debt being used as input to private debt creation. Sunderam 2012 and Greenwood et al. 2010 microfound safety demand through money in the utility, whereas I stick to parsimonious risk aversion heterogeneity. Regarding normative implications, Stein 2012 argues there are too much safe debt due to pecuniary externality, Caballero and Farhi 2013 argues there are not enough safe assets. I argue there are not enough *public safe assets* due to a lack of negative beta assets in the economy.

7.1.4 Asset pricing

I relax the standard assumption in standard portfolio allocation theory of Campbell and Viceira (2002) that the risk free asset is in perfectly elastic supply. In my environment, the price of the safe asset results from its scarcity, and in turn it impacts private leverage. In the field of closed economy asset pricing, Campbell et al. (2013b), Campbell et al. 2013a and Backus and Wright 2007 explain changing betas of US government bonds by changes in inflation expectations (monetary policy stance). They develop Neo-Keynesian models to emphasize the nominal nature of government bonds.

In open economy asset pricing, Coeurdacier and Gourinchas 2013 and Bhamra et al. (2013) insist on the role of bonds to hedge real exchange rate risk. Models in this literature feature complete markets, hence Modigliani-Miller holds and leverage is not determinate.

⁷⁷This paper exogenously assumes that private collateral is more information-sensitive than public collateral.

7.2 Conclusion

In this paper I develop a banking model in which banks' economic role is to provide insurance to risk-averse investors against macroeconomics shocks. I analyze a model of safe assets creation that features risk aversion heterogeneity and incomplete markets. Public debt has an endogenous negative beta by anticipation of flight to safety in the recursive equilibrium. As public debt is used as input to private debt issuance, the model delivers a safety multiplier. The open economy extension introduces heterogeneity in sovereign risk. Sovereign risk weakens the hedging properties of public debt. The open economy model captures redomestication of sovereign debt by the interaction of risk-shifting motives with endogenous leverage. The empirical discussion points towards the shortage of safe public debt as the key driver of the European debt crisis. Europe is more fragile to the safety multiplier due to the limited participation of investors in the risky technology. The spread between public debt and private debt can be used as a financial stability indicator to reveal the extent of the safety multiplier.

The model has crucial normative implications. The private equilibrium is constrained inefficient because private agents do not internalize the beneficial effects of negative beta securities, which makes them issue too much short-term. Issuing public debt is welfare improving, because it is endogenously endowed with negative beta in the recursive equilibrium. This leads to a higher level of safe assets overall in the economy. Issuance of long-term public debt improves welfare as long as it does not flood the economy up to a point at which its hedging properties are weakened. Consistent with the monetary dominance view, public liabilities can be issued to manipulate the price of safe assets. Public debt is therefore an instrument of unconventional monetary policy, which operates through the regulation of safe assets production.

APPENDIX

A Empirical Appendix

A.1 Betas of Eurozone government bonds

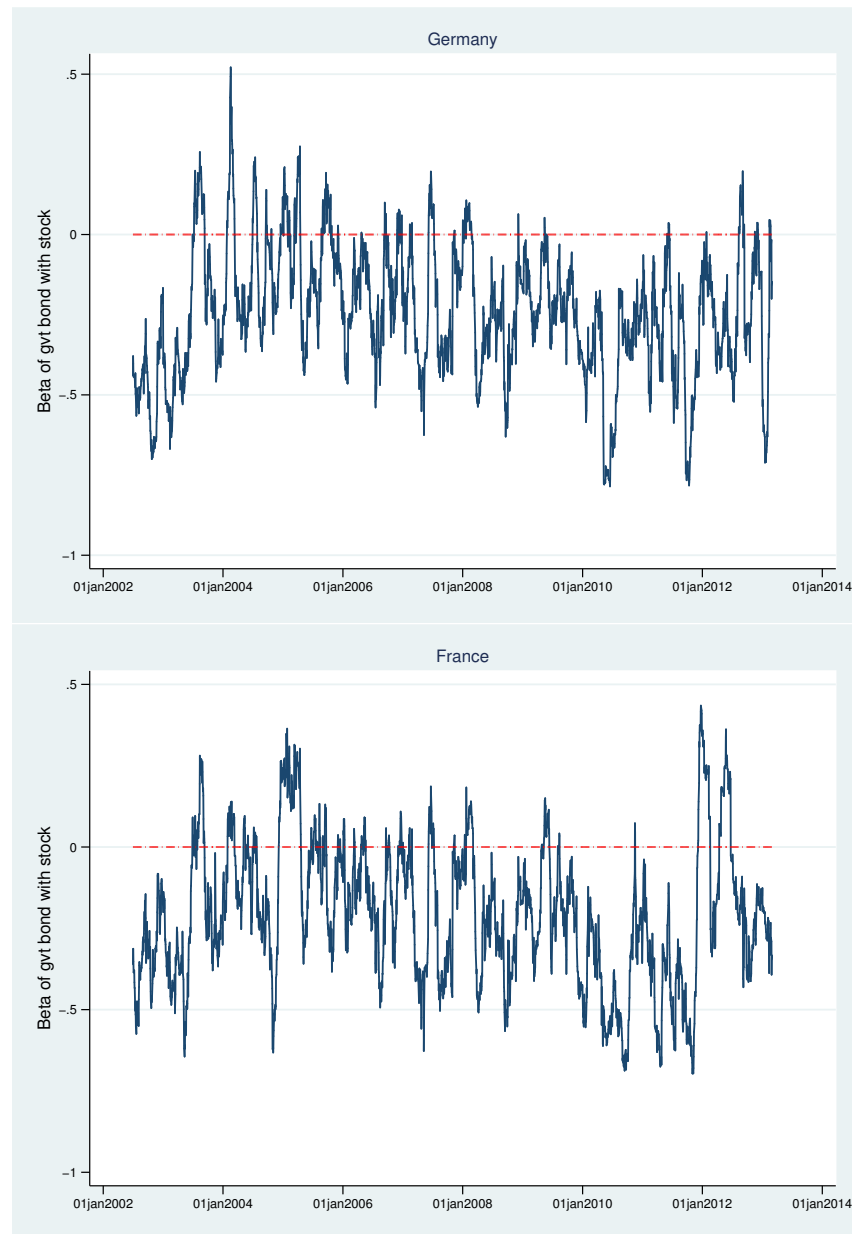


Figure 14: **Prices:** betas of government bonds with DJ EUROSTOXX 50 . Core countries: Germany (top) and France (bottom). Source: Financial Database.

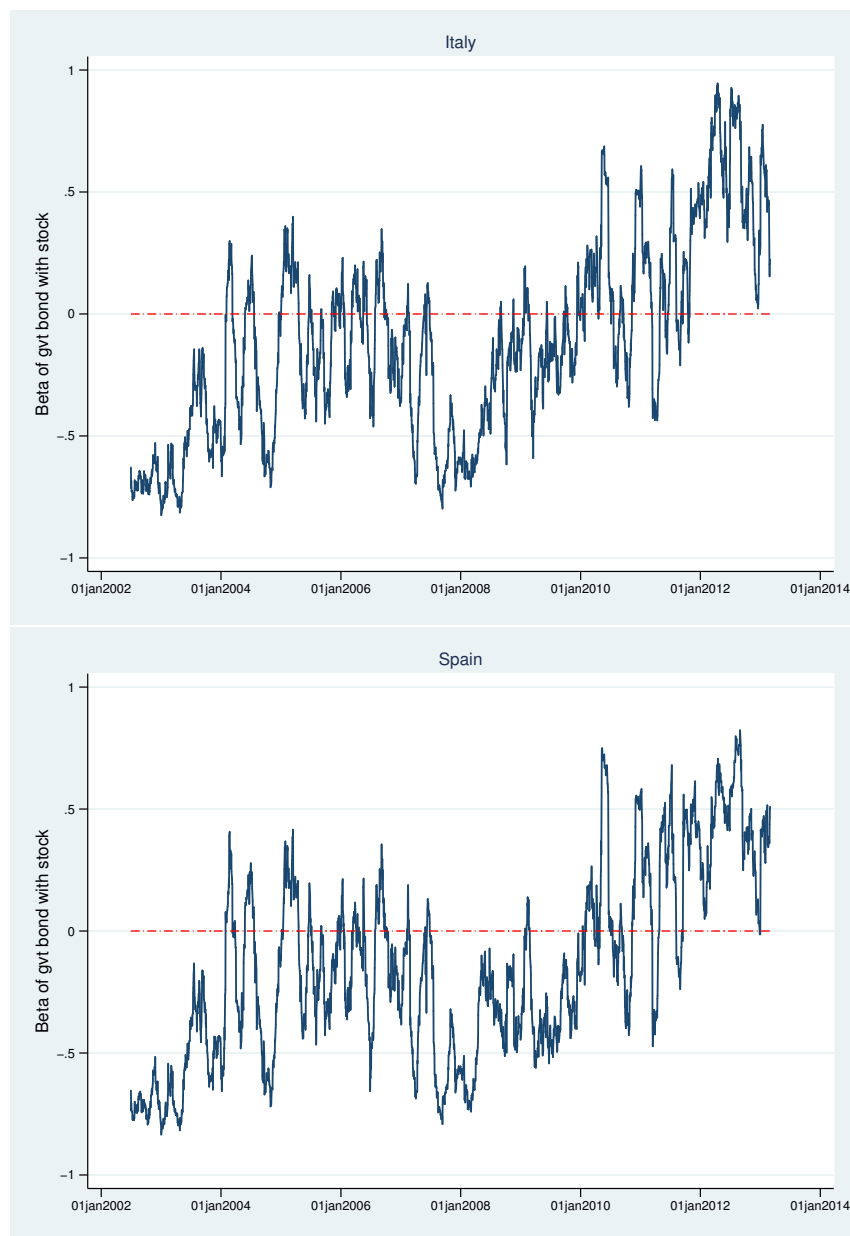


Figure 15: **Prices:** betas of government bonds with DJ EUROSTOXX 50 . Periphery countries: Italy (top) and Spain (bottom). Source: Financial Database.

A.2 Construction of the Eurozone banking balance sheet

A.2.1 Private debt: banking short-term liabilities

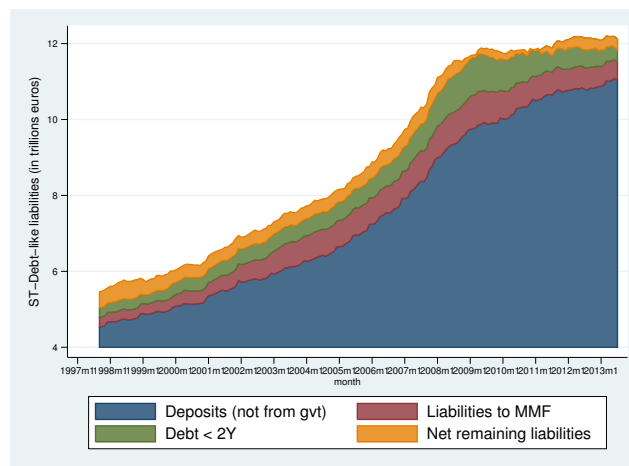


Figure 16: EU banks short-term debt.

A.2.2 Positive vs. Negative beta holdings

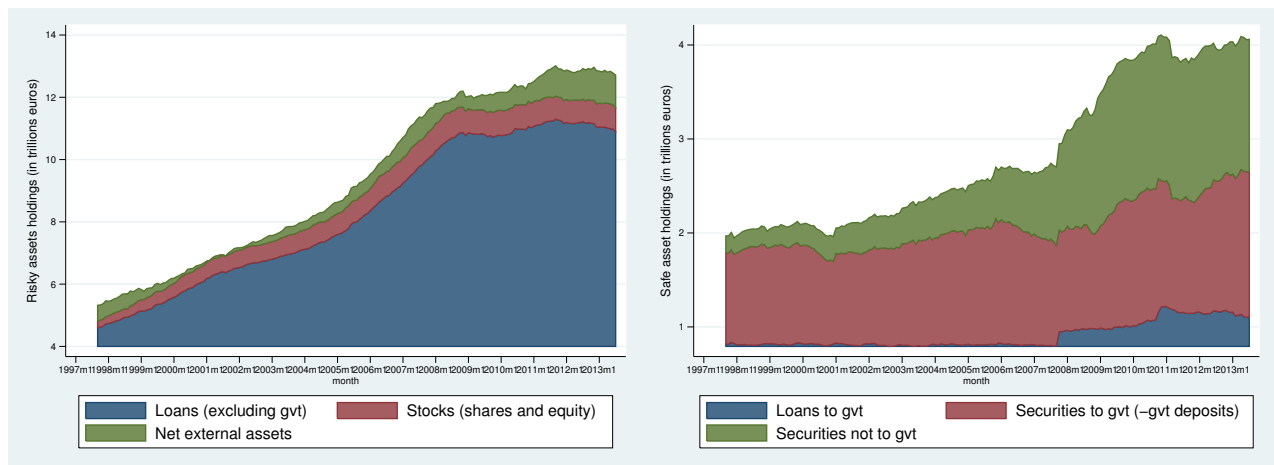


Figure 17: EU banks risky assets holdings ($\beta > 0$) on the left, and safe asset holdings ($\beta < 0$) on the right.

A.3 Concentration of ownership for Eurozone public debt

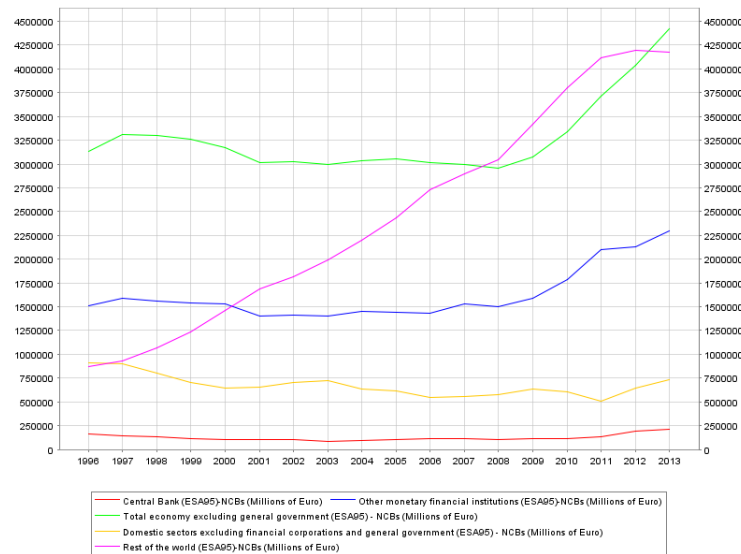


Figure 18: Breakdown of Eurozone public debt by type of holders.

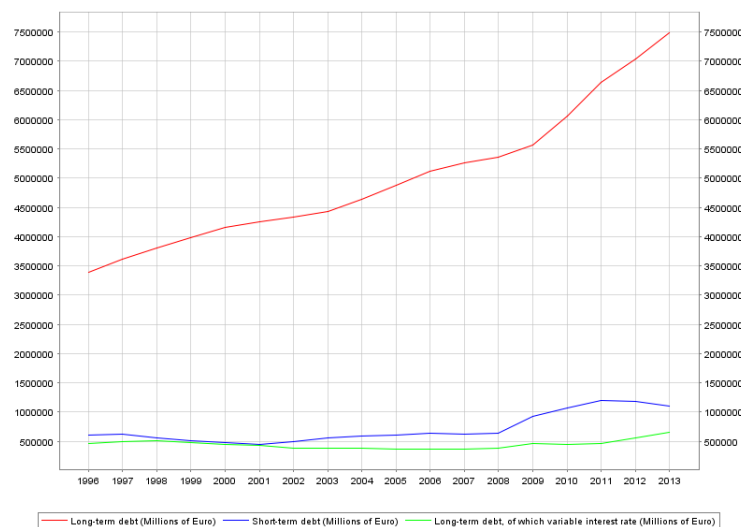


Figure 19: Breakdown of Eurozone public debt by maturity: long-term in red, short-term in blue.

A.4 Calibration

Parameters		
Bankers wealth	$n^B = 5$	
Investors wealth	$n^L = 15$	
Investors risk-aversion	$\gamma_L = 2$	
Risky technology volatility	$\sigma_1 = 0.4$	
Risky technology return	$r^1 = 0.12$	
Public debt beta	$\beta = -0.3$	
Public debt volatility	$\sigma_2 = 0.2$	
Public debt supply	$S^b = 3.5$	
Equilibrium variables		
Bank portfolio expectation	$\mu_u = 0.3$	
Bank portfolio volatility	$\sigma_u = 0.2$	
Observable quantity variables		
	Model	Data
Bank risky assets holdings	$x_1 = 13$	$x_1 = 15$
Bank safe assets holdings	$x_2 = 2$	$x_2 = 4$
Bank debt	$D = 13.5$	$D = 14$
Bank leverage	$D/n^B = 2.7$	$D/n^B = 2.8$
Investors direct risky assets holdings	$y_1 = 0$	$y_1 = 0.5$
Investors direct safe assets holdings	$y_2 = 1.5$	$y_2 = 2$
Observable price variables		
Public debt yield	$r^{safe} = 2\%$	$r^{safe} = 1.8\%$
Bank debt yield	$r^{bank} = 5\%$	$r^{bank} = 4\%$
Bank probability of default	$p^{def} = 6.7\%$	$p^{def} = 4.5\%$

All quantity variables are given in trillion €. Price variables for public debt are from German 10 year bond. Bank default probability is recovered from CDS quotes of the 90 largest Eurozone banks (data source: Markit).

Table 1: Calibration of the static model on the Eurozone economy.

A.5 Result: time-series tests

A.5.1 Safety multiplier D and credit crunch x_1

	(1)	(2)	(3)	(4)	(5)	(6)
	D	D	D	x_1	x_1	x_1
	b/se	b/se	b/se	b/se	b/se	b/se
SbGDP	1.749*** (0.16)			1.889*** (0.17)		
Germany beta		-0.404 (0.87)			-0.265 (0.94)	
Sb_betaGER			0.275 (0.35)			0.203 (0.38)
Constant	-0.464 (0.48)	4.763*** (0.25)	4.641*** (0.26)	-0.593 (0.51)	5.098*** (0.27)	4.981*** (0.28)
N	42	42	42	42	42	42
R-squared	0.76	0.01	0.01	0.76	0.00	0.01

* p<0.10, ** p<0.05, *** p<0.01

Table 2: Quantities test: regression of leverage D and risky holdings x_1 on the supply S^b , the beta β of public safe assets and the interaction between the two $S^b * \text{Max}(-\beta, 0)$. All quantities are scaled by EU GDP.

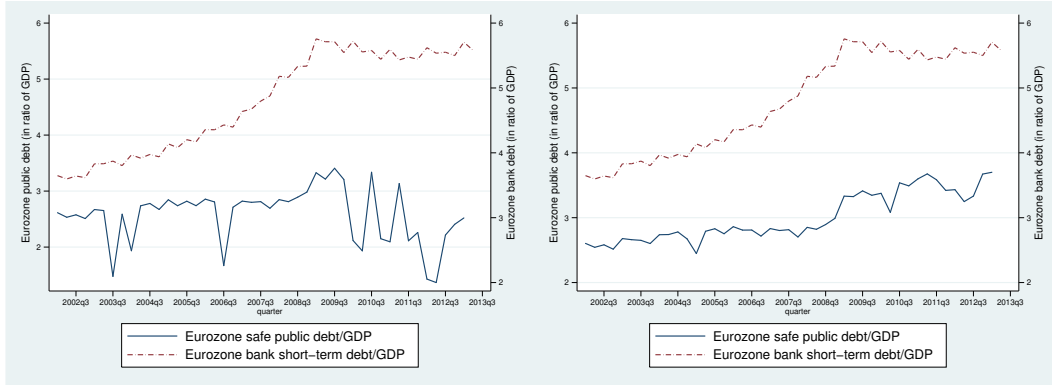


Figure 20: Public and Private safety positive comovement. The one on the left uses the rigorous measure of public safe asset supply: $S^b = \sum_{\text{Eurozone country } k} 1_{\{\beta_k < 0\}} S^k$. Whereas the one on the right uses a smoother version of the measure: $S^b = \sum_{\text{Eurozone country } k} S^k \left(1 - e^{-\text{Max}(0, \beta_k)}\right)$.

A.6 Mechanism: cross-sectional tests

A.6.1 Impact of safe asset holdings on refinancing costs

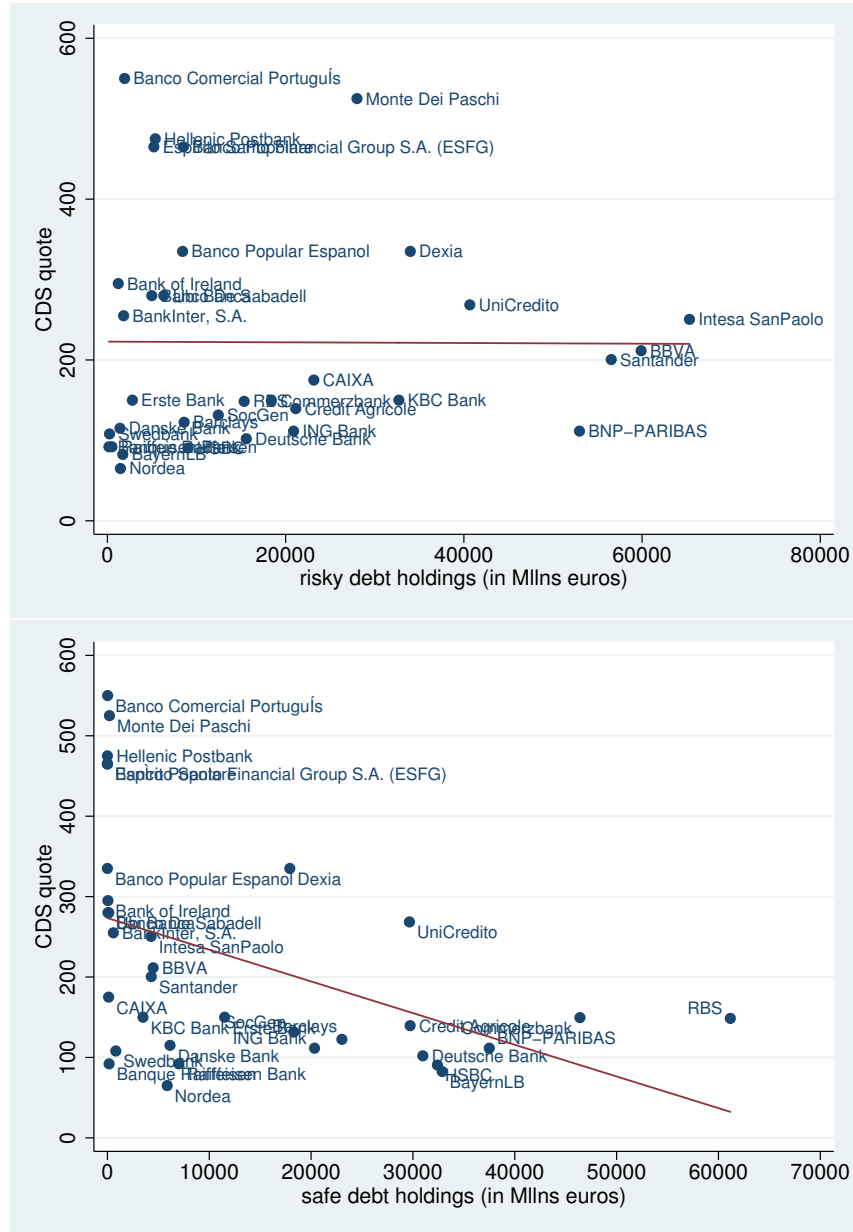


Figure 21: CDS quotes vs. bank holdings of risky (top) and safe (bottom) public debt: $x_{2,i} = \sum_{\text{Eurozone country } k} 1_{\{\beta_k < 0\}} x_{k,i}$. Public debt is assigned in the risky and safe categories according to their beta at the stress test date, 2011q3. Risky public debt ($\beta > 0$) are: Belgium, Greece, Ireland, Italy, Portugal, Spain. Safe public debt ($\beta < 0$) are: Austria, Cyprus, Finland, France, Germany, Luxembourg, Malta, Netherlands, Slovakia, Slovenia.

	(1)	(2)	(3)	(4)
	CDS	CDS	CDS	CDS
	b/se	b/se	b/se	b/se
safe_debt_ALL	-3.940*** (1.38)			
safe_debt_core		-4.437** (1.71)		
risky_debt_ALL			-0.042 (1.32)	
risky_debt_core				0.444 (1.44)
Constant	273.331*** (28.43)	265.650*** (28.05)	222.775*** (33.65)	215.866*** (31.89)
N	33	33	33	33
R-squared	0.21	0.18	0.00	0.00

* p<0.10, ** p<0.05, *** p<0.01

Table 3: Explanatory power of public debt holdings $x_{2,i} = \sum_{Eurozone\ country\ k} 1_{\{\beta_k < 0\}} x_{k,i}$ in the bank CDS cross-section. Safe_debt_all and Risky_debt_all includes all the countries listed in the former figure, whereas Safe_debt_core is only France and Germany, and Risky_debt_core is only Spain and Italy. All quantities are in billion euros and CDS in basis points.

A.7 Asset-pricing tests

A.7.1 Safe asset price q_B



Figure 22: 10-year government bond yields in Europe.

	(1)	(2)	(3)	(4)	(5)	(6)
	rsafe	rsafe	rsafe	rsafe	rsafe	rsafe
	b/se	b/se	b/se	b/se	b/se	b/se
SbcleanGDP	0.455** (0.22)				0.435* (0.22)	
Sb_GERGDP		-5.908*** (0.54)				
Germany beta			0.763 (0.92)		1.171 (0.83)	
Italy beta				-1.917*** (0.21)		
nBGDP						-1.955*** (0.29)
Constant	2.388*** (0.59)	8.362*** (0.44)	3.683*** (0.24)	3.370*** (0.08)	2.709*** (0.62)	7.126*** (0.54)
N	41	41	42	42	41	42
R-squared	0.10	0.76	0.02	0.67	0.14	0.53

* p<0.10, ** p<0.05, *** p<0.01

Table 4: Determinants of the EU the public safety premium: r^{safe} .

A.7.2 Safety Mismatch Index $SMI = r^{safe} - r^{bank}$

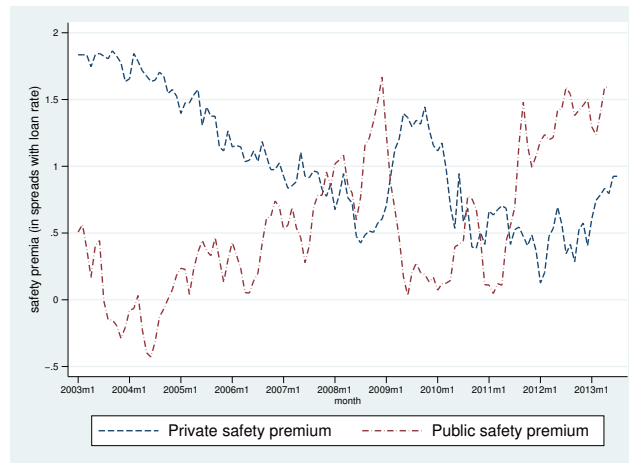


Figure 23: Private and public safety premia. SMI is equal to their spread.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	SMI	SMI	SMI	SMI	SMI	SMI	SMI
	b/se	b/se	b/se	b/se	b/se	b/se	b/se
SbcleanGDP	0.254 (0.28)				0.217 (0.27)		
Sb_GERGDP		-5.453*** (0.98)					
Germany beta			1.901* (1.01)		2.222** (1.01)		17.117 (19.29)
Italy beta				-1.409*** (0.36)			
xratio						-1.278 (6.16)	-10.668 (16.39)
interact							-50.732 (64.43)
Constant	-0.165 (0.74)	4.921*** (0.81)	0.888*** (0.27)	0.355*** (0.13)	0.443 (0.75)	0.830 (1.80)	4.058 (4.82)
N	41	41	42	42	41	42	42
R-squared	0.02	0.44	0.08	0.28	0.13	0.00	0.10

* p<0.10, ** p<0.05, *** p<0.01

Table 5: Determinants of the EU the safety premia spread (SMI): $r^{bank} - r^{safe}$.

A.7.3 Predictive regression of $SMI = r^{safe} - r^{bank}$ on bank leverage D

	(1)	(2)	(3)	(4)
	DGDP	DGDP	x1GDP	x1GDP
	b/se	b/se	b/se	b/se
SMI	-0.564*** (0.08)		-0.658*** (0.08)	
rsafe		-0.497*** (0.11)		-0.578*** (0.12)
Constant	5.156*** (0.08)	6.643*** (0.41)	5.506*** (0.08)	7.232*** (0.42)
N	42	42	42	42
R-squared	0.54	0.32	0.65	0.38

* p<0.10, ** p<0.05, *** p<0.01

Table 6: Predictive regression of the safety premia spread (SMI): $r^{bank} - r^{safe}$ for bank leverage D .

A.7.4 Predictive regression of $SMI = r^{safe} - r^{bank}$ on bank profits Π^B

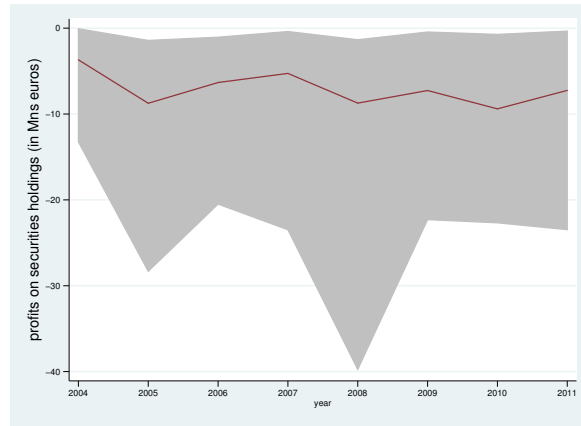


Figure 24: Negative carry trade of banks on safe assets, computed as: $yield * securities\ holdings - (refinancing\ costs + operating\ expenses) * securities\ holding / total\ assets$ (source: Bankscope).

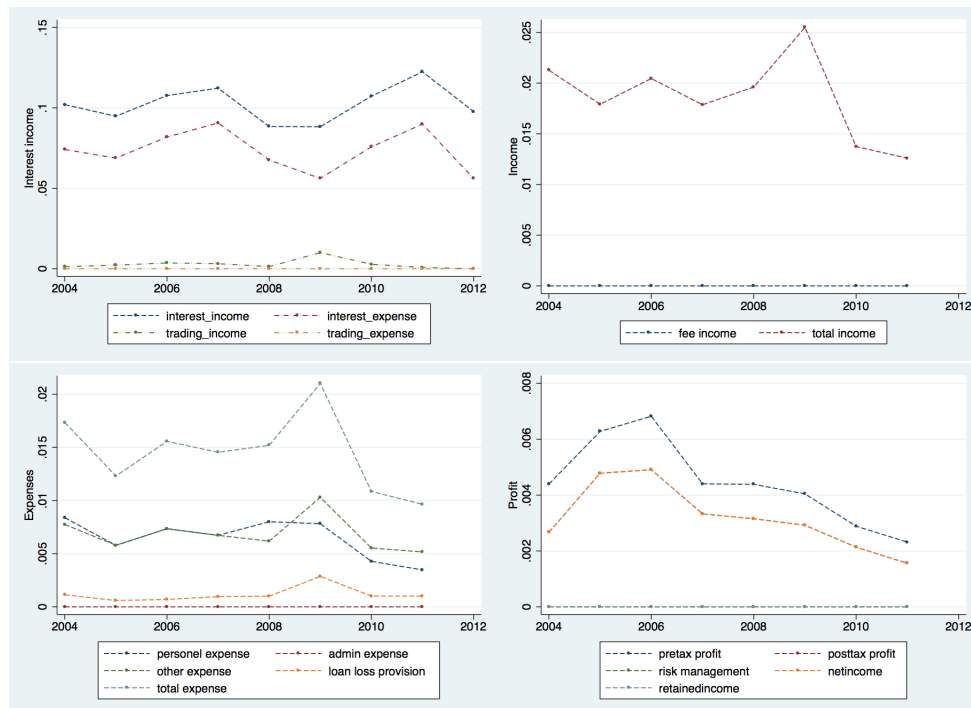


Figure 25: Banks profitability in Europe.

A.8 Open economy

A.8.1 Redomestication of government debt

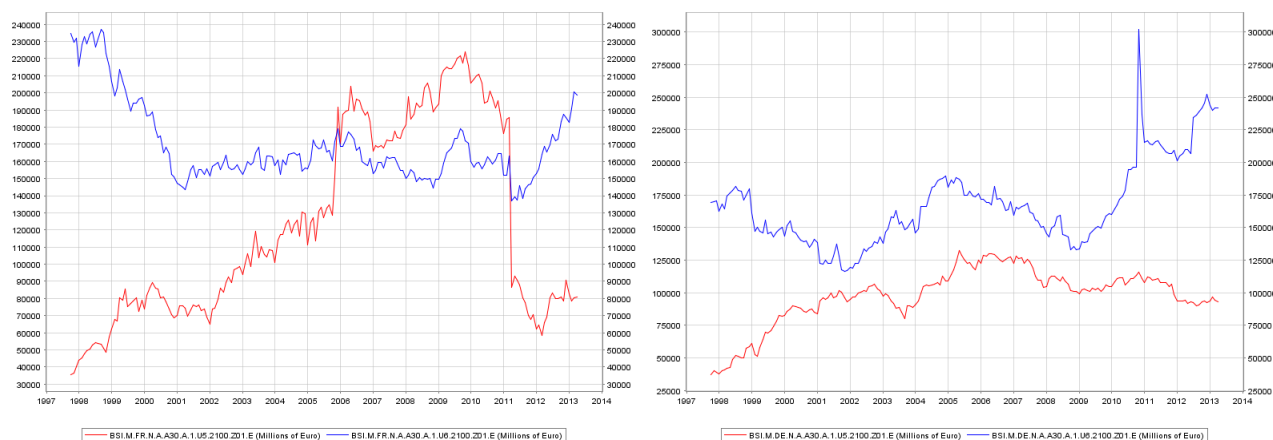


Figure 26: North: France (left) and Germany (right) government bond holdings by MFI. Red foreign debt, blue domestic debt.

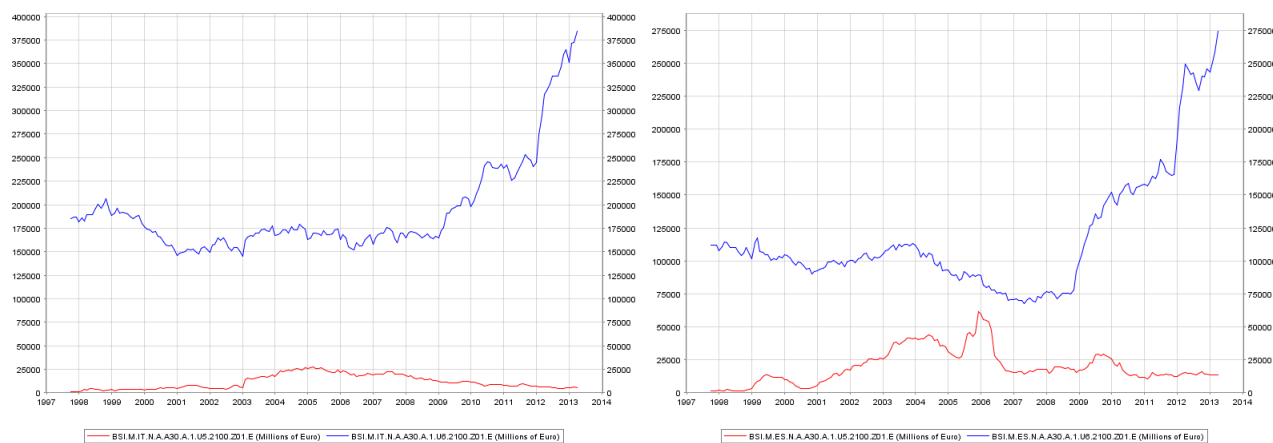


Figure 27: South: Italy (left) and Spain (right) government bond holdings by MFI (red foreign debt, blue domestic debt)

A.8.2 Deleverage in North, stable leverage in South

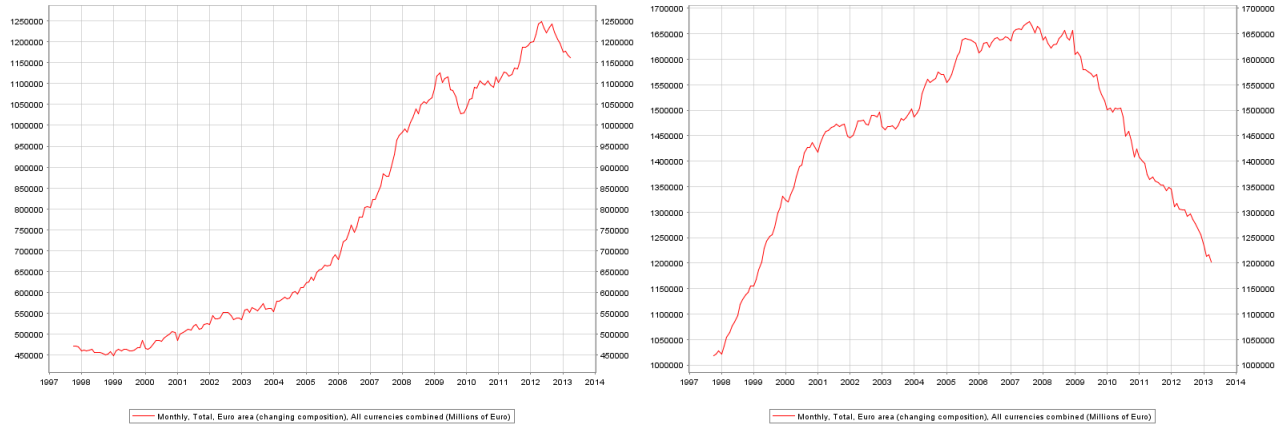


Figure 28: France (left) and Germany (right) MFI short-term debt

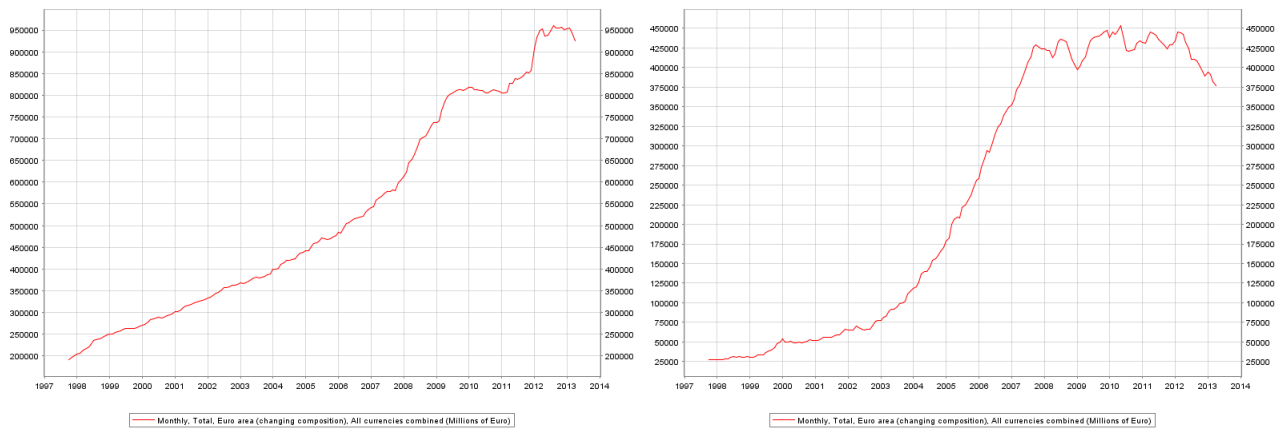


Figure 29: Italy (left) and Spain (right) MFI short-term debt

B Theory appendix

B.1 Toy model with discrete states and no choice of default threshold

Consider four equally plausible states. Assume that a $t = 1$ risky payoff for the technology and consider a public security that is imperfectly negatively correlated with the technology:

$$X_1 = s_K = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \text{ and } X_2 = s_B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}$$

Conjecture an equilibrium in which risk-neutral banks sell securities with promise 1 to mean-variance risk-averse investors, and where the bank is pushed in default if and only if one the two lowest states realize. Introduce the conditional payoffs introduced by default:

$$X_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } X_4 = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \tilde{X}_4 = \begin{bmatrix} 4 \\ 3 \\ 0 \\ 0 \end{bmatrix} \text{ and } X_5 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 3 \end{bmatrix}, \tilde{X}_5 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

In this equilibrium, the bank solves the following program:

$$\begin{aligned} \underset{\{x_K^A, x_B^A, y^A\}}{\text{Max}} \quad W^A &= \mathbb{E}_0\{x_K^A \tilde{X}_4 + x_B^A \tilde{X}_5 - y^A X_3\} \\ \text{s.t.} \quad &x_K^A c_K + x_B^A p_B \leq n^A + y^A q \end{aligned}$$

The three first-order conditions of the bank leads to, denoting λ the Lagrange multiplier on their budget:

$$\lambda = \frac{\mathbb{E}_0[\tilde{X}_4]}{c_K} = \frac{\mathbb{E}_0[\tilde{X}_5]}{p_B} = \frac{\mathbb{E}_0[X_3]}{q}$$

which gives $p_B = \frac{\mathbb{E}_0[\tilde{X}_5]}{\mathbb{E}_0[\tilde{X}_4]} c_K$ and $q = \frac{\mathbb{E}_0[X_3]}{\mathbb{E}_0[\tilde{X}_4]} c_K$. The mean-variance investor P solves:

$$\begin{aligned} \underset{\{x_K^P, x_B^P, y^P\}}{\text{Max}} \quad W^P &= L_{\mathbb{E}_0 - \gamma^P \mathbb{V}_0} \left[x_K^P X_1 + x_B^P X_2 + y^P X_3 + x_K^A X_4 + x_B^A X_5 \right] \\ \text{s.t.} \quad &x_K^P c_K + x_B^P p_B + y^P q \leq n^P \end{aligned}$$

The three first-order conditions of the investor leads to, denoting μ the Lagrange multiplier on their budget and Σ^{ij} the covariance matrix of $\{X_i\}$:

- $c_K \mu = \mathbb{E}_0[X_1] - 2\gamma^P (\Sigma^{11} x_K^P + \Sigma^{12} x_B^P + \Sigma^{13} y^P + \Sigma^{14} x_K^A + \Sigma^{15} x_B^A)$
- $p_B \mu = \mathbb{E}_0[X_2] - 2\gamma^P (\Sigma^{21} x_K^P + \Sigma^{22} x_B^P + \Sigma^{23} y^P + \Sigma^{24} x_K^A + \Sigma^{25} x_B^A)$
- $q \mu = \mathbb{E}_0[X_3] - 2\gamma^P (\Sigma^{31} x_K^P + \Sigma^{32} x_B^P + \Sigma^{33} y^P + \Sigma^{34} x_K^A + \Sigma^{35} x_B^A)$

The market clearings for public debt and private debt are:

$$x_B^A + x_B^P = B \text{ and } y^A = y^P$$

Denote $\Pi = [x_K^P; x_B^P; y^P; x_K^A; x_B^A; \mu]'$ the vector of equilibrium portfolios and:

$$R = [\mathbb{E}_0[X_1]; \mathbb{E}_0[X_2]; \mathbb{E}_0[X_3]; n^P; n^A; B]'$$

The equilibrium is characterized by the following linear system:

$$A\Pi = R$$

$$A = \begin{bmatrix} 2\gamma^P \Sigma^{11} & 2\gamma^P \Sigma^{12} & 2\gamma^P \Sigma^{13} & 2\gamma^P \Sigma^{14} & 2\gamma^P \Sigma^{15} & c_K \\ 2\gamma^P \Sigma^{21} & 2\gamma^P \Sigma^{22} & 2\gamma^P \Sigma^{23} & 2\gamma^P \Sigma^{24} & 2\gamma^P \Sigma^{25} & p_B \\ 2\gamma^P \Sigma^{31} & 2\gamma^P \Sigma^{32} & 2\gamma^P \Sigma^{33} & 2\gamma^P \Sigma^{34} & 2\gamma^P \Sigma^{35} & q \\ c_K & p_B & q & 0 & 0 & 0 \\ 0 & 0 & -q & c_K & p_B & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Applying Cramer's rule, developing along the third column, plugging the prices, γ^P and c_K cancel out by multilinearity in the derivative with respect to B . Finally we obtain:

$$\frac{\partial y}{\partial B} = - \frac{\begin{vmatrix} \Sigma^{11} & \Sigma^{12} & \Sigma^{14} & \Sigma^{15} & 1 \\ \Sigma^{21} & \Sigma^{22} & \Sigma^{24} & \Sigma^{25} & \frac{\mathbb{E}_0[\tilde{X}_5]}{\mathbb{E}_0[\tilde{X}_4]} \\ \Sigma^{31} & \Sigma^{32} & \Sigma^{34} & \Sigma^{35} & \frac{\mathbb{E}_0[\tilde{X}_3]}{\mathbb{E}_0[\tilde{X}_4]} \\ 1 & \frac{\mathbb{E}_0[\tilde{X}_5]}{\mathbb{E}_0[\tilde{X}_4]} & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{\mathbb{E}_0[\tilde{X}_5]}{\mathbb{E}_0[\tilde{X}_4]} & 0 \end{vmatrix}}{\begin{vmatrix} \Sigma^{11} & \Sigma^{12} & \Sigma^{13} & \Sigma^{14} & \Sigma^{15} & 1 \\ \Sigma^{21} & \Sigma^{22} & \Sigma^{23} & \Sigma^{24} & \Sigma^{25} & \frac{\mathbb{E}_0[\tilde{X}_5]}{\mathbb{E}_0[\tilde{X}_4]} \\ \Sigma^{31} & \Sigma^{32} & \Sigma^{33} & \Sigma^{34} & \Sigma^{35} & \frac{\mathbb{E}_0[\tilde{X}_3]}{\mathbb{E}_0[\tilde{X}_4]} \\ 1 & \frac{\mathbb{E}_0[\tilde{X}_5]}{\mathbb{E}_0[\tilde{X}_4]} & \frac{\mathbb{E}_0[\tilde{X}_3]}{\mathbb{E}_0[\tilde{X}_4]} & 0 & 0 & 0 \\ 0 & 0 & -\frac{\mathbb{E}_0[\tilde{X}_3]}{\mathbb{E}_0[\tilde{X}_4]} & 1 & \frac{\mathbb{E}_0[\tilde{X}_5]}{\mathbb{E}_0[\tilde{X}_4]} & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{vmatrix}}$$

With the given correlation structure and denoting $D = yq$, this example features:

$$\frac{\partial D}{\partial B} > 0$$

B.2 Intragenerational equilibrium

Consider the middle-aged generation risk-sharing problem and the generic asset correlation structure as multivariate normal:

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \sim N \left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \right)$$

The portfolio of the borrower is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, the lender's is $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and the promise of the debt contract is \bar{s} .

Introduce the following auxiliary variables: $u = X'S - \bar{s}$ and $v = Y'S + \bar{s}$.

$$\begin{bmatrix} u \\ v \end{bmatrix} \sim N \left(\begin{bmatrix} X'\mu - \bar{s} \\ Y'\mu + \bar{s} \end{bmatrix}, \begin{bmatrix} X\Sigma X' & \\ & Y\Sigma Y' \end{bmatrix} \right)$$

Borrower With the change of variable: $u = X'S - \int x_{\bar{s}}\bar{s}$, we have: $u \sim N(\mu_u, \sigma_u^2)$ with $\mu_u(x_1, x_2, \bar{s}) = x_1\mu_1 + x_2\mu_2 - \int x_{\bar{s}}\bar{s}$ and $\sigma_u^2(x_1, x_2) = x_1^2\sigma_1^2 + 2\rho x_1x_2\sigma_1\sigma_2 + x_2^2\sigma_2^2$. Denote Φ and ϕ respectively the density and the cdf of the standard normal distribution. We can write, using the truncated moment generating function for the normal distribution⁷⁸⁷⁹:

$$\begin{aligned} W^B &= \mathbb{E}_0 \left[\left(X'S - \int x_{\bar{s}}\bar{s} \right) 1_{\{X'S \geq \int x_{\bar{s}}\bar{s}\}} \right] \\ W^B &= \mu_u \Phi \left(\frac{\mu_u}{\sigma_u} \right) + \sigma_u \phi \left(\frac{\mu_u}{\sigma_u} \right) \end{aligned}$$

Lender For the investor, the derivation is more cumbersome in order to take explicitly into account default. With the same change of variable $u = X'S - \int x_{\bar{s}}\bar{s}$ and $v = Y'S + \int y_{\bar{s}}\bar{s}$:

$$\begin{aligned} W^L &= -\mathbb{E}_0 \left[e^{-\gamma_L(Y'S + X'S)} 1_{\{X'S < \int x_{\bar{s}}\bar{s}\}} + e^{-\gamma_L(Y'S + \int y_{\bar{s}}\bar{s})} 1_{\{X'S \geq \int x_{\bar{s}}\bar{s}\}} \right] \\ &= \int_{-\infty}^0 \left(\int_{-\infty}^{+\infty} -e^{-\gamma_L v} f_{M_{u,v}, \Sigma_{u,v}}(v|u) dv \right) e^{-\gamma_L u} f(u) du + \int_0^{+\infty} \left(\int_{-\infty}^{+\infty} -e^{-\gamma_L v} f_{M_{u,v}, \Sigma_{u,v}}(v|u) dv \right) f(u) du \\ &= W^{L def} + W^{L no def} \end{aligned}$$

We have $M_{u,v} = \begin{bmatrix} x_1\mu_1 + x_2\mu_2 - \bar{s} \\ y_1\mu_1 + y_2\mu_2 + \bar{s} \end{bmatrix}$ and $\Sigma_{u,v} = \begin{bmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix}$ with:

$$\begin{aligned} \sigma_u^2 &= x_1^2\sigma_1^2 + 2\rho x_1x_2\sigma_1\sigma_2 + x_2^2\sigma_2^2 \\ \sigma_v^2 &= y_1^2\sigma_1^2 + 2\rho y_1y_2\sigma_1\sigma_2 + y_2^2\sigma_2^2 \\ \rho_{uv} &= \frac{x_1y_1\sigma_1^2 + \rho(x_1y_2 + x_2y_1)\sigma_1\sigma_2 + x_2y_2\sigma_2^2}{\sqrt{(x_1^2\sigma_1^2 + 2\rho x_1x_2\sigma_1\sigma_2 + x_2^2\sigma_2^2)(y_1^2\sigma_1^2 + 2\rho y_1y_2\sigma_1\sigma_2 + y_2^2\sigma_2^2)}} \end{aligned}$$

⁷⁸We have: $\int_{a_1}^{a_2} y f_{\mu, \sigma^2}(y) dy = \frac{d}{dt} \left(e^{\mu t + \frac{1}{2}\sigma^2 t^2} \left\{ \Phi \left(\frac{a_2 - \mu}{\sigma} - \sigma t \right) - \Phi \left(\frac{a_1 - \mu}{\sigma} - \sigma t \right) \right\} \right) |_{t=0}$

⁷⁹In the general case of a CARA- γ_B utility for borrower is (risk-neutrality is recovered in the neighborhood $\gamma_B \sim 0$):

$$W^B = - \left\{ 1 - \Phi \left(\frac{\mu_u}{\sigma_u} \right) \right\} - e^{-\gamma_B \mu_u + \frac{1}{2} \gamma_B^2 \sigma_u^2} \Phi \left(\frac{\mu_u}{\sigma_u} - \sigma_u \gamma_B \right)$$

Using the Moment Generating Function of the normal distribution:

$$-\int_{-\infty}^{+\infty} e^{-\gamma_L v} f_{M_{u,v}, \Sigma_{u,v}}(v|u) dv = -e^{-\gamma_L (\mu_v - \frac{\rho_{uv}\sigma_v}{\sigma_u} \mu_u) + \frac{1}{2} \gamma_L^2 (1 - \rho_{uv}^2) \sigma_v^2 - \gamma_L \frac{\rho_{uv}\sigma_v}{\sigma_u} u}$$

Some algebra leads to:

$$W^L = -e^{-\gamma_L \mu_v + \frac{1}{2} \gamma_L^2 \sigma_v^2} \left\{ e^{-\gamma_L \mu_u + \frac{1}{2} \gamma_L^2 (\sigma_u^2 + 2\rho_{uv}\sigma_v\sigma_u)} \left\{ 1 - \Phi \left(\frac{\mu_u}{\sigma_u} - \gamma_L (\rho_{uv}\sigma_v + \sigma_u) \right) \right\} + \Phi \left(\frac{\mu_u}{\sigma_u} - \gamma_L \rho_{uv}\sigma_v \right) \right\}$$

B.2.1 Optimality conditions

Bank maximization We write their Lagrangian:

$$\text{Max}_{\{X, \{x_s\}\}} L^B = \mathbb{E}_0 \left[\left(X'S - \int x_s \bar{s} \right) 1_{\{X'S \geq \int x_s \bar{s}\}} \right] + \lambda \left[n^B + \int x_s q_s - X'P \right]$$

Its program takes the price of debt securities q_s . As all securities yield the same interest rate, the rate is the price: $r^{bank} = \frac{\bar{s}}{q_s}$ and we can write, denoting $D = \int x_s q_s$ and $\bar{S} = \int x_s \bar{s}$: $D = \bar{S} \frac{1}{r^{bank}}$. So the program can be written:

$$\text{Max}_{\{X, \{x_s\}\}} L^B = \mathbb{E}_0 \left[(X'S - \bar{S}) 1_{\{X'S \geq \bar{S}\}} \right] + \lambda \left[n^B + \bar{S} \frac{1}{r^{bank}} - X'P \right]$$

And the bank f.o.c are: $\frac{dL}{d\bar{S}} = 0 = \frac{\partial W^B}{\partial \bar{S}} + \lambda \frac{1}{r^{bank}}$ and $\frac{dL}{dX} = 0 = \frac{\partial W^B}{\partial x_i} - \lambda p_i$.

Investor maximization Their Lagrangian is:

$$\text{Max}_{\{Y, \{y_s\}\}} L^L = -\mathbb{E}_0 \left[e^{-\gamma_L (Y'S + X'S)} 1_{\{X'S < \int x_s \bar{s}\}} + e^{-\gamma_L (Y'S + y_s \bar{s})} 1_{\{X'S \geq \int x_s \bar{s}\}} \right] + \mu \left[n^L - \int y_s q_s - Y'P \right]$$

So investor f.o.c (taking as given the bank portfolio X) are: $\frac{dL}{d\bar{S}} = 0 = \frac{\partial W^L}{\partial \bar{S}} - \mu D'(\bar{s})$ and $\frac{dL}{dY} = 0 = \frac{\partial W^L}{\partial y_i} - \mu p_i$

Introduce the borrower Marginal Rate of Substitution in the portfolio choice from Asset 1 to Asset 2:

$$MRS_B = \frac{\frac{\partial W^B}{\partial x_2}}{\frac{\partial W^B}{\partial x_1}}$$

The Marginal Rate of Transformation from Asset 1 to private contract (promise \bar{s}):

$$MRT_B = \frac{\frac{\partial W^B}{\partial \bar{s}}}{\frac{\partial W^B}{\partial x_1}}$$

The lender Marginal Rate of Substitution from Asset 2 to private contract (promise \bar{s}):

$$MRS_L = \frac{\frac{\partial W^L}{\partial \bar{s}}}{\frac{\partial W^L}{\partial y_2}}$$

The first-order conditions can be expressed in terms of Marginal Rates of Substitution and of Transformation. In the case of limited participation of risk-averse agents in the risky technology⁸⁰, the equilibrium is characterized by the following five equations:

- Bank portfolio choice:

$$\frac{p_2}{p_1} = MRS_B = \frac{\frac{\partial W^B}{\partial x_2}}{\frac{\partial W^B}{\partial x_1}} = \frac{\mu_2 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_2(\sigma_2 x_2 + \rho \sigma_1 x_1)}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right)}{\mu_1 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_1(\sigma_1 x_1 + \rho \sigma_2 x_2)}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right)} \quad (20)$$

- Bank leverage choice:

$$\frac{D'(\bar{s})}{p_1} = MRT_B = \frac{\frac{\partial W^B}{\partial \bar{s}}}{\frac{\partial W^B}{\partial x_1}} = - \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\mu_1 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_1(\sigma_1 x_1 + \rho \sigma_2 x_2)}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right)} \quad (21)$$

- Investor portfolio choice:

$$\frac{D'(\bar{s})}{p_2} = MRS_L = \frac{\frac{\partial W^L}{\partial \bar{s}}}{\frac{\partial W^L}{\partial y_2}} \quad (22)$$

- The two budget constraints:

$$y_2 p_2 + D \leq n^L \quad (23)$$

$$x_1 p_1 + x_2 p_2 \leq n^B + D \quad (24)$$

B.2.2 Computation of the marginal rates of substitution and transformation

Borrower Using $\frac{\partial W^B}{\partial \mu_u} = \Phi\left(\frac{\mu_u}{\sigma_u}\right)$ and $\frac{\partial W^B}{\partial \sigma_u} = \phi\left(\frac{\mu_u}{\sigma_u}\right)$, we obtain for the marginal benefits of portfolio investment and the marginal cost of leveraging:

$$\begin{aligned} \frac{\partial W^B}{\partial x_i} &= \mu_i \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_i(\sigma_i x_i + \rho \sigma_{-i} x_{-i})}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right) \\ \frac{\partial W^B}{\partial \bar{s}} &= -\Phi\left(\frac{\mu_u}{\sigma_u}\right) \end{aligned}$$

So I obtain closed form expressions for the two B-optimality sufficient statistics MRS_B and MRT_B .

$$\begin{aligned} MRS_B &= \frac{\frac{\partial W^B}{\partial x_2}}{\frac{\partial W^B}{\partial x_1}} = \frac{\mu_2 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_2(\sigma_2 x_2 + \rho \sigma_1 x_1)}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right)}{\mu_1 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_1(\sigma_1 x_1 + \rho \sigma_2 x_2)}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right)} \\ MRT_B &= \frac{\frac{\partial W^B}{\partial \bar{s}}}{\frac{\partial W^B}{\partial x_1}} = - \frac{1}{\mu_1 + \frac{\sigma_1(\sigma_1 x_1 + \rho \sigma_2 x_2)}{\sigma_u} \frac{\phi\left(\frac{\mu_u}{\sigma_u}\right)}{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}} \end{aligned}$$

⁸⁰This assumption follows Basak and Cuoco (1998) and Cuoco and Kaniel (2011). The general case is solved in the same way and features the same properties as long as the asset correlation is higher than -1 . The two cases are compared in section 2.5.4.

Lender The algebraic complexity of investor value function is dramatically simplified under the European feature of limited participation, i.e. investor being restricted access to the risky asset: $y_1 = 0$. The moment of the transformed distribution are now:

$$\begin{aligned}\sigma_v &= y_2 \sigma_2 \\ \rho_{uv} &= \frac{(\rho x_1 \sigma_1 + x_2 \sigma_2)}{\sigma_u}\end{aligned}$$

We have $\frac{\partial \mu_v}{\partial y_2} = \mu_2$, $\frac{\partial \sigma_v}{\partial y_2} = \sigma_2$ and $\frac{\partial \rho_{uv}}{\partial y_2} = 0$. And with respect to the transformed moments:

$$\frac{\partial W^L}{\partial \mu_v} = -\gamma_L W^L$$

Deriving $\frac{\partial W^L}{\partial \sigma_v}$ and $\frac{\partial W^L}{\partial \mu_u}$ delivers closed-form expression for the L-optimality sufficient statistic MRS_L :

$$MRS_L = \frac{num}{den}$$

$$num = (-\gamma_L) \Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L \rho_{uv} \sigma_v\right) + \frac{1}{\sigma_u} \left\{ e^{-\gamma_L \mu_u + \frac{1}{2} \gamma_L^2 (\sigma_u^2 + 2\rho_{uv} \sigma_v \sigma_u)} \phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L (\rho_{uv} \sigma_v + \sigma_u)\right) - \phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L \rho_{uv} \sigma_v\right) \right\}$$

$$\begin{aligned}den &= \left\{ \left(-\mu_2 \gamma_L + \gamma_L^2 \sigma_2 (\sigma_v + \rho_{uv} \sigma_u) \right) e^{-\gamma_L \mu_u + \frac{1}{2} \gamma_L^2 (\sigma_u^2 + 2\rho_{uv} \sigma_v \sigma_u)} \left\{ 1 - \Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L (\rho_{uv} \sigma_v + \sigma_u)\right) \right\} \right\} \\ &+ \left\{ \left(-\mu_2 \gamma_L + \gamma_L^2 \sigma_2 \sigma_v \right) \Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L \rho_{uv} \sigma_v\right) \right\} \\ &+ (\sigma_2 \gamma_L \rho_{uv}) \left\{ e^{-\gamma_L \mu_u + \frac{1}{2} \gamma_L^2 (\sigma_u^2 + 2\rho_{uv} \sigma_v \sigma_u)} \phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L (\rho_{uv} \sigma_v + \sigma_u)\right) - \phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L \rho_{uv} \sigma_v\right) \right\}\end{aligned}$$

B.2.3 Bank portfolio choice

From $MRS_B = \frac{p_2}{p_1}$ and using $MRS_B = \frac{\mu_2 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_2 (\sigma_2 x_2 + \rho \sigma_1 x_1)}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right)}{\mu_1 \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \frac{\sigma_1 (\sigma_1 x_1 + \rho \sigma_2 x_2)}{\sigma_u} \phi\left(\frac{\mu_u}{\sigma_u}\right)}$ we obtain:

$$p_1 \sigma_2 (\sigma_2 x_2 + \rho \sigma_1 x_1) - p_2 \sigma_1 (\sigma_1 x_1 + \rho \sigma_2 x_2) = (p_2 \mu_1 - p_1 \mu_2) \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} \quad (25)$$

B.2.4 Bank leverage choice

From $D'(\bar{s}) = -p_1 MRT_B$ and using $MRT_B = -\frac{1}{\mu_1 + \frac{\sigma_1 (\sigma_1 x_1 + \rho \sigma_2 x_2)}{\sigma_u} \frac{\phi\left(\frac{\mu_u}{\sigma_u}\right)}{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}}$ we get:

$$D'(\bar{s}) = \frac{p_1}{\mu_1 + \frac{\sigma_1 (\sigma_1 x_1 + \rho \sigma_2 x_2)}{\sigma_u} \frac{\phi\left(\frac{\mu_u}{\sigma_u}\right)}{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}} \quad (26)$$

B.2.5 Debt market equilibrium

Combining the two bank optimality conditions (portfolio choice 25 and leverage choice 26) and eliminating μ_u we obtain the price of debt at equilibrium:

$$D'(\bar{s}) = \frac{1}{\frac{\mu_1}{p_1} + \frac{\frac{\mu_1 - \mu_2}{p_1} \frac{p_2}{p_2}}{\frac{p_1}{p_2} \frac{\sigma_2}{\sigma_1} \left(\frac{\sigma_2 x_2 + \rho \sigma_1 x_1}{\sigma_1 x_1 + \rho \sigma_2 x_2} - 1 \right)}} = \frac{p_2}{\mu_2} \frac{1 - \frac{p_1}{p_2} \frac{\sigma_2}{\sigma_1} \frac{\rho + \frac{\sigma_2 x_2}{\sigma_1 x_1}}{1 + \rho \frac{\sigma_2 x_2}{\sigma_1 x_1}}}{1 - \frac{\mu_1}{\mu_2} \frac{\sigma_2}{\sigma_1} \frac{\rho + \frac{\sigma_2 x_2}{\sigma_1 x_1}}{1 + \rho \frac{\sigma_2 x_2}{\sigma_1 x_1}}}$$

This shows that the marginal benefit of an extra unit of leverage (of promise \bar{s}) is *constant along the contract curve*, i.e. does not depend on the actual promise \bar{s} . Using the definition of the price of debt $D'(\bar{s}) = \frac{D}{\bar{s}} = \frac{q}{\bar{s}}$ and denote \hat{MRS}_B the second ratio on the above right hand side:

$$D(\bar{s}) = \frac{p_2}{\mu_2} \hat{MRS}_B \bar{s} = \frac{p_2}{\mu_2} \hat{MRS}_B (x_1 \mu_1 + x_2 \mu_2 - \mu_u)$$

Introduce $r^{bank} = \frac{\bar{s}}{D} = \frac{\mu_2}{p_2} \frac{1}{\hat{MRS}_B}$ and denote $X(x_1, x_2; \rho) = \frac{\sigma_2 x_2 + \rho \sigma_1 x_1}{\sigma_1 x_1 + \rho \sigma_2 x_2} = \frac{\rho + \frac{\sigma_2 x_2}{\sigma_1 x_1}}{1 + \rho \frac{\sigma_2 x_2}{\sigma_1 x_1}}$, it can be written:

$$\hat{MRS}_B = \frac{1 - \frac{p_1}{p_2} \frac{\sigma_2}{\sigma_1} X}{1 - \frac{\mu_1}{\mu_2} \frac{\sigma_2}{\sigma_1} X}$$

The ratio $X = \frac{\sigma_2 x_2 + \rho \sigma_1 x_1}{\sigma_1 x_1 + \rho \sigma_2 x_2} = \frac{\rho + \frac{\sigma_2 x_2}{\sigma_1 x_1}}{1 + \rho \frac{\sigma_2 x_2}{\sigma_1 x_1}}$ is a measure of the effective correlation on bank's balance sheet.

As $X'(\rho) = \frac{1 - \left(\frac{\sigma_2 x_2}{\sigma_1 x_1} \right)^2}{\left(1 + \rho \frac{\sigma_2 x_2}{\sigma_1 x_1} \right)^2}$, in the equilibrium that we look for in which $\frac{\sigma_2 x_2}{\sigma_1 x_1} < 1$ (which is feasible under Assumption 1 $\frac{\sigma_2}{\sigma_1} > \frac{p_2}{p_1} > \frac{\mu_2}{\mu_1}$), we obtain that $X(\rho)$ is increasing, between -1 and 1 . Furthermore, this functional is concave, with the concavity more marked when $\frac{\sigma_2 x_2}{\sigma_1 x_1}$ high. Besides we directly see that $\frac{p_2}{p_1} > \frac{\mu_2}{\mu_1}$ from Assumption 1 implies, as long as $X < 0$ $\hat{MRS}_B < 1$, but when $X > 0$ we get $\hat{MRS}_B > 1$. So the interest rate on bank debt critically depends on the bank balance sheet correlation measure X . We can ascertain that $r^{bank} > 1$ for sure only in the $X < 0$ case. Define the Safety Mismatch Index as the carry trade on public debt (the opposite is the bank credit spread):

$$\begin{aligned} r^{safe} - r^{bank} &= \frac{\mu_2}{p_2} - \frac{\bar{s}}{D} \\ &= \left(1 - \frac{1}{\hat{MRS}_B} \right) \frac{\mu_2}{p_2} \end{aligned}$$

So the sign of the carry trade is the sign of the correlation measure $X(x_1, x_2; \rho)$. Finally:

$$r^{safe} - r^{bank} < 0 \Leftrightarrow \rho < -\frac{\sigma_2 x_2}{\sigma_1 x_1}$$

Furthermore, some algebra delivers the exact dependence:

$$r^{safe} - r^{bank} = \left(r^1 - r^{safe} \right) \frac{\mu_2}{p_1} \left(\frac{1}{1 - \frac{p_1}{p_2} \frac{\sigma_2}{\sigma_1} X} - 1 \right)$$

So taking the equilibrium as given (envelope condition), the carry trade SMI increases with $X(x_1, x_2; \rho)$, which itself increases with ρ . The negative carry trade is even more negative when p_2 increases.

B.2.6 Bank Mean Variance Frontier

The bank budget constraint combined with bank portfolio choice is a quadratic system in (x_1, x_2) as $M\hat{R}S_B$ depends on x_1, x_2 . Solving it gives (x_1, x_2) as functions of μ_u and $\lambda_u = \sigma_u \frac{\Phi(\frac{\mu_u}{\sigma_u})}{\phi(\frac{\mu_u}{\sigma_u})}$.

$$\begin{cases} x_1 p_1 + x_2 p_2 & = n^B + D \\ p_1 \sigma_2 (\sigma_2 x_2 + \rho \sigma_1 x_1) - p_2 \sigma_1 (\sigma_1 x_1 + \rho \sigma_2 x_2) & = (p_2 \mu_1 - p_1 \mu_2) \sigma_u \frac{\Phi(\frac{\mu_u}{\sigma_u})}{\phi(\frac{\mu_u}{\sigma_u})} \end{cases}$$

The portfolio choice directly gives:

$$x_2 = \frac{(p_2 \mu_1 - p_1 \mu_2)}{(\sigma_2^2 p_1 - \rho \sigma_1 \sigma_2 p_2)} \sigma_u \frac{\Phi(\frac{\mu_u}{\sigma_u})}{\phi(\frac{\mu_u}{\sigma_u})} + \frac{(\sigma_1^2 p_2 - \rho \sigma_1 \sigma_2 p_1)}{(\sigma_2^2 p_1 - \rho \sigma_1 \sigma_2 p_2)} x_1 \quad (27)$$

Using the value of debt D and the expression of X then plugging x_2 in the budget constraint leads to a trinomial in x_1 :

$$T(x_1) = \frac{1}{2} x_1^2 + b x_1 - c = 0$$

There is one and only one positive root x_1 (as long as ρ low enough). The *exact* solution is (x_1 increases with c and decreases with b):

$$x_1 = -b + \sqrt{b^2 + 4c}$$

$$b = \frac{\left[(\sigma_2 p_1 - \rho \sigma_1 p_2) \frac{\left\{ n^B p_2 \left(\sigma_1 - \sigma_2 \left\{ \frac{\mu_1}{\mu_2} (1 + \rho) - \frac{p_1}{p_2} \right\} \right) - \frac{\mu_u}{\mu_2} (\sigma_1 p_2 - \rho p_1 \sigma_2) \right\}}{\left(\frac{\mu_1}{\mu_2} - \frac{p_1}{p_2} \right)} + 2 (1 + \rho) p_2 (p_2 \mu_1 - p_1 \mu_2) \sigma_u \frac{\Phi(\frac{\mu_u}{\sigma_u})}{\phi(\frac{\mu_u}{\sigma_u})} \right]}{2 \sigma_1 [\sigma_2^2 p_1^2 + (1 - 2\rho - \rho^2) \sigma_1 \sigma_2 p_1 p_2 + \sigma_1^2 p_2^2]}$$

$$c = \frac{\left[\left(\frac{\sigma_2}{\sigma_1} p_1 - \rho p_2 \right) \frac{n^B \left(\frac{\mu_1}{\mu_2} \frac{\sigma_2}{\sigma_1} - \rho \right) - \frac{\mu_u}{\mu_2} \left(\frac{p_1}{p_2} \frac{\sigma_2}{\sigma_1} - \rho \right)}{\frac{\mu_1}{\mu_2} - \frac{p_1}{p_2}} - \frac{1}{\sigma_1^2} (p_2 \mu_1 - p_1 \mu_2) \sigma_u \frac{\Phi(\frac{\mu_u}{\sigma_u})}{\phi(\frac{\mu_u}{\sigma_u})} \right]}{2 \sigma_1 (1 - \rho) [\sigma_2^2 p_1^2 + (1 - 2\rho - \rho^2) \sigma_1 \sigma_2 p_1 p_2 + \sigma_1^2 p_2^2]} (p_2 \mu_1 - p_1 \mu_2) \sigma_u \frac{\Phi(\frac{\mu_u}{\sigma_u})}{\phi(\frac{\mu_u}{\sigma_u})}$$

Finally I express $\sigma_u^2 = x_1^2 \sigma_1^2 + 2\rho x_1 x_2 \sigma_1 \sigma_2 + x_2^2 \sigma_2^2$ as a function of x_1 and denoting Σ , we obtain:

$$(\sigma_2 p_1 - \rho \sigma_1 p_2)^2 \sigma_u^2 = 2 \Sigma c + \delta_u^2 + 2 \left\{ -b + \sqrt{b^2 + 4c} \right\} \left\{ (1 - \rho^2) \sigma_1^2 p_2 \delta_u - \Sigma b \right\}$$

$$\Sigma = (1 - \rho^2) \sigma_1^2 \left\{ \sigma_2^2 p_1^2 - 2\rho \sigma_1 \sigma_2 p_1 p_2 + \sigma_1^2 p_2^2 \right\}$$

This is the mean-variance frontier, fully solved in the general case. This mapping $\sigma_u \mapsto \mu_u$ is first increasing, then decreasing.

B.2.7 Equilibrium price of debt

The contract curve is defined by $D'(\bar{s}) = \frac{p_2}{\mu_2} \hat{MRS}_L$, i.e.:

$$\hat{MRS}_B = \hat{MRS}_L$$

I develop \hat{MRS}_L in orders of γ_L , using $\Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L \rho_{uv} \sigma_v\right) = \Phi\left(\frac{\mu_u}{\sigma_u}\right) - \phi\left(\frac{\mu_u}{\sigma_u}\right) \gamma_L \rho_{uv} y_2 \sigma_2$ and $1 - \Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L (\rho_{uv} \sigma_v + \sigma_u)\right) = 1 - \Phi\left(\frac{\mu_u}{\sigma_u}\right) + \phi\left(\frac{\mu_u}{\sigma_u}\right) \gamma_L (\rho_{uv} y_2 \sigma_2 + \sigma_u)$ and $e^{-\gamma_L \mu_u + \frac{1}{2} \gamma_L^2 (\sigma_u^2 + 2\rho_{uv} \sigma_v \sigma_u)} = 1 - \gamma_L \mu_u + \frac{1}{2} \gamma_L^2 (\sigma_u^2 + 2\rho_{uv} y_2 \sigma_2 \sigma_u)$. The numerator and denominator simplify to:

$$num = \Phi\left(\frac{\mu_u}{\sigma_u}\right) - \phi\left(\frac{\mu_u}{\sigma_u}\right) \gamma_L \left\{ \left(2 - \frac{\mu_u^2}{\sigma_u^2}\right) \rho_{uv} y_2 \sigma_2 + \left(\frac{1}{2} - \frac{\mu_u^2}{\sigma_u^2}\right) \sigma_u \right\}$$

$$\begin{aligned} den = & \Phi\left(\frac{\mu_u}{\sigma_u}\right) \gamma_L [\sigma_2 \rho_{uv} \sigma_u + \mu_u (\mu_2 - \gamma_L \sigma_2 y_2 \sigma_2 - \gamma_L \sigma_2 \rho_{uv} \sigma_u)] \\ & + \phi\left(\frac{\mu_u}{\sigma_u}\right) \gamma_L \left[\mu_2 - \gamma_L \sigma_2^2 y_2 \sigma_u - \gamma_L \sigma_2 \rho_{uv} \sigma_u (\sigma_u + \rho_{uv} y_2 \sigma_2) - \gamma_L \mu_u \left[(\mu_2 - \gamma_L \sigma_2^2 y_2 - \gamma_L \sigma_2 \rho_{uv} \sigma_u) (\sigma_u + \rho_{uv} y_2 \sigma_2) \right] \right] \\ & + (1 - \gamma_L \mu_u) (\mu_2 - \gamma_L \sigma_2 (y_2 \sigma_2 + \rho_{uv} \sigma_u)) \end{aligned}$$

Finally we obtain, first order in γ_L :

$$\hat{MRS}_L = \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right) - \phi\left(\frac{\mu_u}{\sigma_u}\right) \gamma_L \left\{ \left(2 - \frac{\mu_u^2}{\sigma_u^2}\right) \rho_{uv} y_2 \sigma_2 + \left(\frac{1}{2} - \frac{\mu_u^2}{\sigma_u^2}\right) \sigma_u \right\}}{1 - \gamma_L \left\{ \left(1 - \Phi\left(\frac{\mu_u}{\sigma_u}\right)\right) \mu_u - \Phi\left(\frac{\mu_u}{\sigma_u}\right) \frac{\sigma_2}{\mu_2} \rho_{uv} \sigma_u + \frac{\sigma_2}{\mu_2} (y_2 \sigma_2 + \rho_{uv} \sigma_u) - \phi\left(\frac{\mu_u}{\sigma_u}\right) \right\}}$$

So \hat{MRS}_L increases in γ_L for ρ low enough (to make $\rho_{uv} \ll 1$) and y_2 low enough (i.e. S^b low enough). On the contract curve, we see that implies a higher equilibrium leverage. We also observe that \hat{MRS}_L does not depend on p_2 , so easily get the impact of a higher price p_2 here: it increases the slope MRSB: the MB of leverage has to be higher to match the increased price p_2 .

We can also get a approximate for the debt-pricing functional in the neighborhood of $\gamma_L \sim 0$:

$$\mu_u = \sigma_u \Phi^{-1} \left(\frac{\mu_2}{p_2} \frac{p_1}{\mu_1} \left(1 - \frac{\sigma_1}{\sigma_2} \frac{\frac{p_2}{p_1} - \frac{\mu_2}{\mu_1}}{X - \frac{\mu_2}{\mu_1} \frac{\sigma_1}{\sigma_2}} \right) \right)$$

which is an increasing functional.

B.2.8 Equilibrium price of long-term safe asset

The GE: $x_2 + y_2 = S^b$, and adding the two budget constraints:

$$p_2 S^b + p_1 x_1 (\mu_1; p_2) = n^B + n^L$$

So p_2 is depends on the equilibrium only through x_1 (a key feature of the model):

$$x_1 (\mu_1; p_2) = n^B \left\{ \frac{(1+\rho) \sigma_2 p_2}{\sigma_1 \Sigma} (\sigma_2 p_1 - \rho \sigma_1 p_2) + \left[\frac{\sigma_1}{\Sigma} \frac{1-\rho}{1+\rho} - 2 \frac{(1+\rho) p_2^2}{\sigma_1 \Sigma} \right] \mu_1 \sqrt{\delta} \right\}$$

By the implicit function theorem applied to this resource constraint, we can characterize the safe asset price $p_2 (S^b, \mu_1)$.

$$\frac{dp_2}{dS^b} = -\frac{p_2}{S^b + p_1 \partial_{p_2} x_1}$$

So the demand curve is indeed downward sloping ($\frac{dp_2}{dS^b} < 0$) as long as $-\frac{S^b}{p_1} < \partial_{p_2} x_1$.

Second, I investigate the dependence of p_2 on μ_1 , as a building block for the dynamic model.

$$\frac{dp_2}{d\mu_1} = -\frac{\partial_{\mu_1} F}{\partial_{p_2} F} = -\frac{p_1 \partial_{\mu_1} x_1}{S^b + p_1 \partial_{p_2} x_1}$$

Third, the dynamic model solution requires $\frac{d^2 p_2}{d(\mu_1)^2}$. I compute it through a double application of implicit function theorem to the resource constraint $F(p_2; \mu_1) = 0$. Total differentiating twice in μ_1 :

$$\frac{d^2 p_2}{d(\mu_1)^2} \partial_{p_2} F + \frac{dp_2}{d\mu_1} \left(\frac{dp_2}{d\mu_1} \partial_{p_2 p_2} F + \partial_{\mu_1 p_2} F \right) + \frac{dp_2}{d\mu_1} \partial_{p_2 \mu_1} F + \partial_{\mu_1 \mu_1} F = 0$$

$$\frac{d^2 p_2}{d(\mu_1)^2} \partial_{p_2} F + \left(\frac{dp_2}{d\mu_1} \right)^2 \partial_{p_2 p_2} F + 2 \frac{dp_2}{d\mu_1} \partial_{\mu_1 p_2} F + \partial_{\mu_1 \mu_1} F = 0$$

$$\frac{\frac{d^2 p_2}{d(\mu_1)^2}}{\frac{dp_2}{d\mu_1}} = \frac{\left(\frac{dp_2}{d\mu_1} \right)^2 \partial_{p_2 p_2} F + 2 \frac{dp_2}{d\mu_1} \partial_{\mu_1 p_2} F + \partial_{\mu_1 \mu_1} F}{\partial_{\mu_1} F}$$

$\partial_{\mu_1} F$ and $\partial_{\mu_1 \mu_1} F$ does not depend on S^b . $\partial_{p_2 p_2} F$ by separability. But $\left(\frac{dp_2}{d\mu_1} \right)^2$ does depend on S^b . As we have $\partial_{p_2 p_2} F = p_1 \partial_{p_2 p_2} x_1$, which is < 0 (Σ does not dominate - cf. s.o.c.), then this positive effect can counteract the fact that $\partial_{\mu_1 p_2} F = p_1 \partial_{\mu_1 p_2} x_1 < 0$ and $\frac{dp_2}{d\mu_1}$ increasing in S^b .

The second order condition needs to be verified: $\partial_{p_2 p_2} x_1 >> 0$, despite $\partial_{\mu_1 p_2} x_1 < 0$. Algebra leads to, denoting $u = \frac{p_1 \partial_{\mu_1} x_1}{S^b + p_1 \partial_{p_2} x_1} = u \left(\begin{smallmatrix} S^b \\ (-) \end{smallmatrix} \right)$:

$$-\partial_{\mu_1 p_2} x_1 < -\partial_{p_2 p_2} x_1 u$$

Beware of the elasticities signs: $\partial_{\mu_1 p_2} x_1 < 0$ and $\partial_{p_2 p_2} x_1 < 0$ so the above condition is a lower bound on u , i.e. an upper bound on S^b . The elasticities signs are guaranteed with a high enough γ_L .

So finally the second-order Taylor expansion of $p_2(\mu_1)$ writes:

$$p_2(\mu_1) = \alpha - \beta \mu_1 - \gamma \mu_1^2$$

$$\text{where } \beta = \left| \frac{dp_2}{d\mu_1} \right| = \frac{p_1 \partial_{\mu_1} x_1}{S^b + p_1 \partial_{p_2} x_1}$$

$$\text{and } \gamma = \left| \frac{d^2 p_2}{d(\mu_1)^2} \right| = \frac{\left(\frac{dp_2}{d\mu_1} \right)^2 \partial_{p_2 p_2} F + 2 \frac{dp_2}{d\mu_1} \partial_{\mu_1 p_2} F + \partial_{\mu_1 \mu_1} F}{\partial_{p_2} F} = \frac{\left(\frac{dp_2}{d\mu_1} \right)^2 p_1 \partial_{p_2 p_2} x_1 + 2 \frac{dp_2}{d\mu_1} p_1 \partial_{\mu_1 p_2} x_1 + p_1 \partial_{\mu_1 \mu_1} x_1}{S^b + p_1 \partial_{p_2} x_1}$$

which means that the price functional $\mu_1 \mapsto p_2$ is decreasing concave.

B.2.9 Equilibrium

Equilibrium is therefore characterized by the MVF and the debt pricing curve:

$$\begin{cases} (\sigma_2 p_1 - \rho \sigma_1 p_2)^2 \sigma_u^2 &= 2\Sigma c + \delta_u^2 + 2 \left\{ -b + \sqrt{b^2 + 4c} \right\} \left\{ (1 - \rho^2) \sigma_1^2 p_2 \delta_u - \Sigma b \right\} \\ \frac{\mu_2}{p_2} \frac{p_1}{\mu_1} \left(1 - \frac{\sigma_1}{\sigma_2} \frac{p_2 - \frac{\mu_2}{\mu_1}}{X - \frac{\mu_2}{\mu_1} \frac{\sigma_1}{\sigma_2}} \right) &= \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right) - \phi\left(\frac{\mu_u}{\sigma_u}\right) \gamma_L \left\{ \left(2 - \frac{\mu_u^2}{\sigma_u^2}\right) \rho_{uv} y_2 \sigma_2 + \left(\frac{1}{2} - \frac{\mu_u^2}{\sigma_u^2}\right) \sigma_u \right\}}{1 - \gamma_L \left\{ (1 - \Phi\left(\frac{\mu_u}{\sigma_u}\right)) \mu_u - \Phi\left(\frac{\mu_u}{\sigma_u}\right) \frac{\sigma_2}{\mu_2} \rho_{uv} \sigma_u + \frac{\sigma_2}{\mu_2} (y_2 \sigma_2 + \rho_{uv} \sigma_u) - \phi\left(\frac{\mu_u}{\sigma_u}\right) \right\}} \end{cases}$$

Denoting $\lambda_u = \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}$ the MVF delivers:

$$\begin{aligned} \frac{\left(\frac{\rho \sigma_1}{\mu_1}\right)^2}{(1 - \rho^2)} &= \lambda_u^2 \left[\left(\frac{\mu_1^2}{\mu_2}\right)^2 \left(\frac{1}{n^B}\right)^2 (\sigma_u \lambda_u)^2 + 2 \left\{ 1 - \left(\frac{\mu_1}{\mu_2} \sigma_1 \sigma_2 + \sigma_1^2\right) \right\} \frac{\sigma_1 \mu_1}{\mu_2} \frac{1}{n^B} \frac{\mu_1}{\sigma_1} (\sigma_u \lambda_u) \right] \\ &\quad + \left(\sigma_2 + \sigma_1 \frac{\mu_2}{\mu_1} \right) \frac{\sigma_1 \mu_1}{\mu_2} \left\{ \left(\frac{\mu_1}{\mu_2} \sigma_1 \sigma_2 + \sigma_1^2\right) - 2 \right\} + \frac{1}{(1 - \rho^2)} \end{aligned}$$

The degree 4 polynomial in λ_u on the RHS is increasing so it will cross the positive flat line $(\rho \sigma_1)^2$. And the higher ρ^2 is, the higher λ_u needs to be. The lower ρ^2 , the lower λ_u so the lower the ellipse mapping is. For n^B low enough:

$$\frac{\left(\frac{\rho \sigma_1}{\mu_1}\right)^2}{(1 - \rho^2)} = \lambda_u^2 \left[\left(\frac{\mu_1^2}{\mu_2}\right)^2 \left(\frac{1}{n^B}\right)^2 (\sigma_u \lambda_u)^2 + \left(\sigma_2 + \sigma_1 \frac{\mu_2}{\mu_1} \right) \frac{\sigma_1 \mu_1}{\mu_2} \left\{ \left(\frac{\mu_1}{\mu_2} \sigma_1 \sigma_2 + \sigma_1^2\right) - 2 \right\} + \frac{1}{(1 - \rho^2)} \right]$$

So this a $X (aX + b) - c = 0$. We then get only one positive root:

$$\lambda_u^2 = \frac{-b + \sqrt{b^2 + 4c}}{2a}$$

with $a = \left(\frac{\mu_1^2}{\mu_2}\right)^2 \left(\frac{1}{n^B}\right)^2 \sigma_u^2$, $b = \left(\sigma_2 + \sigma_1 \frac{\mu_2}{\mu_1} \right) \frac{\sigma_1 \mu_1}{\mu_2} \left\{ \left(\frac{\mu_1}{\mu_2} \sigma_1 \sigma_2 + \sigma_1^2\right) - 2 \right\} + \frac{1}{(1 - \rho^2)}$ and $c = \frac{\left(\frac{\rho \sigma_1}{\mu_1}\right)^2}{(1 - \rho^2)}$.

So λ_u eq decreases with σ_u , hence μ_u decreases with σ_u . The mapping is:

$$\lambda_u^2 = \left(-b + \sqrt{b^2 + 4c} \right) \left(\frac{\mu_1^2}{\mu_2} \right)^{-2} \frac{(n^B)^2}{\sigma_u^2}$$

I write $\lambda_u = \sqrt{\delta} \frac{n^B}{\sigma_u}$, with $\sqrt{\delta}$ increasing in $|\rho|$. δ spikes up when ρ tends to -1 . Denoting ξ^{-1} the inverse function of $\xi : u \mapsto \frac{\Phi(u)}{\phi(u)}$ we have:

$$\mu_u = \sigma_u \xi^{-1} \left(\frac{\sqrt{\delta} n^B}{\sigma_u} \right) \quad (28)$$

Now remember the debt pricing curve (in the admissible region $0.5 < \frac{p_2}{\mu_2} \hat{MRS}_B < 1$):

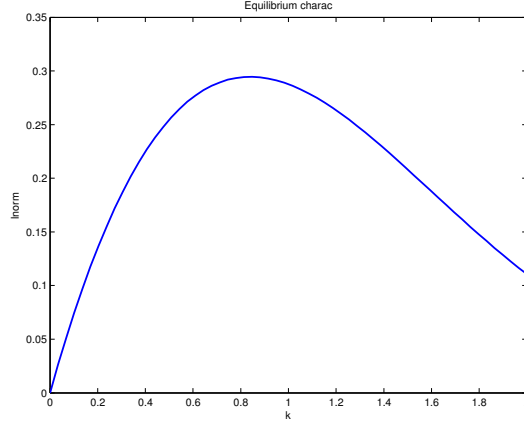
$$\mu_u = \sigma_u \Phi^{-1} \left(\frac{p_2}{\mu_2} \hat{MRS}_B \right) \quad (29)$$

By identifying 28 and 29:

$$\sigma_u = \frac{\sqrt{\delta} n^B}{\xi \left(\Phi^{-1} \left(\frac{p_2}{\mu_2} \hat{MRS}_B \right) \right)}$$

$$\mu_u = \sqrt{\delta} n^B \frac{\Phi^{-1} \left(\frac{p_2}{\mu_2} \hat{M} \hat{R} S_B \right)}{\xi \left(\Phi^{-1} \left(\frac{p_2}{\mu_2} \hat{M} \hat{R} S_B \right) \right)}$$

And $x/\xi(x)$ is plotted below: it increases in x as long as $x < 1$. We do have $\frac{p_2}{\mu_2} < 1$ and $\hat{M} \hat{R} S_B < 1$, but Φ^{-1} can take arbitrarily high values. Therefore to get $\mu_u = f \left(\frac{p_2}{(+)} \right)$ we need to make sure that γ_L is large enough, as we have that $\Phi^{-1} \left(\frac{p_2}{\mu_2} \hat{M} \hat{R} S_B \right)$ is decreasing with γ_L .



B.3 Intergenerational equilibrium

B.3.1 Endogenous beta

In the Markov equilibria with the one state variable $s^t \equiv \mu_1$, the resource constraint gives:

$$p_2^t(s^t) S^b + p_1 x_1(p_2^t(s^t); s^t) = n^B + n^L$$

Appealing to the static model has solved, the safe asset price functional has the following characteristics⁸¹:

$$p_2(s^{t+1}) = \alpha - \beta s^{t+1} - \gamma (s^{t+1})^2$$

Generically we know that for an univariate normal distribution $s \sim N(\mu, \sigma)$ the correlation between s and $p_2 = \alpha - \beta s - \gamma s^2$, first-order in γ/β ⁸²:

$$\text{corr}(s, p_2) = -1 + \left(2\mu \frac{\gamma}{\beta} \right)^2$$

Therefore applied, to the random walk process of s^t : $s^{t+1} \sim N(s^t, \sigma_1)$:

- $\hat{\rho} = -1 + \left(2s^t \frac{\gamma}{\beta} \right)^2$

⁸¹If I only consider the first-order Taylor expansion $p_2 = \alpha - \beta s^t$, then the endogenous beta is equal to -1 . No traction for any comparative statics.

⁸²The exact expression in γ/β is:

$$\sigma(p_2) = \beta \sigma \sqrt{1 + 4\mu \frac{\gamma}{\beta} + 2 \left(\frac{\gamma}{\beta} \right)^2 (\mu^2 + \sigma^2)} \text{ and } \text{corr}(s, p_2) = -\frac{1 + 2\mu \frac{\gamma}{\beta}}{\sqrt{1 + 4\mu \frac{\gamma}{\beta} + 2 \left(\frac{\gamma}{\beta} \right)^2 (\mu^2 + \sigma^2)}}$$

- $\hat{\sigma}_2 = \beta \sigma_1 \sqrt{1 + 4s^t \frac{\gamma}{\beta} + 2 \left(\frac{\gamma}{\beta} \right)^2 (s^{t2} + \sigma_1^2)}$
- $\hat{\mu}_2 = \alpha - \beta s^t - \gamma (s^t)^2$

This is a system of 3 equations, with 3 unknowns $(\hat{\rho}, \hat{\sigma}_2, \hat{\mu}_2)$. The recursion implies that β and γ are themselves functions of $(\hat{\rho}, \hat{\sigma}_2, \hat{\mu}_2)$. σ_1 is an exogenous parameter and the state is $s^t \equiv \mu_1$.

Consider $x_1(p_2; \mu_1)$. In $p_2 = 0$: $\Sigma(0) = \sigma_2^2 p_1^2$ and $x_1(0) = n^B \frac{\sigma_1}{\sigma_2^2 p_1^2} \frac{1-\rho}{1+\rho} \mu_1 \sqrt{\delta}$ so:

- $\partial_{\mu_1} x_1 = n^B \frac{\sigma_1}{\sigma_2^2 p_1^2} \frac{1-\rho}{1+\rho} \sqrt{\delta}$
- $\partial_{\mu_1} \partial_{\mu_1} x_1 = 0$
- $\partial_{p_2} \partial_{\mu_1} x_1 = n^B \partial_{p_2} \left(\frac{\sigma_1^2 (1-\rho) - 2(1+\rho)^2 p_2^2}{\sigma_2^2 p_1^2 + (1-2\rho-\rho^2) \sigma_1 \sigma_2 p_1 p_2 + \sigma_1^2 p_2^2} \right) \frac{1-\rho}{1+\rho} \sqrt{\delta}$
- $\partial_{p_2} \partial_{p_2} x_1 = n^B \frac{1-\rho}{1+\rho} \mu_1 \sqrt{\delta} \frac{1}{1+\rho} \partial_{p_2} \partial_{p_2} \left(\frac{\sigma_1^2 (1-\rho) - 2(1+\rho)^2 p_2^2}{\sigma_2^2 p_1^2 + (1-2\rho-\rho^2) \sigma_1 \sigma_2 p_1 p_2 + \sigma_1^2 p_2^2} \right)$

By inspection we see that $\partial_{p_2} \left(\frac{\sigma_1^2 (1-\rho) - 2(1+\rho)^2 p_2^2}{\sigma_2^2 p_1^2 + (1-2\rho-\rho^2) \sigma_1 \sigma_2 p_1 p_2 + \sigma_1^2 p_2^2} \right) < 0$ and $\partial_{p_2} \partial_{p_2} \left(\frac{\sigma_1^2 (1-\rho) - 2(1+\rho)^2 p_2^2}{\sigma_2^2 p_1^2 + (1-2\rho-\rho^2) \sigma_1 \sigma_2 p_1 p_2 + \sigma_1^2 p_2^2} \right) < 0$, for a given p_1, σ_1, ρ . Denote:

$$\kappa(p_2) = \frac{\sigma_1^2 (1-\rho) - 2(1+\rho)^2 p_2^2}{\sigma_2^2 p_1^2 + (1-2\rho-\rho^2) \sigma_1 \sigma_2 p_1 p_2 + \sigma_1^2 p_2^2}$$

We have, around $\rho = -1$:

$$\begin{aligned} |\partial_{p_2} \kappa| &= \frac{2(1+\rho)^2 p_2 (2\sigma_2^2 p_1^2 + (1-2\rho-\rho^2) \sigma_1 \sigma_2 p_1 p_2) + \sigma_1^2 (1-\rho) ((1-2\rho-\rho^2) \sigma_1 \sigma_2 p_1 + 2\sigma_1^2 p_2)}{\Sigma^2} \\ &= \frac{2\sigma_1^4 p_2}{[\sigma_2^2 p_1^2 + \sigma_1^2 p_2^2]^2} (1-\rho) \end{aligned}$$

$$\begin{aligned} \partial_{p_2} \partial_{p_2} \kappa &= 2(1+\rho)^2 (-2(\sigma_2^2 p_1^2)^2 + 6\sigma_2^2 p_1^2 \sigma_1^2 p_2^2 + 2(1-2\rho-\rho^2) \sigma_1 \sigma_2 p_1 \sigma_1^2 p_2^3) \\ &\quad + 2\sigma_1^2 (1-\rho) \left\{ \left((1-2\rho-\rho^2) \sigma_1 \sigma_2 p_1 \right)^2 + 3 \left((1-2\rho-\rho^2) \sigma_1 \sigma_2 p_1 \right) \sigma_1^2 p_2 + \sigma_1^2 (-\sigma_2^2 p_1^2 + 3\sigma_1^2 p_2^2) \right\} \end{aligned}$$

So finally $\hat{\rho} = -1 + \left(2s^t \frac{\gamma}{\beta} \right)^2$, I just need $\frac{\gamma}{\beta}$. With $\partial_{\mu_1} |\partial_{p_2} x_1| < 0$ we have $\frac{\gamma}{\beta}$ increasing in S^b .

$$\frac{\gamma}{\beta} = \left(\frac{\partial_{\mu_1} x_1}{\frac{S^b}{p_1} + \partial_{p_2} x_1} \right)^2 \frac{\partial_{p_2} \partial_{p_2} x_1}{\partial_{\mu_1} x_1} - 2 \frac{\partial_{\mu_1} \partial_{p_2} x_1}{\frac{S^b}{p_1} + \partial_{p_2} x_1}$$

It results in:

$$\rho = -1 + 4\mu^2 \left(\mu_1 \frac{1}{1+\rho} \frac{\kappa \partial_{p_2} \partial_{p_2} \kappa}{(\partial_{p_2} \kappa)^2} \frac{1}{\left(\frac{S^b}{p_1} \frac{1}{n^B \frac{1-\rho}{1+\rho} \sqrt{\delta} \partial_{p_2} \kappa} + \mu_1 \right)^2} - 2 \frac{1}{\frac{S^b}{p_1} \frac{1}{n^B \frac{1-\rho}{1+\rho} \sqrt{\delta} \partial_{p_2} \kappa} + \mu_1} \right)^2$$

As I have shown above, $\partial_{p_2}\kappa < 0$ and $\partial_{p_2}\partial_{p_2}\kappa < 0$. Using the exact expression of κ :

$$\hat{\rho} = -1 + \left(\frac{S^b [\hat{\sigma}_2^2 p_1^2 + \sigma_1^2 \hat{p}_2^2]^2}{n^B 2\sigma_1^4 p_1^2 \hat{p}_2^2} \right)^2$$

B.4 Results: comparative statics

I first show that the two equilibrium variables are as such $\mu_u \left(\begin{smallmatrix} \rho \\ (-) \end{smallmatrix}, p_2 \right)$ and $\sigma_u \left(\begin{smallmatrix} \rho \\ (-) \end{smallmatrix}, p_2 \right)$: lower ρ make bank lever up and take more risk, whereas better p_2 make bank lever up more and take less risk. Propositions 1,2 and 3 then follow.

Equilibrium σ_u decreases with p_2 . As for dependence in ρ :

$$\frac{\partial \lambda_u^2}{\partial \rho^2} = \frac{1 + \frac{1-\lambda_u^2}{(1-\rho^2)}}{(2a+b)(1-\rho^2)}$$

which is positive as long as $2 - \rho^2 > \lambda_u^2$.

B.4.1 Leverage D

$$D = MRS_B * \bar{s}$$

$$\frac{\partial D}{\partial p_2} = \partial MRS_B * \bar{s} + MRS_B * \partial \bar{s}$$

where $MRS_B = \frac{p_2}{\mu_2} * \frac{1}{1 + \frac{\mu_1 \sigma_2}{\mu_2 \sigma_1}}$. Denote $\theta = \frac{1}{\mu_2 + \frac{\mu_1 \sigma_2}{\sigma_1}} < 1$.

$$\frac{\partial D}{\partial p_2} < 0 \Leftrightarrow \frac{1}{p_2} < \frac{|\partial \bar{s}|}{\bar{s}}$$

So \bar{s} decreases in p_2 as long this $x/\xi(x)$ increases.

$$\begin{aligned} \frac{|\partial \bar{s}|}{\bar{s}} &= \frac{\frac{\partial}{\partial p_2} \left(\frac{\Phi^{-1} \left(\frac{p_2}{\mu_2} \hat{MRS}_B \right)}{\xi \left(\Phi^{-1} \left(\frac{p_2}{\mu_2} \hat{MRS}_B \right) \right)} \right)}{\frac{\mu_1^2}{\mu_2 \sigma_1} \left\{ \sqrt{\delta} \frac{\mu_1}{\sigma_1} - \left(\sigma_2 + \sigma_1 \frac{\mu_2}{\mu_1} \right) \right\} \left(\mu_1 + \mu_2 \frac{\sigma_1}{(-\rho \sigma_2)} \right) + \frac{\mu_1 \mu_2}{(-\rho \sigma_1 \sigma_2)} - \frac{\Phi^{-1} \left(\frac{p_2}{\mu_2} \hat{MRS}_B \right)}{\xi \left(\Phi^{-1} \left(\frac{p_2}{\mu_2} \hat{MRS}_B \right) \right)}} \\ &= \frac{1}{a - p_2} \\ &> \frac{1}{p_2} \end{aligned}$$

Last is true as long as $p_2 > \bar{p}_2 = a/2$. So complementarity (i.e. $D(p_2)$ decreasing) for S^b low enough. Low ρ helps to have the limit a low. Also high γ_L helps: $a \left(\gamma_L \right)$. And finally we need to make sure that y_2 low, which is the case when S^b low enough in PE⁸³. This proves the safety multiplier in Partial Equilibrium.

⁸³Recall $\hat{MRS}_L = \frac{\Phi \left(\frac{\mu_u}{\sigma_u} \right) - \phi \left(\frac{\mu_u}{\sigma_u} \right) \gamma_L \left\{ \left(2 - \frac{\mu_u^2}{\sigma_u^2} \right) \rho_{uv} y_2 \sigma_2 + \left(\frac{1}{2} - \frac{\mu_u^2}{\sigma_u^2} \right) \sigma_u \right\}}{1 - \gamma_L \left\{ \left(1 - \Phi \left(\frac{\mu_u}{\sigma_u} \right) \right) \mu_u - \Phi \left(\frac{\mu_u}{\sigma_u} \right) \frac{\sigma_u^2}{\mu_2^2} \rho_{uv} \sigma_u + \frac{\sigma_u^2}{\mu_2^2} (y_2 \sigma_2 + \rho_{uv} \sigma_u) - \phi \left(\frac{\mu_u}{\sigma_u} \right) \right\}}$: $\rho_{uv} y_2 \sigma_2$ needs to be small enough to make sure the

B.4.2 Risky asset holdings x_1

Assumption 1 allows to work in the very high μ_1 (relative to μ_2 and μ_u) neighborhood. The analytical expression of $x_1 = -b + \sqrt{b^2 + 4c}$ then gives, after algebra:

$$x_1 = n^B \left\{ \frac{(1+\rho)\sigma_2 p_2}{\sigma_1 \Sigma} (\sigma_2 p_1 - \rho \sigma_1 p_2) - \left[2 \frac{(1+\rho)p_2^2}{\sigma_1 \Sigma} - \frac{\sigma_1}{\Sigma} \frac{1-\rho}{1+\rho} \right] \mu_1 \frac{\lambda_u}{n^B} \right\}$$

- The convexity of x_1 with respect to n^B drives the procyclicality of bank leverage in this environment. Using the equilibrium: $\lambda_u = \sqrt{\delta} n^B$:

$$x_1(\mu_1; p_2) = n^B \left\{ \frac{(1+\rho)\sigma_2 p_2}{\sigma_1 \Sigma} (\sigma_2 p_1 - \rho \sigma_1 p_2) + \left[\frac{\sigma_1}{\Sigma} \frac{1-\rho}{1+\rho} - 2 \frac{(1+\rho)p_2^2}{\sigma_1 \Sigma} \right] \mu_1 \sqrt{\delta} \right\}$$

The elasticities of the partial equilibrium risky asset holding are as following:

- $\partial_{\mu_1} x_1 = n^B \left[\frac{\sigma_1}{\Sigma} \frac{1-\rho}{1+\rho} - 2 \frac{(1+\rho)p_2^2}{\sigma_1 \Sigma} \right] \sqrt{\delta} > 0$ for $\rho < 0$

$$\partial_{p_2} x_1 = \frac{1}{\Sigma^2} \left\{ \frac{(1+\rho)\sigma_2}{\sigma_1} (\sigma_2 p_1 - 2\rho \sigma_1 p_2) [\sigma_2^2 p_1^2 - \sigma_1^2 p_2^2] - \left[\sigma_1 \frac{1-\rho}{1+\rho} \partial_{p_2} \Sigma + 2 \frac{(1+\rho)}{\sigma_1} p_2 [2\sigma_2^2 p_1^2 + (1-2\rho-\rho^2)\sigma_1 \sigma_2 p_1 p_2] \right] \mu_1 \sqrt{\delta} \right\}$$

- We already see that $\partial_{p_2} x_1 < 0$ in the neighborhood of $\rho = -1$. Denoting $u = \frac{\sigma_1 p_2}{\sigma_2 p_1}$, this condition writes: $\partial_{p_2} x_1 < 0 \Leftrightarrow \sigma_2 \left(\frac{1}{u} + 2 \right) [1 - u^2] < 4$ which is true as long as $\frac{\sigma_2^2}{2} < u < 1$. So this refinement of Assumption 1 prevents from imposing *any condition* on ρ to get Proposition 2: $\partial_{p_2} x_1 < 0$.
- Finally the second order condition: $\partial_{p_2} \partial_{\mu_1} x_1 < 0$ to check convexity of the problem:

$$\partial_{p_2} \partial_{\mu_1} x_1 = - \left[\sigma_1 \frac{1-\rho}{1+\rho} \partial_{p_2} \Sigma + 2 \frac{(1+\rho)}{\sigma_1} p_2 [2\sigma_2^2 p_1^2 + (1-2\rho-\rho^2)\sigma_1 \sigma_2 p_1 p_2] \right] \sqrt{\delta} < 0$$

B.4.3 Safe asset holdings x_2

At equilibrium:

$$x_2 = \frac{\mu_1}{(-\rho \sigma_1 \sigma_2)} \sqrt{\delta} n^B + \frac{\sigma_1}{(-\rho \sigma_2)} \frac{\mu_1^2}{\mu_2 \sigma_1} \left\{ \frac{\mu_1}{\sigma_1} \sqrt{\delta} - \left(\sigma_2 + \sigma_1 \frac{\mu_2}{\mu_1} \right) \right\} \sqrt{\delta} n^B$$

Inspection of $x_2(\mu_u, \sigma_u)$ gives $x_2 \left(\begin{smallmatrix} \rho \\ (+) \end{smallmatrix}, \begin{smallmatrix} p_2 \\ (+) \end{smallmatrix} \right)$. So in PE safe asset holdings x_2 *increase* with high p_2 .

B.5 Normative analysis

B.5.1 Constrained efficient allocations

Consider the Lagrangian of the social planner under an arbitrary Pareto weights β

denominator dominates on the numerator.

$$\begin{aligned}
L &= \int -e^{-\gamma_L c_s^L} f(s) ds + \lambda \int c_s^B f(s) ds \\
&+ \int \mu_s \left\{ (x_1 + y_1) s - c_s^L - c_s^B \right\} f(s) ds \\
&+ \nu_s^L c_s^L + \nu_s^B c_s^B \\
&+ \mu_0 \left\{ n^L + n^B - p_1 (x_1 + y_1) \right\}
\end{aligned}$$

The f.o.c. in $i = (x_1 + y_1)$ is:

$$\int \mu_s s f(s) ds - \mu_0 p_1 = 0$$

The f.o.c. in c_s^L is:

$$\gamma_L e^{-\gamma_L c_s^L} f(s) ds - \mu_s f(s) ds + \nu_s^L = 0$$

The f.o.c. in c_s^B is:

$$f(s) ds - \mu_s f(s) ds + \nu_s^B = 0$$

Equating μ_s from last two, we get the equality of marginal utility of wealth in state s :

$$\gamma_L e^{-\gamma_L c_s^L} + \frac{\nu_s^L}{f(s) ds} = \lambda + \frac{\nu_s^B}{f(s) ds}$$

We easily derive the constrained first best allocation, denoting $\bar{s} = \frac{p_1}{i_{t-1}} \left[\frac{1}{\gamma} \ln \left(\gamma \frac{\beta^p}{\beta^A} \right) - i_t - \Sigma n^\theta \right]$:

$$\begin{cases} c_s^{L*} = \frac{1}{\gamma} \ln \left(\gamma \frac{\beta^p}{\beta^A} \right) \text{ and } c_s^{B*} = \frac{n^L + n^B}{p_1} s + i_{t-1} \frac{s_t}{p_K} - i_t - \frac{1}{\gamma} \ln \left(\gamma \frac{\beta^p}{\beta^A} \right) & \text{if } s > \bar{s} \\ c_s^{L*} = \frac{n^L + n^B}{p_1} s + i_{t-1} \frac{s_t}{p_K} - i_t \text{ and } c_s^{B*} = 0 & \text{if } s \leq \bar{s} \end{cases}$$

We recognize the risky debt as in the decentralized equilibrium of the within generation model in the case there is no long-term asset. This is the efficient risk-sharing agreement within generation.

The f.o.c. on investment gives:

$$c_s^{L*} \left[1 - \beta \int_{\bar{s}} s f(s) ds \right] = \beta \gamma e^{-\gamma \Sigma n^\theta} e^{\gamma i_t} \int^{\bar{s}} e^{-\gamma i_{t-1}} s f(s) ds$$

Jointly with the definition of \bar{s} and the efficient consumptions, the history $\{i_t\}$ of efficient levels of investment is well defined. As the equilibrium is recursive, we can write $i_t = a s_t$ and solve for a .

The indirect utilities derived with this allocations parametric in λ traces the Pareto frontier:

$$\begin{aligned}
V^L(\lambda) &= W^L \left(\left\{ c_s^{L*} \right\} \right) \\
&= \mathbb{E}_0 \left[\left(-e^{-\gamma_L \left(\frac{n^L + n^B}{p_1} s \right)} \right) 1_{\{s \leq \frac{1}{\gamma_L} \ln \left(\frac{\gamma_L}{\lambda} \right) \frac{p_1}{n^L + n^B}\}} + \left(-e^{-\gamma_L \left(\frac{1}{\gamma_L} \ln \left(\frac{\gamma_L}{\lambda} \right) \right)} \right) 1_{\{s > \frac{1}{\gamma_L} \ln \left(\frac{\gamma_L}{\lambda} \right) \frac{p_1}{n^L + n^B}\}} \right] \\
&= \int_{-\infty}^{\frac{1}{\gamma_L} \ln \left(\frac{\gamma_L}{\lambda} \right) \frac{p_1}{n^L + n^B}} \left(-e^{-\gamma_L \left(\frac{n^L + n^B}{p_1} s \right)} \right) f(s) ds + \int_{\frac{1}{\gamma_L} \ln \left(\frac{\gamma_L}{\lambda} \right) \frac{p_1}{n^L + n^B}}^{+\infty} \left(-e^{-\gamma_L \left(\frac{1}{\gamma_L} \ln \left(\frac{\gamma_L}{\lambda} \right) \right)} \right) f(s) ds
\end{aligned}$$

and

$$\begin{aligned}
V^B(\lambda) &= W^B\left(\left\{c_s^{B*}\right\}\right) \\
&= \mathbb{E}_0\left[(0) 1_{\left\{s \leq \frac{1}{\gamma_L} \ln\left(\frac{\gamma_L}{\lambda}\right) \frac{p_1}{n^L+n^B}\right\}} + \left(\frac{n^L+n^B}{p_1}s - \frac{1}{\gamma_L} \ln\left(\frac{\gamma_L}{\lambda}\right)\right) 1_{\left\{s > \frac{1}{\gamma_L} \ln\left(\frac{\gamma_L}{\lambda}\right) \frac{p_1}{n^L+n^B}\right\}}\right] \\
&= \int_{-\infty}^{\frac{1}{\gamma_L} \ln\left(\frac{\gamma_L}{\lambda}\right) \frac{p_1}{n^L+n^B}} (0) f(s) ds + \int_{\frac{1}{\gamma_L} \ln\left(\frac{\gamma_L}{\lambda}\right) \frac{p_1}{n^L+n^B}}^{+\infty} \left(\frac{n^L+n^B}{p_1}s - \frac{1}{\gamma_L} \ln\left(\frac{\gamma_L}{\lambda}\right)\right) f(s) ds
\end{aligned}$$

B.5.2 Public debt issuance

The Social Planner problem: which financial policy (S^k, τ^B, τ^L) maximizes the welfare of the investors under the competitive equilibrium.

Computation of the indirect utility of risk averse investors

$$W^L = -e^{-\gamma_L \mu_v + \frac{1}{2} \gamma_L^2 \sigma_v^2} \left\{ e^{-\gamma_L \mu_u + \frac{1}{2} \gamma_L^2 (\sigma_u^2 + 2\rho_{uv} \sigma_v \sigma_u)} \left\{ 1 - \Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L (\rho_{uv} \sigma_v + \sigma_u)\right) \right\} + \Phi\left(\frac{\mu_u}{\sigma_u} - \gamma_L \rho_{uv} \sigma_v\right) \right\}$$

Developing in orders of γ_L , we can write:

$$W^L = - \left\{ 1 - \gamma_L \left(y_2 \mu_2 + \bar{s} + \frac{1}{2} \mu_u - \frac{1}{2} \sqrt{\frac{2}{\pi}} \sigma_u \right) \right\}$$

Using lender budget:

$$W^L \propto \mu_2 \frac{n^L - D}{p_2} + \bar{s} + \frac{1}{2} \mu_u - \frac{1}{2} \sqrt{\frac{2}{\pi}} \sigma_u$$

Recall $D = \frac{p_2}{\mu_2} \hat{MRS}_B \bar{s}$:

$$W^L \propto r^{safe} n^L + (r^{bank} - r^{safe}) D + \frac{1}{2} \mu_u - \frac{1}{2} \sqrt{\frac{2}{\pi}} \sigma_u$$

We sign as follows:

$$W^L = \underset{(-)}{r^{safe}} n^L + \underset{(-)}{SMI} \underset{(-)}{D} + \underset{(+)}{0.5} \mu_u - \underset{(+)}{0.4} \sigma_u$$

So there is a trade-off: increasing p_2 worsens the wealth effect and the safety multiplier, but as bank takes less risk, it is also beneficial. There is a direct effect on n^L (a 'wealth effect': the first term $\underset{(-)}{r^{safe}} n^L$), and then there is the indirect SMI/safety creation effect (and additionally there is the cost of default: the two last terms). Both tends to call for more S^b , as long as ρ is low enough. Given that ρ is endogenous in the dynamic model, higher S^b leads to a higher ρ . As a result, there is an interior solution for S^b . Contrary to Lorenzoni-Werning (2013) and Calvo (1988), and in line with common practice, the government picks S^b and not $p_2 S^b$, so there is no Laffer curve, hence no multiple equilibria.

B.6 Extension with sovereign risk

B.6.1 Closed economy

Following the assumption made in the text the public debt market value is: $\tilde{p}_2^t = \kappa s^t 1_{\{s^t < \bar{s}\}} + p_2^t 1_{\{s^t \geq \bar{s}\}}$. We focus on Markov equilibria, defined exactly in the same way as in the main model. The covariance of the *post public default* price \tilde{p}_2^t with s^t is now:

$$\text{cov}(\tilde{p}_2^t, s^t) = \kappa \text{cov}(s^t 1_{\{s^t < \bar{s}\}}, s^t) + \text{cov}(p_2^t 1_{\{s^t \geq \bar{s}\}}, s^t)$$

We derive, denoting $s_1^{t=1} = \mu_1$, $h = \frac{\bar{s} - \mu_1}{\sigma_1}$:

$$\begin{aligned} \text{cov}(s^t 1_{\{s^t < \bar{s}\}}, s^t) &= \sigma_1^2 \Phi(h) - \{\sigma_1 h + \mu_1\} \sigma_1 \phi(h) \\ \text{cov}(p_2^t 1_{\{s^t \geq \bar{s}\}}, s^t) &= -\beta \sigma_1^2 (1 - \Phi(h)) + \{\alpha - \beta \mu_1 - \beta h \sigma_1\} \sigma_1 \phi(h) \end{aligned}$$

Therefore the total covariance is, using the linear approximation $p_2^t(s^t) = \alpha - \beta s^t$:

$$\text{cov}(\tilde{p}_2^t, s^t) = -\beta \sigma_1^2 + (\beta + \kappa) \sigma_1^2 \Phi(h) + \{\alpha - (\beta + \kappa) \mu_1 - (\beta + \kappa) \sigma_1 h\} \sigma_1 \phi(h)$$

We also have $\tilde{\mu}_2 = \alpha (1 - \Phi(h)) - \beta \mu_1 + (\beta + \kappa) \{\mu \Phi(h) - \sigma \phi(h)\}$ and $\tilde{\sigma}_2 = \beta \sigma_1$ in a low sovereign risk approximation.

Now, to solve for the fixed point, use the explicit expression of α and β : $\beta = -\frac{p_1}{S^b} \frac{(\mu_1 \tilde{\sigma}_2 - \tilde{\rho} \mu_2 \sigma_1)}{\tilde{\mu}_2 \sigma_1^3} \sqrt{\delta} n^B$ and $\alpha = \frac{n^B + n^L}{S^b}$ and $\text{cov}(\tilde{p}_2^t, s^t) = \tilde{\rho} \sigma_1 \tilde{\sigma}_2$.

As in the dynamic model with no sovereign risk, we eliminate $\tilde{\mu}_2$ and $\tilde{\sigma}_2$, and collate in $\tilde{\rho}$. In the low sovereign risk approximation to get rid of the ϕ terms:

$$\tilde{\rho} = -1 + \left(1 + \frac{\kappa}{\beta}\right) \Phi(h) \quad (30)$$

The right strategy is to add the sovereign term through the linear term and the quadratic term from the dynamic model, and only then solve for the fixed point. This gives:

$$\tilde{\rho} = -1 + \left(1 + \frac{\bar{\kappa}}{\beta}\right) \Phi(h) + \frac{16\mu^2}{\left(\mu - \frac{S^b}{p_1} \frac{1}{n^B \frac{1-\rho}{1+\rho} \sqrt{\delta} |\partial_{p_2} \kappa|}\right)^2}$$

with $\beta = \left|\frac{dp_2}{d\mu_1}\right| = \frac{p_1 \partial_{\mu_1} x_1}{S^b + p_1 \partial_{p_2} x_1}$ and $\partial_{\mu_1} x_1 = n^B \frac{\sigma_1}{\sigma_2^2 p_1^2} \frac{1-\rho}{1+\rho} \sqrt{\delta}$. So we write the new beta:

$$\tilde{\rho} + 1 = \left(1 + \frac{\bar{\kappa}}{\beta}\right) \Phi(h) + \frac{16\mu^2}{\left(\mu - \frac{S^b}{p_1} \frac{1}{n^B \frac{1-\rho}{1+\rho} \sqrt{\delta} |\partial_{p_2} \kappa|}\right)^2}$$

Doing the same manipulation as in the no sovereign environment, denoting $u = \tilde{\rho} + 1$:

$$u = \left(1 + \frac{\bar{\kappa}}{\beta} \frac{\partial_{p_2} \kappa}{\kappa} \left(\mu_1 - \frac{S^b}{p_1} \frac{1}{n^B \frac{1-\rho}{1+\rho} \sqrt{\delta} |\partial_{p_2} \kappa|}\right)\right) \Phi(h) + \frac{16\mu^2}{\left(\mu - \frac{S^b}{p_1} \frac{1}{n^B \frac{1-\rho}{1+\rho} \sqrt{\delta} |\partial_{p_2} \kappa|}\right)^2}$$

Of the same manner as in the no sovereign risk model:

$$\tilde{\rho} = -1 + \frac{\left(\frac{S^b}{n^B} \frac{[\hat{\sigma}_2^2 p_1^2 + \sigma_1^2 \hat{\rho}_2^2]^2}{2\sigma_1^4 p_1^2 \hat{\rho}_2^2} \right)^2 + \left(1 + \kappa \frac{1}{\tilde{\rho}_2} \left(s^t - \frac{S^b}{n^B} \frac{[\hat{\sigma}_2^2 p_1^2 + \sigma_1^2 \hat{\rho}_2^2]^2}{2\sigma_1^4 p_1^2 \hat{\rho}_2^2} \right) \right) \Phi \left(\frac{\bar{s} - s^t}{\sigma_1} \right)}{1 + \kappa \frac{1}{\tilde{\rho}_2} \frac{S^b}{n^B} \frac{[\hat{\sigma}_2^2 p_1^2 + \sigma_1^2 \hat{\rho}_2^2]^2}{2\sigma_1^4 p_1^2 \hat{\rho}_2^2} \Phi \left(\frac{\bar{s} - s^t}{\sigma_1} \right)}$$

We directly observe that increasing \bar{s} (sovereign risk through h) increases $\tilde{\rho}$ and therefore destroys the endogenous hedging properties of public debt.

B.6.2 Open economy

In the Open Economy setup, we have:

$$\sigma_u^2 = x_1^2 \sigma_1^2 + 2\rho x_1 x_2 \sigma_1 \sigma_2 + x_2^2 \sigma_2^2 + x_3^2 \sigma_3^2 + 2\rho x_1 x_3 \sigma_1 \sigma_3$$

$$\mu_u = x_1 \mu_1 + x_2 \mu_2 + x_3 \mu_3 - \bar{s}$$

It adds a third f.o.c on bank:

$$\begin{aligned} MRS_B^S &= \frac{\frac{\partial W^B}{\partial x_2}}{\frac{\partial W^B}{\partial x_1}} = \frac{\mu_2 \Phi \left(\frac{\mu_u}{\sigma_u} \right) + \sigma_2 \frac{x_2 \sigma_2 + \rho_2 x_1 \sigma_1}{\sigma_u} \phi \left(\frac{\mu_u}{\sigma_u} \right)}{\mu_1 \Phi \left(\frac{\mu_u}{\sigma_u} \right) + \sigma_1 \frac{(x_1 \sigma_1 + \rho_2 x_2 \sigma_2 + \rho_3 x_3 \sigma_3)}{\sigma_u} \phi \left(\frac{\mu_u}{\sigma_u} \right)} \\ MRS_B^N &= \frac{\frac{\partial W^B}{\partial x_3}}{\frac{\partial W^B}{\partial x_1}} = \frac{\mu_3 \Phi \left(\frac{\mu_u}{\sigma_u} \right) + \sigma_3 \frac{x_3 \sigma_3 + \rho_3 x_1 \sigma_1}{\sigma_u} \phi \left(\frac{\mu_u}{\sigma_u} \right)}{\mu_1 \Phi \left(\frac{\mu_u}{\sigma_u} \right) + \sigma_1 \frac{(x_1 \sigma_1 + \rho_2 x_2 \sigma_2 + \rho_3 x_3 \sigma_3)}{\sigma_u} \phi \left(\frac{\mu_u}{\sigma_u} \right)} \end{aligned}$$

So in the OE we get:

$$(p_1 \sigma_2 \rho_2 - p_2 \sigma_1) x_1 \sigma_1 + (p_1 \sigma_2 - p_2 \sigma_1 \rho_2) x_2 \sigma_2 + (0 - p_2 \sigma_1 \rho_3) x_3 \sigma_3 = (p_2 \mu_1 - p_1 \mu_2) \sigma_u \frac{\Phi \left(\frac{\mu_u}{\sigma_u} \right)}{\phi \left(\frac{\mu_u}{\sigma_u} \right)}$$

Similarly on asset 3:

$$(p_1 \sigma_3 \rho_3 - p_3 \sigma_1) x_1 \sigma_1 + (0 - p_3 \sigma_1 \rho_2) x_2 \sigma_2 + (p_1 \sigma_3 - p_3 \sigma_1 \rho_3) x_3 \sigma_3 = (p_3 \mu_1 - p_1 \mu_3) \sigma_u \frac{\Phi \left(\frac{\mu_u}{\sigma_u} \right)}{\phi \left(\frac{\mu_u}{\sigma_u} \right)}$$

Second object is the Marginal Rate of Transformation between Asset 1 and leverage (promise \bar{s}):

$$D'(\bar{s}) = \frac{p_1}{\mu_1 + \sigma_1 \frac{(x_1 \sigma_1 + \rho_2 x_2 \sigma_2 + \rho_3 x_3 \sigma_3)}{\sigma_u} \frac{\phi \left(\frac{\mu_u}{\sigma_u} \right)}{\Phi \left(\frac{\mu_u}{\sigma_u} \right)}}$$

So the asset side correlation metrics is now:

$$X = \frac{(x_2 \sigma_2 + \rho_2 x_1 \sigma_1)}{(x_1 \sigma_1 + \rho_2 x_2 \sigma_2 + \rho_3 x_3 \sigma_3)}$$

and we still have:

$$D'(\bar{s}) = \frac{p_2}{\mu_2} \frac{1 - \frac{p_1}{p_2} \frac{\sigma_2}{\sigma_1} X}{1 - \frac{\mu_1}{\mu_2} \frac{\sigma_2}{\sigma_1} X}$$

So in the open economy model, all the results go through as long as we use the new X . Consider the South bank:

$$\begin{cases} x_1 p_1 + x_2 p_2 + x_3 p_3 & = n^B + \frac{p_2}{\mu_2} \hat{MRS}_B (x_1 \mu_1 + x_2 \mu_2 + x_3 \mu_3 - \mu_u) \\ (p_1 \sigma_2 \rho_2 - p_2 \sigma_1) x_1 \sigma_1 + (p_1 \sigma_2 - p_2 \sigma_1 \rho_2) x_2 \sigma_2 + (0 - p_2 \sigma_1 \rho_3) x_3 \sigma_3 & = (p_2 \mu_1 - p_1 \mu_2) \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} \\ (p_1 \sigma_3 \rho_3 - p_3 \sigma_1) x_1 \sigma_1 + (0 - p_3 \sigma_1 \rho_2) x_2 \sigma_2 + (p_1 \sigma_3 - p_3 \sigma_1 \rho_3) x_3 \sigma_3 & = (p_3 \mu_1 - p_1 \mu_3) \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} \end{cases}$$

Invert the 2x2 system in $(x_2 \sigma_2, x_3 \sigma_3)$:

$$\begin{cases} (p_1 \sigma_2 - p_2 \sigma_1 \rho_2) x_2 \sigma_2 + (0 - p_2 \sigma_1 \rho_3) x_3 \sigma_3 & = (p_2 \mu_1 - p_1 \mu_2) \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} - (p_1 \sigma_2 \rho_2 - p_2 \sigma_1) x_1 \sigma_1 \\ (0 - p_3 \sigma_1 \rho_2) x_2 \sigma_2 + (p_1 \sigma_3 - p_3 \sigma_1 \rho_3) x_3 \sigma_3 & = (p_3 \mu_1 - p_1 \mu_3) \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} - (p_1 \sigma_3 \rho_3 - p_3 \sigma_1) x_1 \sigma_1 \end{cases}$$

Tedious algebra (matrix notation for general N sovereign bonds) leads to:

$$x_2 \sigma_2 = \Sigma_2^u \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} + \Sigma_2^1 x_1 \sigma_1$$

with $\Sigma_2^u = p_2 p_1 (\sigma_3 \mu_1 - \sigma_1 \rho_3) - p_1^2 \sigma_3 + p_1 p_3 \sigma_1 \rho_3$ and $\Sigma_2^1 = p_1 p_2 \sigma_1 \sigma_3 (1 - \rho_3^2) - p_1^2 \sigma_3 \sigma_2 \rho_2 + p_1 p_3 \sigma_1 \sigma_2 \rho_2 \rho_3$.

By symmetry

$$x_3 \sigma_3 = \Sigma_3^u \sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)} + \Sigma_3^1 x_1 \sigma_1$$

Get now back to bank budget and using $\hat{MRS}_B = \frac{1 - \frac{p_1}{p_2} \frac{\sigma_2}{\sigma_1} X}{1 - \frac{\mu_1}{\mu_2} \frac{\sigma_2}{\sigma_1} X}$ and $X = \frac{(x_2 \sigma_2 + \rho_2 x_1 \sigma_1)}{(x_1 \sigma_1 + \rho_2 x_2 \sigma_2 + \rho_3 x_3 \sigma_3)}$, it defines the equilibrium. The equilibrium $\mu_u \sigma_u$ is the same as in the closed economy (the debt-pricing mapping, so far the two banks are symmetric.). This pins down a $x_1(\mu_u, \sigma_u)$.

Perturbation argument. Consider South bond loosing its hedging property: $\rho_3 < \rho_2 = 0$ and higher volatility $\sigma_2 > \sigma_3$. This amounts to a higher γ_L so at the margin, lower μ_u and higher σ_u . It is less expensive to lever up for a South bank. From the eq $x_1(\mu_u, \sigma_u)$, more investment x_1 . From the x_2 and x_3 expressions (as $\sigma_u \frac{\Phi\left(\frac{\mu_u}{\sigma_u}\right)}{\phi\left(\frac{\mu_u}{\sigma_u}\right)}$ is pinned down by the closed economy equilibrium), an increasing x_1 crowds in x_2 and x_3 (to hedge against default). The sensitivity of which one increases more is commanded by $\frac{\Sigma_2^1}{\Sigma_2^u}$ and $\frac{\Sigma_3^1}{\Sigma_3^u}$. To make x_2 increase more, despite σ_2 increasing, it suffices to make $\Sigma_2^1 > \Sigma_3^1$. This is the case as ρ_3^2 decreases, making Σ_2^1 increasing more than Σ_3^1 . As result, x_2^{South} increases more at the margin than x_2^{North} . Starting from the symmetric equilibrium, we obtain redomestication of public debt.

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