Leverage and Reputational Fragility in Credit Markets*

Charles-Henri Weymuller
Harvard University

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Abstract

This paper analyzes the equilibrium on secured debt markets with heterogeneous borrowers. I develop a bargaining model of endogenous leverage when debt capacity depends not only on the nature of collateral but also on borrower's reputation. Agents with higher reputation secure *both* more and cheaper credit. As a result, haircuts and rates positively comove in the cross-section, a prediction that cannot be delivered in a model where the heterogeneity among borrowers is about collateral, such as beliefs. I endogeneize reputation by identifying it to continuation value in a repeated borrowing game. Leverage is procyclical because of the endogeneity of reputation. Leverage is high and stable when agents are sufficiently capitalized. However, as their net worth depletes, reputation vanishes in a non-linear way, and leverage then is limited and unstable. Reputation acts as a stabilization mechanism in good times but as an amplification mechanism in bad times. In this dynamic environment, long-term contracts stabilize leverage by avoiding margin calls in the states in which the borrower has low net worth. Reputation also provides an economic rationale to a long intermediation chain, in order to monetize intermediaries franchise.

The empirical analysis on repo transactions of money market funds confirms the predictions of the model. First, financial intermediaries with high reputation write repo contracts with lower haircuts and lower rates. Second, haircuts are more sensitive to borrower's reputation when borrower's net worth is low. Third, long-term relationships stabilize funding. It provides evidence that reputation and bilateral relationships matter even in secured funding markets.

JEL numbers: G12, G23, G32, D8, L14.

Keywords: leverage cycle, beliefs heterogeneity, secured funding, risk sharing, franchise value, long-term contracts, money market funds.

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1 Introduction

This paper investigates the determinants of the leverage of financial intermediaries through short-term secured funding markets. The sudden freeze of such markets in 2008 was one of the most surprising and unexpected developments of the financial crisis.[1] Secured debt markets were thought to be immune from traditional bank runs, as each contract is collateralized by a distinct pool of assets. I develop a model where the characteristics of collateral interacts with borrowers' reputation and with the history of the borrower-lender relationship. This model rationalizes the cross-section of secured debt contracts, and provides a narrative for sudden freezes in secured debt markets based on the endogeneity of reputation.

Theories of debt capacity are conventionally asset-specific.[2] In Shleifer and Vishny (1992), debt capacity of an asset is its liquidation value. Similarly, in models of endogenous margins such as Geanakoplos (2003) and Simsek (2011), debt capacity only depends on beliefs about the asset which collateralizes the debt contract. However, Figures 1 and 2 suggest that not only the collateral but also broader attributes of the relationship between the borrower and the lender are priced in. The dataset used in this paper includes all the tri-party repo transactions between 2006 and 2012 involving one of the 145 largest money market funds. Repurchase agreements (repo) contracts are secured debt contracts. Their defining feature is that *two* pricing terms are contracted upon: not only the compensation (the interest rate), but also the protection (the haircut). The rate is defined as the ratio of the ex post promise $\bar{s}$ on the amount of cash raised ex ante D: $1 + r = \frac{\bar{s}}{D}$. The haircut is computed as one minus the ratio of ex ante cash raised D on the ex ante market value of collateral p: $m = 1 - \frac{D}{p}$. Figure 1 plots haircuts and rates in the cross-section of borrowers. It illustrates that borrowers whose perceived creditworthiness by the market - i.e. reputation - is better secure repo contracts that are favorable on the two pricing dimensions: lower rates *and* lower haircuts.

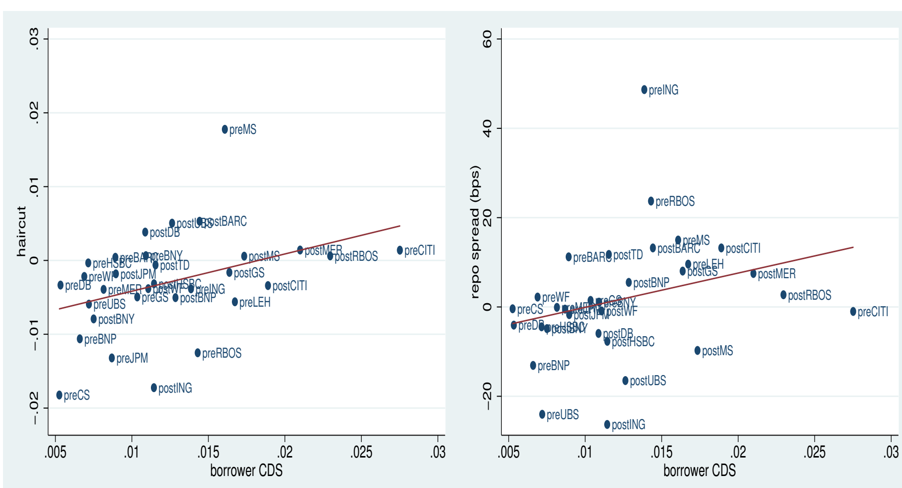

Figure 1: Haircuts and rates in the cross-section of *borrowers*; orthogonalized on time, collateral and lender fixed effects.

[1]See for instance Brunnermeier (2009) and Gorton and Metrick (2010).

[2]Debt capacity and loan-to-value are equivalent concepts, i.e. the amount that can be borrowed against an asset or a security. One minus the haircut (or margin) measures debt capacity.

Figure 2 conducts the same type of exercice in the cross-section of lenders. It illustrates that lenders with more outside options secure repo contracts with more favorable terms: higher rates *and* higher haircuts.

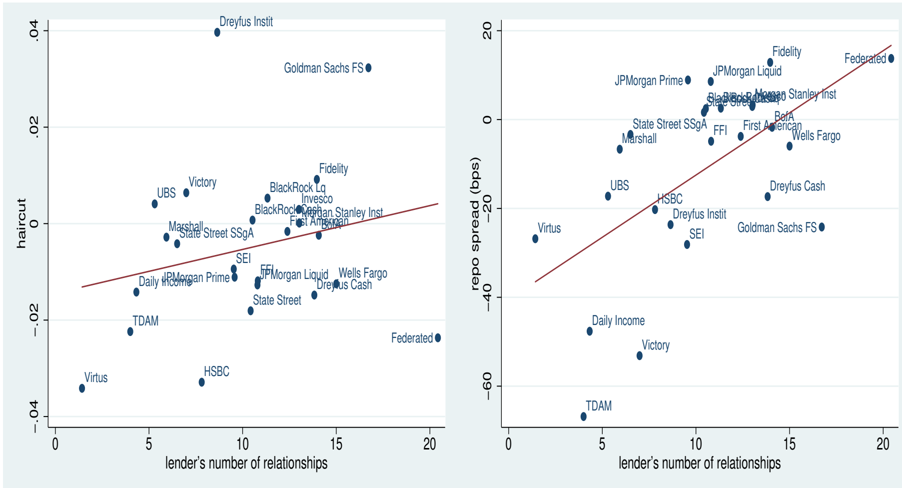

Figure 2: Haircuts and rates in the cross-section of *lenders*; orthogonalized on time, collateral and borrower fixed effects.

I show in this paper that a positive comovement of haircuts and rates in the cross-section of borrowers can only be rationalized by a dimension of heterogeneity that is not about the collateral. Models using a continuum of beliefs generically predict that a negative correlation between haircuts and rates. On the contrary, in an environment that explicitely takes into account reputation heterogeneity, borrowers with high reputation do enjoy lower haircuts and lower rates. To make this point, I develop a model of multilateral Nash bargaining between I optimists and J pessimists, and I consider two cases: one where optimists only differ according to their beliefs on collateral riskiness, and one where optimists only differ according to their reputation. Only the equilibrium of the second case features a positive comovement of haircuts and rates in the cross-section of borrowers.

I then turn to the dynamic version of the model in order to provide foundations for reputation. I identify reputation as the continuation value of any agent in the infinitely repeated version of the static game. In this environment, agents net worth are the only state variables and reputation can then be solved as a function of net worth. The second result of the paper is that *only with endogenous reputation* leverage can be procyclical with respect to net worth. The intuition is as follows. Reputation is a non-linear function of net worth: it vanishes at a fast pace when net worth gets close to zero. As reputation supports agents debt capacity, equilibrium leverage gets slashed for low levels of net worth. Such dynamics provides an explanation for the 'run on repo' documented in Gorton and Metrick (2010) without resorting to any coordination failure narrative. The third result is that long-term contracts are valuable in this environment, as they insulate borrowers from margin calls when they have low net worth, precisely the states where they would need to lever up to replenish their capital.

Finally, the fourth and last theoretical result is delivered by an extension of the model featuring *repo chain*. I consider an environment with three types of agents: optimists without franchise value 'Hedge

Funds', optimists with franchise value 'Broker-Dealer' and pessimists 'Money Markets'. I show that Broker Dealers emerge as financial intermediaries: they rehypothecate collateral, as they are the only ones capable of 'monetizing' their franchise value. I show that Broker-Dealer leverage is more stable than Hedge-Fund leverage in this equilibrium, consistent with the stylized facts.

The empirical analysis of the repo market confirms the first three theoretical results. I use a hand-collected dataset of repurchase agreement transactions between money market funds as lenders and broker dealers as borrowers. The dataset is constructed, following Krishnamurthy, Nagel and Orlov (2011), by parsing the quarterly SEC filings. The only difference on data source is that I parse for the 145 largest prime institutional money market funds but for no securities lenders. This choice is motivated by the focus of the present paper on the price determinants in the cross-section of borrowers, whereas Krishnamurthy, Nagel and Orlov (2011) focuses on the aggregate time-series of repo funding. In the dataset, borrowers with high reputation, proxied by a low CDS, do secure lower rates and lower haircuts (prediction 1). I also identify the reputation channel by the exposure to the European crisis: controlling for collateral, borrowers that were more exposed to Europe did experience higher haircuts even on US collateral. Moreover, in the time-serie, borrowers with low net worth are the ones that face more volatile haircuts (Prediction 2). Finally, borrowers that enjoy long-term relationships with money market funds are more insulated from haircut volatility (Prediction 3). The last two findings can help reconciling Gorton and Metrick (2010), which document dramatic increases in haircuts (with data coming from bilateral repos), with, on the other hand, Copeland, Martin and Walker (2010) which show that haircuts were by and large unresponsive over the crisis (with data coming from tri-party repos).

Related literature. The corporate finance literature traditionally features agency frictions to model limited pledgeability, which is tantamount to a leverage constraint. The agency friction can be moral hazard, such as in Holmstrom and Tirole (1997) and Adrian and Shin (2012), or informational asymmetries about the quality of collateral, such as Dang, Gorton and Holmstrom (2011a). On the contrary, Geanakoplos (2003) and Simsek (2011) rely on beliefs heterogeneity to derive an optimal level of leverage. As I focus in this paper on the debt capacity of financial intermediaries, I choose to cast my model in the latter piece of literature.[3] The key feature of my model is that franchise value cannot be seized by the lenders. It draws from the limited commitment literature, following Hart and Moore (1994) and Albuquerque and Hopenhayn (2004). However, in my model, parties can commit to the debt contract; which is priced taking into account the fact that the franchise value relaxes the default constraint. Due to beliefs heterogeneity, the optimal contracting problem is more akin to an ex ante risk-sharing agreement, in the spirit of Kocherlakota (1996), Lustig (2000) and Alvarez and Jermann (2000). Contrary to the latter, my model features equilibrium default. Beyond beliefs heterogeneity, the only friction in my model is the multilateral Nash bargaining that arises between borrowers and lenders; contractual externalities arise from the collateral constraint, which is shared by all the lenders. Multilateral Nash bargaining is inspired by Stole and Zwiebel (1996),[4] and makes the outside

[3]Given their dispersed lending base, it is hard to think of financial intermediaries as being disciplined by the lenders from a moral hazard friction, or cash stealing friction a la Bolton and Scharfstein (1990) and Bolton and Scharfstein (1996). Similarly, Dang, Gorton and Holmstrom (2011a) microfounds the haircut to prevent information acquisition from a potential lender more sophisticated than the borrower. In the case of repo markets and bank wholesale funding, the ultimate lenders are money market funds and passive money, whose key characteristics seem to be their relative pessimism or risk aversion.

[4]Such friction is also featured in Ausubel and Deneckere (1993), Acemoglu et al. (2007) and Goldberg and Tille (2012) in the trade context. The alternative would have been a search and matching set up. The latter makes sense between segmented capital markets as in (Duffie and Strulovici, forthcoming) but is harder to justify on liquid wholesale funding markets such as repo.

options endogenous. This feature of my model is shared with Krueger and Uhlig (2006), Ghosh and Ray (1996), Phelan (1995), and Rampini and Viswanathan (2012). The optimal long-term contract I derive is a long-term risk-sharing agreement, similar to the labor literature as Harris and Holmstrom (1982), Thomas and Worrall (1988), and not a dynamic contract that helps to mitigate an agency friction as in Biais et al. (2007).

He and Xiong (2012a) also considers a dynamic environment with beliefs heterogeneity and rollover risk to analyze the optimal maturity choice. Differing from their set up, I model rollover risk as being *institution-specific* and not *asset-specific*, and I take into account imperfect competition among lenders. This enables me to capture the idiosyncratic dispersion of repo contracts for a given type of collateral. Geanakoplos and Fostel (2011) also features pricing dispersion of repo contracts, at the cost of introducing a continuum of beliefs types. In my model, two borrowers with the same beliefs but different franchises will have the repo priced differently. Acharya, Gale and Yorulmazer (2011) is a model of dynamic debt capacity of a long-term asset in presence of small but frequent rollover risk.[5] Contrary to their set up, the risky asset I consider can be short-term. Only institution-specific characteristics (franchise and relationships) are long-term. In my model, impaired debt capacity here arises from the endogeneity of borrower franchise value. Oehmke (2012) analyzes the disorderly liquidation of illiquid collateral. He focuses on the dynamics conditional on the default, whereas I analyze the ex ante choice of leverage under imperfect competition. Brunnermeier and Pedersen (2008) develops a theory of margin spirals based on the feedback loop between market liquidity and funding liquidity, based on asymmetryc information. Their mechanism relies on margins set by Value-at-Risk constraints and information extraction for volatility estimates. In my paper, the collateralized debt contract is an optimal contract between heterogeneous agents.[6]

Dynamic models of capital structure traditionally take the size of the balance sheet as exogenous in order to analyze the debt-equity tradeoff.[7] On the contrary, I take the level of net worth (equity) as given and analyze the optimal size of the balance sheet of financial intermediaries. In the dynamic version of the model, the balance sheet size is a control variable and equity is the state variable, i.e. the reverse of traditional models of optimal capital structure. Overall, the present model seems more relevant to analyze the dynamics of leverage of financial institutions whose choice variable is the size of the balance sheet, whereas traditional models of endogenous capital structure seem more relevant for non-financial firms. Finally, the theoretical model developed here implies that high franchise values take on more risk and leverage. This is the reverse of the traditional literature on charter value and reputation building, e.g. Stiglitz and Weiss (1983), Diamond (1989), Keeley (1990), Hellmann et al. (2000), Allen and Gale (2004) and Carletti et al. (2007). In these models, as franchise is destroyed in default, strong franchises take on less risk. Similarly, relationship banking usually hinges on asymmetric information whereas in my model it arises from imperfect competition among lenders.

In the macroeconomics literature, the lender not only cares about the liquidation value he can extract from collateral as in corporate finance, but also about the fact that the borrower cares about the loss of collateral, as in the sovereign debt literature. My model features both aspects of collateral. Gertler and

[5]Other papers analyzing rollover risk are He and Xiong (2012b) and Eisenbach (2011). They usually feature perfect credit markets on banks' liability side. My Industrial Organization set up can be seen as a way to microfound the imperfect credit line, which breaks at a random Poisson random time, in He and Xiong (2012b).

[6]In Brunnermeier-Ohemke (2012) language, the two liquidity spirals, the loss spiral and the haircut spiral, are jointly determined. This model, in its General Equilibrium version (section 6), provides a framework to analyze their interaction.

[7]Following Merton (1974), Leland (1994) and Leland and Toft (1996).

Kiyotaki (2012) introduces rollover risk in a macro model, but features unsecured debt (deposits) and their leverage constraint hinges on an agency friction (cash diversion). The dynamic model developed in this paper delivers procyclical leverage, contrary to the state-independent leverage of Kiyotaki and Moore (1997) and in accordance with empirical evidence of Adrian and Shin (2010).[8] Compared to Adrian and Boyarchenko (2012), I only have one shock that commands both the asset risk and counterparty risk, and the procyclicality of leverage in my model arises from the endogeneity of franchise value, whereas in their model it arises from a risk-sensitive Value-at-risk constraint.

On the empirical side, Gorton and Metrick (2010), Copeland et al. (2010) and Krishnamurthy et al. (2011) document the behavior of repo markets over the crisis.[9] They focus on the aggregate behavior, whereas I analyze the idiosyncratic component of repo contracts. Duffie and Ashcraft (2007), Afonso et al. (2011), Kuo et al. (2012), Soramaki et al. (2010) and Chernenko and Sunderam (2012) analyze funding conditions on unsecured debt markets. My empirical results complement these studies by showing that idiosyncratic components are priced in secured debt. Kacperczyk and Schnabl (2012) demonstrates that money market funds are heterogeneous in their risk-taking behavior, and that concerns for sponsors' franchise mitigate their risk-taking. Their study taken together with the present one suggests an ambiguous effect of franchise along the intermediation chain: intermediaries closer to the ultimate borrowers use their franchise to lever up more and take on more risk, whereas intermediaries closer to the ultimate lenders (households) use their franchise as a commitment device to take on less risk, and thus mitigating agency frictions.

The rest of the paper is organized as follows. Section 2 develops a static model of endogenous leverage with heterogeneous borrowers. Section 3 extends the model to a dynamic environment by endogeneizing the franchise value. Section 4 analyzes the rehypothecation chain. Section 5 turns to the empirical analysis and tests the three key predictions of the model. Section 6 sketches the normative implications and concludes.

2 The static model

The goal of this section is to develop a model of repo markets with endogenous prices and quantities of debt contracts, when there are multiple heterogeneous borrowers that can differ whether through their beliefs or through their reputation. There are 3 periods: $t = 0, 1, 2$.

2.1 Environment

Agents The economy is populated by two types of agents: there is a discrete number I of type-B agents and a discrete number J of type-L agents. There is one risky asset: investing in one unit of the asset costs p at $t = 0$ and yields a stochastic payoff $s \in \mathbb{S}$ at $t = 1$.[10] The type-B agents are risk-neutral optimists, each endowed with net worth n^B. Their prior distribution of beliefs on the dividend is of density $f_B(s)$ and cumulative distribution function $F_B(s)$. The type-L agents are risk-neutral pessimists, each endowed with net worth n^L. Their prior beliefs are noted $f_L(s)$ for the density and $F_L(s)$ their cumulative distribution function. The type-L

[8] Acharya and Viswanathan (2011) also delivers procyclical leverage, but with a moral hazard friction.

[9] Compared to Copeland et al. (2010), my dataset has interest rates information, which turn out to be instrumental for the test of the relationships theory. Compared to Krishnamurthy et al. (2011), I have more MMF families (45 against 19) and not exactly the same funds.

[10] p is first taken exogenous (perfectly elastic supply). The asset price is endogeneized in the General Equilibrium of section 2.3.

agents are pessimists in the sense that $\mathbb{E}_L[s] < \mathbb{E}_B[s]$.[11] I define the subjective expected returns μ^B and μ^L by: $\mathbb{E}_B[s] = p(1+\mu^B)$ and $\mathbb{E}_L[s] = p(1+\mu^L)$. Following Simsek (2011), I impose the following parameter restriction on the nature of beliefs heterogeneity.[12]

Assumption 1. *The heterogeneous beliefs of the agents satisfy the hazard rate property:* $\forall s \in \mathbb{S}\ \frac{f_L(s)}{1-F_L(s)} > \frac{f_B(s)}{1-F_B(s)}$

Assumption 1 states that optimists are even more optimistic compared to pessimists for high states of nature. Although most results hold under the general property of Assumption 1, I focus on normal beliefs: $f_L \sim N\left(p\left(1+\mu^L\right), \sigma\right)$ and $f_B \sim N\left(p\left(1+\mu^B\right), \sigma\right)$ with $\mu^L < \mu^B$ (Figure 6). Such beliefs are tractable and allows for elegant comparative statics with respect to collateral volatility σ.

Financial contracts I restrict the contracting set to collateralized debt: *repo contracts*. A repo contract is a non-state-contingent promise to pay $\bar{s}$ at $t = 2$, made by the seller of the contract (the borrower) to its buyer (the lender). This contract is secured, in the sense that if the borrower does not honor its promise to pay back $\bar{s}$, the lender can push him into default. He then recovers a pre-specified amount of the borrower portfolio in the risky asset. Thus a repo contract has three terms to contract upon: the promise $\bar{s}$, the amount of cash D that is lent at $t = 0$ by the seller to the buyer, and the amount of collateral. By linearity, we can normalize the contracting set to consider that all the repo contracts are collateralized by one unit of the risky asset. The two contract terms $\{\bar{s}, D\}$ are isomorphic to the two observables prices defined below.

Definition 1. The repo spread is: $r = \frac{\bar{s}-D}{D}$ and the haircut on the risky asset is: $m = \frac{p-D}{p}$.

The interest rate r on the contract captures the credit spread. Reciprocally, the value D of the contract can be written: $D = (1-m)\,p$ and the promise: $\bar{s} = (1+r)(1-m)\,p$. Given there is only one risky asset, the haircut m also captures leverage and credit supply.

Timeline The key feature of the model relies in the ability of all agents to pledge long-run revenues at anytime, but at a non attractive interest rate. Although agents will never use this possibility at $t = 0$, they will at $t = 1$ in some states in order to avoid default and save the remainder of their long-run revenues. The *reputation* of an agent is encoded in its ability to pledge long-term cash flows.[13] The exact timing is as follows and is illustrated on Figure 3.

- At $t = 0$, agents write repo contracts among themselves. Each pair of agents $(i, j) \in I \times J$ enters in x_{ij} units of a repo contract, whose terms are $\{r_{ij}, m_{ij}, 1\}$: a rate, a haircut and a number of asset units as collateral. These terms can be reframed as a duplet $\{\bar{s}_{ij}, D_{ij}\}$: a promise and an ex ante value. Prices $\{\bar{s}_{ij}, D_{ij}\}$ and quantities x_{ij} are determined jointly through multilateral Nash bargaining.

[11] This beliefs heterogeneity creates a rationale for contracting between the two types of agents. It a state-dependent utility heterogeneity, isomorphic to heterogeneity in risk aversion when there is only one asset. All theoretical results therefore transpose to heterogeneity in risk aversion.

[12] Only under this assumption, the debt contract traded in equilibrium is risky. The interest rate on the contract is then non-zero as a credit spread, and this enables to carry out the comparative statics on rates.

[13] It can also be thought as the Present Discounted Value of cash-flow generating divisions of the borrower which are not levered but heavily relies on the strength of the franchise value, such as M&A advisory. It can also be thought as the ability to rollover in states of nature.

- At $t = 1$, the asset shock s realizes and is observed by all agents. Agents i that face $\sum_{j \in J} x_{ij} s < \sum_{j \in J} x_{ij} \bar{s}_{ij}$ are in situation of distress. In this interim period, any agent can decide to pledge its long-run revenues V^{B_i}, albeit at an exogenous (high) rate r^*. I define $\beta = \frac{1}{1+r^*}$.
- At $t = 2$, contracts are settled, agents receive the share of long-revenues that have not been pledged at $t = 1$, and consume.

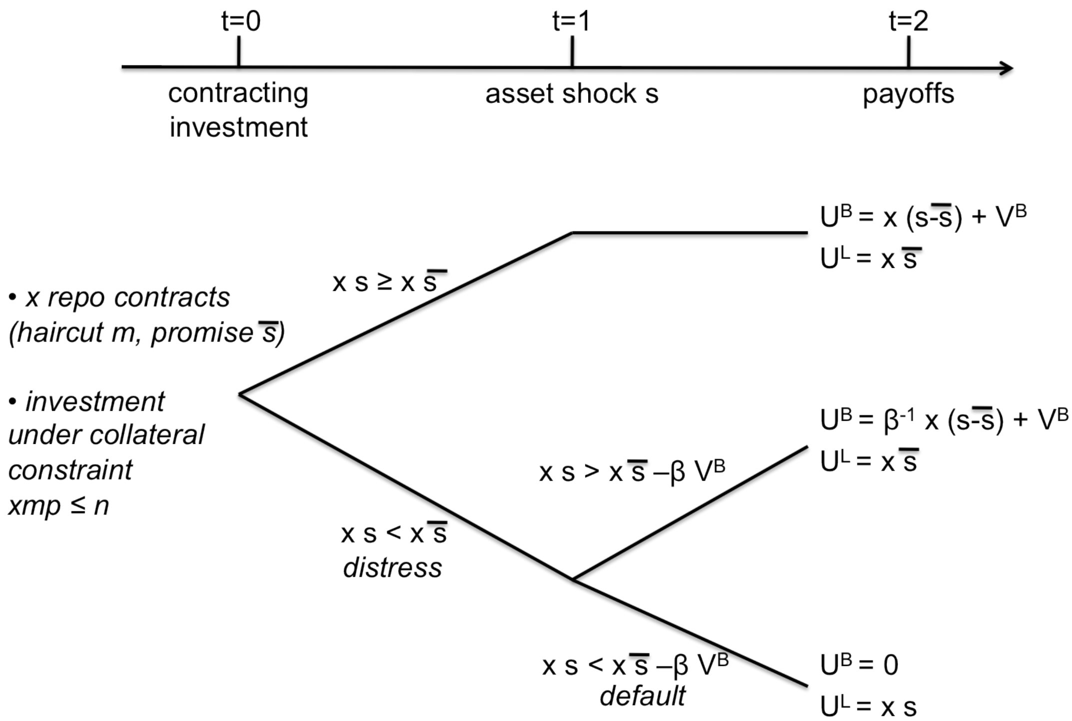

Figure 3: Timeline of the static model.

2.2 Equilibrium with deep pocket lenders

If the number J of lenders is large and deep pockets ($J\, n^L >> I\, n^B$), they Bertrand-compete away any surplus for them. This is equivalent to granting full bargaining to the borrowers.

Default decision At $t = 1$, borrowers have the option whether to default or to pledge part of their long-run revenues V^{B_i} at the rate r^*. This partitions the states into three regions:

- If $\frac{\sum_{j \in J} x_{ij} \bar{s}_{ij}}{\sum_{j \in J} x_{ij}} < s$, the agent i does not default.
- If $\frac{\sum_{j \in J} x_{ij} \bar{s}_{ij}}{\sum_{j \in J} x_{ij}} - \beta V^{B_i} \frac{1}{\sum_{j \in J} x_{ij}} < s < \frac{\sum_{j \in J} x_{ij} \bar{s}_{ij}}{\sum_{j \in J} x_{ij}}$, the agent pledges a part of its long-revenus and stays afloat.
- If $s < \frac{\sum_{j \in J} x_{ij} \bar{s}_{ij}}{\sum_{j \in J} x_{ij}} - \beta V^{B_i} \frac{1}{\sum_{j \in J} x_{ij}}$, the agent is forced into default.

Let denote $\tilde{s} = s^{def}\left(x_{ij}, \bar{s}_{ij}\right) = \frac{\sum_{j\in J} x_{ij}\bar{s}_{ij}}{\sum_{j\in J} x_{ij}} - \beta V^{B_i} \frac{1}{\sum_{j\in J} x_{ij}}$ the effective default threshold ('riskiness' of the contract), and $\left[\tilde{s}; \frac{\sum_{j\in J} x_{ij}\bar{s}_{ij}}{\sum_{j\in J} x_{ij}}\right]$ the light-distress region. In this region, the borrower is under water but can still survive thanks to the strength of its franchise value V^{B_i}. This allows for a region of the state space where the borrower has negative net worth at $t = 1$. This does not trigger default as the borrower is in effect able to run a simili-Ponzi scheme thanks to its franchise value.[14]

A key assumption is that V^B cannot be seized by the lenders in the states of borrower's default. In practice, repurchase agreement contracts are exempt from the automatic stay thanks to the safe harbor provision,[15] but they usually also are *recourse*: after having seized the collateral, the lender possesses an unsecured claim to the counterparty. I assume that the value of this recourse claim is zero:[16] the recourse feature does not imply any additional recovery for the lender, and V^{B_i} evaporates as soon as the borrower enters bankruptcy. The payoff profile of the lender under a repo contract with respect to the realization of the collateral is illustrated in Figure 7.

The following definition will prove useful in the derivations and for the economic interpretation. The wedge in the valuation of the put underpins the rationale for contracting between borrowers and lenders.

Definition 2. The valuations of the limited liability put with strike $\tilde{s}$, respectively by the type-B agents and by the type-L agents, are noted: $\pi^B(\tilde{s}) = \mathbb{E}_B\left[(\tilde{s} - s)^+\right] = \int_{s^{min}}^{\tilde{s}} F_B(s)ds$ and $\pi^L(\tilde{s}) = \mathbb{E}_L\left[(\tilde{s} - s)^+\right] = \int_{s^{min}}^{\tilde{s}} F_L(s)ds$.

Expected payoffs at $t = 0$ The expected utility of any borrower is the levered gains from the carry trade between investing and its borrowing costs. The franchise value introduces a concern to preserve this franchise value, which works as a continuation term. The expected utility of the borrower in the equilibrium, for a given number of contracts x_{ij}, is then:

$$\begin{aligned} U^B_{I,J} &= \sum_J x_{ij}\, \mathbb{E}_B\left[(s - \bar{s})\, 1_{\{no\, distress\}} + \beta^{-1}\,(s - \bar{s})\, 1_{\{light\, distress\}}\right] + \mathbb{E}_B\left[V^B 1_{\{no\, def\}}\right] \\ s.t. &\quad \sum_J x_{ij} m_{ij} p \le n^B \end{aligned} \tag{1}$$

If the borrower does not contract at all, it does not lever up and invests only its net worth in the risky asset, yielding $p(1 + \mu^B)$ under its beliefs:

$$U^B_0 = n^B\left(1 + \mu^B\right) + V^B$$

On the other hand, lenders expected utility are the returns on its portfolio of repo contracts and direct investment in the asset. The expected utility of the lender in the equilibrium, for given contracts x_{ij}, is:

$$U^L_{I,J} = \sum_I x_{ij}\, \mathbb{E}_L\left[1_{\{def\}} s + 1_{\{no\, def\}}\,(\bar{s}) - D\right] + \left(n^L - \sum_I x_{ij} D\right)(1 + \mu^L) \tag{2}$$

[14] This Ponzi-scheme arises rationally in equilibrium as there is a positive probability of recovery. It can be seen as the symmetric counterpart of rational bubbles of Abreu and Brunnermeier (2003).

[15] See for instance the bankruptcy court decision reported in Schweitzer et al. (2008).

[16] This assumption follows Geanakoplos (1997), making repo contracts non-recourse. The effect of reputation analyzed in this paper are robust to the recourse feature of repo contract (i.e. assuming that lenders have access to the rest of the balance sheet of the borrower beyond the pre-specified collateral in case of default), as long as they cannot seize all of it. See for instance Khan (2010). Such treatment of repo as been applied in the case of Lehman bankruptcy, see Valukas (2012).

If a lender does not contract at all, it invests its endowment in the risky asset, valued at $p(1+\mu^L)$:

$$U_0^L = n^L \left(1+\mu^L\right)$$

Under the assumption that lenders are extracting no surplus from the contracting, the following equality holds: $U_{I,J}^L = U_0^L$. This implicitely defines a *contract curve*, plotted on Figure 4 in the haircut-rate space.

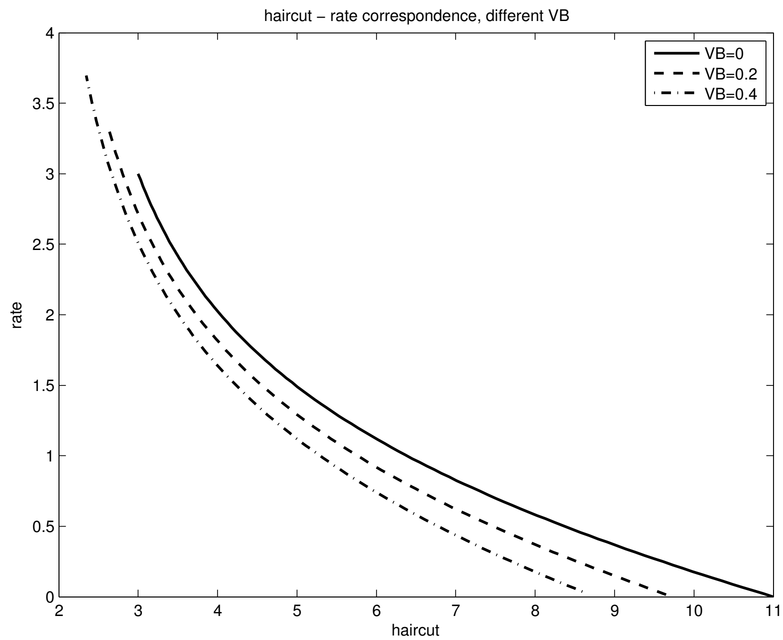

Figure 4: Rate (y axis) - Haircut (x axis) contract curve.

Equilibrium repo contract The equilibrium is then solved by the program 1 of the borrowers, taking the contract curve as a constraint. Its first-order condition leads to the following lemma.

Lemma 1. *For each borrower i, the optimal contract it picks is characterized by its riskiness $\tilde{s}$ that satisfies:*

$$p = -\int_{s^{min}}^{\tilde{s}} s f_L ds + \int_{\tilde{s}}^{s^{max}} s f_B ds \frac{1-F_L}{1-F_B} - V^{B_i} \frac{\left(p - \int_{s^{min}}^{\tilde{s}} s f_L ds - \tilde{s}\left(1-F_L\right)\right)^2}{n^B + \beta V^{B_i}\left(1-F_L\right)} \left(\beta \frac{f_L}{1-F_L} + (1-\beta)\frac{f_B}{1-F_B}\right) \quad (3)$$

The effects of franchise V^{B_i} on the equilibrium are all embedded in one term. The environment features a unique and interior solution for the optimal contract thanks to the continuum of states s, as in Simsek (2011).[17] Proposition 1 then follows easily from the comparative statics.

Proposition 1. ***Franchise Value Collateral.*** *Under Assumption 1, haircuts and rates covary positively in the cross-section of borrowers. A higher franchise secures both lower haircuts and lower rates.*

[17]On the contrary, in Geanakoplos (2009) and He and Xiong (2012a), the borrower program has a knife-edge structure, as their value function is monotonic with respect to the promise $\bar{s}$.

Proof. Appendix proves that higher franchise V^{B_i} leads to: higher promise $\partial_{V^B}\bar{s} > 0$, lower haircuts $\partial_{V^B} m < 0$, lower riskiness $\partial_{V^B}\tilde{s} < 0$ and lower rates $\partial_{V^B} r < 0$. □

It is easy to see that such property cannot be obtained in any model where borrowers heterogeneity is about collateral, such as beliefs heterogeneity or risk aversion heterogeneity among borrowers. Indeed, in the latter case, all borrowers face the same contract curve 2, which is univocally downward sloping. On the contrary, Proposition 1 can be interpreted as Franchise Value being used as intangible collateral in repo borrowing. When endowed with franchise, the borrower is incentivized to lever up more. Despite higher leverage, Proposition 1 shows that refinancing terms are cheaper for the borrower with higher franchise.

An examination of the first-order condition 3 convinces that the effect of franchise on haircuts and rates is amplified when beliefs disagreement between borrowers and lenders is higher, and when the collateral is more volatile.

Corollary 1. *Franchise value matters more when collateral is more volatile and when beliefs disagreement is larger.*

Proof. It follows from , $\frac{\partial}{\partial\sigma}\left(\left|\frac{\partial m}{\partial V^B}\right|\right) > 0$, $\frac{\partial}{\partial\sigma}\left(\left|\frac{\partial r}{\partial V^B}\right|\right) > 0$, $\frac{\partial|\partial_{V^B} m|}{\partial(\mu^B-\mu^L)} > 0$ and $\frac{\partial|\partial_{V^B} r|}{\partial(\mu^B-\mu^L)} > 0$. □

This corollary informs us that it is precisely in times of distress and on volatile collateral that there is more chance of idiosyncratic dispersion in repo prices. The intangible franchise value collateral depends on the volatility of the underlying asset σ. Recall that the equilibrium haircut comes from the wedge in the perceived values of the put option, and this wedge increase with the volatility of collateral. As a result, haircuts increase with the collateral volatility. The cross-partial is more interesting, as it is a form of *volatility paradox*. As the asset becomes more volatile from the lender's viewpoint, he values tangible collateral less relative to the franchise values of the borrower. As a result, the franchise value collateral channel is magnified, to an extent that can make optimal leverage actually higher with more volatile collateral.

The equilibrium haircut is given by the riskiness of the equilibrium contract:

$$m = \frac{1+\mu^\delta}{1+\mu^{\delta\, barg}} - \frac{1}{p\left(1+\mu^{\delta\, barg}\right)}\left(\tilde{s} + \left(1-F_\delta(\tilde{s})\right)V^B - \pi^\delta(\tilde{s})\right)$$

A Taylor expansion in the franchise obtains a closed-form solution of the haircut as a function of the primitive.

$$\begin{aligned} m \quad = \quad & \left(\frac{1+\mu^\delta}{1+\mu^{\delta\, barg}} - \frac{s^{min}}{p\left(1+\mu^{\delta\, barg}\right)}\right) - \frac{1}{p\left(1+\mu^{\delta\, barg}\right)}\left[\tilde{s} - s^{min} + \pi^\delta(\tilde{s})\right] \\ & - \left(V^B\right)^2 \frac{\left(1-F_\delta(\tilde{s})\right)}{p\left(1+\mu^{\delta\, barg}\right)} V^B \left(\frac{1}{\mathbb{E}_B\left[s|s>\tilde{s}\right]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/\left(1-F_B\right)}{h(0)} + \frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})}\right) \end{aligned}$$

This closed-form solution exhibits the different determinants of the equilibrium haircut: the first term s^{min} is the safe component (present in Geanakoplos (2003)), the second is risky debt coming from the wedge in the limited liability put valuation (present in Simsek (2011)). The third new term consists in a direct effect of V^B on m from more promise $\bar{s}$, and an indirect effect from lower riskiness $\tilde{s}$. In the appendix I show that the first direct effect dominates. When $V^B \mapsto \infty$, debt becomes essentially risk-free ($s^{def} = 0$) as in Geanakoplos (2003). When $V^B \mapsto 0$, m tends to the equilibrium margin with no franchise value as in Simsek (2011).

As for interest rate, the closed-form solution is:

$$r = \frac{\tilde{s} + V^B}{p - \frac{1}{\left(1+\mu^{\delta\, barg}\right)}\left(p(1+\mu^\delta) - \tilde{s} - \left(1-F_\delta(\tilde{s})\right)V^B + \pi^\delta(\tilde{s})\right)} - 1$$

As can be seen from the expression of the rate as a function of $\tilde{s}$, there are two distinct effects from V^B on r. The promise $\bar{s} = \tilde{s} + \beta V^{B_i} \frac{1}{\sum_{j \in J} x_{ij}}$ is higher, but D is also higher. Proposition 1 showed $\frac{\partial \tilde{s}}{\partial V^B} < 0$ and $\frac{\partial \bar{s}}{\partial V^B} > 0$. This means that starting from a situation where $V^B = 0$, increasing V^B de-links $\tilde{s}$ from $\bar{s}$, in such a way that the riskiness $\tilde{s}$ decreases and the promise $\bar{s}$ increases.

The second corollary is concerned with the convexity of the mapping $m(V^B)$, a key element of the procyclical leverage result of the dynamic model below.

Corollary 2. *The haircut is stable for high franchises: $m(V^B)$ satisfies $\frac{\partial^2 m}{\partial \left(V^B\right)^2} > 0$.*

This proposition implies that the franchise value collateral channel is more sensitive to innovation on franchise when franchise is already low. When franchise value V^B is high, the equilibrium margin is low and stable, as most of the lending is done against V^B and not against the risky collateral. When franchise value is low, the margin abruptly adjusts to high levels, with the limit of the upper bound $m(0)$.

2.3 Equilibrium with limited lending wealth

I now turn to the general case in which lenders have limited wealth, and show that the main result of positive comovement of haircuts and rates still holds. This relaxation is interesting because Kacperczyk and Schnabl (2012) has empirically shown that the cross-section of money market funds as lenders is highly heterogeneous in their risk attitude. I characterize in this section the equilibrium matching between borrowers with heterogeneous franchise and lenders with heterogeneous pessimism.

In the matching equilibrium, there is not Bertrand competition among the lenders, and the multilateral Nash bargaining is non degenerate. In that case, contractual externalities arise from the co-existence of multiple repo contracts.

Market structure: Multilateral Nash Bargaining at $t = 0$ As observed in the data, repo contracts are highly non-exclusive: in equilibrium, a borrower will contract with several lenders and, reciprocally, a lender will contract with several borrowers.[18] The I optimists and J pessimists are allowed to write repo contracts with any other agent in the economy, and can hold multiple contracts at the same time. The borrower agrees with each lender j on a short-term repo contract, which specifies a value D^j, an interest rate r^j and a haircut m^j. Denote X the set of repo contracts. The set of contracts can be represented as a Cartesian product $X = I \times J \times K$. For $x \in X$, $x(i)$ is the identity of the borrower, $x(j)$ is the identity of the lender and $x(k)$ are the terms of the bilateral contract: $x(k) = \{m, r, colclass, T\}$ where m is the haircut, r is the interest rate, *colclass* is the type of collateral and T is the maturity. I assume for now segmented markets in *colclass* and a unique maturity (overnight), so $x(k) = \{m, r\}$. An *allocation* is a collection of contracts derived as an outcome of the multilateral Nash bargaining. The equilibrium concept is as follows.

[18] This model aims at capturing the decentralized nature of money markets such as repo markets. This also relaxes the matching structure of the intermediation market of He and Krishnamurthy (2012), where matches are identical and exogenously broken at $t + dt$. It can also be seen as microfounding the imperfection of capital markets used by He and Xiong (2012b), which is that borrowers 'have' to rely on a continuum of small creditors. In my static model, borrowers endogenously choose to diversify their creditor structure, and this comes from the endogeneity of the borrowing rate. The multilateral Nash bargaining modelled here captures the pricing of rollover risk in the borrowing rate, a possibility mentioned in their footnote 10.

Definition 3. An allocation is an equilibrium if it is pairwise stable, i.e. if it is not pairwise blocked by any pair of agents. Pairwise blocking consists in a borrower and a lender that would like to add a new joint contract or replace a previous joint contract while not canceling other contracts.

This equilibrium concept draws from Stole and Zwiebel (1996) and the matching literature.[19] The existence of a non-degenerate equilibrium stems from the contractual externalities imposed by borrowers' aggregate collateral constraints: each lender of a given borrower shares the same collateral constraint. In an equilibrium allocation, the surplus of the pairwise bilateral relationship is equal to its Shapley value in the corresponding cooperative game.

$$S_{i,j} = S_{I,J} = U^B_{I,J} - U^B_{I,J-1} + U^L_{I,J} - U^L_{I-1,J}$$

Following Stole and Zwiebel (1996), I assume Nash bargaining with the bilateral relationship. I denote ω the bargaining power of the lender. The contract $x(i,j)$ must satisfy:

$$\omega\left(U^B_{I,J} - U^B_{I,J-1}\right) = (1-\omega)\left(U^L_{I,J} - U^L_{I-1,J}\right)$$

As benchmark, I first characterize the unique symmetric stable allocation and the optimal contract implementing it. Under this allocation, all possible bilateral relationships (i,j) enter in one contract and each contract feature the same terms $x(k)$. An equilibrium with small J can be referred as concentrated financing, whereas an equilibrium with large J can be referred as dispersed financing.

Lemma 2. *The expected values of the relationship under any contract (m,r) for the borrower and for the lender:*

$$\begin{aligned}
U^B_{I,J} - U^B_0 &= \frac{n^B}{m}\left[(1-m)\left(\mu^B - r\right) + \frac{1}{p}\pi^B(\tilde{s}) + \left(\frac{1}{p} - \beta m\right)V^B F_B(\tilde{s})\right] \\
U^L_{I,J} - U^L_0 &= \frac{I}{J}\frac{n^B}{m}\left[(1-m)\left(r - \mu^L\right) - \frac{1}{p}\left(\pi^L(\tilde{s}) + V^B F_L(\tilde{s})\right)\right]
\end{aligned}$$

The expression for the borrowers' expected value tells us that the franchise value has three distinct direct effects. First, it relaxes the default constraint and as such lowers the value of the long put held by the borrower (lower $\pi^B(\tilde{s})$). Second, this is counterbalanced by the states of nature where the borrower does not have to pay back this additional promise V^B: $\frac{1}{mp}V^B F_B(\tilde{s})$. Third, the traditional care about the franchise value also makes expected utility decrease with franchise: $-\beta V^B F_B(\tilde{s})$. These direct effects should make the franchise imply a lower optimal choice of leverage. This is without taking into account the indirect effect of franchise through the price of the loan $r(m)$, i.e. the contract curve.

Under Nash bargaining, the contract curve is slightly modified. I define:

$$\delta = \frac{1}{\frac{1-\omega}{\omega}\frac{I}{J} + 1}$$

δ is a measure of the *effective bargaining power* of the lenders: it is increasing in the bilateral bargaining power ω and it is increasing in the lender intensity J/I. Subsequently, define a weighted average beliefs where each agent is weighted by its effective bargaining power.

[19] The repo market is modelled here as a two-sided many-to-many matching market with contracts, which combines matching and contracting (see Roth (1984), Hatfield and Milgrom (2005) and Klaus and Walzl (2009)). With beliefs heterogeneity and contractual externalities from the collateral constraint, there is a breakdown of full substitutability. This way to model the intermediation market differs from He and Krishnamurthy (2012) where intermediation features a Walrasian equilibrium for risk exposure (equity).

Definition 4. The beliefs F_δ are the average beliefs, weighted by δ: $F_\delta \sim (1-\delta)F_L + \delta F_B$

In the normal case, the beliefs F_δ are such that $F_\delta \sim N\left(\mu^\delta, \sigma\right)$ with $\mu^\delta = (1-\delta)\mu^L + \delta\mu^B$. The beliefs δ and B still satisfy the hazard rate order property, in the same order as L and B.

Lemma 3. *The Nash bargaining implies a rate-haircut contract curve:*

$$r(m) = \mu^\delta + \frac{1}{1-m}\left[\frac{1}{p}\left(\pi^\delta(\tilde{s}) + V^B F_\delta(\tilde{s})\right) - \delta\beta m V^B F_B(\tilde{s})\right] - \delta(1-\omega)\frac{m}{1-m}\left[S_{I,J-1} - S_{I-1,J}\right]$$

The contract curve tells that the rate is the sum of two components: the weighted-average of the agents' means, weighted by their respective bargaining powers, and the weighted-average of the agents' perceived values of the limited liability put. The mapping $r(m)$ is decreasing, which is intuitive: as the the borrower picks lower haircuts, the loan becomes riskier, and this commands higher interest rates in order to compensate the lender for the credit risk. As in the case with deep picket investors, a higher franchise value shifts the correspondence leftwards.

Equilibrium The equilibrium can be expressed as follows. At $t = 0$, the borrower chooses a haircut m, and thus a size of its balance sheet x (leverage) taking into account the bargaining friction.[20] The maximization program of each borrower takes the form:

$$U^B_{I,J} = \underset{\{x,\bar{s}\}}{Max}\left\{\sum_J x_{ij}\,\mathbb{E}_B\left[(s-\bar{s})\,1_{\{no\,distress\}} + \beta^{-1}(s-\bar{s})\,1_{\{light\,distress\}}\right] + \mathbb{E}_B\left[V^B 1_{\{no\,def\}}\right]\right\}$$

$$(collateral\,constraint)\ \ s.t.\ \sum_J x_{ij} m_{ij} p \leq n^B$$

$$(default\,condition)\ \ s.t.\ default\ i.f.f.\ s < \tilde{s}$$

$$(Nash\,bargaining)\ \ s.t.\ \omega\left(U^B_{I,J} - U^B_{I,J-1}\right) = (1-\omega)\left(U^L_{I,J} - U^L_{I-1,J}\right)$$

The borrower leverage x, counted as units of risky asset purchased, is the sum of the $\tilde{J}$ 'micro-leverages' x^j, which is, the number of units of risky assets that can be purchased on margin through each repo contract j: $x = \sum_{j\in\tilde{J}} x^j$. Rewriting the program with only the contract-haircut m and the contract-rate r, we have:

$$U^B = \underset{\{m,r\}}{Max}\left\{n^B\left[\left(\frac{1}{m}-1\right)\left(r^B - r(m)\right) + \frac{1}{mp}\pi^B(\tilde{s}) + \left(\frac{1}{mp} - \beta\right)V^B F_B(\tilde{s})\right]\right\}$$

$$(Nash\,bargaining)\ s.t.\ r(m) = \left[(1-\delta)\bar{r} + \delta r^B\right] + \frac{1}{1-m}\left[(1-\delta)\Pi^L + \delta\Pi^B\right] - \omega(1-\omega)\frac{m}{1-m}\left[S_{I,J-1} - S_{I-1,J}\right]$$

This program is convex thanks to Assumption 1, and the equilibrium can be solved by induction on the number of borrowers and lenders. The optimal contract is determined as a function of the endogenous outside options $S_{I,J-1}$ and $S_{I-1,J}$. I formulate the induction hypotheses: $U^B_{I,J-1} - U^B_0 = (1-\omega)n^B S_{I,J-1}$ and $U^L_{I-1,J} - U^L_0 = \omega n^B S_{I-1,J}$. Endogenous surplus in bilateral relationships are computed by induction:

$$S_{I,J} = n^B\left[\left(1+\mu^\delta\right)\frac{R^{IJ}(\tilde{s})}{R^{\delta\,U}(\tilde{s})} - \left(1+\mu^B\right) + (1-\omega)S_{I,J-1}\left(\omega\frac{R^{IJ}(\tilde{s})}{R^{\delta\,U}(\tilde{s})} - 1\right) - \omega S_{I-1,J}\left((1-\omega)\frac{R^{IJ}(\tilde{s})}{R^{\delta\,U}(\tilde{s})} + 1\right)\right]$$

[20] The equilibrium is given by maximizing over (r,m) the Nash bargaining function $\left(U^B_{I,J} - U^B_{I,J-1}\right)^\omega\left(U^L_{I,J} - U^L_{I-1,J}\right)^{(1-\omega)}$.

where $\frac{R^{IJ}(\tilde{s})}{R^{\delta U}(\tilde{s})} = \frac{\left(\left\{p\left(1+\mu^B\right)-\frac{I}{J}p(1+\mu^L)(1-m)\right\}-\left\{\left(1-\frac{I}{J}\right)\left(\tilde{s}+V^B\right)\right\}+\left\{\left(F_B(\tilde{s})-\frac{I}{J}F_L(\tilde{s})\right)V^B\right\}+\left\{\pi^B(\tilde{s})-\frac{I}{J}\pi^L(\tilde{s})\right\}\right)}{p\left(1+\mu^\delta\right)-\left(\tilde{s}+V^B\right)+F_\delta(\tilde{s})V^B+\pi^\delta(\tilde{s})} > 1$. The closed form solution for the equilibrium haircut is also derived by induction, and is graphed on Figure 8. As in Lemma 1, the characterization of the equilibrium can be characterized by one first-order condition.

Lemma 4. *The optimal contract in the I-J equilibrium is unique and is characterized by its riskiness $\tilde{s}$:*

$$\begin{aligned} p(1+\mu^\delta) &= \kappa_1 V^B\left(\tilde{s}+V^B\right)+F_\delta(\tilde{s})\,\mathbb{E}_L\left[s|s<\tilde{s}\right]+(1-F_\delta(\tilde{s}))\;\kappa_2\,\mathbb{E}_B\left[s|s>\tilde{s}\right] \\ &\quad -\frac{\beta V^B}{n^B\left(1+\mu^{\delta\,barg}\right)}\frac{f_B(\tilde{s})}{1-f_B(\tilde{s})V^B-F_B(\tilde{s})}\left(\mathbb{E}_L\left[1_{\{no\,def\}}\left(s-\bar{s}\right)\right]\right)^2 \end{aligned}$$

Lemma 4 characterizes the riskiness of the loans as a function of the primitives of the model: $0 = F\left(\tilde{s};\mu^\delta,\mu^B,\sigma,V^B,p,\mu^{\delta\,barg},\beta\right)$. Intuitively, the borrower picks the optimal promise trading off the gains from levering up with the borrowing costs increasing with the riskiness. Not only the beliefs about the collateral matter, but also the franchise value V^B and the bargaining structure: the lenders' ability to extract surplus (δ) and the outside options $S_{I,J-1}$ and $S_{I-1,J}$. The next corollary shows that the main result, stated in Proposition 1, is robust to the general case of limited lenders net wealth and lenders heterogeneity.

Corollary 3. *Under multiple Nash bargaining, the Proposition 1 result of positive comovement of haircuts and rates in the cross-section of borrowers still holds.*

Furthermore, the more general environment considered here enables to characterize the equilibrium matching between borrowers of heteregeneous reputation and lenders of heterogeneous risk attitude.

Corollary 4. *Lenders reach-for-yield: in equilibrium, less pessimistic lenders pick lower haircuts and higher rates.*

Proof. For $(1-\delta)\frac{\mu^B-\mu^L}{\sigma} < 1$ (mild beliefs heterogeneity and large lender bargaining power): $\frac{\partial\tilde{s}}{\partial\mu^L} > 0$, $\frac{\partial m}{\partial\mu^L} < 0$ and $\frac{\partial r}{\partial\mu^L} > 0$. For $(1-\delta)\frac{\mu^B-\mu^\delta}{\sigma} > 1$: $\frac{\partial\tilde{s}}{\partial\mu^L} < 0$, $\frac{\partial m}{\partial\mu^L} < 0$ and $\frac{\partial r}{\partial\mu^L} < 0$. □

The effect of beliefs disagreement is measured by $\mu^B-\mu^L$. The relative pessimism of the lenders about the asset can be interpreted as their ability to actually seize the collateral in states of nature in which the borrower defaults. A very pessimist lender will ask for higher promise ceteris paribus, and the promise is more dependent on the franchise value in this case. At the same time, a very pessimist lender overestimates the probability of default according to the borrower, which makes the borrower less wary of loosing his continuation value, thereby mitigating the fear of default. This makes the optimal leverage less dependent on the franchise value V^B. This proposition shows that a borrower will be able to lever up even more when the lender does not have the capacity to seize the collateral efficiently. Indeed in this case the lender values much more the promise $\bar{s}$ in states of no default than the actual collateral in states of default (franchise value collateral is more important). In the tri-party repo market $\mu^B-\mu^L$ is higher (more disagreement about the asset) than in the bilateral repo market, so the franchise value collateral channel matters more in the tri-party repo market. This intangible collateral channel, through which the franchise value of the borrower backs the promise of the borrower and enables to achieve lower haircuts (higher leverage), is magnified when beliefs disagreement is high.[21]

[21] This is equivalent to analyze the pricing of the contract with respect to the respective marginal utilities. When one agent's beliefs is steeper, this agent's marginal utility will be more responsive to the aggregate shock, and as a result will be in less favorable effective bargaining power position against the other agent type.

Finally, the last corollary demonstrates the impact of imperfect competition among lenders on the equilibrium haircuts and rates, and on the franchise value collateral channel.

Corollary 5. *Imperfect competition among lenders temper leverage, but at the same time makes the Franchise Value channel more pivotal.*

Proof. It follows from $\partial_\delta \bar{s} < 0$, $\partial_\delta m > 0$ and $\partial_\delta r > 0$, as well as $\frac{\partial}{\partial \delta}\left(|\frac{\partial m}{\partial V^B}|\right) > 0$ and $\frac{\partial}{\partial \delta}\left(|\frac{\partial r}{\partial V^B}|\right) < 0$. □

Recall that δ captures the effect of imperfect competition among lenders: the higher δ is, the lesser competition there is among lenders. Intuitively, as the competition among lenders becomes more imperfect (I/J higher) δ increases, so the compound beliefs F^δ become closer to F^B. As a result, beliefs heterogeneity about the tangible collateral matters less than franchise value. From the borrower standppoint, having a dispersed creditor structure enhances its effective bargaining power. A surprising feature of the equilibrium haircut is that it depends on the bargaining structure of the credit market: the haircut equilibrium haircut depends on the number I of borrowers and the number J of lenders. As a result, the total leverage and credit supply in this economy is hindered when the *number* of lenders J is small. This is counterintuive, as one might suppose that the total surplus does not depend on the number of lenders. The wedge arises from the fact that the promise $\bar{s}$ does *not* have the same dependence to m when varying I and J. In economic terms, it is because the difference of valuation in the borrower put option is spread out between the two lenders, enabling the borrower to lever up more. The static model captures a type of diversification benefit from having a dispersed financing. With concentrated financing, the *contractual externalities* between borrowers are exacerbated.[22]

2.4 General Equilibrium

I explore here the asset pricing implications of the credit market microstructure. The $t = 0$ price p of the risky asset is endogeneized by relaxing its perfectly elastic supply. Assume now that its supply is fixed, normalized at 1. Combining the optimality conditions arising from the above multilateral Nash bargaining and the risky asset market clearing yields the equilibrium.

Definition 5. The general equilibrium is given by a collection $\{x_{ij}, m_{ij}, r_{ij}\}_{(i,j)\in I\times J}$ of repo contracts, each specifying a number of units, a haircut and a rate, and a $t = 0$ price p for the risky asset, such that:

i) the outcome of the Nash bargaining among the I borrowers and the J lenders is pairwise stable;

ii) the Walrasian market at $t = 0$ for the risky asset clears: $\sum_i \sum_j x_{ij} = 1$.

In general equilibrium, two countervailing forces are at play for the effect of franchise on haircuts and rates. The partial equilibrium effect of Proposition 1 is still present: higher franchise imply lower haircuts and rates. However, lower haircuts imply higher leverage, hence higher demand in the risky asset market. When the latter is in fixed supply the only margin of adjustment is an inflated asset price p. This in turn tempers leverage, as it makes it deteriorates the expected levered returns of optimists. Finally, Proposition 1 still holds. Only the magnitude of the cross-sectional dispersion is weakened.

Corollary 6. *Proposition 1 is robust to the General Equilibrim.*

[22]Even more so with capacity constraints on lenders, in accordance with concentration limits faced by money market funds.

In General Equilibrium, the average haircut $\bar{m} = \frac{\sum_I \sum_J x_{ij} m_{ij}}{\sum_I \sum_J x_{ij}}$ satisfy:[23]

$$\bar{m} = I \frac{n^B}{p}$$

It implies that the distribution of franchises V^{B_i} is priced in the risky asset through a parsimonious sufficient statistics, the average haircut $\bar{m}$. Its effect is graphed on Figure 9 and shows that a distribution skewed towards high franchises props up the asset price p.

In the cross-section of assets, this leads to an endogenous-margin CAPM. With a discrete number of risky assets K, under the same multilateral Nash bargaining , the commonality of borrowers' franchises $\{V^{B_i}\}$ introduces the correlation between assets. I write m^k and $r^k(m^k)$ as the haircuts and repo rates secured by asset k. There is no riskfree asset.

Corollary 7. *Endogenous-margin CAPM: the franchise distribution correlates asset prices.*

The result is not a priori straightforward, as all agents are risk neutrals, hence the traditional CAPM does not hold. The franchises $\{V^{B_i}\}$ act as pricing kernels, and the more so the larger the franchise is. Precisely the endogenous-margin CAPM formula is:

$$\mathbb{E}_B \left[R^k\right] - r^k(m^k) = \alpha_k + \beta_k V^B$$

with $\beta_k = \frac{m_k F_L(\tilde{s})}{(1-m_k)^2}$ and $\alpha_k = \frac{m_k}{(1-m_k)^2} \int_{s^{min}}^{\tilde{s}} u^k f_L(u) du$. It is a generalization of the Ashcraft, Garleanu and Pedersen (2010) margin-CAPM, by endogeneizing the margins and the riskfree rate, and highlight borrowers franchise values as key pricing kernels.[24] The key asset pricing prediction of the model is that asset correlation is higher when franchise values of financial intermediaries are high. This property is consistent with Adrian et al. (2012) which shows the leverage of financial intermediaries, as a single factor, price the cross-section of assets with a R^2 of 0.77. This prediction is contrary to models of fire-sales in which assets are more correlated in bad times.

3 The dynamic model

I endogeneize in this section borrowers' franchise values $\{V^{B_i}\}$, taken as exogenous in the static model. I identify franchise to the continuation value of borrowers in the dynamic version of the static model. This section derives two results. First, haircuts are countercyclical with respect to borrowers' net worth. Although franchise value acts as stabilization in good times, it becomes an amplification force in bad times. The ability of the borrower to lever up today depends on its ability to lever up tomorrow, and this feedback loop creates the high sensitivity of leverage to borrower net worth. Second, this fragility can be mitigated with long-term contracts.

3.1 Dynamic environment

The horizon is infinite and time $t, t+1, t+2, ...$ is discrete.

[23] It follows from $\sum_J x_{ij} m_{ij} p = n^B$ and $\sum_i \sum_j x_{ij} = 1$.

[24] Jurek and Stafford (2010) also prices the cross-section of assets in presence of collateralized lending. They feature risk-aversion but leverage and the haircuts are exogenous in their set up.

Agents The environment is populated by a number I of borrowers and a number J of lenders, which are all infinitely-lived.[25] The economy is endowed with an infinite supply of Lucas trees, with price p which pay i.i.d dividends at the next period $s(t+1)$. Agents have the same beliefs about this dividend as in the static model.[26] At each period, agents invest and contract through the credit market that has been analyzed in detail in the above section. At each period they consume a fixed fraction c of their total wealth. This artefact implies that agents are simply happy to be rich, and allows to avoid to specify a consumption process for them. Otherwise the borrower would never consume to save its way out of the financing constraint and would eternally postpone consumption. Preferences then are:

$$U_t^i = \mathbb{E}_i \left[\sum_{k=0}^{\infty} \rho^k \, 1_{\{no\,def\}} \, c \, n_{t+k}^i \right]$$

Timeline Each period is broken down in 3 stages, which are the exact same steps as in the static model. The stage timing is as follows and illustrated in Figure 5.

- Stage 0 *'evening'*: All the agents enter into multilateral Nash bargaining, and writes the resulting contracts. A bilateral contract (i, j) is the combination of a repo contract $\left\{x_{ij}, m_{ij}, r_{ij}\right\}_{(i,j)\in I\times J}$ (specifying a number of units, a haircut and a rate), plus an unsecured long-term promise. The latter is activated only in the equilibrium with long-term contracts.

- Stage 1 *'night'*: The asset shock is realized overnight. Agents i that face $\sum_{j\in J} x_{ij} s < \sum_{j\in J} x_{ij} \bar{s}_{ij}$ are in situation of distress. During the night, any agent can decide to pledge its continuation value V^{B_i}, albeit at an exogenous (high) rate r^*. I define $\beta = \frac{1}{1+r^*}$.

- Stage 2 *'morning'*: contracts are settled. If agents default, they exit the market with outside utility U^{def}. In case of default, debtholders seize the tangible collateral. If agents do not default, they settle both the short-term repo contracts (promises $\bar{s}_{ij}$) and pay back a fraction of the emergency borrowing if contracted at any prior stage 1. This reduces the principal balance on the emergency loan. Next they consume a constant fraction c of the remaining wealth. Before moving on to the next period, agents decide to stay or not in the long-term bilateral relationships if long-term contracts have been written at any prior stage 0.

3.2 Equilibrium with short-term contracts

In a first step, I rule out long-term contracts. No out-of-the-period promises can be made to lenders; only short-term repo contracts can be written. The policy choices then are the number of contracts x_t^j with each

[25] The equilibrium default feature of the model prevents from the need of beliefs switching or of killing optimists at an exogenous Poisson rate of optimists. The ergodic distribution of wealth is not explosive.

[26] The countercyclicality of haircuts is robust to the introduction of persistent shocks. Beliefs can then be written as $s_{t+1} = a^B + \rho s_t + \sigma\epsilon$ and $s_{t+1} = a^L + \rho s_t + \sigma\epsilon$, $a^L < a^B$ (both perceive the AR(1) for the asset but with different drifts). In the continuous-time representation of the game, the dividend stream follows a random walk (brownian motion with drift μ^i and volatility σ). I rule out learning. The fact that lenders do not learn about the risky asset could be microfounded by a model of rational inattention. Indeed, Dang et al. (2011b) show that under debt and high information acquisition costs, the lender does not learn about the underlying asset. Under a flexible technology of information acquisition, Yang (2012) shows the robustness of such result.

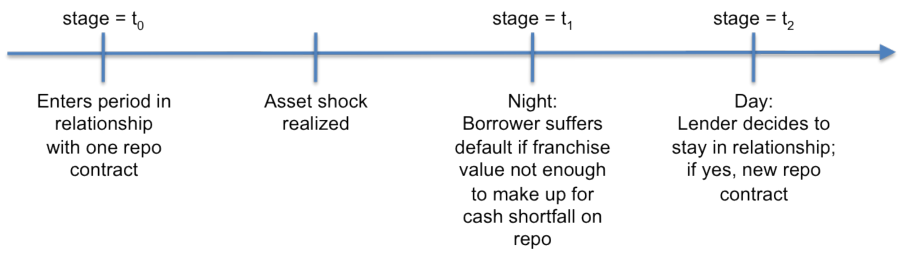

Figure 5: Timing of the dynamic model.

lender j, the haircut m_t^j and the promise $\bar{s}_t^j$ in each of these contracts. The amount of emergency borrowing is also a choice: borrowers decide which fraction ϕ of their continuation value to pledge against immediate liquidity.

The distress region Denote $\bar{s}$ the average level of promises contracted at stage 0: $\bar{s} = \frac{\sum_{j \in J} x_{ij} \bar{s}_{ij}}{\sum_{j \in J} x_{ij}}$. At stage 1 of the dynamic model, any borrower faces the following partition of the state space:

- If $\bar{s} < s$, the agent i does not default.
- If $\tilde{s} < s < \bar{s}$, the agent pledges an endogenous share $\phi(s)$ of its continuation value, and stays afloat.
- If $s < \tilde{s}$, the agent is forced into default.

The state $\tilde{s}$ is an endogenous default barrier that distinguishes the default region from the grace region ('light distress'). In the distress region, borrowers decide how much $\phi \in [0;1]$ to pledge of their continuation value. This share is above what is needed to receive the liquidity that covers exactly the shortfall on short-term promises. Indeed, even the emergency rate r^* is prohibitive,[27] an agent that stays afloat but with zero net worth enjoys zero utility.[28] Hence by choosing ϕ, the distressed borrower also picks a level of net worth $n_{t+1}^{B\,post-grace} > 0$:

$$n_{t+1}^{B\,post-grace}(s) = \sum_{j \in J} x_{ij} s - \sum_{j \in J} x_{ij} \bar{s}_{ij} + \beta \phi(s) U^B \left(n_{t+1}^{B\,post-grace}(s) \right)$$

The policy choice ϕ is determined by this fixed point. Given stage 0 contracting, denote by $\Delta(s)$ the state-contingent cash shortfall on short-term promises (positive in the distress region, and affine decreasing with the state):

$$\Delta_{x,\bar{s}}(s) = \left(\sum_{j \in J} x_{ij} \bar{s}_{ij} \right) - \left(\sum_{j \in J} x_{ij} \right) s = x\bar{s} - xs$$

This yields an expression of the state-contingent $\phi(s)$ as a function of $n_{t+1}^{B\,post-grace}$:

$$\phi(s) = \frac{n_{t+1}^{B\,post-grace}(s) + \Delta(s)}{\beta U^B \left(n_{t+1}^{B\,post-grace}(s) \right)} \tag{4}$$

For $s \in (\tilde{s}; \bar{s})$:

[27] The emergency rate r^* can be thought as exogenous and high, or endogenous and priced by the most pessimistic lender.

[28] Given the consumption process is tied to net worth and agents cannot borrow with zero net worth: as the lenders are all more pessimistic than the optimists, there is no equilibrium in which a pessimist agree to lend to a borrower with zero net worth.

$$n_{t+1}^{B\,post-grace}(s)=\beta\phi(s)U^{B}\left(n_{t+1}^{B\,post-grace}(s)\right)-\Delta(s)$$

Taking the value function $U^{B}\left(n\right)$, as well the contracts $\{x,\bar{s}\}$ of stage 0 as given, the fixed point implicitely defines $n_{t+1}^{B\,post-grace}(s)$. For instance, if the value function was linear: $U^{B}\left(n\right)=\theta n$ and $\theta>1/\beta$ (first-order expansion of the value function), then we obtain:

$$n_{t+1}^{B\,post-grace}(s)=\frac{1}{\beta\phi(s)\theta-1}\Delta(s)$$

Posit that the policy function ϕ is decreasing and interpolates: $\phi(\tilde{s})=1$ and $\phi(\bar{s})=0$. The borrower tries to pledge all its continuation value ($\phi=1$) before entering default. Assume a linear policy function over the grace region:

$$\phi=\frac{\bar{s}-s}{\bar{s}-\tilde{s}}$$

Straightforward algebra on the fixed point yields to:

$$n_{t+1}^{B\,post-grace}(s)=x\frac{(\bar{s}-\tilde{s})}{\beta\theta}\left[1+\frac{1}{\beta\theta\frac{\bar{s}-s}{\bar{s}-\tilde{s}}-1}\right]$$

We see that this mapping with respect to s increases from $n_{t+1}^{B\,post-grace}(\tilde{s})=x\frac{(\bar{s}-\tilde{s})}{\beta\theta-1}$ to the point at which $s=\bar{s}-\frac{1}{\beta\theta}\left(\bar{s}-\tilde{s}\right)$, before reverting monoticity. By value matching at state $\tilde{s}$ between the default region and the grace region:

$$U^{B}\left(n_{t+1}^{B\,post-grace}(\tilde{s})\right)=\theta x\frac{(\bar{s}-\tilde{s})}{\beta\theta-1}=U^{def}$$

This boundary condition characterizes the threshold $\tilde{s}$ between the default region and the grace region:

$$\tilde{s}=\bar{s}-U^{def}\frac{\beta\theta-1}{\theta x}$$

The default boundary at stage 1 involves not only the value function θ, but also the contracts $\{x,\bar{s}\}$ written at stage 0.

Recursive formulation The environment has a recursive structure with one state variable: the net worths of each borrower $\left\{n_{t}^{B_{i}}\right\}_{i}$. Borrowers' value functions then satisfy the following Bellman equation:

$$\begin{aligned}
U^{B}(n_{t}^{B}) &= \underset{\{x_{t},m_{t},\bar{s}_{t},\phi_{t}(s)\}}{Max}\quad \mathbb{E}_{B}\left[1_{\{no\,def\}}c\,n_{t+1}+\rho 1_{\{no\,def\}}U^{B}\left((1-c)\,n_{t+1}^{B}\right)\right]-\mathbb{E}_{B}\left[1_{\{grace\}}\rho\phi_{t}(s)\,U^{B}\left((1-c)\,n_{t+1}^{B}\right)\right]\\
(collateral\ constraint) &\quad \sum_{J}x_{ij}m_{ij}p\leq n^{B}\\
(default\ condition) &\quad default\ i.i.f\ s<\tilde{s}\\
(contract\ curve) &\quad \omega\left(U_{I,J}^{B}-U_{I,J-1}^{B}\right)=(1-\omega)\left(U_{I,J}^{L}-U_{I-1,J}^{L}\right)\\
(law\,of\,motion\,of\,wealth) &\quad n_{t+1}^{B}=\sum_{j\in J}x_{ij}\left(s_{t+1}-\bar{s}_{ij,t}\right)+1_{\{grace\}}\beta\,\phi_{t}(s)U^{B}\left((1-c)\,n_{t+1}^{B}\right)
\end{aligned}$$

In the stationary Markov equilibrium, the optimal short-term repo contracts picked by borrowers can be characterized by their riskiness $\tilde{s}$ as sufficient statistics. The latter is uniquely determined by the following lemma.

Lemma 5. *The optimal repo contracts picked are characterized by their riskiness $\tilde{s}$, which satisfies:*

$$
\begin{aligned}
0 &= -p(1+\mu^{\delta}) + \kappa_1 V^B \left(\tilde{s} + V^B\right) + F_{\delta}(\tilde{s})\,\mathbb{E}_{\delta}\left[s|s<\tilde{s}\right] + \left(1 - F_{\delta}(\tilde{s})\right)\,\kappa_2\,\mathbb{E}_B\left[s|s>\tilde{s}\right] \\
&\quad + \beta \frac{1}{1-\partial_{\tilde{s}} V^B} \frac{\left(R_{\delta}^{unl}\right)^2}{\partial_{\tilde{s}} R_B^{unl}} \int_{\tilde{s}}^{s^{max}} \partial_{\tilde{s}} R_B^{lev}(s,\tilde{s})\,\partial_{n^B} V^B \left(n^B \left(1+\mu^{\delta\,barg}\right) R_B^{lev}(s,\tilde{s})\right) f_B(s) ds
\end{aligned}
$$

Proof. See Appendix. □

This first order condition implicitely defines the riskiness $\tilde{s}$ of the optimal contract:

$$0 = F^{dyn}\left(\tilde{s}; \mu^{\delta}, \mu^B, \sigma, V^B, p, \mu^{\delta\,barg}, \beta\right)$$

The first two terms of the first-order condition are identical to the static model. The third new term arises from the endogeneity of the franchise value in the dynamic environment. V^B is not constant anymore, and the borrower takes into account the impact of $\bar{s}$ on the state-contingent $V^B(n^B_{t+1})$. The term brings in two effects: one is concern about long-term continuation, which tempers leverage. The other one is franchise value collateral, which incentivizes higher leverage. The value function V^B is the fixed point solution of the recursive equilibrium, taking into account two intertemporal interlinkages: the law of motion of wealth, and the franchise value collateral channel.

Compared to the traditional dynamic models of the capital structure,[29] this model features a *grace* region in the dividend space $\mathbb{S}$. When the realization of the divident s is such as $s \in (\tilde{s}_{\bar{s},V^B}; \bar{s})$, the borrower is able not to default. The borrower then has negative net worth, before the credit line cash injection $\beta\,\phi_t(s) U^B\left((1-c)\,n^B_{t+1}\right)$.

The value function is not exactly linear, as the riskiness of the contract depends on the scale of investment x: $\tilde{s} = \bar{s} - U^{def}\frac{\beta\theta-1}{\theta x}$. However this concavity tends asymptotically to linearity as x gets larger and the grace region shrinks to zero measure. The value function is solved on Figure 10. The concavity of the franchise value $V^B(n)$ results in the fragility of the franchise value collateral channel exhibited in the static model. At low levels of borrower net worth, franchise value evaporates. The intuition for the concavity of the value function with respect to net worth is as follows. The first-order effect is linear in net worth, as in macro models with financial frictions[30]. In my model, there is an additional role of the value function, which is to relax the default threshold, thus enhancing the debt capacity of the borrower. This feedback loop from rollover ability on the value function is magnified at low levels of net worth and breaks the linearity.

Compounding the concavity of the value function with respect to net worth and the concavity of haircuts with respect to franchise yields the second result of the paper: haircuts are countercyclical with respect to net worth if and only if there is a franchise value channel.

Proposition 2. ***Countercyclical haircuts and rates.***

When $\beta = 0$ (no franchise value channel), haircuts and rates are procyclical.

When $\beta > 0$ (existence of a franchise value channel), haircuts and rates are countercyclical.

[29] Such as Bolton et al. (forthcoming).

[30] E.g. Brunnermeier and Sannikov (2010) and Gertler and Kiyotaki (2012).

The proposition is illustrated on Figure 11. Countercyclical haircuts imply a procyclical leverage.[31] It needs the concavity of the franchise value in order to counteract the direct effect of low net worth: a greater incentive to lever up. The countervailing force is that, at low levels of net worth, the haircut adjust upwards to their no-franchise value levels. For low borrower wealth levels, we have:

$$m(\underset{(-)}{n^B}) = 1 - \frac{1}{p} D \left(\underset{(-)}{n^B}, \underset{(++)}{V^B(n^B)} \right)$$

Furthermore, a consequence of the concavity of the value function with respect to net worth and of the convexity of haircuts to franchise is *fragility of leverage*: haircuts are convex with respect to net worth ($\frac{\partial^2 m}{\partial (n^B)^2} > 0$). This demonstrates that at high levels of net worth, haircuts are low and stable, whereas at low levels of net worth, haircurs are high and unstable. In other words, at low borrower wealth levels the correlation between asset-risk and counterparty-risk $Corr\left(s, V^B(n^B)\right)$ is very high. **It is not a run**, it is a progressive depleting of the borrower debt capacity, which can be very steep when n^B gets closer to zero. It looks like a run on volumes, but it is actually an abrupt adjustment on prices.

3.3 Equilibrium with long-term contracts

The existence of a bargaining friction brings a rationale for bilateral long-term contracting.[32] I solve here for the optimal long-term contract, and show how long-term contracting helps mitigating the countercyclicality of haircuts. I also show that dispersed financing (i.e. J/I high) undermines this optimal contract.[33]

I now allow the borrowers to choose between a long-term contract with the lender j (*relationship repo*), or staying out of any long-term relationship (*arm's length repo*). In the latter case, he contracts at each period short-term repo contracts. If the borrower enters a long-term contract, he is able to compensate the lender with promised continuation value V^L_{+1}. This additional instrument enables him to secure lower and more stable haircuts and rates. In the timing of the game, only stage 0 is modified as follows.

- Stage 0, *'evening'*: The borrower decides between entering a long-term contract (relationship repo) or staying out. In the former case, the borrower and the lender bargain on the split of the total relationship surplus between the borrower (V^B) and the lender (V^L). This long-term agreement is implemented by a sequence of a short-term (overnight) repo contracts, which specifies a notional D (notional value of debt), an interest rate r and a haircut m, and state-contingent promised continuation values $V^L_{t+1}(s)$. If she decides to stay out, the borrower bargains over short-term contracts as in the static model.[34]

[31] The literature sometimes refers to procyclicality of leverage as with respect to x_t instead of n^B_t. This mechanism of leverage procyclicality is alternative to the 'scary bad news' mechanism of Geanakoplos (2009) and Cao (2011), which relies on an uncertainty shock on the collateral. Their shock is *collateral-specific* whereas my mechanism goes through borrower net worth and is *institution-specific.*

[32] The bargaining friction breaks the Fudenberg, Holmstrom and Milgrom (1990) irrelevance result of long-term contracts. Without commitment, long-term contracting would still improve over the sequence of short-term contracts in an environment of costly search for counterparties. I leave this set up for future research.

[33] As a consequence, an optimal number of creditors J trades off the benefits of diversification with the costs of dispersed financing due to the inability to promise future continuation value to the lender in this case. It can be seen as a Jacklin (1987) critique: more competition among lenders hurts the optimal contract and leverage.

[34] In the extension without commitment, the lender decides to break up the relationship, it searches for a new borrower match, forming expectations about this franchise value with a mean-field approximation, as franchise value V^B is still concave with respect to net worth n^B, we have by Jensen inequality: $\bar{V^B} = \mathbb{E}_L\left[V^B\left(n^B\right)\right] < V^B\left(\mathbb{E}_L\left[n^B\right]\right)$. This raises the issue of more sophisticated contracts in

The continuation values of both the borrower and the lender are state-contingent in the long-term contract. As a result, this contracting problem can be seen as an intermediary case between Kocherlakota (1996) (two-sided lack of commitment with autarky as outside options, under complete markets) and Geanakoplos (2003) (one-side lack of commitment with zero as outside option, under incomplete markets).[35] My set up features equilibrium default even under the optimal contract.

- Expected utility of the borrower:

$$V^B_{I,J,t} - V^B_{I,J-1,t} \quad = \quad \frac{n^B}{m}\left[(1-m)\left(\mu^B - r(m)\right) + \frac{1}{p}\pi^B(\bar{s}) + V^B\frac{F_B(\tilde{s})}{p}\right] + \rho\,\mathbb{E}_B\left[V^B_{I,J,t+1} - V^B_{I,J-1,t+1}\right]$$

- Expected utility of the lender:

$$V^L_{I,J,t} - V^L_{I-1,J,t} \quad = \quad \frac{I}{J}\frac{n^B}{m}\left[(1-m)\left(r(m) - \mu^L\right) - \frac{1}{p}\left(\pi^L(\tilde{s}) + V^B F_L(\tilde{s})\right)\right] + \rho\mathbb{E}_L\left[V^L_{I,J,t+1} - V^L_{I-1,J,t+1}\right]$$

I focus on stationary Markov equilibria.[36] In this case, following Abreu et al. (1990) and Abreu and Pearce (2007),[37] I can use the continuation value of the lender V^L_t as an aditional state variable and write the borrowers' maximization program in a recursive formulation:

$$\begin{aligned}
V^B(n^B_t, V^L_t) \;=\; & \underset{\{x_t, m_t, \bar{s}_t, \phi_t(s)\}}{Max} \quad \mathbb{E}_B\left[1_{\{no\,def\}} c\, n_{t+1} + \rho 1_{\{no\,def\}}\left\{V^B\left((1-c)\,n^B_{t+1}\right) - V^L_{t+1}\right\}\right] \\
& - \mathbb{E}_B\left[1_{\{grace\}}\rho\phi_t(s)\,V^B\left((1-c)\,n^B_{t+1}\right)\right] \\
(collateral\ constraint) \quad & \sum_J x_{ij} m_{ij} p \le n^B \\
(default\ condition) \quad & default\ i.i.f\ s < \tilde{s} \\
(contract\ curve) \quad & \omega\left(V^B_{I,J} - V^B_{I,J-1}\right) = (1-\omega)\left(V^L_{I,J} - V^L_{I-1,J}\right) \\
(law\ of\ motion\ of\ wealth) \quad & n^B_{t+1} = \sum_{j\in J} x_{ij}\left(s_{t+1} - \bar{s}_{ij,t}\right) + 1_{\{grace\}}\beta\,\phi_t(s) V^B\left((1-c)\,n^B_{t+1}\right)
\end{aligned}$$

The continuation value is equal to the promised utility $V^L_{I,J,t+1}$ if the borrower does not default, and to $V^L_{I-1,J,t+1}$ if the borrower defaults after seizng collateral. The borrower designs his optimal long-term contract

which the borrower whishes to signal the quality of its balance sheet. In this extension, heterogeneity in lenders information regarding franchise value explains flights to safety as observed in summer 2011 against European banks. With endogenous information acquisition, the optimal long-term relational contract might then want to prevent information acquisition about collateral but foster information acquisition about franchise value (it would be win-win for both parties). A search cost $\theta = \theta(J/I)$, an increasing function of the ratio J/I, would be a reduced-form to capture the bargaining process outside the relationship. J/I is a measure of the tightness of money markets. When $J >> I$, there are many more lenders J than borrowers I, and therefore the search cost of finding a free borrower is very high. Due to the presence of multiple equilibria, the surplus of a new relationship should take into account the probability of sunspot run on the new borrower. This would deteriorate the outside option of the lender, and as a result strengthens the result of lower haircuts thanks to long-term relationships.

[35]The haircut is another price variable for collateralized debt compared to uncollateralized debt. So if this haircut is made state-contingent, it helps completing the markets. The fact that the borrower has *two* choice variables(repo spread r_t and haircut m_t) in effect completes markets and partially overcomes the non-state contingency of overnight short-term debt contracts.

[36]Thus I rule out more complicated strategies, where some borrowers might ask the lender what the terms of the contract proposed to him by the other borrowers. I also rule out cooperation among borrowers, which could punish lenders that break up with even lower bargaining power at the start of new relationship.

[37]An alternative would be Marcet and Marimon (2011), where the dynamics of co-state variables give insights on the tightness of the constraint.

taking the outside option of the lenders as given. The equilibrium optimal contract is then the fixed point on this outside option. The borrower has now a third instrument, beyond the promise $\bar{s}_t$ and the leverage x_t: the state-contingent long-term promise $V^L_{+1}(s)$. The optimization of this policy variable involves the following trade-off. On the one hand, promising more $V^L_{+1}(s)$ enables the borrower to lever up today at no cost. On the other hand, it diminishes the share of the surplus the borrower can enjoy inside the relationship tomorrow, as $\partial_{V^L} V^B < 0$, as shown in the simulation in Figure 12, Panel A. As long as:

$$\rho\, \mathbb{E}_L \left[1_{\{no\,def\}} \left(1_{\{stays\}} V^L_{t+1} + 1_{\{quits\}} V^{L\,out}_{t+1} \right) + 1_{\{def\}} V^{L\,out}_{t+1} \right] > V^L_t$$

then the borrower is able to lever up more than in the case without long-term relationships: $D^{LT} > D^{ST}$.

Lemma 6. *The f.o.c. of the optimal short-term riskiness in presence of long-term contracts is:*

$$\begin{aligned} 0 \;=\; & -p(1+\mu^{\delta}) \\ & +\kappa_1 V^B \left(\tilde{s} + V^B\right) + F_\delta(\tilde{s})\, \mathbb{E}_\delta \left[s | s < \tilde{s} \right] + \left(1 - F_\delta(\tilde{s})\right) \kappa_2\, \mathbb{E}_B \left[s | s > \tilde{s} \right] \\ & + \frac{\beta}{n^B \left(1 + \mu^{\delta\, barg}\right)} \frac{1}{1 - \partial_{\tilde{s}} V^B} \frac{\left(R^{unl}_\delta\right)^2}{\partial_{\tilde{s}} R^{unl}_B} \int_{\tilde{s}}^{s^{max}} \left[n^B \left(1 + \mu^{\delta\, barg}\right) \partial_{\tilde{s}} R^{lev}_B \partial_{n^B} V^B + \partial_{\tilde{s}} V^L \partial_{V^L} V^B \right] f_B(s) ds \end{aligned}$$

where

$$\mu^{\delta\, barg} = \mu^{\delta} + \tfrac{\delta}{n^B} \left(U^B_{I,J-1,t} - \beta\, \mathbb{E}_B \left[V^B_{I,J,t+1} - V^B_{I,J-1,t+1} \right] \right) - \tfrac{1-\delta}{n^B} \left(V^L_{I-1,J,t} - \beta \mathbb{E}_L \left[V^L_{I,J,t+1} - V^L_{I-1,J,t+1} \right] \right).$$

The optimal long-term contract substitutes continuation value for haircuts ($\frac{\partial m}{\partial V^L_{I,J,t+1}(s)} < 0$). The interpretation is that both V^B and V^L are supporting the promise $\bar{s}$ (*intangible collateral*). The relationship value V^L helps mitigating the countercyclicality of V^B compared to the equilibrium with only short-term contracts.

Proposition 3. ***Stability of leverage in long-term relationships.***

Leverage under the long-term contract is less volatile than in the sequence of short-term contracts.

Proof. Appendix derives $0 < \partial_{n^B} \tilde{s}^{LT} < \partial_{n^B} \tilde{s}^{ST}$. □

In long-term relationships, the continuation value is used as a **haircut waiver**, and as a result repo funding is more stable in volumes and in prices (i.e. haircuts and rates). It avoids margin calls exactly in the states where borrower net worth is low, the states where he would like to lever up in order to replenish capital. At low levels of net worth, under short-term contracts, the haircut adjusts abruptly upwards. Under long-term contracts, there is a surplus gain to grant a haircut waiver to the borrower: the lender is ready to maintain a low haircut against the promise of more long-term continuation value.[38] Indeed, the concavity and the

[38] If we add precautionary motive and occasionally binding collateral constraint, this margin waiver is even more valuable in the states of the world in which the collateral constraint binds: it economizes on margin spirals. Moreover, in the framework of Oehmke (2012), waiving a haircut call avoids disordely liquidation of illiquid collateral, and as such avoid cost of illiquidity and this is translated in repo spread.

Without commitment and costly search, when the lender is far from its participation constraint, the lender is more entranched, and so optimal contract features higher endogenous surplus, and this alters policy functions x_t and $\bar{s}_t$. As such it enhances the endogenous franchise value V^B and achieve lower margins. Moreover, in this case, the margin is less responsive. This a result of the dynamics of the relationship a la Thomas and Worrall (1988): as long as the (IR) do not bind, the optimal contract does not them into account. I conjecture that the lender will not be willing to quit the contract or renegotiate as in the outside option, due to the newnesss of the relationship, the fragility of V^B are mitigated for high levels of promised continuation value V_t^L, as illustrated in Panel B of Figure 12. The continuation values of the relationship for both parties in excess of their outside option commands not only the current pricing of the repo contract, but also the volatility of the relationship to shocks.

A continuous-time version of the environment and the introduction of persistent shocks enables to derive the following expression for the sensitivity ξ of the lender continuation value to asset innovations:

$$\xi = -\frac{\partial_{nV^L} V^B}{\partial_{V^L V^L} V^B} \frac{n\,\sigma}{mp} > 0$$

When the mapping $V^L \mapsto V^B(V^L)$, we obtain that: $\xi < \sigma$, where σ is the fundamental volatility of the asset. This can be interpreted as long-term relationships mitigating the volatility of asset markets. Borrowers then engage in *volatility transformation*. It is another justification of financial intermediation: insulate the final lender from the shocks on the underlying collateral. The intuition for this insurance result is that it is optimal for both parties to insure the lender against the aggregate shock even in a risk-neutral environment. Long-term relationships enjoy more stable financing. On the contrary, the pricing terms and volume of short-term relationships are more volatile. Therefore, it is optimal for the borrower to concentrate its financing, provided they can commit to long-term contracts.

4 Extension: the rehypothecation chain

The goal of this part is to provide an economic rationale to repo chains. I model the pyramiding arrangement consisting of having money market funds lending to broker dealers, which in turn lend to hedge funds. The two lending agreements are secured by the same collateral (rehypothecation). I show how the franchise value of the broker dealer is priced in the optimal contract on both sides of its balance sheet.

I keep only two types of beliefs,[39] but I break down optimists into two sub-groups: the no-franchise optimists ('Hedge Funds': HF) and the optimists that are endowed with franchise value ('Broker Dealers': BD). The pessimists are 'Money Market Funds': MMF. I assume rehypothecation of collateral.[40] I construct an equilibrium in which MMF lends to BD (the *tri-party repo* debt) and in turn BD lends to HF (the *bilateral repo* debt). x^{tri}, m^{tri} and r^{tri} are the number of contracts (each collaterized by one unit of the risky asset), the haircut and the rate of the first transaction ($\bar{s}^{tri}$ is the promise and D^{tri} the value of each conract). x^{bil}, m^{bil}, r^{bil}, $\bar{s}^{bil}$ and D^{bil} are the respective quantities for the second transaction. Only BD enjoys franchise value: $\tilde{s}^{bil} = \bar{s}^{bil}$ and $\tilde{s}^{tri} = \bar{s}^{tri} - V^B$. Figure 13 gives the $t = 1$ contractual payoffs.

The balance sheet constraint of HF binds: $pm^{bil}x^{bil} = n^{HF}$. The balance sheet constraint (BS) of the BD is: $x^{bil}D^{bil} \leq n^{HF} + x^{bil}D^{bil}$. Cash raised by the BD non-invested in HF debt is invested in the risky asset. The

continuation values are not as high and so the haircuts required to the borrower will be higher than in the current contract. This feature of the optimal contract (higher margins are required in new relationships) endogenously prevent from the borrower from exiting the contract ex-post. This required high margin in a new relationship outside option acts as an endogenous glue (no need of an exogenous cost of breaking up the relationship) to make the optimal contract sustainable.

[39] In this environment, the introduction of a 3^{rd} type M with moderate priors and a second state-variable, reputation capital f_t^i, would make the moderates type emerge as Financial Intermediaries, as the one with the relative most accute incentive to build up franchise value.

[40] If the Broker Dealer debt is collateralized by Hedge Fund debt, the analysis is more tedious as the BD debt needs to be priced as a *put on put*, using Geske formula, but the qualitative results of the repo chain are similar.

rehypothecation collateral constraint (CC) imposes: $x^{tri} \leq x^{bil}$. There are two regimes, depending on which of the constraint (CC) or (BS) binds. The $t = 0$ expected payoffs of the agents are as following, denoting the collar $\Delta\pi^B(m^{bil}; m^{tri}) = \pi^B(\tilde{s}^{bil}) - \pi^B(\tilde{s}^{tri})$:

$$\begin{cases} U^{HF} - U_0^{HF} & = \frac{n^{HF}}{m^{bil}p}\left[\left(1 - m^{bil}\right)\left(\mu^B - r^{bil}\right) + \pi^B(\tilde{s}^{bil})\right] \\ U^{BD} - U_0^{BD} & = \frac{n^{HF}}{m^{bil}}\left[r^{bil}(m^{bil}; m^{tri}) - r^{tri}(m^{bil}; m^{tri}) - \Delta\pi^B(m^{bil}; m^{tri}) - m^{tri}r^C p\right] \\ U^{MMF} - U_0^{MMF} & = x^{tri}\left[\left(1 - m^{tri}\right)\left(r^{tri} - \mu^L\right)p - \pi^L(\tilde{s}^{tri}) - V^B F_B(\tilde{s}^{tri})\right] \end{cases}$$

Solving jointly for the two bargaining processes (HF-BD and BD-MMF) delivers that BD engages in a positive carry trade on repo rates.

Lemma 7. *The Broker-Dealer earns a positive repo rate spread:*

$$r^{bil}(m^{bil}; m^{tri}) - r^{tri}(m^{bil}; m^{tri}) = \frac{1}{1 - \omega_{tri}(1 - \omega_{bil})}\left[\omega_{bil}(1 - \omega_{tri})\left(\mu^B + \frac{1}{1 - m^{bil}}\pi^B(\tilde{s}^{bil}) - \mu^L - \frac{1}{1 - m^{tri}}\pi^L(\tilde{s}^{tri})\right)\right]$$

From the BD first order conditions in $(m^{bil}; m^{tri})$, the rehypothecation chain features even lower haircuts.

Proposition 4. *A high Broker-Dealer franchise value lowers both the bilateral and the tri-party haircuts.*

Proof. Appendix shows $\frac{\partial m^{tri}}{\partial V^B} < 0$ *and* $\frac{\partial m^{bil}}{\partial V^B} < 0$. □

Even if both BD and HF are equally optimistic, they will find a rationale to contract secured debt. The haircut spread $m^{bil} - m^{tri}$ can be negative, and this is sustainable in equilibrium as the BD is compensated through a positive rate spread $r^{bil} - r^{tri}$. It rationalizes bilateral haircuts lower than triparty haircuts in normal times, while observing bilateral haircuts higher than tri party haircuts in stress times.

The introduction of a role for franchise value in the ability to lever up provides a justification for financial intermediation, alternative to the threat of runs as disciplining device as in Diamond and Rajan (2001a) and Diamond and Rajan (2001b), or the returns to scale in monitoring costs as in Diamond (1984) and Holmstrom and Tirole (1997). In my approach, financial intermediaries has a superior ability to develop franchise value, and this helps mitigating the bargaining frictions on both sides of the balance sheet of the broker dealer. It delivers in equilibrium the two-tiered structure: $HF \mapsto Broker - Dealer \mapsto MMF$.[41] Broker-dealers, on their asset side, are more able to seize the collateral than pessimists. At the same time, on their liability side, they are more able to lever up their franchise value than hedge funds. The first array is backed by tangible collateral: so more responsive but also more robust. The second array is backed by intangible franchise value, and therefore less responsive to collateral shocks.

Moreover, the comparative statics of the haircut spread being positive with respect to volatility, it rationalizes why, with an increase in uncertainty σ, bilateral haicuts are more reponsive than tri-party haricuts. This also delivers a more volatile (and procyclical in general equilibrium) leverage for HF than for BD, consistent with Krishnamurthy (2010) evidence. Furthermore, the **haircut spread** can turn negative.

In General equilibrium, when endogeneizing the price p of the asset, p is equal to the sum of its fundamental value and of *two* collateral values: the one enjoyed by HF and the one enjoyed by BD. This

[41] This mechanism is reminiscent of the industry practice, especially by universal banks, to *wrap up the collateral* with some of its own credit risk/franchise value.

happens from rehypothecation practice, a *collateral multiplier* effect. Bringing this repo chain to dynamics requires two state variables: net worths of the BD and of the HF, and derive the value function $V^B(n^B; n^{HF})$ along the lines of section 3. This set up delivers a leverage more procyclical for HF than BD at high levels of BD net worth. On the other hand, at low levels of BD let worth, the concavity of $V^B(n^B; n^{HF})$ makes both HF *and* BD leverage procyclical, a state that can be thought of systemic crisis.

5 Empirical analysis

The purpose of this section is to provide support to the 3 key predictions of the theoretical model:[42]

1. Proposition 1: in the cross-section, high franchise value borrowers secure lower haircuts *and* rates.

2. Proposition 2: in the time-series, haircuts are more sensitive to borrower's franchise at low net worth.

3. Proposition 3: haircuts and rates are lower and more stable in long-term relationships.

Taken together, the tests reject the hypothesis that repo markets are perfectly competitive and provide evidence that relationships matter even in secured funding. Repo markets involve four types of prices and volumes determinants: collateral specific (the type of the underlying security of the repo), borrower i specific (its franchise value V^B), lender j specific (its bargaining power ω and its risk attitude), and relationship ij specific (long-term relationship value V^L).

5.1 Data

I use a hand-collected dataset of repo transactions (*repurchase agreements)* contracted over the last six years by money market funds.[43] My dataset includes 27,172 repo transactions extracted from the quarterly SEC filings of the universe of the 145 largest Prime Institutional Money Market Funds. Money Market Funds (MMF) compose the largest volume of repo lending. According to the September 2012 Flow of Fund, US Money Market Funds hold \$508.4 *bn* outstanding in repo contracts for 2012Q2, which represents 65% of total volume of repo lending in the US to banks and broker dealers on this quarter.[44] In turn the repo holdings of the MMFs in the sample account for \$280 *bn*, i.e. 55% of the total MMF holdings. As the sample is composed of the prime institutional money market funds, I argue that the selection bias of the sample works against the tested hypothesis of screening with respect to borrower franchise and relationships. Indeed, smaller funds would have an even more pronounced incentive to trust franchise and relationships over collateral. I merge this dataset with broker-dealers characteristics: CDS from Markit and balance-sheet quantities from Y9-C call reports.

[42] Moreover, haircuts and rates are more sensitive to franchise value when the collateral is illiquid/volatile (Corollary 1), so the effects should be stronger on illiquid collateral. The static model has the additional prediction that haircuts and rates are more sensitive to franchise value with higher effective bargaining power of the lender (Corollary 4). The dynamic model also predicts that haircuts are low and stable at high borrower net worth and high and fragile at low borrower net worth.

[43] Most transactions in the dataset are US tri-party (using JP Morgan Chase and BNY Mellon as a clearing bank).

[44] The ratio is computed from Table L207 of Sept. 2012 Flow of Funds. MMF holdings are the liability line 'Money market mutual funds' and total repo lending to banks and broker dealers are the sum of the following asset lines: 'U.S. chartered depository institutions', 'Foreign banking offices in the U.S.', 'Credit unions' and 'Security brokers and dealers'. The aggregate time-series of other source of funding of broker dealers (commercial paper and fed funds) show that these markets do not perfectly substitute to secured funding.

5.1.1 Data from Money Market Fund filings

The identity and CIK numbers of the 145 largest Prime Institutional Money Market Funds is obtained from Peter Crane intelligence. Prime Money Market Funds are a recent financial innovation which allegedly offers higher returns with no risk, and are allowed to invest in non-government securities. Following the procedure of Krishnamurthy et al. (2011), I parse with a Perl script all the quaterly filings of the last 6 years of these 145 MMF (24 quarterly filings for each MMF: forms N-Q, N-CSR and N-CSRS available on SEC Edgar website).[45] MMFs of the same family concatenate their filings in the same html file. I collapse these MMF in one lender identity j, in order to wash out substitution effects from one fund to another within the same family. The haircut can be computed from the collateral fair value and the notional, and the repo rate can be computed from the repurchase amount and the notional.[46] I categorize the collateral described as free-entry text in MMF filings into 9 categories: Treasuries, Agencies, Municipals, Commercial Paper, Corporate Debt, Foreign Debt, Equities, Structured Finance and MixedPool. This follows the topography of collateral used in custodian contracts of tri-party agreements. My dataset contains all the repo transactions reported in these SEC filings, and details for each of those: the volume of the transaction, the rate, the haircut, the maturity, the collateral type and the identities of the borrower and the lender. The counterparty identity is manually screened and replaced by the relevant franchise name (e.g. Barclays for Barclay's Capital or any subsidiary of Barclays). I compute the repo spread, as the difference between the repo rate and the Fed fund rate of the same maturity.

5.1.2 Complementary data source

I use specific characteristics of borrowers and of lenders as regressors on repo prices and volumes. For borrowers, I match the broker-dealers included in the repo dataset with Y9-C call reports items: *goodwill* (Y9C item BHCK3163) as proxy for the franchise value V^B and *total equity capital* (Y9C items BHCK3300 and BHCK2948) as proxy for borrower net worth n_t^B. I also fetch their respective exposure to the different funding markets: *commercial paper* (Y9C item BHCK2309) and *fed funds* (Y9C item BHDMB993), in order to control for substitution effects between these markets. I also match the borrowers with market-based measures of their franchise: CDS and CDS lagged 3 months (from Markit).[47] Regarding the lenders, I use CRSP Mutual Fund database to construct a measure of their risk attitude based on Inflows and Yield their experienced in 2008, following the procedure of Kacperczyk and Schnabl (2012) and Chernenko and Sunderam (2012). For collateral, I use volatility index (VIX and TED)) to capture the volatility of the underlying collateral. Finally, to assess the persistence of relationships between Hedge Funds and Broker Dealers, I use prime brokerage information in TASS dataset. For robustness, I also explore the cross-section of haircuts and rates in a sample not from money market funds but from pension funds, which are not regulated by Rule $2a - 7$. Data have been obtained under the Freedom of Information Act.

[45] I carry out robustness checks on N-MFP forms, filed monthly by the MMF since November 2010.

[46] For joint repurchase agreements, the haircuts and rates are computed over the entire collateral pool, and assigned to each repo transaction included in the joint repurchase agreement. This type of joint contract usually involves MMF from the same family.

[47] CDS of 5 year tenure (the most liquid). I manually take into account franchise mergers (HVB taken over by Unicredit, Wamu by BoA, Wachovia by Wells Fargo).

5.2 Summary statistics

5.2.1 Aggregate volumes

Figure 16 plots the time-serie of aggregate repo volume from the dataset by collateral class. It follows a pattern analogous to the Flow of Fund repo lending to banks and broker dealers, albeit less dramatic. The discrepancy therefore comes from repo not contracted by money market funds, e.g. the *rest of the world* lending category in Flow of Fund which sharply contracted over the crisis. This is consistent with the hypothesis of the existence of relationships between banks and broker dealers and money market funds that helped sustain a stable level of repo funding over the crisis along these relationships. The right-hand side panel plots the same time-serie excluding Treasuries & Agencies. Consistent with Krishnamurthy et al. (2011) and (Martin, 2012), it documents a volatile level of repo funding for structured finance. I use this segment of the repo market to gain power in the test of the franchise value channel. The time-serie of aggregate volume has a semestrial spiky shape.[48] This might be due to difference in data reporting between forms N-Q (q2 and q4), and forms N-CSRS/N-CSRS (q1 and q3). This is not of a concern given the cross-sectional analysis.

5.2.2 Pricing terms

Table 1 gives the summary statistics of haircuts. For pricing terms (haircut and rate), the dataset is winsorized at the 1% and 99% levels to dismiss reporting mistakes. The majority of repo transactions collateralized by Treasuries command a haircut of 2%. However, some are not, and these are mostly term repos with long-term maturities. Figure 15 documents that there is a higher haircut dispersion for more volatile collateral, consistent with Proposition 4 of the model. Summary statistics of repo spreads are given in Table 2 and dispersion in Figure 16. Even more than haircuts, repo spreads exhibit dispersion around the mean for more volatile collateral. Maturities are the third pricing variable of a repo transaction and its summary statistics are given in Table 3. In the specifications presented in the followong, I focus on overnight repos (maturity of 1 day) to make sure the results are not driven by maturity risk. Across the three pricing variables, the sample documents an aggregate time-varying funding premium, especially in the Lehman episode and over the European debt crisis. I investigate in the following their idiosyncratic component.

5.2.3 Microstructure: network of relationships

In Figure 17, I provide a graphical representation of the network of bilateral repo transactions for two key quarters: 2007q3 (start of the crisis) and 2008q3 (midst of the crisis). The dataset features 40 borrowers (broker-dealers) and 45 lenders (the 145 money market funds grouped by family).

One static feature of the network relies in agents' heterogeneity in their *concentration* of financing. On the borrower side, nodes like Merrill, Citi and Goldman Sachs secure repo funding from a variety of sources. On the contrary, Deutsche Bank, Barclays and JP Morgan exhibit concentrated financing. I construct quantitative measures of financing dispersion. For a node $i \in I$ the universe of borrowers, with D_{ij} the repo volume in the relationship between borrower i and lender j over the given quarter, I define the following metrics. The first one uses the extensive margin of relationship existence, the second one is inspired by the Herfindahl index of atomicity:

[48] The relative decline of volume at each end-of-year quarter hints to some window-dressing practices. Although this is not a first-order issue in the present idiosyncratic analysis, I run robustness regressions excluding all quarters q4.

$$nbrel_i = \frac{\sum_j \#relationships_{ij}}{\sum_i \sum_j \#relationships_{ij}}$$

$$networkscope_i = \frac{1}{1-\frac{1}{\sum_j \#relationships_{ij}}}\left(1-\left(\frac{\sum_j D_{ij}}{\sum_i \sum_j D_{ij}}\right)^2\right)$$

$$reposhare_i = \frac{\sum_j D_{ij}}{\sum_i \sum_j D_{ij}}$$

One dynamic feature consists in the *persistence* of bilateral edges, especially the ones involving a borrower exhibiting concentrated financing. These bilateral edges flag the potential existence of a long-term relationship between the two agents. I construct quantitative measures to capture the persistence of bilateral connections:[49]

$$persistenceratio_{i,t} = \frac{\sum_j \#relationships_{ij}|existing\ at\ t-1}{\sum_j \#relationships_{ij,t}}$$

And a bilateral-specific measures of long-term relationships:

$$persist\ rel_{ij,t} = 1_{\{link\ ij\ t-1\}}$$

$$history\ rel_{ij,t} = \sum_{t_-} 1_{\{link\ ij\ t_-\}}$$

Table 4 presents the summary statistics of these relationship metrics for the 40 borrowers in the sample, along with the balance sheet characteristics from call reports. Figure 18 illustrates the time-serie pattern of the heterogeneity of the $persistenceratio_{i,t}$ metrics. Table 5 presents the summary statistics of the symmetic metrics for the lenders (grouped in 40 families), along with MMF characteristics obtained from Peter Crane intelligence.

5.3 Test of the model

First, I reject the null hypothesis that repo markets are perfectly competitive. Under this hypothesis, characteristics related to the identity of the borrower and the lender should not matter in repo funding whose price should be solely determined by the nature of collateral. Second, I investigate the power of borrower-specific, lender-specific and bilateral-relationship-specific characteristics to explain repo pricing and volumes.

The first stage is therefore to extract the component in haircuts and repo rates that is not explained by the nature of collateral and aggregate conditions. I carry this out by computing the residual of the OLS of haircuts and repo spreads on quarterly time fixed effects and collateral type fixed effects(specification (0a)):[50]

$$m_{l,ijt} = \mu_t^m + \sum_k \beta_k^m 1_{\{col\ k\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \sum_k \beta_k^r 1_{\{col\ k\}} + \epsilon_{l,t}^r$$

[49] Simarly to the static metrics, the first one uses information on the extensive margin (prior existence of the relationship), whereas the second one uses the continuous information of repo flows.

[50] Arguably, on the US tri-party repo market, for a given collateral type, *the lender does not care about which exact security collateralizes the repo*. Indeed, the lender delegates to the clearing bank the responsibility to check that the collateral posted by the borrower enters in the collateral type agreed upon by the two party, according to the collateral topography of the custodian agreement. Therefore, collateral type fixed effects are sufficient to absorb all the collateral-specific component from the dependent variables.

The quarter and collateral class coefficients of the first-stage regression are given in Table 6. The coefficients on each collateral class fixed effect are in line with model, which implies that more volatile collateral command higher haircuts and higher rates. The quarterly time fixed-effect coefficients document aggregate change in repo funding over the sample. Interestingly, the OLS does not find an aggregate time-effect on haircuts, but shows a significant positive aggregate coefficient of $44bps$ for 2007q4 at Lehman crisis, immediately followed by significant coefficients from 2008q1 to 2008q4, from $-57bps$ to $-27bps$. Taken together with the Flow of Fund evidence that repo funding volume progressively declined over this period are evidence that the 2007q4 fuding stress episode was a negative supply shock (less repo supply), followed by a protracted negative demand shock (less repo demand). The residuals $\hat{m}_{l,t}$ and $\hat{r}_{l,t}$ are the idiosyncratic components I am investigating in the following.

In order to flag the identity of which borrowers and which lenders get consistent idiosyncratic pricing, I run are the following dummy specification (0b):[51]

$$m_{l,ijt} = \mu_t^m + \textstyle\sum_k \beta_k^m 1_{\{col\,k\}} + \sum_i \beta_i^m 1_{\{borrower\,i\}} + \sum_j \beta_j^m 1_{\{lender\,j\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \textstyle\sum_k \beta_k^r 1_{\{col\,k\}} + \sum_i \beta_i^r 1_{\{borrower\,i\}} + \sum_j \beta_j^r 1_{\{lender\,j\}} + \epsilon_{l,t}^r$$

Results are reported Tables 7 and 8, columns 1 and 3. Several dummy variables are omitted due to colinearity. Testing for $\beta_i^m = \beta_j^m = 0$ rejects the null hypothesis of perfect competition. Moreover, dummy coefficient already show that strong franchise borrowers are securing repo with lower spreads and MMF with high bargaining power are getting higher spreads.

5.3.1 Test of the effect of borrowers franchise value

Test of Proposition 1: $\frac{\partial m}{\partial V^B} < 0$ and $\frac{\partial r}{\partial V^B} < 0$ To test the franchise value channel, I run a panel regression of haircut and rate on measures of borrower franchise as regressors, still including collateral dummies and time fixed effects (specification (1a)):

$$m_{l,ijt} = \mu_t^m + \beta_B^m V^B + \textstyle\sum_k \beta_k^m 1_{\{col\,k\}} + \sum_j \beta_j^m 1_{\{lender\,j\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \beta_B^r V^B + \textstyle\sum_k \beta_k^r 1_{\{col\,k\}} + \sum_j \beta_j^r 1_{\{lender\,j\}} + \epsilon_{l,t}^r$$

For measures of V^B I use the following characteristics: CDS, CDS lagged, book value of equity and goodwill.[52] Table 9 reports the results. The coefficient β_B^r is statistically and economically significant: $43bps$ has to be compared to the mean gross yield of repo contracts: $20bps$. The coeefficient β_B^m is also significant when use CDS lagged one quarter, and net worth computed from Y9-C call reports. I find support that stronger franchise (lower CDS) secure lower haircuts, at the expense of higher rates, consistent with the model.

Compared to Krishnamurthy et al. (2011) which find no effect from borrower idendity, two characteristics of the dataset are in order to explain the different results. First, as I parse also smaller money market funds (but not securities lenders), I get more more borrowers in the dataset. Moreover, smaller money market funds might be more prone to trust franchise beyond collateral.

[51] Robustness checks I ran include ading an interaction term $1_{\{col\,k\}} * 1_{\{borrower\,i\}}$ to control for borrowers riskier only because they are more exposed to volatile collateral.

[52] CDS spreads capture risk-neutral default probabilities and a recovery rate from the debtholder standpoint. Therefore it contains a collateral component and a franchise component. The colateral dummies filter out the former component.

Identification strategy: the European sovereign debt crisis I use the European sovereign debt crisis as an exogenous shock on the franchise value channel. Arguably, this crisis is a shock uncorrelated to the underlying US collateral (US structured finance, US corporate debt), and I take advantage of the fact that European banks act as US financial intermediaries: borrowing from US MMF before lending to US households. The US subsidiaries of European banks are heavily reliant as borrowers on US tri-party repo market, as shown by network graphs (figures 20 and 21) and heavy lenders to US shadow banking sector. The European debt crisis is an exogenous shock to the franchiseof European borrowers i: borrower ticker equal to DB, CS, UBS, ABN, HSBC, HVB, DRSDNR, SOCGEN, BARC, CALYON, CMZB, BNP, ING, FORT. [53]

I run the following Difference-in-Difference specification (1b):[54]

$$m_{ijt} = \mu_t^m + \beta_1^m * 1_{Eur\,crisis} + \beta_2^m * 1_{Eur\,crisis} + \beta_3^m * 1_{Eur\,crisis} * 1_{Eur\,bank} + \epsilon_{l,t}^m$$

$$r_{ijt} = \mu_t^r + \beta_1^r * 1_{Eur\,crisis} + \beta_2^r * 1_{Eur\,crisis} + \beta_3^r * 1_{Eur\,crisis} * 1_{Eur\,bank} + \epsilon_{l,t}^r$$

The test of the franchise value channel is: $\beta_3^m, \beta_3^r > 0$. Table 10 reports the results of this specification. In line with prediction 5, the results are highly significant when using the illiquid collateral sample. The coefficients $\beta_1^m, \beta_1^r < 0$ are also interesting and document a negative demand shock in repo funding over the European crisis.

Test of Corollary 1: $\frac{\partial^2 m}{\partial\sigma\partial V^B} < 0$ and $\frac{\partial^2 r}{\partial\sigma\partial V^B} < 0$ A first piece of evidence is that there is more *dispersion* in haircuts and repo rates for illiquid collateral than for liquid collateral, and during quarters with high volatility than quiet quarters. This is illustrated by the 10^{th} and 90^{th} percentiles on Figures 15 and 16. Results of the previous specification are much more significant when excluding Treasuries and Agencies from the sample. I also run the dummy specification with interaction terms:

$$m_{l,t} = \mu_t^m + \sum_i \sum_k \beta_{ik}^m 1_{\{col\,k\}} * 1_{\{borrower\,i\}} + \epsilon_{l,t}^m$$

$$r_{l,t} = \mu_t^r + \sum_i \sum_k \beta_k^r 1_{\{col\,k\}} * 1_{\{borrower\,i\}} + \epsilon_{l,t}^r$$

5.3.2 Test of the effect of lenders risk attitude

Corollary 4 predicts that lenders with more pessimistic beliefs should obtain lower haircuts and lower rates, and also should trust more the franchise value. The first piece of evidence comes from the lenders coefficients in specification (1) (Table 8): preeminent money market funds secure higher haircuts and rates. It leads to investigate how the haircut and the repo spread comove in the cross-section of lenders. To this end, I run the following preliminary regression:

$$r_{l,ijt} = \mu_t^r + \alpha\, m_{l,ijt} + \sum_k \beta_k^r 1_{\{col\,k\}} + \sum_j \beta_i^r 1_{\{borrower\,i\}} + \epsilon_{l,t}^r$$

The cofficient obtained is positive and significant: $\alpha = 295$ with a standard error (clustered at the lender level) of 28 . This suggests that, *in the market for secured funding*, the MMF risk-return trade off demonstrated

[53] If I had data on European lenders investing in US tri-party as hinted by repo volume from Flow of Fund, i.e. j belonging to Europe, I could use the European debt crisis as an exogenous shock on the *relationship value* V^L, symmetrically to what I do here on V^B.

[54] Due to the short time period of the sample, robustness checks include to run the specification of $1_{Eur\,bank}$ on symmetric pre- and crisis samples separately, following Bertrand et al. (2004).

by Kacperczyk and Schnabl (2012) is complemented by another type of heterogeneity, the one on the borrower side documented above. I investigate two plausible explanations consistent with this positive coefficient α: lenders heterogeneity in pessimism or in relationship value.

To investigate further the origin of the effect of this heterogeneity on repo lending, I test lender characteristics on both the haircut and the rate (specification (2a)):

$$m_{l,ijt} = \mu_t^m + \beta_L^m V^L + \textstyle\sum_k \beta_k^m 1_{\{col\,k\}} + \sum_j \beta_j^m 1_{\{borrower\,i\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \beta_L^r V^L + \textstyle\sum_k \beta_k^r 1_{\{col\,k\}} + \sum_j \beta_j^r 1_{\{borrower\,i\}} + \epsilon_{l,t}^r$$

I use for V^L the following MMF characteristics, proxying pessimism and concentration limits: $RepoExp_j$. Results are reported in Table 11. $\beta_L^m > 0$ and $\beta_L^r > 0$ are both economically and statistically significant, showing that MMF with higher bargaining power secure both higher haircuts and higher rates. Moreover, when use for V^L metrix characterizing MMF risk-taking attitude / pessimism (Inflows08 and Yield08), I find $\beta_L^m > 0$ and $\beta_L^r > 0$, consistent with the results on unsecured funding volumes of Kacperczyk and Schnabl (2012) and Chernenko and Sunderam (2012).

5.3.3 Test of the effects of long-term relationships

Proposition 3 states that repo funding is more stable (less countercyclical) for borrowers enjoying long-term relationships. Long-term relationships should enjoy lower and more stable haircuts and rates. I measure the existence of long-term relationships by more *concentrated financing* and *longer history of the relationship*.

Results on pricing variables: haircuts and rates The first exercice is to complement the dummy OLS with relationship dummies. The specification I run are (specification (3a)):

$$m_{l,ijt} = \mu_t^m + \textstyle\sum_i \beta_i^m 1_{\{borrower\,i\}} + \sum_j \beta_j^m 1_{\{lender\,j\}} + \beta_{ij}^m 1_{\{borrower\,i\}} 1_{\{lender\,j\}} + \sum_k \beta_k^m 1_{\{col\,k\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \textstyle\sum_i \beta_i^r 1_{\{borrower\,i\}} + \sum_j \beta_j^r 1_{\{lender\,j\}} + \beta_{ij}^r 1_{\{borrower\,i\}} 1_{\{lender\,j\}} + \sum_k \beta_k^r 1_{\{col\,k\}} + \epsilon_{l,t}^r$$

Even for the second set of specification, bilateral relationship coefficients are significant, especially on the rates (β_{ij}^r). This dummy regression elicits which (i, j) pair between a broker-dealer borrower and a MMF lender is an actual long-term relationship. The sign of the coefficients β_{ij}^m and β_{ij}^r informs in which direction the bilateral relationship is more in favor. Results of this voluminous dummy regression are available upon demand.

I now investigate how the respective bargaining powers of borrowers and lenders affect the haircut and rate pricing. I run the following specifications with continuous dependent variables (specification (3a) - can be run on first differences):

$$m_{l,ijt} = \mu_t^m + \beta^m Rel_t^B + \gamma^m Rel_t^L + \delta^m history\,rel_{ij} + \textstyle\sum_k \beta_k^m 1_{\{col\,k\}} + \epsilon_{l,t}^m$$

$$r_{l,ijt} = \mu_t^r + \beta^r Rel_t^B + \gamma^r Rel_t^L + \delta^r history\,rel_{ij} + \textstyle\sum_k \beta_k^r 1_{\{col\,k\}} + \epsilon_{l,t}^r$$

For Rel_t^B and Rel_t^L, I use different measures of the impact of relationships on respectively the borrowers and the lenders: $RepoShare_i$ and $\#relationships_i$ and $ScopeL$, and the symmetric measures for lenders bargaining powers δ^j.[55] The test of Corollary 4 is $\beta^m, \beta^r > 0$ and $\gamma^m, \gamma^r < 0$. Results on the sample excluding Treasuries and Agencies are reported Table 12. The results are more conclusive for repo spreads than for haircuts. This is due to the empirical fact that haircuts are not negotiated on a daily basis, but set in the custodian agreement. We observe a negative significant effect of Rel_t^L both for haircuts and rates. The results of the OLS give some significance for the Rel_t^L . In line with Corollary 1, results on the sample excluding Treasuries and Agencies are even more significant. The results using the history of the relationships are conclusive for $PersistentRel$: persistent relationships are able to achieve lower rates for the borrower.[56]

Result on volume variables First I analyze the stability of the network structure. I use two metrics to capture the stability of bilateral connections. On the intensive margin, $\Delta Volij = |\Delta RepoVolij|$ is the absolute value of the change, from one quarter to another, in the repo volume of bilateral relationship between borrower i and lender j, normalized to the total quarterly repo volume. On the extensive margin, $1_{rel\,ij,t}$ is a dummy variable equal to 1 if the bilateral connection ij at the quarter t. The test is done via probit. The two regressors are $HistoryRel_{ij,t} = \sum_{t_-} 1_{\{link\,ij\,t_-\}}$ and $PersistentRel_{ij,t} = 1_{\{link\,ij\,t_-\}}$. Thus specification (3b) is:

$$|\Delta RepoVolij|_{ijt} = \alpha^{int} + \beta^{int}\, HistoryRel_{ij} + \epsilon_{ij,t}$$

$$E\left[1_{\{link\,ij,t\}}\right] = 1/1 + exp\left(\alpha^{ext} + \beta^{ext} HistoryRel_{ij}\right)$$

The test of the model is $\beta^{int} < 0$ and $\beta^{ext} > 0$. Table 13 reports the results and finds significant coefficients consistent with the model. The probit coefficient can be interpreted (dprobit) as: one more quarter of history of the relationship increases by 12% the probability of existence of the relationship in quarter t.

Finally, I test the effect of the existence of long-term relationships on the stability of secured funding volume for borrowers. $\Delta Repo = |\Delta RepoVolB|$ is the absolute value of the change, from one quarter to another, in borrower B repo funding normalized to the total quarterly repo volume. $Repo/ST = Repo/STfunding$ is the ratio, for borrower B, between its repo funding (in \$) and the \$ sum of all its short-term funding sources: fed funds + repo + short-term deposits + commercial paper + short-term liabilities (data from Y9-C call reports). This proxies for the easiness of access to repo for borrower B and captures potential substitution from other funding sources, in case of repo funding difficulties. $VBtotMMFrisk$ is a measure of MMF risk-taking behavior, aggregated at the Borrower level. $NbPersRel$ is the number of bilateral relationships that the borrower already had at the previous quarter. $NbLTRel$ is the number of long-term relationships the borrower enjoys (a long-term relationship is defined when the number of quarter of existence of the relationship is above the median of the universe of relationships). The last two lines are interaction terms to test that long-term relationships help stabilize repo funding.

Borrowers with long-term relationships enjoy more stable funding. It is tested in specification (3c):

$$|\Delta RepoVolB|_{i,t} = \alpha + \beta\, NbLTRel_{i,t} + \epsilon_{i,t}$$

Table 14 reports the results of this specification, as well results of specification with potential borrower repo volume stability as regressors. I find that the existence of Long-Term relationships mitigates the

[55] Even if these regressors are highly endogenous, they capture the bargaining power δ information according to my model (see also Lee and Fong (2012) for a model with endogenous network formation where this property is also true).

[56] The existence of previous relationship is still an endogenous variable. One could instrument the prior existence of relationship by lending to first-time borrowers. This would be instruments of an exogenous variation in relationships.

sensitivity of repo funding to the quality of the franchise (measured by $-CDSquarter$). The existence of long-term relationships provides an explanation of why repo lending was fairly stable over the crisis. It is consistent with Hrung and Sarkar (2012), which finds an autocorrelation of 0.95 in the volume of borrower repo funding, in the daily tri-party repo data of the Fed.

6 Conclusion

This paper develops a model of decentralized markets for secured funding in which the franchise value of borrowers and long-term relationships matter. Continuation values substitute for collateral and sustain low and stable levels for both the haircuts and the rates. Due to the endogeneity of borrower franchise value, haircuts are countercyclical. This can be mitigated by establishing long-term bilateral relationships. This channel is magnified when the collateral is volatile and when lenders competition is imperfect. Franchise value of broker-dealers can also rationalize long intermediation chains as the efficient arrangement to monetize franchise.

The empirical analysis on a hand-collected dataset of repo transactions rejects the null hypothesis that repo markets are perfectly competitive. It demonstrates that relationships are priced even in secured funding, and are instrumental in the stability of refinancing. It shows that this stability is conditional on the existence of franchise and relationships, which can unravel quite abruptly. Even if small compared to ABCP, repo markets can therefore be quite destabilizing for the financial system.

Eventhough constrained efficient, the present model delivers policy recommendations. It enables to compare the *effectiveness* of different ex post policy instruments to alleviate a credit crunch. Franchise value is concave with respect to borrower net worth. This advocates for *equity injections* (e.g. second Paulson plan) in order to jump start franchise values and restore confidence in secured funding markets. Alternatively, *lending facilities* (LTRO, TALF, PDCF, TSLF, CPFF, AMLF) also enable financial intermediaries leverage by short-circuiting the bargaining friction in private secured funding markets. In the model though, it has a negative side-effect of breaking up welfare-improving long-term relationships in private secured funding markets. On the other hand, the model does not provide any support for policies of *asset purchases* (TARP), as in general equilibrium it would merely have a crowding-out effect of private investment.

Regarding ex-ante policies, the existence of a franchise value channel supports *institution-level* regulation of haircuts and leverage, and not at an asset-level. Finally, the model of the rehypothecation chain advocates for more transparency in prime brokerage, as bargaining outcome on $r^{tri}(m^{tri})$ instead of $r^{tri}(m^{bil}; m^{tri})$ leads to suboptimal choice of haircuts.

An interesting extension of the model is to endow borrowers and lenders with a costly capacity to learn about the collateral, in a rational inattention framework. Under the presence of franchise, the lender cares *less* about the collateral, and as a result learns less about it. This predicts that there is less learning on a specific class of collateral if one strong franchise value is the marginal buyer in this market.

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APPENDIX

A Static model

A.1 Optimal contract

There are I borrowers and J lenders. We assume there are no inactive borrower or lender in equilibrium. We focus on symmetric pairwise stable equilibria. The collateral constraint of each borrower is: $\sum_J x_{ij} m_{ij} p \leq n^B$ so $x_{ij} = \frac{1}{J} \frac{n^B}{m_{ij} p}$. We note $m_{ij} = m$.

Expected utility of each borrower The expected utility of the borrower, for given contracts x_{ij}, is:

$$U^B_{I,J} = \sum_J x_{ij} \, \mathbb{E}_B \left[1_{\{no\,def\}} \left(s - \bar{s} \right) \right] + \beta \mathbb{E}_B \left[1_{\{no\,def\}} V^B \right] \text{ with } \sum_J x_{ij} m_{ij} p \leq n^B$$

Using $\tilde{s} = \bar{s} - V^B$ the default threshold, recalling $mp = p - D$ and $\pi^B(\tilde{s}) = \int_{s^{min}}^{\tilde{s}} (\tilde{s} - s) \, f_B(s) ds$ (with s^{min} and s^{max} potentially equal to $\pm\infty$):

$$\mathbb{E}_B \left[1_{\{no\,def\}} \left(s - \bar{s} \right) \right] = \int_{\bar{s} - V^B}^{s^{max}} \left(s - \bar{s} \right) ds = p \left(1 + \mu^B \right) - \tilde{s} - \left(1 - F_B(\tilde{s}) \right) V^B + \pi^B(\tilde{s})$$

$$\left(U^B_{I,J} - U^B_0 \right)(\tilde{s}, D) = \frac{n^B}{p - D} \left(p \left(1 + \mu^B \right) - \tilde{s} - \left(1 - F_B(\tilde{s}) \right) V^B + \pi^B(\tilde{s}) \right) + \beta V^B \left(1 - F_B(\tilde{s}) \right) - n^B \left(1 + \mu^B \right) + \beta V^B$$

The expected utility of the borrower can also be written as a function of r and m only:

$$\left(U^B_{I,J} - U^B_0 \right)(r, m) = \frac{n^B}{m} \left[(1 - m) \left(\mu^B - r \right) + \frac{1}{p} \pi^B(\tilde{s}) + \left(1 - \frac{\beta}{n^B} mp \right) V^B \frac{1}{p} F_B(\tilde{s}) \right]$$

Expected utility of each lender The expected utility of each lender, for given contracts x_{ij}, is[57]:

$$\left(U^L_{I,J} - U^L_0 \right)(\tilde{s}, D) = \sum_i x_{ij} \mathbb{E}_L \left[1_{\{def\}} s + 1_{\{no\,def\}} \bar{s} \right] + \left(n^L - \sum_i x_{ij} D \right) (1 + \mu^L) - n^L (1 + \mu^L)$$

With $\mathbb{E}_L \left[1_{\{def\}} s + 1_{\{no\,def\}} \bar{s} \right] = \int_{s^{min}}^{\tilde{s}} s \, f_L(s) ds + \left(\tilde{s} + V^B \right) \left(1 - F_L(\tilde{s}) \right) = \tilde{s} + \left(1 - F_L(\tilde{s}) \right) V^B - \pi^L(\tilde{s})$:

$$\left(U^L_{I,J} - U^L_0 \right)(\tilde{s}, D) = \frac{I}{J} \frac{n^B}{p - D} \left(\tilde{s} + \left(1 - F_L(\tilde{s}) \right) V^B - \pi^L(\tilde{s}) - (1 + \mu^L) D \right)$$

We can also write it as a function of r and m only:

$$\left(U^L_{I,J} - U^L_0 \right)(r, m) = \frac{I}{J} \frac{n^B}{m} \left[(1 - m) \left(r - \mu^L \right) - \left(\frac{1}{p} \pi^L(\tilde{s}) + V^B \frac{1}{p} F_L(\tilde{s}) \right) \right]$$

[57] The extension where lenders have mean-variance preferences adds a penalty term quadratic in x_{ij}^2. This delivers extra diversification benefits with respect to J. Lender heterogeneity can also be thought along the γ dimension.

$$\left(U^B_{I,J} - U^B_0 \right)(r, m) = \frac{I}{J} \frac{n^B}{m} \left[(1 - m) \left(r - \mu^L \right) - \Pi^L(\tilde{s}) \right] - \frac{I}{J^2} \gamma \left(\frac{n^B}{mp} \right)^2$$

Multilateral Nash bargaining Denoting ω the bargaining power of the lender, the surplus of each bilateral relationship is shared according to:

$$\omega\left(U^B_{I,J}-U^B_{I,J-1}\right)=(1-\omega)\left(U^L_{I,J}-U^L_{I-1,J}\right)$$

I formulate the induction hypothesis: $U^B_{I,J-1}-U^B_0=(1-\omega)n^B S_{I,J-1}$ and $U^L_{I-1,J}-U^L_0=\omega n^B S_{I-1,J}$. The bargaining does not into account he continuation term of the borrower (static bargaining).

$$\omega\left\{\frac{n^B}{p-D}\mathbb{E}_B\left[1_{\{no\,def\}}\left(s-\tilde{s}\right)\right]-n^B\left(1+\mu^B\right)\right\} \quad = \quad (1-\omega)\left\{\frac{I}{J}\frac{n^B}{p-D}\left(\mathbb{E}_L\left[1_{\{def\}}s+1_{\{no\,def\}}\tilde{s}\right]-(1+\mu^L)D\right)\right\}$$

I define:

$$\delta=\frac{\omega}{(1-\omega)\frac{I}{J}+\omega}\text{ and }1-\delta=\frac{(1-\omega)\frac{I}{J}}{(1-\omega)\frac{I}{J}+\omega}\text{ and }\delta^{IJ}=\frac{(1-\omega)\omega}{(1-\omega)\frac{I}{J}+\omega}=\delta(1-\omega)$$

$\delta\in[0;1]$ is a measure of the *lenders effective bargaining power*. Rearranging terms yields the value D of the contract obtained from the bargaining:

$$D=\frac{(1-\delta)\mathbb{E}_L\left[1_{\{def\}}s+1_{\{no\,def\}}\tilde{s}\right]+\delta\left(\left(1+\mu^B\right)p-\mathbb{E}_B\left[1_{\{no\,def\}}(s-\tilde{s})\right]\right)+\delta^{IJ}\left(S_{I,J-1}-S_{I-1,J}\right)p}{(1-\delta)(1+\mu^L)+\delta\left(1+\mu^B\right)+\delta^{IJ}\left(S_{I,J-1}-S_{I-1,J}\right)}$$

I define the levered return (per unit of net worth) perceived by the borrower:

$$R^{lev}_B=\frac{\mathbb{E}_B\left[1_{\{no\,def\}}(s-\tilde{s})\right]}{p-D}$$

I note $\mu^\delta=(1-\delta)\mu^L+\delta\mu^B$ and $\mu^{\delta\,barg}=\mu^\delta+\delta^{IJ}\left(S_{I,J-1}-S_{I-1,J}\right)$. Given the value of the contract D derived from the bargaining:

$$R^{lev}_B=\left(1+\mu^{\delta\,barg}\right)\frac{\mathbb{E}_B\left[1_{\{no\,def\}}(s-\tilde{s})\right]}{(1-\delta)\left(1+\mu^L\right)p-(1-\delta)\mathbb{E}_L\left[1_{\{def\}}s+1_{\{no\,def\}}\tilde{s}\right]+\delta\mathbb{E}_B\left[1_{\{no\,def\}}(s-\tilde{s})\right]}$$

I now introduce the compound distribution: $F_\delta(s)=(1-\delta)F_L(s)+\delta F_B(s)$. It is a linear combination of normal distributions, thus: $F_\delta\sim N\left(\mu^\delta;\sigma\right)$. The beliefs δ and B still satisfy the hazard rate order property:

$$\forall\tilde{s},\ \frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})}>\frac{f_B(\tilde{s})}{1-F_B(\tilde{s})}$$

Using the expressions of $\mathbb{E}_L\left[1_{\{def\}}s+1_{\{no\,def\}}\tilde{s}\right]$ and $\mathbb{E}_B\left[1_{\{no\,def\}}\left(s-\tilde{s}\right)\right]$ as a function of the put perceived valuation $\pi^B(\tilde{s})$ and $\pi^L(\tilde{s})$, and denoting $\pi^\delta(\tilde{s})$ the put valuation under the compound beliefs F_δ, I obtain the following expression for R^{lev}_B, introducing $R^{unl}_\delta(\tilde{s})$ and $R^{unl}_B(\tilde{s})$:

$$R^{lev}_B(\tilde{s})=\left(1+\mu^{\delta\,barg}\right)\frac{p\left(1+\mu^B\right)-\tilde{s}-(1-F_B(\tilde{s}))V^B+\pi^B(\tilde{s})}{p(1+\mu^\delta)-\tilde{s}-(1-F_\delta(\tilde{s}))V^B+\pi^\delta(\tilde{s})}=\left(1+\mu^{\delta\,barg}\right)\frac{R^{unl}_B(\tilde{s})}{R^{unl}_\delta(\tilde{s})}$$

As a result I have expressed the borrower expected utility as a function of the contract riskiness $\tilde{s}$ only:

$$\left(U^B_{I,J}-U^B_0\right)(\tilde{s}) \quad = \quad n^B\left(1+\mu^{\delta\,barg}\right)\left(\frac{p\left(1+\mu^B\right)-\tilde{s}-\left(1-F_B(\tilde{s})\right)V^B+\pi^B(\tilde{s})}{p(1+\mu^\delta)-\tilde{s}-\left(1-F_\delta(\tilde{s})\right)V^B+\pi^\delta(\tilde{s})}-\frac{\beta V^B}{n^B\left(1+\mu^{\delta\,barg}\right)}F_B(\tilde{s})\right)-n^B\left(1+\mu^B\right)$$

Borrower maximization program The borrower solves for the optimal riskiness $\tilde{s}$:

$$\underset{\{\tilde{s}\}}{Max}\ \left(U^B_{I,J} - U^B_0\right)(\tilde{s}) \propto \underset{\{\tilde{s}\}}{Max}\left(\frac{R^{unl}_B(\tilde{s})}{R^{unl}_\delta(\tilde{s})} - \frac{\beta V^B}{n^B\left(1+\mu^{\delta\, barg}\right)}F_B(\tilde{s})\right)$$

The first order condition delivers:

$$0 = -R^{unl}_\delta + R^{unl}_B \frac{\partial_{\tilde{s}} R^{unl}_\delta}{\partial_{\tilde{s}} R^{unl}_B} - \frac{\beta V^B}{n^B\left(1+\mu^{\delta\, barg}\right)} \frac{\left(R^{unl}_\delta\right)^2}{\partial_{\tilde{s}} R^{unl}_B} f_B$$

Rearranging terms:

$$\begin{aligned}
\left(1+\mu^\delta\right)p &= \left[(1-F_\delta(\tilde{s})) - \frac{1-f_\delta(\tilde{s})V^B - F_\delta(\tilde{s})}{1-f_B(\tilde{s})V^B - F_B(\tilde{s})}(1-F_B(\tilde{s}))\right]V^B \\
&\quad + \left[\tilde{s} - \pi^\delta(\tilde{s})\right] + \frac{1-f_\delta(\tilde{s})V^B - F_\delta(\tilde{s})}{1-f_B(\tilde{s})V^B - F_B(\tilde{s})}\left[p\left(1+\mu^B\right) - \tilde{s} + \pi^B(\tilde{s})\right] \\
&\quad - \frac{\beta V^B}{n^B\left(1+\mu^{\delta\, barg}\right)} \frac{f_B(\tilde{s})}{1-f_B(\tilde{s})V^B - F_B(\tilde{s})}\left(\mathbb{E}_\delta\left[1_{\{no\, def\}}(s-\bar{s})\right]\right)^2
\end{aligned}$$

Introducing the auxiliary functions measuring the distortion of beliefs with V^B:

$$\kappa_1(\tilde{s};V^B) = \frac{f_\delta(\tilde{s})(1-F_B(\tilde{s})) - f_B(\tilde{s})(1-F_\delta(\tilde{s}))}{1-F_B(\tilde{s}) - f_B(\tilde{s})\,V^B} \text{ and } \kappa_2(\tilde{s};V^B) = \frac{1-V^B\frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})}}{1-V^B\frac{f_B(\tilde{s})}{1-F_B(\tilde{s})}}$$

$$\kappa_3(\tilde{s};V^B) = \frac{1-F_\delta(\tilde{s}) - f_\delta(\tilde{s})\,V^B}{1-F_B(\tilde{s}) - f_B(\tilde{s})\,V^B} = \frac{1-F_\delta(\tilde{s})}{1-F_B(\tilde{s})}\kappa_2(\tilde{s};V^B)$$

By the hazard rate property we have $0 \leq \kappa^1 \leq 1$ and $0 \leq \kappa^3 \leq 1$. κ_1 and κ_3 are related by: $(1-F_\delta(\tilde{s})) - \kappa_3(1-F_B(\tilde{s})) = V^B\kappa_1$. I then write the f.o.c:

$$\begin{aligned}
p(1+\mu^\delta) &= \kappa_1 V^B\left(\tilde{s} + V^B\right) + F_\delta(\tilde{s})\,\mathbb{E}_\delta\left[s|s<\tilde{s}\right] + (1-F_\delta(\tilde{s}))\ \kappa_2\,\mathbb{E}_B\left[s|s>\tilde{s}\right] \\
&\quad - \frac{\beta V^B}{n^B\left(1+\mu^{\delta\, barg}\right)} \frac{f_B(\tilde{s})}{1-f_B(\tilde{s})V^B - F_B(\tilde{s})}\left(\mathbb{E}_\delta\left[1_{\{no\, def\}}(s-\bar{s})\right]\right)^2
\end{aligned}$$

This first order condition implicitely defines the riskiness $\tilde{s}$ of the optimal contract:

$$0 = F\left(\tilde{s};\mu^\delta,\mu^B,\sigma,V^B,p,\mu^{\delta\, barg},\beta\right)$$

A.2 Comparative statics

For any given parameter $a \in \left\{\mu^\delta,\mu^B,\sigma,V^B,p,\mu^{\delta\, barg},\beta\right\}$, the implicit function theorem gives: $\partial_a\tilde{s} = -\frac{\partial_a F}{\partial_{\tilde{s}} F}$.

I first compute $\partial_{\tilde{s}} F$ (for $\beta \sim 0$: weak care for continuation):

$$\partial_{\tilde{s}} F = \partial_{\tilde{s}}\kappa^1 V^B\left(\tilde{s} + V^B\right) + \kappa^1 V^B + \tilde{s} f_\delta(\tilde{s}) + \partial_{\tilde{s}}\kappa_3\,(1-F_B(\tilde{s}))\,\mathbb{E}_B\left[s|s>\tilde{s}\right] - \kappa_3\tilde{s} f_B(\tilde{s})$$

We can verify that:

$$\partial_{\tilde{s}}\kappa_3 = -\kappa_3\left[\frac{\left(f_\delta + V^B f'_\delta\right)}{(1-F_\delta(\tilde{s}) - V^B f_\delta(\tilde{s}))} - \frac{\left(f_B + V^B f'_B\right)}{(1-F_B(\tilde{s}) - V^B f_B(\tilde{s}))}\right]$$

Let $h : V^B \mapsto h(V^B) = \frac{\left(f_\delta(\tilde{s})+V^B f'_\delta(\tilde{s})\right)}{\left(1-F_\delta(\tilde{s})-V^B f_\delta(\tilde{s})\right)} - \frac{\left(f_B(\tilde{s})+V^B f'_B(\tilde{s})\right)}{\left(1-F_B(\tilde{s})-V^B f_B(\tilde{s})\right)}$. With normal beliefs:

$$h(V^B) = \frac{\left(f_\delta(\tilde{s}) - \frac{\tilde{s}-\mu^L}{\sigma} V^B f_\delta(\tilde{s})\right)}{(1-F_\delta(\tilde{s}) - V^B f_\delta(\tilde{s}))} - \frac{\left(f_B(\tilde{s}) - \frac{\tilde{s}-\mu^B}{\sigma} V^B f_B(\tilde{s})\right)}{(1-F_B(\tilde{s}) - V^B f_B(\tilde{s}))}$$

We show that $\forall V^B, h(V^B) > 0$. We have $h(0) = \frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})} - \frac{f_B(\tilde{s})}{1-F_B(\tilde{s})} > 0$ by the hazard rate order property. We also have $h(+\infty) \sim -\frac{f'_\delta(\tilde{s})}{f_\delta(\tilde{s})} + \frac{f'_B(\tilde{s})}{f_B(\tilde{s})} = \frac{\tilde{s}-\mu^\delta}{\sigma} - \frac{\tilde{s}-\mu^B}{\sigma} = \frac{\mu^B-\mu^\delta}{\sigma} > 0$. After rearranging terms, we get:

$$\left(1 - \frac{f_B}{(1-F_B)} V^B\right)\left(1 - \frac{f_\delta}{(1-F_\delta)} V^B\right) h(V^B) = h(0) - \frac{\tilde{s}-\mu^\delta}{\sigma^2} h(0) V^B - \frac{\mu^B-\mu^\delta}{\sigma^2} \frac{f_B}{(1-F_B)} V^B \left(1 - \frac{f_\delta}{(1-F_\delta)} V^B\right)$$

The result obtains by monoticity of function h: $\partial_{\tilde{s}} \kappa_3 = -\kappa_3 h(\tilde{s}; V^B) < 0$.

Similarly, with $(1-F_\delta(\tilde{s})) - \kappa_3 (1-F_B(\tilde{s})) = V^B \kappa_1$ we get:

$$\partial_{\tilde{s}} \left(V^B \kappa_1\right) = -f_\delta(\tilde{s}) + \kappa_3 f_B(\tilde{s}) + (1-F_B(\tilde{s})) \kappa_3 h(\tilde{s}; V^B)$$

$$\partial_{\tilde{s}} F = -(1-F_B) \kappa_3 h(V^B) \left[\mathbb{E}_B [s|s>\tilde{s}] - \tilde{s} - V^B\right]$$

As $\mathbb{E}_B [s|s>\tilde{s}] > \tilde{s}$, we obtain: $\partial_{\tilde{s}} F < 0$ for small V^B.

A.2.1 With respect to franchise value V^B (borrower heterogeneity)

$$\partial_{V^B} F = \partial_{V^B} \kappa_1 V^B \left(\tilde{s} + V^B\right) + \kappa_1 \left(\tilde{s} + 2V^B\right) + \partial_{V^B} \kappa_3 (1-F_B(\tilde{s})) \mathbb{E}_B [s|s>\tilde{s}]$$

Using $\partial_{V^B} \kappa_1 = \frac{f_B(\tilde{s})}{1-F_B(\tilde{s})-f_B(\tilde{s})V^B} \kappa_1$ and $\partial_{V^B} \kappa_3 = -\frac{1}{1-F_B(\tilde{s})-f_B(\tilde{s})V^B} \kappa_1$, it implies:

$$\partial_{V^B} F = -\frac{\kappa_1(1-F_B)}{1-F_B-f_B V^B} \left[\mathbb{E}_B [s|s>\tilde{s}] - \tilde{s} - 2V^B + \frac{f_B}{(1-F_B)} \left(V^B\right)^2\right]$$

I obtain the following equation for the comparative statics $\partial_{V^B} \tilde{s}$, using:

$$\frac{h(0)}{\left(1 - \frac{f_B}{(1-F_B)} V^B\right)\left(1 - \frac{f_\delta}{(1-F_\delta)} V^B\right) h(V^B)} = \frac{1}{1 - \frac{\tilde{s}-\mu^\delta}{\sigma^2} V^B - \frac{\mu^B-\mu^\delta}{\sigma^2} \frac{f_B/(1-F_B)}{h(0)} V^B \left(1 - \frac{f_\delta}{(1-F_\delta)} V^B\right)}$$

$$\partial_{V^B} \tilde{s} = \frac{1}{1 - \frac{\tilde{s}-\mu^\delta}{\sigma^2} V^B - \frac{\mu^B-\mu^\delta}{\sigma^2} \frac{f_B/(1-F_B)}{h(0)} V^B \left(1 - \frac{f_\delta}{(1-F_\delta)} V^B\right)} \left(-1 + \frac{V^B \left(1 - \frac{f_B}{(1-F_B)} V^B\right)}{\mathbb{E}_B [s|s>\tilde{s}] - \tilde{s} - V^B}\right)$$

By Taylor expanding first-order in V^B:

$$\partial_{V^B} \tilde{s} = -1 + \left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2} \frac{f_B/(1-F_B)}{h(0)}\right) V^B$$

At this order $\partial_{V^B} \tilde{s} < 0$. The comparative statics of contract characteristics all derive from this expression for $\partial_{V^B} \tilde{s}$. The value of the contract D can be expressed as a function of the riskiness $\tilde{s}$:

$$D = \frac{1}{1+\mu^{\delta\, barg}} \left(p \left(\mu^{\delta\, barg} - \mu^\delta\right) + \tilde{s} + (1-F_\delta(\tilde{s})) V^B - \pi^\delta(\tilde{s})\right)$$

- Promise $\bar{s} = s^{def} + V^B$. The first-order Taylor expansion in V^B gives:

$$\partial_{V^B}\bar{s} = \partial_{V^B}\tilde{s} + 1 = \left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}\right) V^B$$

Thus starting from $V^B = 0$ the promise increases. It also shows $-1 \leq \partial_{V^B}\tilde{s} < 0$. We also notice the convexity and the null derivative at $V^B = 0$.

- Haircut: $m = 1 - \frac{D}{p}$:

$$m = \frac{1}{p\left(1+\mu^{\delta\, barg}\right)}\left(p(1+\mu^\delta) - \tilde{s} - (1-F_\delta(\tilde{s}))\, V^B + \pi^\delta(\tilde{s})\right)$$

We obtain, as $-1 \leq \partial_{V^B}\tilde{s} < 0$ and $0 \leq 1 - V^B\frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})} < 1$:

$$\partial_{V^B}m = -\frac{1-F_\delta}{p\left(1+\mu^{\delta\, barg}\right)}\left[1 + \partial_{V^B}\tilde{s}\left(1 - V^B\frac{f_\delta}{1-F_\delta}\right)\right]$$

First-order Taylor expansion in V^B:

$$\partial_{V^B}m = -\frac{(1-F_\delta(\tilde{s}))}{p\left(1+\mu^{\delta\, barg}\right)}V^B\left(\frac{1}{\mathbb{E}_B\left[s|s>\tilde{s}\right]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/\left(1-F_B\right)}{h(0)} + \frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})}\right)$$

Thus $\partial_{V^B}m < 0$. We also notice the concavity of $m(V^B)$ in the neighborhood of $V^B = 0$:

$$\partial_{V^BV^B}m_{\{V^B=0\}} = -\frac{1-F_\delta}{p\left(1+\mu^{\delta\, barg}\right)}\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}} < 0$$

- Rate: $r = \frac{\tilde{s}+V^B}{D} - 1$

$$\partial_{V^B}r = \frac{1}{D^2}\left[\left(1+\partial_{V^B}\tilde{s}\right)D - \left(\tilde{s}+V^B\right)\partial_{V^B}D\right]$$

Using the expression of contract value D:

$$\partial_{V^B}r = \frac{1}{D^2\left(1+\mu^{\delta\, barg}\right)}\left[\left(1+\partial_{V^B}\tilde{s}\right)\left(p\left(\mu^{\delta\, barg}-\mu^\delta\right) - \pi^\delta(\tilde{s}) + F_\delta(\tilde{s})\tilde{s}\right) + \partial_{V^B}\tilde{s}\left(\tilde{s}+V^B\right)V^B f_\delta(\tilde{s})\right]$$

From above, we have $-1 \leq \partial_{V^B}\tilde{s} < 0$, denoting $\mu^{\delta\, barg} - \mu^\delta = barg$ and by integration by parts, $F_\delta(\tilde{s})\tilde{s} - \pi^\delta(\tilde{s}) = F_\delta(\tilde{s})\,\mathbb{E}_\delta\left[s|s<\tilde{s}\right]$:

$$\partial_{V^B}r = \frac{V^B}{D^2\left(1+\mu^{\delta\, barg}\right)}\left[-\left(\tilde{s}+V^B\right)f_\delta(\tilde{s}) + \partial_{V^B}\bar{s}\left(\left(\tilde{s}+V^B\right)V^B f_\delta(\tilde{s}) + p\, barg + F_\delta(\tilde{s})\,\mathbb{E}_L\left[s|s<\tilde{s}\right]\right)\right]$$

As a result we have $\partial_{V^B}r_{\{V^B=0\}} = 0$. By Taylor expanding first-order in V^B:

$$\partial_{V^B}r = \frac{V^B}{D^2\left(1+\mu^{\delta\, barg}\right)}\left[-\tilde{s}f_\delta(\tilde{s}) + \left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}\right)V^B\left(p\, barg + F_\delta(\tilde{s})\,\mathbb{E}_L\left[s|s<\tilde{s}\right]\right)\right]$$

Thus for $V^B = 0$ we have $\partial_{V^B}r = 0$. For V^B small, $\partial_{V^B}r < 0$. However the comparative statics is reversed as V^Bgrows larger (but still $V^B < \mu^B$).

A.2.2 With respect to drifts μ^L (lender heterogeneity) and μ^B, and asset price p

Using $\partial_{\mu^L} F_L(\tilde{s}) = -\frac{1}{\sigma} F_L(\tilde{s})$ and $\partial_{\mu^L} \pi_L(\tilde{s}) = -\frac{1}{\sigma} \pi_L(\tilde{s})$, we derive (first-order in V^B):

$$\partial_{\mu^L} F = -(1-\delta)\left[p - \frac{1}{\sigma} F_\delta(\tilde{s}) \left(\mathbb{E}_B\left[s|s > \tilde{s}\right] - \mathbb{E}_\delta\left[s|s < \tilde{s}\right]\right)\right]$$

We have $\mathbb{E}_B\left[s|s > \tilde{s}\right] > p(1+\mu^B)$ and $\mathbb{E}_\delta\left[s|s < \tilde{s}\right] < p(1+\mu^\delta)$. Using the first-order condition again: $F_\delta(\tilde{s}) \left(\mathbb{E}_B\left[s|s > \tilde{s}\right] - \mathbb{E}_\delta\left[s|s < \tilde{s}\right]\right) = \mathbb{E}_B\left[s|s > \tilde{s}\right] - p(1+\mu^\delta) - \kappa_1 V^B\left(\tilde{s} + V^B\right)$

$$\partial_{\mu^L} F = -(1-\delta)\left[p - \frac{1}{\sigma}\left(\mathbb{E}_B\left[s|s > \tilde{s}\right] - p(1+\mu^\delta) - \kappa_1 V^B\left(\tilde{s} + V^B\right)\right)\right]$$

As we have $\mathbb{E}_B\left[s|s > \tilde{s}\right] > \mu^B$:

$$\left(\mathbb{E}_B\left[s|s > \tilde{s}\right] - p(1+\mu^\delta) - \kappa_1 V^B\left(\tilde{s} + V^B\right)\right) > p(\mu^B - \mu^\delta) - \kappa_1 V^B\left(\tilde{s} + V^B\right)$$

so for $\frac{\mu^B - \mu^\delta}{\sigma} < 1$ we have $\partial_{\mu^L} \tilde{s} > 0$: a less pessimist lender implies a higher optimal riskiness.
Similarly, $\partial_{\mu^B} F = (1-\delta)\left(\frac{1-F_L(\tilde{s})}{1-F_B(\tilde{s})}\right) p > 0$ so $\partial_{\mu^B} \tilde{s} > 0$.
On the haircut:

$$\begin{aligned}
\partial_{\mu^B} m &= (1-\delta)\,\partial_{\mu^\delta}\left(\frac{1}{p\left(1+\mu^{\delta\,barg}\right)}\right)\left(p(1+\mu^\delta) - \tilde{s} - (1 - F_\delta(\tilde{s}))\,V^B + \pi^\delta(\tilde{s})\right) \\
&\quad + \frac{1}{p\left(1+\mu^{\delta\,barg}\right)}\left(p\,(1-\delta) - \partial_{\mu^B}\tilde{s}(1 - F_\delta(\tilde{s}) - V^B f_\delta(\tilde{s}))\right)
\end{aligned}$$

So $\partial_{\mu^L} m < 0$ as soon as $\frac{\mu^B - \mu^\delta}{\sigma} < 1$ (mild beliefs heterogeneity) and similarly $\partial_{\mu^B} m < 0$.
On the rate:

$$\partial_{\mu^L} r = -\left(\tilde{s} + V^B\right)(1-\delta)\,p + \partial_{\mu^B}\tilde{s}\left(p\,barg - \pi^\delta + 2\left(\tilde{s} + V^B\right)\left(1 - F_\delta - V^B f_\delta\right)\right)$$

So for δ large enough and mild beliefs heterogeneity ($\frac{\mu^B - \mu^\delta}{\sigma} < 1$): $\partial_{\mu^L} r > 0$.

As for the comparative statics w.r.t the asset price p: $\partial_p F = -(1+\mu^\delta)$ so $\partial_p \tilde{s} < 0$, the lower the asset price the riskier the loan is chosen.

$$\partial_p m = \frac{\left(\tilde{s} + (1 - F_\delta(\tilde{s}))\,V^B - \pi^\delta(\tilde{s})\right)}{p^2\left(1+\mu^{\delta\,barg}\right)} - \partial_p\tilde{s}\left(1 - F_\delta(\tilde{s}) - V^B f_\delta(\tilde{s})\right) > 0$$

$$\partial_p r \propto \left(\frac{barg}{1+\mu^{\delta\,barg}}\right)\left(\partial_p\tilde{s}\,p - 1\right) + \partial_p\tilde{s}\left[\mathbb{E}_\delta\left[s|s < \tilde{s}\right] F_\delta(\tilde{s}) + \left(\tilde{s} + V^B\right) V^B f_\delta(\tilde{s}))\right] < 0$$

A.2.3 Interactions: with respect to asset volatility σ

We analyze the sensitivity of the franchise value collateral channel with respect the volatility of the risky asset σ. There is a non-zero effect, despite risk-neutrality. Using standard properties of the Gaussian distribution: $\partial_\sigma F_\delta(\tilde{s}) = \frac{\mu^\delta - \tilde{s}}{\sigma^2} f_\delta(\tilde{s})$, which is negative as long as $\tilde{s} > \mu^\delta$, and $\partial_\sigma f_\delta(\tilde{s}) = \frac{\left(\mu^\delta - \tilde{s}\right)^2}{\sigma^3} f_\delta(\tilde{s}) > 0$. At small V^B, $\partial_{V^B}\tilde{s} = -1$ so the volatility σ has no effect on the franchise value channel. The expression for $\partial_{V^B}\tilde{s}$ first-order in V^B:

$$\partial_{V^B}\tilde{s} = -1 + \left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}\right)V^B$$

As a result, using $\partial_\sigma \mathbb{E}_B\left[s|s>\tilde{s}\right] < 0$ as $\tilde{s} < \mu^B$: $\partial_\sigma|\partial_{V^B}\tilde{s}| > 0$. Subsequently, in first-order in V^B:

$$|\partial_{V^B}m| = \frac{(1-F_\delta(\tilde{s}))}{p\left(1+\mu^{\delta\, barg}\right)}V^B\left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)} + \frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})}\right)$$

The three dependences $\mathbb{E}_B\left[s|s>\tilde{s}\right]^{-1}$, $-F_\delta(\tilde{s})$ and σ^{-2} are all positive so we obtain: $\partial_\sigma|\partial_{V^B}m| > 0$.

$$|\partial_{V^B}r| = \frac{V^B}{D^2\left(1+\mu^{\delta\, barg}\right)}\left[\tilde{s}f_\delta(\tilde{s}) - \left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}\right)V^B\left(p\, barg + F_\delta(\tilde{s})\,\mathbb{E}_L\left[s|s<\tilde{s}\right]\right)\right]$$

Given that $\partial_\sigma f_\delta > 0$, we obtain in the neighborhood of $V^B = 0$: $\partial_\sigma|\partial_{V^B}r| > 0$.

A.2.4 Interactions: with respect to bargaining parameter δ

δ captures all the effect of imperfect competition among lenders.

$$\partial_\delta F = -\left(\mu^B - \mu^L\right)p + \partial_\delta\kappa_1 V^B(\tilde{s}+V^B) + \partial_\delta\left(F_\delta(\tilde{s})\,\mathbb{E}_\delta\left[s|s<\tilde{s}\right]\right) + \partial_\delta\kappa_3\,\left(1-F_B(\tilde{s})\right)\mathbb{E}_B\left[s|s>\tilde{s}\right]$$

We have $\partial_\delta\kappa_1 = -\frac{f_L-f_B}{1-V^B\frac{f_B}{1-F_B}}$ and $\partial_\delta\kappa_3 = \frac{\left(F_L-F_B+V^B(f_L-f_B)\right)}{1-F_B-V^Bf_B}$. First-order in V^B: $\partial_\delta F = -F^L(\tilde{s}^\delta)$.

Where the last F^L is the characterization function of optimal promise $\tilde{s}^\delta$ (under parameter $\delta = 0$ but evaluated at δ). As we have $\partial_{\tilde{s}}F < 0$ and $F^L(\tilde{s}^0) = 0$, I can conclude: $\partial_\delta F < 0$ and therefore $\partial_\delta\tilde{s} < 0$.

$$\begin{aligned}\partial_\delta m &= \partial_\delta\left(\frac{1}{1+\frac{barg}{1+\mu^\delta}}\right) + \partial_\delta\left(-\frac{1}{p\left(1+\mu^\delta+barg\right)}\right)\left(\tilde{s} + \left(1-F_\delta(\tilde{s})\right)V^B - \pi^\delta(\tilde{s})\right)\\ &\quad+\frac{1}{p\left(1+\mu^{\delta\, barg}\right)}\left(-\partial_\delta\tilde{s}\right)\left(1-F_\delta(\tilde{s}) - V^Bf_\delta(\tilde{s})\right)\end{aligned}$$

$$\partial_\delta r \propto \left(\mu^B-\mu^L\right)\left(\frac{barg}{\left(1+\mu^{\delta\, barg}\right)^2} - \frac{1}{(1-F_\delta)h(0)\mu^B}\left[p\left(\frac{barg}{1+\mu^{\delta\, barg}}\right) + \mu^LF_\delta(\tilde{s}) + \left(\tilde{s}+V^B\right)V^Bf_\delta(\tilde{s})\right]\right)$$

as the second bracket is an increasing function of beliefs disagreement $\mu^B - \mu^L$. So we obtain:

$$\partial_\delta m > 0 \text{ and } \partial_\delta r > 0$$

- Haircut:

From $|\partial_{V^B}m| = \frac{(1-F_\delta(\tilde{s}))}{p\left(1+\mu^{\delta\, barg}\right)}V^B\left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)} + \frac{f_\delta(\tilde{s})}{1-F_\delta(\tilde{s})}\right)$, for σ high enough and mild beliefs disagreement:

$$\partial_\delta|\partial_{V^B}m| \propto \frac{barg\,(F_L-F_B)+(1-F_B)(1+\mu^L)-(1-F_L)(1+\mu^B)}{\left(1+\mu^{\delta\, barg}\right)^2} + \frac{(1-F_\delta(\tilde{s}))}{\left(1+\mu^{\delta\, barg}\right)}\left(-\partial_\delta\tilde{s}\right)\left(\frac{\tilde{s}f_B(\tilde{s})}{\left(F_B(\tilde{s})\pi^B(\tilde{s})\right)^2}\right) > 0$$

- Rate:

From $|\partial_{V^B}r| = \frac{V^B}{D^2\left(1+\mu^{\delta\, barg}\right)}\left[\tilde{s}f_\delta(\tilde{s}) - \left(\frac{1}{\mathbb{E}_B[s|s>\tilde{s}]-\tilde{s}} - \frac{\tilde{s}-\mu^\delta}{\sigma^2} - \frac{\mu^B-\mu^\delta}{\sigma^2}\frac{f_B/(1-F_B)}{h(0)}\right)V^B\left(p\, barg + F_\delta(\tilde{s})\,\mathbb{E}_L\left[s|s<\tilde{s}\right]\right)\right]$, and first-order in V^B:

$$\partial_\delta|\partial_{V^B}r| \quad\propto\quad \partial_\delta\left(\frac{V^B}{D^2\left(1+\mu^{\delta\, barg}\right)}\tilde{s}f_\delta(\tilde{s})\right)$$

The three dependences w.r.t. δ are decreasing ($\tilde{s}$, f_δ and $\left(\mu^{\delta\, barg}\right)^{-1}$) therefore we derive the surprising result that when competition is more imperfect among lenders there is *less* screening with respect the franchise value: $\partial_\delta |\partial_{V^B} r| < 0$.

- With respect to endogenous outside options $barg$. From the f.o.c: $\partial_{barg} F = 0$ so $\partial_{barg}\tilde{s} = 0$. As we have:

$$m = \frac{1}{p\left(1+\mu^\delta + barg\right)}\left(p(1+\mu^\delta) - \tilde{s} - \left(1-F_\delta(\tilde{s})\right)V^B + \pi^\delta(\tilde{s})\right)$$

$$\begin{aligned}
\partial_{barg} r &\propto -\partial_{barg}\left(\frac{barg}{1+\mu^\delta + barg}\right) \\
&= -\frac{1}{\left(1+\mu^\delta+barg\right)^2}\left(1+\mu^\delta\right)
\end{aligned}$$

As a result $\partial_{barg} m < 0$ and $\partial_{\mu^{\delta\, barg}} r < 0$.

A.3 Endogenous bilateral surplus

The two last comparative statics shows that the optimal riskiness $\tilde{s}$ does depend on δ (and thus of I and J), but not on $barg$ (and thus not on the outside options $barg = \omega(1-\omega)\left(S_{I,J-1} - S_{I-1,J}\right)$). However the optimal haircut does depend on both δ and $barg$. We can compute the surplus by induction.

$$\begin{aligned}
S_{I,J} &= \left(U^B_{I,J} - U^B_{I,J-1}\right) + \left(U^L_{I,J} - U^L_{I-1,J}\right) \\
&= \left(U^B_{I,J} - U^B_0\right) - \left(U^B_{I,J} - U^B_0\right) + \left(U^L_{I,J} - U^L_0\right) - \left(U^L_{I,J} - U^L_0\right)
\end{aligned}$$

We formulate the induction hypothesis: $U^B_{I,J-1} - U^B_0 = (1-\omega)n^B S_{I,J-1}$ and $U^L_{I-1,J} - U^L_0 = \omega n^B S_{I-1,J}$.

$R^{unl}_{IJ}(\tilde{s}) = p\left(1+\mu^B\right) - \frac{I}{J}p(1+\mu^L)(1-m) - \left(1-\frac{I}{J}\right)\left(\tilde{s}+V^B\right) + \left(F_B(\tilde{s}) - \frac{I}{J}F_L(\tilde{s})\right)V^B + \pi^B(\tilde{s}) - \frac{I}{J}\pi^L(\tilde{s})$

$$\frac{R^{unl}_{IJ}(\tilde{s})}{R^{unl}_{\delta}(\tilde{s})} = \frac{\left(\left\{p\left(1+\mu^B\right) - \frac{I}{J}p(1+\mu^L)(1-m)\right\} - \left\{\left(1-\frac{I}{J}\right)\left(\tilde{s}+V^B\right)\right\} + \left\{\left(F_B(\tilde{s}) - \frac{I}{J}F_L(\tilde{s})\right)V^B\right\} + \left\{\pi^B(\tilde{s}) - \frac{I}{J}\pi^L(\tilde{s})\right\}\right)}{p\left(1+\mu^\delta\right) - \left(\tilde{s}+V^B\right) + F_\delta(\tilde{s})V^B + \pi^\delta(\tilde{s})}$$

This ratio is above 1. Plugging $m_{IJ}p = \frac{1}{\left(1+\mu^\delta+\omega(1-\omega)\left(S_{I,J-1}-S_{I-1,J}\right)\right)}R^{\delta\, U}(\tilde{s})$ in the definition of the surplus:

$$\begin{aligned}
S_{I,J} &= \frac{n^B}{m_{IJ}p}R^{IJ}(\tilde{s}) - n^B\left(1+\mu^B\right) - (1-\omega)n^B S_{I,J-1} - \omega n^B S_{I-1,J} \\
S_{I,J} &= n^B\left[\left(1+\mu^\delta\right)\frac{R^{IJ}(\tilde{s})}{R^{\delta\, U}(\tilde{s})} - \left(1+\mu^B\right) + (1-\omega)S_{I,J-1}\left(\omega\frac{R^{IJ}(\tilde{s})}{R^{\delta\, U}(\tilde{s})} - 1\right) - \omega S_{I-1,J}\left((1-\omega)\frac{R^{IJ}(\tilde{s})}{R^{\delta\, U}(\tilde{s})} + 1\right)\right]
\end{aligned}$$

This recursively characterizes the endogenous surplus of the bilateral relationship (I, J). The surplus depends on how it is shared (ω), and is equal to zero if beliefs are identical.

B Dynamic model in discrete time

B.1 Equilibrium with short-term contracts

We verify Blackwell sufficiency conditions of the Bellman equation show existence and uniqueness of function value V^B. There are no multiplicity of equilibria where each lender lends to all borrowers. However there are other asymmetric equilibria, for instance in which no lender lends to a given borrower, and as such the franchise of the latter is low, reinforcing the equilibrium property of not lending to this given borrower.

The only difference is that agents now takes into account the endogeneity of the franchise value V^B with respect to net worth n^B_{t+1}, and as a result the impact of the choice of $\tilde{s}$ on the value of its franchise tomorrow.

$$\begin{aligned}\left(U^B_{I,J}-U^B_0\right)(\tilde{s}) &= n^B\left[\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(\tilde{s})-\left(1+\mu^B\right)\right]\\ &\quad+\beta\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s})\right)-V^B\left(n^Bs\right)\right]\end{aligned}$$

The optimal riskiness comes from the following maximization:

$$\underset{\{\tilde{s}\}}{Max}\left(n^B\left[\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(\tilde{s})-\left(1+\mu^B\right)\right]+\beta\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^L_B(s,\tilde{s})\right)-V^B\left(n^Bs\right)\right]\right)$$

I note the continuation component:

$$w(\tilde{s})=\beta\frac{1}{n^B\left(1+\mu^{\delta\,barg}\right)}\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s})\right)-V^B\left(n^Bs\right)\right]$$

The f.o.c can be written, using $\partial_{V^B}R^{lev}_B(\tilde{s})=-\frac{(1-F_\delta)R^{\delta\,unl}-(1-F_B)R^{B\,unl}}{\left(R^{\delta\,unl}\right)^2}$ (in the static model, $\partial_{\tilde{s}}R^{lev}_B(\tilde{s})=0$):

$$0=-R^{unl}_\delta+R^{unl}_B\frac{\partial_{\tilde{s}}R^{unl}_\delta}{\partial_{\tilde{s}}R^{unl}_B}+\frac{\beta}{n^B\left(1+\mu^{\delta\,barg}\right)}\frac{1}{1-\partial_{\tilde{s}}V^B}\frac{\left(R^{unl}_\delta\right)^2}{\partial_{\tilde{s}}R^{unl}_B}\partial_{\tilde{s}}\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s})\right)\right]$$

I compute $\partial_{\tilde{s}}\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s})\right)\right]$ with static bargaining:

$$\begin{aligned}\partial_{\tilde{s}}\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s})\right)\right] &= -f_B(\tilde{s})V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s^{def},\tilde{s})\right)\\ &\quad+\int_{\tilde{s}}^{s^{max}}\partial_{\tilde{s}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s})\right)f_B(s)ds\end{aligned}$$

As we have $V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^L_B(s^{def},\tilde{s})\right)=0$ in the dynamic model:

$$\partial_{\tilde{s}}\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s})\right)\right]=n^B\left(1+\mu^{\delta\,barg}\right)\int_{\tilde{s}}^{s^{max}}\partial_{\tilde{s}}R^{lev}_B(s,\tilde{s})\,\partial_{n^B}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s})\right)dF_B$$

So the f.o.c can be written, rearranging terms:

$$\begin{aligned}0 &= -p(1+\mu^\delta)+\kappa_1V^B\left(\tilde{s}+V^B\right)+F_\delta(\tilde{s})\,\mathbb{E}_\delta\left[s|s<\tilde{s}\right]+\left(1-F_\delta(\tilde{s})\right)\,\kappa_2\,\mathbb{E}_B\left[s|s>\tilde{s}\right]\\ &\quad+\beta\frac{1}{1-\partial_{\tilde{s}}V^B}\frac{\left(R^{unl}_\delta\right)^2}{\partial_{\tilde{s}}R^{unl}_B}\int_{\tilde{s}}^{s^{max}}\partial_{\tilde{s}}R^{lev}_B(s,\tilde{s})\,\partial_{n^B}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s})\right)f_B(s)ds\end{aligned}$$

Concavity of the value function $V^B(n_t)$ Differentiating this f.o.c w.r.t n^B (optimal riskiness does not move first-order by the envelope condition), using $\partial_{\tilde{s}}V^B=\partial_{\tilde{s}}R^{lev}_B(\tilde{s})\,n^B\left(1+\mu^{\delta\,barg}\right)\partial_{n^B}V^B$:

$$\begin{aligned}0 &= \left(1-\partial_{\tilde{s}}V^B\right)\kappa_1\partial_{n^B}V^B\left(\tilde{s}+2V^B\right)+\left(-\partial_{\tilde{s}}R^{lev}_B(\tilde{s})\left(1+\mu^{\delta\,barg}\right)\left(n^B\,\partial_{n^Bn^B}V^B+\partial_{n^B}V^B\right)\right)\kappa_1V^B\left(\tilde{s}+V^B\right)\\ &\quad+\beta\frac{\left(R^{unl}_\delta\right)^2}{\partial_{\tilde{s}}R^{unl}_B}\int_{\tilde{s}}^{s^{max}}\partial_{\tilde{s}}R^{lev}_B(s,\tilde{s})\,\partial_{n^Bn^B}V^B\left(n^B_{t+1}\right)f_B(s)ds\end{aligned}$$

Developing first-order in n^B:

$$-\left(1-\partial_{\tilde{s}} V^B\right) \kappa_1 \partial_{n^B} V^B \left(\tilde{s}+2V^B\right) \quad = \quad \beta \frac{\left(R_\delta^{unl}\right)^2}{\partial_{\tilde{s}} R_B^{unl}} \int_{\tilde{s}}^{s^{max}} \partial_{\tilde{s}} R_B^{lev}(s,\tilde{s})\, \partial_{n^B n^B} V^B \left(n_{t+1}^B\right) f_B(s) ds$$

Using $\partial_{n^B} V^B > 0$, this equation implies: $\partial_{n^B n^B} V^B < 0$.

Comparative statics of m w.r.t n_t^B The first-order condition delivers:

$$\begin{aligned}
\partial_{n^B} F^{dyn} \quad = \quad & \kappa_1 \partial_{n^B} V^B \left(\tilde{s}+2V^B\right) + \left(-\partial_{\tilde{s}} R_B^{lev}(\tilde{s}) \left(1+\mu^{\delta\, barg}\right) \left(n^B\, \partial_{n^B n^B} V^B + \partial_{n^B} V^B\right)\right) \frac{1}{1-\partial_{\tilde{s}} V^B} \kappa_1 V^B \left(\tilde{s}+V^B\right) \\
& + \beta \frac{1}{1-\partial_{\tilde{s}} V^B} \frac{\left(R_\delta^{unl}\right)^2}{\partial_{\tilde{s}} R_B^{unl}} \int_{\tilde{s}}^{s^{max}} \partial_{\tilde{s}} R_B^{lev}(s,\tilde{s})\, \partial_{n^B n^B} V^B \left(n_{t+1}^B\right) f_B(s) ds
\end{aligned}$$

At low n^B: $\partial_{n^B n^B} V^B \left(n_{t+1}^B\right) < 0$ and $\left(n^B\, \partial_{n^B n^B} V^B + \partial_{n^B} V^B\right) > 0$. Therefore $\partial_{n^B} F^{dyn} < 0$, which, along with $\partial_{\tilde{s}} F^{dyn} > 0$, results in $\partial_{n^B} \tilde{s} > 0$.

- Haircut m: $m = \frac{1}{p\left(1+\mu^{\delta\, barg}\right)} \left(p(1+\mu^\delta) - \tilde{s} - (1-F_\delta(\tilde{s}))\, V^B + \pi^\delta(\tilde{s})\right)$

So $\partial_{n^B} m = -\frac{1}{p\left(1+\mu^{\delta\, barg}\right)} \partial_{n^B} \tilde{s}\, \left[1-F^\delta(\tilde{s}) - V^B f^\delta(\tilde{s})\right]$ implies $\partial_{n^B} m < 0$

Fragility: $\partial_{n^B n^B} m = -\frac{1}{p\left(1+\mu^{\delta\, barg}\right)} \left(\partial_{n^B n^B} \tilde{s}\, \left[1-F^\delta(\tilde{s}) - V^B f^\delta(\tilde{s})\right] - (\partial_{n^B} \tilde{s})^2 \left[f^\delta(\tilde{s}) \left(1 - V^B \frac{\tilde{s}-\mu^\delta}{\sigma}\right)\right]\right)$

At low net worth levels n^B: $\partial_{n^B n^B} m > 0$

$$\begin{aligned}
\partial_{n^B} \tilde{s} \quad &= \quad -A \left(\partial_{n^B} V^B + n^B\, \partial_{n^B n^B} V^B\right) \\
\partial_{n^B n^B} \tilde{s} \quad &= \quad -A \left(2\partial_{n^B n^B} V^B + n^B\, \partial_{n^B n^B n^B} V^B\right) \\
\partial_{n^B n^B} \tilde{s} \quad &\propto \quad -2A \partial_{n^B n^B} V^B
\end{aligned}$$

- Rate r: $r = \frac{\tilde{s}+V^B}{p\left(\frac{barg}{1+\mu^{\delta\, barg}}\right) + \tilde{s} + (1-F_\delta(\tilde{s}))V^B - \pi^\delta(\tilde{s})} - 1$

$$\partial_{\tilde{s}} r = \frac{1}{\left(p\left(\frac{barg}{1+\mu^{\delta\, barg}}\right) + \tilde{s} + (1-F_\delta(\tilde{s}))\, V^B - \pi^\delta(\tilde{s})\right)^2} \left[p\left(\frac{barg}{1+\mu^{\delta\, barg}}\right) + E_\delta\left[s|s<\tilde{s}\right] F_\delta(\tilde{s}) + \left(\tilde{s}+V^B\right) V^B f_\delta(\tilde{s}))\right]$$

So $\partial_{n^B} r = \partial_{n^B} \tilde{s} \times \partial_{\tilde{s}} r + \partial_{n^B} V^B \times \partial_{V^B} r$ with $\partial_{V^B} r < 0$.

At low levels of n_t^B at which $\partial_{n^B} V^B$ high enough: $\partial_{n^B} r < 0$.

B.2 Equilibrium with long-term contracts

Following Abreu-Pearce-Stacchetti (1980), we can use the promised utility to the agent (who is the lender here) as state variable. We now have two state variables: the borrower net worth n_t^B and the lender continuation value V^L.

- Expected utility of the borrower:

$$U_{I,J,t}^B - U_{I,J-1,t}^B \quad = \quad \frac{n^B}{m}\left[(1-m)\left(\mu^B - r(m)\right) + \frac{1}{p}\pi^B(\tilde{s}) + V^B \frac{F_B(\tilde{s})}{p}\right] + \beta\, \mathbb{E}_B\left[V_{I,J,t+1}^B - V_{I,J-1,t+1}^B\right]$$

- Expected utility of the lender:

$$V^L_{I,J,t} - V^L_{I-1,J,t} \quad = \quad \frac{I}{J}\frac{n^B}{m}\left[(1-m)\left(r(m)-\mu^L\right) - \frac{1}{p}\left(\pi^L(\tilde{s}) + V^B F_L(\tilde{s})\right)\right] + \beta\mathbb{E}_L\left[V^L_{I,J,t+1} - V^L_{I-1,J,t+1}\right]$$

- The bilateral Nash bargaining delivers a contract value D:

$$D = \frac{(1-\delta)\mathbb{E}_L\left[1_{\{def\}}s+1_{\{no\,def\}}\bar{s}\right]+\delta\left(\left(1+\mu^B\right)p-\mathbb{E}_B\left[1_{\{no\,def\}}(s-\bar{s})\right]\right)+\frac{1}{n^B}\left(\delta\left(U^B_{I,J-1,t}-\beta\,\mathbb{E}_B\left[V^B_{I,J,t+1}-V^B_{I,J-1,t+1}\right]\right)-(1-\delta)\left(V^L_{I-1,J,t}-\beta\mathbb{E}_L\left[V^L_{I,J,t+1}-V^L_{I-1,J,t+1}\right]\right)\right)p}{(1-\delta)(1+\mu^L)+\delta\left(1+\mu^B\right)+\frac{1}{n^B}\left(\delta\left(U^B_{I,J-1,t}-\beta\,\mathbb{E}_B\left[V^B_{I,J,t+1}-V^B_{I,J-1,t+1}\right]\right)-(1-\delta)\left(V^L_{I-1,J,t}-\beta\mathbb{E}_L\left[V^L_{I,J,t+1}-V^L_{I-1,J,t+1}\right]\right)\right)}$$

So the only change compared to the dynamic problem with static bargaining is the value of $\mu^{\delta\,barg}$:

$$\mu^{\delta\,barg} = \mu^\delta + \tfrac{1}{n^B}\left(\delta\left(U^B_{I,J-1,t} - \beta\,\mathbb{E}_B\left[V^B_{I,J,t+1} - V^B_{I,J-1,t+1}\right]\right) - (1-\delta)\left(V^L_{I-1,J,t} - \beta\mathbb{E}_L\left[V^L_{I,J,t+1} - V^L_{I-1,J,t+1}\right]\right)\right)$$

- The borrower maximization program is now:

$$\begin{aligned} \underset{\{\tilde{s},V^L_{I,J,t+1}\}}{Max} \qquad & n^B\left[\left(1+\mu^{\delta\,barg}(V^L_{I,J,t+1})\right)R^{lev}_B(\tilde{s}) - \left(1+\mu^B\right)\right] \\ & +\beta\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^L_B(s,\tilde{s}), V^L_{I,J,t+1}\right) - V^B\left(n^B s\right)\right] \end{aligned}$$

- And the dynamic f.o.c. still holds:

$$0 = -R^{unl}_\delta + R^{unl}_B\frac{\partial_{\tilde{s}}R^{unl}_\delta}{\partial_{\tilde{s}}R^{unl}_B} + \frac{\beta}{n^B\left(1+\mu^{\delta\,barg}\right)}\frac{1}{1-\partial_{\tilde{s}}V^B}\frac{\left(R^{unl}_\delta\right)^2}{\partial_{\tilde{s}}R^{unl}_B}\partial_{\tilde{s}}\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s})\right)\right]$$

$$\partial_{\tilde{s}}\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s})\right)\right] \quad = \quad \int_{\tilde{s}}^{s^{max}}\left[n^B\left(1+\mu^{\delta\,barg}\right)\partial_{\tilde{s}}R^{lev}_B\partial_{n^B}V^B + \partial_{\tilde{s}}V^L\partial_{V^L}V^B\right]f_B(s)ds$$

So the f.o.c can be written, rearranging terms:

$$\begin{aligned} 0 \quad = \quad & -p(1+\mu^\delta) \\ & +\kappa_1 V^B\left(\tilde{s}+V^B\right) + F_\delta(\tilde{s})\,\mathbb{E}_\delta\left[s|s<\tilde{s}\right] + \left(1-F_\delta(\tilde{s})\right)\,\kappa_2\,\mathbb{E}_B\left[s|s>\tilde{s}\right] \\ & +\beta\frac{1}{1-\partial_{\tilde{s}}V^B}\frac{\left(R^{unl}_\delta\right)^2}{\partial_{\tilde{s}}R^{unl}_B}\int_{\tilde{s}}^{s^{max}}\left[\partial_{\tilde{s}}R^{lev}_B\partial_{n^B}V^B + \frac{1}{n^B\left(1+\mu^{\delta\,barg}\right)}\partial_{\tilde{s}}V^L\partial_{V^L}V^B\right]f_B(s)ds \end{aligned}$$

I show now how the optimal riskiness $\tilde{s}$ depends on the continuation value $V^L_{I,J,t+1}$ promised to the lender. The countercyclicality of the haircut comes from $\partial_{\tilde{s}}R^{lev}_B\partial_{n^B}V^B$ in the continuation term, which by variational argument as a positive impact on optimal riskiness $\tilde{s}$ (same effet as of minus price $-p$). The optimal long-term contract counteracts this countercycality by the term $\frac{1}{n^B\left(1+\mu^{\delta\,barg}\right)}\partial_{\tilde{s}}V^L\partial_{V^L}V^B$: as n^B decreases, this term increases as long as $\frac{\partial_{\tilde{s}}V^L\partial_{V^L}V^B}{\left(1+\mu^{\delta\,barg}\right)}$ does not decrease too fast. The impact of the continuation term is dampened, making the variation of optimal riskiness $\tilde{s}$ less sensitive to the state variable n^B.

- The f.o.c in promised continuation value delivers $V^L_{I,J,t+1}$:

$$\begin{aligned} \underset{\{\tilde{s},V^L_{I,J,t+1}\}}{Max} \qquad & n^B\left[\left(1+\mu^{\delta\,barg}(V^L_{I,J,t+1})\right)R^{lev}_B(\tilde{s}) - \left(1+\mu^B\right)\right] \\ & +\beta\mathbb{E}_B\left[1_{\{no\,def\}}V^B\left(n^B\left(1+\mu^{\delta\,barg}\right)R^{lev}_B(s,\tilde{s}), V^L_{I,J,t+1}\right) - V^B\left(n^B s\right)\right] \end{aligned}$$

$$0 = n^B \partial_{V^L_{I,J,t+1}} \mu^{\delta\, barg} R^{lev}_B(\tilde{s}) + \beta \mathbb{E}_B \left[1_{\{no\, def\}} \partial_{V^L_{I,J,t+1}} V^B \right]$$

Using $\partial_{V^L_{I,J,t+1}} \mu^{\delta\, barg} = \frac{\beta(1-\delta)}{n^B}$: $0 = (1-\delta) R^{lev}_B(\tilde{s}) + \mathbb{E}_B \left[1_{\{no\, def\}} \partial_{V^L_{I,J,t+1}} V^B \right]$

The intuition is that increasing the long-term promise $V^L_{I,J,t+1}$ to the lender by one unit increase the short-term gain for the borrower by $(1-\delta) R^{lev}_B(\tilde{s})$ but decreases its long-term expectation by $\mathbb{E}_B \left[1_{\{no\, def\}} |\partial_{V^L_{I,J,t+1}} V^B| \right]$. Under the optimal long-term contract, the two legs are equalized.

$$\partial_{V^L_{I,J}} V^B < 0 \text{ and } |\partial_{V^L_{I,J}} V^B| = (1-\delta) R^{lev}_B(\tilde{s})$$

- Finally I show formally that $0 < \partial_{n^B} \tilde{s}^{LT} < \partial_{n^B} \tilde{s}^{ST}$.

$$\begin{aligned} \partial_{\tilde{s}} F^{dyn} &= -(1-F_B)\, \kappa_3 h(V^B) \left[\mathbb{E}_B [s | s > \tilde{s}] - \tilde{s} - V^B \right] \\ &\quad + \beta \frac{1}{1 - \partial_{\tilde{s}} V^B} \frac{\left(R^{unl}_\delta \right)^2}{\partial_{\tilde{s}} R^{unl}_B} \int_{\tilde{s}}^{s^{max}} [\partial_{\tilde{s}\tilde{s}} R^{lev}_B(s,\tilde{s})\, \partial_{n^B} V^B \left(n^B_{t+1} \right) \\ &\quad + \left(\partial_{\tilde{s}} R^{lev}_B(s,\tilde{s}) \right)^2 n^B \left(1 + \mu^{\delta\, barg} \right) \partial_{n^B n^B} V^B \left(n^B_{t+1} \right) + \frac{1}{n^B \left(1 + \mu^{\delta\, barg} \right)} \partial_{\tilde{s}} V^L \partial_{V^L} V^B] f_B(s) ds \end{aligned}$$

$$\begin{aligned} \partial_{n^B} F^{dyn} &= \kappa_1 \partial_{n^B} V^B \left(\tilde{s} + 2V^B \right) + \left(-\partial_{\tilde{s}} R^{lev}_B(\tilde{s}) \left(1 + \mu^{\delta\, barg} \right) \left(n^B \partial_{n^B n^B} V^B + \partial_{n^B} V^B \right) \right) \frac{1}{1 - \partial_{\tilde{s}} V^B} \kappa_1 V^B \left(\tilde{s} + V^B \right) \\ &\quad + \beta \frac{1}{1 - \partial_{\tilde{s}} V^B} \frac{\left(R^{unl}_\delta \right)^2}{\partial_{\tilde{s}} R^{unl}_B} \int_{\tilde{s}}^{s^{max}} \left[\partial_{\tilde{s}} R^{lev}_B(s,\tilde{s})\, \partial_{n^B n^B} V^B \left(n^B_{t+1} \right) - \frac{1}{\left(n^B \right)^2 \left(1 + \mu^{\delta\, barg} \right)} \partial_{\tilde{s}} V^L \partial_{V^L} V^B \right] f_B(s) ds \end{aligned}$$

As $\partial_{V^L} V^B < 0$ the term $-\frac{1}{\left(n^B\right)^2 \left(1+\mu^{\delta\, barg}\right)} \partial_{\tilde{s}} V^L \partial_{V^L} V^B$ is positive and therefore counteracts the negativity of $\partial_{n^B n^B} V^B \left(n^B_{t+1} \right) < 0$. Using $|\partial_{V^L_{I,J}} V^B| = (1-\delta) R^{lev}_B(\tilde{s})$ (f.o.c in $V^L_{I,J,t+1}$) this counteract effect on $\partial_{n^B} F^{dyn}$ is larger than the effect of $\frac{1}{n^B \left(1+\mu^{\delta\, barg}\right)} \partial_{\tilde{s}} V^L \partial_{V^L} V^B$ on $\partial_{\tilde{s}} F^{dyn}$.

$$\begin{aligned} \partial_{n^B} \tilde{s}^{LT} &= \frac{\left(-\partial_{n^B} F^{dyn\, LT} \right)}{\partial_{\tilde{s}} F^{dyn\, LT}} \\ &= \frac{\left(-\partial_{n^B} F^{dyn\, ST} \right) - \beta \frac{1}{1-\partial_{\tilde{s}} V^B} \frac{\left(R^{unl}_\delta\right)^2}{\partial_{\tilde{s}} R^{unl}_B} \int_{\tilde{s}}^{s^{max}} \frac{1}{\left(n^B\right)^2 \left(1+\mu^{\delta\, barg}\right)} \partial_{\tilde{s}} V^L \left(-\partial_{V^L} V^B \right) f_B(s) ds}{\partial_{\tilde{s}} F^{dyn\, ST} + \beta \frac{1}{1-\partial_{\tilde{s}} V^B} \frac{\left(R^{unl}_\delta\right)^2}{\partial_{\tilde{s}} R^{unl}_B} \int_{\tilde{s}}^{s^{max}} \frac{1}{n^B \left(1+\mu^{\delta\, barg}\right)} \partial_{\tilde{s}} V^L \partial_{V^L} V^B f_B(s) ds} \\ &< -\frac{\partial_{n^B} F^{dyn\, ST}}{\partial_{\tilde{s}} F^{dyn\, ST}} \\ &= \partial_{n^B} \tilde{s}^{ST} \end{aligned}$$

Optimal riskiness is less sentive to net worth under long-term contracts than under short-term contracts.

C The rehypothecation chain

We stack two I-J set ups described above. There is one multilateral Nash bargaining between the HF and the BD and one multilateral Nash bargaining between the BD and the MMF. This implies two $r(m)$ mappings, which are inter-related.

Expected utilities of each agent

- Hedge Fund expected value (its balance sheet constraint always binds - no cash hoarding - so $pm^{bil}x^{bil} = n^{HF}$):

$$U^{HF} - U_0^{HF} = \frac{n^{HF}}{m^{bil}p}\left[\left(1-m^{bil}\right)\left(\mu^B - r^{bil}\right) + \pi^B(\tilde{s}^{bil})\right]$$

- Broker Dealer expected value (its balance sheet constraint does not always bind: $p\left(1-m^{bil}\right)x^{bil} \leq n^{BD} + p\left(1-m^{tri}\right)x^{tri}$):

$$\begin{aligned} U^{BD} - U_0^{BD} &= x^{bil}\left[\left(1-m^{bil}\right)\left(1+r^{bil}\right)p - \pi^B(\tilde{s}^{bil})\right] \\ &\quad -x^{tri}\left[\left(1-m^{tri}\right)\left(1+r^{tri}\right)p + \pi^B(\tilde{s}^{tri}) + \left(1-\beta n^B/x^{tri}\right)V^B F_B(\tilde{s}^{tri})\right] \\ &\quad -n^{BD}\left[\left(1-m^{bil}\right)\left(1+r^{bil}\right)p - \pi^B(\tilde{s}^{bil})\right] \end{aligned}$$

- Money Market Fund expected value:

$$U^{MMF} - U_0^{MMF} = x^{tri}\left[\left(1-m^{tri}\right)\left(r^{tri}-\mu^L\right)p - \pi^L(\tilde{s}^{tri}) - V^B F_B(\tilde{s}^{tri})\right]$$

1/ Scarce collateral regime Assume we are in a scarce collateral regime. In this case:

- The BD collateral constraint binds: $x^{tri} = x^{bil}$
- The BD balance sheet constraint is slack: $p\left(1-m^{bil}\right)x^{bil} < n^{BD} + p\left(1-m^{tri}\right)x^{tri}$

Combining with $pm^{bil}x^{bil} = n^{HF}$ we get: $x^C = n^B + n^{HF}\left(1 - \frac{m^{tri}}{m^{bil}}\right)$

In this regime, we can write the BD expected utility as:

$$U^{BD} - U_0^{BD} = \frac{n^{HF}}{m^{bil}}\left[r^{bil}(m^{bil};m^{tri}) - r^{tri}(m^{bil};m^{tri}) - \Delta\pi^B(m^{bil};m^{tri}) - m^{tri}r^C p\right]$$

where we introduce the value of the collar: $\Delta\pi^B(m^{bil};m^{tri}) = \pi^B(\tilde{s}^{bil}) - \pi^B(\tilde{s}^{tri}) = \int_{\tilde{s}^{tri}-V^B}^{\tilde{s}^{bil}} F_B(s)ds$. The first term can be called the *repo spread* (carry trade from lending at a higher rate than borrowing). The second term (the collar) arises from the composition of the two put options the borrower bears (long with MMF, short with HF) and would be traced to a *haircut spread.* The third is cash gains arising from collateral management (high haircuts secured against HF, low haircuts against MMF).

Derivation of the two mappings $r^{bil}(m^{bil};m^{tri})$ and $r^{tri}(m^{bil};m^{tri})$ Denote ω_{bil} the bargaining power of the lender BD in the HF-BD bilateral repo and ω_{tri} the bargaining power of the lender MMF in the BD-MMF triparty repo. These two Nash bargainings imply:

$$(1-\omega_{bil})\left(U^{BD} - U_0^{BD}\right) = \omega_{bil}\left(U^{HF} - U_0^{HF}\right)$$

$$(1-\omega_{tri})\left(U^{MMF} - U_0^{MMF}\right) = \omega_{tri}\left(U^{BD} - U_0^{BD}\right)$$

Following the static bargaining from the I-J model, we get:

$r^{bil}\left(m^{bil};m^{tri}\right) = \omega_{bil}\mu^B + (1-\omega_{bil})r^{tri}\left(m^{bil};m^{tri}\right) + \omega_{bil}\frac{1}{1-m^{bil}}\pi^B(\tilde{s}^{bil}) + (1-\omega_{bil})\frac{1}{1-m^{bil}}\Delta\pi^B(\tilde{s}^{bil};\tilde{s}^{tri})$

$r^{tri}\left(m^{bil};m^{tri}\right)=\omega_{tri}r^{bil}\left(m^{bil};m^{tri}\right)+(1-\omega_{tri})\mu^L-\omega_{tri}\frac{1}{1-m^{tri}}\Delta\pi^B(\tilde{s}^{bil};\tilde{s}^{tri})+(1-\omega_{tri})\frac{1}{1-m^{tri}}\pi^L(\tilde{s}^{tri})$
The solution of the linear system, by Cramer's rule, is:

$$\begin{cases} r^{bil}\left(m^{bil};m^{tri}\right) & =\frac{1}{1-\omega_{tri}(1-\omega_{bil})} \\ & \left[\omega_{bil}\left(\mu^B+\frac{1}{1-m^{bil}}\pi^B(\tilde{s}^{bil})\right)+(1-\omega_{bil})(1-\omega_{tri})\left(\mu^L+\frac{1}{1-m^{tri}}\pi^L(\tilde{s}^{tri})+\frac{1}{1-m^{tri}}\Delta\pi^B(\tilde{s}^{bil};\tilde{s}^{tri})\right)\right] \\ r^{tri}\left(m^{bil};m^{tri}\right) & =\frac{1}{1-\omega_{tri}(1-\omega_{bil})} \\ & \left[\omega_{bil}\omega_{tri}\left(\mu^B+\frac{1}{1-m^{bil}}\pi^B(\tilde{s}^{bil})-\frac{1}{1-m^{tri}}\Delta\pi^B(\tilde{s}^{bil};\tilde{s}^{tri})\right)+(1-\omega_{tri})\left(\mu^L+\frac{1}{1-m^{tri}}\pi^L(\tilde{s}^{tri})\right)\right] \end{cases}$$

Maximization program of the Broker Dealer The equilibrium is given by the BD program:

$$\begin{aligned} \underset{\{x^{BD},x^{MMF},\tilde{s}^{BD},\tilde{s}^{MMF}\}}{Max} & \{x^{BD}\left(\tilde{s}^{BD}-\pi^B(\tilde{s}^{BD})\right)-x^{MMF}\left(\tilde{s}^{MMF}-\pi^B(\tilde{s}^{MMF})+V^B(1-F_B(\tilde{s}^{MMF}))\right) \\ & +x^c\bar{r}+\beta n\,V^B(1-F_B(\tilde{s}^{MMF}))\} \\ (BD\,balance\,sheet\,constraint) & \;s.t.\; x^c+x^{BD}D^{HF}\le n^{BD}+x^{MMF}D^{BD}(\tilde{s}^{MMF}) \\ (BD\,collateral\,constraint) & \;s.t.\; x^{MMF}\le x^{BD} \\ (BD\,default\,condition) & \;s.t.\; default\; i.f.f.\; s<\tilde{s}^{MMF} \\ (Nash\,bargainings) & \;s.t.\; r^{bil}(m^{bil};m^{tri})\; and\; r^{tri}(m^{bil};m^{tri}) \end{aligned}$$

This can be written as only functions of m^{bil} and m^{tri}, using the solution of the joint Nash bargainings.

$$\underset{\{m^{bil};m^{tri}\}}{Max}U^{BD}-U_0^{BD} \;=\; \frac{n^{HF}}{m^{bil}}\left[r^{bil}(m^{bil};m^{tri})-r^{tri}(m^{bil};m^{tri})-\Delta\Pi^B(m^{bil};m^{tri})-m^{tri}r^Cp\right]$$

where:

$$r^{bil}(m^{bil};m^{tri})-r^{tri}(m^{bil};m^{tri}) \;=\; \frac{1}{1-\omega_{tri}(1-\omega_{bil})}\left[\omega_{bil}(1-\omega_{tri})\left(\mu^B+\frac{1}{1-m^{bil}}\pi^B(\tilde{s}^{bil})-\mu^L-\frac{1}{1-m^{tri}}\pi^L(\tilde{s}^{tri})\right)\right]$$

As a result the BD maximizes the following functional $U^{BD}-U_0^{BD}$:

$$\underset{\{m^{bil};m^{tri}\}}{Max}\frac{n^{HF}}{m^{bil}}\left[\frac{\omega_{bil}(1-\omega_{tri})}{1-\omega_{tri}(1-\omega_{bil})}\left(\mu^B+\frac{1}{1-m^{bil}}\pi^B(\tilde{s}^{bil})-\mu^L-\frac{1}{1-m^{tri}}\pi^L(\tilde{s}^{tri})\right)-\Delta\Pi^B(m^{bil};m^{tri})-m^{tri}r^Cp\right]$$

- Fo.c. with respect to m^{tri} (uniquely defines m^{tri}, independently from m^{bil}):

$$\begin{aligned} 0 &= H^{tri}(m^{tri};\omega_{bil},\omega_{tri},V^B,\mu^L) \\ &= \frac{\omega_{bil}(1-\omega_{tri})}{1-\omega_{tri}(1-\omega_{bil})}\left(\frac{1}{1-m^{tri}}\right)^2\left[\mathbb{E}_L\left[s|s<\tilde{s}^{tri}\right]+V^B\right]F_L(\tilde{s}^{tri})-(1+r^{bil})pF_L(\tilde{s}^{tri})-\mu^Lp \end{aligned}$$

Comparative statics:

$$\begin{aligned} \partial_{m^{tri}}H^{tri} &= 2\left(\frac{1}{1-m^{tri}}\right)^3\frac{\omega_{bil}(1-\omega_{tri})}{1-\omega_{tri}(1-\omega_{bil})}\left[\mathbb{E}_L\left[s|s<\tilde{s}^{tri}\right]+V^B\right]F_L(\tilde{s}^{tri}) \\ & -\left(\tilde{s}^{tri}f_L(\tilde{s}^{tri})F_L(\tilde{s}^{tri})+\left[\mathbb{E}_L\left[s|s<\tilde{s}^{tri}\right]+V^B\right]f_L(\tilde{s}^{tri})-(1+r^{bil})pf_L(\tilde{s}^{tri})\right) \end{aligned}$$

So $\partial_{m^{tri}}H^{tri}>0$. Similarly $\partial_{\omega_{bil}}H^{tri}>0$ and $\partial_{\omega_{tri}}H^{tri}<0$ and $\partial_{V^B}H^{tri}>0$ and $\partial_{\mu^L}H^{tri}<0$, so $\partial_{\omega_{bil}}m^{tri}<0$ and $\partial_{\omega_{tri}}m^{tri}>0$ and $\partial_{\mu^L}m^{tri}>0$ and $\partial_{V^B}m^{tri}<0$

- Fo.c. with respect to m^{bil}:

$$
\begin{aligned}
0 &= H^{bil}(m^{tri}; \omega_{bil}, \omega_{tri}, V^B, \mu^L) \\
&= \frac{1}{n^{HF}}\left[U^{BD} - U_0^{BD}\right] + \frac{\omega_{bil}(1-\omega_{tri})}{1-\omega_{tri}(1-\omega_{bil})}\left(\frac{1}{1-m^{bil}}\right)^2 \left[\int_{s^{min}}^{\tilde{s}^{bil}} u f_B(u) du\right] - (1+r^{bil}) p F_B(\tilde{s}^{bil})
\end{aligned}
$$

$$
\partial_{V^B} F = \left[\frac{\omega_{bil}(1-\omega_{tri})}{1-\omega_{tri}(1-\omega_{bil})} F_L(\tilde{s}^{tri}) - F_B(\tilde{s}^{bil})\right]
$$

For high ω_{tri} and low ω_{bil}: $\partial_{V^B} H^{bil} < 0$. We also have: $\partial_{m^{bil}} H^{bil} < 0$, $\partial_{\omega_{bil}} H^{bil} > 0$ and $\partial_{\omega_{tri}} H^{bil} < 0$ and $\partial_{V^B} H^{bil} < 0$. As a result: $\partial_{\omega_{bil}} m^{bil} > 0$, $\partial_{\omega_{tri}} m^{bil} < 0$ and $\partial_{V^B} m^{bil} < 0$.

Tables and Figures

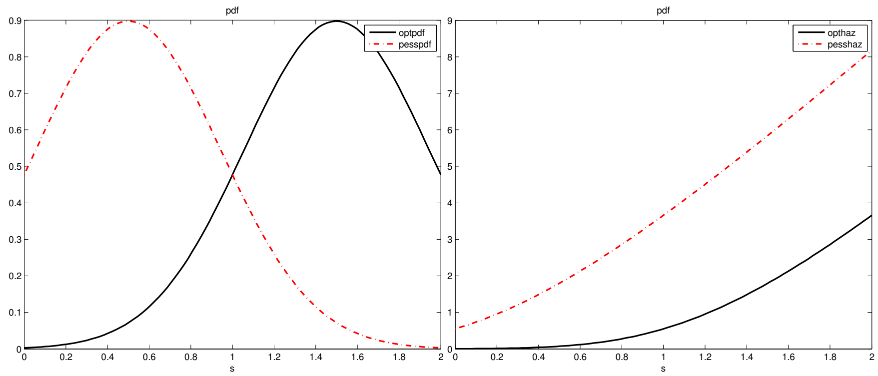


Figure 6: Heterogeneous beliefs satisfying the hazard rate property

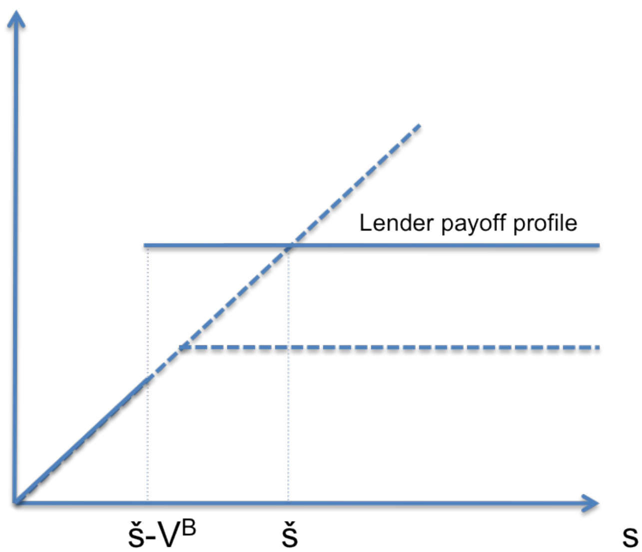


Figure 7: Payoff profile of the lender with borrower franchise value

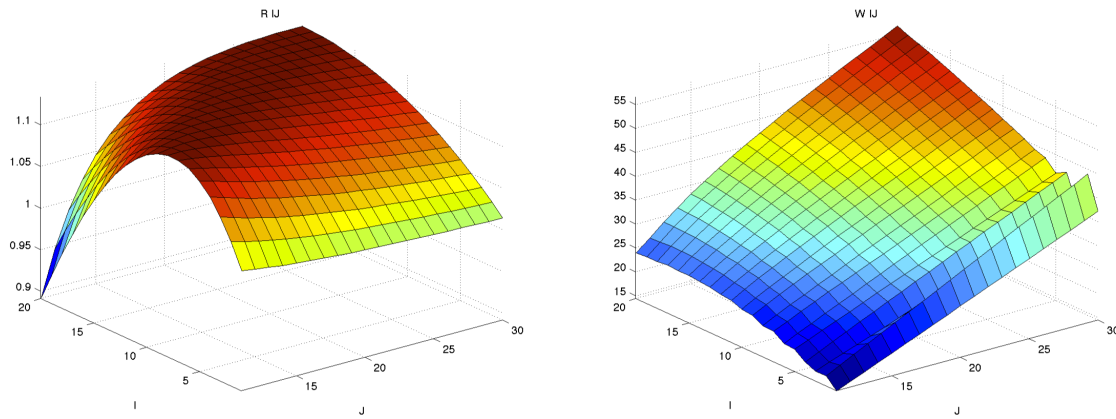


Figure 8: Levered return and welfare, as function of the number of borrowers I and numbers of lenders J

Figure 9: Franchise value in General Equilibrium

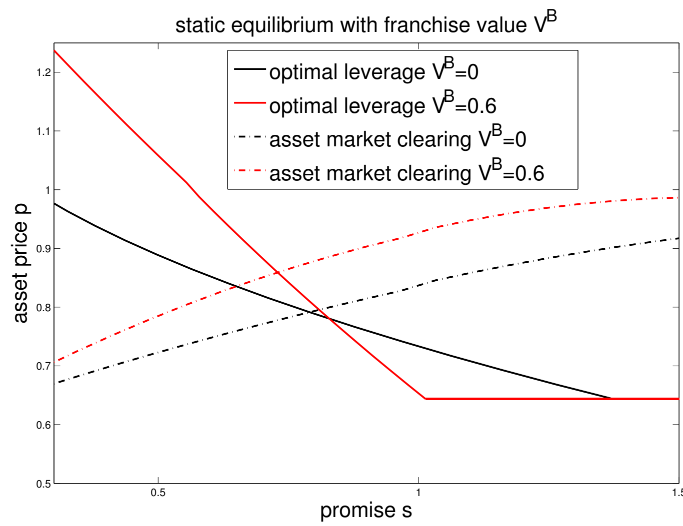
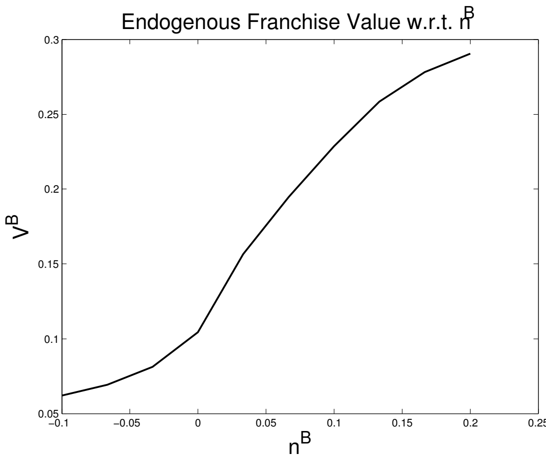


Figure 10: Endogenous franchise value V^B
Parameter choice: $\beta = 0.85$, $\sigma = 0.25$, $\mu^B = 1.5$, $\mu^L = 0.5$



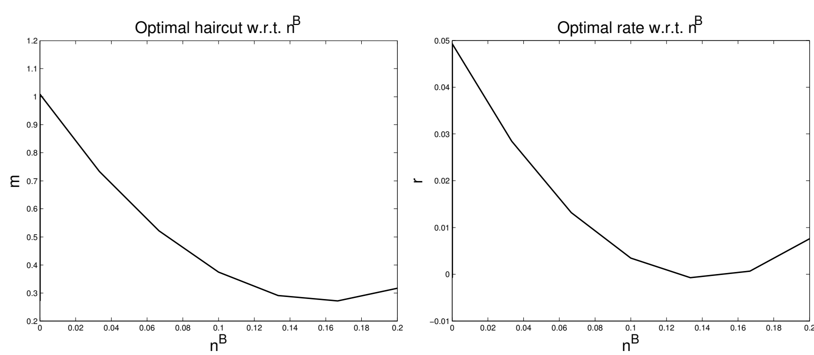


Figure 11: Optimal haircut m and rate r with endogenous franchise value

Figure 12: Endogenous franchise value with long-term relationships

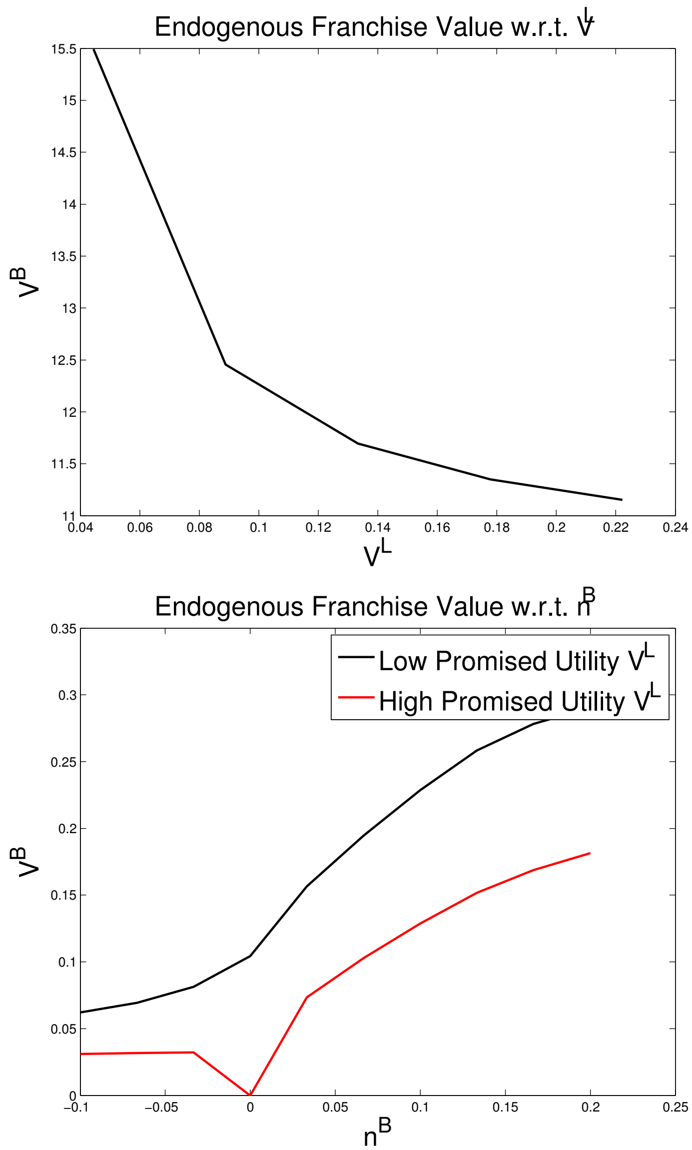


Figure 13: Repo chain with rehypothecation of collateral

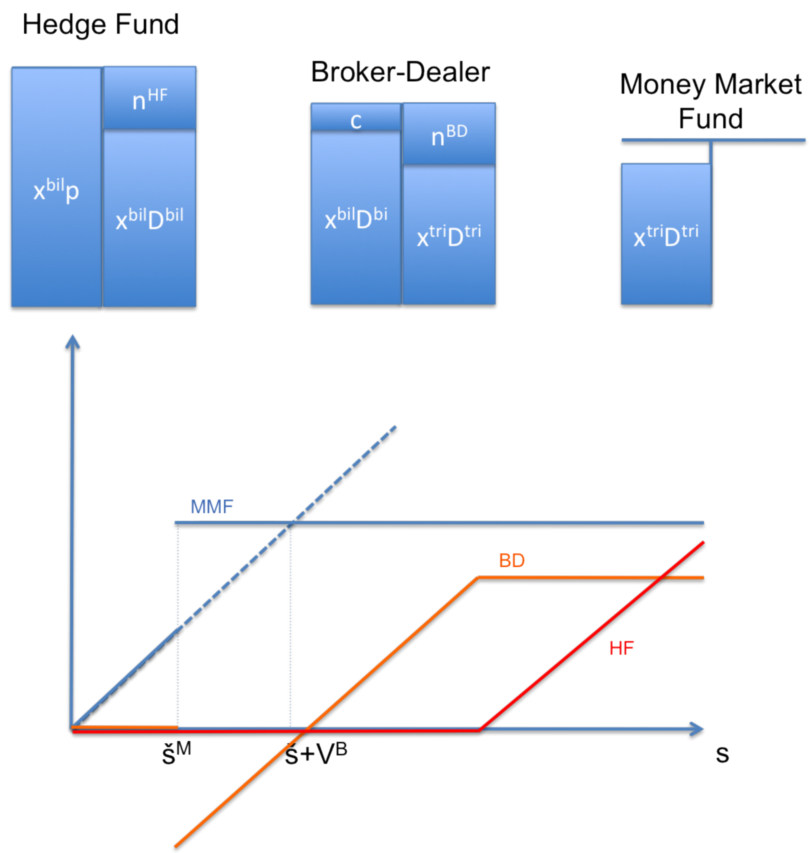


Figure 14: Aggregate repo volume by collateral class

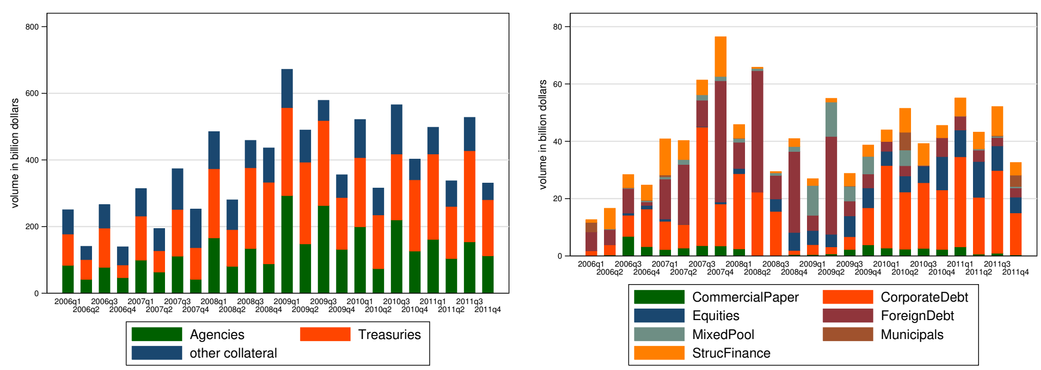
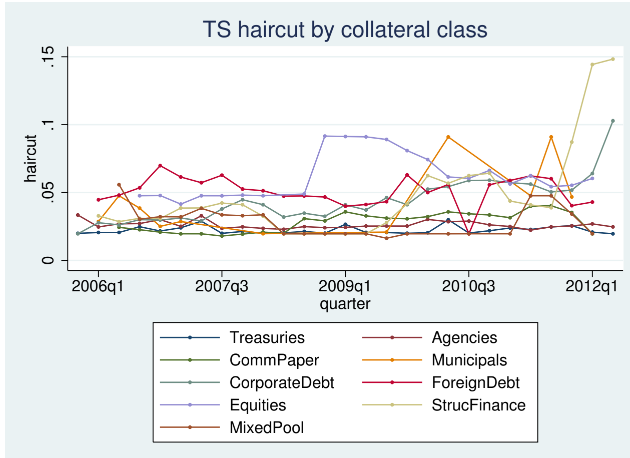


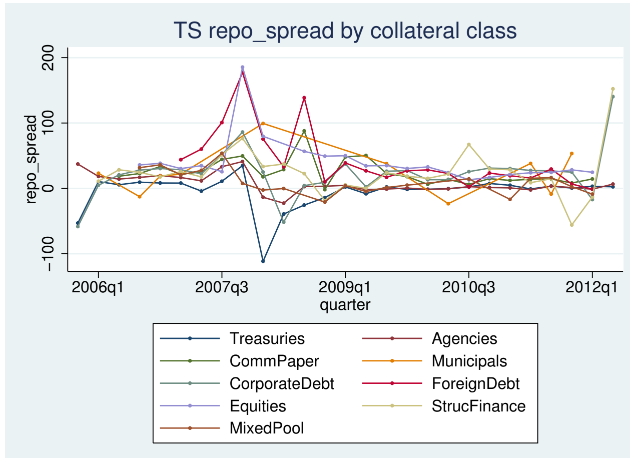
Table 1: Summary statistics of haircuts by collateral class



col_class	mean	sd	p10	p25	p50	p75	p90
1-Treasuries	.023	.021	.02	.02	.02	.02	.029
2-Agencies	.026	.03	.02	.02	.02	.029	.029
3-CommPaper	.029	.01	.02	.02	.029	.033	.048
4-Municipals	.039	.021	.02	.02	.038	.048	.065
5-CorporateDebt	.048	.033	.02	.029	.048	.056	.074
6-ForeignDebt	.055	.036	.029	.03	.048	.053	.092
7-Equities	.063	.02	.048	.048	.048	.077	.091
8-StrucFinance	.046	.036	.02	.02	.032	.071	.077
9-MixedPool	.027	.013	.02	.02	.02	.035	.048
Total	.03	.03	.02	.02	.02	.029	.048

Table 2: Summary statistics of repo spreads by collateral class

The repo spread is spread between the repo rate and the Fed fund rate of the same maturity. It is annualized and given in basis points.



col_class	mean	sd	p10	p25	p50	p75	p90
1-Treasuries	-3.91	43.9	-21	-5.89	1.23	6.25	16.9
2-Agencies	4.14	30	-15.3	-2.7	4.17	14	25
3-CommPaper	21.1	20.3	5.33	11.9	16.4	27.7	49.3
4-Municipals	31.9	31.2	-12.7	16.6	29.7	38.7	81.8
5-CorporateDebt	24.8	42.6	-11.9	12.5	22	40.8	70.1
6-ForeignDebt	55.3	56.9	10.4	20.1	35.9	51.9	171
7-Equities	29.3	29.3	-8.99	18.2	27.7	39.2	53.3
8-StrucFinance	29.5	45	-10.7	2.92	24	44.9	83.4
9-MixedPool	9.47	34.1	-16.8	-3.14	10.5	31.7	36
Total	7.08	42.4	-15.9	-2.9	4.31	17.5	38.9

Figure 15: Dispersion in haircuts, by collateral class

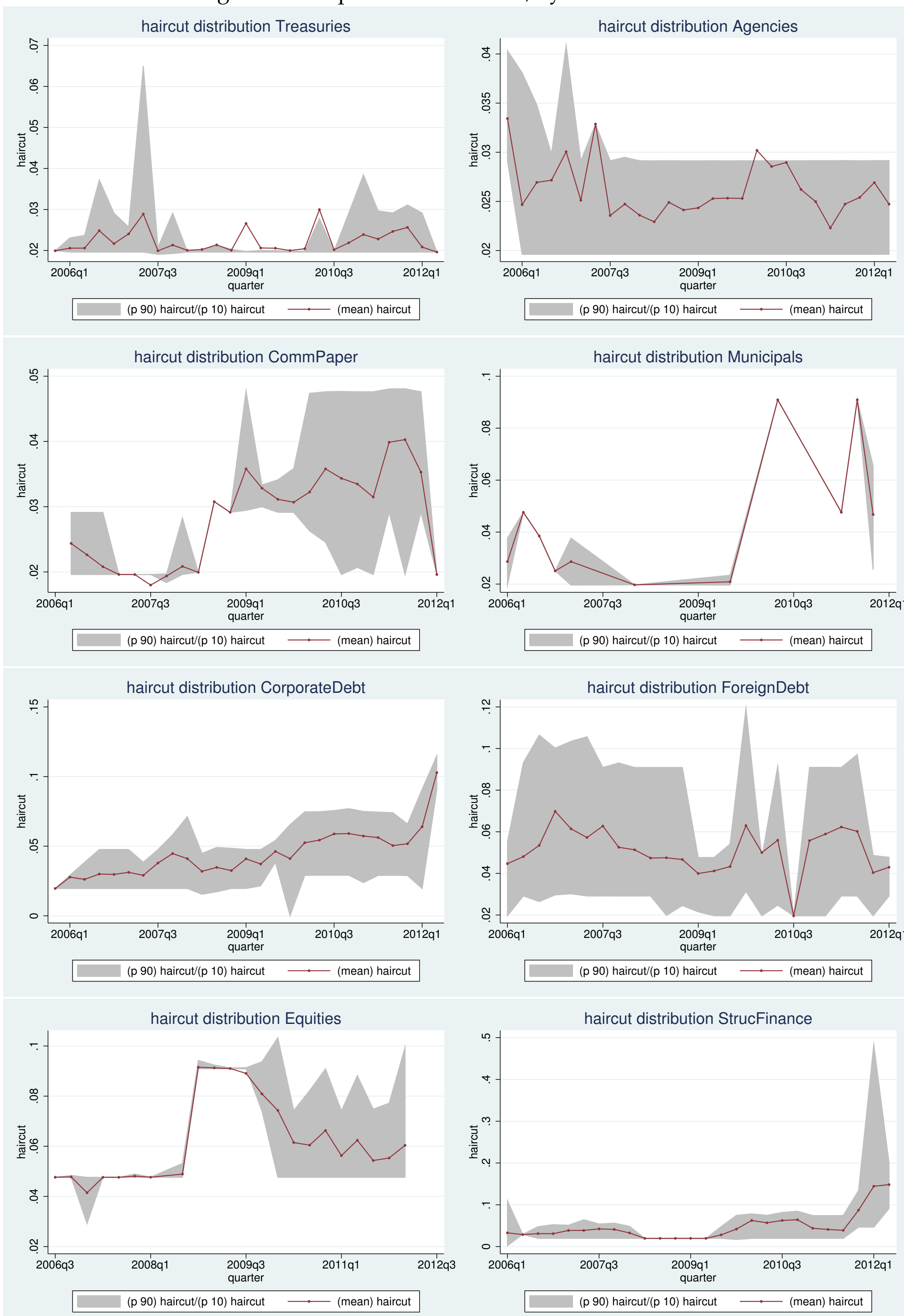


Figure 16: Dispersion in repo spreads, by collateral class

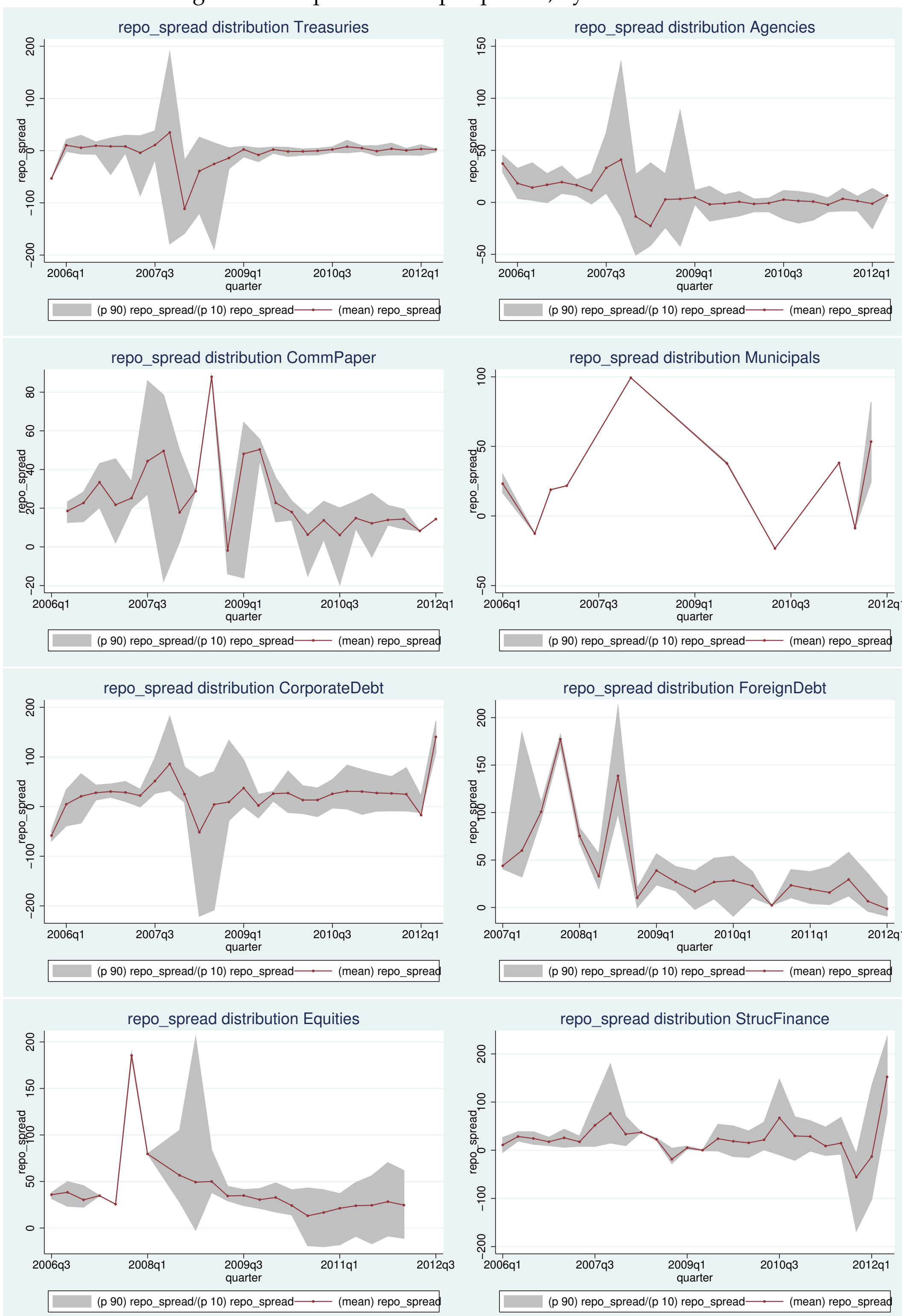
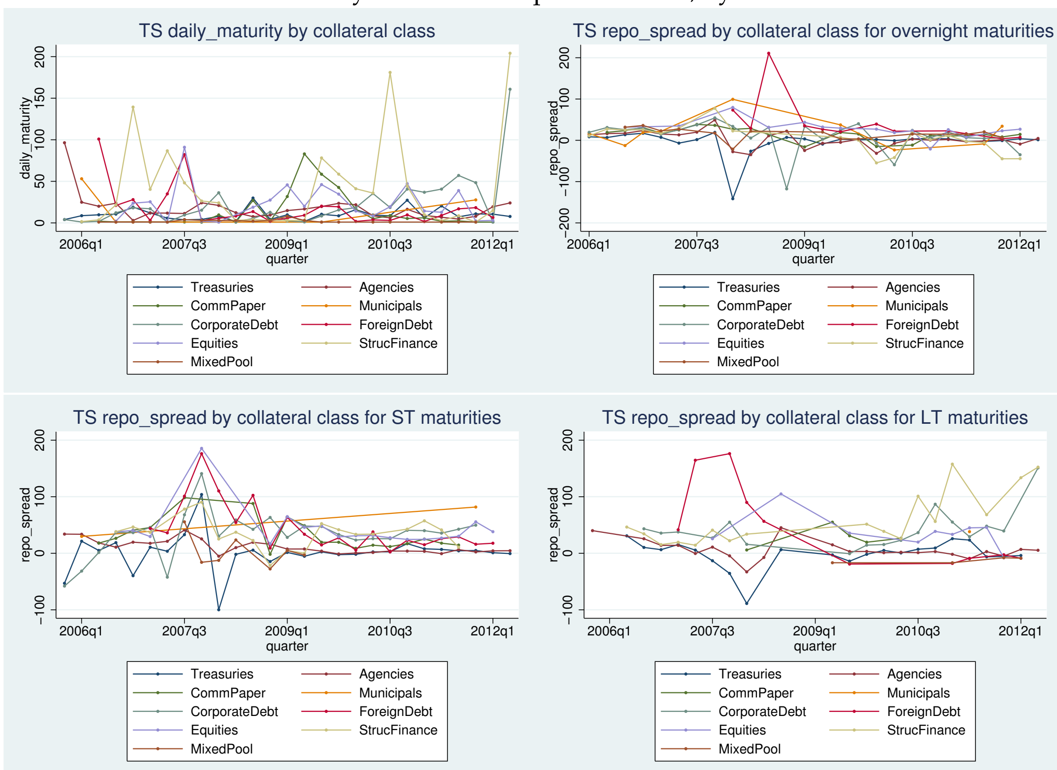


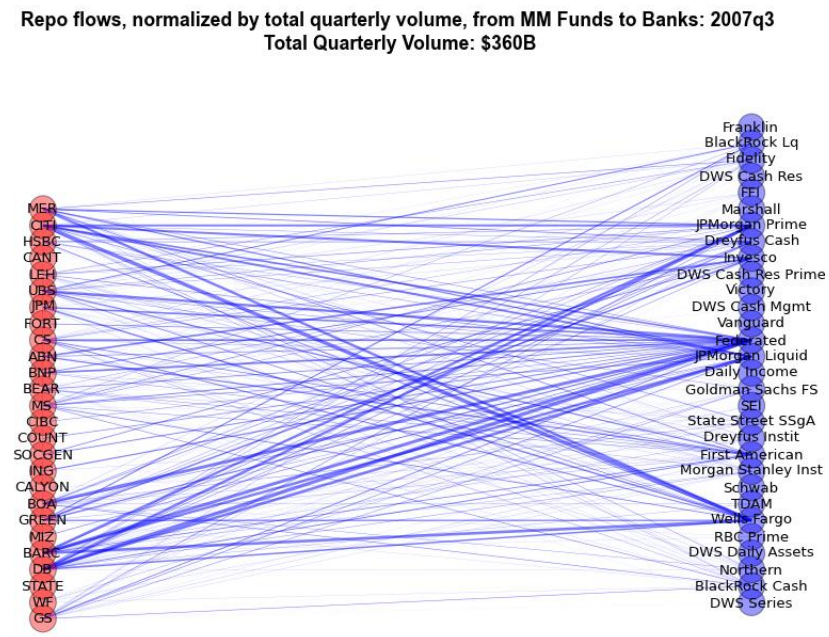
Table 3: Summary statistics of repo maturities, by collateral class



col_class	mean	p50	p75	p90
1-Treasuries	9.71	1	3	24
2-Agencies	11.6	3	3	32
3-CommPaper	18.2	1	6	90
4-Municipals	19.9	1	32	90
5-CorporateDebt	24.4	3	31	90
6-ForeignDebt	14.3	2	3	30
7-Equities	23	3	7.5	90
8-StrucFinance	48.1	3	90	182
9-MixedPool	2.36	2	3	4
Total	13.1	2	3	35

Figure 17: **Network of repo transactions**

Borrowers are vertically aligned on the left of the bipartite graph. Lenders (collapsed by MMF family) are vertically aligned on the right. Each edge connecting one borrower with one lender documents the existence of a bilateral relationship. The thickness of the edge is given by the oustanding repo volume of the bilateral relationship, normalized by the total outstanding volume in the dataset for the quarter 2007q3. The upper panel is 2007q3, the bottom panel is 2008q3.



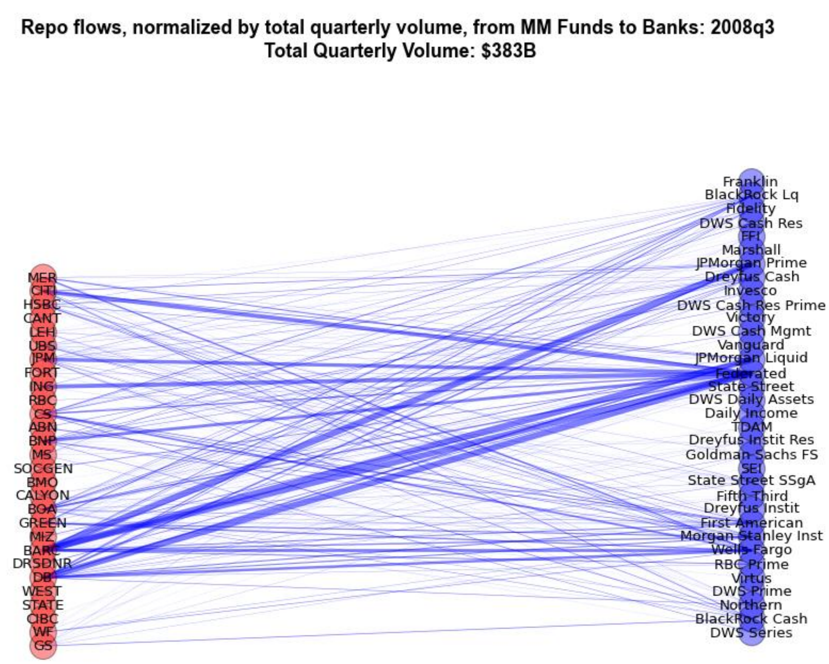


Table 4: **Summary statistics of borrowers**

Name is the identity of the borrower. The following quantities are averaged over the 24 quarters of the sample: Repo is the volume of funding raised by the borrower in the sample ($\$Bil$), Assets are the total assets from Y9-C call reports ($\$Bil$), Networth is total assets minus total liabilities ($\$Bil$), CapRatio is $(totalassets - totalliabilities)\,/totalassets$, CDS is 5 year tenure CDS spread on the borrower name, Scope is the scope of the network of the borrower as defined in the text, PersistNb is the number of relationships of the borrower that already existed in at the previous quarter, NbRel is the number of all relationships of the borrower, NbLTRel is the number of Long-Term relationships, Concentr is equal to one if the borrower has relative concentrated financing (i.e. number of LT relationships superior to the median of number of LT relationships in the sample of borrowers).

Name	Repo	Assets	Networth	CapRatio	CDS	Scope	PersistNb	NbRel	NbLTRel	Concentr
BARC	66	359	21	0.06	0.87	0.89	22	23	25	1
DB	47	21	-7	-0.35	0.62	0.92	27	23	32	1
BNP	31				0.58	0.78	14	17	24	1
BOA	30	991	98	0.07	0.34	0.87	11	20	13	1
RBOS	26	43	25	0.60	1.02	0.91	15	17	16	1
CS	23				0.32	0.86	12	15	12	1
GS	22	702	42	0.06	1.03	0.89	16	20	23	1
CITI	18	1060	-3	-0.02	1.04	0.88	9	13	17	1
MS	17	606	57	0.09	1.29	0.87	11	15	8	1
UBS	17				0.65	0.87	19	16	14	1
MER	16				1.22	0.86	7	14	14	1
SOCGEN	16				0.73	0.80	4	8	8	1
JPM	13	1005	131	0.11	0.71	0.81	9	14	40	1
HSBC	12	192	-62	-0.34	0.55	0.91	8	12	7	1
GREEN	10					0.84	4	10	3	0
LEH	10				3.85	0.75	7	11	0	0
BNY	9	185	87	0.46	0.85	0.47	1	2	1	0
BEAR	9					0.85	7	14	1	0
CALYON	7				0.41	0.92	2	4	6	0
ING	6				0.67	0.66	5	4	6	0
ABN	6	78	-7	-0.08	1.65	0.69	3	6	3	0
RBC	6	69	16	0.23		0.83	4	8	8	1
WF	4	349	73	0.12	0.61	0.74	3	5	7	1
FORT	4				0.70	0.57	2	4	0	0
BMO	3	63	27	0.43	0.56	0.71	1	2	4	0
COUNT	3	110	-25	-0.23		0.51	1	3	2	0
SCOT	3	0	0	0.19	0.45	0.75	0	2	8	1
TD	2	112	71	0.64	0.49	0.88	2	4	8	1
MIZ	2				0.47	0.83	1	2	2	0
DRSDNR	1				0.36	0.47	1	2	0	0
WAMU	1				1.39		0	1	0	0
ABBEY	1				0.34		0	1	0	0
NAT	1				1.56		1	1	2	0
CIBC	0	1	-1	-1.77	0.53	0.74	1	2	3	0
MITSU	0						1	1	0	0
WEST	0						1	1	0	0
NYFED	0						0	1	0	0
CANT	0						1	1	0	0
STATE	0	139	85	0.60		0.37	2	3	0	0

Table 5: **Summary statistics of lenders**

Name is the identity of the lender. The following quantities are averaged over the 24 quarters of the sample: Repo is the volume of funding raised by the borrower in the sample (\$$Bil$), RepoExp is the ratio of repo holdings to Total Net Assets of the MMF, NbRel is the number of all relationships of the borrower, Scope is the scope of the network of the lender as defined in the text, Persist is the ratio of relationships of the borrower that already existed in at the previous quarter. Inflows08 is the cumulative monthly flows over 2008 where $flows_t = \frac{TNA_t - TNA_{t-1}}{TNA_{t-1}}$, InflowsCum is the same quantity cumulated over the 24 quarters of the sample, Yield08 is the cumulative monthly yields over 2008 of the MMF, YieldCum is the same quantity cumulated over the 24 quarters of the sample.

Name	Repo	RepoExp	Inflows08	InflowsCum	Yield08	YieldCum	NbRel	Scope	Persist
Federated	66	0.07	0.22	1.54	-0.00	5.68	19	0.94	0.84
Fidelity	38	0.13	-0.03	0.62	0.07	5.89	12	0.68	0.72
Wells Fargo	34	0.02	1.32	4.58	-0.16	5.15	12	0.95	0.83
JPMorgan Liquid	32	0.00	0.50	0.96	0.04	-12.57	10	0.91	0.88
Invesco	32	0.01		-0.06		-27.43	10	0.89	0.60
JPMorgan Prime	28	0.06	0.44	1.19	-0.01	5.01	9	0.90	0.72
Goldman Sachs FS	20	0.12	-0.20	0.44	-0.31	5.17	8	0.65	0.65
BlackRock Cash	20	0.01	0.53	-0.59	-0.13	-24.51	11	0.93	0.86
First American	20	0.01	1.26	1.36	-0.12	5.20	13	0.97	0.90
BlackRock Lq	16	0.08	0.51	0.26	-0.01	5.61	10	0.90	0.91
Morgan Stanley Inst	14	0.03	-0.51	0.58	-0.09	6.01	12	0.93	0.87
BofA	12	0.03	0.23	1.11	-0.15	5.18	13	0.98	0.77
Dreyfus Cash	11	0.00	1.76	2.66	-0.14	4.37	10	0.95	0.57
Schwab	8	0.01	-0.17	0.20	-0.13	3.90	7	0.86	0.38
Northern	8	0.04	-0.25	0.06	-0.88	3.21	12	0.93	0.82
TDAM	8	0.00	1.01	3.48	4.35	-7.77	3	0.38	0.82
State Street SSgA	8	0.06	-0.12	0.54	-0.18	5.11	11	0.74	0.74
UBS	8	0.02	0.22	0.17	-0.13	5.29	6	0.86	0.70
FFI	6						10	0.91	0.75
State Street	5	0.13	0.25	4.08	4.81	10.66	10	0.84	0.71
HSBC	3	0.01	1.45	2.05	-0.23	5.20	5	0.81	0.53
Dreyfus Instit	3	0.03	0.17	1.32	-0.07	5.73	5	0.90	0.58
Franklin	2	0.04	0.56	1.18	-0.71	3.09	5	0.79	0.69
RBC Prime	2	0.01	0.01	1.27	-0.66	0.74	5	0.87	0.69
Fifth Third	2	0.00	-0.02	0.46	0.14	3.82	6	0.90	0.70
Western Asset Cash	2	0.01	0.86	1.16	-0.00	-12.68	2	0.63	0.69
Dreyfus Instit Res	2	0.00	0.07	-0.75	-1.76	-26.16	6	0.96	0.80
Invesco Premier	2	0.01		0.50		-27.46	8	0.92	0.53
SEI	1	0.01	0.26	0.88	-0.27	4.99	6	0.69	0.80
Marshall	1	0.00	0.23	0.58	0.04	5.75	6	0.82	0.94
Daily Income	1						5	0.94	0.82
DWS Cash Res	1	0.00	-0.09	0.23	-0.21	5.24	3	0.63	0.46
DWS Cash Mgmt	1	0.00	0.07	0.13	-0.27	4.69	3	0.63	0.49
DWS Cash Res Prime	1	0.00	-0.32	-0.76	-0.31	-15.97	3	0.73	0.45
DWS Series	1	0.00	0.31	0.64	-0.16	5.72	3	0.55	0.45
DWS Prime	0	0.00	-0.20	-0.94	-0.28	3.43	4	0.74	0.60
Virtus	0	0.00	-0.15	-1.53	-0.14	5.55	2	0.83	0.58
DWS Daily Assets	0						2	0.80	0.45
Victory	0	0.01	0.15	-0.98	-0.68	3.41	5	0.95	0.73
Vanguard	0	0.00	0.28	1.70	-0.04	5.95	8	0.93	0.70

Figure 18: Dynamic metrics of network structure, for borrowers and lenders

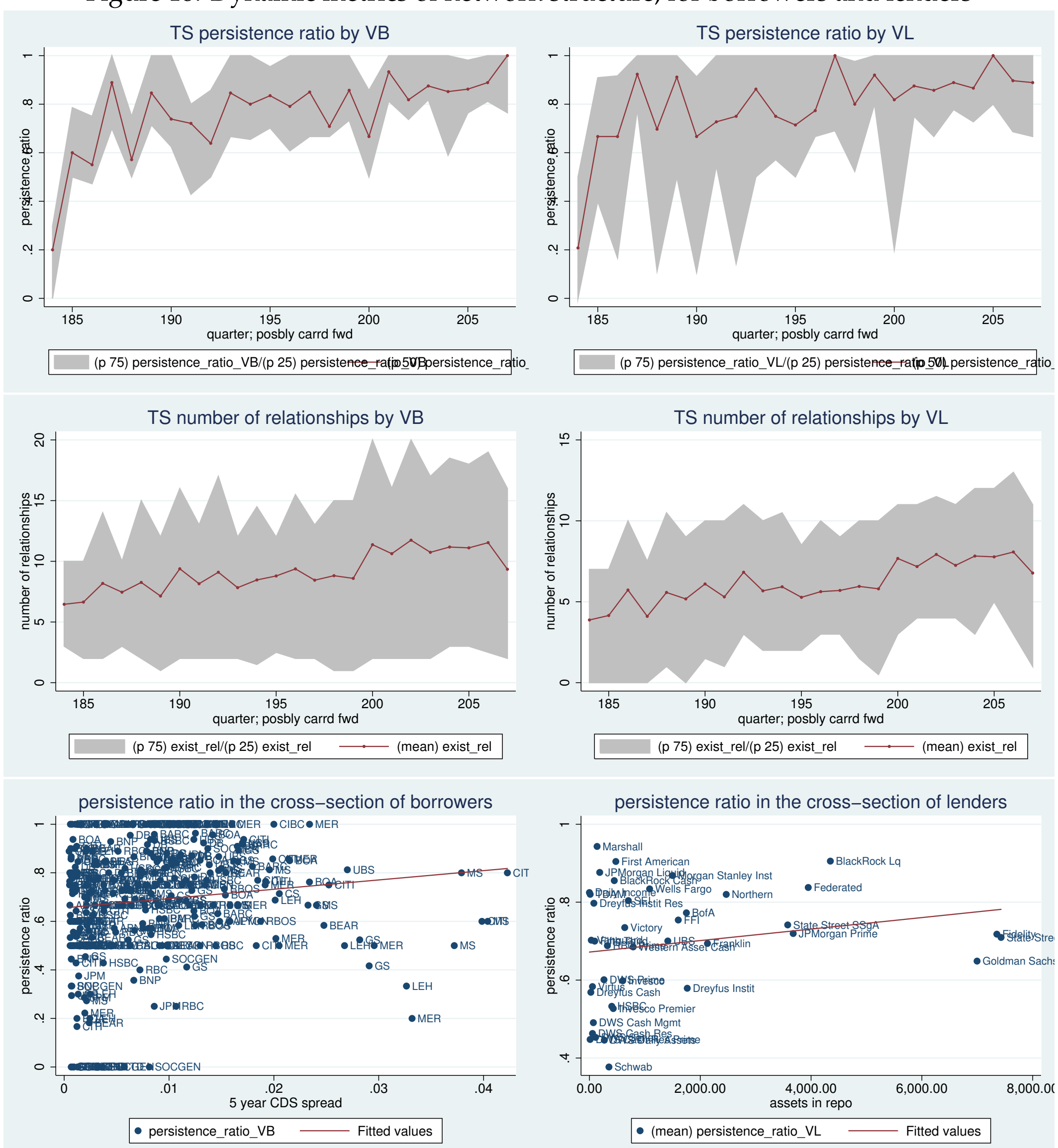


Table 6: **First stage (0a): Haircuts and Rates on Collateral Class and Quarters**

The omitted collateral class is Treasuries, and the omitted quarter is 2006q1. I only report quarterly time-fixed effects that are significant.

	haircut	repo_spread
col_class==2-Agencies	0.00***	7.65***
	(0.00)	(0.63)
col_class==3-CommPaper	0.01***	21.69***
	(0.00)	(2.47)
col_class==4-Municipals	0.02***	34.10***
	(0.00)	(8.19)
col_class==5-CorporateDebt	0.03***	28.17***
	(0.00)	(1.02)
col_class==6-ForeignDebt	0.03***	57.80***
	(0.00)	(1.28)
col_class==7-Equities	0.05***	32.89***
	(0.00)	(2.09)
col_class==8-StrucFinance	0.02***	25.23***
	(0.00)	(1.80)
col_class==9-MixedPool	0.01***	11.66**
	(0.00)	(3.87)
quarter==2007q3	-0.01*	12.35
	(0.00)	(7.42)
quarter==2007q4	-0.00	44.50***
	(0.00)	(7.43)
quarter==2008q1	-0.01*	-57.03***
	(0.00)	(7.40)
quarter==2008q2	-0.01*	-45.07***
	(0.00)	(7.42)
quarter==2008q3	-0.01*	-22.48**
	(0.00)	(7.40)
quarter==2008q4	-0.01*	-27.20***
	(0.00)	(7.40)
N	16,602	16,602
R-squared	0.44	0.30

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 7: **Specification (0b): borrowers dummies**

Borrowers fixed-effects. The specification (0b) is ran on the entire sample (16387 transactions). The smallest borrowers (according to the ranking of Table 4) are not displayed. Omitted borrower is ABN Amro.

	haircut	repo_spread
borrower_ticker==BARC	0.01***	4.27
	(0.00)	(3.46)
borrower_ticker==BNP	0.00**	-1.89
	(0.00)	(3.54)
borrower_ticker==BOA	0.00***	2.05
	(0.00)	(3.47)
borrower_ticker==CITI	0.01***	3.96
	(0.00)	(3.56)
borrower_ticker==CS	0.00***	0.87
	(0.00)	(3.49)
borrower_ticker==DB	0.00***	-0.91
	(0.00)	(3.43)
borrower_ticker==GS	0.00***	-4.72
	(0.00)	(3.48)
borrower_ticker==HSBC	0.00**	-3.53
	(0.00)	(3.69)
borrower_ticker==JPM	0.00*	-1.22
	(0.00)	(3.52)
borrower_ticker==LEH	-0.00	-2.74
	(0.00)	(3.93)
borrower_ticker==MER	0.00***	-3.43
	(0.00)	(3.62)
borrower_ticker==MS	0.01***	-1.20
	(0.00)	(3.54)
borrower_ticker==RBOS	0.01***	4.02
	(0.00)	(3.69)
borrower_ticker==SOCGEN	0.00**	1.62
	(0.00)	(3.88)
borrower_ticker==UBS	0.01***	-2.85
	(0.00)	(3.48)
Collateral FE	Y	Y
Quarter FE	Y	Y
N	16,387	16,387
R-squared	0.86	0.38

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 8: **Specification (0b): lenders dummies**

Lenders fixed-effects. The specification (0b) is ran on the entire sample (16387 transactions). The smallest lenders (according to the ranking of Table 4) are not displayed. Omitted lender is Blackrock Cash.

	haircut	repo_spread
MMF_name==BlackRock Lq	0.00***	-8.33***
	(0.00)	(1.77)
MMF_name==BofA	-0.00***	-14.48***
	(0.00)	(1.93)
MMF_name==Dreyfus Cash	-0.01***	-13.54***
	(0.00)	(1.98)
MMF_name==Federated	0.00	-12.66***
	(0.00)	(1.40)
MMF_name==Fidelity	0.01***	11.51***
	(0.00)	(1.72)
MMF_name==First American	-0.00***	-15.25***
	(0.00)	(2.03)
MMF_name==Goldman Sachs FS	0.00***	-50.50***
	(0.00)	(2.11)
MMF_name==Invesco	-0.00***	-14.50***
	(0.00)	(2.00)
MMF_name==JPMorgan Liquid	-0.00***	-6.89***
	(0.00)	(1.77)
MMF_name==Morgan Stanley Inst	0.00	-12.07***
	(0.00)	(1.63)
MMF_name==Schwab	0.01***	19.24***
	(0.00)	(5.41)
MMF_name==State Street	-0.00***	-21.60***
	(0.00)	(2.18)
MMF_name==UBS	-0.00**	-14.91***
	(0.00)	(2.30)
MMF_name==Wells Fargo	0.00**	-5.01*
	(0.00)	(2.15)
Collateral FE	Y	Y
Quarter FE	Y	Y
N	16387	16387
R-squared	0.86	0.38

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 9: **Specification (1a): continuous OLS test of borrower franchise value**

This table reports the results of the regression of the pricing variables (haircut and rates) on different proxies of franchise value: $-CDS$, and its determinant in the dynamic model: borrower net worth n^B. CDS is the 5year tenture CDS contract obtained from Markit: $CDS\,day$ is the CDS on the date of the repo transaction, $CDS\,quarter$ is the CDS averaged on the quarter of the repo transaction, and $CDS\,quarter\,lagged$ is the CDS averaged on the quarter leading the repo transaction. Net worth is computed from the bank holding company Y9-C call report. Time and Collateral Class fixed effects are included in each regression. Repo collateralized by Treasuries or Agencies are excluded from the sample. Standard errors are robust and clustered at the borrower level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	haircut	haircut	haircut	haircut	haircut	rate	rate	rate	rate	rate
- CDS day	-0.013					-43.240**				
	(0.01)					(15.15)				
- CDS quarter		-0.017					-26.140			
		(0.02)					(20.75)			
- CDS quarter lagged			-0.044*					-46.373*		
			(0.02)					(23.34)		
capital ratio				-0.013					18.391***	
				(0.02)					(5.15)	
net worth					-0.035***					20.506
					(0.01)					(12.66)
Lender FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Collateral FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Quarter FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
N	2670	2998	3001	1686	1686	2670	2998	3001	1686	1686
R-squared	0.87	0.87	0.88	0.86	0.87	0.72	0.70	0.70	0.80	0.80

* p<0.10, ** p<0.05, *** p<0.01

Table 10: **Specification (1b): Difference-in-Difference on European borrowers**

This table presents the results of a Difference-in-Difference specification that uses the European sovereign debt crisis as an instrument of European borrowers. Repo transactions contracted on the US tri-party are collateralized by US securities uncorrelated to the European shock. $1_{European\,crisis}$ is a dummy variable equal to one from 2010q1 to 2012q2, and equal to zero from 2006q1 to 2009q4. $1_{European\,bank}$ is a dummy variable equal to one for the following borrowers: DB, CS, UBS, ABN, HSBC, HVB, DRSDNR, SOCGEN, BARC, CALYON, CMZB, BNP, ING, FORT, and equal to zero for the other borrowers. The first two regressions are run on the whole sample, the last two ones are ran on the sample excluding repo transactions collateralized by Treasuries and Agencies.

	(1)	(2)	(3)	(4)
	haircut	repo_spread	haircut	repo_spread
$1_{Europeancrisis}$	0.00***	0.71	0.01***	-16.16***
	(0.00)	(0.85)	(0.00)	(2.08)
$1_{Europeanbank}$	-0.00	-1.14	-0.01***	-7.70**
	(0.00)	(0.85)	(0.00)	(2.45)
$1_{Europeancrisis} * 1_{Europeanbank}$	-0.00	1.98	0.01***	10.01**
	(0.00)	(1.23)	(0.00)	(3.29)
Collateral FE	Y	Y	Y	Y
N	16702	16702	3608	3608
R-squared	0.17	0.14	0.10	0.10

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 11: **Specification (2a): continuous OLS test of lender pessimism / risk-taking**

This table tests in the cross-section of lenders for the hypothesis of heterogeneity in pessimism (state-dependent utility, isomorphic to risk-aversion). RepoExp is the ratio of repo holding to total asset at the MMF level (proxy for bargaining power). Inflows2008 and Yield08 are respectively the inflows and the yield experienced by the MMF in 2008, used as proxy for MMF risk-taking. Time, Collateral Class and Borrower fixed effects are included. Repo collateralized by Treasuries or Agencies are excluded from the sample. Standard errors are robust and clustered at the Lender level.

	(1)	(2)	(3)	(4)	(5)	(6)
	haircut	haircut	haircut	repo_spread	repo_spread	repo_spread
RepoExp	0.016***			22.907***		
	(0.00)			(6.44)		
Inflows2008		-0.048*			50.835	
		(0.03)			(39.87)	
Yield08			0.001			13.380
			(0.00)			(12.38)
Borrower FE	Y	Y	Y	Y	Y	Y
Collateral FE	Y	Y	Y	Y	Y	Y
Quarter FE	Y	Y	Y	Y	Y	Y
N	2982	3411	3411	2982	3411	3411
R-squared	0.89	0.87	0.87	0.50	0.65	0.66

* p<0.10, ** p<0.05, *** p<0.01

Table 12: **Specification (3a): relationship pricing**

This table tests for relationship characteristics priced in haircuts and rates. The number of relationships *NbRelB* and *NbRelL* proxy for the concentration of financing for the borrower, and for the concentration of the lending base for the lender. Repo volumes *RepoVolumeB* and *RepoVolumeL* proxy for the respective bargaining powers. *ScopeB* and *ScopeL* is an Herfindhal index measuring the atomicity of the respective counterparty bases: it is high when the counterparties are uniformely dispersed. *PersistentRel* and *HistoryRel* are relationship-specific variables: *PersistentRel* is a dummy variable equal to one if the bilateral connection also exists at the previous quarter, *HistoryRel* counts the number of quarters in which the bilateral connection was existing up to the date of the repo transaction. Time, Collateral Class and Borrower and Lender fixed effects are included. Repo collateralized by Treasuries or Agencies are excluded from the sample. Standard errors are robust and clustered at the relationship level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	haircut	haircut	haircut	haircut	haircut	rate	rate	rate	rate	rate
NbRelB	0.000					0.395				
	(0.00)					(0.26)				
NbRelL	0.001***					0.295				
	(0.00)					(0.49)				
RepoVolumeB		-0.001					22.369*			
		(0.01)					(11.35)			
RepoVolumeL		-0.019**					-40.176***			
		(0.01)					(15.07)			
ScopeB			0.009					14.301		
			(0.01)					(13.75)		
ScopeL			-0.012**					-0.912		
			(0.01)					(11.33)		
PersistentRel				0.003					-5.762**	
				(0.00)					(2.81)	
HistoryRel					0.000					-0.215
					(0.00)					(0.34)
Borrower FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Lender FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Collateral FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Quarter FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
N	3497	3497	3493	3497	3497	3497	3497	3493	3497	3497
R-squared	0.88	0.88	0.88	0.88	0.88	0.73	0.73	0.73	0.73	0.73

* p<0.10, ** p<0.05, *** p<0.01

Table 13: **Specification (3b): Long-term relationships and borrower stable network**

This table tests the effect of long-term relationships on the stability of the network of bilateral connections. $\Delta Volij = |\Delta RepoVolij|$ is the absolute value of the change, from one quarter to another, in the repo volume of bilateral relationship between borrower i and lender j, normalized to the total quarterly repo volume. $1_{rel\,ij,t}$ is a dummy variable equal to 1 if the bilateral connection ij at the quarter t. $NbRelB$ is the number of relationships a borrower has at a given quarter. $ScopeB$ is an Herfindhal index measuring the atomicity of the counterparty base. The two latter regression are run via probit, and the R^2 given for these regressions is McFadden's pseudo R^2.

	(1)	(2)	(3)	(4)
	ΔVolij	ΔVolij	$1_{relij,t}$	$1_{relij,t}$
NbRelB	-0.000***			
	(0.00)			
ScopeB		-0.004**		
		(0.00)		
HistoryRel:$\sum_{s<t} 1_{relij,s}$			0.160***	
			(.0012)	
PersistRel$1_{relij,t-1}$				0.081**
				(.002)
N	6668	6571	527	2095
R-squared	0.18	0.18	0.47	0.22

* p<0.10, ** p<0.05, *** p<0.01

Table 14: **Specification (3c): Long-term relationships and borrower stable funding volume**

This table the effect of the long-term relationships on the stability of secured funding *volumes* for one given borrower. $\Delta Repo = |\Delta RepoVolB|$ is the absolute value of the change, from one quarter to another, in borrower B repo funding normalized to the total quarterly repo volume. $Repo/ST = Repo/STfunding$ is the ratio, for borrower B, between its repo funding (in $) and the $ sum of all its short-term funding sources: fed funds + repo + short-term deposits + commercial paper + short-term liabilities (data from Y9-C call reports). This proxies for the easiness of access to repo for borrower B and captures potential substitution from other funding sources, in case of repo funding difficulties. $VBtotMMFrisk$ is a measure of MMF risk-taking behavior, aggregated at the Borrower level. $NbPersRel$ is the number of bilateral relationships that the borrower already had at the previous quarter. $NbLTRel$ is the number of Long-Term relationships the borrower enjoys (a Long-Term relationship is defined when the number of quarter of existence of the relationship is above the median of the universe of relationships). The last two lines are interaction terms to test that Long-Term relationships help stabilize repo funding. Standard errors are robust and clustered at the borrower level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	ΔRepo	ΔRepo	ΔRepo	ΔRepo	ΔRepo	ΔRepo	ΔRepo	Repo/ST	Repo/ST
- CDS quarter	-153.297*					-108.332	-376.478***		
	(80.92)					(78.23)	(102.25)		
CapRatio		-0.013*							
		(0.01)							
Net worth			-0.056						
			(0.03)						
VBtotMMFrisk				-0.000***				0.000***	
				(0.00)				(0.00)	
NbPersRel					-0.004***				0.018***
					(0.00)				(0.00)
(- CDSq)*NbLTRel						-32.515***			
						(8.25)			
(- CDSq)*NbPersRel							25.627***		
							(5.07)		
N	448	238	238	669	669	448	448	246	246
R-squared	0.01	0.01	0.01	0.01	0.07	0.02	0.10	0.14	0.39

* p<0.10, ** p<0.05, *** p<0.01