Which is better?

Alex and Morgan were asked to simplify $\sqrt{24}$

Alex's "prime factor" way	Morgan's "recognize the biggest perfect squares" way
$\sqrt{24}$	$\sqrt{24}$
$\downarrow$	$\downarrow$
$\sqrt{2\cdot 2\cdot 2\cdot 3}$	$\sqrt{4\cdot 6}$
$\downarrow$	$\downarrow$
$\sqrt{2^2}\cdot\sqrt{2\cdot 3}$	$\sqrt{4}\cdot\sqrt{6}$
$\downarrow$	$\downarrow$
$2\sqrt{6}$	$2\sqrt{6}$

First I wrote down the prime factors of 24 under the radical.

Then I found the pairs of prime factors and pulled them out.

I simplified the square root that I could (2 squared is 4, and the square root of 4 is 2) and multiplied the numbers in the other square root. Here is my answer.

First I factored 24 with a perfect square as one of the factors.

Then I split up the square root into two separate radicals.

Then I simplified the square root of 4, which is 2. Here is my answer.

*** Why did Alex write the prime factors of 24 as a first step?**
*** How did Morgan simplify the expression?**
*** What are some similarities between the two methods?**
*** Which method is better for this problem? Why? Is there another case where you would use the other method?**

Which is better?

Alex and Morgan were asked to simplify $\sqrt{24}$

Alex's "prime factor" way

Morgan's "recognize the biggest perfect squares" way

$\sqrt{24}$

$\sqrt{24}$

First I wrote dow... the prime factors... 24 under the radical.

Then I f... pai...

I simplifie... square roo... could (2... is 4, a... squ... 2)... the numbers ... other square root. Here is my answer.

First I factored 24 with a perfect square as one of the factors.

...I split up ...root

...ied the ...oot of ... 2.

There is often more than one way to simplify expressions with radicals. Sometimes using prime factors may be the easiest, but on other problems looking for the biggest perfect square may be better.

Before you start simplifying, you can look at the problem first and try to see which way will be easier.

*** Why did Alex write the prime ... 24 as...**
*** How did Morgan simplify the ...ssion?**
*** What are some similarities be...een the two met...**
*** Which method is better for this problem? Why? Is ... another case where you would use the other method?**

Student Worksheet 10.1.1

1 Why did Alex write the prime factors of 24 as a first step?

2 How did Morgan simplify the expression?

3 What are some similarities between the two methods?

4 Which method is better for this problem? Why?

5 Is there another case where you would use the other method?

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{44}$

Alex's "divide by 2" way

$$\sqrt{44}$$

$$\downarrow$$

$$22$$

I took half of 44. It is 22.

Morgan's "take the square root" way

$$\sqrt{44}$$

$$\downarrow$$

$$\sqrt{2 \cdot 2 \cdot 11}$$

$$2\sqrt{11}$$

I wrote the prime factorization of 44.

I simplified the expression.

*** How did Alex simplify the expression?**
*** How did Morgan simplify the expression?**
*** Whose answer is correct, Alex's or Morgan's? How do you know?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?**

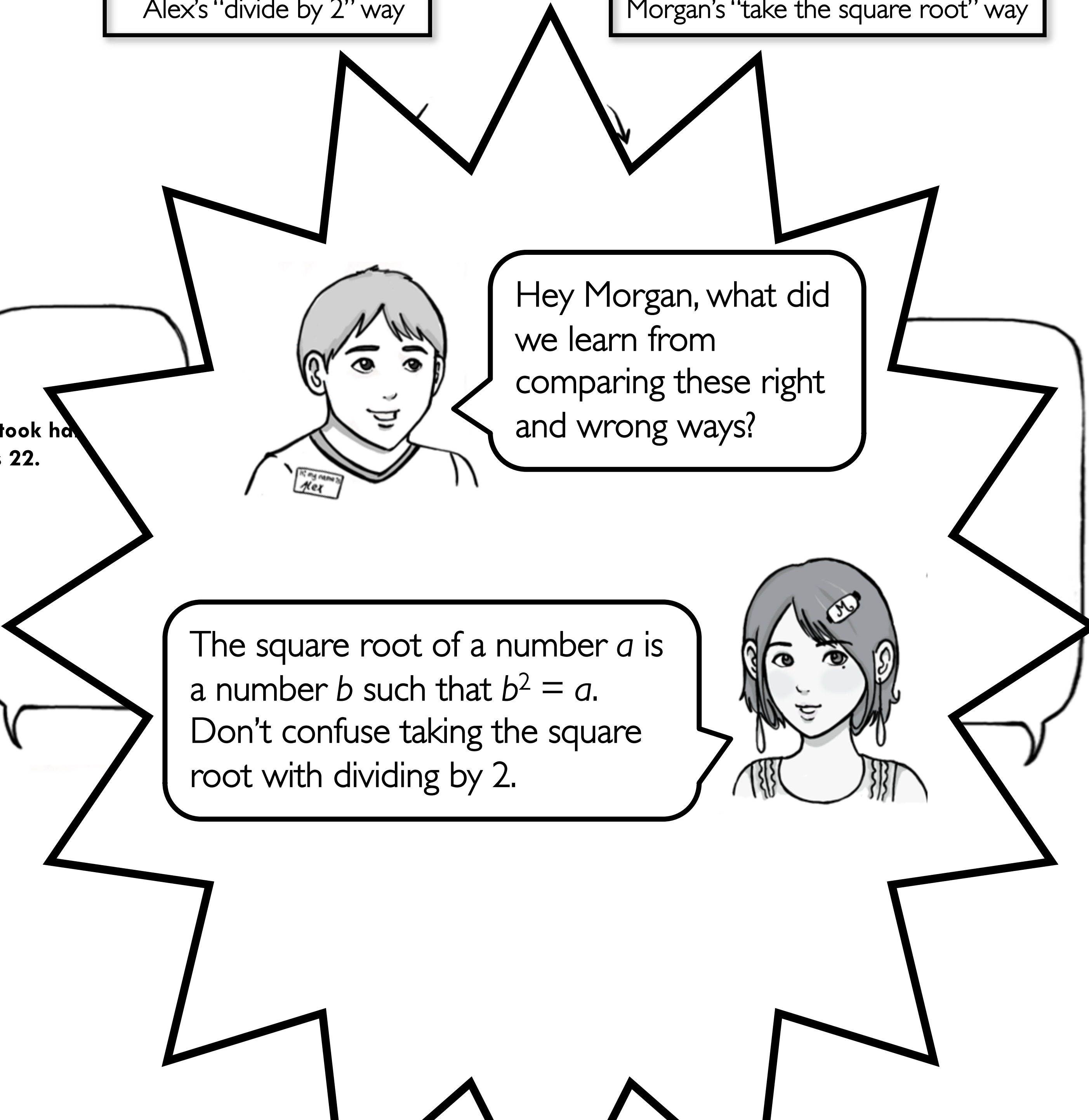
Which is correct?

Alex and Morgan were asked to simplify $\sqrt{44}$

Alex's "divide by 2" way

Morgan's "take the square root" way

I took ha
is 22.



* **How did Alex simplify the express**
* **How did Morgan simplify the expr on?**
* **Whose answer is correct, Alex's or Morgan's? How do know?**
* **What are some similarities and differences between Al 's and Morgan's ways?**
* **Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?**

Student Worksheet 10.1.2

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{50}$

Alex's "rewrite terms outside the radical" way

$$\sqrt{50} \rightarrow \sqrt{25 \cdot 2} \rightarrow \sqrt{5 \cdot 5 \cdot 2} \rightarrow 25\sqrt{2}$$

I began to factor 50.

I continued to factor until I got the prime factorization of 50.

I rewrote 5 times 5 outside the radical. Here is my answer.

Morgan's "take the square root" way

$$\sqrt{50} \rightarrow \sqrt{25 \cdot 2} \rightarrow \sqrt{5 \cdot 5 \cdot 2} \rightarrow 5\sqrt{2}$$

I began to factor 50.

I continued to factor until I got the prime factorization of 50.

I rewrote the square root of 5 times 5 as 5 outside the radical. Here is my answer.

*** How did Alex simplify the expression?**
*** How did Morgan simplify the expression?**
*** Whose answer is correct, Alex's or Morgan's? How do you know?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?**

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{50}$

Alex's "rewrite terms outside the radical" way	Morgan's "take the square root" way

I began to factor 50.

I continued t… factor until I g… the prime factorization … 50.

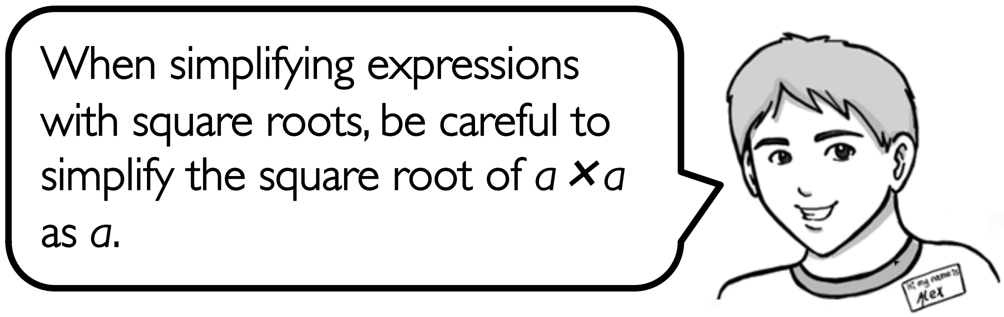
I rewrot… outside the radical. Here is … answer.

…n to factor

…ot

…f

…o as …e radical. …swer.



*** How did Alex simplify the express…**
*** How did Morgan simplify the expre…**
*** Whose answer is correct, Alex's or M…n's? How d… …ow?**
*** What are some similarities and diffe…nces between A… …nd Morgan's w…s?**
*** Can you state a general rule that describes what you h… learned from comparing Alex's and Morgan's ways of simplifying this expression?**

Student Worksheet 10.1.3

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

Why does it work?

Alex and Morgan were asked to estimate the value of $\sqrt{20}$

Alex's "use a common root" way

$$\sqrt{20}$$

$$\downarrow$$

First I wrote the prime factorization of 20.

$$\sqrt{2 \cdot 2 \cdot 5}$$

$$\downarrow$$

I simplified the expression.

$$2\sqrt{5}$$

$$\downarrow$$

I know that the square root of 5 is about 2.2. I multiplied 2 times 2.2, and I got approximately 4.4.

$$2 \cdot 2.2 \approx 4.4$$

Morgan's "compare to known roots" way

I don't know the square root of 20, but I do know the square roots of some numbers that are near 20.

$$\sqrt{20}$$

$$\downarrow$$

I know that the square root of 25 is 5.

$$\sqrt{25} = 5$$

$$\downarrow$$

The square root of 16 is 4.

$$\sqrt{16} = 4$$

$$\downarrow$$

20 is between 16 and 25. So I think the square root of 20 is between 4 and 5.

$$16 < 20 < 25$$

$$4 < \sqrt{20} < 5$$

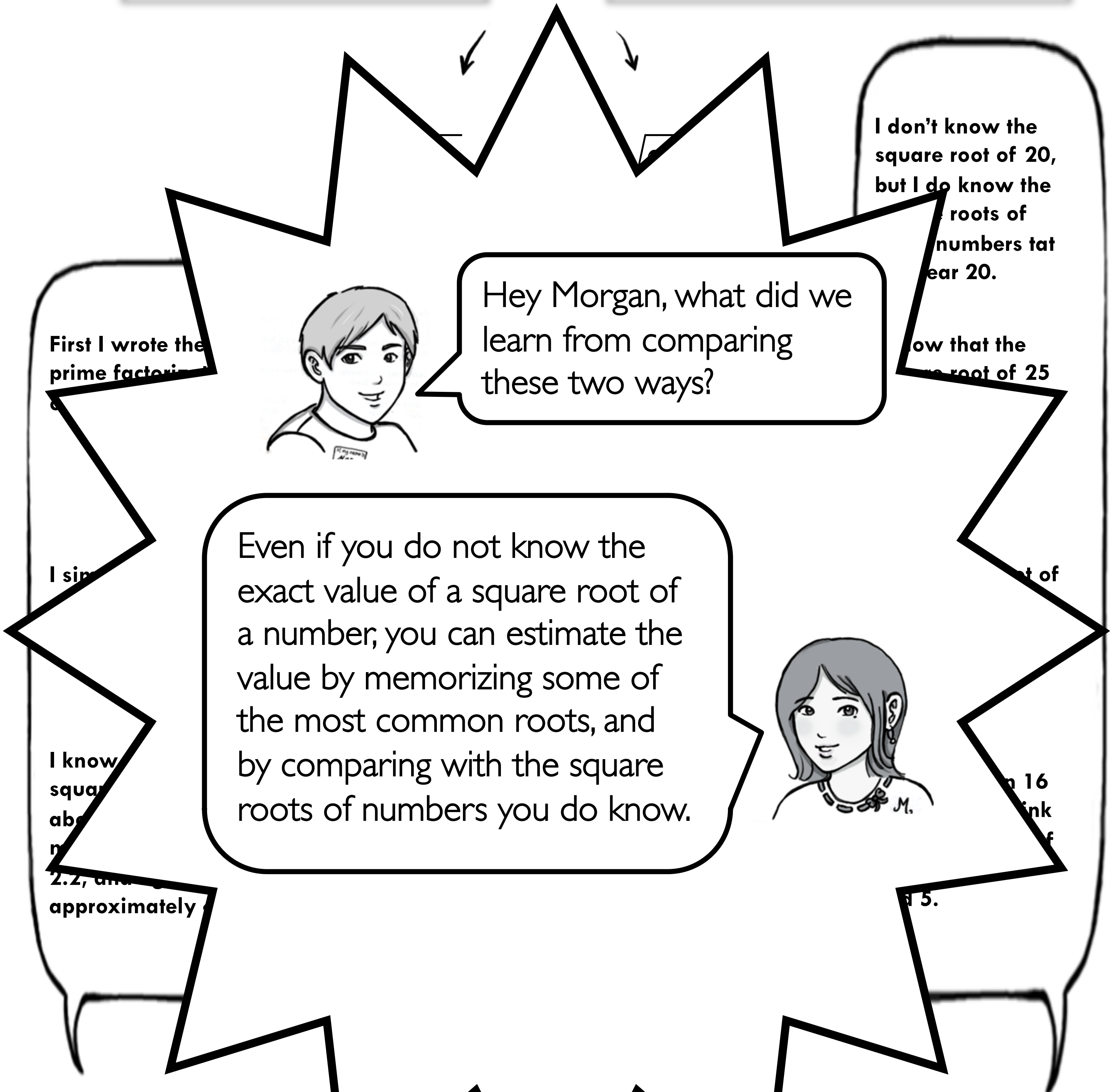
*** How did Alex estimate the value of the square root?**
*** How did Morgan estimate the value of the square root?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Morgan thinks that if *b* is between *a* and *c*, then the square root of *b* is between the square root of *a* and the square root of *c*. Is her idea reasonable? Do you think she is right?**
*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?**

Why does it work?

Alex and Morgan were asked to estimate the value of $\sqrt{20}$

Alex's "use a common root" way

Morgan's "compare to known roots" way



*** How did Alex estimate the v sq**

*** How did Morgan estimate th of the s**

*** What are some similarities a ifferences be lex's and M 's ways?**

*** Morgan thinks that if *b* is bet ween *a* and *c*, the square root of *b* between the square root of *a* and the square root of *c*. Is her idea reasonable? D you think she is right?**

*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?**

Student Worksheet 10.1.4

1a How did Alex estimate the value of the square root?

1b How did Morgan estimate the value of the square root?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 Morgan thinks that if *b* is between *a* and *c*, then the square root of *b* is between the square root of *a* and the square root of *c*. Is her idea reasonable? Do you think she is right?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

How do they differ?

Alex and Morgan were asked to determine whether $\sqrt{n}$ is greater or less than n

Alex's "test an integer" way

To figure out whether the square root of a number is greater or less than the number itself, I decided to test a number 4.

$$\text{I'll try } n = 4.$$

$$\downarrow$$

$$\sqrt{n}$$

$$\downarrow$$

I substituted 4 for *n*.

$$\sqrt{4}$$

$$\downarrow$$

I took the square root of 4. It is 2.

$$2$$

$$\downarrow$$

2 is less than 4.

$$2 < 4$$

$$\downarrow$$

So I think the square root of a number is less than the number.

$$\text{I think } \sqrt{n} < n$$

Morgan's "test a number between 0 and 1" way

To figure out whether the square root of a number is greater or less than the number itself, I decided to test a number 1/4.

$$\text{I'll try } n = \frac{1}{4}.$$

$$\downarrow$$

$$\sqrt{n}$$

$$\downarrow$$

I substituted 1/4 for *n*.

$$\sqrt{\frac{1}{4}}$$

$$\downarrow$$

I took the square root of 1/4. It is 1/2.

$$\frac{1}{2}$$

$$\downarrow$$

1/2 is greater than 1/4.

$$\frac{1}{2} > \frac{1}{4}$$

$$\downarrow$$

So I think the square root of a number is greater than the number.

$$\text{I think } \sqrt{n} > n$$

* How did Alex try to figure out whether the square root of a number is greater or less than the number?
* How did Morgan try to figure out whether the square root of a number is greater or less than the number?
* What are some similarities and differences between Alex's and Morgan's ways?
* Why do you think Alex and Morgan reached different conclusions? Is one right and the other wrong?
* When $n = 0$ or $n = 1$, who is correct, Alex or Morgan?
* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

How do they differ?

Alex and Morgan were asked to determine whether $\sqrt{n}$ is greater or less than n

Alex's "test an integer" way	Morgan's "test a number between 0 and 1" way

To figure out whether the square root of a number is g ter or less than t number itself, decided to tes number. I decid to test 4.

n.

I t

2 is less

square root of number is less the number.

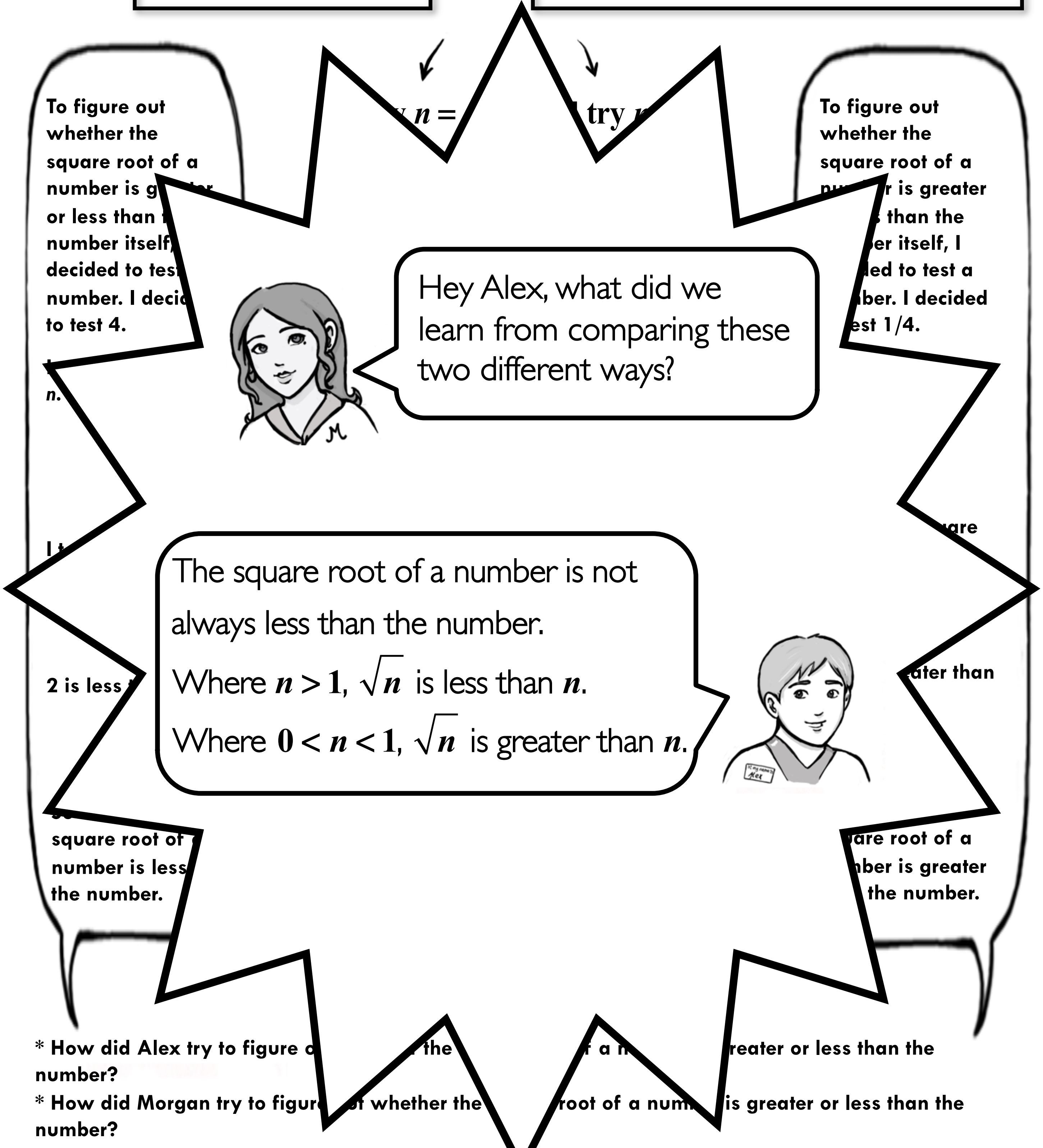
y n = try

To figure out whether the square root of a nu r is greater than the er itself, I ded to test a ber. I decided est 1/4.

are

ater than

are root of a ber is greater the number.



*** How did Alex try to figure o the f a n reater or less than the number?**
*** How did Morgan try to figur t whether the root of a num is greater or less than the number?**
*** What are some similarities and differences betw n Alex's and Morgan's ways?**
*** Why do you think Alex and Morgan reached different conclusions? Is one right and the other wrong?**
*** When n = 0 or n = 1, who is correct, Alex or Morgan?**
*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?**

Student Worksheet 10.1.5

1a How did Alex try to figure out whether the square root of a number is greater or less than the number?

1b How did Morgan try to figure out whether the square root of a number is greater or less than the number?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 Why do you think Alex and Morgan reached different conclusions? Is one right and the other wrong?

4 When $n = 0$ or $n = 1$, who is correct, Alex or Morgan?

5 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

Why does it work?

Alex and Morgan were asked to simplify $\sqrt{18x^2y^5}$

Alex's "find the biggest perfect squares" way

$$\sqrt{18x^2y^5}$$

First I factored the number and variables into the largest perfect squares I could find.

$$\sqrt{2\cdot 3^2\cdot x^2\cdot y^4\cdot y}$$

I split up the radical for each of the perfect squares and the 2y that was left over.

$$\sqrt{3^2}\cdot\sqrt{x^2}\cdot\sqrt{y^4}\cdot\sqrt{2\cdot y}$$

I simplified the square roots. Here is my answer.

$$3xy^2\sqrt{2y}$$

Morgan's "factor completely" way

$$\sqrt{18x^2y^5}$$

First I factored the number and variables into as many factors as possible without using 1.

$$\sqrt{2\cdot 3\cdot 3\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y}$$

I took out the pairs of factors in separate radicals because these are perfect squares.

$$\sqrt{3^2}\cdot\sqrt{x^2}\cdot\sqrt{y^2}\cdot\sqrt{y^2}\cdot\sqrt{2\cdot y}$$

I simplified the radicals with perfect squares and multiplied what was left over.

$$3xyy\sqrt{2y}=3xy^2\sqrt{2y}$$

* Explain Alex's method to a new student in the class.

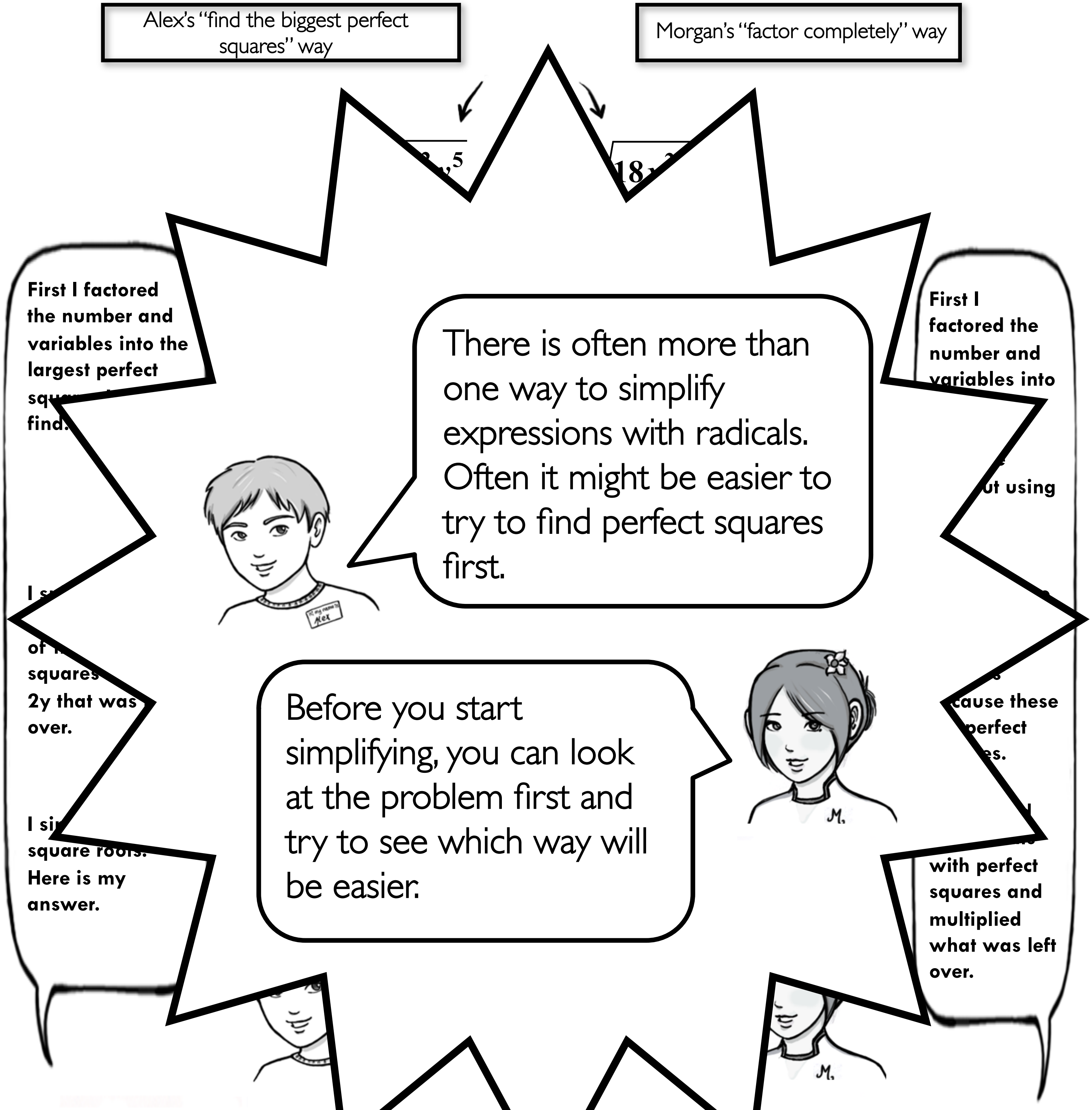
* How did Morgan simplify the expression?

* Describe two differences between Alex's way and Morgan's way.

* If the number inside the radical were 216 instead of 18, how would Morgan simplify the radical? How would Alex simplify it? Can you think of another way to simplify it?

Why does it work?

Alex and Morgan were asked to simplify $\sqrt{18x^2y^5}$



*** Explain Alex's method to a new student in the class.**
*** How did Morgan simplify the expression?**
*** Describe two differences between Alex's way and Morgan's way.**
*** If the number inside the radical were 216 instead of 18, how would Morgan simplify the radical? How would Alex simplify it? Can you think of another way to simplify it?**

Student Worksheet 10.1.6

1 Explain Alex's method to a new student in the class.

2 How did Morgan simplify the expression?

3 Describe two differences between Alex's way and Morgan's way.

4 If the number inside the radical were 216 instead of 18, how would Morgan simplify the radical? How would Alex simplify it? Can you think of another way to simplify it?

Which is better?

Alex and Morgan were asked to simplify $\sqrt{3}\cdot\sqrt{15}$

Alex's "multiply first" way

$$\sqrt{3}\cdot\sqrt{15}$$

$$\downarrow$$

First I multiplied the numbers inside the square root.

$$\sqrt{45}$$

$$\downarrow$$

Then I found the biggest perfect square in 45, which was 9.

$$\sqrt{9\cdot 5}$$

$$\downarrow$$

I used this to simplify the answer.

$$3\sqrt{5}$$

Morgan's "simplify first" way

$$\sqrt{3}\cdot\sqrt{15}$$

$$\downarrow$$

First I split up 15 into 3 times 5, because there were two factors of $\sqrt{3}$.

$$\sqrt{3}\cdot\sqrt{3\cdot 5}$$

$$\downarrow$$

Then I multiplied the two square roots of 3.

$$\left(\sqrt{3}\right)^2\cdot\sqrt{5}$$

$$\downarrow$$

I simplified $\left(\sqrt{3}\right)^2$ as 3, and I got my answer.

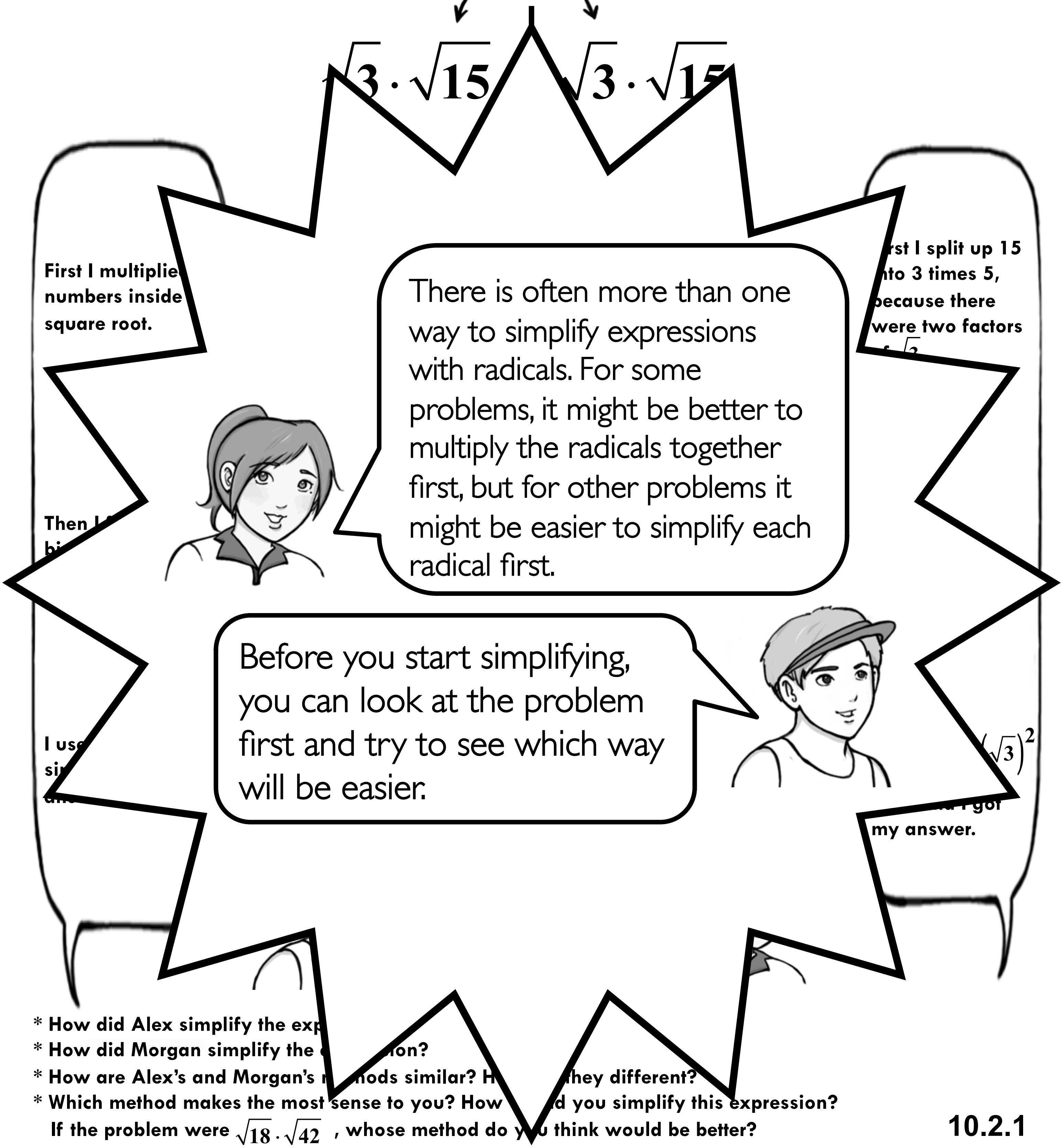
$$3\sqrt{5}$$

* **How did Alex simplify the expression?**
* **How did Morgan simplify the expression?**
* **How are Alex's and Morgan's methods similar? How are they different?**
* **Which method makes the most sense to you? How would you simplify this expression? If the problem were $\sqrt{18}\cdot\sqrt{42}$, whose method do you think would be better?**

Which is better?

Alex and Morgan were asked to simplify $\sqrt{3} \cdot \sqrt{15}$

Alex's "multiply first" way	Morgan's "simplify first" way
$\sqrt{3} \cdot \sqrt{15}$	$\sqrt{3} \cdot \sqrt{15}$
First I multiplie... numbers inside... square root.	**...rst I split up 15 ...nto 3 times 5, ...because there were two factors ...**
Then I ... bi...	
I use... si... an...	**... $\left(\sqrt{3}\right)^2$ ... I got my answer.**



*** How did Alex simplify the exp...**
*** How did Morgan simplify the ...ion?**
*** How are Alex's and Morgan's m...ods similar? H... they different?**
*** Which method makes the most sense to you? How ...d you simplify this expression?**
If the problem were $\sqrt{18} \cdot \sqrt{42}$, whose method do y...u think would be better?

Student Worksheet 10.2.1

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 How are Alex's and Morgan's methods similar? How are they different?

3 Which method makes the most sense to you? How would you simplify this expression? If the problem were $\sqrt{18} \cdot \sqrt{42}$, whose method do you think would be better?

Why does it work?

Alex and Morgan were asked to simplify $\sqrt{9 \cdot 25}$

Alex's "use the product property of radicals" way	Morgan's "multiply terms in the radicand first" way
$\sqrt{9 \cdot 25}$	$\sqrt{9 \cdot 25}$
$\downarrow$	$\downarrow$
$\sqrt{9} \cdot \sqrt{25}$	$\sqrt{225}$
$\downarrow$	$\downarrow$
$3 \cdot 5$	15
$\downarrow$	
15	

I rewrote the radical expression as the product of radical terms.

I took the square root of each term.

I multiplied 3 times 5, and I got 15.

First I multiplied the terms in the radicand.

Then I took the square root of the product. I got 15.

*** How did Alex simplify the expression?**
*** How did Morgan simplify the expression?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Even though Alex and Morgan used different ways, why did they both get the same answer?**

Why does it work?

Alex and Morgan were asked to simplify $\sqrt{9 \cdot 25}$

Alex's "use the product property of radicals" way	Morgan's "multiply terms in the radicand first" way

I rewrote the radical expres as the product radical terms

multiplied the s in the radicand.

I took th root of ea

he square product. I

I multip 5, and I

Hey Morgan, what did we learn from comparing these two different ways?

The square root of a product is equal to the product of the square roots of the terms being multiplied (in other words, $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$, as long as *x* and *y* are non-negative).

*** How did Alex simplify the sion?**
*** How did Morgan simplify expression?**
*** What are some similarities and differences b en Alex's and Morgan's ways?**
*** Even though Alex and Morgan used different ys, why did they both get the same answer?**

Student Worksheet 10.2.2

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 Even though Alex and Morgan used different ways, why did they both get the same answer?

Why does it work?

Alex and Morgan were asked to simplify $\sqrt{64 \div 16}$

Alex's "quotient of radical terms" way		Morgan's "divide terms in the radicand first" way	
	$\sqrt{64 \div 16}$	$\sqrt{64 \div 16}$	
I rewrote the radical expression as the quotient of radical terms.	$\sqrt{64} \div \sqrt{16}$	$\sqrt{4}$	**First I divided the terms in the radicand.**
I took the square root of each term.	$8 \div 4$	2	**Then I took the square root of the quotient. I got 2.**
I did 8 divided by 4, and I got 2.	2		

*** How did Alex simplify the expression?**
*** How did Morgan simplify the expression?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Even though Alex and Morgan used different ways, why did they both get the same answer?**

Why does it work?

Alex and Morgan were asked to simplify $\sqrt{64 \div 16}$

Alex's "quotient of radical terms" way

Morgan's "divide terms in the radicand first" way

I rewrote the radical expres... as the quotient... radical terms

...divided the ...in the radicand.

I took t... root of e...

...e square ...uotient. I

I did 8 ... 4, and I ...

*** How did Alex simplify the ...sion?**
*** How did Morgan simplify ...expression?**
*** What are some similarities and differences b...en Alex's and Mo...an's ways?**
*** Even though Alex and Morgan used different ...ys, why did they both get the same answer?**

Student Worksheet 10.3.1

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 Even though Alex and Morgan used different ways, why did they both get the same answer?

Which is better?

Alex and Morgan were asked to simplify $\sqrt{\frac{15}{18}}$

Alex's "simplify the fraction first" way

First I simplified the fraction inside the square root. 15/18 is equivalent to 5/6.

$$\sqrt{\frac{15}{18}}$$

$$\sqrt{\frac{5}{6}}$$

$$\downarrow$$

Then I split up the square root into the numerator and denominator.

$$\frac{\sqrt{5}}{\sqrt{6}}$$

$$\downarrow$$

Next I multiplied the numerator and denominator by the square root of 6 to rationalize the denominator.

$$\frac{\sqrt{5}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$

$$\downarrow$$

I multiplied to get my answer.

$$\frac{\sqrt{30}}{6}$$

Morgan's "split up the square root first" way

First I split up the square root into a radical in the numerator and the denominator.

$$\sqrt{\frac{15}{18}}$$

$$\frac{\sqrt{15}}{\sqrt{18}}$$

$$\downarrow$$

I knew that you could simplify the square root of 18, so I wrote the prime factors.

$$\frac{\sqrt{15}}{\sqrt{2 \cdot 3^2}}$$

$$\downarrow$$

I factored the 3 out of the radical.

$$\frac{\sqrt{15}}{3\sqrt{2}}$$

$$\downarrow$$

I multiplied the numerator and denominator by the square root of 2 to rationalize the denominator.

$$\frac{\sqrt{15}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\downarrow$$

Then I simplified the denominator to get my answer.

$$\frac{\sqrt{30}}{3 \cdot 2}$$

$$\frac{\sqrt{30}}{6}$$

* How did Alex simplify the expression? How did Morgan simplify the expression?
* What are some similarities and differences between Alex's and Morgan's approaches?
* Can you think of a case where Morgan's method would be better than Alex's?
* What are some advantages of Morgan's method? Of Alex's method?

Which is better?

Alex and Morgan were asked to simplify $\sqrt{\frac{15}{18}}$

Alex's "simplify the fraction first" way

Morgan's "split up the square root first" way

First I simplified the fraction inside the square root. 15/18 is equivalent to 5/6.

Then I split up the square root into the numerator and d

Next I … the num… denomina… the squ… 6 t…

I m… my ans…

$\sqrt{\frac{15}{18}}$

$\sqrt{\frac{15}{18}}$

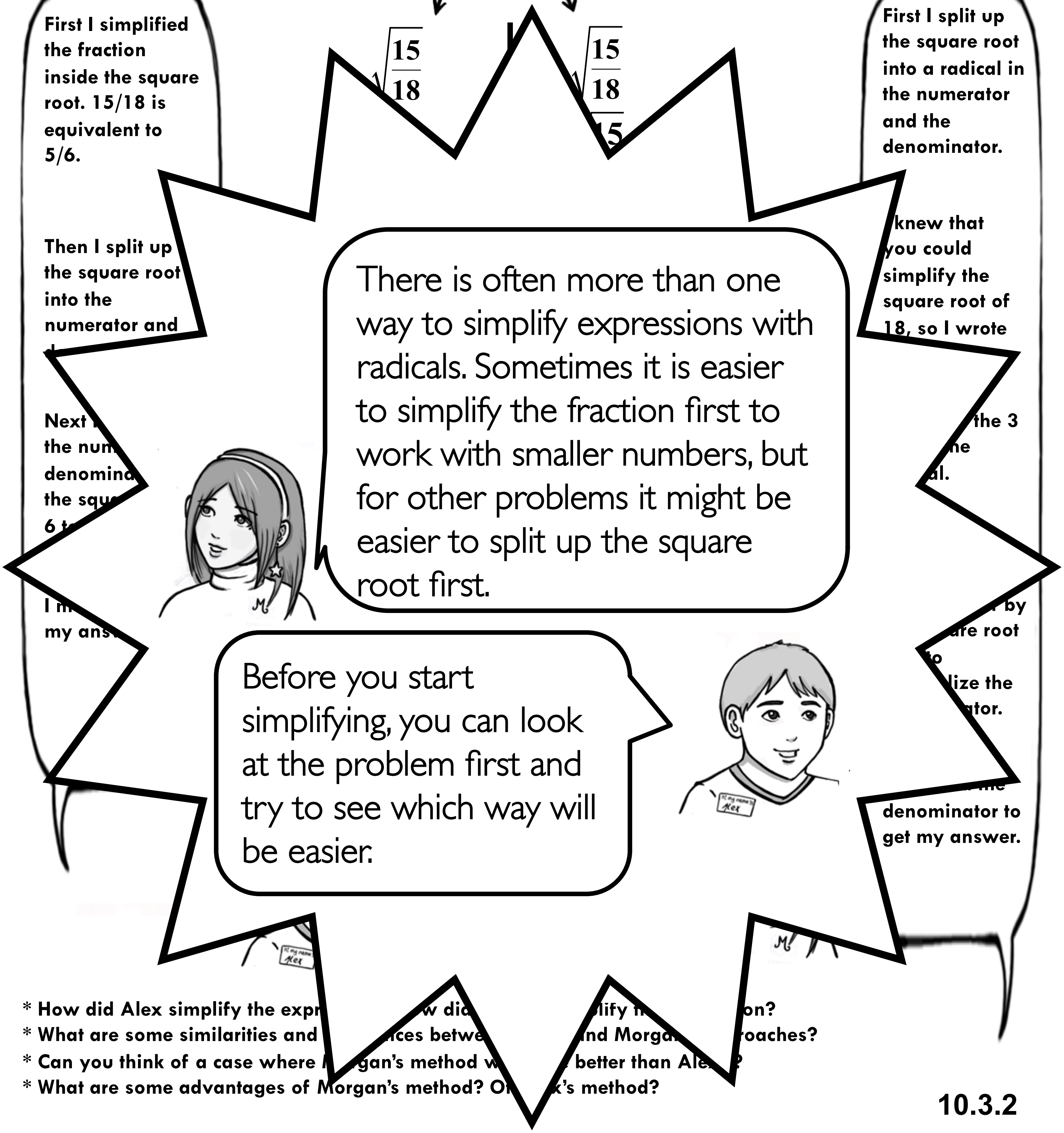
First I split up the square root into a radical in the numerator and the denominator.

…knew that you could simplify the square root of 18, so I wrote

… the 3 … he … al.

… by … are root … to … lize the … tor.

… the denominator to get my answer.



* **How did Alex simplify the expr… w di… lify … on?**
* **What are some similarities and … ces betwe… nd Morga… roaches?**
* **Can you think of a case where M…gan's method w… better than Ale…?**
* **What are some advantages of Morgan's method? O… x's method?**

Student Worksheet 10.3.2

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's approaches?

3 Can you think of a case where Morgan's method would be better than Alex's?

4 What are some advantages of Morgan's method? Of Alex's method?

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{16+4}$

Alex's "take the sum of the square roots" way		Morgan's "take the square root of the sum" way	
	$\sqrt{16+4}$	$\sqrt{16+4}$	
	$\downarrow$	$\downarrow$	
First I separated the expression into two radical terms.	$\sqrt{16}+\sqrt{4}$	$\sqrt{20}$	**First I added 16 plus 4. I got 20.**
	$\downarrow$	$\downarrow$	
Then I took the square root of each term.	$4+2$	$\sqrt{2\cdot 2\cdot 5}$	**Then I wrote the prime factorization of 20.**
	$\downarrow$	$\downarrow$	
I added the terms together, and I got 6.	6	$2\sqrt{5}$	**I simplified the expression.**

*** How did Alex simplify the expression?**
*** How did Morgan simplify the expression?**
*** Whose answer is correct, Alex's or Morgan's? How do you know?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?**

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{16+4}$

Alex's "take the sum of the square roots" way

Morgan's "take the square root of the sum" way

First I separated
expr
ra

Then I too
root o

I added th
together

ded 16 plus

f

If $x \neq 0$ and $y \neq 0$, $\sqrt{x+y} \neq \sqrt{x}+\sqrt{y}$. (In words, the square root of a sum is not equal to the sum of the square roots.)

Don't confuse sums and products – the square root of a product is equal to the product of the square roots of the multipliers (in other words, $\sqrt{xy} = \sqrt{x}\cdot\sqrt{y}$, as long as x and y are non-negative).

*** How did Alex simplify th   n?**
*** How did Morgan simplify   ression?**
*** Whose answer is correct, A s or Morgan's   do you know**
*** What are some similarities and differences b   en Alex's and Mo gan's ways?**
*** Can you state a general rule that describes wh   you have learned from comparing Alex's and Morgan's ways?**

Student Worksheet 10.4.1

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{20}+\sqrt{45}$

Alex's "add the radicands together first" way	Morgan's "simplify each radical term first" way
$\sqrt{20}+\sqrt{45}$	$\sqrt{20}+\sqrt{45}$
$\downarrow$	$\downarrow$
$\sqrt{65}$	$\sqrt{2\cdot 10}+\sqrt{9\cdot 5}$
	$\downarrow$
	$\sqrt{2\cdot 2\cdot 5}+\sqrt{3\cdot 3\cdot 5}$
	$\downarrow$
	$2\sqrt{5}+3\sqrt{5}$
	$\downarrow$
	$5\sqrt{5}$

I added 20 plus 45. Here is my answer.

I started to factor each radicand.

I continued factoring so that I got the prime factorization of each radicand.

I simplified the expression.

I added the terms together. Here is my answer.

*** How did Alex simplify the expression?**
*** How did Morgan simplify the expression?**
*** Whose answer is correct, Alex's or Morgan's? How do you know?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?**

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{20} + \sqrt{45}$

Alex's "add the radicands together first" way	Morgan's "simplify each radical term first" way

$\sqrt{4}$ … $\sqrt{20} +$ …

I added 20 plus
Here is my answ

ed to factor each
and.

oring
he prime
n of each

ms
my

*** How did Alex simplify th n?**
*** How did Morgan simplify ression?**
*** Whose answer is correct, A s or Morgan's do you know**
*** What are some similarities and differences b en Alex's and Morgan's ways?**
*** Can you state a general rule that describes wh you have learned from comparing Alex's and Morgan's ways?**

Student Worksheet 10.4.2

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{2}+\sqrt{2}+\sqrt{2}$

Alex's "add the radicands together first" way	Morgan's "combine like radical terms" way
$\sqrt{2}+\sqrt{2}+\sqrt{2}$	$\sqrt{2}+\sqrt{2}+\sqrt{2}$
$\downarrow$	$\downarrow$
$\sqrt{6}$	$3\sqrt{2}$

I added 2 plus 2 plus 2. Here is my answer.

I combined like terms by adding the coefficients of the radical terms. Here is my answer.

*** How did Alex simplify the expression?**
*** How did Morgan simplify the expression?**
*** Whose answer is correct, Alex's or Morgan's? How do you know?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?**

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{2}+\sqrt{2}+\sqrt{2}$

Alex's "add the radicands together first" way	Morgan's "combine like radical terms" way

I a
2. H
answe

ms
the
s. Here is

If $x \neq 0$ and $y \neq 0$, then
$\sqrt{x+y} \neq \sqrt{x}+\sqrt{y}$.
(In other words, the square root of a sum is not equal to the sum of the square roots.)

When adding or subtracting terms with radicals, the terms with the radical signs operate in ways similar to terms with variables. When simplifying expressions with radicals, combine like radical terms.

*** How did Alex simplify th n?**
*** How did Morgan simplify ression?**
*** Whose answer is correct, A s or Morgan's do you know**
*** What are some similarities and differences b en Alex's and Morgan's ways?**
*** Can you state a general rule that describes wh you have learned from comparing Alex's and Morgan's ways?**

Student Worksheet 10.4.3

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{x+3}+\sqrt{4x+12}$

Alex's "cannot be simplified" way	Morgan's "factor the radicand" way

Alex's "cannot be simplified" way

$$\sqrt{x+3}+\sqrt{4x+12}$$

$$\downarrow$$

Cannot be simplified

These are unlike terms, so I can't combine them.

Morgan's "factor the radicand" way

$$\sqrt{x+3}+\sqrt{4x+12}$$

$$\downarrow$$

$$\sqrt{x+3}+\sqrt{4(x+3)}$$

First I factored out a common factor from the radicand of the second term.

$$\downarrow$$

$$\sqrt{x+3}+\sqrt{4}\cdot\sqrt{(x+3)}$$

Then I rewrote the second term as the product of two radicals.

$$\downarrow$$

$$\sqrt{x+3}+2\sqrt{(x+3)}$$

I took the square root of 4. It is 2.

$$\downarrow$$

$$3\sqrt{(x+3)}$$

I added the like terms. Here is my answer.

*** How did Alex simplify the expression?**
*** How did Morgan simplify the expression?**
*** Whose answer is correct, Alex's or Morgan's? How do you know?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?**

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{x+3}+\sqrt{4x+12}$

Alex's "cannot be simplified" way

Morgan's "factor the radicand" way

$\sqrt{x}$ ... $+12$

These are unli

irst I factored out

factor

nd

ewrote the

as the

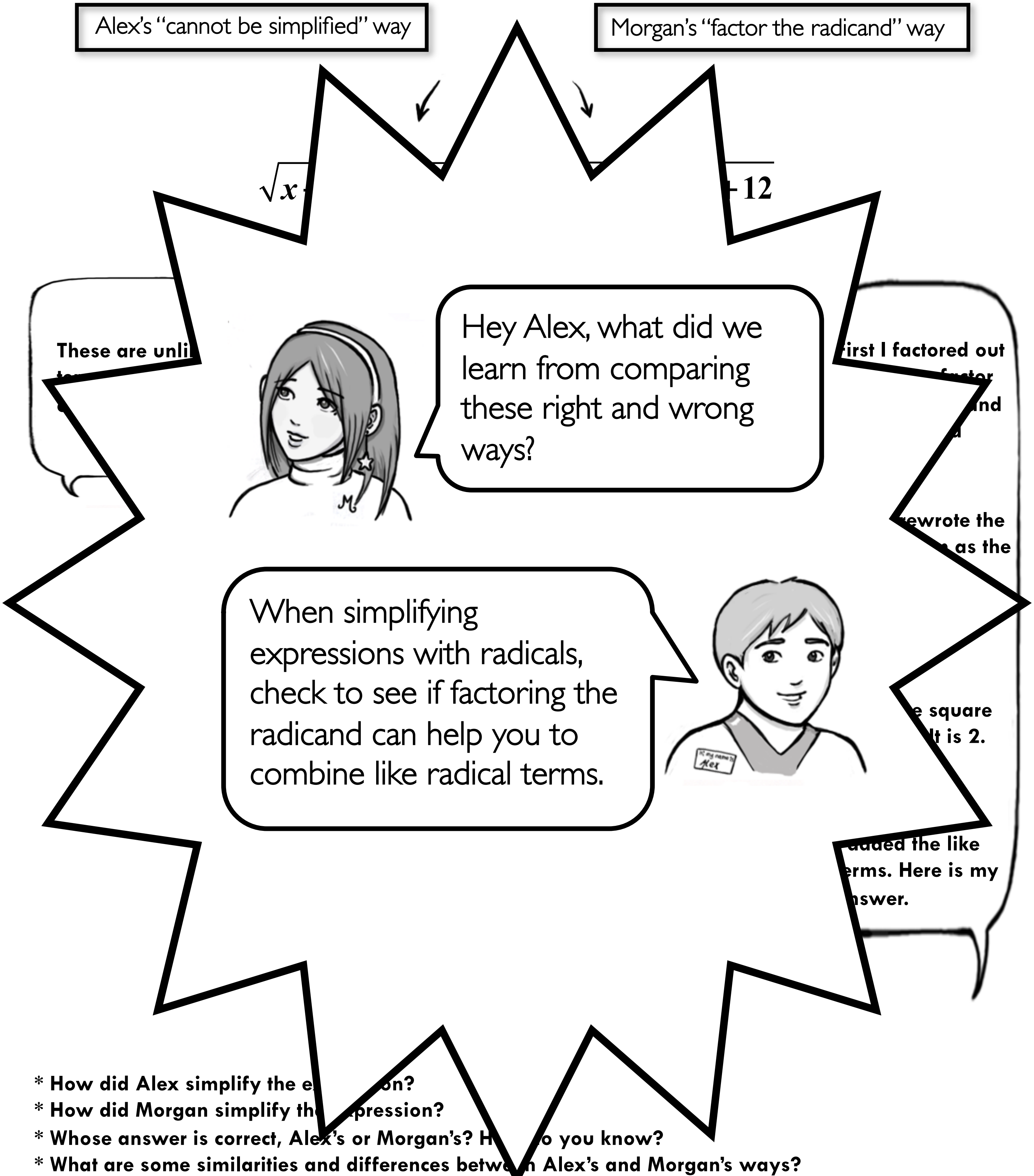
e square

lt is 2.

added the like

erms. Here is my

nswer.



*** How did Alex simplify the e on?**
*** How did Morgan simplify th xpression?**
*** Whose answer is correct, Alex's or Morgan's? H o you know?**
*** What are some similarities and differences betw n Alex's and Morgan's ways?**
*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?**

Student Worksheet 10.4.4

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

What are some similarities and differences between Alex's and Morgan's ways?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{81-49}$

Alex's "take the difference of the square roots" way

$$\sqrt{81-49}$$

$$\downarrow$$

$$\sqrt{81}-\sqrt{49}$$

First I separated the expression into two radical terms.

$$\downarrow$$

$$9-7$$

Then I took the square root of each term.

$$\downarrow$$

$$2$$

I subtracted the terms, and I got 2.

Morgan's "take the square root of the difference" way

$$\sqrt{81-49}$$

$$\downarrow$$

$$\sqrt{32}$$

First I subtracted 81 minus 49.

$$\downarrow$$

$$\sqrt{4\cdot 8}$$

Then I started to factor 32.

$$\downarrow$$

$$\sqrt{2\cdot 2\cdot 2\cdot 2\cdot 2}$$

I continued factoring so that I got the prime factorization of 32.

$$\downarrow$$

$$2\cdot 2\sqrt{2}$$

I simplified the expression.

$$\downarrow$$

$$4\sqrt{2}$$

I further simplified the expression. Here is my answer.

*** How did Alex simplify the expression?**
*** How did Morgan simplify the expression?**
*** Whose answer is correct, Alex's or Morgan's? How do you know?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?**

Which is correct?

Alex and Morgan were asked to simplify $\sqrt{81 - 49}$

Alex's "take the difference of the square roots" way	Morgan's "take the square root of the difference" way

First I separat
expression int
radical terms.

The
root

cted 81

mplified the
Here is my

If $x \neq 0$ and $y \neq 0$, $\sqrt{x - y} \neq \sqrt{x} - \sqrt{y}$. (In words, the square root of a difference is not equal to the difference of the square roots.)

Don't confuse differences and products: the square root of a product *is* equal to the product of the square roots of the numbers being multiplied (in other words, $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$, as long as *x* and *y* are non-negative).

*** How did Alex simplify th on?**
*** How did Morgan simplify pression?**
*** Whose answer is correct, A x's or Morgan's do you know?**
*** What are some similarities and differences be en Alex's and Morgan's ways?**
*** Can you state a general rule that describes wh you have learned from comparing Alex's and Morgan's ways?**

Student Worksheet 10.4.5

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

How do they differ?

Alex was asked to simplify $\sqrt{36+4}$, and Morgan was asked to simplify $\sqrt{36 \cdot 4}$

Alex's "don't 'split up' the radical first" way

$$\sqrt{36+4}$$

First I added the terms in the radicand.

$$\sqrt{40}$$

I started to factor 40.

$$\sqrt{4 \cdot 10}$$

I continued to factor so that I got the prime factorization of 40.

$$\sqrt{2 \cdot 2 \cdot 5 \cdot 2}$$

I simplified the expression.

$$2\sqrt{10}$$

Morgan's "'split up' the radical first" way

$$\sqrt{36 \cdot 4}$$

First I rewrote the expression as the product of two radical terms.

$$\sqrt{36} \cdot \sqrt{4}$$

Then I took the square root of each term.

$$6 \cdot 2$$

Then I multiplied 6 times 2. This is my answer.

$$12$$

*** How did Alex simplify the expression?**

*** How did Morgan simplify the expression?**

*** What are some similarities and differences between Alex's and Morgan's problems?**

*** Could Alex have used Morgan's "split up the radical" way in his second step (in other words, could he have rewritten his expression as $\sqrt{36} + \sqrt{4}$)? Why or why not?**

*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's problems?**

How do they differ?

Alex was asked to simplify $\sqrt{36+4}$, and Morgan was asked to simplify $\sqrt{36 \cdot 4}$

Alex's "don't 'split up' the radical first" way	Morgan's "'split up' the radical first" way
$\sqrt{\ldots+4}$	$\sqrt{36\ldots}$
First I added t... terms in the radicand.	**...rewrote the ...ssion as the ...uct of two radical ...s.**
I start... 40.	**...e square ...term.**
... so th... prime fa... of 40.	**...my**
I s...	

Be careful not to confuse the processes for simplifying radical expressions with sums in the radicand versus products in the radicand.

$\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$ (as long as x and y are non-negative), but $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$ (as long as x and y are not equal to zero).

*** How did Alex simplify the e...**
*** How did Morgan simplify th...ion?**
*** What are some similarities a... ...erences b... ...ex's and M... ...'s problems?**
*** Could Alex have used Morg...s "split up the r... ...way in his sec...d step (in other words, could he have rewritten his expression as $\sqrt{36}+\ldots$ Why or why not?**
*** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's problems?**

Student Worksheet 10.5.1

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's problems?

3 Could Alex have used Morgan's "split up the radical" way in his second step (in other words, could he have rewritten his expression as $\sqrt{36}+\sqrt{4}$)? Why or why not?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's problems?

Which is better?

Alex and Morgan were asked to simplify $\dfrac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}}$

Alex's "rationalize the denominator first" way	Morgan's "factor out a common factor first" way

Alex's way

$$\frac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}}$$

$$\downarrow$$

$$\frac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

First I multiplied the numerator and the denominator by the square root of 2.

$$\downarrow$$

$$\frac{2+10}{2}$$

I simplified the expression.

$$\downarrow$$

$$\frac{12}{2}$$

I added 2 plus 10, and I got 12.

$$\downarrow$$

$$6$$

I simplified the expression.

Morgan's way

$$\frac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}}$$

$$\downarrow$$

$$\frac{\sqrt{2}(1+5)}{\sqrt{2}}$$

First I factored out a common factor from the numerator.

$$\downarrow$$

$$\frac{\sqrt{\cancel{2}}(1+5)}{\sqrt{\cancel{2}}}$$

Then I simplified the expression.

$$\downarrow$$

$$6$$

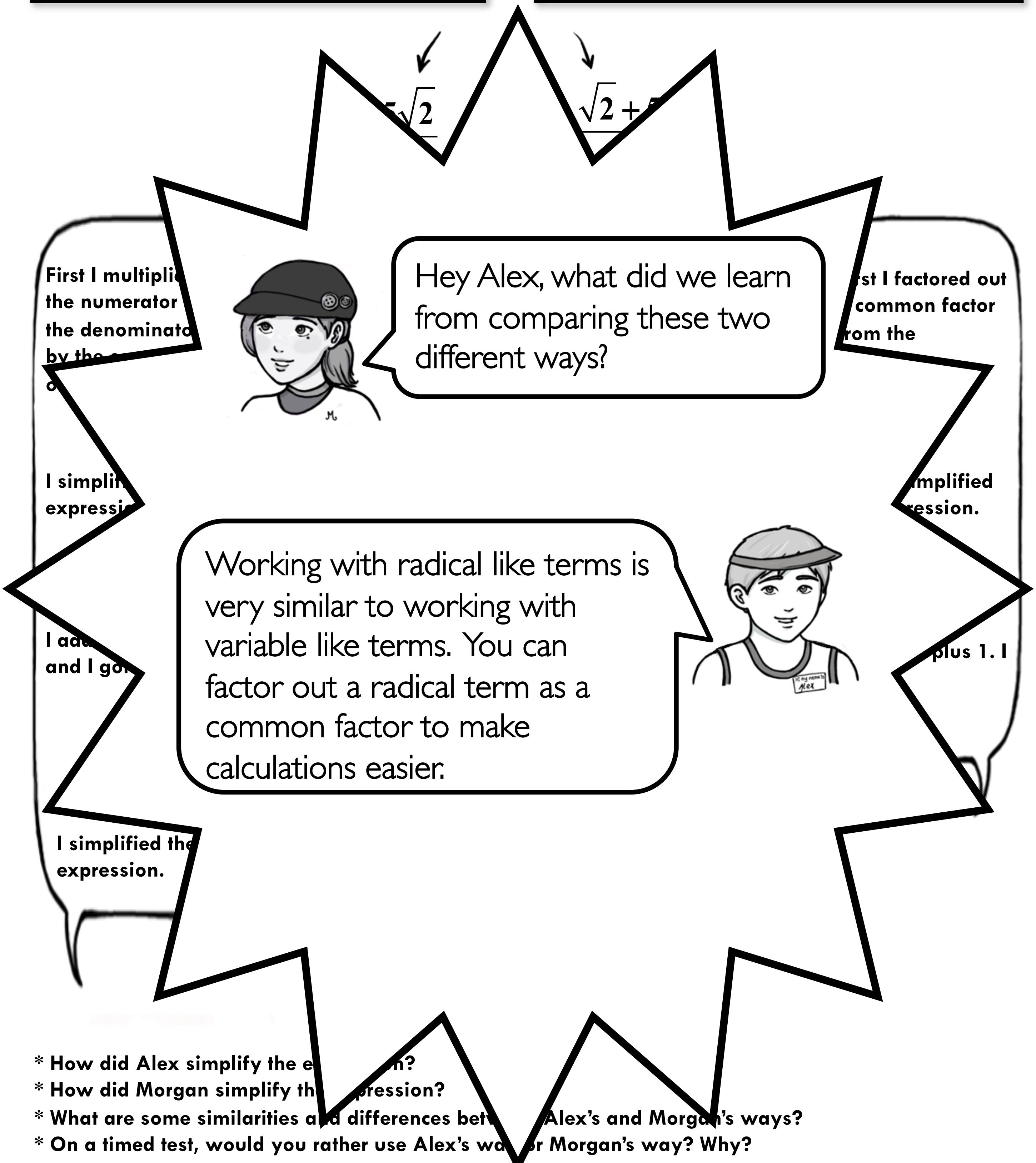
I added 5 plus 1. I got 6.

- **How did Alex simplify the expression?**
- **How did Morgan simplify the expression?**
- **What are some similarities and differences between Alex's and Morgan's ways?**
- **On a timed test, would you rather use Alex's way or Morgan's way? Why?**

Which is better?

Alex and Morgan were asked to simplify $\dfrac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}}$

Alex's "rationalize the denominator first" way	Morgan's "factor out a common factor first" way
First I multipli… the numerator … the denominato… by the …	**…rst I factored out …common factor …rom the**
I simpli… expressi…	**…mplified …ression.**
I ad… and I go…	**…plus 1. I**
I simplified the… expression.	



*** How did Alex simplify the e… …n?**
*** How did Morgan simplify th… …pression?**
*** What are some similarities a… differences bet… Alex's and Morga…'s ways?**
*** On a timed test, would you rather use Alex's wa… …r Morgan's way? Why?**

Student Worksheet 10.6.1

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 On a timed test, would you rather use Alex's way or Morgan's way? Why?

Which is better?

Alex and Morgan were asked to simplify $\sqrt[3]{\left(\frac{8}{125}\right)^2}$

Alex's "exponentiate first" way

$$\sqrt[3]{\left(\frac{8}{125}\right)^2}$$

$$\downarrow$$

$$\sqrt[3]{\frac{8^2}{125^2}}$$

First I applied the exponent to the numerator and the denominator.

$$\downarrow$$

$$\sqrt[3]{\frac{64}{15{,}625}}$$

Then I squared each term.

$$\downarrow$$

$$\frac{\sqrt[3]{64}}{\sqrt[3]{15{,}625}}$$

I rewrote the expression as the cube root of the numerator and the cube root of the denominator.

$$\downarrow$$

$$\frac{4}{25}$$

I took the cube root of the numerator and of the denominator. Here is my answer.

Morgan's "rewrite the expression using perfect squares first" way

$$\sqrt[3]{\left(\frac{8}{125}\right)^2}$$

$$\downarrow$$

$$\sqrt[3]{\frac{(2^3)^2}{(5^3)^2}}$$

First I rewrote 8 and 125 as terms with exponents.

$$\downarrow$$

$$\sqrt[3]{\frac{2^6}{5^6}}$$

I applied the exponents.

$$\downarrow$$

$$\left(\frac{2^6}{5^6}\right)^{\frac{1}{3}}$$

I rewrote the cube root as the exponent 1/3.

$$\downarrow$$

$$\frac{2^2}{5^2}$$

I applied the exponent.

$$\downarrow$$

$$\frac{4}{25}$$

I simplified the numerator and the denominator. Here is my answer.

- **How did Alex simplify the expression?**
- **How did Morgan simplify the expression?**
- **What are some similarities and differences between Alex's and Morgan's ways?**
- **Can you think of any other ways to simplify the expression?**
- **On a timed test, would you rather use Alex's way or Morgan's way for this problem? Why?**

Which is better?

Alex and Morgan were asked to simplify $\sqrt[3]{\left(\frac{8}{125}\right)^2}$

Alex's "exponentiate first" way

Morgan's "rewrite the expression using perfect squares first" way

First I applied t
exponent to the
numerat

Then I s
term.

I r
express
root of th
and the
the d

I
the numera
the denominato
is my answer.

ote 8 and
ms with

ents.

root
3.

e numerator
inator.
Here is my wer.

Hey Morgan, what did we learn from comparing these two different ways?

Finding the prime factorization of terms in an expression first can sometimes help us to simplify expressions more easily, as in this case where leaving terms in factored form made the calculations easier.

*** How did Alex simplify the sion?**
*** How did Morgan simplify expression?**
*** What are some similarities and differences b en Alex's and Morgan's ways?**
*** Can you think of any other ways to simplify t expression?**
*** On a timed test, would you rather use Alex's way or Morgan's way for this problem? Why?**

Student Worksheet 10.6.2

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 Can you think of any other ways to simplify the expression?

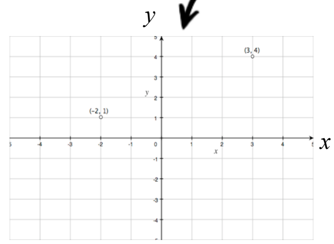
4 On a timed test, would you rather use Alex's way or Morgan's way for this problem? Why?

Why does it work?

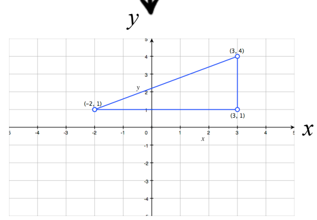
Alex and Morgan were asked to find the distance between (-2,1) and (3,4)

Alex's "Pythagorean theorem" way

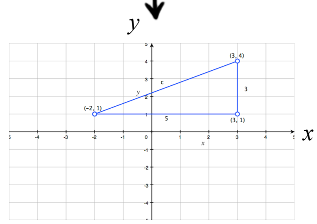
First I plotted the two points on the coordinate plane.



I drew a right triangle.



I subtracted 4 – 1 = 3 and 3 – (-2) = 5 to get the lengths of the two legs.



I plugged the lengths of the legs into the formula for the Pythagorean theorem and solved for c.

$$a^2 + b^2 = c^2$$
$$5^2 + 3^2 = c^2$$
$$25 + 9 = c^2$$
$$34 = c^2$$
$$\sqrt{34} = \sqrt{c^2}$$
$$c = \sqrt{34} \approx 5.83$$

Morgan's "use the distance formula" way

First I wrote the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

I assigned (x_1, y_1) to be (-2,1) and (x_2, y_2) to be (3,4).

$$(x_1, y_1) = (-2, 1)$$
$$(x_2, y_2) = (3, 4)$$

I substituted my ordered pairs into the distance formula.

$$d = \sqrt{[3 - (-2)]^2 + (4 - 1)^2}$$

I performed the operations inside the parentheses, then squared the results.

I added the numbers inside the radical sign together, and I got the square root of 34 as my answer, which is around 5.83.

$$d = \sqrt{(5)^2 + (3)^2}$$
$$d = \sqrt{25 + 9}$$
$$d = \sqrt{34} \approx 5.83$$

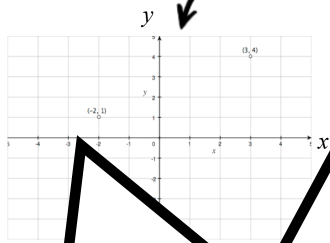
* How did Alex find the distance between the two points?
* How did Morgan find the distance between the two points?
* What are some similarities and differences between Alex's and Morgan's ways?
* Why did Alex subtract 4 – 1 and 3 – (-2) in step #3?
* Why did Morgan choose (-2,1) to be (x_1, y_1) and (3,4) to be (x_2, y_2)?
* In looking at the similarities and differences between Alex's and Morgan's ways, can you explain why the distance formula works?

Why does it work?

Alex and Morgan were asked to find the distance between (-2,1) and (3,4)

Alex's "Pythagorean theorem" way

Morgan's "use the distance formula" way



$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

First I plotted the two points on the coordinate plane.

I drew a right triangle.

I s…
4 – …
3 – (-2…
get the l…
of the two…

legs …
formula …
Pythagor…
theore…
solve…

First I wrote the distance formula.

…igned (x_1, y_1)
…be (-2,1) and
(x_2, y_2) to be (3,4).

…into
…e

…the
…le

…the
…side
…ign
…got
…f
…as my answer,
which is around
5.83.

Hey Morgan, what did comparing these two examples help us to see?

These examples help us see why the distance formula works. It works because it is derived from the Pythagorean theorem.

* How did Alex find the distan…
* How did Morgan find the dis…
* What are some similarities a… …ences be… …and M… …ays?
* Why did Alex subtract 4 – 1 a… – (-2) in step…
* Why did Morgan choose (-2,1) to be (x_1, y_1) and (… …) be (x_2, y_2)?
* In looking at the similarities and differences betwe… Alex's and Morgan's ways, can you explain why the distance formula works?

Student Worksheet 10.7.1

1a How did Alex find the distance between the two points?

1b How did Morgan find the distance between the two points?

2 What are some similarities and differences between Alex's and Morgan's *ways*?

3 Why did Alex subtract 4 – 1 and 3 – (-2) in step #3?

4 Why did Morgan choose (-2,1) to be (x_1, y_1) and (3,4) to be (x_2, y_2)?

5 In looking at the similarities and differences between Alex's and Morgan's ways, can you explain why does the distance formula works?