

Which is better?

Alex and Morgan were asked to simplify  $\left(\frac{5}{12}\right)\left(\frac{4}{25}\right)$

Alex's "multiply numerators and denominators first" way

First I multiplied the numerator of the first fraction times the numerator of the second fraction, and the denominator of the first fraction times the denominator of the second fraction.

Then I simplified the resulting fraction by dividing the numerator and denominator by 10.

Then I simplified the last fraction by dividing the numerator and denominator by 2. I got  $1/15$ , which can't be simplified any more.

$$\left(\frac{5}{12}\right)\left(\frac{4}{25}\right)$$

$$\downarrow$$
$$\frac{5 \cdot 4}{12 \cdot 25} = \frac{20}{300}$$

$$\downarrow$$

$$\frac{2}{30}$$

$$\downarrow$$

$$\frac{1}{15}$$

Morgan's "eliminate common factors first" way

First I eliminated common factors in the numerators and denominators. I eliminated a common factor of 5 from 5 and 25, and a common factor of 4 from 4 and 12. This left me with the problem  $1/3$  times  $1/5$ .

Then I multiplied the remaining factors in the numerator and the denominator. I got  $1/15$ .

$$\left(\frac{5}{12}\right)\left(\frac{4}{25}\right)$$

$$\downarrow$$
$$\begin{array}{cc} 1 & 1 \\ \left(\frac{\cancel{5}}{1\cancel{2}}\right) & \left(\frac{\cancel{4}}{2\cancel{5}}\right) \\ 3 & 5 \end{array}$$

$$\downarrow$$

$$\left(\frac{1}{3}\right)\left(\frac{1}{5}\right)$$

$$\downarrow$$

$$\frac{1}{15}$$



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Even though Alex and Morgan did different first steps, why did they both get the same answer?
- \* Which way is easier, Alex's way or Morgan's way?

Which is better?

Alex and Morgan were asked to simplify  $\left(\frac{5}{12}\right)\left(\frac{4}{25}\right)$

Alex's "multiply numerators and denominators first" way

Morgan's "eliminate common factors first" way

First I multiplied the numerator of the first fraction times the numerator of the second fraction and the denominator of the first fraction times the denominator of the second fraction.

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divi  
and a  
10.

dene  
got 1/  
be r



When multiplying fractions, eliminating common factors in the numerators and denominators can make the problem easier.

First I eliminated common factors in the numerators and denominators.

or of 5  
25, and a  
ctor of 4  
12. This



Before you start solving a problem, you can look at the problem first and try to see which way will be easier.

- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Even though Alex and Morgan did different steps, why did they both get the same answer?
- \* Which way is easier, Alex's way or Morgan's way?

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's *ways*?

3 Even though Alex and Morgan did different first steps, why did they both get the same answer?

4 Which way is easier, Alex's way or Morgan's way?

How do they differ?

Alex was asked to simplify  $6 \div 2$ , and Morgan was asked to simplify  $6 \cdot \frac{1}{2}$

Alex's "division" way

I did 6 divided by 2.

It is 3.

✓  
 $6 \div 2$



3



Morgan's "multiplication by a reciprocal" way

I did 6 times  $\frac{1}{2}$ .

I rewrote 6 as an improper fraction, which is  $\frac{6}{1}$ .

I multiplied the numerator times the numerator and the denominator times the denominator.

I simplified the answer. It is 3.

↓  
 $6 \cdot \frac{1}{2}$



$\frac{6}{1} \cdot \frac{1}{2}$



$\frac{6}{2}$



3



- \* How did Alex simplify his expression?
- \* How did Morgan simplify her expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* What is the relationship between 2 and  $\frac{1}{2}$ ?
- \* What can you conclude about how multiplication and division are related from this problem?

How do they differ?

Alex was asked to simplify  $6 \div 2$ , and Morgan was asked to simplify  $6 \cdot \frac{1}{2}$

Alex's "division" way

Morgan's "multiplication by a reciprocal" way

I did 6 divided by 2.

I did 6 times  $\frac{1}{2}$ .

It is 3.

as an fraction,  $\frac{6}{2}$ .

Hey Alex, what did comparing these two examples help us to see?

Dividing by a number and multiplying by its reciprocal are equivalent.



- \* How did Morgan's way differ from Alex's way?
- \* What are some similarities between the two ways?
- \* What is the relationship between division and multiplication by a reciprocal?
- \* What can you conclude about how division and multiplication by a reciprocal are related from this problem?

1a How did Alex simplify his expression?

1b How did Morgan simplify her expression?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 What is the relationship between 2 and  $\frac{1}{2}$ ?

4 What can you conclude about how multiplication and division are related from this problem?

How do they differ?

Alex was asked to simplify  $5 \cdot 2$ , and Morgan was asked to simplify  $5 \div \frac{1}{2}$

Alex's "multiply" way

I multiplied 5 times 2.

It is 10.



$$5 \cdot 2$$



$$10$$

Morgan's "division by a reciprocal" way

I did 5 divided by  $\frac{1}{2}$ .

I wrote the division as multiplication by the reciprocal, because dividing is the same as multiplying by the reciprocal.

I rewrote 5 as an improper fraction.

I multiplied the numerator times the numerator and the denominator times the denominator.

I simplified my answer.  
It is 10.

$$5 \div \frac{1}{2}$$



$$5 \cdot \frac{2}{1}$$



$$\frac{5}{1} \cdot \frac{2}{1}$$



$$\frac{10}{1}$$



$$10$$



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's problems?
- \* What is the relationship between 2 and  $\frac{1}{2}$ ?
- \* From this example, what can you conclude about how the operations of multiplication and division are related?

How do they differ?

Alex was asked to simplify  $5 \cdot 2$ , and Morgan was asked to simplify  $5 \div \frac{1}{2}$

Alex's "multiply" way

Morgan's "division by a reciprocal" way

I multiplied 5 times 2.

It is 10.



Hey Morgan, what did comparing these two examples help us to see?

I did 5 divided by  $\frac{1}{2}$ .

I wrote the division as  $5 \cdot \frac{2}{1}$  because the same as  $5 \cdot 2$

erator and the

Multiplying by a number and dividing by its reciprocal are equivalent.



- \* How do you simplify  $5 \cdot 2$  and  $5 \div \frac{1}{2}$ ?
- \* How did Morgan simplify  $5 \div \frac{1}{2}$ ?
- \* What are some similarities between  $5 \cdot 2$  and  $5 \div \frac{1}{2}$ ?
- \* What is the relationship between multiplication and division?
- \* From this example, what do you conclude about the relationship between multiplication and division?



1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's *problems*?

3 What is the relationship between 2 and  $\frac{1}{2}$ ?

4 From this example, what can you conclude about how the operations of multiplication and division are related?

How do they differ?

Alex and Morgan were asked to simplify  $\frac{3}{5} \div \frac{6}{7}$

Alex's "divide" way

Morgan's "multiply by the reciprocal" way

↙

$$\frac{3}{5} \div \frac{6}{7}$$

↓

$$\frac{3}{\frac{5}{\frac{6}{7}}}$$

↓

$$\frac{3}{\frac{5}{\frac{6}{7}}} \cdot \frac{7}{6}$$

↓

$$\frac{21}{30} \downarrow \frac{7}{10}$$

First I rewrote the division expression.

Then I multiplied the numerator and the denominator by the reciprocal of the denominator.

I simplified the expression.



↘

$$\frac{3}{5} \div \frac{6}{7}$$

↓

$$\frac{3}{5} \cdot \frac{7}{6}$$

↓

$$\frac{21}{30}$$

↓

$$\frac{7}{10}$$

First I rewrote the expression as multiplication by the reciprocal.

I simplified the expression.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?

How do they differ?

Alex and Morgan were asked to simplify  $\frac{3}{5} \div \frac{6}{7}$

Alex's "divide" way

Morgan's "multiply by the reciprocal" way

$$\frac{3}{5} \div \frac{6}{7}$$

$$\frac{3}{5} \div \frac{6}{7}$$

First I rewrote the division expression.



Hey Alex, what did we learn from comparing these two ways?

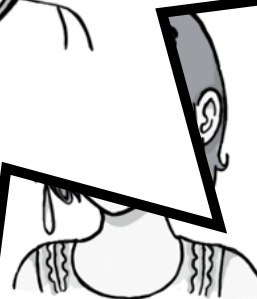
rewrote the division as

Then I multiplied the numerator by the denominator and simplified the expression.

Dividing is equivalent to multiplying by the reciprocal.



I simplified the expression.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?

**1a** How did Alex simplify the expression?

**1b** How did Morgan simplify the expression?

**2** What are some similarities and differences between Alex's and Morgan's ways?

**3** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?

Which is correct?

Alex and Morgan were asked to simplify  $\frac{5+3}{5}$

Alex's "cancel terms" way

$$\begin{array}{c} \swarrow \\ \frac{5+3}{5} \\ \downarrow \\ \frac{\cancel{5}+3}{\cancel{5}} \\ \downarrow \\ 3 \end{array}$$

First I canceled out the 5's, because there is one in the numerator and one in the denominator.

I got 3.



Morgan's "add terms" way

$$\begin{array}{c} \swarrow \\ \frac{5+3}{5} \\ \downarrow \\ \frac{8}{5} \end{array}$$

I added the terms in the numerator. This is my answer.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* Whose answer is correct, Alex's or Morgan's? How do you know?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

Which is correct?

Alex and Morgan were asked to simplify  $\frac{5+3}{5}$

Alex's "cancel terms" way

Morgan's "add terms" way

$$\frac{5+3}{5}$$

$$\frac{5+3}{5}$$

First I canceled out the 5's, because there is one in the numerator and one in the denominator.

I got 3.



Hey Morgan, what did we learn from comparing these right and wrong ways?

I added the terms in the numerator. This is my answer.

You can "cancel" common factors in the numerator and denominator of a fraction, but you cannot "cancel" terms that are being added or subtracted in the numerator or denominator of a fraction in the same way.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* Whose answer is correct, Alex's or Morgan's? How do you know?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

Why does it work?

Alex and Morgan were asked to find the value of the expression  $5 + 3 + 7$

Alex's way

$$5 + 3 + 7$$



$$(5 + 3) + 7$$



$$8 + 7$$



$$15$$

I can't add three numbers together at the same time; I have to pick two to start with. So I added the first two numbers together.

5 plus 3 is 8, and then I added 8 plus 7.

My answer is 15.



Morgan's way

$$5 + 3 + 7$$



$$5 + (3 + 7)$$



$$5 + 10$$



$$15$$

I can't add three numbers together at the same time; I have to pick two to start with. So I added the last two numbers together.

3 plus 7 is 10, and then I added 5 plus 10.

My answer is 15.



- \* How did Alex find the value of the expression? How did Morgan find the value of the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this type of expression?



Why does it work?

Alex and Morgan were asked to find the value of the expression  $5 + 3 + 7$

Alex's way

Morgan's way

I can't add three numbers together at the same time; I have to pick two to start. I added 5 and 3.

5 plus 3 is 8, then I added 7.

My answer



Hey Alex, what did comparing these two examples help us to see?

I can't add three numbers together at the same time; I have to pick two to start. I added 5 and 3.

5 plus 3 is 8, then I added 7.

My answer is 15.



Addition is associative—terms that are being added together can be grouped and added in any order with the same result.

- \* How did Alex find the value of the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways of finding the value of the expression?
- \* Can you state a general rule that describes what you learned from comparing Alex's and Morgan's ways of simplifying this type of expression?

1a

How did Alex find the value of the expression?

1b

How did Morgan find the value of the expression?

2

What are some similarities and differences between Alex's and Morgan's *ways*?

3

Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this type of expression?

Why does it work?

Alex and Morgan were asked to simplify the expression  $7 - 5 - 1$

Alex's way

Morgan's way

$$7 - 5 - 1$$

$$7 - 5 - 1$$

First I rewrote the subtraction as addition of the opposite.

I decided to add together the first two terms first. I added 7 plus -5.

I got 2. Then I added 2 plus -1.

I got 1.

$$7 + (-5) + (-1)$$

$$[7 + (-5)] + (-1)$$

$$[2] + (-1)$$

$$1$$

$$7 + (-5) + (-1)$$

$$7 + [(-5) + (-1)]$$

$$7 + (-6)$$

$$1$$

First I rewrote the subtraction as addition of the opposite.

I decided to add together the second two numbers first. I added -5 plus -1.

I got -6. Then I added 7 plus -6.

I got 1.



- \* How did Alex simplify the expression? How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Would Alex's or Morgan's ways have worked if they had not rewritten subtraction as addition of the opposite?
- \* In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

Why does it work?

Alex and Morgan were asked to simplify the expression  $7 - 5 - 1$

Alex's way

Morgan's way

First I rewrote the subtraction as addition of the opposite.

I decided to add the terms first. plus -5.

2 plus

I got 1.



Hey Morgan, what did comparing these two examples help us to see?

First I rewrote the subtraction as addition of the

add the second numbers first. I plus -1.



To use the associative property, subtraction in an expression must be rewritten as addition of the opposite. Once you rewrite subtraction as addition of the opposite, you can group the terms together in any order and evaluate.

- \* How did Alex simplify the expression? How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Would Alex's or Morgan's ways have worked if the subtraction was not rewritten subtraction as addition of the opposite?
- \* In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's *ways*?

3 Would Alex's or Morgan's ways have worked if they had not rewritten subtraction as addition of the opposite?

4 In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

Which is correct?

Alex and Morgan were asked to simplify  $5 - 7 - 3$

Alex's "subtract 3 from 7 first" way

↙

$$5 - 7 - 3$$

↓

$$5 - 4$$

↓

$$1$$

First I subtracted 7 minus 3.

Then I subtracted 5 minus 4.



Morgan's "rewrite the expression using addition first" way

↘

$$5 - 7 - 3$$

↓

$$5 + (-7) + (-3)$$

↓

$$5 + (-10)$$

↓

$$-5$$

First I rewrote the expression using addition.

Then I added the negative terms together. I added -7 plus -3, and I got -10.

Last, I added 5 plus -10, and I got -5.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Whose answer is correct, and how do you know?
- \* For the person who got the wrong answer, why did he or she get the wrong answer?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?

Which is correct?

Alex and Morgan were asked to simplify  $5 - 7 - 3$

Alex's "subtract 3 from 7 first" way

Morgan's "rewrite the expression using addition first" way

$$7 - 3$$

$$5 - 7$$

First I subtracted  
minus 3.



Hey Alex, what did we  
learn from comparing  
those two examples?

First I rewrote the  
expression using  
addition.

Then I subtracted  
minus 4.

When subtracting, you must  
subtract from left to right. If you  
subtract in a different order, you  
might get a different answer. In  
this way, subtracting is different  
from adding (even adding  
negative numbers), because for  
adding you can apply the  
associative property and perform  
the addition in any order you  
choose and still get the same  
answer.



the  
terms  
I added  
and I

and I got

- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Whose answer is correct, and how do you know?
- \* For the person who got the wrong answer, why did she get the wrong answer?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's *ways*?

3 Whose answer is correct, and how do you know?

4 For the person who got the wrong answer, why did he or she get the wrong answer?

5 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?



*Why does it work?*

**Alex and Morgan were asked to find the value of the expression  $3 \cdot 4 \cdot 2$**

Alex's way

Morgan's way

I can't multiply three numbers at the same time; I have to pick two to start with. So I multiplied the first two numbers together.

3 times 4 is 12, and then I multiplied 12 by 2.

My answer is 24.



$$3 \cdot 4 \cdot 2$$



$$3 \cdot 4 \cdot 2$$



$$12 \cdot 2$$



$$24$$

$$3 \cdot 4 \cdot 2$$



$$3 \cdot 4 \cdot 2$$



$$3 \cdot 8$$



$$24$$

I can't multiply three numbers at the same time; I have to pick two to start with. So I multiplied the last two numbers together.

4 times 2 is 8, and then I multiplied 3 by 8.

My answer is 24.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this type of expression?

Why does it work?

Alex and Morgan were asked to find the value of the expression  $3 \cdot 4 \cdot 2$

Alex's way

Morgan's way

I can't multiply three numbers at the same time; I have to start with multiplying two numbers together.



Hey Morgan, what did comparing these two examples help us to see?

I can't multiply three numbers at the same time; I have to start with multiplying two numbers together.

Multiplication is associative: you can multiply the terms in any order you choose and still get the same answer.



- \* How did Alex solve the problem?
- \* How did Morgan solve the problem?
- \* What are some similarities between Alex's and Morgan's ways?
- \* Can you state a general rule about multiplying three numbers that you learned from comparing Alex's and Morgan's ways of solving this type of problem?

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's *ways*?

3 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this type of expression?

Which is correct?

**Alex and Morgan were asked to simplify  $8 \div 4 \div 2$**

Alex's "divide from left to right" way

Morgan's "'group' the second two terms first" way

I decided to simplify this expression from left to right, by grouping the first two terms first.

So I did 8 divided by 4, and I got 2.

Then I did 2 divided by 2, and I got 1.

✓  
 $8 \div 4 \div 2$



$$(8 \div 4) \div 2$$



$$2 \div 2$$



$$1$$

↓  
 $8 \div 4 \div 2$



$$8 \div (4 \div 2)$$



$$8 \div 2$$



$$4$$

I decided to simplify this expression by grouping the second two terms first.

So I did 4 divided by 2, and I got 2.

Then I divided 8 by 2, and I got 4.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* Which answer is correct, Alex's or Morgan's? How do you know?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

Which is correct?

**Alex and Morgan were asked to simplify  $8 \div 4 \div 2$**

Alex's "divide from left to right" way

Morgan's "'group' the second two terms first" way

I decided to simplify this expression from left to right, by grouping the first two terms first

So I did  $8 \div 4$ , and I got

$$8 \div 4 \div 2$$

$$8 \div 4 \div 2$$

I decided to simplify this expression by grouping the second two terms first

led by

Hey Alex, what did we learn from comparing those two examples?

When simplifying expressions with division, you must perform the division from left to right. If you divide in a different order, you might get a different answer. In this way, dividing is different from multiplying, because for multiplying, you can multiply the terms in any order and still get the same result.



- \* Alex's way
- \* How did Morgan's way differ from Alex's?
- \* Which answer is correct?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* In thinking about similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Which answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

Which is correct?

**Alex and Morgan were asked to simplify  $3 - 5$**

Alex's way

Morgan's way

I rewrote the subtraction (minus 5) as addition of the opposite or additive inverse (plus the opposite of 5).

Then I rearranged the terms. It's okay for me to do that because of the commutative property.

I added -5 plus 3 and I got -2.

$$3 - 5$$

↓

$$3 + -5$$

↓

$$-5 + 3$$

↓

$$-2$$

$$3 - 5$$

↓

$$5 - 3$$

↓

$$2$$

First I rearranged the terms. It's okay for me to do that because of the commutative property.

I got 2.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* Which answer is correct, Alex's or Morgan's? How do you know?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this type of expression?

Which is correct?

## Alex and Morgan were asked to simplify $3 - 5$

Alex's way

Morgan's way

I rewrote the  
(minus 5) as  
the opposite of  
inverse (plus the  
opposite of 5).

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term  
do the  
commu



Hey Morgan, what did we  
learn from comparing  
these right and wrong  
ways?

changed the  
okay for me  
because of  
mutative

Subtraction is not commutative. If  
you want to rearrange the order  
of terms (by using the  
commutative property) to  
simplify an expression with  
subtraction, you'll need to rewrite  
the subtraction as addition of the  
opposite. Subtraction is not  
commutative, but addition  
(including addition of negative  
numbers) is.



- \* How did Alex simplify
- \* How did Morgan simplify
- \* Which answer is correct
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this type of expression?



1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

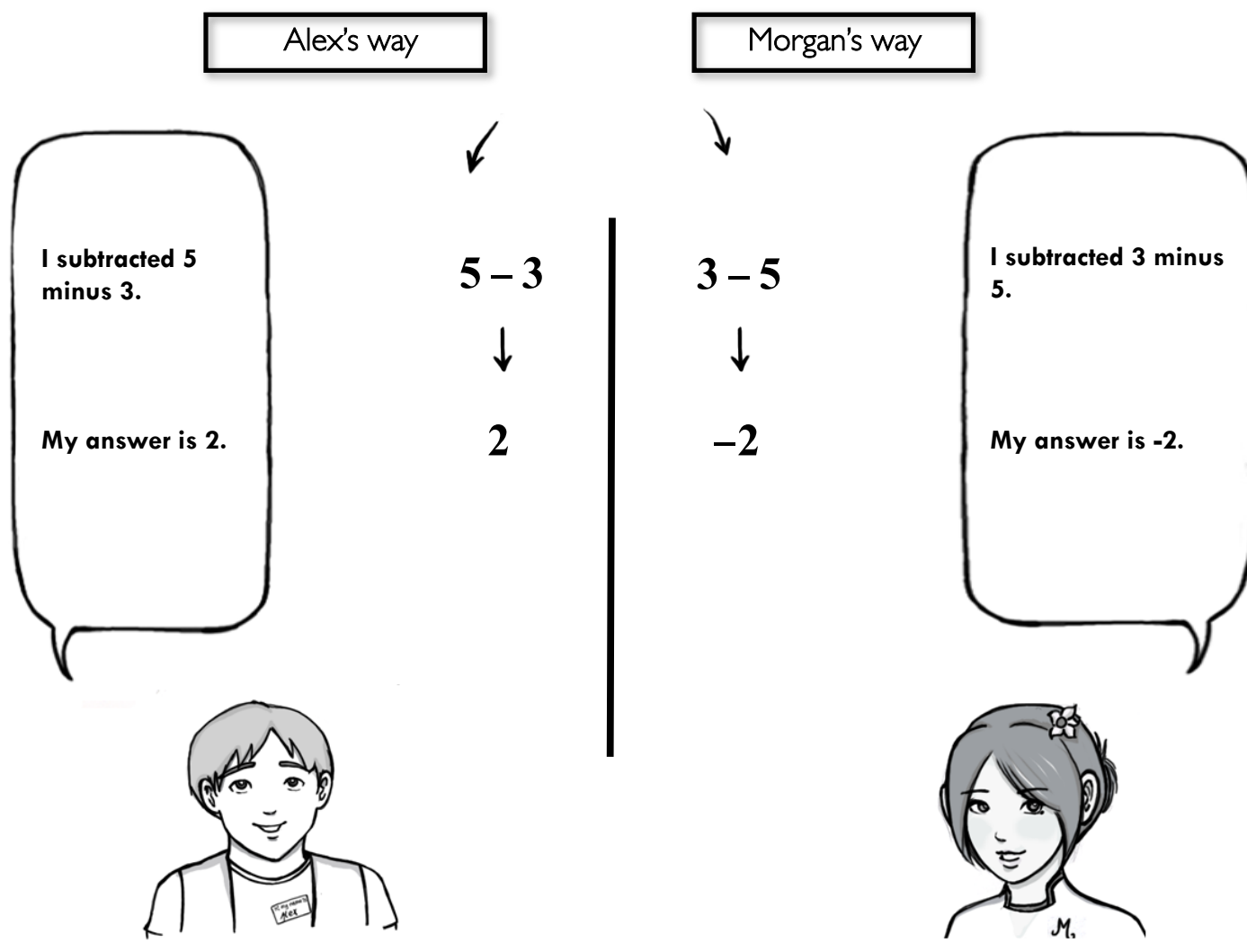
2 Which answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this type of expression?

*How do they differ?*

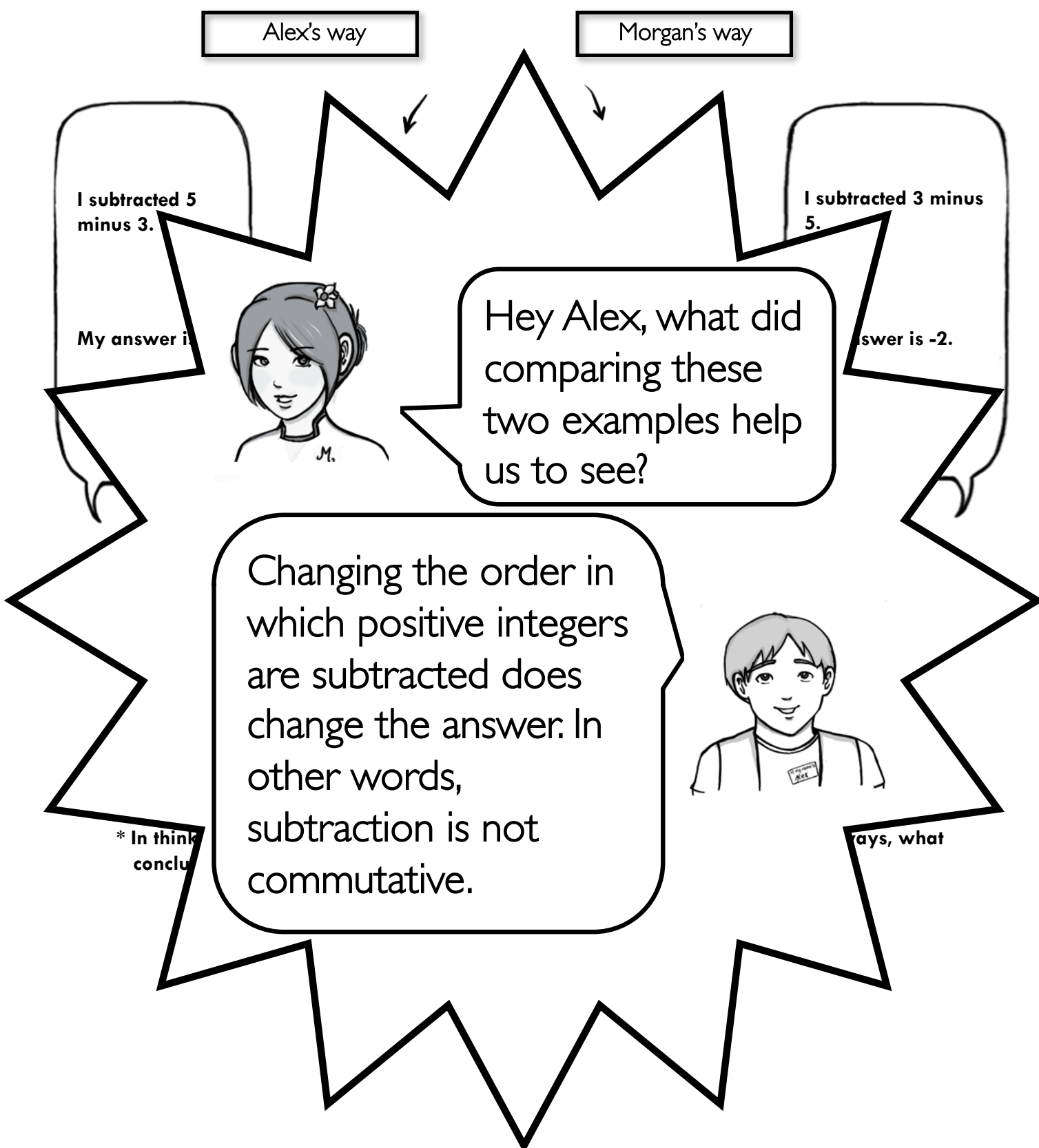
**Alex was asked to simplify  $5 - 3$ , and Morgan was asked to simplify  $3 - 5$**



- \* Did Alex correctly subtract  $5 - 3$ ? Did Morgan correctly subtract  $3 - 5$ ?
- \* What are some similarities and differences between Alex's and Morgan's problems?
- \* In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify these types of expressions?

*How do they differ?*

**Alex was asked to simplify  $5 - 3$ , and Morgan was asked to simplify  $3 - 5$**



1a Did Alex correctly subtract  $5-3$ ?

1b Did Morgan correctly subtract  $3-5$ ?

2 What are some similarities and differences between Alex's and Morgan's *problems*?

3 In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify these types of expressions?

*Why does it work?*

**Alex and Morgan were asked to simplify the expression  $2(3 + 5)$**

Alex's "order of operations" way

Morgan's "distributive property" way

$$2(3 + 5)$$



$$2(8)$$



$$16$$

$$2(3 + 5)$$



$$2 \cdot 3 + 2 \cdot 5$$



$$6 + 10$$



$$16$$

First I added 3 plus 5, and I got 8.

Then I multiplied 8 times 2. I got 16.

First I distributed the 2, so I got 2 times 3 plus 2 times 5.

I multiplied 2 times 3 and got 6, and 2 times 5 and got 10.

Then I added 6 plus 10. I got 16.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some differences between Alex's and Morgan's ways?
- \* Morgan's way doesn't follow the order of operations. Is this OK?

Why does it work?

**Alex and Morgan were asked to simplify the expression  $2(3 + 5)$**

Alex's "order of operations" way

Morgan's "distributive property" way

First I added  
5, and I got

Then  
times



Hey Morgan, what  
did comparing these  
two examples help  
us to see?

I distributed the  
2 times 3  
times 5.

3  
times

In order to multiply a term  
by an expression in  
parentheses, you can either  
(1) simplify the expression  
in parentheses first or (2)  
use the distributive property  
first. Both ways are OK and  
yield the same answer.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some differences between the two methods?
- \* Morgan's way does not require the distributive property.

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some differences between Alex's and Morgan's *ways*?

3 Morgan's way doesn't follow the order of operations. Is this OK?

Which is correct?

Alex and Morgan were asked to simplify  $2(x + 5)$  by applying the distributive property

Alex's "distribute over both terms in the parentheses" way

Morgan's "remove the parentheses" way

$$2(x + 5)$$

$$2(x + 5)$$

$$2(x) + 2(5)$$

$$2x + 5$$

$$2x + 10$$

I multiplied 2 times  $x$  and 2 times 5.

2 times  $x$  is  $2x$ , and 2 times 5 is 10. So I got  $2x + 10$ .

To distribute the 2, I multiplied 2 times  $x$  to get  $2x$  and then removed the parentheses.



- \* Describe Alex's way to a new student in your class.
- \* Describe Morgan's way to a new student in your class.
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Which answer is correct, Alex's or Morgan's? How do you know?



Which is correct?

Alex and Morgan were asked to simplify  $2(x + 5)$  by applying the distributive property

Alex's "distribute over both terms in the parentheses" way

Morgan's "remove the parentheses" way



Hey Alex, what did we learn from comparing these right and wrong ways?

When using the distributive property, don't forget to distribute to each of the terms in the parentheses. If you only distribute to some terms and not others, you can get an incorrect result.



- \* Describe Alex's way for simplifying  $2(x + 5)$ .
- \* Describe Morgan's way for simplifying  $2(x + 5)$  and how students might have arrived at this answer.
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Which answer is correct, Alex's or Morgan's? How do you know?

1 Describe Alex's way to a new student in your class.

2 Describe Morgan's way to a new student in your class.

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 Which answer is correct, Alex's or Morgan's? How do you know?

Which is correct?

**Alex and Morgan were asked to simplify  $3(2x)$**

Alex's "distribute over the parentheses" way

Morgan's "multiply only" way

$$3(2x)$$



$$3(2) \cdot 3(x)$$



$$6 \cdot 3x$$



$$18x$$

$$3(2x)$$



$$6x$$

I distributed the 3 over the terms in the parentheses.

I multiplied 3 times 2 and 3 times x.

I got  $18x$ .

I multiplied 3 times 2 and got 6. I multiplied that by x and I got  $6x$ .



- \* Describe Alex's way to a new student in your class.
- \* Describe Morgan's way to a new student in your class.
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Which answer is correct, Alex's or Morgan's? How do you know?

Which is correct?

**Alex and Morgan were asked to simplify  $3(2x)$**

Alex's "distribute over the parentheses" way

Morgan's "multiply only" way

$3(2x)$

$3(2x)$

I distributed  
over the term  
the parenthe



Hey Morgan, what did we learn from comparing these right and wrong ways?

d 3 times 2  
. I multiplied  
and I got  $6x$ .

When terms inside parentheses are multiplied together, a term outside the parentheses should not be "distributed" to each term inside. Remember that the official name for the distributive property is the "distributive property of multiplication over addition," and it's only appropriate for terms inside the parentheses that are added together.



- \* Describe Alex's way to a new student in your class.
- \* Describe Morgan's way to a new student in your class.
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Which answer is correct, Alex's or Morgan's? How do you know?

1 Describe Alex's way to a new student in your class.

2 Describe Morgan's way to a new student in your class.

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 Which answer is correct, Alex's or Morgan's? How do you know?

Which is correct?

Alex and Morgan were asked to simplify  $|-5| + |30|$

Alex's "add first" way

Morgan's "take absolute values first" way

I added -5 plus 30.

I got  $|25|$ .

It is 25.



$$|-5| + |30|$$



$$|25|$$



$$25$$

$$|-5| + |30|$$



$$5 + 30$$



$$35$$

I took the absolute value of each number first. I got 5 plus 30.

I added them together, and I got 35.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* Whose answer is correct, Alex's or Morgan's? How do you know?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

Which is correct?

Alex and Morgan were asked to simplify  $|-5| + |30|$

Alex's "add first" way

Morgan's "take absolute values first" way

I added  $-5 + 30$ .

It is 25.



Hey Morgan, what did we learn from comparing these right and wrong ways?

expression

$|a| + |b|$  is only equal to  $|a + b|$

if  $a$  and  $b$  are 0 or positive numbers.

If either  $a$  or  $b$  is negative, then

$|a| + |b| \neq |a + b|$ .



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* Whose answer is correct, Alex's or Morgan's? Do you know why?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?

**1a** How did Alex simplify the expression?

**1b** How did Morgan simplify the expression?

**2** Whose answer is correct, Alex's or Morgan's? How do you know?

**3** What are some similarities and differences between Alex's and Morgan's ways?

**4** Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways?



Which is correct?

**Alex and Morgan were asked to simplify  $-5 - (-2)$**

Alex's "rewrite subtraction as addition of the opposite" way

Morgan's "subtract" way

↙

$$-5 - (-2)$$

↘

$$-5 - (-2)$$

↓

$$-5 + 2$$

↓

$$-7$$

I rewrote the expression as -5 plus 2.

I got -7.

I got -3.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* Whose answer is correct, Alex's or Morgan's? How do you know?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?

Which is correct?

**Alex and Morgan were asked to simplify  $-5 - (-2)$**

Alex's "rewrite subtraction as addition of the opposite" way

Morgan's "subtract" way



Hey Alex, what did we learn from comparing these right and wrong ways?

Subtracting the opposite of a number is not the same as subtracting the number itself. You may rewrite subtraction of the opposite as addition.



I rewrote the expression as  $-5 + 2$ .

I got

- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* Whose answer is correct, Alex's or Morgan's? How do you know?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?