Which is better?

Alex and Morgan were asked to solve $8-4(x-3)=40$

Alex's "distribute first" way

First I distributed the -4 to (*x* - 3).

$$8-4(x-3)=40$$

Then I combined like terms.

$$8-4x+12=40$$

$$-4x+20=40$$

Next I subtracted 20 from both sides.

$$\begin{array}{r} -4x+20=40 \\ -20\ -20 \\ \hline -4x=20 \end{array}$$

Last, I divided both sides by -4 to get the answer.

$$\frac{-4x}{-4}=\frac{20}{-4}$$

$$x=-5$$

Morgan's "subtract first" way

First I subtracted 8 from both sides.

$$\begin{array}{r} 8-4(x-3)=40 \\ -8 \qquad\qquad -8 \\ \hline -4(x-3)=32 \end{array}$$

Next I divided both sides by -4.

$$\frac{-4(x-3)}{-4}=\frac{32}{-4}$$

$$x-3=-8$$

Then I added 3 to both sides to get the answer.

$$\begin{array}{r} x-3=-8 \\ +3\ \ +3 \\ \hline x=-5 \end{array}$$

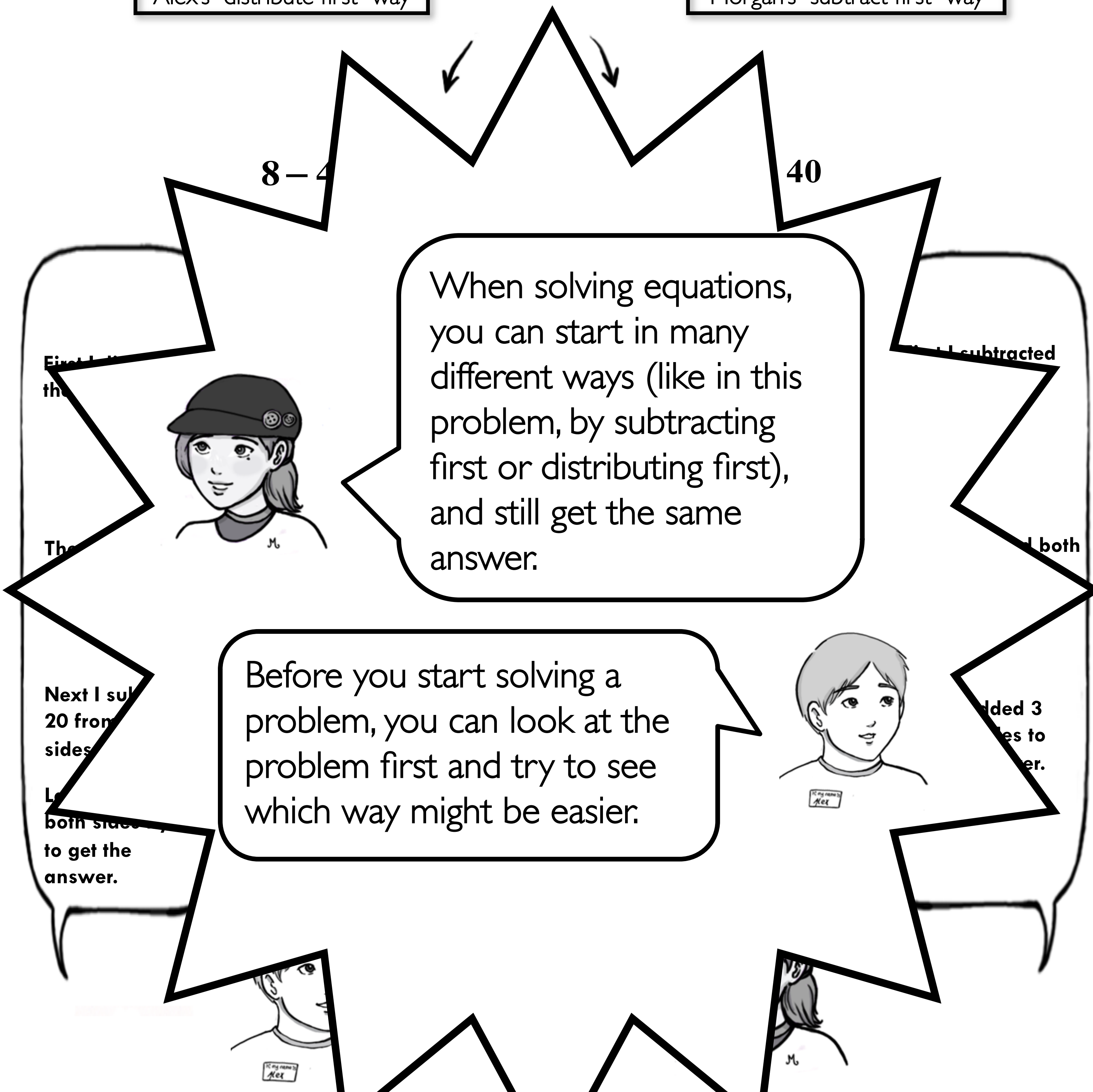
* How did Alex solve the equation?
* How did Morgan solve the equation?
* Describe two ways that Alex's and Morgan's ways are similar.
* Describe two ways that Alex's and Morgan's ways are different.
* On a timed test, would you rather use Alex's way or Morgan's way for this problem?

Which is better?

Alex and Morgan were asked to solve $8 - 4(x - 3) = 40$

Alex's "distribute first" way

Morgan's "subtract first" way



*** How did Alex solve the equation?**
*** How did Morgan solve the equation?**
*** Describe two ways that Alex's and Morgan's ways are similar.**
*** Describe two ways that Alex's and Morgan's ways are different.**
*** On a timed test, would you rather use Alex's way or Morgan's way for this problem?**

Student Worksheet 3.1.1

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 Describe two ways that Alex's and Morgan's ways are similar.

3 Describe two ways that Alex's and Morgan's ways are different.

4 On a timed test, would you rather use Alex's way or Morgan's way for this problem?

Which is better?

Alex and Morgan were asked to solve $\frac{x}{4} - \frac{x}{5} = -2$

Alex's *"eliminate the fractions"* way

$$\frac{x}{4} - \frac{x}{5} = -2$$

$$\downarrow$$

$$20\left(\frac{x}{4} - \frac{x}{5}\right) = -2(20)$$

$$\downarrow$$

$$5x - 4x = -40$$

$$\downarrow$$

$$x = -40$$

First I multiplied both sides of the equation by the least common multiple of the denominators, which is 20.

Then I simplified both sides of the equation.

Then I combined like terms to get the answer.

Morgan's *"find common denominators"* way

$$\frac{x}{4} - \frac{x}{5} = -2$$

$$\downarrow$$

$$\frac{5x}{20} - \frac{4x}{20} = -2$$

$$\downarrow$$

$$\frac{x}{20} = -2$$

$$\downarrow$$

$$(20)\frac{x}{20} = -2(20)$$

$$\downarrow$$

$$x = -40$$

First I gave the two fractions the same denominator.

Then I subtracted the fractions.

Then I multiplied by 20 on both sides.

I simplified both sides of the equation to get the answer.

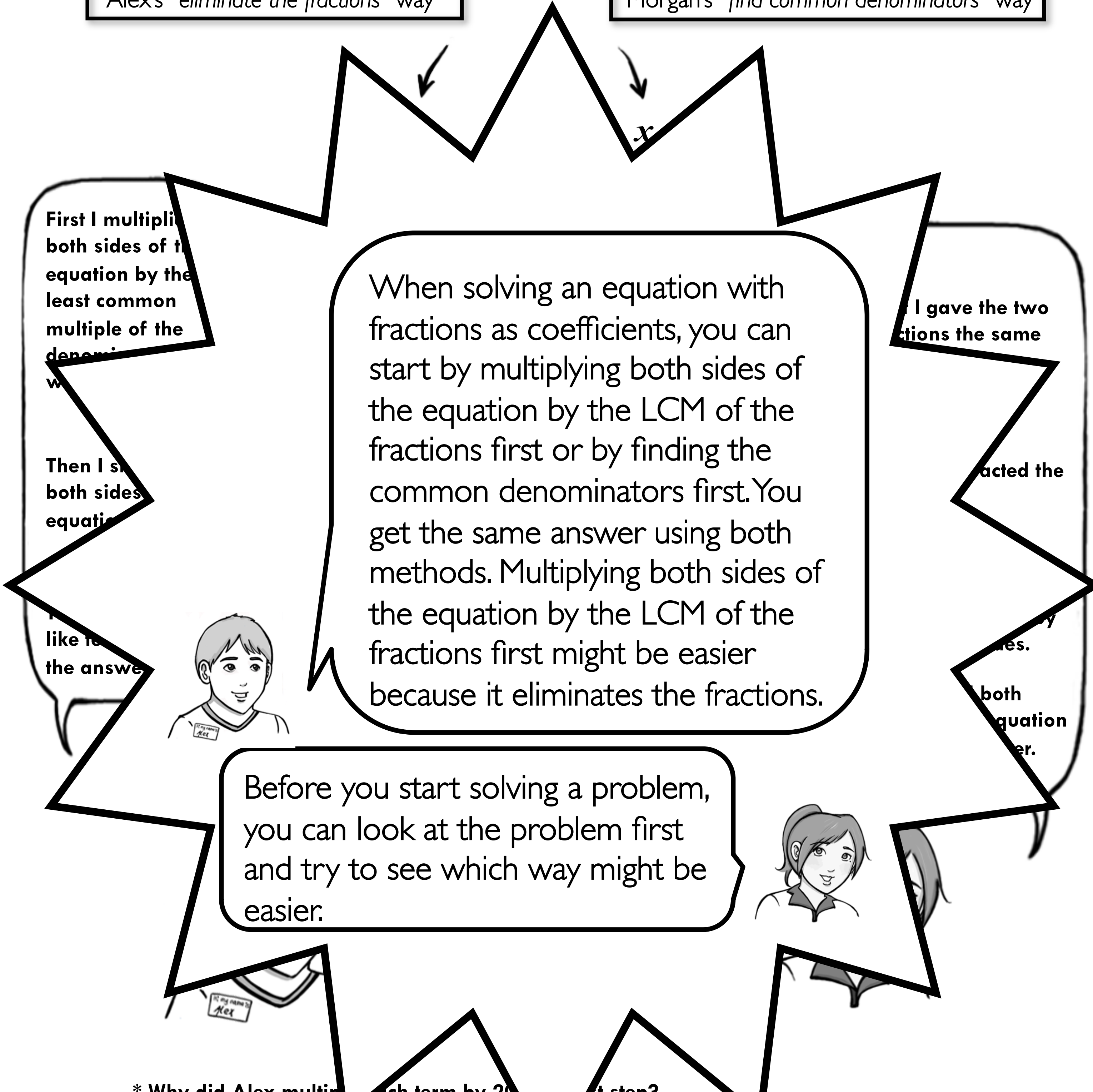
* Why did Alex multiply each term by 20 as a first step?
* Why did Morgan find a common denominator as a first step?
* What are some similarities and differences between Alex's and Morgan's *ways*?
* Which way is easier, Alex's way or Morgan's way? Why?

Which is better?

Alex and Morgan were asked to solve $\frac{x}{4} - \frac{x}{5} = -2$

Alex's *"eliminate the fractions"* way

Morgan's *"find common denominators"* way



*** Why did Alex multip ach term by 20 st step?**
*** Why did Morgan find a common denom as a first step?**
*** What are some similarities and differences between Alex's and Morgan's *ways*?**
*** Which way is easier, Alex's way or Morgan's way? Why?**

Student Worksheet 3.1.2

1a Why did Alex multiply each term by 20 as a first step?

1b Why did Morgan find a common denominator as a first step?

2 What are some similarities and differences between Alex's and Morgan's *ways*?

3 Which way is easier, Alex's way or Morgan's way? Why?

Which is correct?

Alex and Morgan were asked to solve $5(x+3)=20$

Alex's "divide first" way	Morgan's "subtract first" way
$5(x+3)=20$	$5(x+3)=20$
First I divided by 5 on both sides of the equation. $\dfrac{5(x+3)}{5}=\dfrac{20}{5}$	**First I subtracted on both sides of the equation.** $\begin{array}{r} 5(x+3)=20 \\ -3 \quad -3 \\ \hline 5x=17 \end{array}$
$x+3=4$	**Then I divided on both sides.** $\dfrac{5x}{5}=\dfrac{17}{5}$
Then I subtracted on both sides of the equation. $\begin{array}{r} x+3=4 \\ -3-3 \\ \hline x=1 \end{array}$	**Here is my answer.** $x=\dfrac{17}{5}$

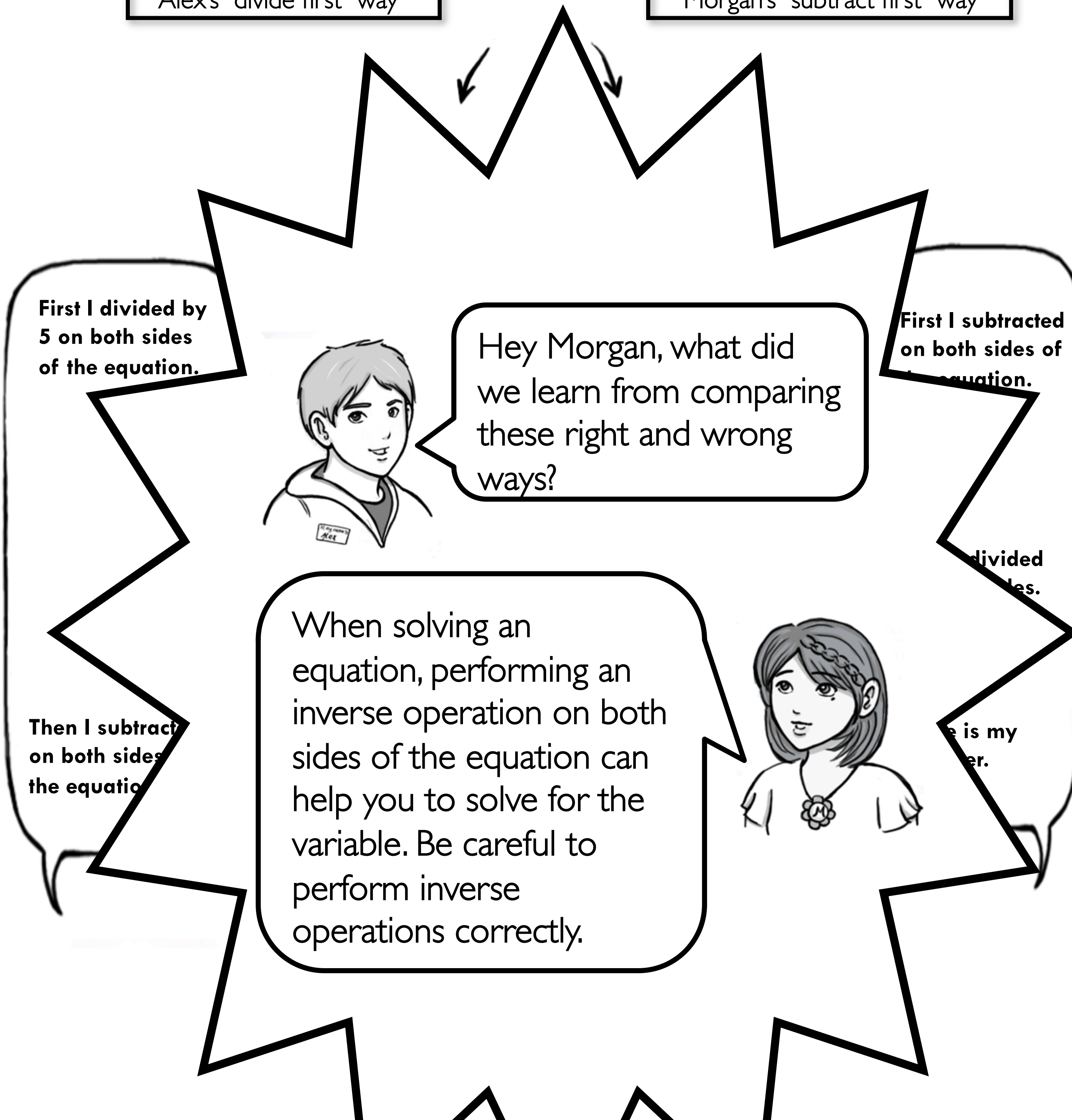
*** How did Alex solve the equation? How did Morgan solve the equation?**
*** Whose answer is correct, Alex's or Morgan's? How do you know?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Alex and Morgan both performed the same operations on both sides of the equation, yet one of them got the wrong answer. Why? Where was the mistake?**

Which is correct?

Alex and Morgan were asked to solve $5(x+3)=20$

Alex's "divide first" way

Morgan's "subtract first" way



* How did Alex solve the equatio... did M... the eq...?

* Whose answer is correct, Alex's ... organ's? H... ou know?

* What are some similarities and ...ifferences betw... lex's and Morga...'s ways?

* Alex and Morgan both performed the same opera...ns on both sides of the equation, yet one of them got the wrong answer. Why? Where was the mistake?

Student Worksheet 3.1.3

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 Alex and Morgan both performed the same operations on both sides of the equation, yet one of them got the wrong answer. Why? Where was the mistake?

Which is better?

Alex and Morgan were asked to solve $\frac{t}{3}-1=12$

Alex's "add first" way

$$\frac{t}{3}-1=12$$

$$\downarrow$$

First I added 1 on both sides of the equation.

$$\begin{array}{rl} \frac{t}{3}-1 &= 12 \\ +1 & \ \ +1 \\ \hline \frac{t}{3} &= 13 \end{array}$$

$$\downarrow$$

Then I multiplied by 3 on both sides.

$$(3)\frac{t}{3}=13(3)$$

$$\downarrow$$

Here is my answer.

$$t=39$$

Morgan's "find a common denominator first" way

$$\frac{t}{3}-1=12$$

$$\downarrow$$

First I rewrote the terms on the left side of the equation so they had a common denominator.

$$\frac{t}{3}-\frac{3}{3}=12$$

$$\downarrow$$

Then I combined the terms on the left into one fraction with a common denominator.

$$\frac{t-3}{3}=12$$

$$\downarrow$$

I multiplied by 3 on both sides of the equation.

$$(3)\frac{t-3}{3}=12(3)$$

$$\downarrow$$

Then I added 3 on both sides of the equation. Here is my answer.

$$\begin{array}{rl} t-3 &= 36 \\ +3 & \ \ +3 \\ \hline t &= 39 \end{array}$$

* How did Alex solve the equation?
* How did Morgan solve the equation?
* What are some similarities and differences between Alex's and Morgan's ways?
* What are some advantages of Alex's way? Of Morgan's way?

Which is better?

Alex and Morgan were asked to solve $\frac{t}{3} - 1 = 12$

Alex's "add first" way	Morgan's "find a common denominator first" way

$-1 = 12$ $-1 = 12$

First I added 1 on b
sides of the equatio

Then I multipli
3 on both si

Here is my ans

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non denominator.

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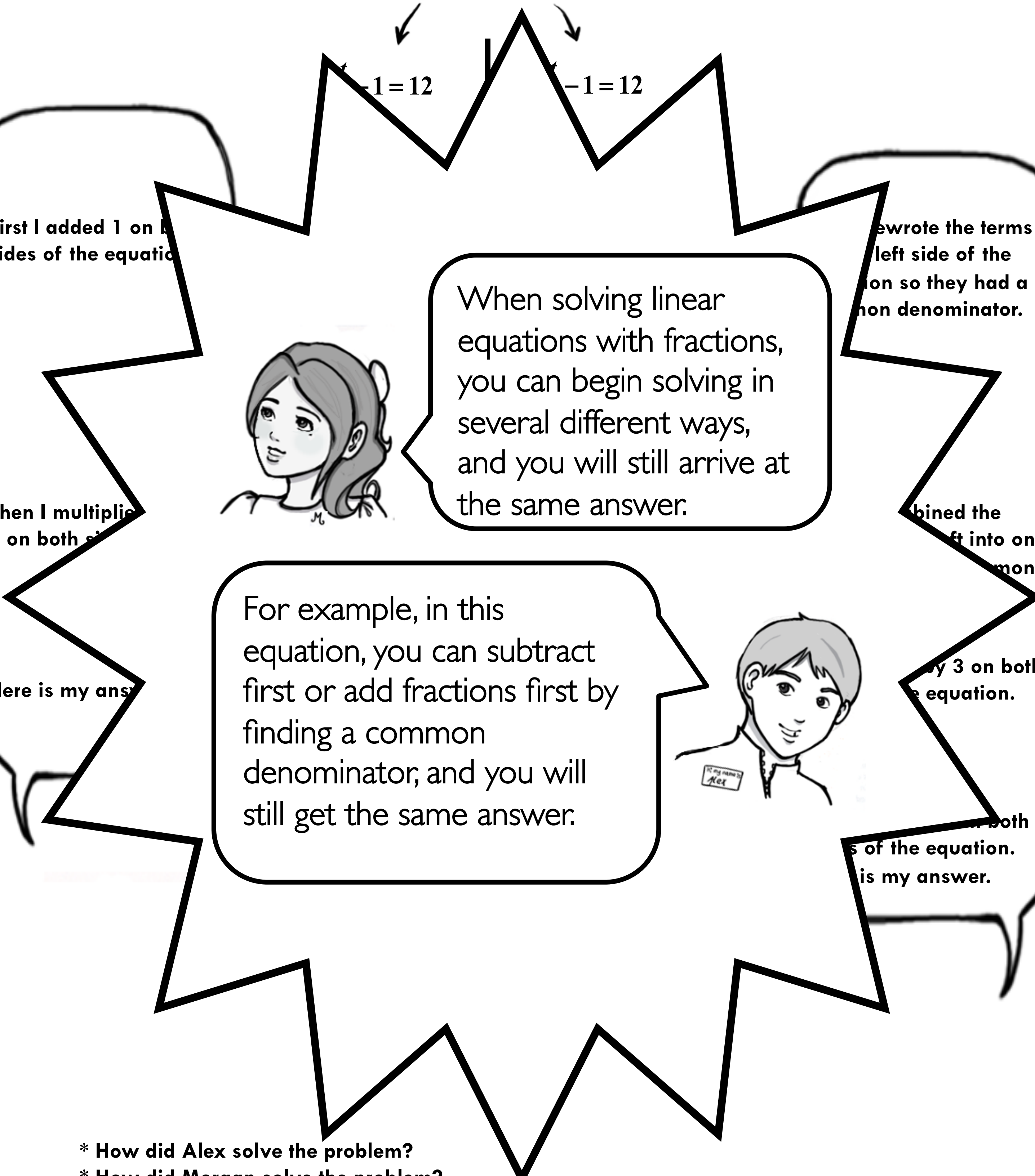
ft into one
mon

y 3 on both
e equation.

both
s of the equation.
is my answer.



*** How did Alex solve the problem?**
*** How did Morgan solve the problem?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** What are some advantages of Alex's way? Of Morgan's way?**

Student Worksheet 3.1.4

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 What are some advantages of Alex's way?

4 What are some advantages of Morgan's way?

Why does it work?

Alex and Morgan were asked to solve $\frac{x}{5} = 20$

Alex's "cross-multiply first" way

$$\frac{x}{5} = 20$$

$$\downarrow$$

$$\frac{x}{5} = \frac{20}{1}$$

$$\downarrow$$

$$x = 100$$

First I rewrote 20 as $\frac{20}{1}$.

Then I cross-multiplied.

I got x = 100.

Morgan's "multiply on both sides first" way

$$\frac{x}{5} = 20$$

$$\downarrow$$

$$(5)\frac{x}{5} = 20(5)$$

$$\downarrow$$

$$x = 100$$

First I multiplied by 5 on both sides.

I got x = 100.

*** How did Alex solve the equation?**
*** How did Morgan solve the equation?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Even though Alex and Morgan used different steps, they both got the same answer. Why?**

Why does it work?

Alex and Morgan were asked to solve $\frac{x}{5} = 20$

Alex's "cross-multiply first" way	Morgan's "multiply on both sides first" way

First I rewrote 20 as $\frac{20}{1}$.

I cross-multi

I got x = 100.

First I multiplied ... both

I got x = 100.

Hey Morgan, what did we learn from comparing these two different ways?

Cross-multiplying and multiplying both sides of the equation by the same value in order to eliminate the denominator are equivalent.

*** How did Alex solve the equation?**
*** How did Morgan solve the equation?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Even though Alex and Morgan used different steps, they both got the same answer. Why?**

Student Worksheet 3.1.5

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 Even though Alex and Morgan used different steps, they both got the same answer. Why?

Which is better?

Alex and Morgan were asked to solve $3(x+2)=15$

Alex's "distribute first" way

$$3(x+2)=15$$

$\downarrow$

First I distributed across the parentheses.

$$3x+6=15$$

$\downarrow$

Then I subtracted on both sides.

$$\begin{array}{r} 3x+6=15 \\ -6\ \ -6 \\ \hline 3x=9 \end{array}$$

$\downarrow$

I divided on both sides. Here is my answer.

$$\frac{3x}{3}=\frac{9}{3}$$

$$x=3$$

Morgan's "divide first" way

$$3(x+2)=15$$

$\downarrow$

First I divided on both sides.

$$\frac{3(x+2)}{3}=\frac{15}{3}$$

$\downarrow$

$$x+2=5$$

$\downarrow$

Then I subtracted on both sides. Here is my answer.

$$\begin{array}{r} x+2=5 \\ -2-2 \\ \hline x=3 \end{array}$$

* **How did Alex solve the equation?**
* **How did Morgan solve the equation?**
* **What are some similarities and differences between Alex's and Morgan's ways?**
* **On a timed test, would you rather use Alex's way or Morgan's way? Why?**
* **If the problem were changed to $3(x + 2) = 17$, would Alex's way or Morgan's way be better? Why?**

Which is better?

Alex and Morgan were asked to solve $3(x+2)=15$

Alex's "distribute first" way

Morgan's "divide first" way

$3 \quad 2) = 15 \qquad + 2) = 1$

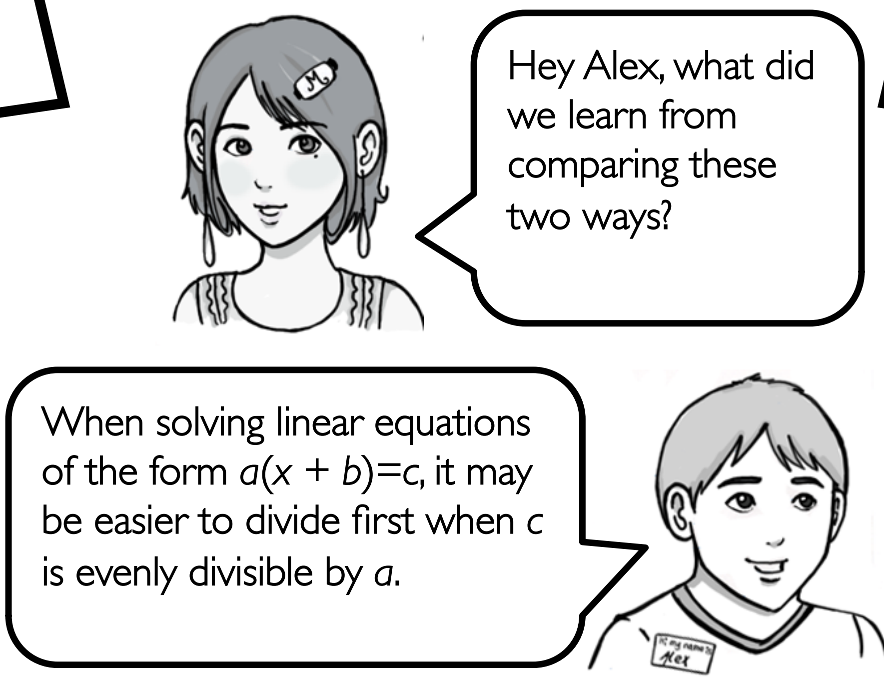
First I distributed across the parentheses.

Then I subtracte on both sides.

I divided on bot sides. Here is answer.

I divided on sides.

tracted on Here is



*** How did Alex solve the equation?**
*** How did Morgan solve the equation?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** On a timed test, would you rather use Alex's way or Morgan's way? Why?**
*** If the problem were changed to $3(x + 2) = 17$, would Alex's way or Morgan's way be better? Why?**

Student Worksheet 3.1.6

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 On a timed test, would you rather use Alex's way or Morgan's way? Why?

4 If the problem were changed to $3(x + 2) = 17$, would Alex's way or Morgan's way be better? Why?

Which is better?

Alex and Morgan were asked to solve $\dfrac{1}{4}(x+3)=2$

Alex's "distribute first" way

$$\frac{1}{4}(x+3)=2$$

First I distributed across the parentheses.

$$\frac{1}{4}x+\frac{3}{4}=2$$

Then I subtracted on both sides.

$$\begin{array}{r} \frac{1}{4}x+\frac{3}{4}=2 \\ -\frac{3}{4}-\frac{3}{4} \\ \hline \frac{1}{4}x=\frac{5}{4} \end{array}$$

Then I multiplied on both sides. Here is my answer.

$$(4)\frac{1}{4}x=\frac{5}{4}(4)$$

$$x=5$$

Morgan's "multiply first" way

$$\frac{1}{4}(x+3)=2$$

First I multiplied on both sides.

$$(4)\frac{1}{4}(x+3)=2(4)$$

$$x+3=8$$

Then I subtracted on both sides. Here is my answer.

$$\begin{array}{r} x+3=8 \\ -3-3 \\ \hline x=5 \end{array}$$

* How did Alex solve the equation?
* How did Morgan solve the equation?
* What are some similarities and differences between Alex's and Morgan's ways?
* Which way do you think is easier for this problem, Alex's way or Morgan's way? Why?

Which is better?

Alex and Morgan were asked to solve $\frac{1}{4}(x+3)=2$

Alex's "distribute first" way

Morgan's "multiply first" way

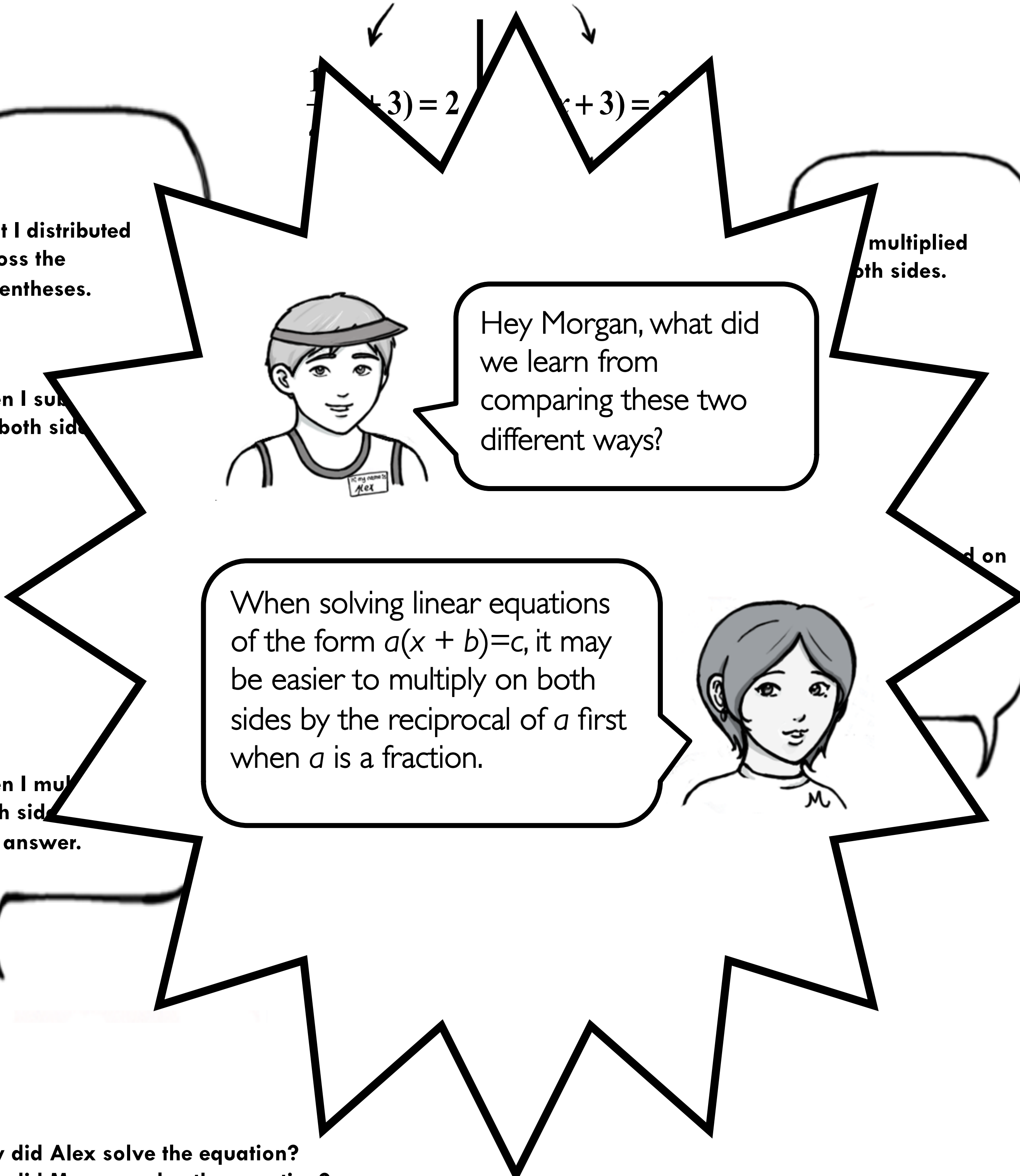
First I distributed across the parentheses.

Then I sub... on both sid...

Then I mul... both sid... my answer.

...multiplied ...oth sides.

...d on



*** How did Alex solve the equation?**
*** How did Morgan solve the equation?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** Which way do you think is easier for this problem, Alex's way or Morgan's way? Why?**

Student Worksheet 3.1.7

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 Which way do you think is easier for this problem, Alex's way or Morgan's way? Why?

Which is better?

Alex and Morgan were asked to solve $2x-(2+3x)=-4x$

Alex's "subtract from both sides first" way			Morgan's "distribute first" way
	$2x-(2+3x)=-4x$	$2x-(2+3x)=-4x$	
I first subtracted 2*x* from both sides.	$-(2+3x)=-6x$	$2x-2-3x=-4x$	**First I used the distributive property to rewrite the equation.**
Then I multiplied both sides of the equation by -1.	$2+3x=6x$	$-x-2=-4x$	**Then I combined the like terms.**
Then I subtracted 3x from both sides.	$2=3x$	$-2=-3x$	**Then I added *x* to both sides.**
Then I divided both sides by 3 to get the answer.	$\frac{2}{3}=x$	$\frac{2}{3}=x$	**Then I divided both sides by -3 and simplified the fraction to get the answer.**

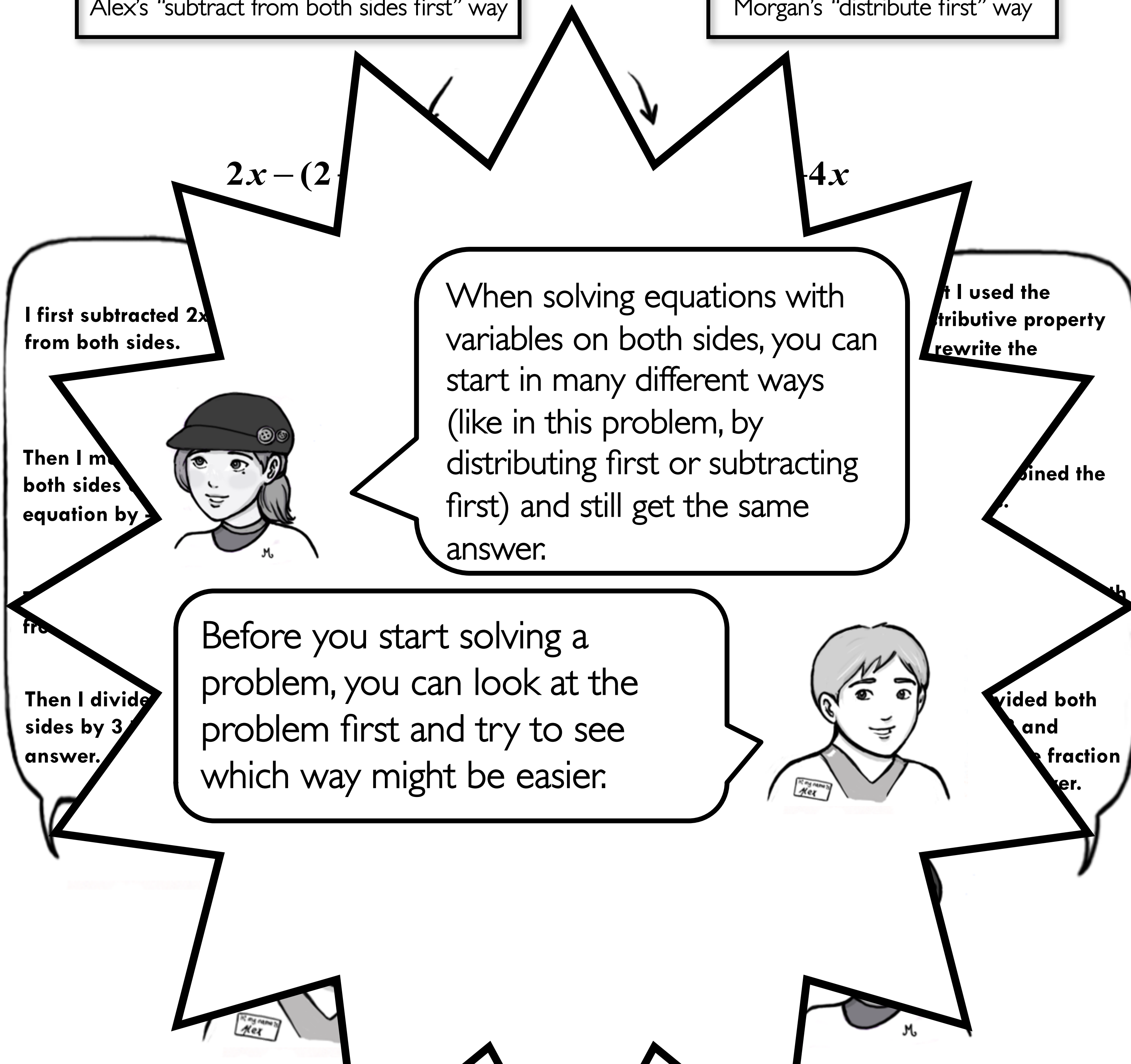
*** How did Alex solve the equation?**
*** Why did Morgan distribute as a first step?**
*** Describe two ways that Alex's and Morgan's ways are similar.**
*** Describe two ways that Alex's and Morgan's ways are different.**
*** What are some advantages of Alex's way? Of Morgan's way?**

Which is better?

Alex and Morgan were asked to solve $2x-(2+3x)=-4x$

Alex's "subtract from both sides first" way

Morgan's "distribute first" way



* How did Alex solve t... ...ion?
* Why did Morgan distr... ...e as a first step...
* Describe two ways th... Alex's and Morga... ...ays are similar.
* Describe two ways that Alex's and Morgan... ways are different.
* What are some advantages of Alex's way? Of Morgan's way?

Student Worksheet 3.2.1

1 How did Alex solve the equation?

2 Why did Morgan distribute as a first step?

3 Describe two ways that Alex's and Morgan's ways are similar.

4 Describe two ways that Alex's and Morgan's ways are different.

5 What are some advantages of Alex’s way? Of Morgan’s way?

Which is correct?

Alex and Morgan were asked to solve $45y + 90 = 60y$

Alex's "combine like terms" way

I first combined like terms on the left side of the equation.

$$45y + 90 = 60y$$

$$\downarrow$$

$$135y = 60y$$

$$\downarrow$$

Then I subtracted both sides by 60*y*.

$$\begin{array}{rr} 135y = & 60y \\ -60y & -60y \\ \hline 75y = & 0 \end{array}$$

$$\downarrow$$

Then I divided both sides by 75 to get the answer.

$$\frac{75y}{75} = \frac{0}{75}$$

$$y = 0$$

Morgan's "combine like terms" way

First I subtracted 45*y* on either side; 60*y* – 45*y* is 15*y*.

$$\begin{array}{rr} 45y + 90 = & 60y \\ -45y \qquad & -45y \\ \hline 90 = & 15y \end{array}$$

$$\downarrow$$

Then I divided both sides by 15 to get the answer.

$$\frac{90}{15} = \frac{15y}{15}$$

$$6 = y$$

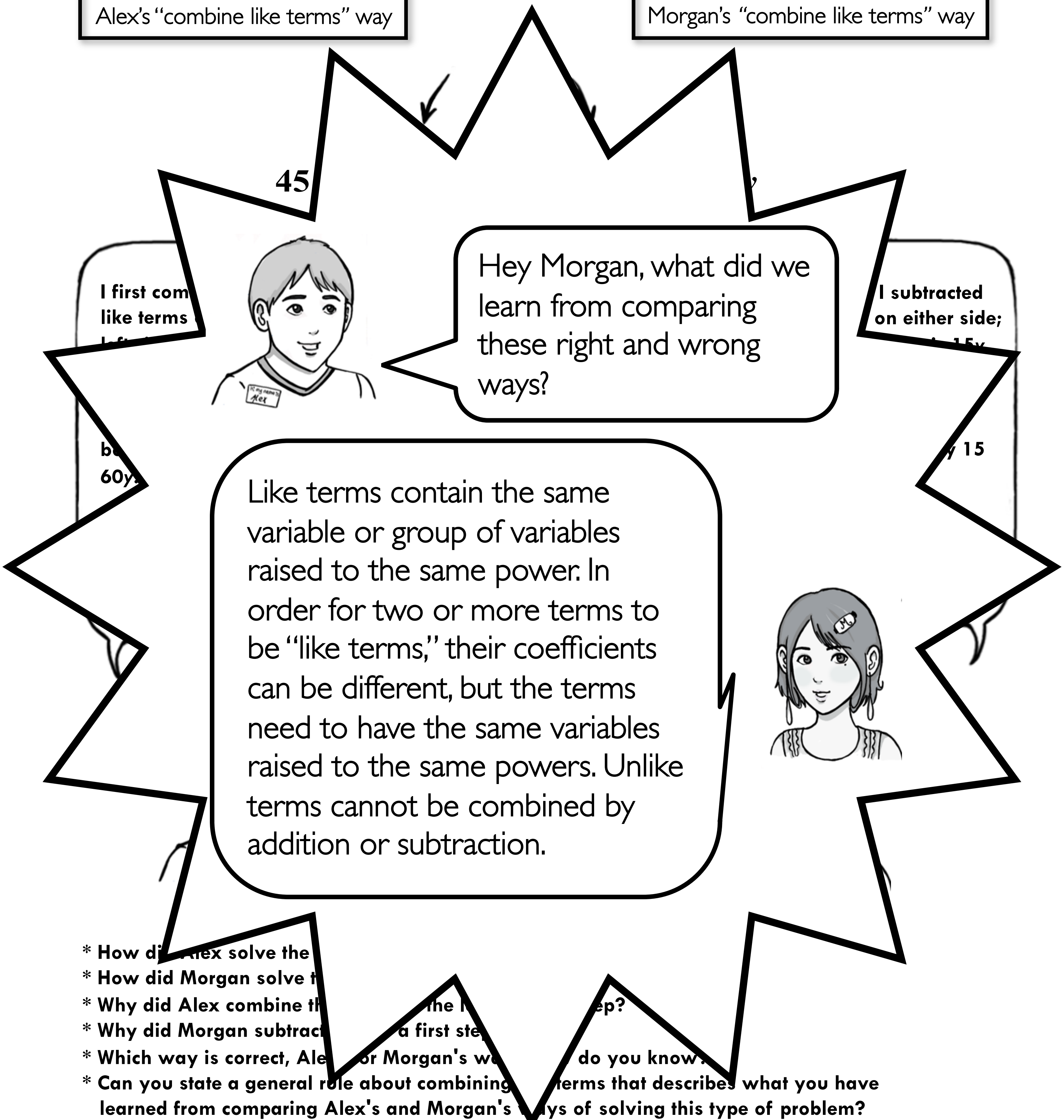
* How did Alex solve the equation?
* How did Morgan solve the equation?
* Why did Alex combine the terms on the left as a first step?
* Why did Morgan subtract 45y as a first step?
* Which way is correct, Alex's or Morgan's way? How do you know?
* Can you state a general rule about combining like terms that describes what you have learned from comparing Alex's and Morgan's ways of solving this type of problem?

Which is correct?

Alex and Morgan were asked to solve $45y + 90 = 60y$

Alex's "combine like terms" way

Morgan's "combine like terms" way



*** How di… Alex solve the …**
*** How did Morgan solve t…**
*** Why did Alex combine th… the l… …ep?**
*** Why did Morgan subtrac… a first ste…**
*** Which way is correct, Ale… or Morgan's w… do you know?**
*** Can you state a general r…le about combining … terms that describes what you have learned from comparing Alex's and Morgan's w…ys of solving this type of problem?**

Student Worksheet 3.2.2

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 Why did Alex combine the terms on the left as a first step?

3 Why did Morgan subtract 45*y* as a first step?

4 Which way is correct, Alex's or Morgan's way? How do you know?

5 Can you state a general rule about combining like terms that describes what you have learned from comparing Alex's and Morgan's ways of solving this type of problem?

Which is correct?

Alex and Morgan were asked to solve $2x-5=5x+7+3x$

Alex's "subtract on both sides first" way

$$2x-5=5x+7+3x$$

I first subtracted 5x from both sides.

$$\begin{array}{rcl} 2x-5 &=& 5x+7+3x \\ -5x & & -5x \qquad -5x \end{array}$$

2x - 5x is -3x, and 3x - 5x is -2x.

$$-3x-5=7-2x$$

Then I added 3x to both sides.

$$\begin{array}{rcl} -3x-5 &=& 7-2x \\ +3x & & \qquad +3x \end{array}$$

I simplified: -3x + 3x is 0, and -2x + 3x is just x. Then I subtracted 7 from both sides.

$$-5=7+x$$

I simplified to get my answer, *x* = -12.

$$-12=x$$

Morgan's "combine like terms first" way

$$2x-5=5x+7+3x$$

I first combined the like terms on the right side of the equal sign; 5x + 3x is 8x.

$$2x-5=8x+7$$

Then I subtracted 2x from both sides of the equation, and then I simplified.

$$\begin{array}{rcl} 2x-5 &=& 8x+7 \\ -2x & & -2x \end{array}$$

Then I subtracted 7 from both sides.

$$-5=6x+7$$

Then I divided both sides by 6.

$$\frac{-12}{6}=\frac{6x}{6}$$

I simplified both sides to get the answer, *x* = -2.

$$-2=x$$

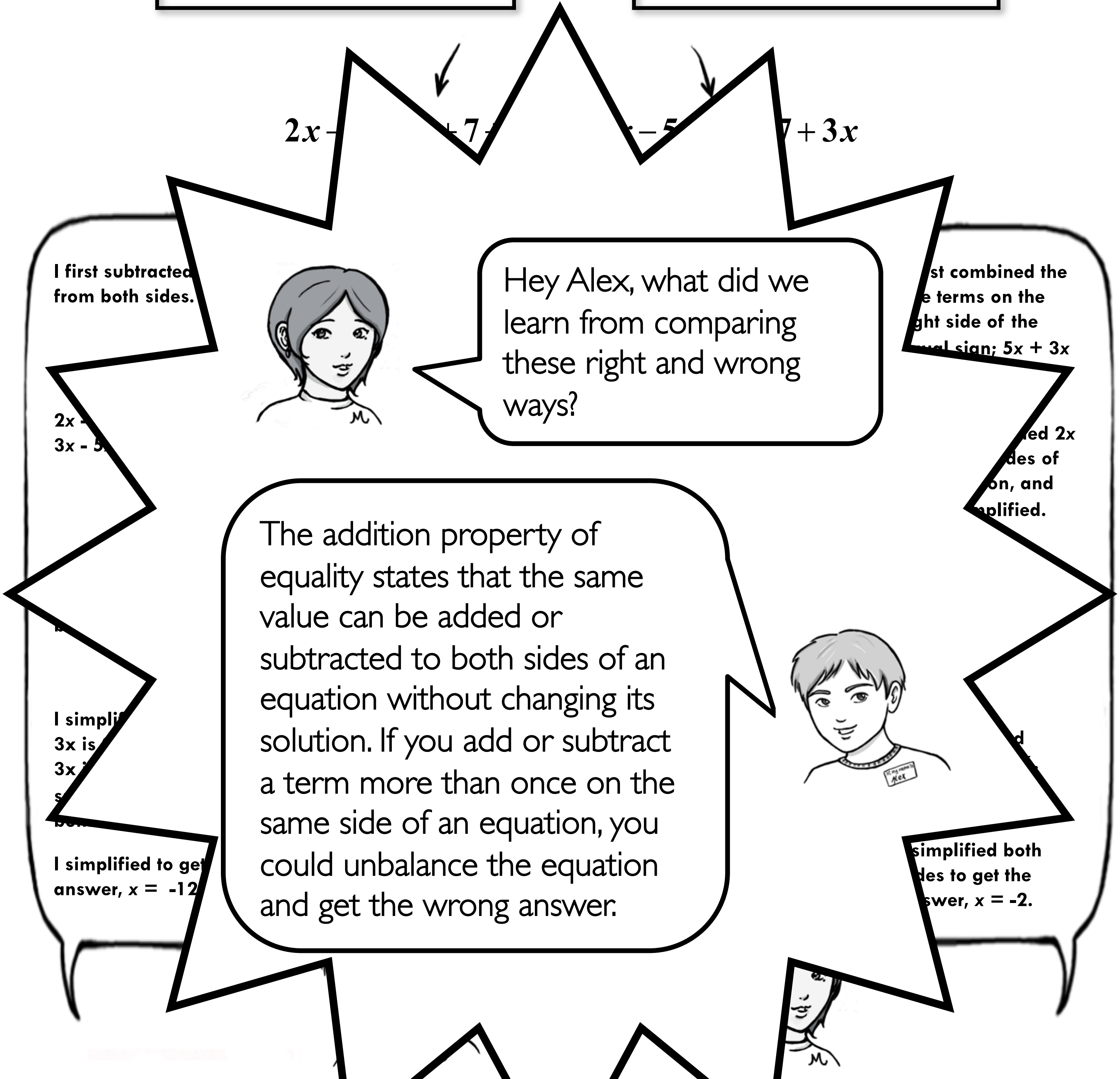
* Describe Alex's way to a new student in your class. Describe Morgan's way to a new student in your class.
* What are some similarities and differences between Alex's and Morgan's ways?
* Which answer is correct, Alex's or Morgan's? How do you know?
* In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of problem?

Which is correct?

Alex and Morgan were asked to solve $2x - 5 = 5x + 7 + 3x$

Alex's "subtract on both sides first" way

Morgan's "combine like terms first" way



*** Describe Alex's way to a new ... dent in your c... cribe Morgan ... ay to a new student in your class.**
*** What are some similarities a... differences betw... ex's and Morgan ... ways?**
*** Which answer is correct, Alex's or Morgan's? How ... you know?**
*** In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of problem?**

Student Worksheet 3.2.3

1 Describe Alex's way to a new student in your class.

2 Describe Morgan's way to a new student in your class.

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 Which answer is correct, Alex's or Morgan's? How do you know?

5 In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of problem?

Which is better?

Alex and Morgan were asked to solve $2(m+3) = -4(m+3) + 12$

Alex's "distribute first" way		Morgan's "shortcut" way	
	$2(m+3) = -4(m+3) + 12$	$2(m+3) = -4(m+3) + 12$	
First I distributed across the parentheses.	$2m + 6 = -4m + -12 + 12$	$\begin{array}{r} 2(m+3) = -4(m+3)+12 \\ \underline{+4(m+3) \quad +4(m+3)} \\ 6(m+3) = 12 \end{array}$	**First I combined terms with common factors.**
Then I combined like terms.	$2m + 6 = -4m$	$\dfrac{6(m+3)}{6} = \dfrac{12}{6}$	**Then I divided by 6 on both sides.**
I subtracted on both sides.	$\begin{array}{r} 2m + 6 = -4m \\ \underline{-2m \qquad -2m} \\ 6 = -6m \end{array}$	$m + 3 = 2$	**Here is the result.**
Then I divided on both sides. Here is my answer.	$\begin{array}{r} \underline{6} = \underline{-6m} \\ -6 \quad -6 \\ -1 = m \end{array}$	$\begin{array}{r} m + 3 = 2 \\ \underline{-3 - 3} \\ m = -1 \end{array}$	**Then I subtracted on both sides. Here is my answer.**

*** How did Alex solve the equation?**
*** How did Morgan solve the equation?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** On a timed test, would you rather use Alex's way or Morgan's way? Why?**

Alex and Morgan were asked to solve $2(m+3) = -4(m+3) + 5$

Alex's "distribute first" way	Morgan's "shortcut" way
$2(m+3) =$ ... $)+5$	$3) =$ ... $)+5$

First I distributed across the parentheses.

Then I combine

like terms.

I added on

sides.

Then I subtr

on both sid

Then I divided on both sides. Here is my answer.

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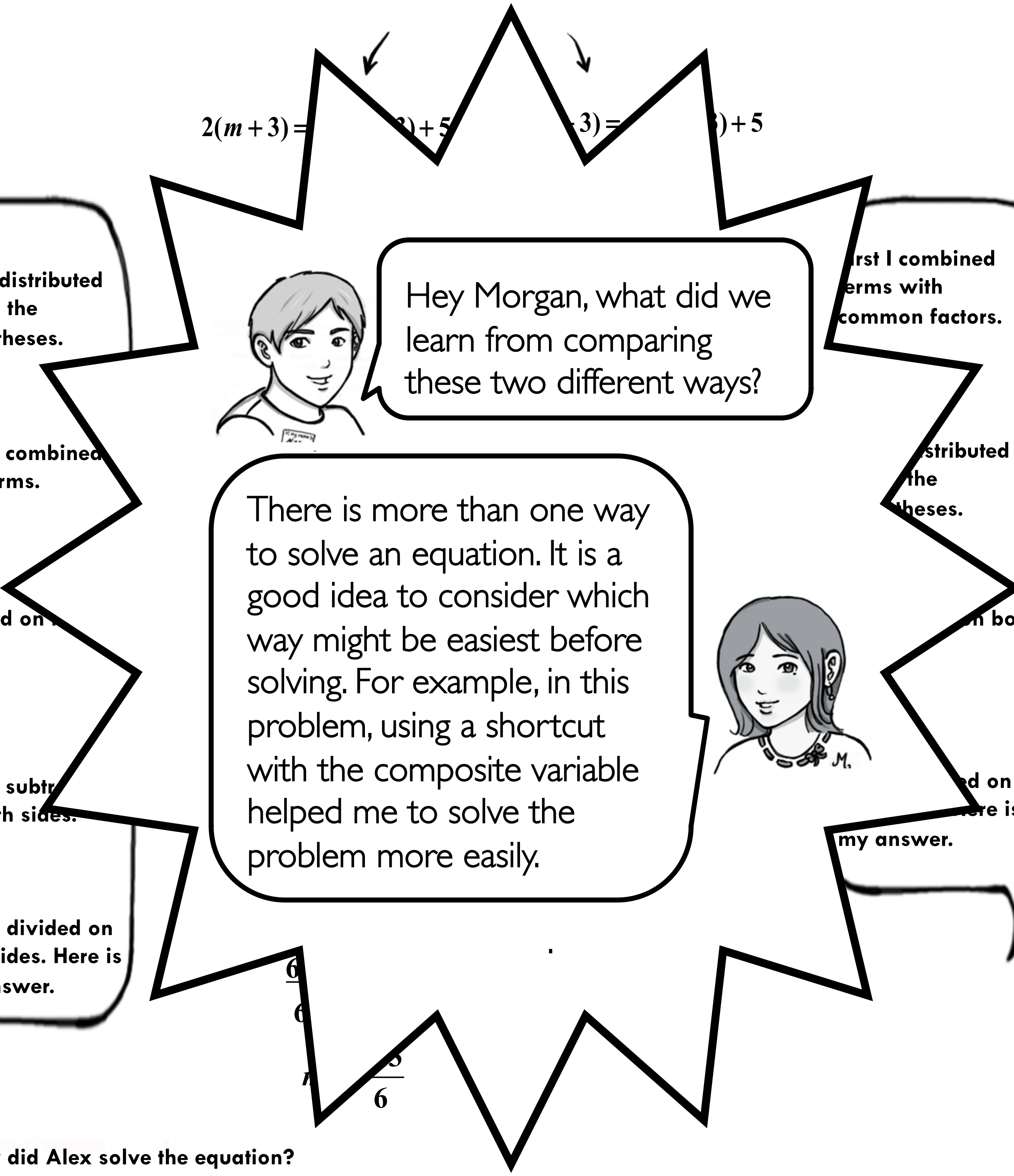


*** How did Alex solve the equation?**
*** How did Morgan solve the equation?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** On a timed test, would you rather use Alex's way or Morgan's way? Why?**

Student Worksheet 3.2.4

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 On a timed test, would you rather use Alex's way or Morgan's way? Why?

Which is better?

Alex and Morgan were asked to solve $2(g+3)=h$ for g

Alex's "divide first" way

$$2(g+3)=h$$

$$\downarrow$$

$$g+3=\frac{h}{2}$$

$$\downarrow$$

$$g=\frac{h}{2}-3$$

First I divided both sides by 2.

Then I subtracted 3 from both sides to get the answer.

Morgan's "distribute first" way

$$2(g+3)=h$$

$$\downarrow$$

$$2g+6=h$$

$$\downarrow$$

$$2g=h-6$$

$$\downarrow$$

$$g=\frac{h}{2}-3$$

First I distributed the 2 on the left side of the equation.

Next I subtracted both sides by 6.

Then I divided by two on both sides to get the answer.

*** How did Alex solve the equation?**
*** How did Morgan solve the equation?**
*** Describe two ways that Alex's and Morgan's ways are similar.**
*** Describe two ways that Alex's and Morgan's ways are different.**
*** What are some advantages of Alex's way? Of Morgan's way? Which way do you think is better for this problem?**
*** How would your answer change if particular values were substituted for *h*?**

Which is better?

Alex and Morgan were asked to solve $2(g+3)=h$ for g

Alex's "divide first" way

Morgan's "distribute first" way

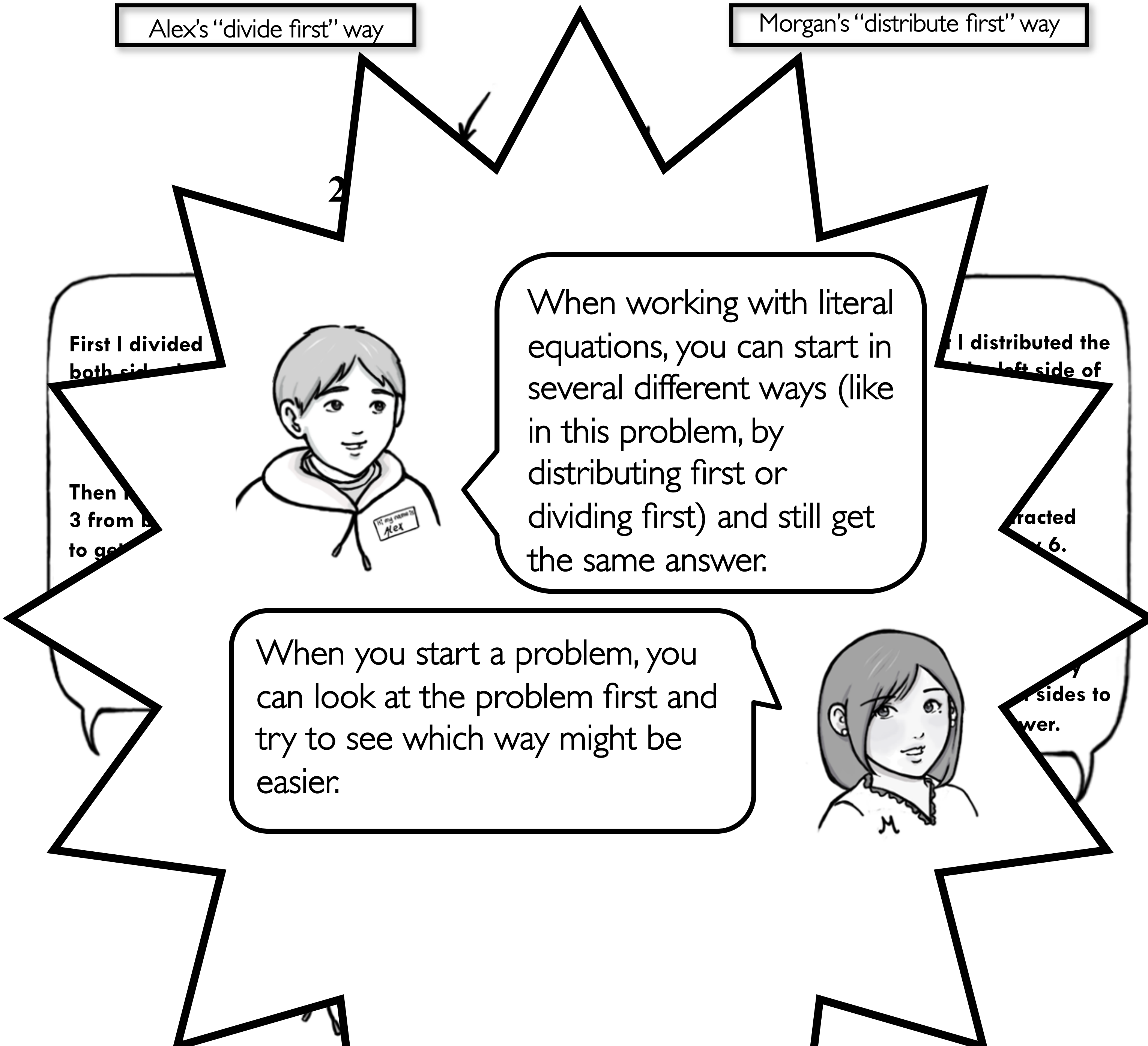
First I divided both sid

Then I 3 from b to get

I distributed the left side of

racted y 6.

sides to wer.



*** How did Alex solve th**
*** How did Morgan solve ion?**
*** Describe two ways tha s and Morg are simila**
*** Describe two ways that ex's and Morga s are different.**
*** Even though Alex and Morgan did different steps, why did they get the same answer?**
*** Which way do you think is better, Alex's way r Morgan's way?**
*** What are some advantages of Alex's way? Of Morgan's way?**

Student Worksheet 3.3.1

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 Describe two ways that Alex's and Morgan's ways are similar.

3 Describe two ways that Alex's and Morgan's ways are different.

4 What are some advantages of Alex's way? Of Morgan's way? Which way do you think is better for this problem? How would your answer change if particular values were substituted for *b*?

Which is better?

Alex and Morgan were asked to solve $\dfrac{m}{n}=\dfrac{x}{y}$ for n

Alex's "step by step" way

$$\frac{m}{n}=\frac{x}{y}$$

$$\downarrow$$

First I multiplied both sides by *n*.

$$(n)\frac{m}{n}=\frac{x}{y}(n)$$

$$\downarrow$$

And then I simplified the equation.

$$m=\frac{xn}{y}$$

$$\downarrow$$

Then I multiplied both sides by *y*.

$$(y)m=\frac{xn}{y}(y)$$

$$\downarrow$$

And then I simplified the equation.

$$my=xn$$

$$\downarrow$$

Then I multiplied both sides by 1/x.

$$(\frac{1}{x})my=xn(\frac{1}{x})$$

$$\downarrow$$

And then I simplified the equation, which is now solved for *n*.

$$\frac{my}{x}=n$$

Morgan's "all at once" way

$$\frac{m}{n}=\frac{x}{y}$$

$$\downarrow$$

I multiplied both sides by (*ny*/*x*).

$$(\frac{ny}{x})\frac{m}{n}=\frac{x}{y}(\frac{ny}{x})$$

$$\downarrow$$

And then I simplified the equation, which is now solved for *n*.

$$\frac{my}{x}=n$$

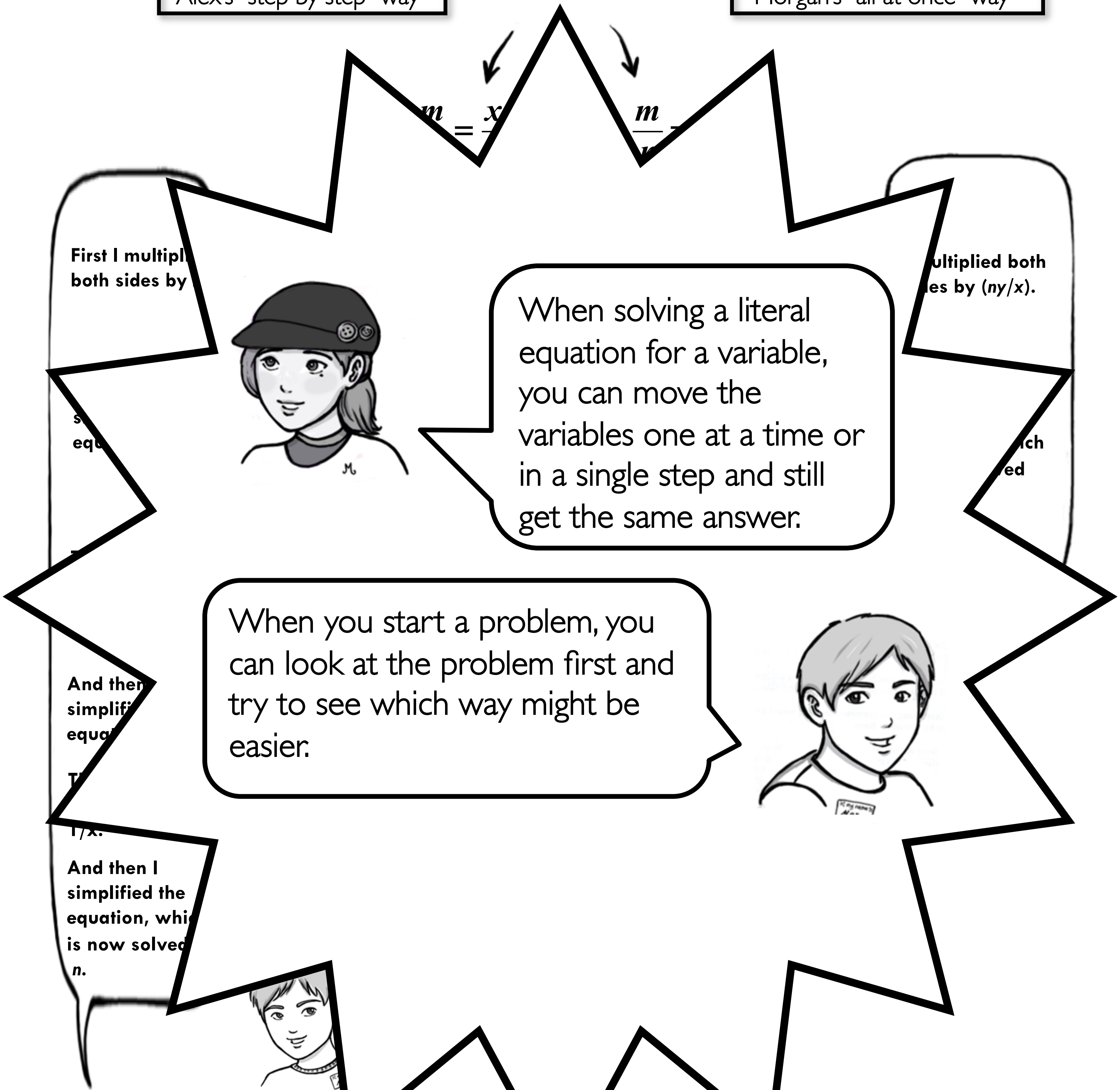
*** Describe Alex's way to a new student in your class. Describe Morgan's way to a new student in your class.**
*** What are some similarities and differences between Alex's and Morgan's *ways*?**
*** Why did Alex and Morgan get the same answer, even though they both did different steps?**
*** Which way is better, Alex's or Morgan's way?**
*** Can you think of another way to solve this problem correctly?**

Which is better?

Alex and Morgan were asked to solve $\frac{m}{n} = \frac{x}{y}$ for *n*

Alex's "step by step" way

Morgan's "all at once" way



*** Describe Alex's way to a new student in your c… scribe Morgan's …y to a new student in your class.**
*** What are some similarities and differences betw… lex's and Morgan's *ways*?**
*** Why did Alex and Morgan get the same answer, e…n though they both did different steps?**
*** Which way is better, Alex's or Morgan's way?**
*** Can you think of another way to solve this problem correctly?**

Student Worksheet 3.3.2

1 Describe Alex's way to a new student in your class.

2 Describe Morgan's way to a new student in your class.

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 Why did Alex and Morgan get the same answer, even though they both did different steps?

5 Which way is better, Alex's or Morgan's way?

6 Can you think of another way to solve this problem correctly?

Why does it work?

Alex and Morgan were asked to solve $2a+14=b$ for a

Alex's "subtract first" way	Morgan's "divide first" way
$2a+14=b$	$2a+14=b$
↓	↓
$2a=b-14$	$a+7=\dfrac{b}{2}$
↓	↓
$a=\dfrac{b-14}{2}$	$a=\dfrac{b}{2}-7$

First I subtracted 14 from both sides of the equation.

Then I divided both sides by 2 to get the answer.

First I divided both sides by 2.

Then I subtracted both sides by 7 to get the answer.

* How did Alex solve the equation?
* How did Morgan solve the equation?
* What are some similarities and differences between Alex's and Morgan's ways?
* Did Alex and Morgan get the same answer? If so, how do you know?
* When solving an equation, do we always get the same answer when we perform the same steps but in a different order? Why or why not?

Why does it work?

Alex and Morgan were asked to solve $2a + 14 = b$ for *a*

Alex's "subtract first" way

Morgan's "divide first" way

First I subt
from both sid
the equation

Then
sides by 2
the answer.

both

to

er.

When transforming literal equations, you can start in several different ways (like in this problem, by subtracting first or dividing first) and still get the same answer.

Before you start a problem, you can look at the problem first and try to see which way might be easier.

*** How did Alex solve the equ**
*** How did Morgan solve the**
*** What are some similarities a ferences be x's and Mo ways?**
*** Did Alex and Morgan get the ame answer? If do you know?**
*** When solving an equation, do we always get the e answer when we perform the same steps but in a different order? Why or why not?**

Student Worksheet 3.3.3

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 What are some similarities and differences between Alex's and Morgan's *ways*?

3 Did Alex and Morgan get the same answer? If so, how do you know?

4 When solving an equation, do we always get the same answer when we perform the same steps but in a different order? Why or why not?

Why does it work?

Alex and Morgan were asked to solve $\frac{3x}{4} = \frac{16}{11}$

Alex's "multiplication" way

$$\frac{3x}{4} = \frac{16}{11}$$

First I multiplied both sides of the equation by 4.

$$(4)\frac{3x}{4} = \frac{16}{11}(4)$$

$$3x = \frac{64}{11}$$

Then I multiplied both sides of the equation by 11.

$$(11)3x = \frac{64}{11}(11)$$

$$33x = 64$$

Then I divided on both sides of the equation by 33. Here is my answer.

$$\frac{33x}{33} = \frac{64}{33}$$

$$x = \frac{64}{33}$$

Morgan's "cross-multiplication" way

$$\frac{3x}{4} = \frac{16}{11}$$

First I cross-multiplied.

$$3x(11) = 16(4)$$

$$33x = 64$$

Then I divided both sides by 33.

$$\frac{33x}{33} = \frac{64}{33}$$

Here is my answer.

$$x = \frac{64}{33}$$

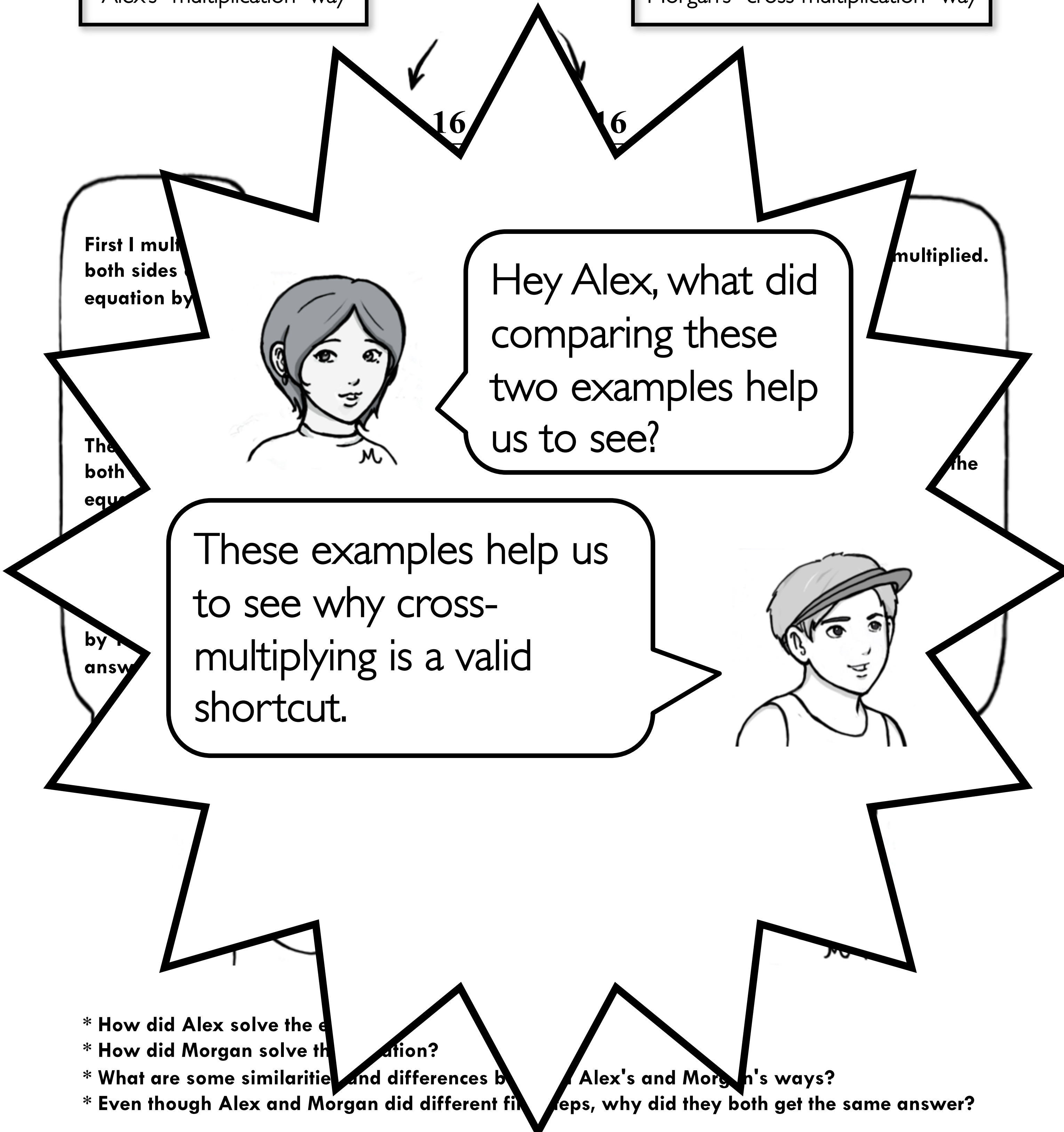
* How did Alex solve the equation?
* How did Morgan solve the equation?
* Describe two ways that Alex's and Morgan's ways are similar.
* Describe two ways that Alex's and Morgan's ways are different.
* Even though Alex and Morgan did different first steps, why did they both get the same answer?

Why does it work?

Alex and Morgan were asked to solve $\frac{3x}{4} = \frac{16}{11}$

Alex's "multiplication" way

Morgan's "cross-multiplication" way



*** How did Alex solve the e**

*** How did Morgan solve th ation?**

*** What are some similaritie and differences b Alex's and Morg n's ways?**

*** Even though Alex and Morgan did different fi eps, why did they both get the same answer?**

Student Worksheet 3.4.1

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 Describe two ways that Alex's and Morgan's ways are similar.

3 Describe two ways that Alex's and Morgan's ways are different.

4 Even though Alex and Morgan did different first steps, why did they both get the same answer?

Which is better?

Alex and Morgan were asked to solve the proportion $\frac{4}{5} = \frac{24}{n}$

Alex's "find equivalent fractions" way

$$\frac{4}{5} = \frac{24}{n}$$

$$\downarrow$$

$$\frac{4 \xrightarrow{\text{times what equals}}}{5} = \frac{24}{n}$$

$$\downarrow$$

$$\frac{4^{\times 6 =}}{5_{\times 6 =}} = \frac{24}{30}$$

$$\downarrow$$

$$n = 30$$

First I asked, what number multiplied by 4 equals 24?

It's 6. So I multiplied 5 times 6 in the denominator and I got 30.

Morgan's "cross-multiply" way

$$\frac{4}{5} = \frac{24}{n}$$

$$\downarrow$$

$$4n = 24 \cdot 5$$

$$\downarrow$$

$$\frac{4n}{4} = \frac{120}{4}$$

$$\downarrow$$

$$n = 30$$

First I cross-multiplied.

After rewriting 24 x 5 as 120, I then divided by 4 on both sides.

I got n = 30.

* How did Alex solve the proportion? How did Morgan solve the proportion?
* What are some similarities and differences between Alex's and Morgan's ways?
* Which way is easier for this problem, Alex's way or Morgan's way? Why?
* If the problem were changed to $\frac{7}{5} = \frac{24}{n}$, would Alex's way or Morgan's way be easier?
* Can you make up a general rule for when Alex's way is better and when Morgan's way is better?

Which is better?

Alex and Morgan were asked to solve the proportion $\frac{4}{5} = \frac{24}{n}$

Alex's "find equivalent fractions" way

Morgan's "cross-multiply" way

$\frac{4}{5} = \frac{24}{n}$ $\frac{4}{5} = \frac{24}{n}$

First I asked, what number multiplied by 4

It's 6 multipl times 6 de

rst I cross- ultiplied.

ting s 120, ided

30.

When solving proportion problems, you can start in several different ways (like by cross-multiplying or using the equivalent fractions strategy) and you will get the same answer. In fact, the equivalent fractions strategy may be easier or faster than cross-multiplying in some problems.

Before you start a problem, you can look at the problem first and try to see which way might be easier.

*** How did Alex solve the p... n?**

*** What are some similaritie... ways?**

*** Which way is easier for th... , Alex... rgan's... y?**

*** If the problem were chang...** $\frac{\ }{5} = \frac{24}{n}$ **, w... way or M... way be easier?**

*** Can you make up a general rule for when Alex... ay is better and when Morgan's way is better?**

Student Worksheet 3.4.2

1a How did Alex solve the proportion?

1b How did Morgan solve the proportion?

2 What are some similarities and differences between Alex's and Morgan's *ways*?

3 Which way is easier for this problem, Alex's way or Morgan's way? Why?

4 If the problem were changed to $\frac{7}{5} = \frac{24}{n}$, would Alex's way or Morgan's way be easier?

5 Can you make up a general rule for when Alex's way is better and when Morgan's way is better?

Which is better?

Alex and Morgan were asked to solve the proportion $\dfrac{2}{24} = \dfrac{3}{n}$

Alex's "cross-multiply" way

$$\frac{2}{24} = \frac{3}{n}$$

$$\downarrow$$

First I cross-multiplied.

$$2n = 3 \cdot 24$$

$$\downarrow$$

After rewriting 3 x 24 as 72, I then divided by 2 on both sides.

$$2n = 72$$

$$\downarrow$$

I got n = 36.

$$n = 36$$

Morgan's "unit rate" way

$$\frac{2}{24} = \frac{3}{n}$$

$$\downarrow$$

First I asked, what number multiplied by 2 is 24? It is 12.

$$\frac{2}{24 \underset{\times 12}{\downarrow}} = \frac{3}{n \downarrow}$$

$$\downarrow$$

So I multiplied 3 times 12 to get n.

$$3 \cdot 12 = n$$

$$\downarrow$$

It is 36.

$$n = 36$$

*** How did Alex solve the proportion? How did Morgan solve the proportion?**

*** What are some similarities and differences between Alex's and Morgan's ways?**

*** Which way is easier for this problem, Alex's way or Morgan's way? Why?**

*** If the problem were changed to $\dfrac{7}{5} = \dfrac{24}{n}$, would Alex's way or Morgan's way be easier?**

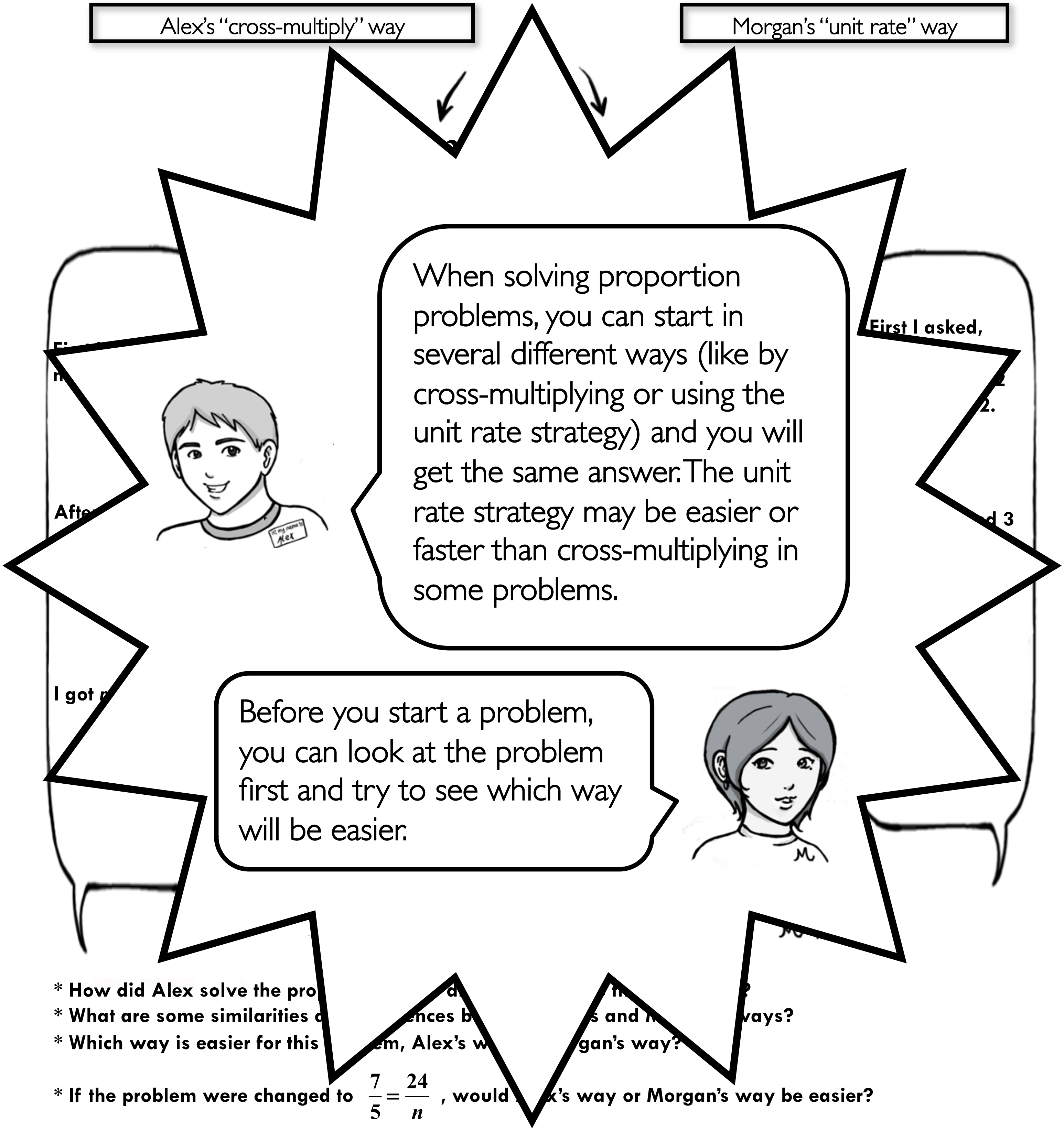
*** Can you make up a general rule for when Alex's way is better and when Morgan's way is better?**

Which is better?

Alex and Morgan were asked to solve the proportion $\frac{2}{24} = \frac{3}{n}$

Alex's "cross-multiply" way

Morgan's "unit rate" way



First I asked,

*** How did Alex solve the pro... ?**

*** What are some similarities a...ences b...s and M...ways?**

*** Which way is easier for this ...em, Alex's w...gan's way?**

*** If the problem were changed to** $\frac{7}{5} = \frac{24}{n}$ **, would ...x's way or Morgan's way be easier?**

*** Can you make up a general rule for when Alex's way is better and when Morgan's way is better?**

Student Worksheet 3.4.3

1a How did Alex solve the proportion?

1b How did Morgan solve the proportion?

2 What are some similarities and differences between Alex's and Morgan's *ways*?

3 Which way is easier for this problem, Alex's way or Morgan's way? Why?

4 If the problem were changed to $\frac{7}{5} = \frac{24}{n}$, would Alex's way or Morgan's way be easier?

5 Can you make up a general rule for when Alex's way is better and when Morgan's way is better?

Which is correct?

Alex and Morgan were asked to simplify $\frac{2x}{5} \cdot \frac{8}{10}$

Alex's "cross-multiply" way

$$\frac{2x}{5} \cdot \frac{8}{10}$$

First I cross-multiplied.

$$2x(10) = 8(5)$$

$$20x = 40$$

Then I divided on both sides.

$$\frac{20x}{20} = \frac{40}{20}$$

Here is my answer.

$$x = 2$$

Morgan's "multiply the numerators and the denominators" way

$$\frac{2x}{5} \cdot \frac{8}{10}$$

First I multiplied the numerator times the numerator and the denominator times the denominator.

$$\frac{2x \cdot 8}{5 \cdot 10}$$

Then I simplified the expression.

$$\frac{16x}{50}$$

Here is my answer.

$$\frac{8x}{25}$$

* How did Alex simplify the expression?
* How did Morgan simplify the expression?
* Whose answer is correct, Alex's or Morgan's? How do you know?
* What are some similarities and differences between Alex's and Morgan's ways?
* Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?

Which is correct?

Alex and Morgan were asked to simplify $\dfrac{2x}{5} \cdot \dfrac{8}{10}$

Alex's "cross-multiply" way

Morgan's "multiply the numerators and the denominators" way

First I cross-multiplied.

Then I divided both sides.

Here answer.

st I multiplied e numerator

he imes ator.

ified

y

Hey Alex, what did we learn from comparing these right and wrong ways?

Don't confuse multiplying fractions with solving a proportion. When multiplying fractions, multiply the numerator times the numerator and the denominator times the denominator.

*** How did Alex simplify the express**
*** How did Morgan simplify the expr ?**
*** Whose answer is correct, Alex's or organ's? How d know?**
*** What are some similarities and differences between A s and Morgan's ways?**
*** Can you state a general rule that describes what you h ve learned from comparing Alex's and Morgan's ways of simplifying this expression?**

Student Worksheet 3.4.4

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 Can you state a general rule that describes what you have learned from comparing Alex's and Morgan's ways of simplifying this expression?