Which is correct?

Alex and Morgan were asked to solve $-5d > 25$

Alex's "divide by -5" way	Morgan's "divide and flip the inequality sign" way
$-5d > 25$	$-5d > 25$
$\dfrac{-5d}{-5} > \dfrac{25}{-5}$	$\dfrac{-5d}{-5} < \dfrac{25}{-5}$
$d > -5$	$d < -5$

First, I divided both sides by -5.

My answer is $d > -5$.

First, I divided both sides by -5, and flipped the inequality sign.

My answer is $d < -5$.

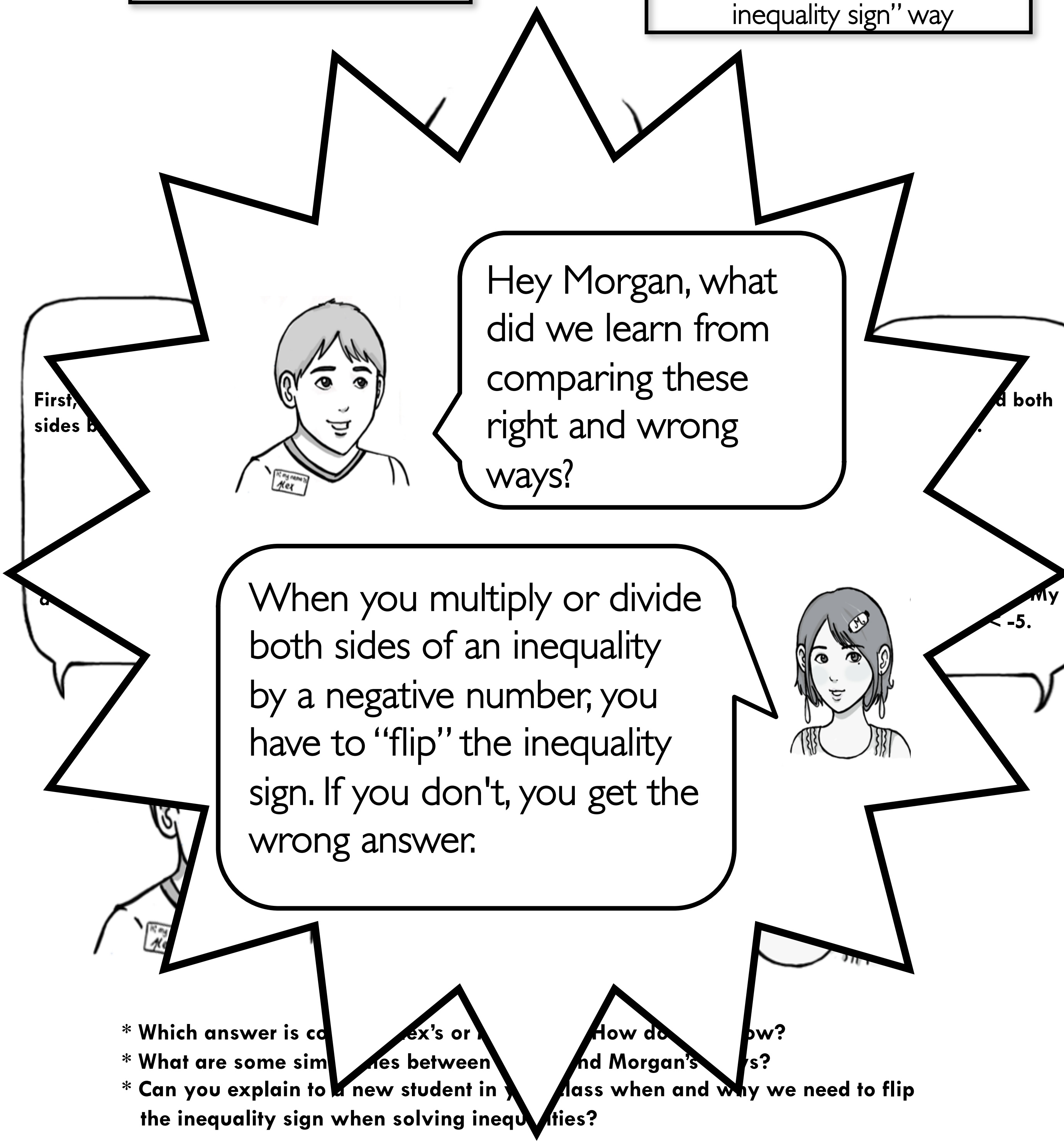
*** Which answer is correct, Alex's or Morgan's? How do you know?**
*** What are some similarities between Alex's and Morgan's ways?**
*** Can you explain to a new student in your class when and why we need to flip the inequality sign when solving inequalities?**

Which is correct?

Alex and Morgan were asked to solve $-5d > 25$

Alex's "divide by -5" way

Morgan's "divide and flip the inequality sign" way



* **Which answer is co... ex's or ... How do ... ow?**
* **What are some sim... ies between ... nd Morgan's ... s?**
* **Can you explain to a new student in y... class when and why we need to flip the inequality sign when solving inequ...ities?**

Student Worksheet 5.1.1

1 Which answer is correct, Alex's or Morgan's? How do you know?

2 What are some similarities between Alex's and Morgan's *ways*?

3 Can you explain to a new student in your class when and why we need to flip the inequality sign when solving inequalities?

Why does it work?

Alex and Morgan were asked to solve $-3r < 9$

Alex's "avoid dividing by a negative" way

$$-3r < 9$$

First, I added 3*r* to both sides.

$$-3r + 3r < 9 + 3r$$
$$0 < 9 + 3r$$

Then, I subtracted 9 from both sides.

$$0 - 9 < 9 + 3r - 9$$

$$-9 < 3r$$

Finally, I divided both sides by 3.

$$\frac{-9}{3} < \frac{3r}{3}$$

This is my answer -- that *r* is greater than -3.

$$-3 < r$$

Morgan's "divide by a negative and flip the inequality" way

$$-3r < 9$$

First, I divided both sides by -3.

$$\frac{-3r}{-3} > \frac{9}{-3}$$

Then, because I divided by a negative number, I flipped the direction of the inequality sign. My answer is *r* > -3.

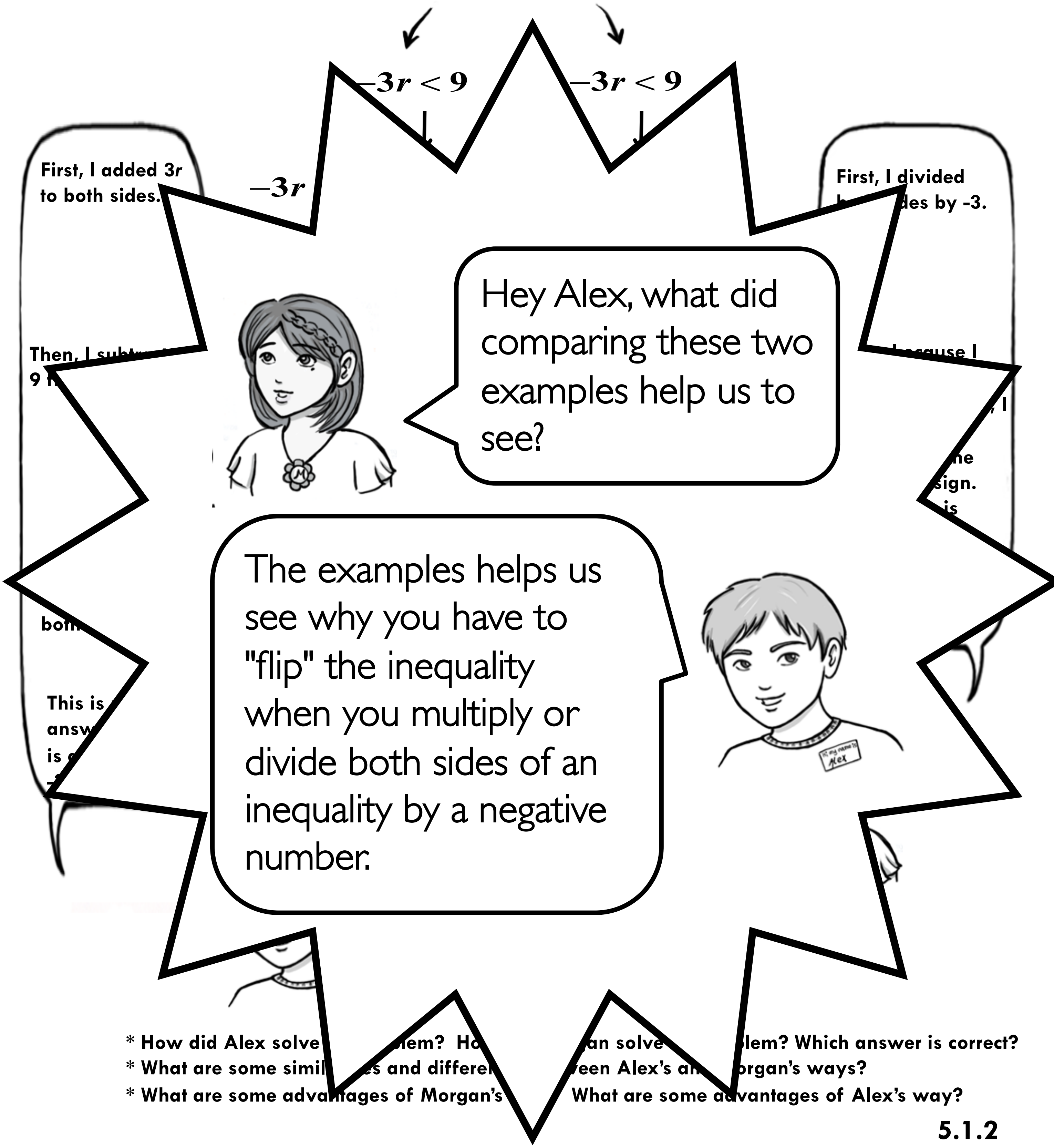
$$r > -3$$

*** How did Alex solve the problem? How did Morgan solve the problem? Which answer is correct?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** What are some advantages of Morgan's way? What are some advantages of Alex's way?**

Why does it work?

Alex and Morgan were asked to solve $-3r < 9$

Alex's "avoid dividing by a negative" way	Morgan's "divide by a negative and flip the inequality" way
$-3r < 9$	$-3r < 9$



*** How did Alex solve [...]lem? Ho[...]an solve [...]lem? Which answer is correct?**
*** What are some simil[...]es and differe[...]een Alex's a[...]organ's ways?**
*** What are some adva[...]ages of Morgan's [...] What are some advantages of Alex's way?**

Student Worksheet 5.1.2

1a How did Alex solve the problem?

1b How did Morgan solve the problem?

2 Which answer is correct?

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 What are some advantages of Morgan's way? What are some advantages of Alex's way?

Which is correct?

Alex and Morgan were asked to solve $5x < 10$

Alex's "divide" way

$$5x < 10$$

$$\frac{5x}{5} < \frac{10}{5}$$

$$x < 2$$

First I divided by 5 on both sides of the inequality.

This is my answer.

Morgan's "divide and 'flip' the inequality" way

$$5x < 10$$

$$\frac{5x}{5} < \frac{10}{5}$$

$$x > 2$$

First I divided by 5 on both sides of the inequality.

Since I divided on both sides of the inequality, I flipped the inequality sign. This is my answer.

*** How did Alex solve the inequality?**
*** How did Morgan solve the inequality?**
*** Whose answer is correct, Alex's or Morgan's? How do you know?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of inequality?**

Which is correct?

Alex and Morgan were asked to solve $5x < 10$

Alex's "divide" way

Morgan's "divide and 'flip' the inequality" way

First I divided by 5 on both sides of the inequality.

This is my a

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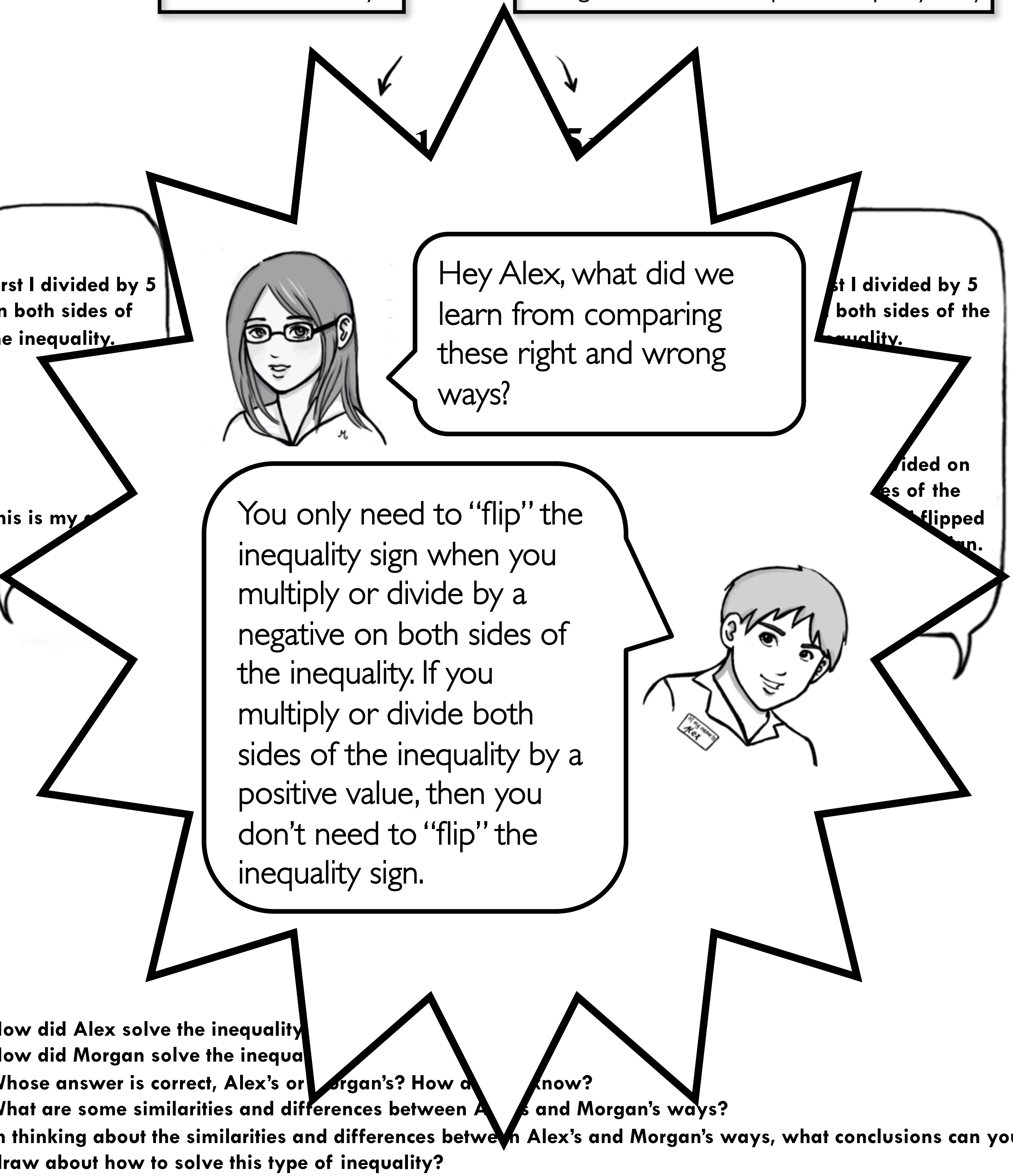
ided on es of the flipped n.



*** How did Alex solve the inequality**
*** How did Morgan solve the inequa**
*** Whose answer is correct, Alex's or organ's? How d know?**
*** What are some similarities and differences between A s and Morgan's ways?**
*** In thinking about the similarities and differences betwe n Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of inequality?**

Student Worksheet 5.1.3

1a How did Alex solve the inequality?

1b How did Morgan solve the inequality?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

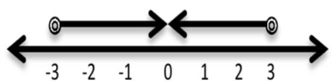
4 In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of inequality?

Why does it work?

Alex and Morgan were asked to solve $|n| < 3$

Alex's "number line" way

$$|n| < 3$$



$$n < 3 \quad and \quad n > -3$$

First, I drew a number line. The inequality says that *n* is any number whose distance from 0 is smaller than 3. I marked these values with arrows.

Using the shaded values in my number line, I wrote a compound inequality to indicate my solution. I found that *n* < 3 and *n* > -3.

Morgan's "positive and negative" way

$$|n| < 3$$

$$n < 3 \quad and \quad -n < 3$$

$$n < 3 \quad and \quad n > -3$$

First, I rewrote the absolute value equation from the original problem as a compound inequality. If the absolute value of *n* is smaller than 3, this means that either *n* is smaller than 3 or the opposite of *n* is smaller than 3.

Then, I solved the second part of the inequality for *n* by dividing both sides by (-1). My solution is *n* < 3 and *n* > -3.

*** Why did Alex draw a number line and mark all of the values within three units of zero as a first step?**
*** How did Morgan solve this problem?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** What are some advantages of Alex's way? Of Morgan's way?**

Why does it work?

Alex and Morgan were asked to solve $|n| < 3$

Alex's "number line" way

Morgan's "positive and negative" way

First, I drew
number line.
inequality say
that *n* is any
number whose
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n > -3.

First, I rewrote the
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ved the
f the

Hey Morgan, what did comparing these two examples help us to see?

You can solve an inequality with absolute value by graphing the possible solutions on a number line, or by using algebra (rewriting the expression as a compound inequality without the absolute value sign, then simplifying). These examples help us see why the algebraic way of solving absolute value inequalities works.

* Why d lex draw a n ee units of 2 as a first step?
* How did Morgan solve
* What are some similarit feren ex's n's ways?
* What are some advanta lex's wa an's way

Student Worksheet 5.2.1

1 Why did Alex draw a number line and mark all of the values within three units of zero as a first step?

2 How did Morgan solve this problem?

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 What are some advantages of Alex's way? Of Morgan's way?

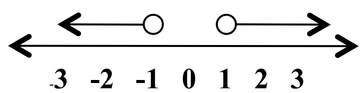
Why does it work?

Alex and Morgan were asked to solve $|w|>1$

Alex's "number line" way

$|w|>1$

↓



↓

$w>1 \quad or \quad w<-1$

First, I drew a number line. The inequality says that *w* is any number whose distance from 0 is bigger than 1. I marked these values using arrows.

Using the shaded values in my number line, I wrote a compound inequality to indicate my solution. I found that *w* > 1 or *w* < -1.

Morgan's "positive and negative" way

$|w|>1$

↓

$w>1 \quad or \quad -w>1$

↓

$w>1 \quad or \quad w<-1$

First, I rewrote the absolute value from the original problem as a compound inequality. If the absolute value of *w* is greater than 1, this means that either *w* is greater than 1 or the opposite of w is greater than 1.

Then, I solved the second part of the inequality for *w* by dividing by sides by (-1). My solution is *w* > 1 or *w* < -1.

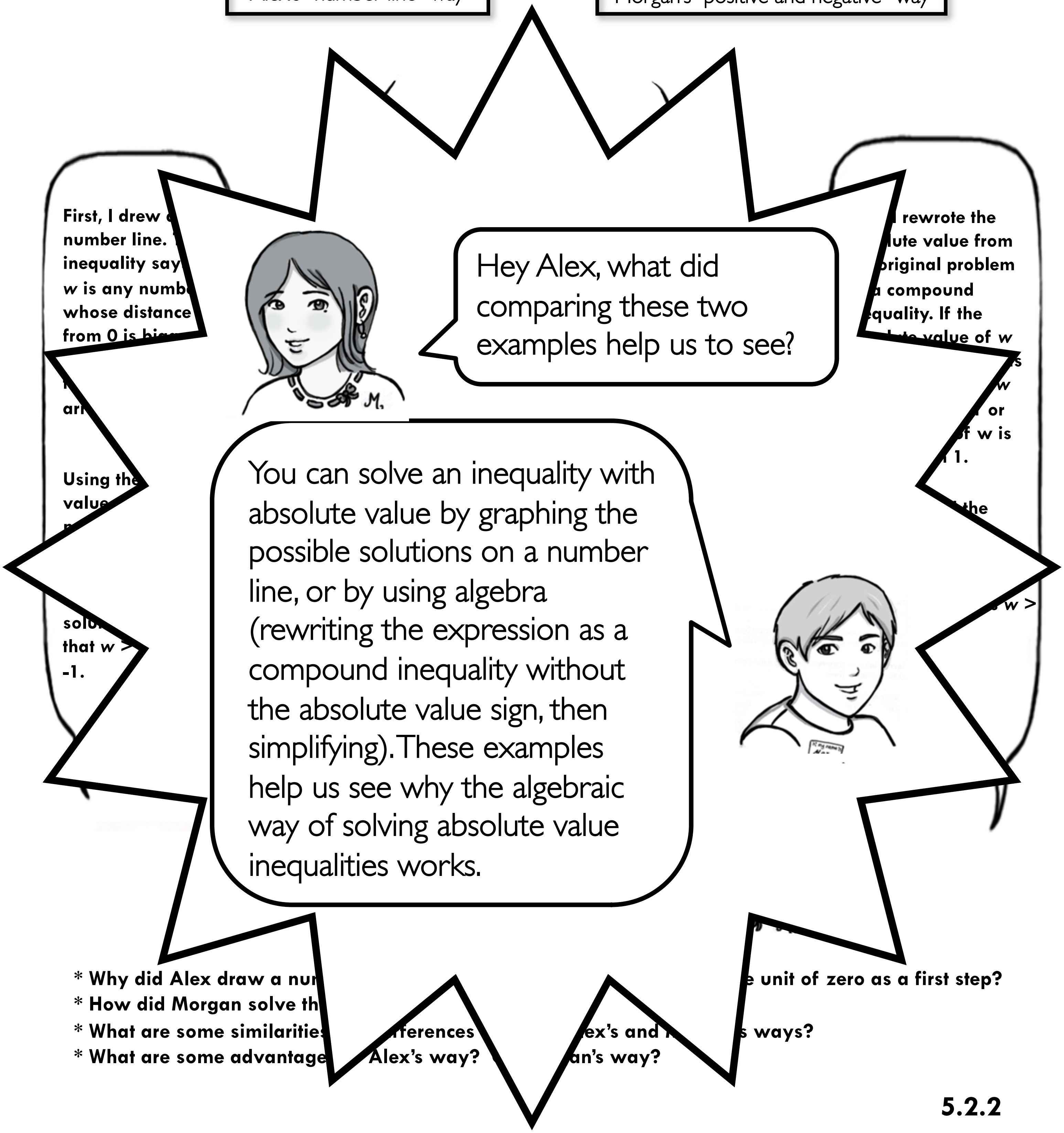
*** Why did Alex draw a number line and mark all of the values beyond one unit of zero as a first step?**
*** How did Morgan solve this problem?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** What are some advantages of Alex's way? Of Morgan's way?**

Why does it work?

Alex and Morgan were asked to solve $|w|>1$

Alex's "number line" way

Morgan's "positive and negative" way



* Why did Alex draw a nu… …e unit of zero as a first step?
* How did Morgan solve th…
* What are some similaritie… …fferences … …ex's and … …s ways?
* What are some advantage… Alex's way? … …an's way?

Student Worksheet 5.2.2

1 Why did Alex draw a number line and mark all of the values beyond one unit of zero as a first step?

2 How did Morgan solve this problem?

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 What are some advantages of Alex's way? Of Morgan's way?

Why does it work?

Alex and Morgan were asked to solve $|a+8|>1$

Alex's "compound inequality" way

First, I rewrote the absolute value inequality as a compound inequality without the absolute value.

$$|a+8|>1$$

$$a+8>1 \quad \text{or} \quad -(a+8)>1$$

Then, I considered a + 8 > 1.

$$a+8>1$$

I solved this inequality.

$$a>-7$$

Then, I considered −(a + 8) > 1.

$$-(a+8)>1$$

$$\frac{-(a+8)}{-1}<\frac{1}{-1}$$

$$a+8<-1$$

$$a+8-8<-1-8$$

I solved this inequality.

$$a<-9$$

Here is my solution.

$$a<-9 \quad or \quad a>-7$$

Morgan's "number line" way

Absolute value inequalities like this one mean that the distance between *a* and *b* is bigger than *c*.

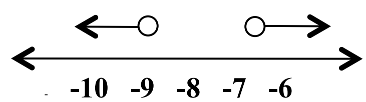
$$|a+8|>1$$

$$|a-b|>c$$

To work with this particular inequality, I first rewrote the value within the absolute value marks as the difference of two values.

$$|a-(-8)|>1$$

Then I drew a number line marking all the possible values for a that are greater than one unit away from (-8).



Here is my solution.

$$a<-9 \quad or \quad a>-7$$

* How did Alex solve this absolute value inequality?
* How did Morgan solve this absolute value inequality?
* Describe at least two ways that Alex's and Morgan's ways are different.
* What are some advantages of Alex's way? Of Morgan's way?

Why does it work?

Alex and Morgan were asked to solve $|a+8|>1$

Alex's "compound inequality" way

Morgan's "number line" way

$|a+8|>1$

$|a+8|>1$

First, I rewrote the absolute value inequality as a compound inequality with the absolute val

Then, I considered a + 8 > 1.

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Then, I con

I s i

Here is my solution.

Absolute value inequalities like this one mean that the distance between *a* and *b* is bigger than *c*.

with this ar inequality, I wrote the value the absolute value s as the difference o values.

er e for a that n one unit (-8).

Hey Morgan, what did comparing these two examples help us to see?

You can solve an inequality with absolute value by graphing the possible solutions on a number line, or by using algebra (rewriting the expression as a compound inequality without the absolute value sign, then simplifying). These examples help us see why the algebraic way of solving absolute value inequalities work.

*** How did Alex so absolute ality?**

*** How did Morgan ve this absolut nequality?**

*** Describe at least two ways that Alex Morgan's ways are different.**

*** What are some advantages of Alex's w y? Of Morgan's way?**

Student Worksheet 5.2.3

1a How did Alex solve this absolute value inequality?

1b How did Morgan solve this absolute value inequality?

2 Describe at least two ways that Alex's and Morgan's ways are different.

3 What are some advantages of Alex's way? Of Morgan's way?

Which is correct?

Alex and Morgan were asked to graph $4x - 3y > 12$

Alex's "shade by choosing a test point" way

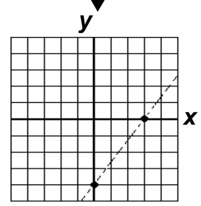
I first subtracted 4x from both sides.

$$4x - 3y > 12$$
$$-3y > -4x + 12$$

Then I divided both sides by -3.

$$y < \frac{4}{3}x - 4$$

Then I graphed the line y = 4/3x - 4 using its slope (4/3) and its y-intercept (-4), using a dashed line.

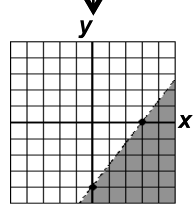


Then I tested the point (0,0) to see if it made the inequality true. I plugged 0 in for x and 0 in for y in the inequality.

$$y < \frac{4}{3}x - 4$$
$$0 < \frac{4}{3}(0) - 4$$
$$0 < -4 \quad \text{False}$$

I got 0 is less than -4, which is not true.

So I shaded the region that did not include the point (0,0), which is below the line on my graph.



Morgan's "shade by looking at the inequality sign" way

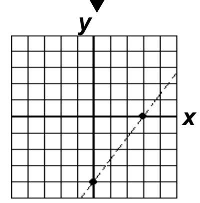
First I found the x-intercept.

$$4x - 3(0) = 12$$
$$x = 3$$
x-intercept is $(3, 0)$

Then I found the y-intercept.

$$4(0) - 3y = 12$$
$$y = -4$$
y-intercept is $(0, -4)$

Then I plotted the two points and connected them with a dashed line, to graph the line 4x - 3y = 12.

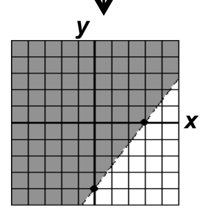


Now I need to find out where to shade for the inequality.

$$4x - 3y > 12$$

The original problem has the greater than sign.

So I shaded above the line, because greater than means above.



* How did Alex graph the inequality? How did Morgan graph the inequality?
* Which answer is correct, Alex's or Morgan's? How do you know?
* Why did Alex change the direction of the inequality sign?
* What are some similarities and differences between Alex's and Morgan's *ways?*
* In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of problem?

Which is correct?

Alex and Morgan were asked to graph $4x-3y>12$

Alex's "shade by choosing a test point" way	Morgan's "shade by looking at the inequality sign" way

I first subtracted 4x from both sides.

$4x-3y>12$

$-3y>$ ... $+12$

Then I divided bo... sides by -3.

Then I graphed the line y = 4/3x - 4 using its slope (4/3) and it... (-4), ... line.

Then I tes... (0,0) to see... the inequality... plugged 0... 0 in f...

than ... is not true.

So I shade... region th... not inc... poin... is ... on my graph.

First I found the x-intercept.

$4x-3(0)=12$

$x=3$

...intercep... ,0)

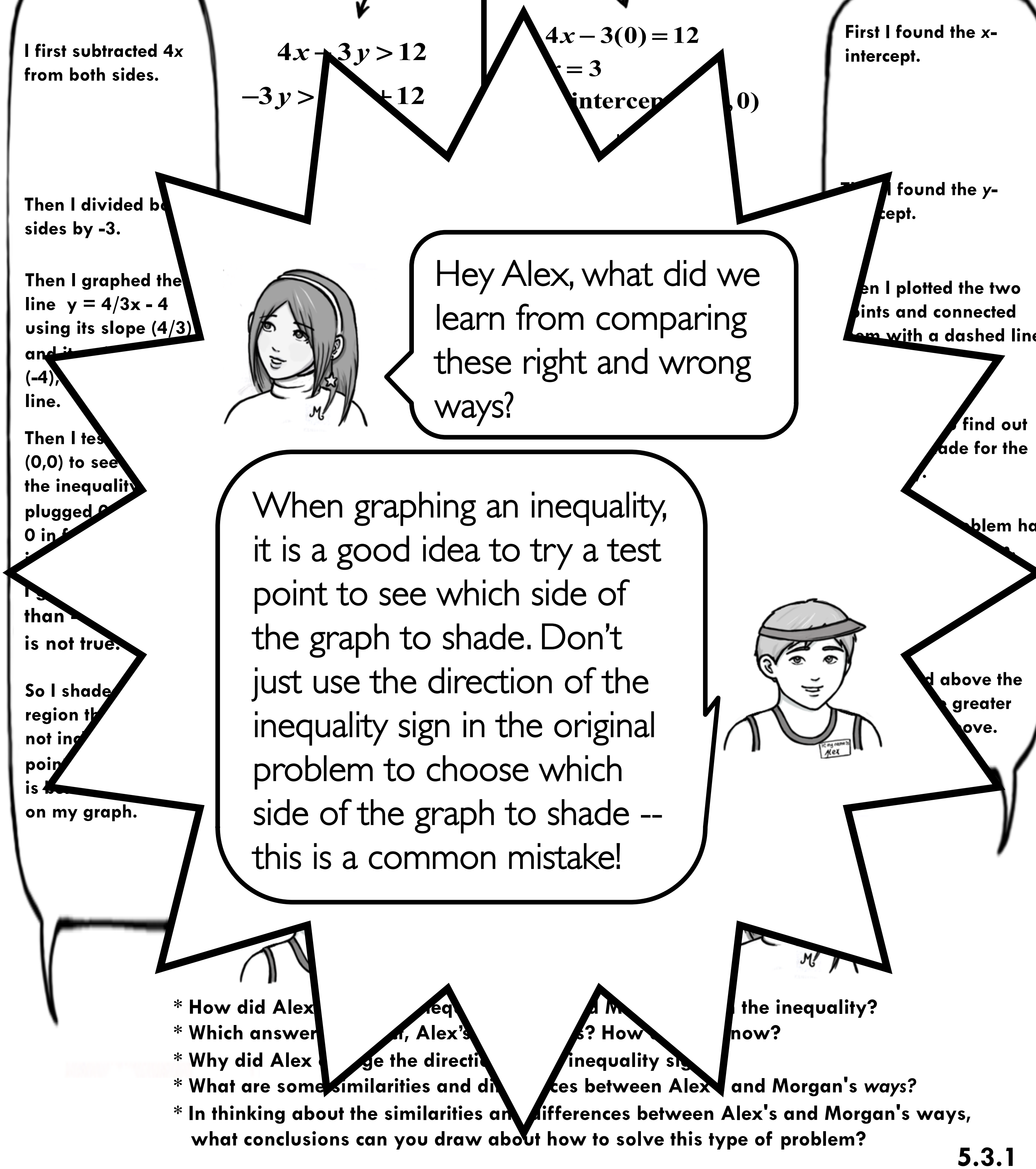
... found the y-... cept.

...en I plotted the two ...ints and connected ...em with a dashed line,

... find out ...ade for the ...

...blem has ...

... above the ... greater ...ove.



* How did Alex ... ineq... M... the inequality?
* Which answe... Alex's ...s? How ... now?
* Why did Alex c...ge the directi... inequality sig...
* What are some similarities and di...ces between Alex... and Morgan's *ways?*
* In thinking about the similarities an... differences between Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of problem?

Student Worksheet 5.3.1

1a How did Alex graph the inequality?

1b How did Morgan graph the inequality?

2 Which answer is correct, Alex's or Morgan's? How do you know?

3 Why did Alex change the direction of the inequality sign?

4 What are some similarities and differences between Alex's and Morgan's *ways?*

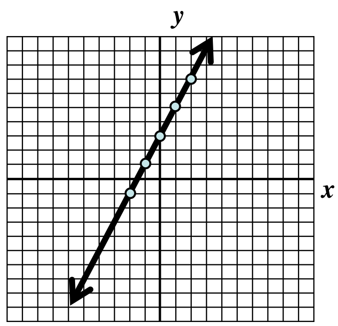
5 In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of problem?

How do they differ?

Alex was asked to graph the equation $y = 2x + 3$, and Morgan was asked to graph the inequality $y > 2x + 3$.

Alex's "graph $y = 2x + 3$" way

$$y = 2x + 3$$

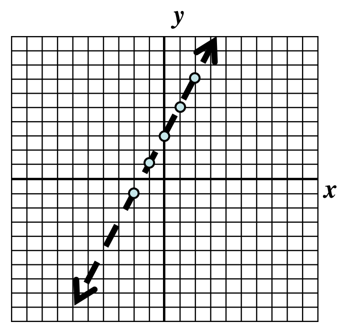


First I graphed the y-intercept, (0,3) and counted up 2, right 1 and down 2, left 1 to plot other points on the line.

I connected the points to draw the graph of the line.

Morgan's "graph $y > 2x + 3$" way

$$y > 2x + 3$$



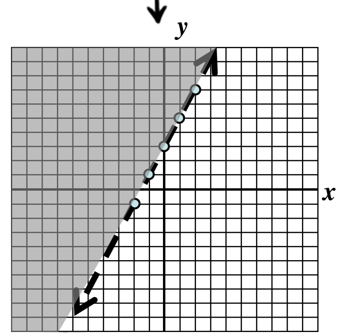
$$(-5, 6)$$

$$y > 2x + 3$$

$$6 > 2(-5) + 3$$

$$6 > -10 + 3$$

$$6 > -7 \quad \text{True}$$



First I graphed the y-intercept, (0,3) and counted up 2, right 1 and down 2, left 1 to plot other points on the line.

I connected the points to draw the graph of the line. I drew a dotted line, since my graph will not include the points on the line.

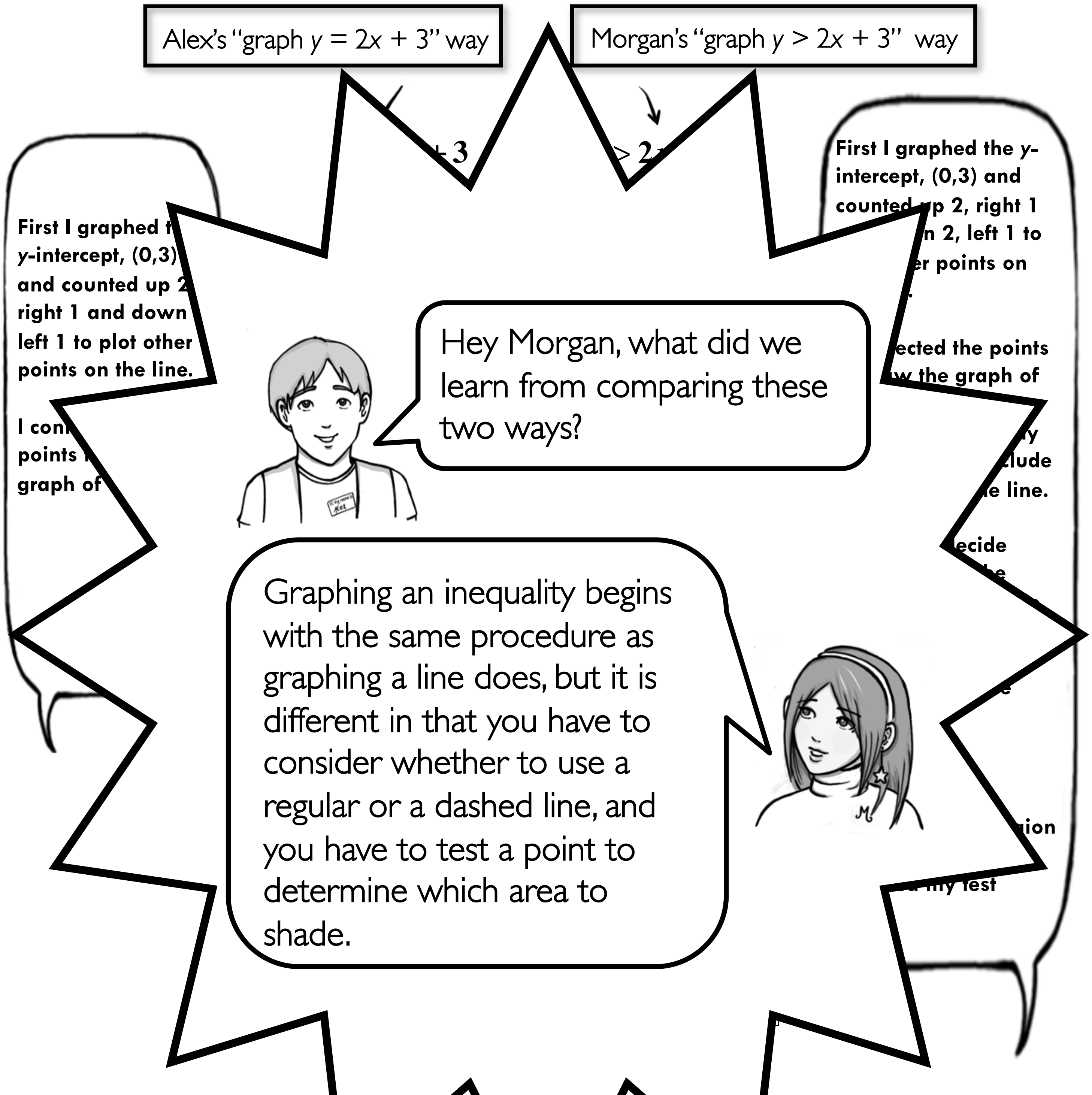
In order to decide which side of the dashed line to shade, I tested a point. I chose (-5,6). That point satisfied the inequality.

So I shaded the region of the graph that included my test point.

*** How did Alex graph his equation? How did Morgan graph her inequality?**
*** What are some similarities and differences between Alex's and Morgan's problems?**
*** What are some similarities and differences between Alex's and Morgan's ways?**
*** When you graph an equation, which points are you graphing? When you graph an inequality, which points are you graphing?**

How do they differ?

Alex was asked to graph the equation $y = 2x + 3$,
and Morgan was asked to graph the inequality $y > 2x + 3$.



*** How did Alex graph his e… …ow … …raph … …ality?**
*** What are some similaritie… …ifference… …Alex's an… …an's problems?**
*** What are some similarities …nd differences … …n Alex's and M… …an's ways?**
*** When you graph an equation, which points a… …ou graphing? When you graph an inequality, which points are you graphing?**

Student Worksheet 5.3.2

1a How did Alex graph his equation?

1b How did Morgan graph her inequality?

2 What are some similarities and differences between Alex's and Morgan's problems?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 When you graph an equation, which points are you graphing? When you graph an inequality, which points are you graphing?

How do they differ?

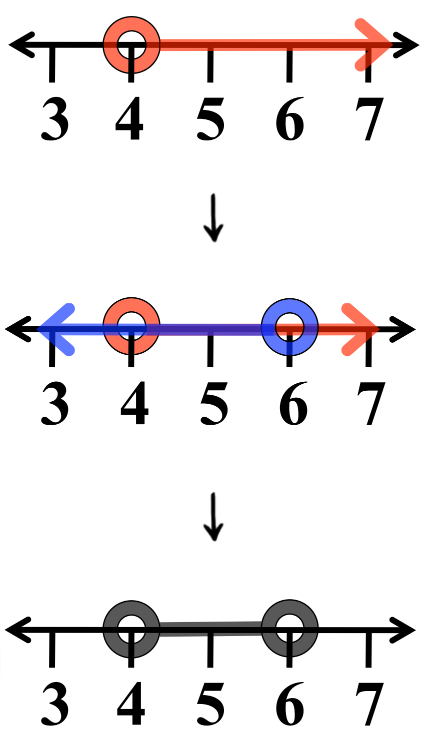
Alex was asked to graph all of the *x* values for which $x > 4$ and $x < 6$, and Morgan was asked to graph all of the *x* values for which $x > 4$ or $x < 6$.

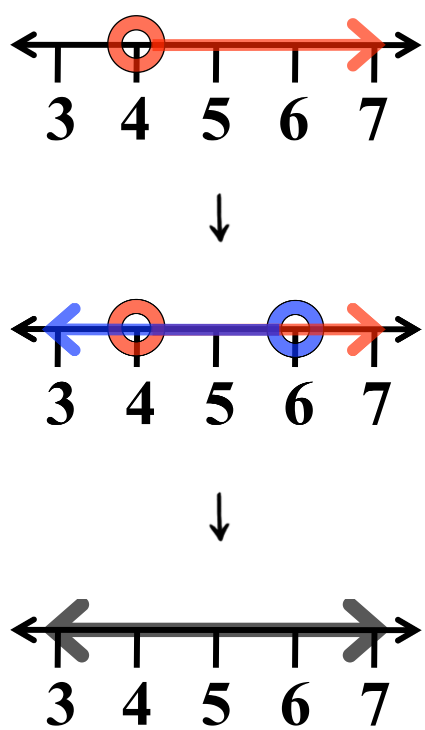
Alex's "graph an inequality with '*and*'" way	Morgan's "graph an inequality with '*or*'" way
$x > 4$ and $x < 6$	$x > 4$ or $x < 6$

First I graphed all of the values for which *x* is greater than 4.

Then I graphed all of the values for which *x* is less than 6.

The overlapping values on the two lines are the values that satisfy both $x > 4$ and $x < 6$. So those are the values I included on my graph.





First I graphed all of the values for which *x* is greater than 4.

Then I graphed all of the values for which *x* is less than 6.

The values that are included on either of the two lines are the values that satisfy either $x > 4$ or $x < 6$. This includes all numbers on the number line. So those are the values I included on my graph.

*** How did Alex graph the solution to his problem? How did Morgan graph the solution to her problem?**

*** What are some similarities and differences between Alex's and Morgan's problems?**

*** What are some similarities and differences between Alex's and Morgan's ways?**

*** How is graphing all of the values that satisfy both one inequality *and* another different from graphing all of the values that satisfy either one inequality *or* another?**

How do they differ?

Alex was asked to graph all of the *x* values for which $x > 4$ and $x < 6$, and Morgan was asked to graph all of the *x* values for which $x > 4$ or $x < 6$.

Alex's "graph an inequality with '*and*'" way

Morgan's "graph an inequality with '*or*'" way

$x > 4$ ... $x < 6$ ... > 4 or ... 6

First I graphed all of the values for which *x* is greater than 4.

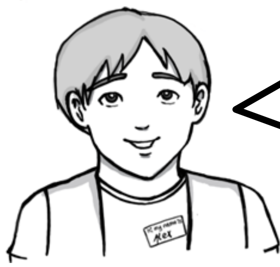
Then ... all of th... for which ... less than 6.

... two ... the value... satisfy both $x > 4$ and ... So thos... value... inc... graph.

... graphed ... the ...es for ...ch *x* is ...eater than 4.

...d ...lues ...x is ... 6.

...re ...es that ...er ...6. ...e ... on my ...aph.



*** How did Alex graph the s... ...aph the solution to her problem?**
*** What are some similaritie... differences ... Alex's and ...an's problems?**
*** What are some similarities and differences b... Alex's and Mo...an's ways?**
*** How is graphing all of the values that satisfy ...h one inequality *and* another different from graphing all of the values that satisfy either one inequality *or* another?**

Student Worksheet 5.3.3

1a How did Alex graph the solution to his problem?

1b How did Morgan graph the solution to her problem?

2 What are some similarities and differences between Alex's and Morgan's problems?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 How is graphing all of the values that satisfy both one inequality *and* another different from graphing all of the values that satisfy either one inequality *or* another?