

Which is better?

Alex and Morgan were asked to simplify $(7b^3 + 3b + 1) + (2b^2 + 4b - 3)$

Alex's "vertical" way

Morgan's "horizontal" way

$$(7b^3 + 3b + 1) + (2b^2 + 4b - 3)$$

$$(7b^3 + 3b + 1) + (2b^2 + 4b - 3)$$

I first rewrote the two polynomials vertically, lining up like terms.

$$\begin{array}{r} 7b^3 \quad + 3b + 1 \\ + \quad 2b^2 + 4b - 3 \\ \hline \end{array}$$

Then I filled in the missing terms with "0" and the appropriate degree (e.g., b^2).

$$\begin{array}{r} 7b^3 + (0b^2) + 3b + 1 \\ + (0b^3) + 2b^2 + 4b - 3 \\ \hline \end{array}$$

Lastly, I added each of the pairs of like terms to get the answer.

$$7b^3 + 2b^2 + 7b - 2$$



$$(7b^3) + (2b^2) + (3b + 4b) + (1 - 3)$$

$$7b^3 + 2b^2 + 7b - 2$$

First, I grouped like terms together using the associative property.

Then I combined like terms to get the answer.



- * Why did Alex leave some "gaps" as a first step?
- * Describe Morgan's way to a new student in your class.
- * Describe two ways that Alex's and Morgan's ways are different.
- * On a timed test, would you rather use Alex's way or Morgan's way for this problem?
- * Even though Alex and Morgan did different first steps, why did they both get the same answer?

Which is better?

Alex and Morgan were asked to simplify $(7b^3 + 3b + 1) + (2b^2 + 4b - 3)$

Alex's "vertical" way

Morgan's "horizontal" way

$$(7b^3 + 3b + 1)$$

$$- 3)$$



Hey Alex, what did comparing these two examples help us to see?

These examples show us that polynomials can be added vertically or horizontally. Both ways use grouping and combining like terms, so even though the two methods look different, they are the same and yield the same answer.



I find two polynomials added vertically up like terms.

The terms are grouped and the appropriate degree (e.g., b^2).

Lastly, of the terms to get the answer.

- * Why did Alex leave some "grouping" as a first step?
- * Describe Morgan's way to a new student in your own words.
- * Describe two ways that Alex's and Morgan's ways are different.
- * On a timed test, would you rather use Alex's way or Morgan's way for this problem?
- * Even though Alex and Morgan did different first steps, why did they both get the same answer?

1 Why did Alex leave some “gaps” as a first step?

2 Describe Morgan's way to a new student in your class.

3 Describe two ways that Alex's and Morgan's ways are different.

4 On a timed test, would you rather use Alex's way or Morgan's way for this problem?

5 Even though Alex and Morgan did different first steps, why did they both get the same answer?

Which is better?

Alex and Morgan were asked to simplify $(4k^3 - 8k - 1) - (7k^2 - 3)$

Alex's "vertical" way

Morgan's "horizontal" way

$$(4k^3 - 8k - 1) - (7k^2 - 3)$$

$$(4k^3 - 8k - 1) - (7k^2 - 3)$$

First I rewrote the terms by lining them up vertically. Then I filled in the empty spaces using zero terms.

$$\begin{array}{r} (4k^3 + 0k^2 - 8k - 1) \\ - (0k^3 + 7k^2 + 0k - 3) \\ \hline \end{array}$$

$$4k^3 - 8k - 1 - 7k^2 + 3$$

Next I distributed the negative sign using the distributive property in order to eliminate the parentheses.

$$\begin{array}{r} 4k^3 + 0k^2 - 8k - 1 \\ - 0k^3 - 7k^2 - 0k + 3 \\ \hline \end{array}$$

$$4k^3 - 7k^2 - 8k - 1 + 3$$

Then I combined each of the like terms to get the answer.

$$4k^3 - 7k^2 - 8k + 2$$

$$4k^3 - 7k^2 - 8k + 2$$

First I distributed the negative sign using the distributive property so that I could eliminate the parentheses.

Next I rearranged the terms using the associative property to put the like terms together.

Then I combined the like terms to get the answer.



- * Describe Alex's way to a new student in your class.
- * Describe Morgan's way to a new student in your class.
- * Describe how Alex's and Morgan's ways are similar.
- * Describe how Alex's and Morgan's ways are different.
- * What are some advantages of Alex's way? Of Morgan's way?

Which is better?

Alex and Morgan were asked to simplify $(4k^3 - 8k - 1) - (7k^2 - 3)$

Alex's "vertical" way

Morgan's "horizontal" way

$$(4k^3 - 8k - 1) - (7k^2 - 3)$$

Hey Morgan, what did comparing these two examples help us to see?

First I rewrote terms by lining

First I distributed the negative sign using

These examples show us that polynomials can be subtracted vertically or horizontally. Both ways use grouping and combining like terms, so even though the two methods look different, they are the same and yield the same answer.

I arranged the terms

I combined the terms to get the

- * Describe Alex's way to a new student in your class.
- * Describe Morgan's way to a new student in your class.
- * Describe how Alex's and Morgan's ways are similar.
- * Describe how Alex's and Morgan's ways are different.
- * What are some advantages of Alex's way? Of Morgan's way?

1 Describe Alex's way to a new student in your class.

2 Describe Morgan's way to a new student in your class.

3 Describe how Alex's and Morgan's ways are similar.

4 Describe how Alex's and Morgan's ways are different.

5 What are some advantages of Alex's way? Of Morgan's way?

Why does it work?

Alex and Morgan were asked to multiply $(x + 5)(x + 7)$

Alex's "area model" way

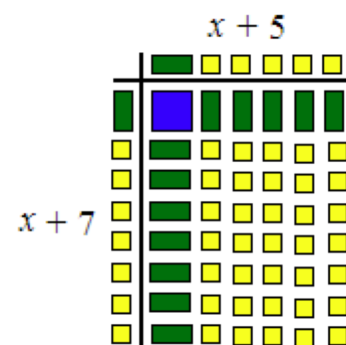
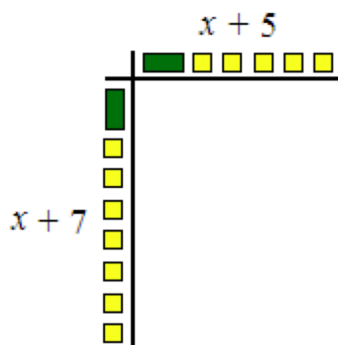
Morgan's "distributive property" way

First I set up an area model representing the two binomials I want to multiply. The green bar represents one x and each yellow square represents 1.

Then I filled in the table by lining up the tiles that would fit straight across and up and down.

Then I combined the like rectangles and squares to get the answer: 1 x^2 tile, 12 x tiles, and 35 ones tiles.

$$(x + 5)(x + 7)$$



$$x^2 + 12x + 35$$



$$(x + 5)(x + 7)$$

$$(x + 5)(x) + (x + 5)(7)$$

$$x^2 + 5x + 7x + 35$$

$$x^2 + 12x + 35$$



First, I distributed $(x+7)$.

Then I had to use the distributive property again to multiply out all the terms.

Next I simplified the expression to get my answer.

- * How did Alex multiply the binomials? How did Morgan multiply the binomials?
- * What are some similarities and differences between Alex's and Morgan's ways?
- Would Morgan have gotten the same answer if Morgan had distributed the $(x + 5)$ as a first step? Why or why not?

Why does it work?

Alex and Morgan were asked to multiply $(x + 5)(x + 7)$

Alex's "area model" way

Morgan's "distributive property" way

$$(x + 5)(x + 7)$$

$$(x + 5)(x + 7)$$

First I set up an area model representing two binomials I want to multiply. The green bar represents one



Hey Alex, what did comparing these two examples help us to see?

First, I distributed $(x+7)$.

had to use distributive property on the

These examples help us see why we can use the distributive property when we multiply binomials.



ified expression to get my answer.

tiles.

- * How did Alex multiply the binomials? How did Morgan multiply the binomials?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Would Morgan have gotten the same answer if Morgan had distributed $(x + 5)$ as a first step? Why or why not?

1 How did Alex multiply the binomials?

2 How did Morgan multiply the binomials?

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 Would Morgan have gotten the same answer if Morgan had distributed the $(x + 5)$ as a first step? Why or why not?

Why does it work?

Alex and Morgan were asked to multiply $(3x + 1)(2x + 5)$

Alex's "distributive property" way

Morgan's "FOIL" way

I first distributed the $(3x + 1)$.

Then I distributed again for each of the terms being multiplied.

Next I used the order of operations to multiply.

Lastly, I combined the like terms to get the answer.

$$(3x + 1)(2x + 5)$$



$$(3x + 1)(2x) + (3x + 1)(5)$$



$$(3x)(2x) + (2x) + (3x)(5) + (5)$$



$$6x^2 + 2x + 15x + 5$$



$$6x^2 + 17x + 5$$

$$(3x + 1)(2x + 5)$$



$$(3x)(2x) + (3x)(5) + (1)(2x) + (1)(5)$$



$$6x^2 + 15x + 2x + 5$$



$$6x^2 + 17x + 5$$

To use FOIL, I wrote down the multiplication of the First terms from each binomial, the two Outside terms, the two Inside terms, and the two Last terms in each binomial, and added each product together.

Next I used the order of operations to multiply.

Lastly, I combined the like terms to get the answer.



- * How did Alex simplify the expression?
- * Why did Morgan multiply all the terms as a first step?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Even though Alex and Morgan did different first steps, why did they both get the same answer?

Why does it work?

Alex and Morgan were asked to multiply $(3x + 1)(2x + 5)$

Alex's "distributive property" way

Morgan's "FOIL" way

$$(3x + 1)(2x + 5) \quad (3x + 1)(2x + 5)$$

I first distributed the $(3x + 1)$.

Then I distributed again of the $(2x + 5)$ being multiplied.

on the operation multiply.

Lastly, I combined like terms.

answer.



Hey Morgan, what did comparing these two examples help us to see?

These examples help us see that the distributive property and FOIL are really the same thing. FOIL is just using the distributive property.

To use FOIL, I wrote down the multiplication of the First terms from each binomial, the two Outside terms, the two Inside terms, and the two Last terms in each binomial, and added each other.

I used the order of operations to simplify.

I combined the like terms to get the answer.



- * How did Alex simplify the expression?
- * Why did Morgan multiply all terms as a first step?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Even though Alex and Morgan did different first steps, why did they both get the same answer?

1 How did Alex simplify the expression?

2 Why did Morgan multiply all the terms as a first step?

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 Even though Alex and Morgan did different first steps, why did they both get the same answer?

Which is correct?

Alex and Morgan were asked to simplify $(x + 2)(x + 3)$

Alex's "multiply the first and last terms" way

Morgan's "distribute to each term" way

$$(x + 2)(x + 3)$$

$$(x + 2)(x + 3)$$

I multiplied x times x and 2 times 3.
Here is my answer.

$$x^2 + 6$$

$$x^2 + 3x + 2x + 6$$

I distributed each term in the first parentheses to each term in the second parentheses.

I combined like terms. Here is my answer.

$$x^2 + 5x + 6$$



- * How did Alex simplify the expression?
- * How did Morgan simplify the expression?
- * Whose answer is correct, Alex's or Morgan's? How do you know?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

Which is correct?

Alex and Morgan were asked to simplify $(x + 2)(x + 3)$

Alex's "multiply the first and last terms" way

Morgan's "distribute to each term" way

$$(x + 2)(x + 3)$$

I multiplied x times x and 2 times 3. Here is



Hey Morgan, what did we learn from comparing these right and wrong ways?

I distributed each term in the first parentheses to each term in the second

I'd like to try my

When multiplying polynomial expressions, remember to distribute each term in the first parentheses to each term in the second parentheses.



- * How did Alex simplify the expression?
- * How did Morgan simplify the expression?
- * Whose answer is correct, Alex's or Morgan's? How do you know?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's ways?

4 In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to simplify this type of expression?

Which is correct?

Alex and Morgan were asked to multiply $\frac{b^3 + 9}{b - 3}$

Alex's "don't include placeholders with coefficient 0" way

Morgan's "include placeholders with coefficient 0" way

First I rewrote the expression in the format for long division.

I saw that b divides into $b^3 b^2$ times, and (-3) divides into 9 (-3) times. So my answer is $b^2 - 3$.

$$b - 3 \overline{) b^3 + 9}$$

↓

$$b - 3 \overline{) b^3 + 9} \quad b^2 - 3$$



$$b - 3 \overline{) b^3 + 0b^2 + 0b + 9}$$

↓

$$\begin{array}{r} b^2 + 3b + 9 + \frac{36}{b - 3} \\ b - 3 \overline{) b^3 + 0b^2 + 0b + 9} \\ \underline{-b^3 + 3b^2} \\ 3b^2 + 0b \\ \underline{-3b^2 + 9b} \\ 9b + 9 \\ \underline{-9b + 27} \\ 36 \end{array}$$



First I rewrote the expression in the format for long division. I put placeholders with coefficients of 0 for the missing terms, b^2 and b .

I did the long division, and got my answer.

- * How did Alex simplify the expression?
- * How did Morgan simplify the expression?
- * Which answer is correct, Alex's or Morgan's? How do you know?
- * What are some differences between Alex's and Morgan's methods?
- * In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of problem?

Which is correct?

Alex and Morgan were asked to multiply $\frac{b^3 + 9}{b - 3}$

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First I rewrote the expression in the format for long division.

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Hey Alex, what did we learn from comparing these right and wrong ways?

First I rewrote the expression in the format for long division. I put placeholders with coefficients of 0 for the missing terms, b^2 and b .

ing
and got
wer.

When dividing polynomials, don't forget to put in placeholders with coefficients of 0 where there are missing terms. Try to avoid this common mistake!



- * How did Alex solve the equation?
- * How did Morgan solve the equation?
- * Which answer is correct, Alex's or Morgan's? How do you know?
- * What are some differences between Alex's and Morgan's methods?
- * In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of problem?

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 Which answer is correct, Alex's or Morgan's? How do you know?

3 What are some differences between Alex's and Morgan's *methods*?

4 In thinking about the similarities and differences between Alex's and Morgan's ways, what conclusions can you draw about how to solve this type of problem?

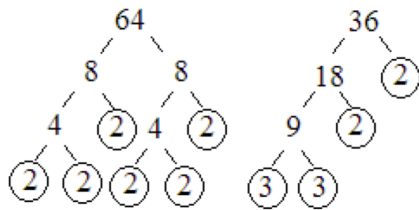
Which is better?

Alex and Morgan were asked to find the GCF of 64 and 36

Alex's "factor tree" way

Morgan's "product pairs" way

First I found the prime factorization of the two numbers using factor trees. I circled the prime factors.



Then I identified the prime factors that were common to 64 and 36.

64 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
36 is $3 \cdot 3 \cdot 2 \cdot 2$
Each number has two 2s.

I multiplied the common factors to get the answer.

GCF = 4



64,1	36,1
32,2	18,2
16,4	12,3
8,8	9,4
	6,6

GCF = 4

First I listed all of the product pairs for 64 and 36.

Then I found the largest number that was part of a product pair for both 64 and 36. I see that 4 is the biggest number that is a factor of both 64 and 36.



- * How did Alex find the GCF?
- * How did Morgan find the GCF?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Which way is easier, Alex's way or Morgan's way? Why?

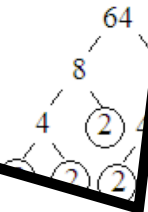
Which is better?

Alex and Morgan were asked to find the GCF of 64 and 36

Alex's "factor tree" way

Morgan's "product pairs" way

First I found the prime factorization of the two numbers using factor trees. I circled the prime factors.



Then I listed the prime factors that were common to 64 and 36.



You can find the prime factorization of two numbers using a factor tree, or by listing all of the product pairs for each number. Both methods give the right answer, but for many problems using a factor tree might be easier.

First I listed all of the product pairs for 64 and 36.

64, 1
32, 2
16, 4
8, 8

1
2
3
4
6
8
12
16
24
32
48
64

I remember that one of the pairs for both 64 and 36. I see that the largest common factor is 8.

I multiplied the common factors to get the answer.

Before you start solving a problem, you can look at the problem first and try to see which way will be easier.



- * How did Alex find the GCF?
- * How did Morgan find the GCF?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Which way is easier, Alex's way or Morgan's way? Why?

1a	How did Alex find the GCF?

1b	How did Morgan find the GCF?

2	What are some similarities and differences between Alex's and Morgan's <i>ways</i> ?

3	Which way is easier, Alex's way or Morgan's way? Why?

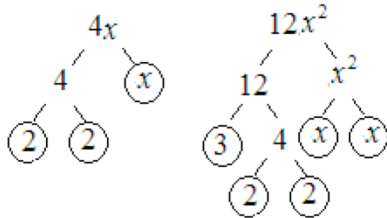
Which is better?

Alex and Morgan were asked to find the GCF of $4x$ and $12x^2$

Alex's "factor tree" way

Morgan's "product pairs" way

First I found the prime factorizations of the two terms using a factor tree. I circled the prime factors.



Then I identified the factors that were common to $4x$ and $12x^2$.

$4x$ is $2 \cdot 2 \cdot x$
 $12x^2$ is $2 \cdot 2 \cdot 3 \cdot x \cdot x$

Each term has two 2s and one x in common.

I multiplied the common factors to get the answer.

$$\text{GCF} = 2 \cdot 2 \cdot x = 4x$$



$1, 4, x$ $12, 1, x, x$
 $2, 2, x$ $6, 2, x, x$
 $4, 3, x, x$
 $4, 3, x^2$

$$\text{GCF} = 2 \cdot 2 \cdot x = 4x$$

First I listed product pairs for $4x$ and $12x^2$.

Then I found the biggest number and power of the variable that was part of a product pair for both $4x$ and $12x^2$ to get the answer.



- * How did Alex find the GCF? How did Morgan find the GCF?
- * Describe ways that Alex's and Morgan's ways are similar or different.
- * Which way do you think is better, Alex's or Morgan's?

Which is better?

Alex and Morgan were asked to find the GCF of $4x$ and $12x^2$

Alex's "factor tree" way

Morgan's "product pairs" way

First I found the prime factorizations of the two terms using a factor tree. I circled the prime factors.

Then I found the common factors to $4x$.

I multiplied the common factors to get the GCF.



You can find the prime factorization of two terms that include variables by using a factor tree, or by listing all of the product pairs for each term. Both methods give the right answer, but for many problems using a factor tree might be easier.

First I listed product pairs for $4x$ and $12x^2$.

Then I found the number and of the that was



Before you start solving a problem, you can look at the problem first and try to see which way will be easier.

- * How did Alex find the GCF? How did Morgan find it?
- * Describe ways that Alex's and Morgan's ways are different.
- * Which way do you think is better? Alex's or Morgan's?

1a	How did Alex find the GCF?

1b	How did Morgan find the GCF?

2	Describe ways that Alex's and Morgan's ways are similar or different.

3	Which way do you think is better, Alex's or Morgan's?

Why does it work?

Alex and Morgan were asked to simplify the expression

$$3x(5x + 2) + 4(5x + 2)$$

Alex's way

Morgan's way

$$3x(5x + 2) + 4(5x + 2)$$

$$3x(5x + 2) + 4(5x + 2)$$

First I expanded the expression using the distributive property.

Then I simplified the expression.

$$15x^2 + 6x + 20x + 8$$

$$15x^2 + 26x + 8$$



$$(3x + 4)(5x + 2)$$

$$15x^2 + 6x + 20x + 8$$

$$15x^2 + 26x + 8$$



First I factored the expression.

Then I expanded the expression.

Then I simplified the expression.

- * How did Alex simplify the expression? How did Morgan simplify the expression?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Is Morgan's way OK to do? Why or why not?

Why does it work?

Alex and Morgan were asked to simplify the expression

$$3x(5x + 2) + 4(5x + 2)$$

Alex's way

Morgan's way

$$3x(5x + 2)$$

5

$$4(5x + 2)$$

First I expanded the expression using the distributive property.

Then I simplified the expression.



Hey Alex, what did we learn from comparing these two ways?

First I factored the expression.

Then I expanded the expression.

Like expressions enclosed by grouping symbols, such as parentheses, can be combined as like terms are combined.



- * How did Alex simplify the expression? How did Morgan simplify the expression?
- * What are some similarities and differences between Alex and Morgan's ways?
- * Is Morgan's way OK to do? Why or why not?

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 Is Morgan's way OK to do? Why or why not?

Which is better?

Alex and Morgan were asked to simplify the expression

$$3(2k + 1) + 5(2k + 1) + 4(2k + 1)$$

Alex's way

Morgan's way

$$3(2k + 1) + 5(2k + 1) + 4(2k + 1)$$

$$3(2k + 1) + 5(2k + 1) + 4(2k + 1)$$

First I expanded the expression using the distributive property.

$$6k + 3 + 10k + 5 + 8k + 4$$

Then I simplified the expression.
This is my answer.

$$24k + 12$$



$$12(2k + 1)$$

$$24k + 12$$

First I combined like terms.

Then I expanded the expression.
Here is my answer.



- * How did Alex simplify the expression?
- * How did Morgan simplify the expression?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * On a timed test, would you rather use Alex's way or Morgan's way for this problem? Why?

Which is better?

Alex and Morgan were asked to simplify the expression

$$3(2k + 1) + 5(2k + 1) + 4(2k + 1)$$

Alex's way

Morgan's way

$$3(2k + 1) + 5$$

$$2k +$$

$$) + 5$$

$$2k + 1)$$

First I expand
expression using
the distributive
property.



Hey Morgan, what did
we learn from
comparing these two
ways?

factored the
expression.

Then I simplified
the expression.
This is the result.

Like expressions enclosed by
grouping symbols, such as
parentheses, can be combined
as like terms.



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n.
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- * How did Alex simplify the expression?
- * How did Morgan simplify the expression?
- * What are some similarities between the two ways?
- * On a timed test, would you prefer using Alex's way or Morgan's way for this problem? Why?

Morgan's ways?
ay for this problem? Why?

1a How did Alex simplify the expression?

1b How did Morgan simplify the expression?

2 What are some similarities and differences between Alex's and Morgan's ways?

3 On a timed test, would you rather use Alex's way or Morgan's way for this problem? Why?

How do they differ?

Alex was asked to factor $x^2 + 11x + 18$, and Morgan was asked to factor $x^2 + 11xy + 18y^2$

Alex's "factor a trinomial in one variable" way

Morgan's "factor a trinomial in two variables" way

First I needed to find which numbers added up to 11 and multiplied out to 18.

I made a table of all the possible factor pairs for 18, and checked to see which pair added up to 11.

2 plus 9 equals 11, so I chose that pair.

Here is my answer.

I checked my answer. I got the same answer as my original problem, so I know it's right.

$$x^2 + 11x + 18$$



Factors of 18	Sums of Factors
1, 18	$1 + 18 = 19$
2, 9	$2 + 9 = 11$
3, 6	$3 + 6 = 9$



$$(x + 2)(x + 9)$$



$$(x + 2)(x + 9)$$

$$x^2 + 9x + 2x + 18$$

$$x^2 + 11x + 18$$



$$x^2 + 11xy + 18y^2$$



Factors of $18y^2$	Sums of Factors
$1y, 18y$	$1y + 18y = 19y$
$2y, 9y$	$2y + 9y = 11y$
$3y, 6y$	$3y + 6y = 9y$



$$(x + 2y)(x + 9y)$$



$$(x + 2y)(x + 9y)$$

$$x^2 + 9xy + 2xy + 18y^2$$

$$x^2 + 11xy + 18y^2$$



First I needed to find which numbers added up to 11 and multiplied out to 18.

I made a table of possible factor pairs for $18y^2$, and checked to see which pair added up to $11y$.

$2y$ plus $9y$ equals $11y$, so I chose that pair.

Here is my answer.

I checked my answer. I got the same answer as my original problem, so I know it's right.

- * How did Alex factor the expression? How did Morgan factor the expression?
- * What are some similarities and differences between Alex's and Morgan's problems?
- * Can you use Alex's and Morgan's ways to help you factor $x^2 + 5xy + 6y^2$? How about $x^2 + 11xy^2 + 18y^4$?

How do they differ?

Alex was asked to factor $x^2 + 11x + 18$, and Morgan was asked to factor $x^2 + 11xy + 18y^2$

Alex's "factor a trinomial in one variable" way

Morgan's "factor a trinomial in two variables" way

First I needed to find which numbers added up to 11 and multiplied out to 18.

I made a table of all the possible factor pairs for 18, and checked to see which pair

2 plus 9 equals 11, so I used that pair.

Here is

I checked the answer. It is the same answer as the original problem.

x^2

18

11

y^2

First I needed to find which numbers added up to 11 and multiplied out to 18.

I made a table of possible factor pairs for $18y^2$, and checked to see which pair

2 plus 9 equals 11, so I used that pair.

Here is

I checked the answer. It is the same answer as the original problem.

Hey Morgan, what did comparing these two examples help us to see?

These examples help us see that the ways that we factor problems of the form $ax^2 + bx + c = 0$ also can be used for problems of the form $ax^2 + bxy + cy^2 = 0$.



- * How did Alex factor the expression? How did Morgan factor the expression?
- * Can you use Alex's and Morgan's ways to help you factor $x^2 + 5xy + 6y^2$? How about $x^2 + 11xy^2 + 18y^4$?

1a How did Alex factor the expression?

1b How did Morgan factor the expression?

2 What are some similarities and differences between Alex's and Morgan's *problems*?

3 Can you use Alex's and Morgan's ways to help you factor $x^2 + 5xy + 6y^2$? What about $x^2 + 11xy^2 + 18y^4$?

Which is better?

Alex and Morgan were asked to factor $2x^2 + 18x + 28$

Alex's "factor by trial and error"
way

Morgan's "factor out a common factor
first" way

I made a table of possible factor pairs for 2 and 28. Then I wrote the factorization and checked to see what the middle term (b) would be for each pair. I wanted to find a factorization that would give me 18 for the coefficient of the middle term.

I found that the factorization for the factor pairs (2,1) and (14,2) gives 18 for b.

Here is my answer.

$$2x^2 + 18x + 28$$



Factors of 2	Factors of 28	Factorization	Middle term
1, 2	1, 28	$(x + 1)(2x + 28)$	$28x + 2x = 30x$
1, 2	28, 1	$(x + 28)(2x + 1)$	$1x + 56x = 57x$
1, 2	2, 14	$(x + 2)(2x + 14)$	$14x + 4x = 18x$
1, 2	14, 2	$(x + 14)(2x + 2)$	$2x + 28x = 30x$
1, 2	4, 7	$(x + 4)(2x + 7)$	$7x + 8x = 15x$
1, 2	7, 4	$(x + 7)(2x + 4)$	$4x + 14x = 18x$
2, 1	1, 28	$(2x + 1)(x + 28)$	$56x + 1x = 57x$
2, 1	28, 1	$(2x + 28)(x + 1)$	$2x + 28x = 30x$
2, 1	2, 14	$(2x + 2)(x + 14)$	$28x + 2x = 30x$
2, 1	14, 2	$(2x + 14)(x + 2)$	$4x + 14x = 18x$



$$(2x + 14)(x + 2)$$



$$2x^2 + 18x + 28$$



$$2(x^2 + 9x + 14)$$



Factors of 14	Sum of Factors
1, 14	$1 + 14 = 15$
2, 7	$2 + 7 = 9$



$$2(x + 2)(x + 7)$$



First I factored out a 2.

Then I made a table to find two numbers that multiplied out to 14 and added up to 9.

I got 2 and 7.

Here is my answer.

- * How did Alex factor the expression? How did Morgan factor the expression?
- * One person's answer is not completely correct. Who is it? What is wrong with that person's answer? How could that person fix his or her answer and still get the problem right?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Whose way is better, Alex's or Morgan's?
- * Based on what you have learned from comparing Alex's and Morgan's ways, can you state a general rule that will apply when solving problems of this type?

Which is better?

Alex and Morgan were asked to factor $2x^2 + 18x + 28$

Alex's "factor by trial and error"
way

Morgan's "factor out a common factor
first" way

I made a table of possible factor pairs for 2 and 28. Then I wrote the factorization and checked to see what the middle term (b) would be for each pair. I wanted to find a factorization that would give me 18 for the middle term.

I found that the factorization (2x + 4)(x + 7) gives the middle term 18.

Here is the answer.



When you are asked to factor a trinomial with a coefficient greater than 1, check to see if you can factor out any common factors first; this will likely make factoring easier.

Before you start factoring, you can look at the problem first and try to see which way will be easier.



First I factored out a 2.

a

to added up

17.

swer.

- * How did Alex factor the expression?
- * One person's answer is not correct. How could that person fix his or her answer? How could you get the right answer?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Whose way is better, Alex's or Morgan's?
- * Based on what you have learned from comparing Alex's and Morgan's ways, can you state a general rule that will apply when solving problems of this type?

1a How did Alex factor the expression?

1b How did Morgan factor the expression?

2 One person's answer is not completely correct. Who is it? What is wrong with that person's answer? How could that person fix his or her answer and still get the problem right?

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 Whose way is better, Alex's or Morgan's?

5 Based on what you have learned from comparing Alex's and Morgan's ways, can you state a general rule that will apply when solving problems of this type?

Which is better?

Alex and Morgan were asked to factor $8x^2 + 10x - 3$

Alex's "factor by trial and error"
way

$$8x^2 + 10x - 3$$

First I made a table of possible factor pairs for 8 and -3. I wrote down what the factorization would look like, and I figured out what the middle term would be. I wanted to find a factorization where the middle term would be $10x$.

From my chart, I found that the factor pairs of (4, 2) for 8 and (-1, 3) for -3 gave me $10x$ for the middle term.

Here is my answer.

Factors of 8	Factors of -3	Factorization	Middle term
1, 8	1, -3	$(x + 1)(8x - 3)$	$-3x + 8x = 5x$
1, 8	-1, 3	$(x - 1)(8x + 3)$	$3x - 8x = -5x$
8, 1	1, -3	$(8x + 1)(x - 3)$	$-24x + 1x = -23x$
8, 1	-1, 3	$(8x - 1)(x + 3)$	$24x - 1x = 23x$
2, 4	1, -3	$(2x + 1)(4x - 3)$	$-6x + 4x = -2x$
2, 4	-1, 3	$(2x - 1)(4x + 3)$	$6x - 4x = 2x$
4, 2	1, -3	$(4x + 1)(2x - 3)$	$-12x + 2x = -10x$
4, 2	-1, 3	$(4x - 1)(2x + 3)$	$12x - 2x = 10x$

$$(4x - 1)(2x + 3)$$



Morgan's "factor by splitting the
middle term" way

$$ax^2 + bx + c$$

$$8x^2 + 10x - 3$$

$$8 \cdot -3 = -24$$

First I needed to find two numbers whose product equaled -24 (8×-3) and whose sum equaled 10.

I made a table of possible factor pairs for -24, and checked to see which pair added up to 10.

-2 plus 12 equals 10, so I chose that pair.

I rewrote the original equation with $(12x - 2x)$ as my new middle term.

I grouped the terms.

I factored out a $4x$ from the first parentheses and a -1 from the second parentheses.

I simplified the resulting expression. Here is my answer.

Factors of -24	Sum of Factors
1, -24	$1 + -24 = -23$
-1, 24	$-1 + 24 = 23$
2, -12	$2 + -12 = -10$
-2, 12	$-2 + 12 = 10$
3, -8	$3 + -8 = -5$
-3, 8	$-3 + 8 = 5$
4, -6	$4 + -6 = -2$
-4, 6	$-4 + 6 = 2$

$$8x^2 + 12x - 2x - 3$$

$$(8x^2 + 12x) + (-2x - 3)$$

$$4x(2x + 3) + -1(2x + 3)$$

$$(4x - 1)(2x + 3)$$



- * How did Alex factor the expression?
- * How did Morgan factor the expression?
- * How did Morgan decide what to put in the table?
- * Whose answer is correct, Alex's or Morgan's? How do you know?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Whose way do you think is better, Alex's way or Morgan's way? Why?
- * Is Morgan's way O.K. to do?

Which is better?

Alex and Morgan were asked to factor $8x^2 + 10x - 3$

Alex's "factor by trial and error"
way

Morgan's "factor by splitting the
middle term" way

$$8x^2 + 10x - 3$$

$$ax^2 + bx + c$$

$$8x^2 + 10x - 3$$

$$8 \cdot -3$$

First I made a table of possible factor pairs for 8 and -3. I wrote down what the factorization would look like, and I figured out what the middle term would be. I wanted to find a factorization where the middle term would be $10x$.

From found pairs of and $(-1, 3)$ for -3 , I got $10x$ for the middle term. Here



When you are factoring a trinomial with lead coefficient greater than 1, you can factor by using trial and error or by splitting the middle term into two terms and grouping. Both methods give the correct answer.

First I needed to find two numbers whose product equaled -24 (8×-3) and whose sum equaled 10 .

I made a table of possible factor pairs for -24 , and checked to see which pair added up to 10 .

I also noted that

I wrote the original equation $(8x^2 - 2x) - 3$ as

I factored out a $4x$ from the first two terms and a second

I simplified the resulting expression. Here is my answer.



Before you start factoring, you can look at the problem first and try to see which way will be easier.



- * How did Alex factor the expression? How did Morgan factor the expression?
- * How did Morgan decide what to put in the table?
- * Whose answer is correct, Alex's or Morgan's?
- * Is Morgan's way okay to do?

1a How did Alex factor the expression?

1b How did Morgan factor the expression?

2 How did Morgan decide what to put in the table?

3 Whose answer is correct, Alex's or Morgan's? How do you know?

4 What are some similarities and differences between Alex's and Morgan's *ways*?

5 Whose way do you think is better, Alex's way or Morgan's way? Why?

6 Is Morgan's way O.K. to do?

Which is better?

Alex and Morgan were asked to solve $3x^2 + 18x + 24 = 0$

Alex's "forget to factor out a common factor first" way

Morgan's "factor out a common factor first" way

I made a table of possible factor pairs for 3 and 24. Then I wrote the factorization and checked to see what the middle term (b) would be for each pair. I wanted to find a factorization that would give me $18x$ for the middle term.

I found that the factorization for the factor pairs (3, 1) and (12, 2) gives 18 for b.

So I rewrote the original expression in factored form.

I set each of the expressions in parentheses equal to zero and solved for x.

The solutions of this equation are -2 and -4.



$$3x^2 + 18x + 24 = 0$$

Factors of 3	Factors of 24	Factorization	Middle term
3, 1	1, 24	$(3x + 1)(x + 24)$	$72x + x = 73x$
3, 1	24, 1	$(3x + 24)(x + 1)$	$3x + 24x = 27x$
3, 1	12, 2	$(3x + 12)(x + 2)$	$6x + 12x = 18x$

$$(3x + 12)(x + 2) = 0$$

$$(3x + 12)(x + 2) = 0$$

$$(3x + 12) = 0$$

$$3x = -12$$

$$x = -4$$

$$(x + 2) = 0$$

$$x = -2$$

$$3x^2 + 18x + 24 = 0$$

$$3(x^2 + 6x + 8) = 0$$

Factors of 8	Sum of Factors
1, 8	$1 + 8 = 9$
2, 4	$2 + 4 = 6$

$$3(x + 2)(x + 4) = 0$$

$$(x + 2) = 0$$

$$x = -2$$

$$(x + 4) = 0$$

$$x = -4$$

First I factored out a 3.

Then I factored $x^2 + 6x + 8$.

I made a table to find two numbers that multiplied out to 8 and added up to 6. I got 4 and 2.

So I rewrote the original expression in factored form.

I set each of the expressions in parentheses equal to zero and solved for x.

The solutions of this equation are -2 and -4.



* How did Alex solve the equation? How did Morgan solve the equation?

* Whose answer is correct, Alex's or Morgan's? How do you know?

* What are some similarities and differences between Alex's and Morgan's ways?

* Whose way is better, Alex's or Morgan's?

* Based on what you have learned from comparing Alex's and Morgan's ways, can you state a general rule that will apply when solving problems of this type?

Which is better?

Alex and Morgan were asked to solve $3x^2 + 18x + 24 = 0$

Alex's "forget to factor out a common factor first" way

Morgan's "factor out a common factor first" way

I made a table of possible factor pairs for 3 and 24. Then I wrote the factorization and checked to see what the middle term (b) would be for each pair. I wanted to find a factorization that would give me $18x$ for the middle term.

I found that the factor pairs (1, 24) and (3, 8) gives $18x$ for the middle term.

So I rewrote the original expression in factored form.

expressed in factored form. I set each factor equal to zero and solved for x .

The solutions to this equation are $x = -2$ and $x = -4$.

$$3x^2 + 18x + 24 = 0$$

$$3(x^2 + 6x + 8) = 0$$

First I factored out a 3.

Then I factored $x^2 + 6x + 8$.

I made a table to find two numbers that multiplied out to 8 and added to 6.

I wrote the factored expression in factored form.

I set each factor equal to zero and solved for x .

The solutions to the equation are $x = -2$ and $x = -4$.

When you are asked to solve a polynomial equation with a lead coefficient greater than 1, check to see if you can factor out a common factor first. This will likely make factoring easier.

Before you start factoring, you can look at the problem first and try to see which way will be easier.

- * How did Alex solve the equation?
- * Whose answer is correct, Alex's or Morgan's? How do you know?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Whose way is better, Alex's or Morgan's?
- * Based on what you have learned from comparing Alex's and Morgan's ways, can you state a general rule that will apply when solving problems of this type?

1a How did Alex solve the equation?

1b How did Morgan solve the equation?

2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities and differences between Alex's and Morgan's *ways*?

4 Whose way is better, Alex's or Morgan's?

5 Based on what you have learned from comparing Alex's and Morgan's ways, can you state a general rule that will apply when solving problems of this type?

Which is better?

Alex and Morgan were asked to factor $4x^2 + 18x + 14$

Alex's "factor by splitting the middle term first" way

Morgan's "factor out a common factor first" way

First I needed to find two numbers whose product equaled 56 (4×14) and whose sum equaled 18.

I made a table of possible factor pairs for 56, and checked to see which pair added up to 18.

4 plus 14 equals 18, so I chose that pair.

I rewrote the original equation with $(14x + 4x)$ as my new middle term.

I grouped the terms.

I factored a $2x$ from the first parentheses and a 2 from the second parentheses.

I simplified the expression.

I factored out a 2 from the first parentheses. Here is my answer.

$$\begin{array}{c} \downarrow \\ ax^2 + bx + c \\ 4x^2 + 18x + 14 \end{array}$$

$$4 \cdot 14 = 56$$

Factors of 56	Sum of Factors
1, 56	$1 + 56 = 57$
2, 28	$2 + 28 = 30$
4, 14	$4 + 14 = 18$

$$\begin{array}{c} \downarrow \\ 4x^2 + 14x + 4x + 14 \end{array}$$

$$\begin{array}{c} \downarrow \\ (4x^2 + 14x) + (4x + 14) \end{array}$$

$$\begin{array}{c} \downarrow \\ 2x(2x + 7) + 2(2x + 7) \end{array}$$

$$\begin{array}{c} \downarrow \\ (2x + 2)(2x + 7) \end{array}$$

$$\begin{array}{c} \downarrow \\ 2(x + 1)(2x + 7) \end{array}$$



$$\begin{array}{c} \downarrow \\ 4x^2 + 18x + 14 \\ 2(2x^2 + 9x + 7) \end{array}$$

Factors of 14	Sum of Factors
1, 14	$1 + 14 = 15$
2, 7	$2 + 7 = 9$

$$\begin{array}{c} \downarrow \\ 2(2x^2 + 7x + 2x + 7) \end{array}$$

$$\begin{array}{c} \downarrow \\ 2[(x)(2x + 7) + (2x + 7)] \end{array}$$

$$\begin{array}{c} \downarrow \\ 2(x + 1)(2x + 7) \end{array}$$



First I factored out a 2 from the expression.

I made a table of possible factor pairs for 14 (2×7), and checked to see which pair added up to 9 (the new middle term).

2 plus 7 equals 9, so I chose that pair.

I rewrote the original equation with $(7x + 2x)$ as my new middle term.

I factored out an x from the first parentheses.

I simplified the resulting expression. Here is my answer.

- * How did Alex factor the expression? How did Morgan factor the expression?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * On a timed test, would you rather do Alex's way or Morgan's way? Why?

Which is better?

Alex and Morgan were asked to factor $4x^2 + 18x + 14$

Alex's "factor by splitting the middle term first" way

Morgan's "factor out a common factor first" way

First I needed to find two numbers whose product equaled 56 (4×14) and whose sum equaled 18.

I made a table of possible factor pairs for 56, and checked to see which pair added up to 18.

4 plus 14 equals 18, so that's my pair.

I rewrote the original equation with $(14x + 4x)$ as my new middle term.

I then

I factored the first parentheses 2 from the parentheses.

I simplified the expression.

I factored out a 2 from the first parentheses. Here is my answer.



$$\begin{array}{ccc} \swarrow & & \searrow \\ ax^2 + bx + c & & 4x^2 + 18x + 14 \\ 4x + 18x + & & 2(2x^2 + 9x + 7) \end{array}$$

First I factored out a 2 from the expression.

I made a table of possible factor pairs for 14 (2×7), and checked to see which pair added up to 9 (the new middle term).

2 plus 7 equals 9, so that's my pair.

I then simplified the original equation $(7x + 2x)$ as my new middle term.

I then

I simplified the expression. Here is my answer.



Before you start factoring, you can look at the problem first and try to see which way will be easier.



- * How did Alex factor the expression? How did Morgan factor the expression?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * On a timed test, would you rather do Alex's way or Morgan's way? Why?

1a How did Alex factor the expression?

1b How did Morgan factor the expression?

2 What are some similarities and differences between Alex's and Morgan's *ways*?

3 Do you think Morgan could have solved the problem in a faster way?

4 On a timed test, would you rather do Alex's way or Morgan's way? Why?